Massive Gravity And Cosmic Acceleration

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Università di Roma La Sapienza June 23, 2011

Motivation

Is it possible to construct a massive extension of GR ? The basic question of FT

Massive gravity with m ~ Today's Hubble (~10^(-33) eV) can naturally lead to the observed cosmic acceleration. The mechanism is technically natural!

Degravitation: screening of large-wavelength sources, such as vacuum energy. Can potentially address the Cosmological Constant problem

Arkani-Hamed, Dimopoulos, Dvali, Gabadadze '02 ; Dvali, Hofmann, Khoury '07

Outline

Introduction and summary

Construction of models with ghost-free “decoupling limit”

Phenomenology: self-acceleration, degravitation, etc.

Fierz-Pauli model '39

The unique ghost-free and tachyon-less linear theory of massive gravity in 4D

$$\mathcal{L} = -\frac{1}{2} M_P^2 h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma} - \frac{1}{4} M_P^2 m^2 (h^{\mu\nu} h_{\mu\nu} - h^2) + \frac{1}{2} h^{\mu\nu} T_{\mu\nu}$$

$$h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$$

Propagates the right # of dof's of a massive 4D graviton (helicities ± 2, ± 1 and 0), as required by Poincare group

However

Even an infinitesimal mass, which would naively play role only in far IR, causes an O(1) modification of gravity at all scales: physical predictions are <u>not</u> continuous in the m → 0 limit!
(van Dam & Veltman, Zakharov '70)

Most easily seen via Stueckelberg treatment of the theory: "restore" gauge invariance by introducing extra fields,

$$h_{\mu\nu} = \frac{\bar{h}_{\mu\nu}}{M_p} + \frac{\partial_\mu V_\nu}{M_p\, m^2} + \frac{\partial_\nu V_\mu}{M_p\, m^2}, \quad V_\mu = m A_\mu + \partial_\mu \pi$$

Diffs $\quad \bar{h}_{\mu\nu} \to \bar{h}_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad A_\mu \to A_\mu - m\, \xi_\mu$

Additional U(1) $\quad A_\mu \to A_\mu + \partial_\mu \zeta, \qquad \pi \to \pi - m\, \zeta$

In the m → 0 limit, the FP model reduces to the following theory,

$$\mathcal{L}_{m=0} = -\frac{1}{2}\bar{h}^{\mu\nu}\mathcal{E}^{\rho\sigma}_{\mu\nu}\bar{h}_{\rho\sigma} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \bar{h}^{\mu\nu}(\partial_\mu\partial_\nu\pi - \eta_{\mu\nu}\Box\pi) + \frac{1}{M_P}\bar{h}^{\mu\nu}T_{\mu\nu}$$

The scalar-tensor mixing can be eliminated by the conformal transformation

$$\bar{h}_{\mu\nu} = \tilde{h}_{\mu\nu} + \eta_{\mu\nu}\pi,$$

The scalar doesn't decouple!

$$\mathcal{L}_{m=0} = -\frac{1}{2}\tilde{h}^{\mu\nu}\mathcal{E}^{\rho\sigma}_{\mu\nu}\tilde{h}_{\rho\sigma} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \pi\Box\pi + \frac{1}{M_P}\tilde{h}^{\mu\nu}T_{\mu\nu} + \frac{1}{M_P}\pi T$$

This leads to an O(1) difference with GR in the predictions for the amount of light bending by the Sun - readily ruled out by Solar System tests.

This does not mean the graviton mass is mathematically 0!

Vainshtein '72; Deffayet, Dvali, Gabadadze, Vainshtein '02

For a generic nonlinear generalization of the FP model, the decoupling limit reads as

$$M_P \to \infty, \quad m \to 0, \quad \Lambda_5 \equiv (M_P m^4)^{(1/5)} = \text{fixed}$$

$$\mathcal{L} = \frac{1}{2}\pi\Box\pi + \frac{(\Box\pi)^3}{\Lambda_5^5} + \frac{1}{M_P}\pi T;$$

Nonlinearities screen the scalar contribution to the gravitational potential within the Vainshtein radius

$r_* = (M/M_P^2 m^4)^{1/5}$ ~ size of the Milky Way for a solar mass source and Hubble scale graviton

However

The same higher-derivative self-interactions lead to an additional dof in the scalar sector, which is inevitably a ghost!

$$\mathcal{L} = -(\partial\tilde{\phi})^2 + (\partial\psi)^2 + \Lambda_5^{5/2}\psi^{3/2} + \frac{1}{M_P}\tilde{\phi}T + \frac{1}{M_P}\psi T.$$

Boulware and Deser '72
Arkani-Hamed,Georgi and Schwartz '03
Deffayet, Rombouts '05

Stueckelbergization of gravity, strong coupling scale, ghosts, etc.

Arkani-Hamed, Georgi and Schwartz '03

Warm-up with spin-1

Consider a SU(N) gauge field with mass = g f

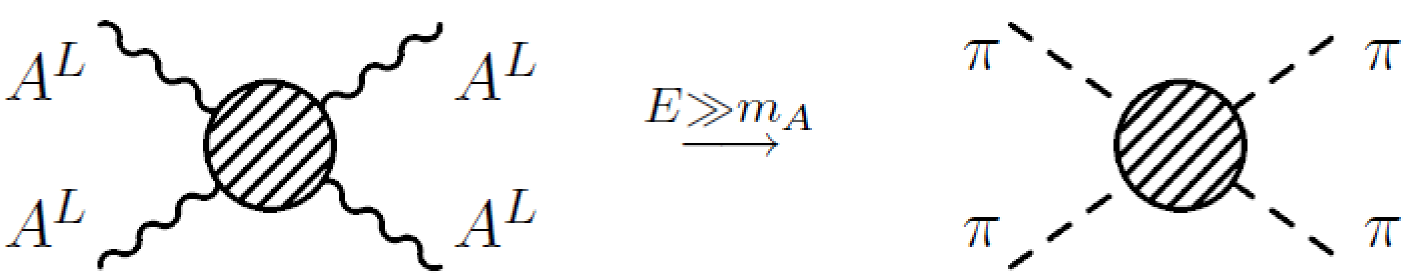
$$\mathcal{L} = -\frac{1}{g^2}\mathrm{tr}F^2 + f^2\mathrm{tr}A^2 + \ldots \to \quad -\frac{1}{g^2}\mathrm{tr}F^2 + f^2\mathrm{tr}|D_\mu U|^2 + \cdots$$

$$U = e^{i\pi} \qquad D_\mu U = \partial_\mu U - U A_\mu \qquad A_\mu \to G A_\mu G^\dagger + G\partial_\mu G^\dagger \qquad U \to U G^\dagger$$

Naively, the 2 -> 2 scattering amplitude grows as $\sim g^2(E/m_A)^4$,

leading to the strong coupling scale $\sim \frac{m_A}{\sqrt{g}}$

$\epsilon_\mu \sim k_\mu/m$



However, going to the limit

$$g \to 0, \qquad m_A \to 0, \qquad f = \text{fixed}$$

completely decouples the fields.

$$\mathcal{L} \supset f^2 \mathrm{tr}\,(\partial_\mu U)^2 + ...$$

The strong coupling scale is evidently ~ f , in distinction to naïve expectations!

The case of spin-2

$$\mathcal{L} = M_P^2 \sqrt{g} R - \frac{1}{4} M_P^2 m^2 (h_{\mu\nu}^2 - h^2 + c_1' h_{\mu\nu}^3 + c_2' h h_{\mu\nu}^2 + c_3' h^4 + \ldots)$$

The most convenient way of restoring diffs is via introducing four scalars

Arkani-Hamed, Georgi, Schwartz '03

$$h_{\mu\nu} \rightarrow H_{\mu\nu} = g_{\mu\nu} - \eta_{ab} \partial_\mu \varphi^a \partial_\nu \varphi^b$$

Consider a covariant theory with the most general potential for $H_{\mu\nu}$

$$\mathcal{L} = M_{\rm Pl}^2\sqrt{-g}R - \frac{M_{\rm Pl}^2 m^2}{4}\sqrt{-g}\left(U_2(g,H) + U_3(g,H) + U_4(g,H) + U_5(g,H)\cdots\right)$$

$$\begin{aligned}
U_2(g,H) &= H_{\mu\nu}^2 - H^2\,, \\
U_3(g,H) &= c_1 H_{\mu\nu}^3 + c_2 H H_{\mu\nu}^2 + c_3 H^3\,, \\
U_4(g,H) &= d_1 H_{\mu\nu}^4 + d_2 H H_{\mu\nu}^3 + d_3 H_{\mu\nu}^2 H_{\alpha\beta}^2 + d_4 H^2 H_{\mu\nu}^2 + d_5 H^4\,, \\
U_5(g,H) &= f_1 H_{\mu\nu}^5 + f_2 H H_{\mu\nu}^4 + f_3 H^2 H_{\mu\nu}^3 + f_4 H_{\alpha\beta}^2 H_{\mu\nu}^3 \\
&+ f_5 H (H_{\mu\nu}^2)^2 + f_6 H^3 H_{\mu\nu}^2 + f_7 H^5\,.
\end{aligned}$$

One can always choose the gauge $\varphi^a = x^\alpha \delta_\alpha^a$, resulting in $H_{\mu\nu} = h_{\mu\nu}$ and get back the theory of a single graviton with a generic potential.

One defines the Goldstone fields as follows

$$\varphi^a = \left(x^\alpha - \pi^\alpha\right)\delta^a_\alpha \quad , \qquad H_{\mu\nu} = \frac{h_{\mu\nu}}{M_{\rm Pl}} + \partial_\mu\pi_\nu + \partial_\nu\pi_\mu - \eta_{\alpha\beta}\partial_\mu\pi^\alpha\partial_\nu\pi^\beta$$

All interesting physics is almost exclusively connected to the helicity-0 graviton

$$\pi_a = \partial_a\pi/\Lambda_3^3$$

Any guiding principle for choosing the coefficients of various terms in the graviton potential?

Yes. We demand the absence of dangerous higher derivative scalar self - interactions (at least) in the decoupling limit.

This is possible

de Rham, Gabadadze '10 ; de Rham, Gabadadze, Tolley '10

The infinite number of terms in the potential can be resummed into a few.

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu} \equiv -\sum_{n=1}^{\infty} \frac{(2n)!}{(1-2n)(n!)^2 4^n} (H^n)^\mu_\nu$$

The key point is that for $h_{\mu\nu} = 0$,

$$\mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi$$

The most general lagrangian, avoiding the decoupling limit with $(\partial^2\pi)^3$ - type operators is then given as follows

$$
\begin{aligned}
\mathcal{L} \;&=\; M_P^2\sqrt{g}\Big[R - m^2\left(\mathcal{K}^\mu_\nu\mathcal{K}^\nu_\mu - \mathcal{K}^2 + b_2\varepsilon_{\mu\alpha\rho\tau}\varepsilon^{\nu\beta\sigma\tau}\mathcal{K}^\mu_\nu\mathcal{K}^\alpha_\beta\mathcal{K}^\rho_\sigma\right.\\
&+\; \left. b_3\varepsilon_{\mu\alpha\rho\tau}\varepsilon^{\nu\beta\sigma\gamma}\mathcal{K}^\mu_\nu\mathcal{K}^\alpha_\beta\mathcal{K}^\rho_\sigma\mathcal{K}^\tau_\gamma\right)\Big]
\end{aligned}
$$

As a result, one obtains an EFT with a raised cutoff

$$
\Lambda_3 = (M_P m^2)^{1/3}
$$

The decoupling limit lagrangian, characterized by keeping the latter scale fixed, is given as follows

$$
\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Pi]
$$

$$
\begin{aligned}
X^{(1)}_{\mu\nu}[\Pi] &= \varepsilon_\mu{}^{\alpha\rho\sigma}\varepsilon_\nu{}^\beta{}_{\rho\sigma}\Pi_{\alpha\beta},\\
X^{(2)}_{\mu\nu}[\Pi] &= \varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma}{}_\gamma\Pi_{\alpha\beta}\Pi_{\rho\sigma},\\
X^{(3)}_{\mu\nu}[\Pi] &= \varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma\delta}\Pi_{\alpha\beta}\Pi_{\rho\sigma}\Pi_{\gamma\delta}\,.
\end{aligned}
\qquad\qquad \Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\pi
$$

The decouping limit theory:

a) Is ghost free: equations of motion are at most second-order in time derivatives.

c) QM does not renormalize its classical part (unless external dynamical sources are involved) at any order in perturbation theory

c) Admits Vainshtein mechanism

d) Naturally gives rise to “Galileons” (**Luty, Porrati and Rattazzi '03**; **Nicolis, Rattazzi and Trincherini '08**) upon a simple tensor field redefinition

Self-acceleration and degravitation

de Rham, Heisenberg, Gabadadze, DP '10

Even without any external source, there is a solution, for which the metric describes local acceleration of the universe

$$\partial_\alpha \partial_\beta h^{\mu\nu} \left(a_1 \varepsilon_\mu{}^{\alpha\rho\sigma} \varepsilon_\nu{}^\beta{}_{\rho\sigma} + 2\frac{a_2}{\Lambda_3^3} \varepsilon_\mu{}^{\alpha\rho\sigma} \varepsilon_\nu{}^{\beta\gamma}{}_\sigma \Pi_{\rho\gamma} + 3\frac{a_3}{\Lambda_3^6} \varepsilon_\mu{}^{\alpha\rho\sigma} \varepsilon_\nu{}^{\beta\gamma\delta} \Pi_{\rho\gamma} \Pi_{\sigma\delta} \right) = 0$$

$$-\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}[\Pi] = 0$$

$$\mathrm{d}s^2 = [1 - \frac{1}{2} H^2 x^\alpha x_\alpha] \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu \qquad \pi = \frac{1}{2} q \Lambda_3^3 x^\alpha x_\alpha + b\Lambda_3^2 t + c\Lambda_3$$

The perturbation lagrangian

$$h_{\mu\nu} = h^b_{\mu\nu} + \chi_{\mu\nu}, \quad \pi = \pi^b + \phi,$$

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}\chi^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta} + 6(a_2 + 3a_3 q)\frac{H^2 M_{\rm Pl}}{\Lambda_3^3}\phi\Box\phi - 3a_3\frac{H^2 M_{\rm Pl}}{\Lambda_3^6}(\partial_\mu\phi)^2\Box\phi \\ &+ \frac{a_2 + 3a_3 q}{\Lambda_3^3}\chi^{\mu\nu}X^{(2)}_{\mu\nu}[\Phi] + \frac{a_3}{\Lambda_3^6}\chi^{\mu\nu}X^{(3)}_{\mu\nu}[\Phi] + \frac{\chi^{\mu\nu}T_{\mu\nu}}{M_{\rm Pl}},\end{aligned}$$

No conformal mixing any more! The scalar drives the acceleration and completely decouples from fluctuations! Novel mechanism for hiding the fifth force. Predictions are very close to those of \Lambda CDM!

Add an arbitrary cosmological constant

$$T_{\mu\nu} = -\lambda \eta_{\mu\nu}$$

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + h^{\mu\nu} \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}[\Pi] + \frac{1}{M_{\rm Pl}} h^{\mu\nu} T_{\mu\nu}$$

there exists a solution, for which

$$H = 0 \qquad \text{and} \qquad \pi = \frac{1}{2} q_0 \Lambda_3^3 x^2$$

for a broad range of parameters (e.g for nonzero a_3, it always exists)

However, the strong coupling scale of fluctuations on such backgrounds is huge!

$$\mathcal{L}^{(2)} = -\frac{1}{2} h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \frac{1}{2} h^{\mu\nu} \left(X_{\mu\nu}^{(1)}[\Phi] + \frac{\tilde{a}_2}{\tilde{\Lambda}^3} X_{\mu\nu}^{(2)}[\Phi] + \frac{\tilde{a}_3}{\tilde{\Lambda}^6} X_{\mu\nu}^{(3)}[\Phi] \right) + \frac{1}{M_{\rm Pl}} h^{\mu\nu} \tau_{\mu\nu}$$

where $\tilde{\Lambda}_3 \sim (\lambda / M_{\rm Pl})^{1/3}$

Solar system tests then imply an upper limit on the magnitude of cosmological constant that can be degravitated – just the value, that does not need to be $\sim (10^{-3}\ {\rm eV})^4$

One needs to think more about the ways of bypassing this constraint.

Thank you!