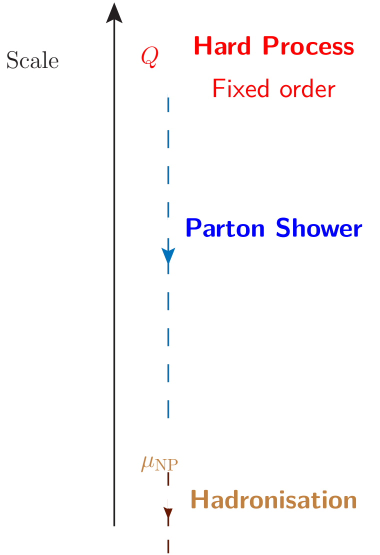
Towards NNLL Accurate Parton Showers

Jack Helliwell

with:

M. Van Beekveld, M. Dasgupta, B. El-Menoufi, S. Ferrario Ravasio, K. Hamilton, A. Karlberg, P. Monni, G. Salam, L. Scyboz, A. Soto-Ontoso, G. Soyez

BOOST 2024



Parton showers play a key role in our interpretation of collider data, evolving states generated in hard scatterings down to the low scale of hadronisation.

We expect logs between disparate scales, e.g

$$\alpha_s^n \ln^{2n} Q/\mu_{NP}$$
$$\alpha_s^n \ln^{2n-1} Q/\mu_{NP}$$

...

Would like to re-sum these logs

$\rightarrow$ **Logarithmic accuracy:**

Well defined

Systematically improvable

Logarithmic accuracy

For event shapes:

$$\Sigma(L) = \Bigg(1 + \underbrace{\frac{\alpha_s}{2\pi} C_1}_{\text{NNLL}}\Bigg) \exp\left[\underbrace{-\frac{1}{\alpha_s}\, g_1(\alpha_s\, L)}_{\text{LL}} - \underbrace{g_2(\alpha_s\, L)}_{\text{NLL}} - \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} \ \ldots\right]$$

- Broadly speaking NLL implies control of terms $\alpha_s^n L^n$
- with NNLL implying control of terms $\alpha_s^n L^{n-1}$
- We expect that NNLL will be needed for percent level accuracy at LHC energies

For event shapes:

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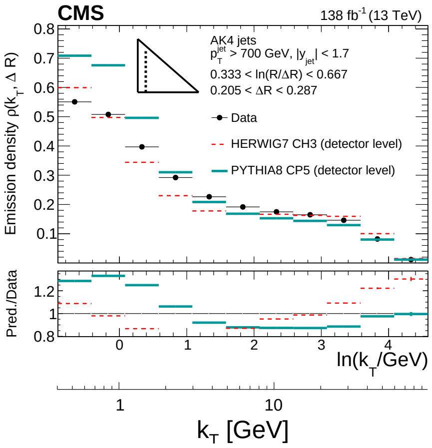
- 1988 - First dipole shower (LL) Gustafson Pettersson
- ... Elements of NLL
- 2020 - First NLL shower * Dasgupta et. al. 2020
- Now ... Towards NNLL parton showers

* Full NLL in 2021 arxiv:2111.01161, Hamilton et. al.

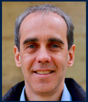
- Broadly speaki
- with NNLL im
- We expect that NNLL will be needed for percent level accuracy at LHC energies

- The widely used LL showers are extremely successful tools, but have room for improvement.
- Particularly in very differential measurements, we can find variations of around 25% between LL showers.
- A slice in the Lund plane is likely indicative of such variations in jet substructure observables.
- Such variations may translate into machine learning algorithms learning un-physical features from Monte Carlo.



2312.16343:CMS

Oxford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PanScales current members

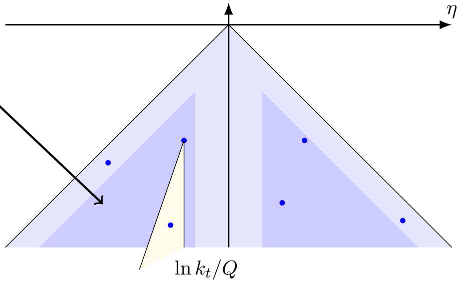
A project to bring logarithmic understanding and accuracy to parton showers

Start with an NLL shower

- The shower should reproduce the correct emission rate at NLO for soft-collinear emissions (included through the CMW scheme)

$$\alpha_s^{\mathrm{CMW}} = \alpha_s \left(1 + \frac{\alpha_s K^{\mathrm{CMW}}}{2\pi}\right)$$

- The shower must produce the correct matrix element for soft and/or collinear emissions which are far apart on the Lund plane



arxiv:1805.09327, Dasgupta, Dreyer, Hamilton, Monni, Salam. arxiv:2002.11114, + Soyez

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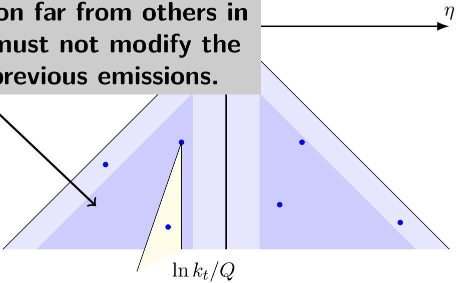
- The shower must produce the correct matrix element for soft and/or collinear emissions which are far apart on the Lund plane

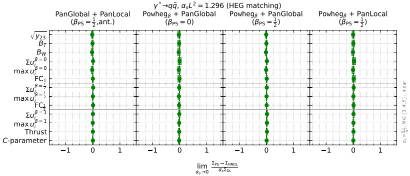
Adding an emission far from others in the Lund plane must not modify the kinematics of previous emissions.



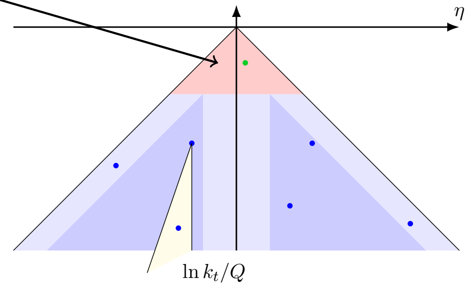
arxiv:1805.09327, Dasgupta, Dreyer, Hamilton, Monni, Salam. arxiv:2002.11114, + Soyez

NLO Matching
without breaking NLL

NNDL accuracy tests



2301.09645: Hamilton, Karlberg, Salam, Scyboz, Verheyen

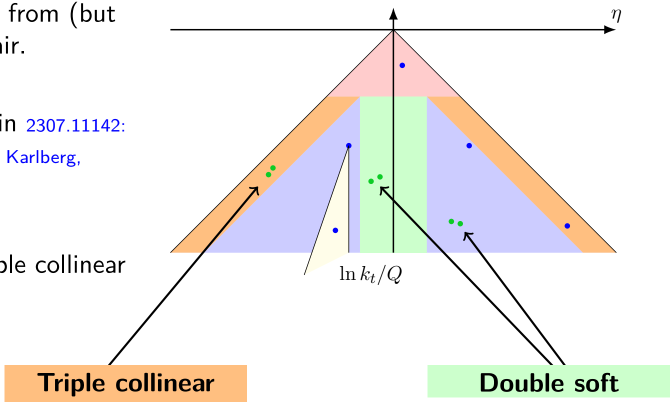


$$\Sigma(L) = \Big(1 + \underbrace{\frac{\alpha_s}{2\pi} C_1}_{\text{NNLL}}\Big) \exp\Big[\underbrace{-\frac{1}{\alpha_s} g_1(\alpha_s L)}_{\text{LL}} - \underbrace{g_2(\alpha_s L)}_{\text{NLL}} - \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} \ldots\Big]$$

What is needed for NNLL?

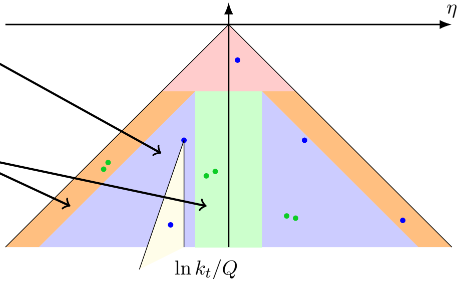
Correct matrix element (ME) for pairs of soft and/or collinear emissions that are close on the Lund plane.

- The matrix element should be correct for emissions from (but not close to) that pair.
- Double soft treated in 2307.11142: Ferrario Ravasio, Hamilton, Karlberg, Salam, Scyboz, Soyez
- Progress towards triple collinear later this talk



Virtual corrections, included through the inclusive emission rate

- Inclusive emission rate at NNLO in the soft and collinear region.
- Inclusive emission rate at NLO for soft or collinear emissions. (Soft region treated in 2307.11142: Ferrario Ravasio, Hamilton, Karlberg, Salam, Scyboz, Soyez)



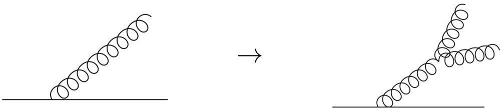
We can write the showers emission probability as

$$d\mathcal{P}_i = \frac{\alpha_s^{\text{eff}}}{\pi}\frac{dv_i}{v_i}dz_i\frac{d\phi_i}{2\pi}P\exp\left[-\int_{v_i}^{v_{i-1}}\frac{\alpha_s^{\text{eff}}}{\pi}\frac{dv}{v}dz\frac{d\phi}{2\pi}P\right]$$

$$\alpha_s^{\text{eff}} = \alpha_s(v_i)\left[1+\frac{\alpha_s(v_i)}{2\pi}K^{(1)}(k_i)+\frac{\alpha_s(v_i)}{2\pi}K^{(2)}\right]$$

- The $K^{(n)}$ encode the probability of producing an emission, inclusive over the virtual corrections and subsequent branchings that are correlated with the presence of that emission.

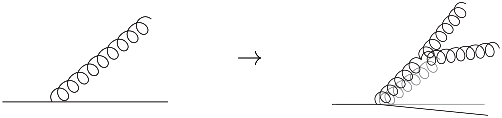
In an analytical resummation, for the gluon branching channel, this corresponds to integrating over the gluon decay products keeping the kinematics of the parent fixed e.g



Combining with the relevant virtual correction, and removing strongly ordered contributions, one finds

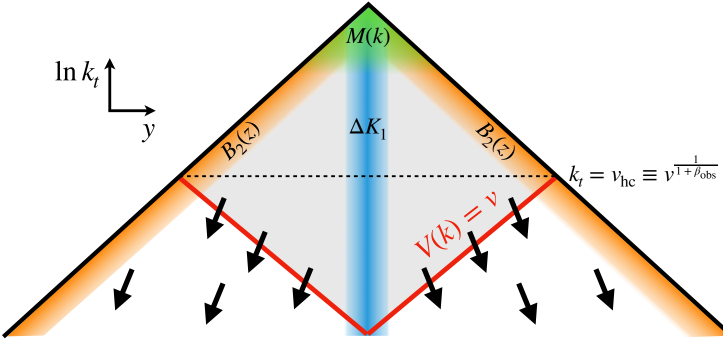
$$P(z)K^{(1)}(k_i) = \frac{2C_F}{1-z}K^{\mathrm{CMW}} + B^{(2)}(z)$$

The shower does not preserve the parent kinematics when it adds an emission, which we need to account for when calculating the inclusive emission probability e.g



$\rightarrow$ We make a scheme change corresponding to integrating the reals over the shower phase space, using the shower's kinematic map.

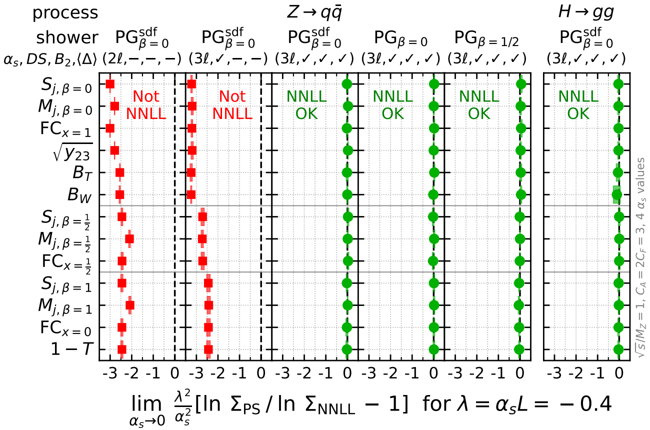
$$P(z)K^{(1)}(k_i) = \frac{2C_F}{1-z}K^{\mathrm{CMW}} + \delta K^{(1)}(y) + B^{(2)}(z) + \delta B^{(2)}(z)$$



A general and intuitive way to understand this is that adding an emission close in phase space to a previous emission causes the first emission to *drift* in phase space, which $\delta K^{(1)}(y)$ and $\delta B^{(2)}(z)$ account for.

arxiv:2406.02661, M. Van Beekveld, M. Dasgupta, B. El-Menoufi, S. Ferrario Ravasio, K. Hamilton, JH, A. Karlberg, P. Monni, G. Salam, L. Scyboz, A. Soto-Ontoso, G. Soyez

NNLL accuracy tests



- For the showers which are not NNLL accurate, we see that coefficient of the spurious NNLL terms is $\mathcal{O}(2-3)$, suggesting a sizeable phenomenological impact.

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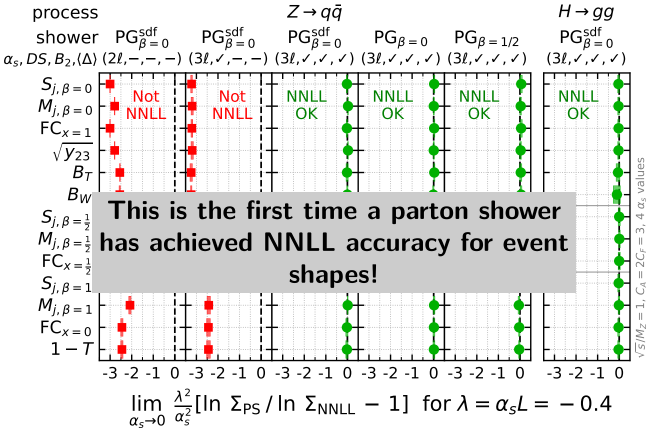

This is the first time a parton shower has achieved NNLL accuracy for event shapes!

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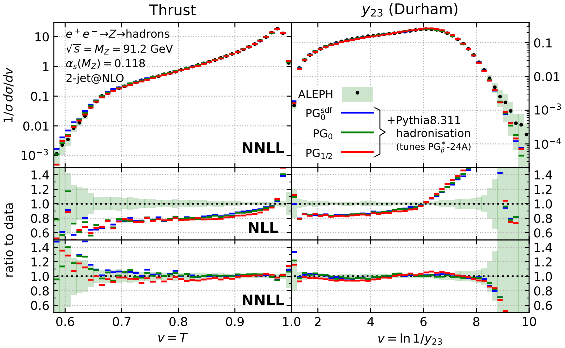
Comparison to LEP data

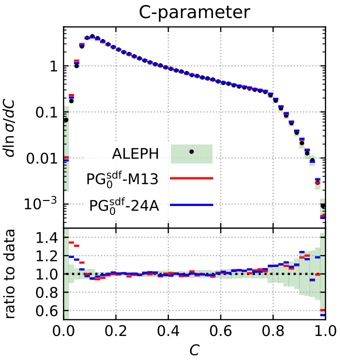


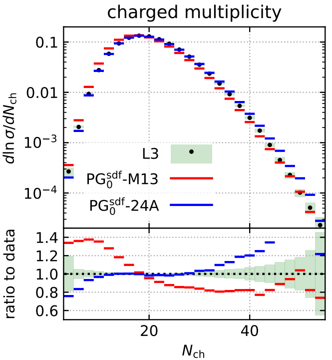
$$\alpha_s(m_Z) = 0.118$$

Colour is handled using the NODS scheme which gives full colour accuracy at NLL for global observables (includes those shown)

- Inclusion of NNLL potentially resolves the issue of needing an anomalously large value of $\alpha_s(m_Z)$ to achieve good agreement with LEP data. ($\alpha_s(m_Z) = 0.137$ in Pythia's Monash 13 tune * arxiv:1404.5630, Skands, Carrazza, Rojo)
- Some caution needed as no 3-jet NLO matching, which is known to be relevant away from the 2-jet region.
- A comprehensive study of shower uncertainties is still to be done.

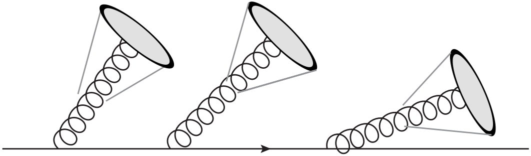
*This should be taken as an average α_s^{eff} not an $\alpha_s^{\overline{MS}}$





- We start from the Monash 13 tune for the non-perturbative parameters
- IRC safe observables (e.g see left plot) are largely insensitive to tuning of non-perturbative parameters as expected.
- Charged multiplicity (see right) and other IRC unsafe observables depend strongly on tuning

- The next step towards general NNLL accuracy is the consistent inclusion of triple collinear corrections
- This will bring NSL accuracy for fragmentation functions, small radius jet spectra and many jet substructure observables
- We have recently developed a non-singlet collinear shower algorithm demonstrating the inclusion of the Abelian triple collinear splitting functions and corresponding virtual corrections. arxiv:2408.xxxxx, van Beekveld, Dasgupta, El-Menoufi, JH, Monni, Salam



- Having accepted a first emission, we use the following shower matrix element

$$P = J(\Phi_i, \Phi_p)\frac{p_{1\to 3}(\Phi_i, \Phi_p)}{P_{qq}(z_p)}\Theta(v_{g_i q_i} < v_{g_p q_i})$$

- Applying this for successive emissions, the shower will correctly reproduces the matrix elements for pairs of emissions, strongly ordered with respect to other pairs.
- The ordering condition (step function) must be symmetric between the two emissions to properly account for the $1/2!$ symmetry factor in the matrix element.
- Always taking the previously generated emission as the *parent* (p) would not be IRC safe. We effectively take the closest emission in the Lund plane to be the parent.

We can deduce the inclusive emission probability by considering the process $e^+e^- \to q\bar{q}$, but the final result (in the infra-red) is **process independent**.

We are inclusive over the virtual corrections and a subsequent emission

$$\frac{\alpha_s}{2\pi}K(z_g) = \overbrace{\frac{V_{q\bar{q}g}}{B_{q\bar{q}g}} + \int_0^{\tilde{v}_g} \frac{d\Phi_{q\bar{q}ij}}{d\Phi_{q\bar{q}g}} \frac{B_{q\bar{q}g_1 g_2}}{B_{q\bar{q}g}}} \underbrace{- \int_0^{v_g} \frac{d\Phi_{q\bar{q}g'}}{d\Phi_{q\bar{q}}} \frac{B_{q\bar{q}g'}}{B_{q\bar{q}}} - \frac{V_{q\bar{q}}}{B_{q\bar{q}}}}$$

and subtract the components of the virtual and subsequent emission probability that factorise from the emission of the first gluon

*A similar equation appears in the context of embedding NLO 3-jet with NLO 2-jet in a shower 1303.4974, 1611.00013, 2108.07133; Li, Skands et.al .

NSL collinear parton shower - Inclusive emission probability

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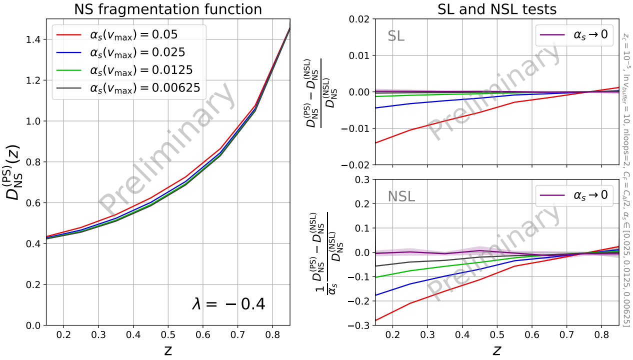
and subtract the components of the virtual and subsequent emission probability that factorise from the emission of the first gluon

- This can be formulated directly in terms of the process independent one-loop $1 \to 2$ and tree level $1 \to 2$ and $1 \to 3$ splitting functions.
- It is crucial that this is evaluated using the shower kinematic map, so that the shower Sudakov will cancel against the real terms down to the cutoff scale.
- This definition is consistent with the drift picture discussed earlier

*A similar equation appears in the context of embedding NLO 3-jet with NLO 2-jet in a shower 1303.4974, 1611.00013, 2108.07133; Li, Skands et.al .

NSL collinear parton shower - Tests of logarithmic accuracy

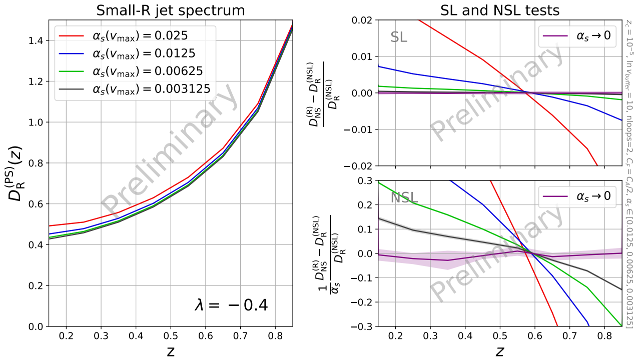
- Non-singlet partonic fragmentation function evolution
- Comparison between shower and HOPPET DGLAP evolution code
- Zero in the $\alpha_s \to 0$ limit signals NSL agreement



$^*C_F = C_A/2 = 3/2$ to avoid the N_c suppressed $q \to q\bar{q}q$ contribution

NSL collinear parton shower - Tests of logarithmic accuracy

- Non-singlet small radius jet spectrum
- same as fragmentation function at SL but distinct at NSL
- NSL prediction derived from arxiv:2402.05170, Van Beekveld, Dasgupta, El-Menoufi, JH, Karlberg, Monni and implemented in HOPPET



$^*C_F = C_A/2 = 3/2$ to avoid the N_c suppressed $q \to q\bar{q}q$ contribution

- As of version 0.2 PanGlobal showers have NNLL accuracy for final state event shapes, NNDL accuracy for sub-jet multiplicities and NSL accuracy for soft dominated non-global observables (e.g the energy flow into a rapidity slice). Available from `https://gitlab.com/panscales/panscales-0.X`

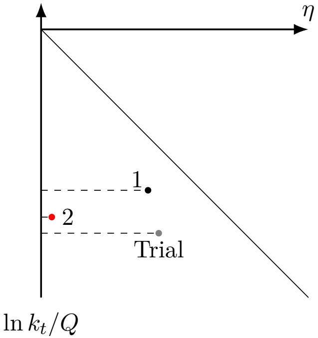
- The phenomenological impact of NNLL corrections can be significant and potentially resolves the tension that showers typically need an unphysically large α_s to achieve good agreement with data.

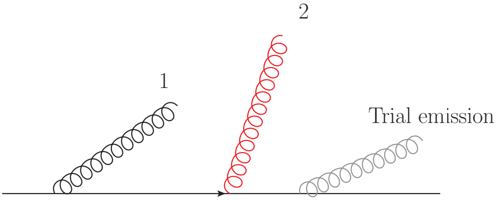
- Work towards triple collinear corrections is ongoing. Once completed the PanScale showers will arguably have the main elements of general NNLL accuracy for final state showers.

Backup slides

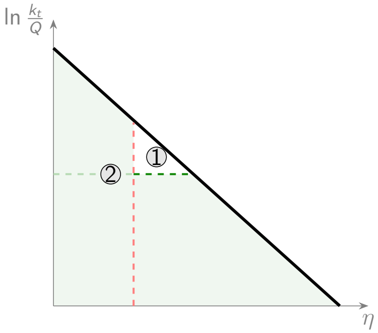
- The correct triple collinear configuration is the quark, emission 1 and the trial emission, because the soft wide angle emission factorises from the production of the trial emission.

- If we took just the previous emission as the parent, we would be taking emission 2, which would not be IRC safe as 2 can be arbitrarily soft.





- We must also take care not to cover the same phase space twice.
- We only trial emissions from a particular parent once at any given scale



To test NSL accuracy of the shower, we construct

$$\frac{1}{\alpha_s}\left(\frac{D_{\mathrm{NS}}^{(\mathrm{PS})}(z, v_{\mathrm{min}}, v_{\mathrm{max}})}{D_{\mathrm{NS}}^{(\mathrm{NSL})}(z, v_{\mathrm{min}}, v_{\mathrm{max}})} - 1\right) .$$

and take the $\alpha_s \to 0$ limit.

For NNLL event shapes, the relevant quantity is

$$\frac{\lambda^2}{\alpha_s^2}\left(\frac{\ln \Sigma^{(\mathrm{PS})}(\lambda)}{\ln \Sigma^{(\mathrm{NNLL})}(\lambda)} - 1\right) .$$

with $\lambda = \alpha_s L$

- We start the shower at scale v_{max} with a quark with momentum fraction $z = 1$.
- Run the shower down to v_{min} and measure the energy distribution of quarks.

$$\lambda = \alpha_s(v_{\mathrm{max}}) \ln(v_{\mathrm{min}}/v_{\mathrm{max}})$$

$$D_{\mathrm{NS}}^{(\mathrm{NSL})}(z, v_{\mathrm{min}}, v_{\mathrm{max}}) = C(v_{\mathrm{min}}) \otimes \exp\left[\int_{v_{\mathrm{min}}^2}^{v_{\mathrm{max}}^2} \frac{dv^2}{v^2} \hat{P}(v)\right] \otimes C^{-1}(v_{\mathrm{max}})$$

The coeficient function accounts for the scheme change between $\overline{\mathrm{MS}}$ fragmentation function, using dimensional regularisation, and the shower scheme, using a cuttoff regularisation scheme.

- Start the shower at sufficiently high scale with a quark with momentum fraction $z = 1$, so that, for any $z \in [z_c, 1 - z_c]$, the angular scale R_0 can be generated by the shower.

- Run the shower down to a low scale, so that for any $z \in [z_c, 1 - z_c]$, the angular scale R can be generated by the shower.

- Veto emissions with angle larger than R_0, so as to mimic starting with a jet of radius R_0.

- Cluster jets with radius R, and study the energy spectrum of jets containing the quark.

Tests of Logarithmic accuracy - small R jets

$$D_{\mathrm{R}}^{(\mathrm{NSL})}(z, ER, ER_0) = C^{(R)}(ER) \otimes \exp\left[2\int_{ER}^{ER_0} \frac{d\mu}{\mu}\hat{P}^{(R)}(\mu, ER)\right] \otimes \left[C^{(R)}(ER_0)\right]^{-1},$$

- As we don't implement matching in the non-single collinear shower, the hard matching coefficient is replaced by $\left[C^{(R)}(ER_0)\right]^{-1}$ which accounts for starting with a jet of radius R_0.

The small R anomalous dimension can be expressed as

$$\hat{P}_{ik}\left(z,\mu,E\,R\right)=\frac{\alpha_s(\mu^2)}{2\pi}\left(\hat{P}_{ik}^{(0)}(z)+\frac{\alpha_s(\mu^2)}{2\pi}\hat{P}_{ik}^{(1),\,\mathrm{AP}}(z)-\frac{\alpha_s(E^2R^2)}{2\pi}\delta\hat{P}_{ik}^{(1)}\right)$$

so that:

$$\frac{dD_k^{\mathrm{jet}}\left(z,\mu,E\,R\right)}{d\ln\mu^2}=\sum_i\int_z^1\frac{d\xi}{\xi}\hat{P}_{ik}\left(\frac{z}{\xi},\mu,E\,R\right)D_i^{\mathrm{jet}}\left(\xi,\mu,E\,R\right)\,.$$

- This can be derived using an NSL generating functional approach
  arxiv:2307.15734, M. van Beekveld, M. Dasgupta, B. El-Menoufi, JH, P. Monni
- The coupling scale in the highlighted term emerges as a consequence of the change in energy of the quark over the course of the evolution