

*One-loop gauge invariant amplitudes with a space-like gluon
in hybrid k_T -factorization*

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Motivation for k_T -factorization in QCD

- One of the main active fields of research in QCD is the study of its behavior in the high energy limit, often referred to as low- x limit
- In this limit, one has to use the **k_T -factorization** scheme [1,2] and the k_T -dependent PDFs, the so called TMDs, which follow the BFKL evolution equations [3,4,5]

[1] E.A. Kuraev, L. N. Lipatov and V.S. Fadin, JETP 44 (1976) 443

[2] L. N. Lipatov Sov. J. Nucl. Phys. 23 (1976) 338

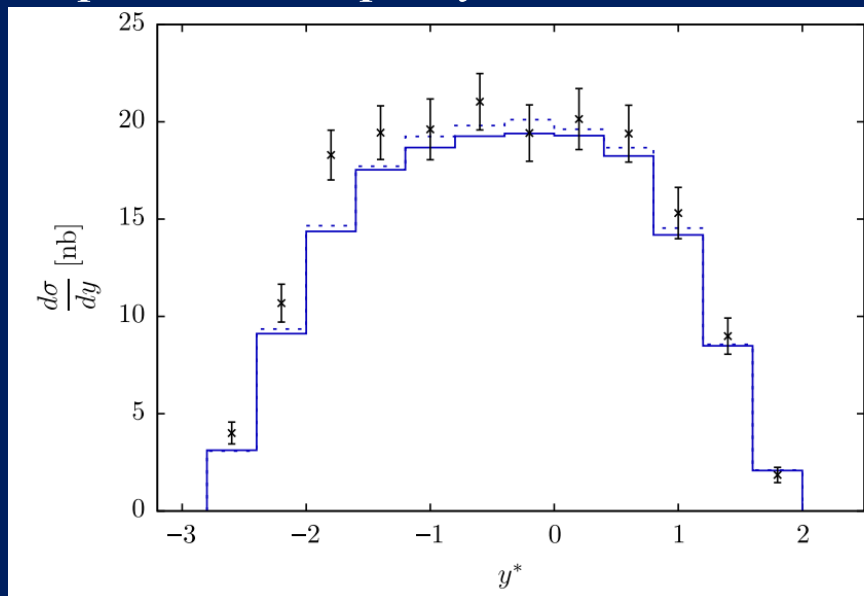
[3] L. N. Lipatov, Phys. Rept. 286 (1997) 131

[4] S. Catani, M. Ciafaloni, and F. Hautmann, Nucl. Phys. B, 366 (1991), 135-188

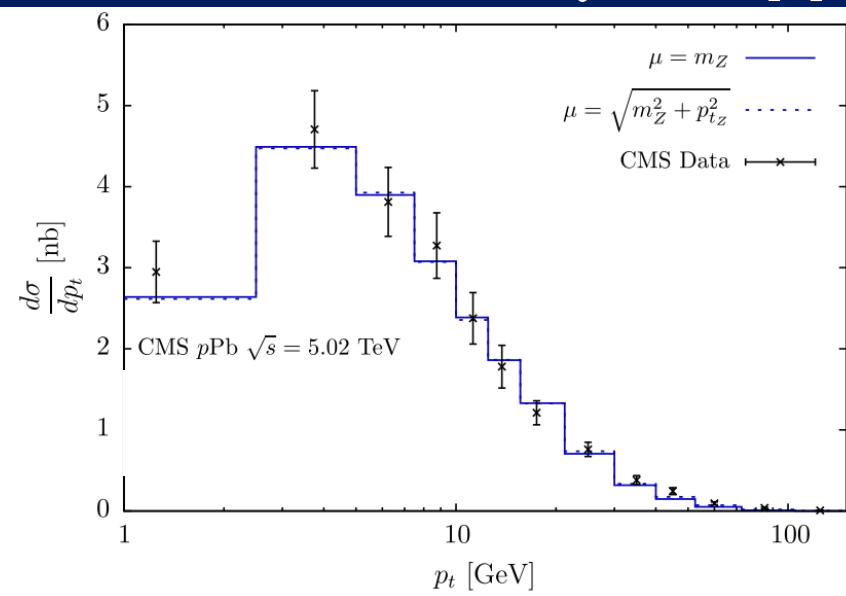
[5] J. C. Collins and R. K. Ellis, Nucl. Phys. B, 360(1) (1991), 3-30

Motivation for k_T -factorization in QCD

Example 1: in collinear factorization the experimental transverse momentum spectrum of the Z boson measured by CMS [1] cannot be well described by a fixed order calculation, and a resummation to all orders of soft gluon radiation is needed (it is possible but very challenging). In **k_T -factorization** one is able to predict the Z boson experimental rapidity and transverse momentum distributions **already at LO** [2] in agreement with the data



Z boson (center of mass) rapidity



Z boson p_T distribution

$$p\text{Pb} \rightarrow (Z/\gamma^*) \rightarrow \ell\bar{\ell}$$

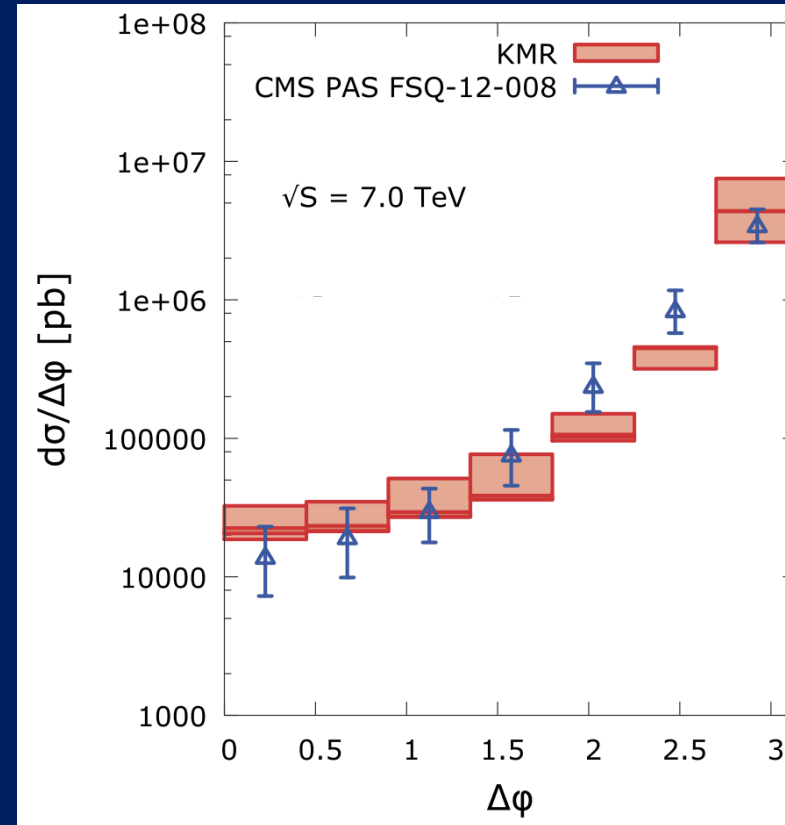
Theoretical prediction reported in these plots are given using PBnCTEQ15FullNuc_208_82 Set2 TMDs

[1] V. Khachatryan et al. (CMS Collaboration), Phys. Lett. B 759, 36 (2016)

[2] E. Blanco et al., Phys. Rev. D 100 (2019) 5, 054023

Motivation for k_T -factorization in QCD

Example 2: the forward-forward dijet correlations measured by the ATLAS collaboration for proton-proton and proton-lead collisions [3] cannot be reproduced in collinear factorization but they are successfully described within the k_T -factorization scheme at LO [4]



[3] M. Aaboud et al., (ATLAS Collaboration) Phys. Rev. C 100 (2019) 034903

[4] A. V. Hameren et al., Phys. Lett. B 795 (2019) 511-515

- IN THE FOLLOWING, I will focus on the **hybrid k_T -factorization** answering to the following questions:
- 1) What is the **hybrid k_T -factorization** ?
- 2) What is the **AUXILIARY PARTON METHOD** ?

Finally, I will explain why the formalism that we developed in [1,2] is a key step to calculate the production cross sections at NLO in **hybrid k_T -factorization**

[1] E. Blanco, A. G., A. van Hameren, P. Kotko, Nucl. Phys. B 995 (2023).

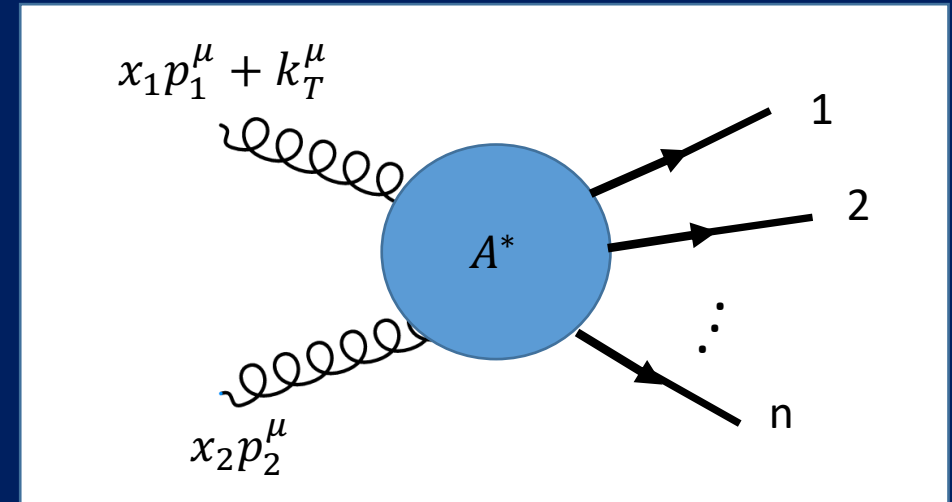
[2] A. G., A. van Hameren and G. Ziarko, *J. High Energ. Phys.* 2024, 167 (2024)

What is hybrid k_T -factorization ?

the **hybrid k_T -factorization** [1] is the case in which **ONLY ONE** initial state parton carries a transverse momentum component

This factorization becomes phenomenologically relevant for scattering events in which the final-state products are boosted towards one direction, implying that one of the collinear momentum fractions was much larger than the other.

hybrid k_T -factorization has already been applied with success at LO [2,3]



the idea is similar to that of the collinear factorization theorem, in the sense that we factorize the process into two parts:

- the part that can be extracted from universal fit to the data, the TMDs
- the part that can be calculated perturbatively, the **parton level cross section**,

[1] A. Van Hameren, P. Kotko and K. Kutak, JHEP 01 (2013) 078

[2] E. Blanco et al., Phys. Rev. D 100 (2019) 5, 054023

[3] A. V. Hameren et al., Phys. Lett. B 795 (2019) 511-515

If one initial-state parton momentum, k , contains a transverse momentum component, k_T , then

$$k^\mu = xp^\mu + k_T^\mu \quad \longrightarrow \quad k^2 \neq 0 \quad \text{the initial-state parton is not on-mass shell}$$

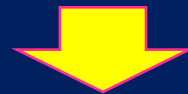
$k_T \cdot p = 0$

fraction of the hadron momentum carried by the scattering parton

light-like momentum of the colliding hadron

the calculation of the **partonic cross section** in hybrid **k_T -factorization** is not straightforward

In [1] it has been shown that a naive application of the QCD Feynman rules to calculate amplitudes with off-shell partons brings to not gauge invariant results



New theoretical tools are needed!

In the literature, this problem was solved at tree-level by the Lipatov's effective action [1,2], and by other methods which restore gauge invariance, Ward identities [3], straight infinite Wilson lines [4], and **the auxiliary parton method**, originally proposed to calculate off-shell scattering amplitudes at LO [5], **and successfully extended to NLO for the virtual [*] and real contributions [**]**

[1] L. N. Lipatov, Nucl. Phys. B 452 (1995) 369

[2] E. N. Antonov, L. N. Lipatov, E. A. Kuraev and I. O. Cherednikov, Nucl. Phys. B 721 (2005)

[3] A. Van Hameren P. Kotko and K. Kutak, JHEP 12 (2012) 029

[4] P. Kotko, JHEP 12 (2012) 029

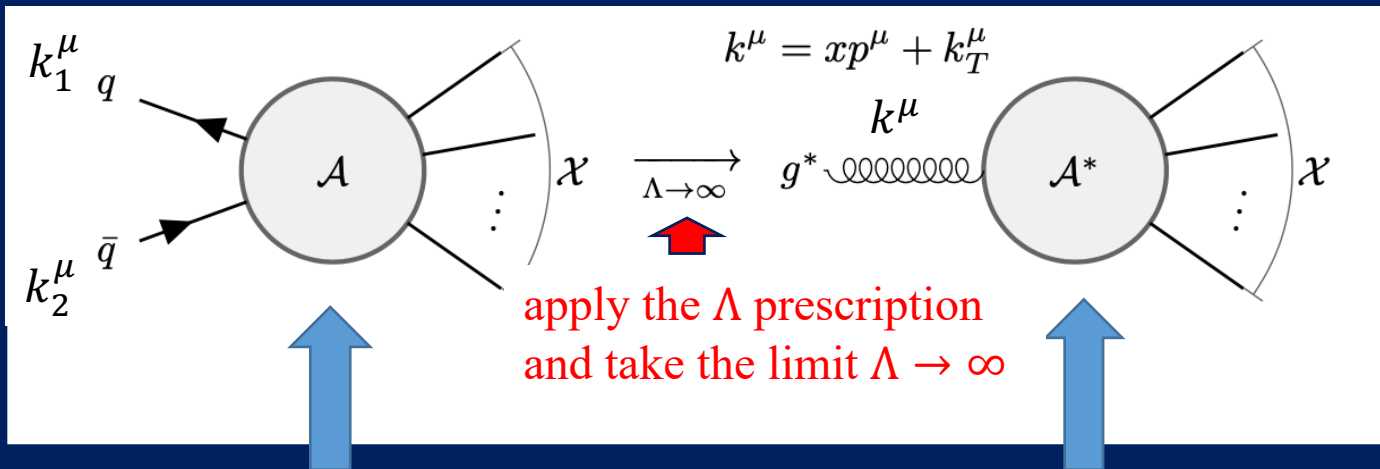
[5] A. Van Hameren, P. Kotko and K. Kutak, JHEP 01 (2013) 078

[*] E. Blanco, A. G., A. van Hameren, P. Kotko, Nucl. Phys. B 995 (2023).

[**] A. G., A. van Hameren and G. Ziarko, *J. High Energ. Phys.* 2024, 167 (2024)

the auxiliary parton method

The idea of the auxiliary parton method relies on embedding the off-shell process into a new process in which the off-shell parton is replaced by an auxiliary quark anti-quark pair or by an auxiliary gluon pair, whose momenta are written as a function of a dimensionless parameter called Λ



The amplitude that we have

The amplitude that we wish to calculate

Λ prescription for the auxiliary parton momenta using Sudakov parametrization:

$$k_1^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu,$$

$$k_2^\mu = k^\mu - k_1^\mu,$$

$k_1^\mu + k_2^\mu = k^\mu$ by construction. Imposing the on-shell condition for the momenta, $k_1^2 = k_2^2 = 0$, one finds the values of α and β

$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda(p+q)^2}, \quad \beta = \frac{1}{1 + \sqrt{1 - 1/\Lambda}}$$

The embedding process is on-shell \rightarrow the constructed amplitude is gauge invariant for any value of Λ .

At LO, in the infinite Λ limit, the constructed amplitude is the desired off-shell scattering amplitude of the original process [1]

[1] A. Van Hameren, P. Kotko and K. Kutak, JHEP 01 (2013) 078

More precisely [1]:

$$\frac{1}{g_s C_{\text{aux-q}}} \lim_{\Lambda \rightarrow \infty} \frac{x|k_T|}{\Lambda} \mathcal{A}(q(k_1)\bar{q}(k_2) \chi) = \mathcal{A}^*(g^*(k) \chi)$$



Using an auxiliary quark pair

At Leading Order (LO) one can use equivalently auxiliary quarks or auxiliary gluons, provided to properly take into account the color factors:

$$\frac{1}{g_s C_{\text{aux-g}}} \lim_{\Lambda \rightarrow \infty} \frac{x|k_T|}{\Lambda} \mathcal{A}(g(k_1)g(k_2) \chi) = \mathcal{A}^*(g^*(k) \chi)$$



Using an auxiliary gluon pair

$$C_{\text{aux-q}} = \frac{N^2 - 1}{2N}, \quad C_{\text{aux-g}} = N$$

we call this *auxiliary parton universality* and we will see that it does not hold anymore at NLO.

The main goal of this research has been to study how to generalize the auxiliary parton method to NLO (see next slide)

AUXILIARY PARTON METHOD IN HYBRID kT -FACTORIZATION

The NLO corrections to the cross section contains both the real and the virtual contributions:

$$\sigma_{\text{NLO}} = \sigma_{\text{NLO}}^{\text{Real}} + \sigma_{\text{NLO}}^{\text{Virtual}}$$

In Ref. [1] Van Hameren et al. identified the explicit structure of the virtual and real contributions based on a conjecture regarding the Λ -dependent pieces.

- We calculated the off-shell scattering amplitudes for the **virtual corrections** in the $0 \rightarrow g^* gg, 0 \rightarrow g^* q\bar{q}, 0 \rightarrow g^* gH$ and $0 \rightarrow g^* q\bar{q} e^+ e^-$ processes for all the helicity configurations using both auxiliary quarks and auxiliary gluons [*]
- In this way, we verified the conjecture proposed in [1] and we also established the formalism to calculate the NLO virtual contributions with the auxiliary parton method (see next slide) [*]
- We presented a subtraction scheme for the calculation of the **real-radiation** contribution [**]

[1] A. van Hameren, L. Motyka and G. Ziarko, JHEP 11 (2022) 103.

[*] E. Blanco, A. G., A. van Hameren, P. Kotko, Nucl. Phys. B 995 (2023).

[**] A. G., A. van Hameren and G. Ziarko, J. High Energ. Phys. 2024, 167 (2024)

AUXILIARY PARTON METHOD IN HYBRID kT -FACTORIZATION

We found that, for each process, the leading color-ordered off-shell scattering amplitudes, \mathcal{A}_1 , depend on the type of the auxiliary partons but in a UNIVERSAL way (i.e. PROCESS INDEPENDENT) [*]:

$$\mathcal{A}_1^{\text{aux-g}} = \mathcal{A}_1 + c_\Gamma \mathcal{A}^{\text{tree}} \left(\frac{\mu^2}{|k_T^2|} \right)^\epsilon \left[\frac{1}{\epsilon} (2 \ln \Lambda - i\pi) - \frac{1}{\epsilon^2} + \frac{\pi^2}{3} \right],$$

← Extra term obtained if auxiliary gluons are used

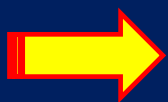
$$\mathcal{A}_1^{\text{aux-q}} = \mathcal{A}_1 + c_\Gamma \mathcal{A}^{\text{tree}} \left(\frac{\mu^2}{|k_T^2|} \right)^\epsilon \left[\frac{1}{\epsilon} (2 \ln \Lambda - i\pi) + \frac{1}{\epsilon} \left(\frac{13}{6} + \frac{\pi^2}{3} + \frac{83 - 3\delta_R}{18} \right) + \frac{1}{N_c^2} \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{7 + \delta_R}{2} \right) - \frac{n_f}{N_c} \left(\frac{2}{3\epsilon} + \frac{10}{9} \right) \right]$$

← Extra term obtained if auxiliary quarks are used

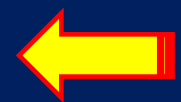
• $\delta_R = 1$: 't Hooft-Veltman scheme
 $\delta_R = 0$: four-dimensional helicity scheme

One loop constant: $c_\Gamma = \frac{1}{(4\pi)^{2-\epsilon}} \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$

dimensional regularization parameter: $\epsilon = \frac{4 - \text{dim}}{2}$



**AUXILIARY PARTON UNIVERSALITY AT NLO IS VIOLATED
 BUT
 THESE VIOLATIONS ARE UNIVERSAL AND SO THEY ARE UNDER CONTROL !**



[*] E. Blanco, A. G., A. van Hameren, P. Kotko, Nucl. Phys. B 995 (2023).

Spinor helicity method

In presenting our results we use the spinor helicity formalism, which is the basis language of modern scattering amplitude calculations.

Let $u(p)$ and $v(p)$ denote the positive and negative energy solutions of the massless Dirac equation, respectively

$$\not{p}u(p) = \not{p}v(p) = 0$$

$$\not{p} \equiv \gamma_\mu p^\mu$$

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$$

Clifford
Algebra

We can use the matrix γ_5 to construct the projection operators onto the upper and lower parts the four-component Dirac spinors $u(p)$ and $v(p)$:

$$u_\pm(p) = \frac{1 \pm \gamma_5}{2} u(p), \quad \text{and} \quad v_\pm(p) = \frac{1 \mp \gamma_5}{2} v(p),$$

Where u_\pm and v_\pm are the solutions of the massless Dirac equation with definite helicity

Spinor helicity method

In the massless limit

$$\mathbf{u}_+(\mathbf{p}) = \mathbf{v}_-(\mathbf{p}) \text{ and } \mathbf{u}_-(\mathbf{p}) = \mathbf{v}_+(\mathbf{p})$$

so it is useful to introduce the following short-hand notation

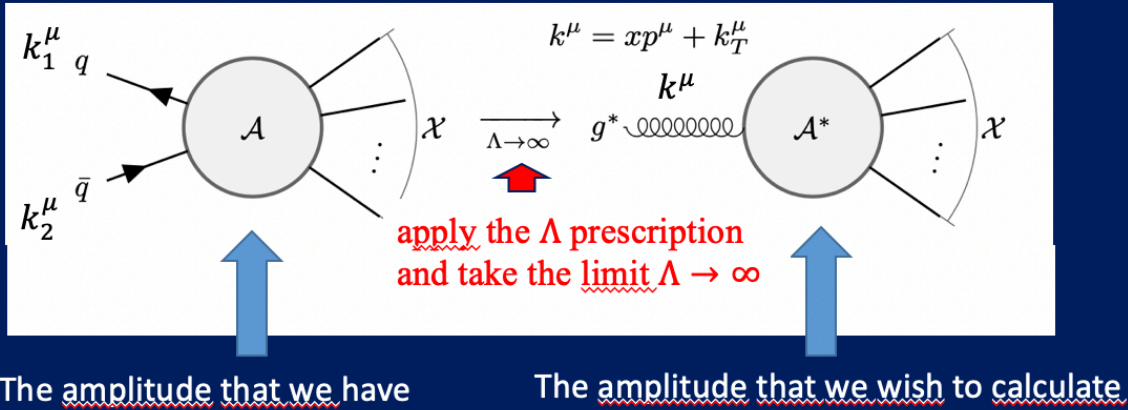
$$|\mathbf{p}] \equiv \mathbf{u}_-(\mathbf{p}) = \begin{pmatrix} L(\mathbf{p}) \\ 0 \end{pmatrix} \text{ and } |\mathbf{p}\rangle \equiv \mathbf{u}_+(\mathbf{p}) = \begin{pmatrix} 0 \\ R(\mathbf{p}) \end{pmatrix}$$

$$L(\mathbf{p}) = \frac{1}{\sqrt{|\mathbf{p}_0 + \mathbf{p}_3|}} \begin{pmatrix} -\mathbf{p}_1 + i\mathbf{p}_2 \\ \mathbf{p}_0 + \mathbf{p}_3 \end{pmatrix}, \quad R(\mathbf{p}) = \frac{\sqrt{|\mathbf{p}_0 + \mathbf{p}_3|}}{|\mathbf{p}_0 + \mathbf{p}_3|} \begin{pmatrix} \mathbf{p}_0 + \mathbf{p}_3 \\ \mathbf{p}_1 + i\mathbf{p}_2 \end{pmatrix}$$

The “dual” Weyl spinors are defined as

$$[\mathbf{p}| = ((\varepsilon L(\mathbf{p}))^T, 0), \quad \langle \mathbf{p}| = (0, (\varepsilon^T R(\mathbf{p}))^T), \quad \text{where } \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The Λ prescription



$$\frac{1}{\Lambda} \mathcal{M}(q(p_A), \bar{q}(p_B), \dots) \xrightarrow{\Lambda \rightarrow \infty} \mathcal{M}(g^*(p + k_T), \dots)$$

1) $\langle AB \rangle \rightarrow -\kappa^*$, $[AB] \rightarrow -\kappa$, $p_A^\mu + p_B^\mu \rightarrow k^\mu$

$$\kappa = \frac{\langle q|k|p \rangle}{\langle qp \rangle} , \quad \kappa^* = \frac{\langle p|k|q \rangle}{[pq]}$$

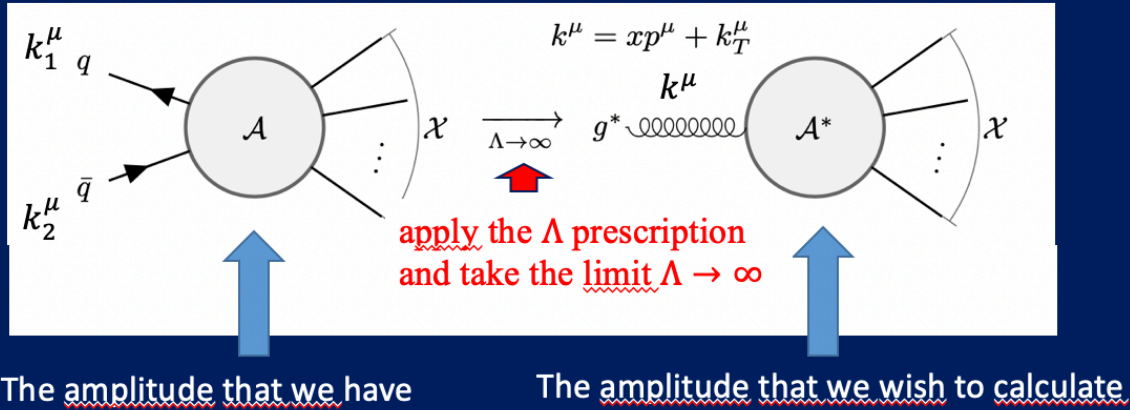
The latter implies also

$$s_{AB} = (p_A + p_B)^2 \rightarrow k^2 = -\kappa\kappa^* , \quad t_{ABi} = (p_A + p_B + p_i)^2 \rightarrow (k + p_i)^2 = s_{ki}$$

etc.

$$\kappa\kappa^* = |k_T^2| .$$

The Λ prescription

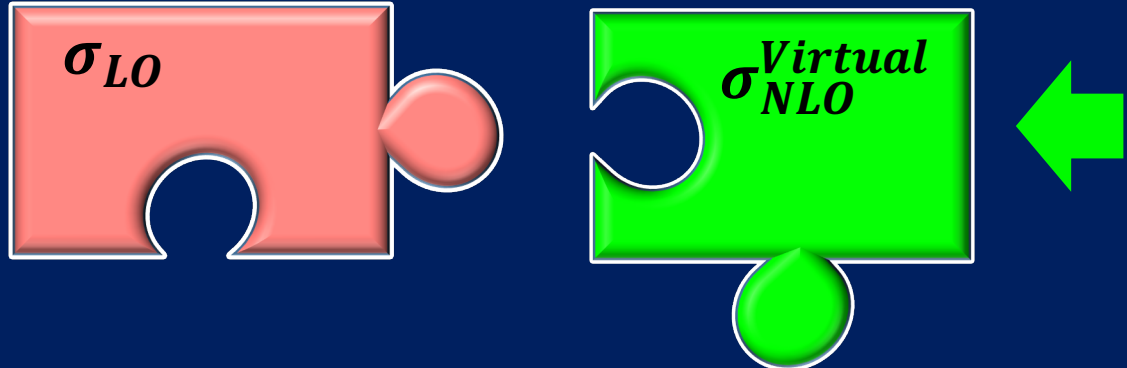


$$2) \quad p_A^\mu \rightarrow \Lambda p^\mu \quad , \quad p_B^\mu \rightarrow -\Lambda p^\mu$$

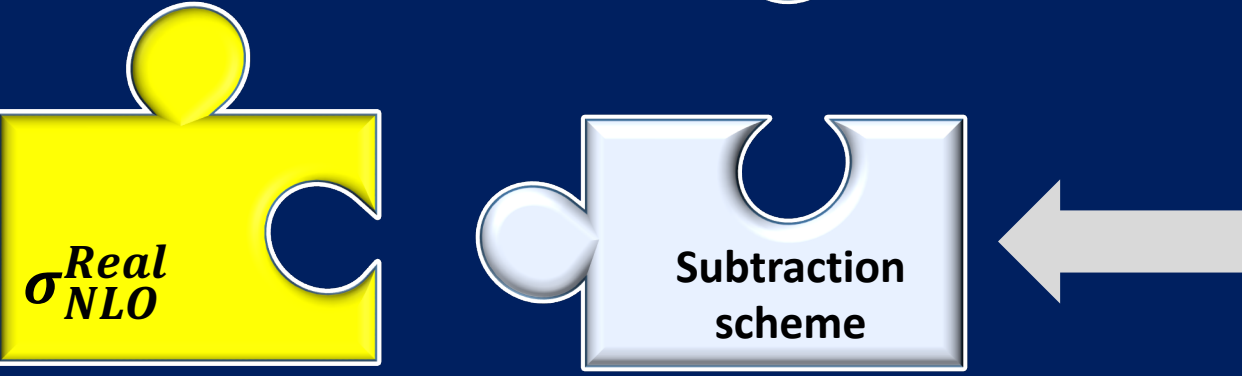
$$s_{Ai} \rightarrow \Lambda s_{pi} \quad , \quad s_{Bi} \rightarrow -\Lambda s_{pi} \quad ,$$

$$3) \quad |A\rangle \rightarrow \sqrt{\Lambda} |p\rangle \quad , \quad |A] \rightarrow \sqrt{\Lambda} |p] \quad , \quad |B\rangle \rightarrow \sqrt{\Lambda} |p\rangle \quad , \quad |B] \rightarrow -\sqrt{\Lambda} |p]$$

$$\sigma^{NLO} = \sigma_{LO} + \sigma_{NLO}^{Real} + \sigma_{NLO}^{Virtual}$$



The Λ dependent contributions have UNIVERSAL structure: this is a fundamental step to calculate NLO cross sections in hybrid kT-factorization! [*]



We presented a subtraction scheme for the calculation of the real-radiation contribution. We did implement the scheme and performed calculations for all processes relevant for 2-jet production as NLO, and found that the subtracted real-radiation integrals indeed converge [**]



[*] E. Blanco, A. G., A. van Hameren, P. Kotko, Nucl. Phys. B 995 (2023).
[**] A. G., A. van Hameren and G. Ziarko, J. High Energ. Phys. 2024, 167 (2024)

CONCLUSIONS

We calculated the off-shell scattering amplitudes for the virtual corrections in the $0 \rightarrow g^* g g, 0 \rightarrow g^* q \bar{q}, 0 \rightarrow g^* g H$ and $0 \rightarrow g^* q \bar{q} e^+ e^-$ processes for all the helicity configurations using both auxiliary quarks and auxiliary gluons

The core of our investigation was to show that the hybrid kT-factorization can be successfully extended to NLO within the auxiliary parton method

We developed the formalism to bridge the gap between the LO and the NLO calculations and we gave the operative prescriptions to calculate the off-shell scattering amplitudes in this formalism

Many thanks for your attention!



Back-up slides

The Λ prescription

- These operations give the correct result under the condition that all terms in an expression to which the operations are applied exhibit at most the leading power behavior of Λ^p with $p = 1$.

If this is not the case, one should use the following exact expressions for the spinors

$$\begin{aligned} |A\rangle &= \sqrt{\Lambda} |p\rangle - \frac{\beta\kappa^*}{\sqrt{\Lambda} \langle qp\rangle} |q\rangle & , & \quad [A] = \sqrt{\Lambda} |p] - \frac{\beta\kappa}{\sqrt{\Lambda} [pq]} |q] & , \\ |B\rangle &= \sqrt{\Lambda - 1} |p\rangle + \frac{\beta\kappa^*}{\sqrt{\Lambda} \langle qp\rangle} |q\rangle & , & \quad [B] = -\sqrt{\Lambda - 1} |p] - \frac{\beta\kappa}{\sqrt{\Lambda} [pq]} |q] \end{aligned}$$

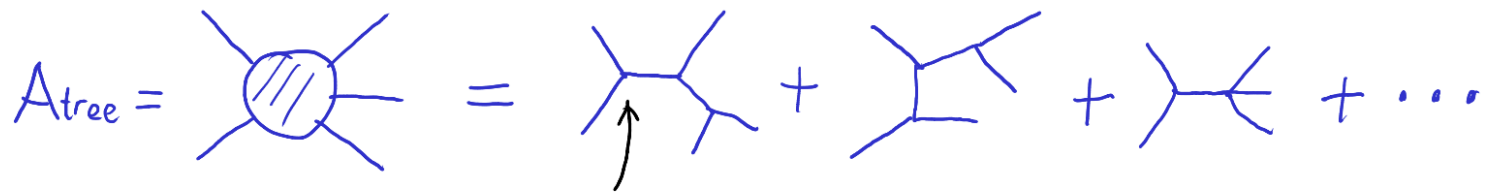
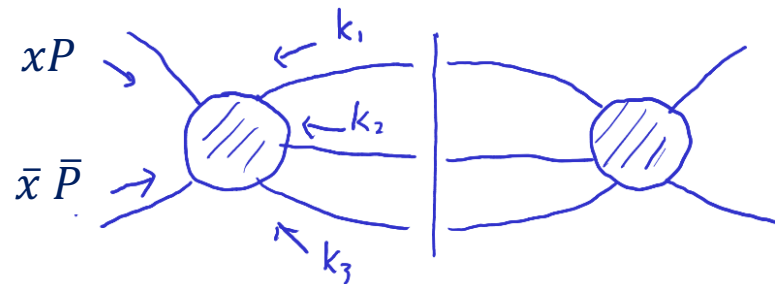
Let us have a closer look at the partonic cross section:

Leading order

Initial state variables

Final state momenta

$$d\hat{\sigma}^{Born}(x\bar{x}; \{k_i\}_{i=1}^n) = \prod_{i=1}^n d^4 k_i \delta(k_i^2) \delta^4 \left(xP + \bar{x}\bar{P} - \sum_i^n k_i \right) \frac{1}{4x\bar{x} P \cdot \bar{P}} \frac{1}{h} \sum_{\text{color}} \sum_{\text{spin}} |A_{\text{tree}}(xP, \bar{x}\bar{P}, \{k_i\}_{i=1}^n)|^2$$



small coupling allowing for
perturbation theory

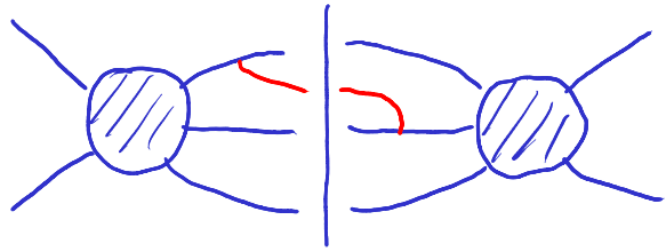
$$\text{Y-vertex} \leftrightarrow \sqrt{\alpha_s}$$

$$\text{X-vertex} \leftrightarrow \alpha_s$$

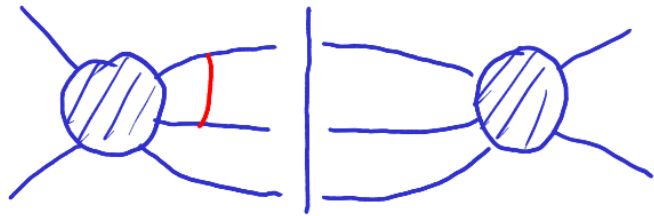
Including higher orders in expansion improves the accuracy of our predictions and reduces the dependence on non-physical parameters such as the factorization scale μ . At NLO we have more coupling and so we have more vertices:

Next-to-leading order

increase coupling power by adding vertices



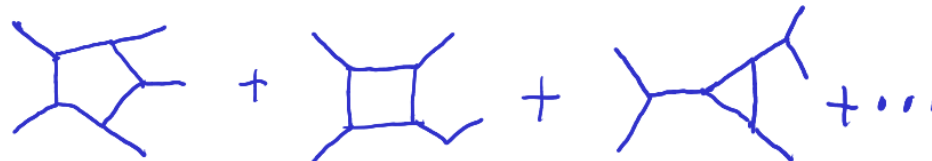
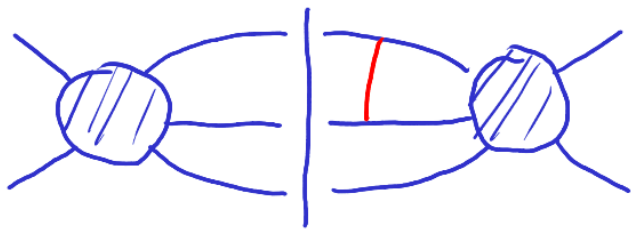
real contribution
add external line to graphs



virtual contribution (and its
complex conjugate)

add internal line to graphs

→ 1-loop graphs



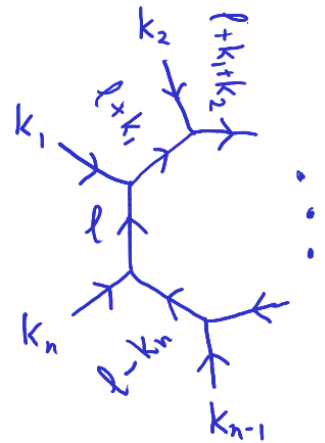
At tree level, the momentum conservation implies that for each internal line the momentum is fixed. This is not true anymore at the NLO there is an extra degree of freedom which is the loop momentum and that has to be integrated.



Virtual contributions

One-Loop Amplitude

involves an extra integration



$$k_1^M + k_2^M + \dots + k_n^M = 0$$

$$= \text{const} * \int d^{4+\epsilon} l \frac{N(l)}{D_0(l) D_1(l) D_2(l) \dots D_{n-1}(l)}$$

$$D_i(l) = (l + k_1 + k_2 + \dots + k_i)^2 + i\eta \leftarrow \text{small}$$

dimensional regularization
 $\epsilon < 0$ for UV divergences
 $\epsilon > 0$ for IR divergences
divergencies appear
as poles $1/\epsilon^2, 1/\epsilon$

If one goes to higher orders one gets extra coupling which imply extra integrals that need to be calculated.

These integrals are in general divergent in 4 dimensions and so they need to be regulated.

$N(l)$ polynomial in l . Degree may be too large for convergence leading to Ultra-Violet divergences \rightarrow renormalization


Denominator factors may vanish leading to Infra-Red divergences \rightarrow cancel (KLM theorem) against similar real divergences

Real radiation

$|A_{\text{tree}}(k_{\text{in}}, k_{\text{in}}, \{k_i\}_{i=1}^n)|^2$ is singular whenever any $k_i \cdot k_j \xrightarrow{\text{collinear}} 0$
or any $E_i = k_i^0 \xrightarrow{\text{soft}} 0$

singularities are protected by the jet definition:

LO \rightarrow each external parton corresponds to a jet
 \rightarrow all k_i must be well-defined and well-separated

NLO \rightarrow one more parton than desired number of jets 
 \rightarrow one pair of partons may correspond to a jet (collinear),
or one parton may become arbitrarily soft
 \rightarrow IR divergencies, can be dealt with in dim. reg. $\rightarrow \frac{1}{\epsilon^2}, \frac{1}{\epsilon}$
cancel (KLM theorem) against virtual ones

More complicated to isolate divergencies than for 1-loop amplitudes, but a solved problem.

In order to use the helicity method, we need to express k_T^μ in terms of spinors. It can be decomposed as follows

$$k_T^\mu = -\bar{\kappa}e^\mu - \bar{\kappa}^*e_*^\mu, \quad (9)$$

with

$$e^\mu = \frac{1}{2}\langle p|\gamma^\mu|q\rangle, \quad e_*^\mu = \frac{1}{2}\langle q|\gamma^\mu|p\rangle \quad (10)$$

and

$$\bar{\kappa} = \frac{\kappa}{[pq]} = \frac{\langle q|\not{k}|p\rangle}{2p \cdot q}, \quad \bar{\kappa}^* = \frac{\kappa^*}{\langle qp\rangle} = \frac{\langle p|\not{k}|q\rangle}{2p \cdot q}. \quad (11)$$

Realize that k_T^μ is a four-vector with a negative square, and we have

$$k_T^2 = -\kappa\kappa^*. \quad (12)$$

The spinors of k_1^μ and k_2^μ can be decomposed into those of p^μ and q^μ following

$$|1\rangle = \sqrt{\Lambda}|p\rangle - \frac{\beta\bar{\kappa}^*}{\sqrt{\Lambda}}|q\rangle, \quad |1] = \sqrt{\Lambda}|p] - \frac{\beta\bar{\kappa}}{\sqrt{\Lambda}}|q] \quad (13)$$

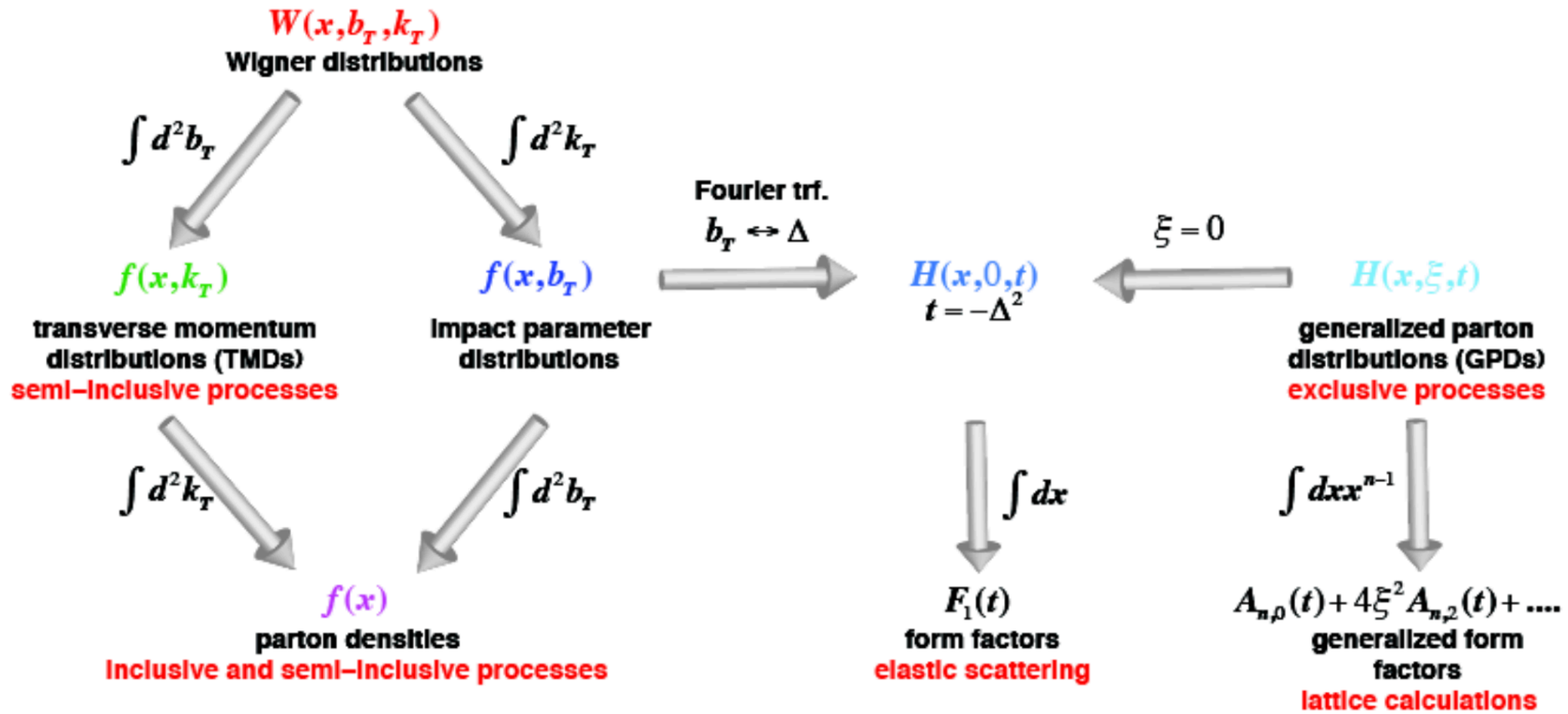
$$|2\rangle = \sqrt{\Lambda-x}|p\rangle + \frac{\beta\bar{\kappa}^*}{\sqrt{\Lambda}}|q\rangle, \quad |2] = -\sqrt{\Lambda-x}|p] - \frac{\beta\bar{\kappa}}{\sqrt{\Lambda}}|q]. \quad (14)$$

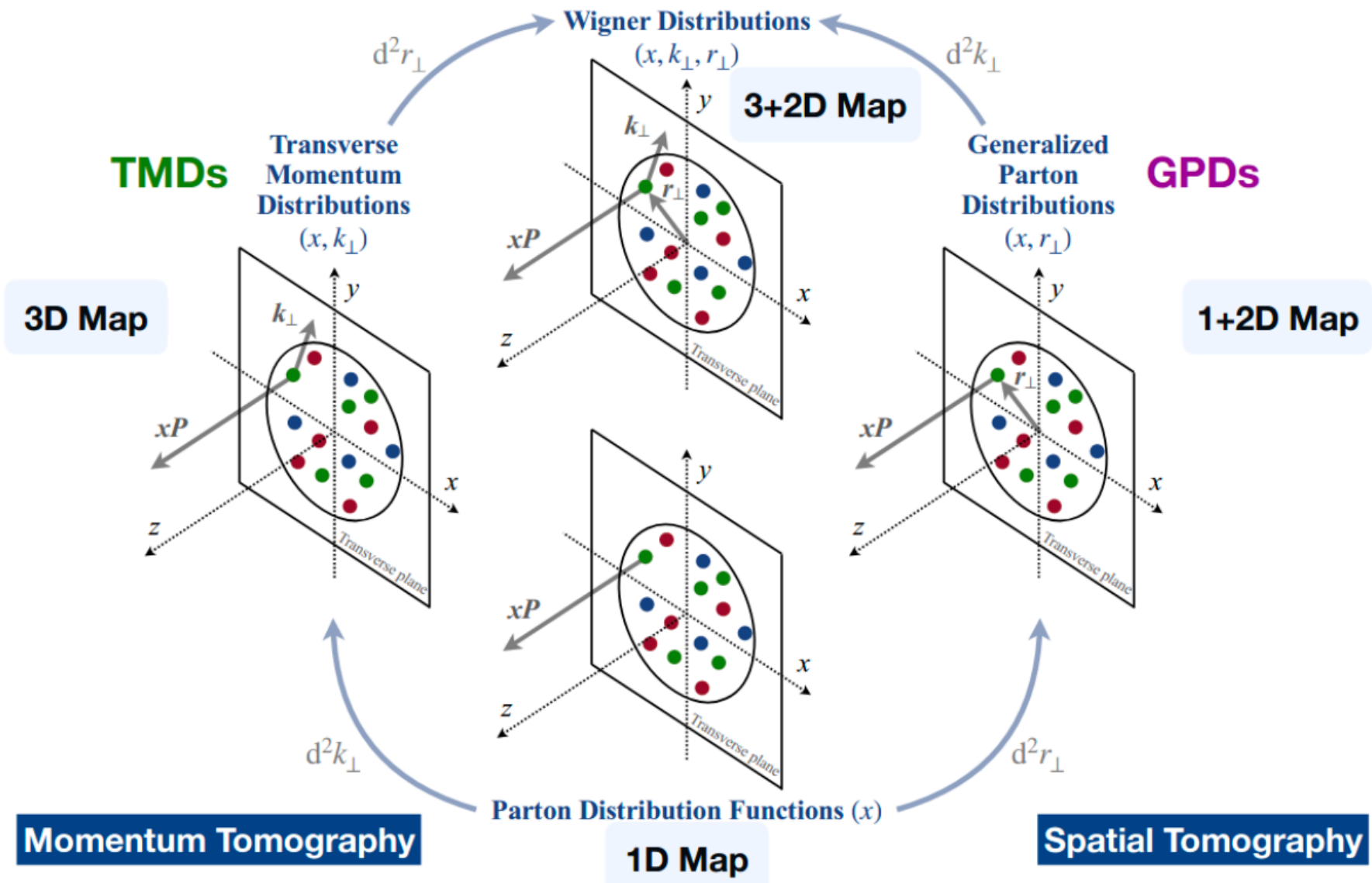
Notice that $\sqrt{(\Lambda-x)/\Lambda}\beta = 1 - \beta$. We see that the spinor products

$$\langle 12\rangle = -\kappa^*, \quad [12] = -\kappa \quad (15)$$

are independent of Λ . Further, the spinors for auxiliary quarks behave for large Λ as

$$|1\rangle \rightarrow \sqrt{\Lambda}|p\rangle, \quad |1] \rightarrow \sqrt{\Lambda}|p], \quad |2\rangle \rightarrow \sqrt{\Lambda}|p\rangle, \quad |2] \rightarrow -\sqrt{\Lambda}|p]. \quad (16)$$





Parton	uPDF/TMD set	identifier	$\Lambda_{qcd}^{(4)}$	k_t^{cut} [GeV]	Q_0 [GeV]	Ref.
Gluon	ccfm-JS-2001	101000	0.25	0.25	1.4	[1]
	ccfm-setA0	101010	0.25	1.3	1.3	[1]
	ccfm-setA0+	101011	0.25	1.3	1.3	[1]
	ccfm-setA0-	101012	0.25	1.3	1.3	[1]
	ccfm-setA1	101013	0.25	1.3	1.3	[1]
	ccfm-setB0	101020	0.25	0.25	1.3	[1]
	ccfm-setB0+	101021	0.25	0.25	1.3	[1]
	ccfm-setB0-	101022	0.25	0.25	1.3	[1]
	ccfm-setB1	101023	0.25	0.25	1.3	[1]
	ccfm-JH-set 1	101001	0.25	1.33	1.33	[2]
	ccfm-JH-set 2	101002	0.25	1.18	1.18	[2]
	ccfm-JH-set 3	101003	0.25	1.35	1.35	[2]
	ccfm-JH-2013-set1	101201	0.2	2.2	2.2	[3]
	ccfm-JH-2013-set2	101301	0.2	2.2	2.2	[3]
	GBWlight	200001	–	–	–	[4]
	GBWcharm	200002	–	–	–	[4]
KS-2013-linear	400001	0.3	–	–	[5]	
KS-2013-non-linear	400002	0.35	–	–	[5]	
Quark	ccfm-setA0	–	0.25	1.3	1.3	[1]
Quark	ccfm-JH-2013-set1	–	0.2	2.2	2.2	[3]
	ccfm-JH-2013-set2	–	0.2	2.2	2.2	[3]
	SBRS-2013-TMDPDFs	300001	–	–	1.55	[6]

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