Energy Correlators Beyond Angles:

Energy-Weighted Observable Correlations (EWOCs)

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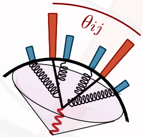
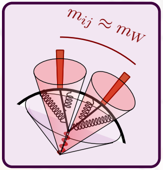
Overview

Motivation:

Energy Correlators and Mass Extraction

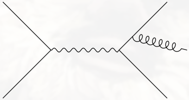


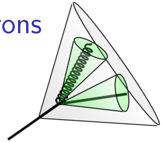
The Mass Energy-Weighted Observable Correlation (**EWOC**)



Calculation:

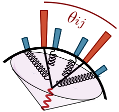
The Mass EWOC in e^+e^- to hadrons





What is the EEC?

The **Energy-Energy Correlator** (**EEC**)
(is a function of θ)



(Roughly)
An answer to the question:

How much of the energy in an event/jet is separated by an angle θ?

What is the EEC?

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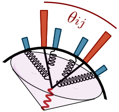

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An answer to the question:

How much of the energy in an event/jet is separated by an angle θ?

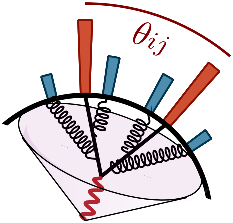
$$\frac{d\Sigma(\theta)}{d\theta} = \frac{1}{\sigma} \sum\!\!\!\!\!\!\int d\sigma \sum_{\substack{\textbf{particles}\\ i,j}} z_i\, z_j\, \delta\left(\theta - \theta_{ij}\right)$$

More common: $z := (1 - \cos\theta)/2$

- z_i is an energy or p_T fraction;
- θ_{ij} is an angular measure (opening angle, ΔR).

The EEC: What makes it special?

The **Energy-Energy Correlator** (**EEC**) has become a standard tool in QCD phenomenology.



- Experimentally practical/physically intuitive definition;

$$\frac{d\Sigma}{d\theta} \sim \left\langle \sum_{\substack{\text{pairs } (i,j) \text{ of ``detectors''} \\ \text{separated by an angle } \theta}} E_i \, E_j \right\rangle$$

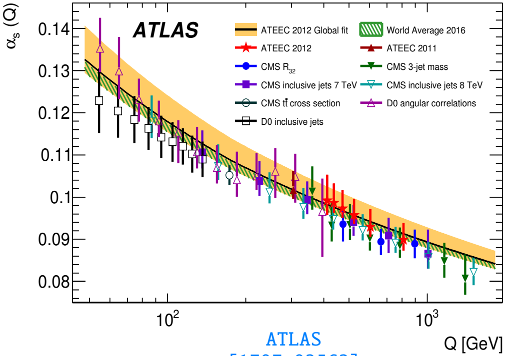
- Isolates high-energy information;
- Soft physics is suppressed and more easily understood.

The EEC: What makes it special?

The **EEC** addresses several important goals of QCD phenomenology, including:

$$\left\langle \sum_{\substack{\text{pairs } (i,j):\\ \theta_{ij}=\theta}} E_i \; E_j \right\rangle$$

- Measurements of α_s;



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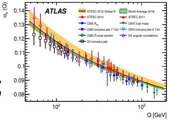
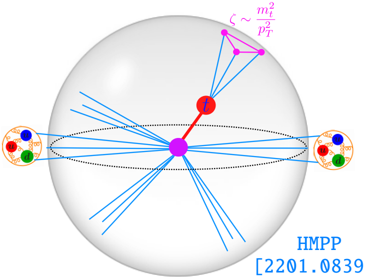


ATLAS [1707.02562]

- **Proposed measurements of m_t;**



HMPP [2201.08393]

Holguin-Moult-Pathak-Procura: [2201.08393]

HMPP-Shöfbeck-Schwartz: [2311.02157]

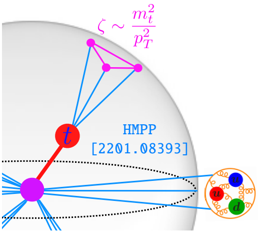
HMPPSS: [2311.14389]

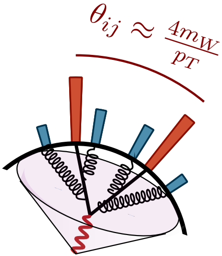
Xiao-Ye-Zhu: [2405.20001]

The EEC: Extraction of Mass Scales

In this talk, **look for m_W instead** (proof-of-concept):

Top mass extraction: **W mass extraction:**



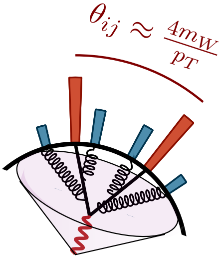


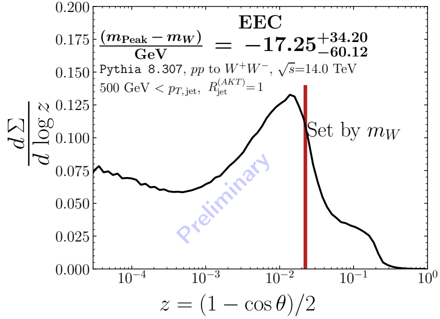
Two-pronged W decays play better with the **pair-wise** EEC:

$$\frac{d\Sigma}{d\theta} \sim \left\langle \sum_{\substack{\textbf{pairs } (i,j) \text{ of particles} \\ \text{separated by an angle } \theta}} E_i \, E_j \right\rangle$$

The EEC: Mass Scales from Angles

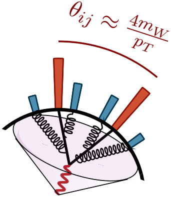
$$\left\langle \sum_{\substack{\textbf{pairs } (i,j) \text{ of particles} \\ \text{separated by an angle } \theta}} E_i \, E_j \right\rangle$$

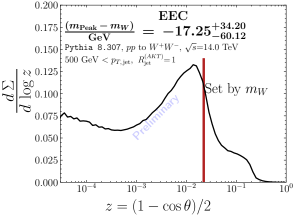




The EEC: Mass Scales from Angles

$$\left\langle \sum_{\substack{\textbf{pairs }(i,j):\\ \theta_{ij}=\theta}} E_i\, E_j \right\rangle$$





Two complications for achieving sharper resolution:

- The EEC involves a sum on *all pairs* of particles, when only some are at an angle dictated by m_W;
- Dealing with masses *indirectly* through angles. $\theta \sim m_W/p_T$, and p_T may not be sharply known.

⏩ At **particle-level**, required to use angles by collinear safety

The EEC: Mass Scales and Subjets

To extract energy-weighted **mass correlations** in

W **boson decays**, we need to go **beyond particle-level:**

> Which **collective** degrees of freedom
> reveal the **two-pronged** W decay shape?

The EEC: Mass Scales and Subjets

To extract energy-weighted **mass correlations** in

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> Which **collective** degrees of freedom
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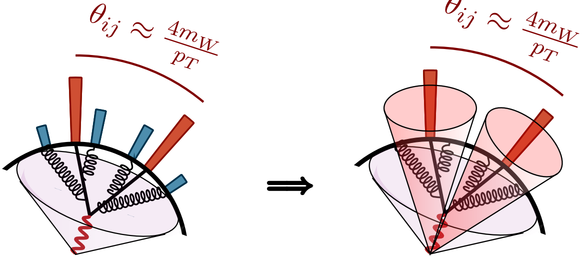
Subjets!

Subjet def dependence



Different collective d.o.f.s

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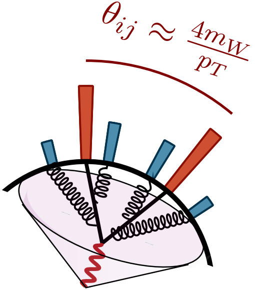
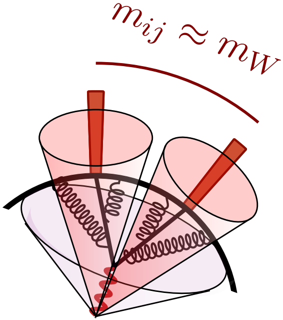
Introducing: The Mass EWOC



$\Longrightarrow$



$$\frac{d\Sigma}{d\theta} \sim \left\langle \sum_{\substack{\textbf{particle pairs } (i,j): \\ \theta_{ij} = \theta}} E_i \, E_j \right\rangle$$

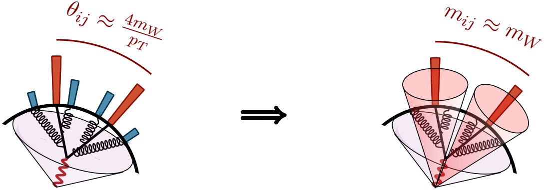
$\Longrightarrow$

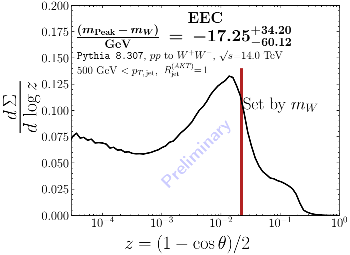
$$\frac{d\Sigma}{dm} \sim \left\langle \sum_{\substack{\textbf{subjet pairs } (i,j): \\ m_{ij} = m}} E_i \, E_j \right\rangle$$

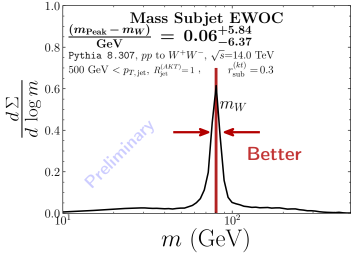
More formal · Generic EWOCs

The EEC and the Mass EWOC

Particle-Level EEC: **Mass EWOC:**

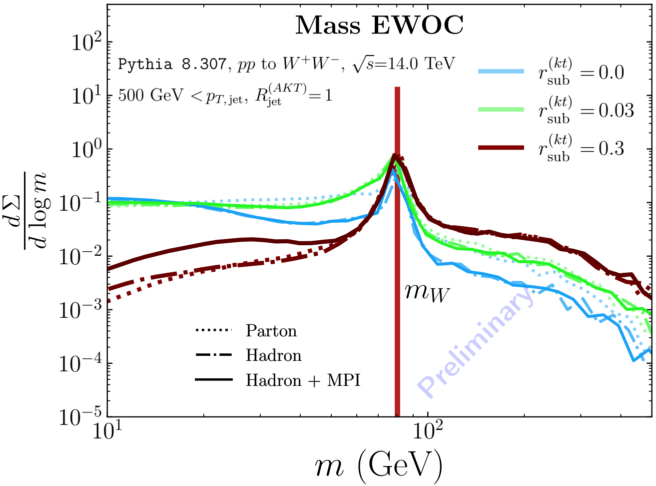


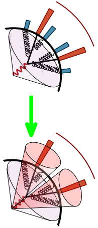




Using **subjets *and* directly probing mass correlations** improves accuracy and precision!

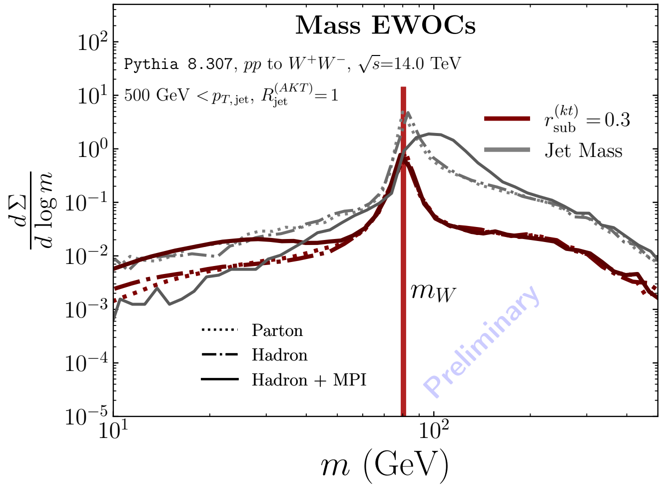
The Mass EWOC: Tuning Subjet Radius

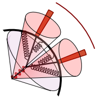




- Tune the subjet radius to isolate **two-pronged structure**;
- Larger subjets **less** sensitive to hadronization;
- Larger subjets have **more** underlying event at small m.

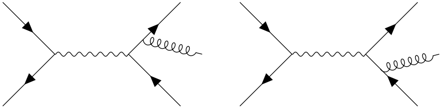
The Mass EWOC: Comparison to Jet Mass





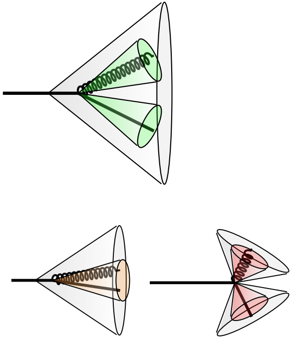
The **EWOC peak remains at m_W** even with underlying event (multi-parton interactions/MPI)

The Mass EWOC in $e^+e^- \to$ hadrons



$$\left\langle \sum_{\substack{\text{subjets } (i,j): \\ m_{ij}=m}} E_i \, E_j \right\rangle$$

LO/Collinear Limit: (anti-)quark jets

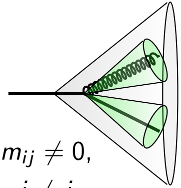


Quark/gluon can be in:

- Different subjets
- Same subjet
- Different jets

The Mass EWOC in $e^+e^- \to$ hadrons

LO: (anti-)quark jets



$m_{ij} \neq 0$,
$i \neq j$

Quark/gluon can be:

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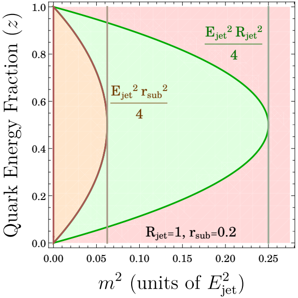
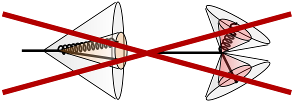
- Same subjet
- Different jets

LO Phase Space





Don't contribute

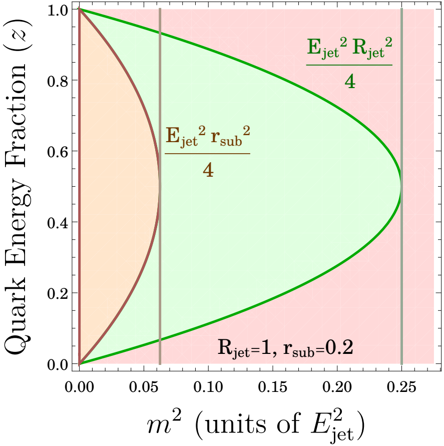
Single subjet per jet,
$m_{ii} = 0$ (WTA $|p|$ scheme)

Only **single jets with two widely separated partons** contribute to the mass EWOC at LO

The Mass EWOC in $e^+e^- \to$ hadrons

Quark/gluon can be in:

- Different subjets
- Same subjet
- Different jets



phase space + **energy weighting** + QCD splitting function:

$$\frac{d\Sigma}{dm} \sim \frac{a_s C_F}{m} \times \left(\textbf{height of green region} \text{ at fixed } m \right)$$

The Mass EWOC in $e^+e^- \to$ hadrons

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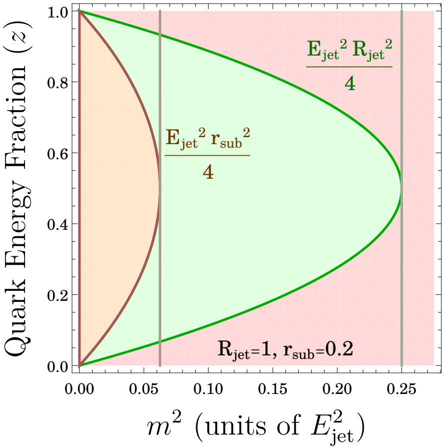
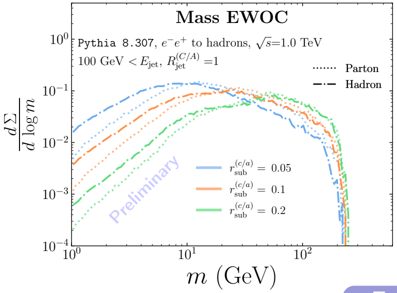
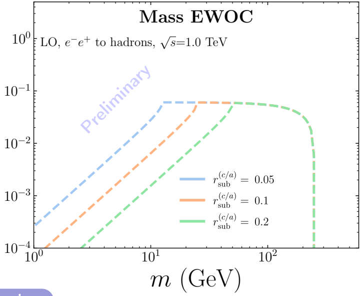


$$\frac{d\Sigma}{dm} \sim \frac{a_s C_F}{m}\left(\mathrm{pol}(m)\sqrt{1-\frac{16m^2}{s\,R_{\rm jet}^2}} - \mathrm{pol}(m)\sqrt{1-\frac{16m^2}{s\,r_{\rm sub}^2}}\right)$$

$e^+e^- \to$ hadrons: Pythia vs. LO





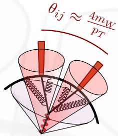
▸▸ Together

- LO features at the correct scales: $\frac{\sqrt{s}\,R_{\rm jet}}{4}$ and $\frac{\sqrt{s}\,r_{\rm sub}}{4}$;
- Square roots in LO result $\implies$ cusps at $\sqrt{s}\,r_{\rm sub}/4$;
- Higher (log.) accuracy needed for accurate comparison.

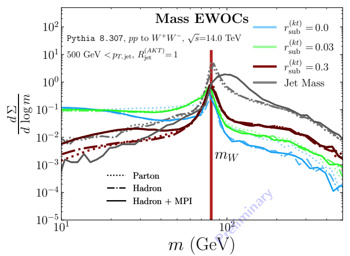
▸▸ Many scales $\implies$ Subtle resummation

EWOC Review

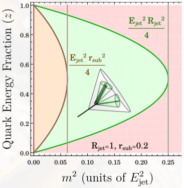
- Subjets isolate collective phenomena + scales
  - More than subjet radius

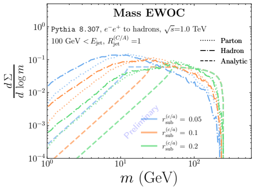


- Subjets $\implies$ **EWOCs**; **EWOCs** isolate *observables* of interest
  - More than mass



- Calculation in $e^+e^- \to$ hadrons
  - With subtleties to explore





What is the EEC? in QCD pheno

The **Energy-Energy Correlator** (**EEC**)
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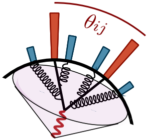

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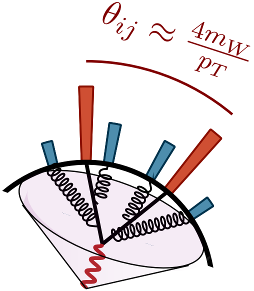
EEC Definition

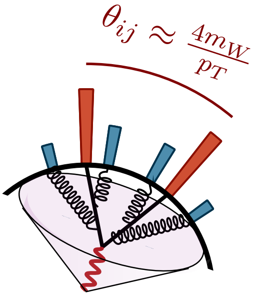
The **Energy-Energy Correlator** (**EEC**) ◂ in QCD pheno



$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\theta} = \left\langle \sum_{\substack{\textbf{particles} \\ i,j}} \frac{E_i\, E_j}{Q^2}\, \delta\left(\theta - \theta_{ij}\right) \right\rangle_{\text{many events}}$$

EEC Definition

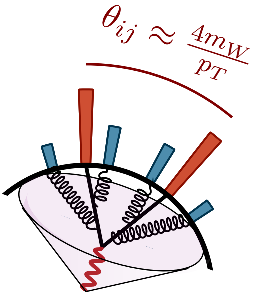
The **Energy-Energy Correlator** (**EEC**) ◂ in QCD pheno



$$\frac{d\Sigma(\chi)}{d\chi} = \left\langle \sum_{\substack{\textbf{particles} \\ i,j}} \frac{E_i E_j}{Q^2} \, \delta\left(\chi - \frac{1 - \cos\theta_{ij}}{2}\right) \right\rangle_{\text{many events}}$$

EEC Definition

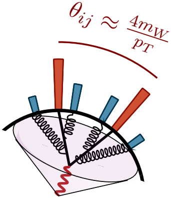
The **Energy-Energy Correlator** (**EEC**) ◂ in QCD pheno

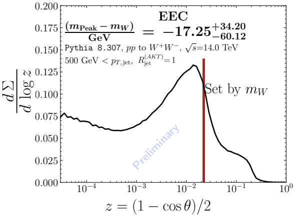


$$\frac{\mathrm{d}\Sigma(\chi)}{\mathrm{d}\chi} = \frac{1}{\sigma} \sum\!\!\!\!\!\!\int \mathrm{d}\sigma \sum_{\substack{\textbf{particles}\\ i,j}} z_i\, z_j\, \delta\left(\chi - \frac{1-\cos\theta_{ij}}{2}\right)$$

The EEC: Mass Scales from Angles

$$\left\langle \sum_{\substack{\textbf{pairs}\ (i,j):\\ \theta_{ij}=\theta}} E_i\ E_j \right\rangle$$



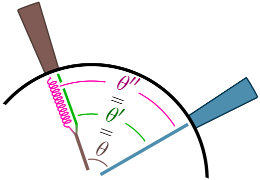


◂ Mass extraction

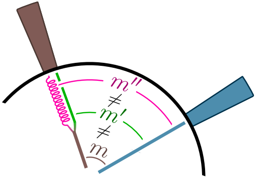
Can't probe mass correlations directly to extract m_W.

At **particle-level**, required to use angles for collinear safety:

Angular correlations:



Mass correlations:



Generic Pairwise EWOCs

Definition: Mass EWOC

The mass Energy-Weighted Observable Correlator (**EWOC**) is a distribution of the form

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}m}(m, R_{\mathrm{jet}}, r_{\mathrm{sub}}) \triangleq$$

$$\int \frac{\mathrm{d}\sigma}{\sigma} \sum_{J \in \mathrm{Jets}} \sum_{s_1, s_2 \in \mathrm{Subjets}} z_1 \, z_2 \, \delta\left(m - \left|(p_{s_1} + p_{s_2})^2\right|\right) .$$

◂ Back to the pheno

Generic Pairwise EWOCs

- Let $\mathcal{O}\,(\cdot,\cdot)$ be a pairwise observable on pairs of pseudo-jets;

Definition: Pairwise EWOC

A pairwise Energy-Weighted Observable Correlator (**EWOC**) is a distribution of the form

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\chi}(\chi, R_{\text{jet}}, r_{\text{sub}}; \mathcal{O}] \triangleq \int \frac{\mathrm{d}\sigma}{\sigma_0} \sum_{J \in \text{Jets}} \sum_{s_1, s_2 \in \text{Subjets}} z_1^n \, z_2^m \, \delta\left(\chi - \mathcal{O}(s_1, s_2)\right).$$

◂ Back to the pheno

Generic Pairwise EWOCs

- Let $\mathcal{O}(\cdot,\cdot)$ be a pairwise observable on pairs of pseudo-jets;
- Additional dependence on energy weights and (sub)jet definition (jet algorithm and recombination scheme).

Definition: Pairwise EWOC

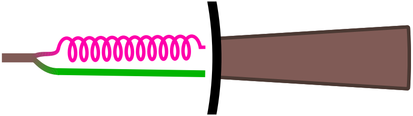
A pairwise Energy-Weighted Observable Correlator (**EWOC**) is a distribution of the form

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\chi}(\chi, R_{\text{jet}}, r_{\text{sub}}; \mathcal{O}] \triangleq \int \frac{\mathrm{d}\sigma}{\sigma_0} \sum_{J \in \text{Jets}} \sum_{s_1, s_2 \in \text{Subjets}} z_1^n\, z_2^m\, \delta\left(\chi - \mathcal{O}(s_1, s_2)\right).$$

Back to the pheno

Now IRC Safe: Energy Weights

- **Particle-Level**: energy fractions with $n \neq 1$ are collinear unsafe even for the EEC (let alone EWOCs!):



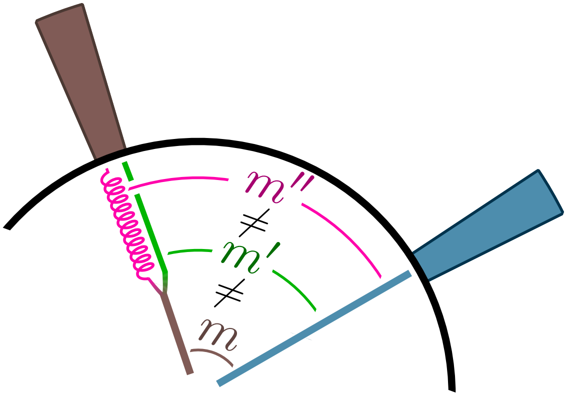
$$z_1^n + z_2^n \neq (z_1 + z_2)^n$$

$$\sum_{\substack{\text{particles } (i,j) \\ \text{after splitting}}} z_i^n z_j^m \delta(\theta - \theta_{ij}) \neq \sum_{\substack{\text{particles } (i,j) \\ \text{before splitting}}} z_i^n z_j^m \delta(\theta - \theta_{ij})$$

- **Collinear safe subjet algorithms** yield collinear safe subjets $\implies$ collinear safe sums!

⏩ *cf* Lund EEC to remove underlying event at RHIC by [2312.12527]

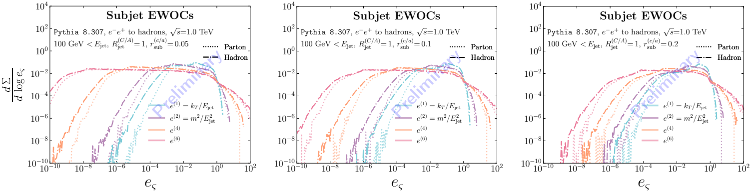
Collinear Unsafety: Safe at LO



- Collinear unsafety of **particle-level EWOCs** only manifests with three or more final-state particles.
- Misleadingly, **particle-level EWOCs** (within jets) can be computed at LO.

Angularity EWOCs

$$\mathcal{O}_{ij} \to e_{ij}^{(\varsigma)} = \frac{\min(E_i, E_j)}{E_{\text{jet}}} \theta_{ij}^{\varsigma}$$



Similar to Mass EWOC ($\varsigma = 2$), now with a tunable parameter

Alternatives to Subjets: Lund EEC

Back to the subjets

Lund EEC: [2312.12527]

- EEC based on Lund declusterings;
- Collinear safe $\rightarrow$ energy weighting now possible for suppression of underlying event cf Subjets
- Similar to subjets which satisfy certain criteria:

  - Undo the last-clustering step to generate two subjets, j_1 and j_2.
  - Calculate their relative k_t defined as $k_t = \min(x_1, x_2)\Delta R_{12}$, where the concrete definitions of x (an energy-like variable) and ΔR_{12} (an angular-like variable) depend on the collision system. For e^+e^-, $x_i = E_i$ and $\Delta = \theta_{ij}$, while in pp in $x_i = p_{ti}$ and $\Delta R_{ij} = \sqrt{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}$.
  - Only when $k_t > k_{t,\text{cut}}$, record the softest branch, so-called primary Lund declustering.
  - Repeat from step 1 following only the hardest subjet, i.e., the primary branch.
  - Once there is nothing left to decluster, calculate the EEC as

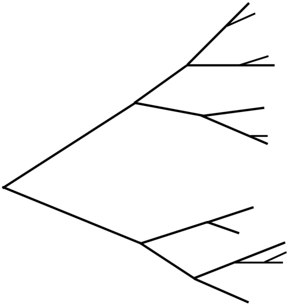
$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\chi} = \frac{1}{\sigma} \sum_{\{i,j\}\in\text{declust.}} \int_0^1 \mathrm{d}z \frac{\mathrm{d}\sigma}{\mathrm{d}\theta_{ij}\mathrm{d}z} z^n (1-z)^n \delta\left(\chi - \frac{\theta_{ij}}{R}\right) \Theta(k_t > k_{t,\text{cut}}) , \qquad (5.1)$$

where the sum runs over all primary Lund declusterings.

FastEEC: [2406.08577]

From “Energy Correlators & Beyond,” Waalewijn, 2024 MITP Workshop

- Recluster jets with C/A



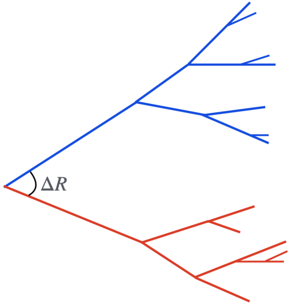
Alternatives to Subjets: FastEEC

Back to the subjets

FastEEC: [2406.08577]

From "Energy Correlators & Beyond," Waalewijn, 2024 MITP Workshop

- Recluster jets with C/A
- Take first split, separation ΔR



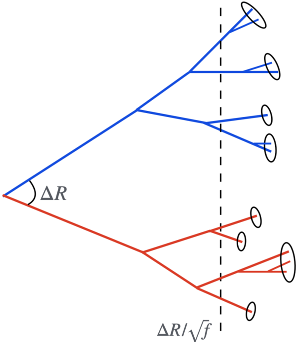
Alternatives to Subjets: FastEEC

◂ Back to the subjets

FastEEC: [2406.08577]

From “Energy Correlators & Beyond,” Waalewijn, 2024 MITP Workshop

- Recluster jets with C/A
- Take first split, separation ΔR
- Decluster until subjets with radius $r = \Delta R/\sqrt{f}$



Alternatives to Subjets: FastEEC

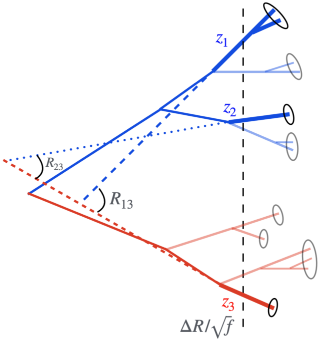
◂ Back to the subjets

FastEEC: [2406.08577]

From "Energy Correlators & Beyond," Waalewijn, 2024 MITP Workshop

- Recluster jets with C/A
- Take first split, separation ΔR
- Decluster until subjets with radius $r = \Delta R/\sqrt{f}$
- Obtain correlator for terms involving both sides of the split:

$$\sum_i z_i \sum_j z_j \, \delta(R_L - R_{ij})$$



Alternatives to Subjets: FastEEC

Back to the subjets

FastEEC: [2406.08577]

From “Energy Correlators & Beyond,” Waalewijn, 2024 MITP Workshop

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- Take first split, separation ΔR
- Decluster until subjets with radius $r = \Delta R/\sqrt{f}$
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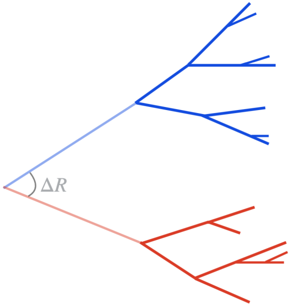
$$\sum_i z_i \sum_j z_j \, \delta(R_L - R_{ij})$$

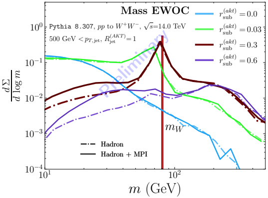
- Recurse on each branch to get correlations at smaller scales.



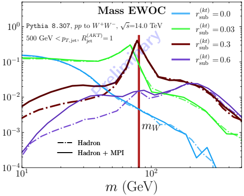
Subjet Algorithms in pp collisions

Back to the Mass EWOC

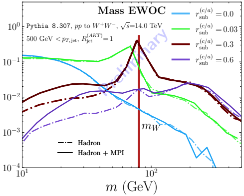
Anti-k_t



k_t



C/A

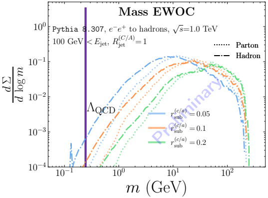


- Very weak algorithm dependence, greatest at large r_{sub};
- Even at large r_{sub}, k_t and C/A subjets behave nearly identically.

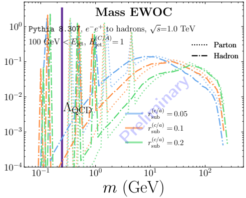
Recombination Schemes in e^+e^- collisions

However, **strong** dependence on *recombination scheme*:

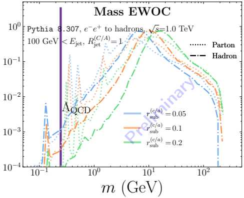
WTA $|p|$ Scheme, $ms = 0$



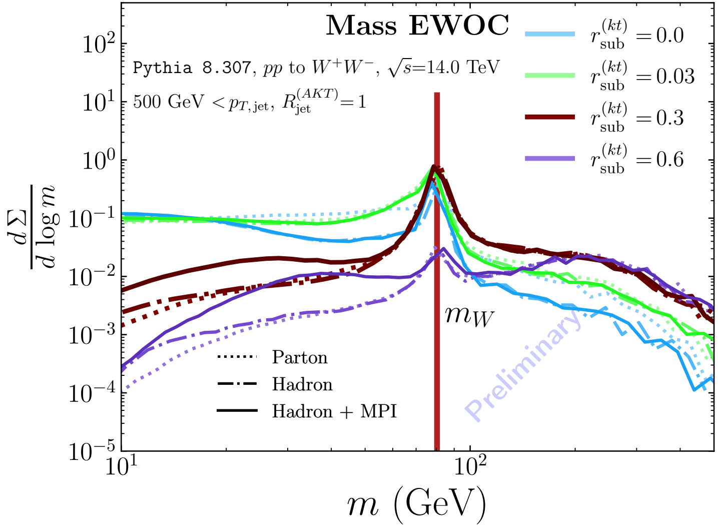
WTA $|p|$ Scheme, $ms \neq 0$



E Scheme, $ms \neq 0$



MPI in proton-proton collisions

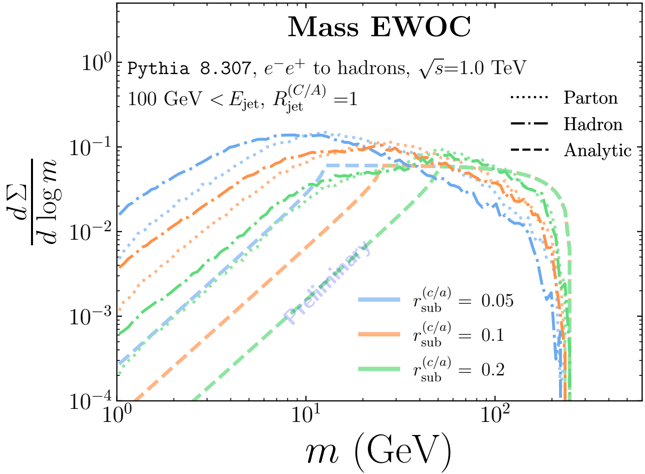


$e^+e^- \to$ hadrons: Pythia vs. LO

LO vs. Pythia:

◂ Back to LO

(massless WTA $|p|$ scheme)



Resummation

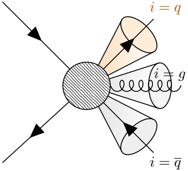
Resummation depends on the separation of scales: Back to LO

$$m \ll Q r_{\mathsf{sub}} \ll Q R_{\mathsf{jet}}$$

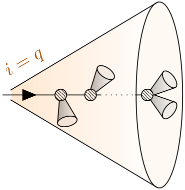
or

$$Q r_{\mathsf{sub}} \ll m \ll Q R_{\mathsf{jet}}$$

For $m \ll Q R_{\mathsf{jet}}$, we can **factorize** the EWOC calculation:

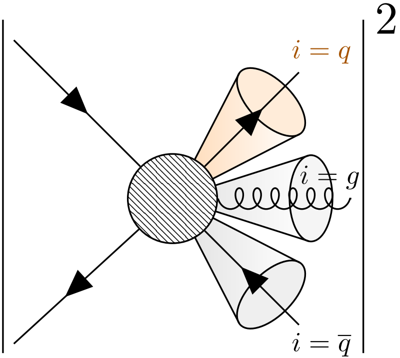


- The **hard function** conveys the details of jet production in the hard process;



- The **jet function** conveys the details of jet substructure, and the EWOC within a single jet.

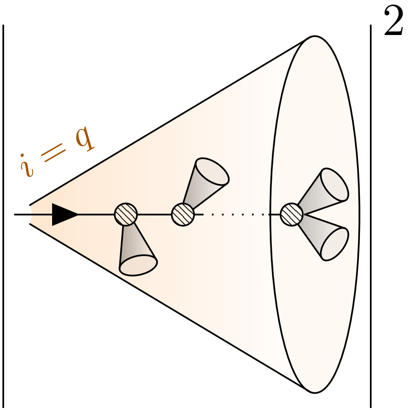
Resummation: Hard Function



$$H_i \sim \frac{\mathrm{d}n_i}{\mathrm{d}x}$$

The **hard function** H_i is, roughly, the number density (per $x = E_{\text{jet}}/\sqrt{s}$) of jets of flavor i emerging from the hard process.

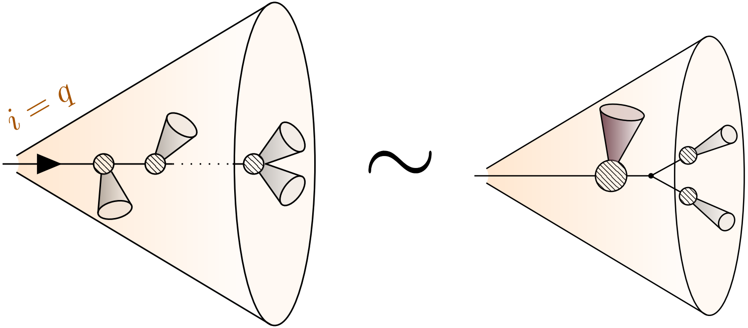
Resumption: Jet Function



$$J_i^{\mathcal{O}} \sim \frac{\Sigma_{\mathcal{O}}}{\mathrm{d}n_i}$$

The **jet function** $J_i^{\mathcal{O}}$ is, roughly, the contribution to the $\mathcal{O}$-EWOC from a jet of partonic flavor i.

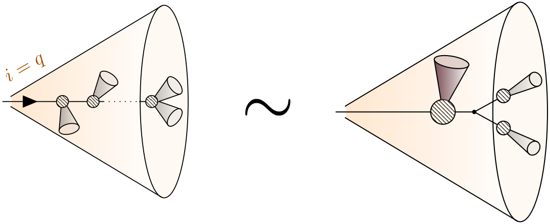
Resummation: Jet Function at LL



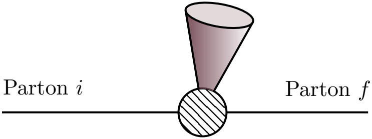
In the collinear limit, the jet function is dominated by correlations between splittings near the end of the jet's history;

We can then **factorize the jet function even further**.

Jet Function: Fragmentation



Parton-to-parton fragmentation

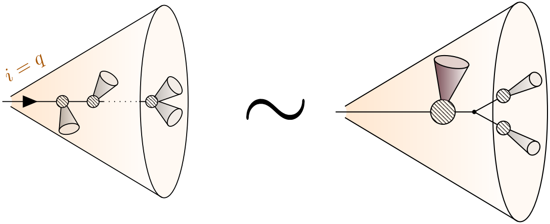


Semi-inclusive fragmenting jet function: [1606.07063]

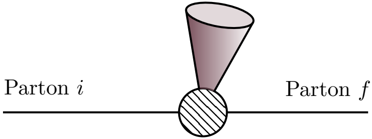
followed by **parton-to-subjet fragmentation**

Semi-inclusive jet function: [1606.06732]

Jet Function: Fragmentation



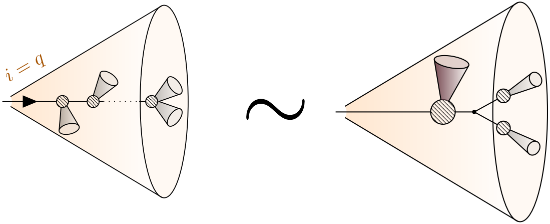
Parton-to-parton fragmentation: Solution to DGLAP



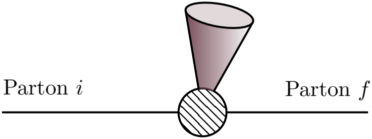
Semi-inclusive fragmenting jet function: [1606.07063]

$$F_{f\leftarrow i}\left(z = \frac{E_f}{E_i};\ R_{\text{jet}} \leftarrow R_{\text{jet}}\right) = \delta\,(1-z)\ \delta_{ij}$$

Jet Function: Fragmentation



Fragmentation in Mellin space: Solution to DGLAP



Semi-inclusive fragmenting jet function: [1606.07063]

$$\hat{F}_{f\leftarrow i}(m;\, \theta_f \leftarrow R_{\text{jet}}) \overset{\text{LL}}{\approx} \left(\frac{\alpha_s\left(Q\,\theta_f\right)}{\alpha_s\left(Q\,R_{\text{jet}}\right)}\right)^{\frac{\hat{p}(m)}{2\pi\beta_0}}_{f\leftarrow i}$$