Multifractal dimension of a QCD superinclusive variable

Daniele Atzori*, Stefano Forte*, Simone Marzani**

**Università degli Studi di Milano, **Università di Genova*

ABSTRACT

In this research, we consider an electron-positron scattering process and introduce a so-called superinclusive observable, suggested to us by Giorgio Parisi. This observable allows one to study the energy flow of an event due to QCD final-state radiation. The aim of the research is to provide a theoretical prediction of its behaviour in the collinear limit, testing the hypothesis that it follows a multifractal law.

METHODS

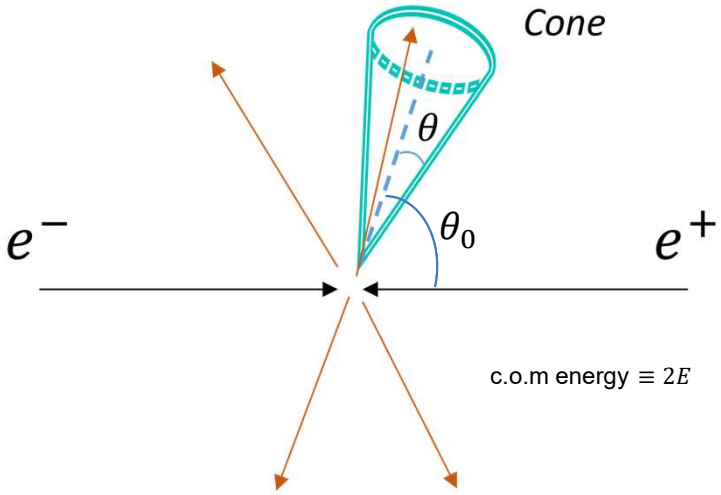
The estimation of the behaviour of the observable is obtained through an analytical calculation at the Leading Logarithm (LL) level.

The procedure is divided into four steps:

1. Definition of the observable in terms of the energies and the momenta of the final-state partons.
2. Fixed-order calculation of the observable.
3. Resummation of LL contributions.
4. Calculation with the running coupling constant.

The observable is defined as the final state energy detected within a cone, raised to the power of a real factor b. We narrow the focus to events due to QCD final-state radiation.

We perform a calculation in the limit where the angular radius of the cone is small, using the collinear approximation and introducing the Altarelli-Parisi splitting functions.



RESULTS

In fixed order calculations the observable depends on powers of collinear logarithms, so the resummed quantity follows a multifractal law. The multifractal dimension is related to the anomalous dimensions of the Altarelli-Parisi splitting functions.

This behaviour is still present if we consider the running coupling constant.

Observable at LL — Multifractal law

$$E_{LL}(\theta, b) = C(\theta_0, b, E) \sum_{i=1}^{2} \left[\theta^{\Phi(b)}\right]_{ij} v^j$$

$$\Phi_{ij}(b) \equiv 2\delta_{ij} - \frac{\alpha_S}{\pi}\left(\gamma^{b+1}\right)_{ij}$$

Multifractal dimension — Anomalous dimensions

CONCLUSIONS

An analytical calculation shows that the hypothesis is correct, at least at LL. The multifractal law is essentially a consequence of the renormalization of the QCD.

This research could be improved by testing the hypothesis at all orders and analyzing the matching between partonic and hadronic levels. This last point leads to considerations about the possibility of experimental measurements of the observable, which can represent an innovative way to measure the strong coupling constant without using PDFs.

REFERENCES

1. G. Altarelli, G. Parisi, *Asymptotic freedom in parton language*, 12 April 1977
2. G. Parisi, *Superinclusive cross sections*, January 1978
3. G. Gustafson, A. Nilsson, *Multifractal dimensions in QCD cascades*, 15 April 1991
4. D. M. Hofman, J. Maldacena, *Conformal collider physics: energy and charge correlations*, 4 May 2008
5. M. Jankowiak, A. J. Larkoski, *Angular scaling in jets*, 6 April 2012
6. S. Marzani, G. Soyez, M. Spannowsky, *Looking inside jets*, 9 July 2022

Contact: daniele.atzori00@gmail.com