



Energy Correlators, Heavy Flavor, & Precision QCD

Evan Craft — Yale University
BOOST 2024



Based on **work** with K. Lee, B. Mecaj, I. Moulton, & M. Gonzalez



Yale University

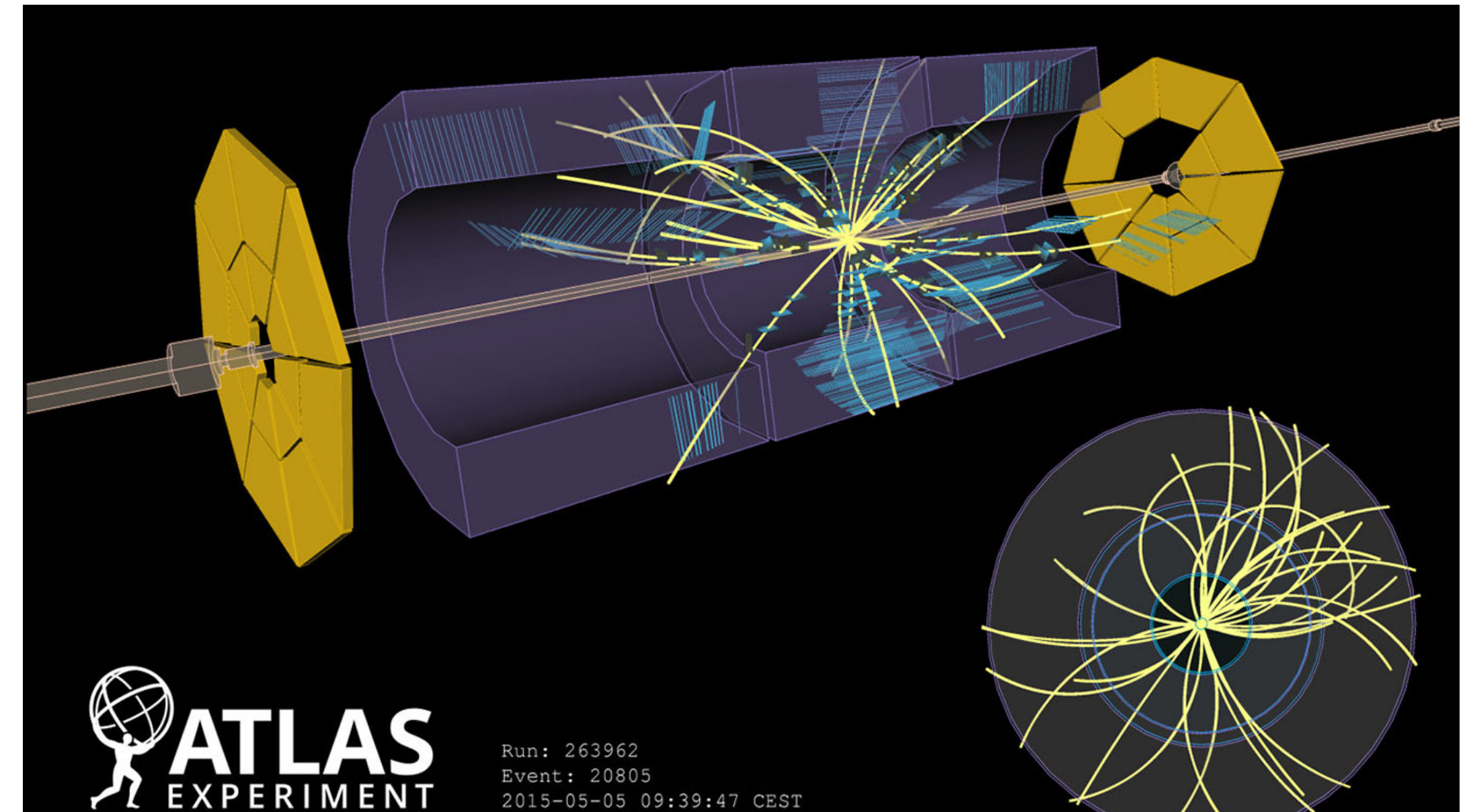
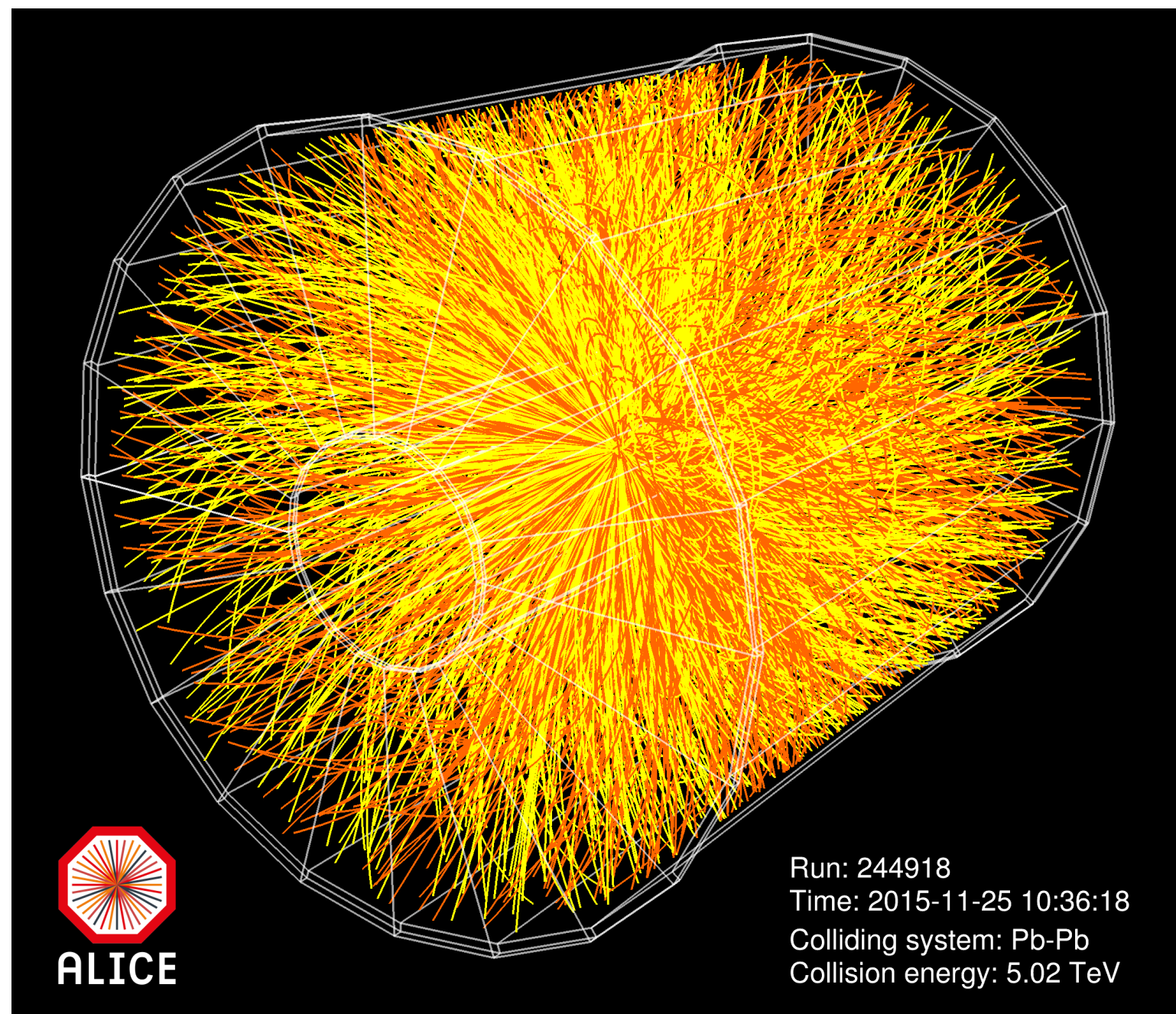


MIT

Collider Experiments

Many important questions have been addressed at **collider experiments**

→ Great historical success in verifying properties of the standard model



→ But the detailed structure of QCD produces immensely complicated datasets.

→ Need new tools for future success

A unique frontier for novel collaborations between both **theory and experiment**

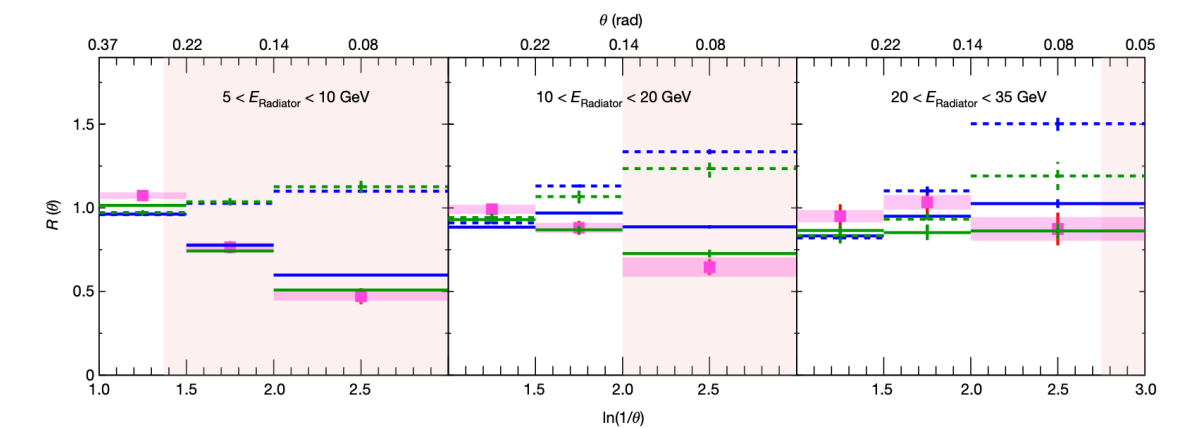
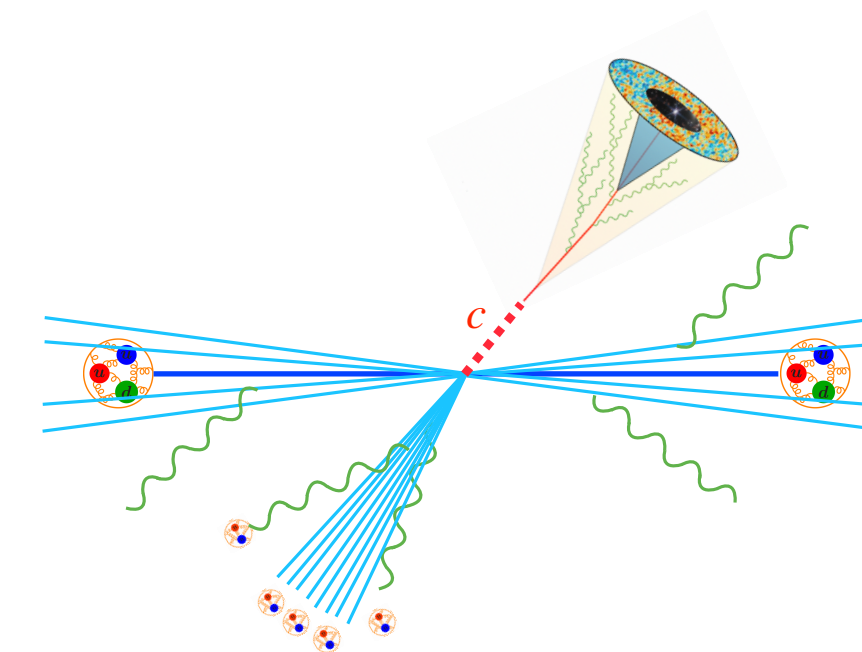
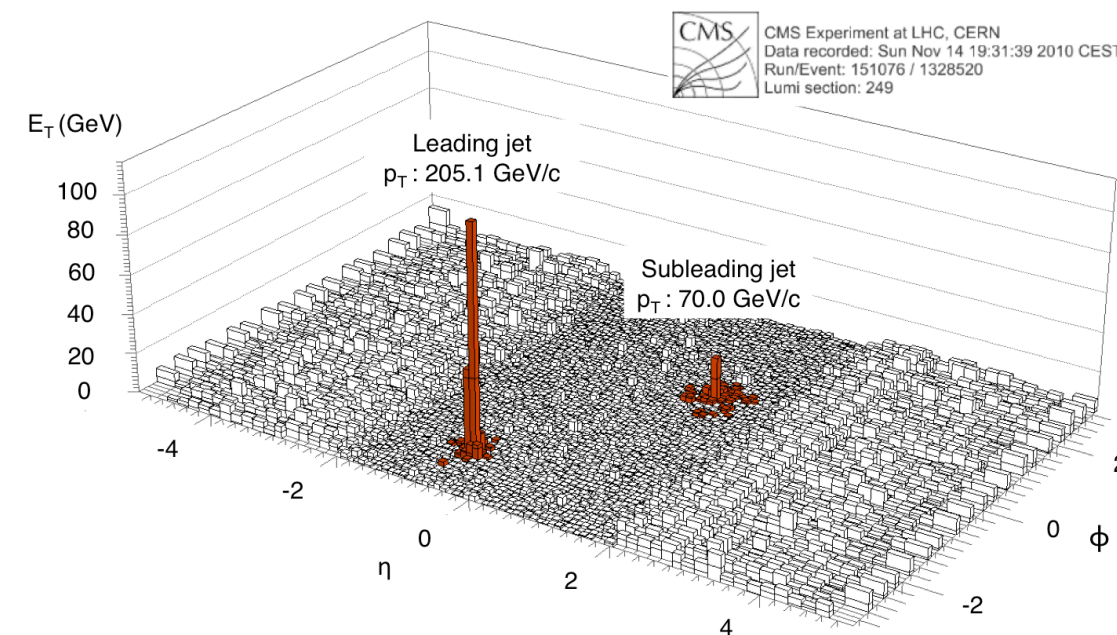
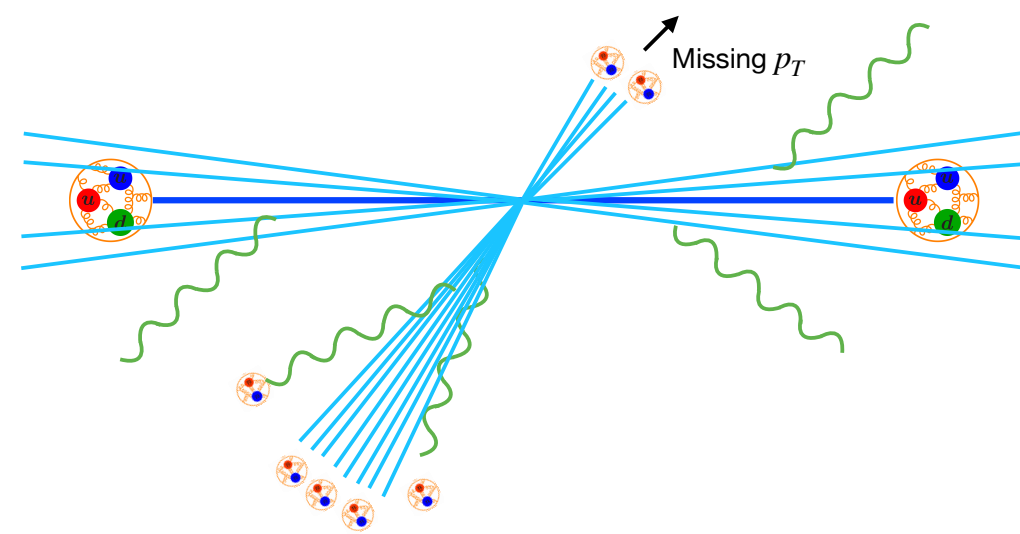
How do we study collisions?

Along with **many** more exciting observations!

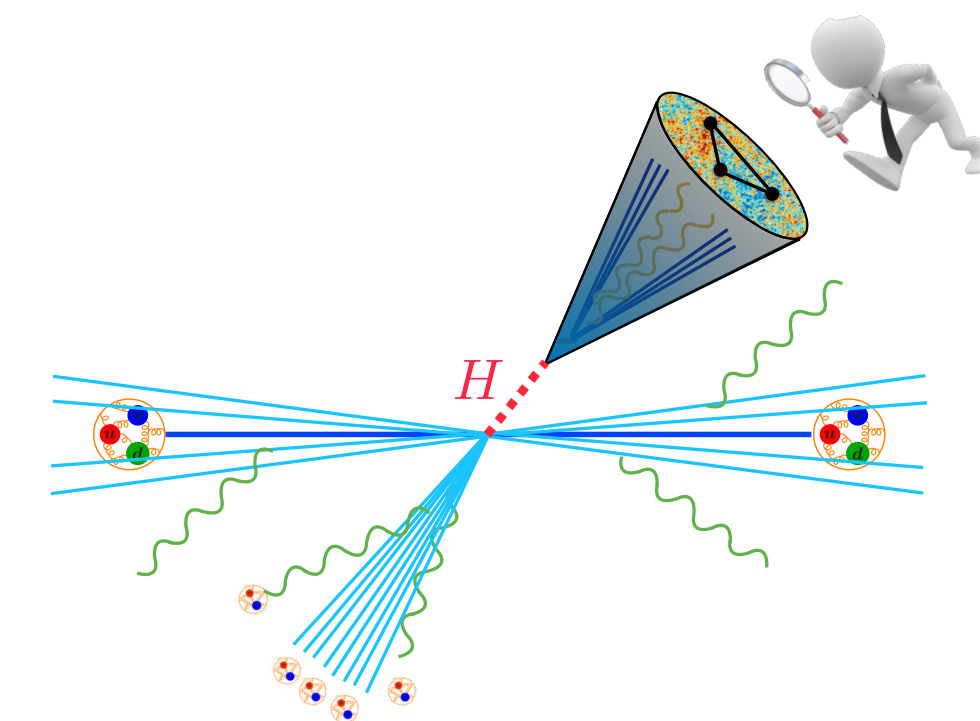
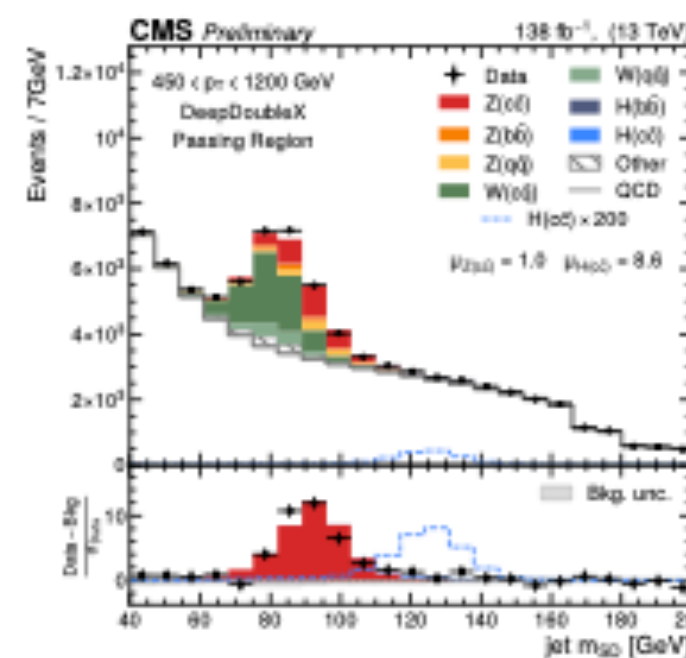
Several remarkable observations made by studying **jets**

→ provided initial evidence for the **existence** of the Quark Gluon Plasma

→ allowed first ever observation of the **“dead cone”** effect of QCD



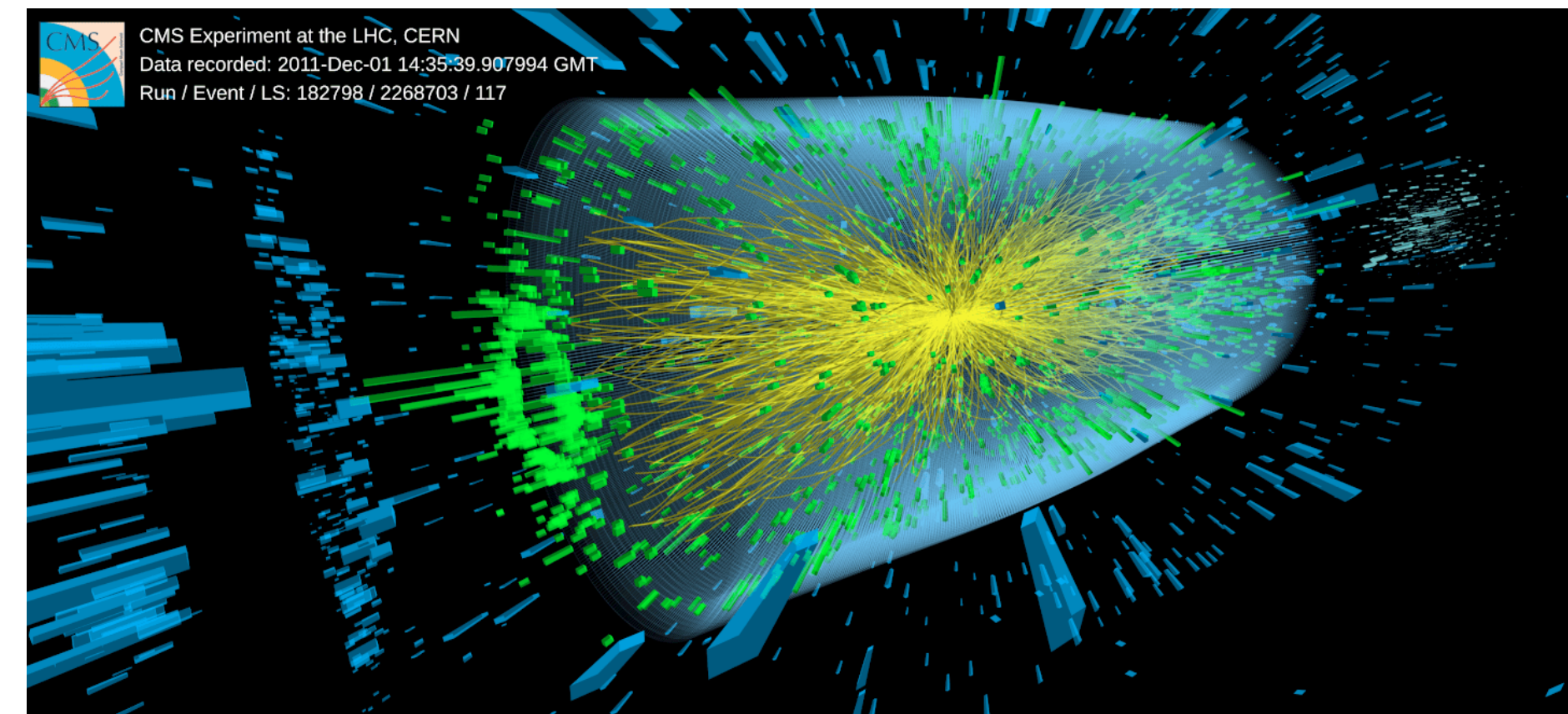
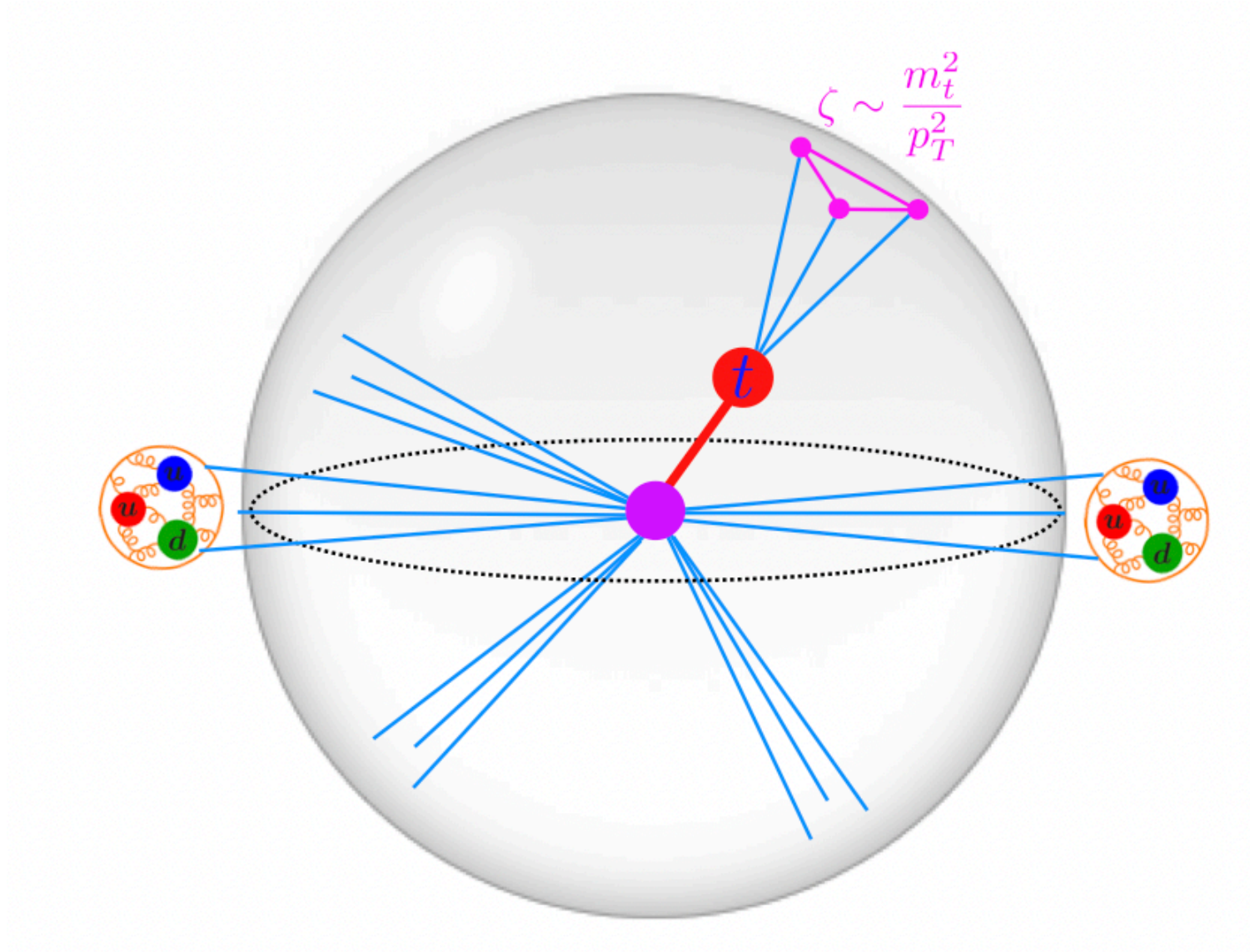
→ provides the most stringent bounds on **charm** Yukawa couplings



From Searches to Measurements

To fully take advantage of the LHC, it is necessary to bolster our current physics searches with **first principles theory calculations**

→ Many interesting opportunities to study QCD at high energies: understanding confinement, precision measurements, ...



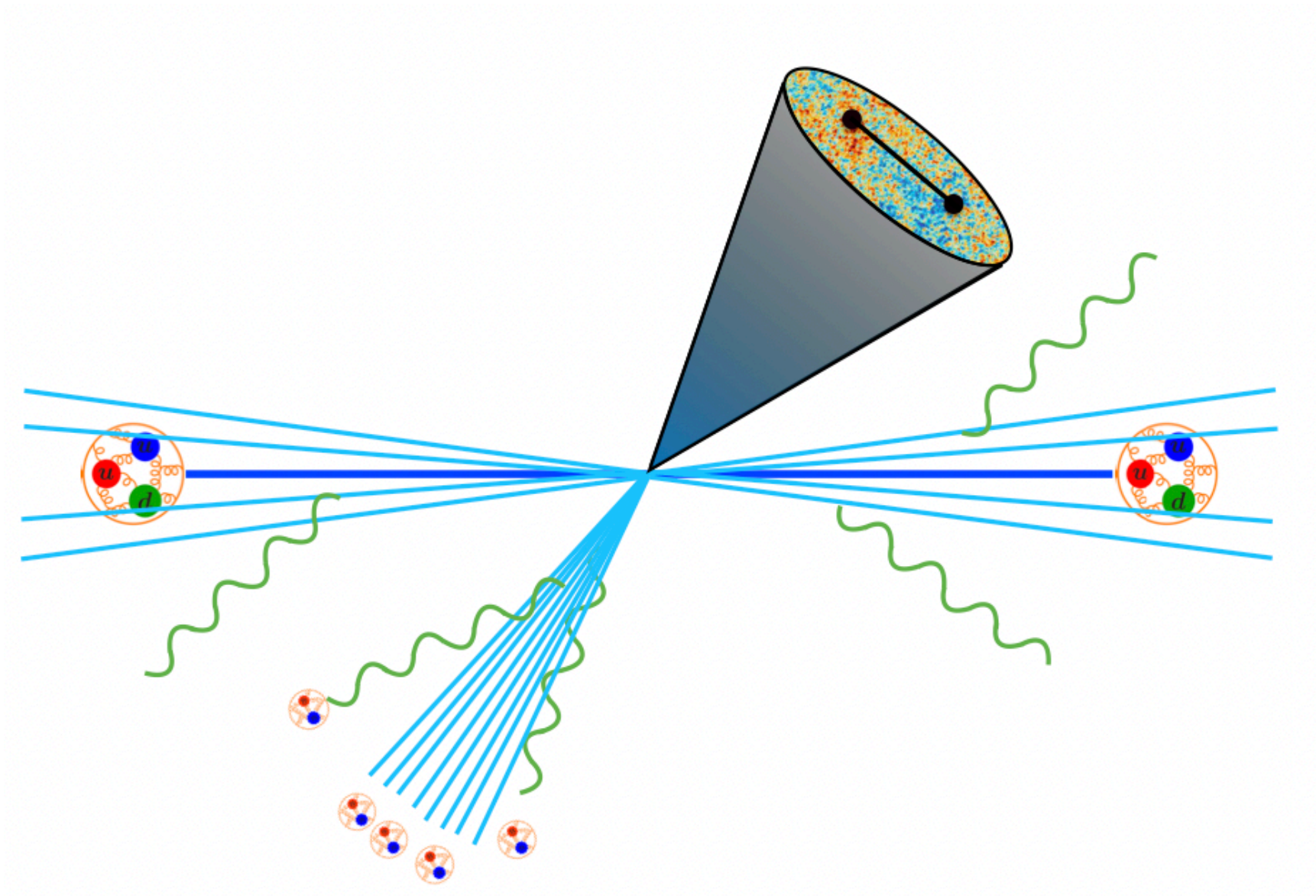
Requires the development of a **new set of theoretical tools**

Energy Flow Operators

Field Theoretic Foundations

Energy Flow Operators

From the perspective of QFT, jet substructure is the study of **correlation functions** of energy flow operators



$$\mathcal{E}(\vec{n}) = \lim_{t \rightarrow \infty} t^3 \int_0^1 dv v^2 \left[n^i T_i^0(t, tv \vec{n}) \right]$$

Sveshnikov, Tkachov (1995)

→ “Energy Flow Operator”

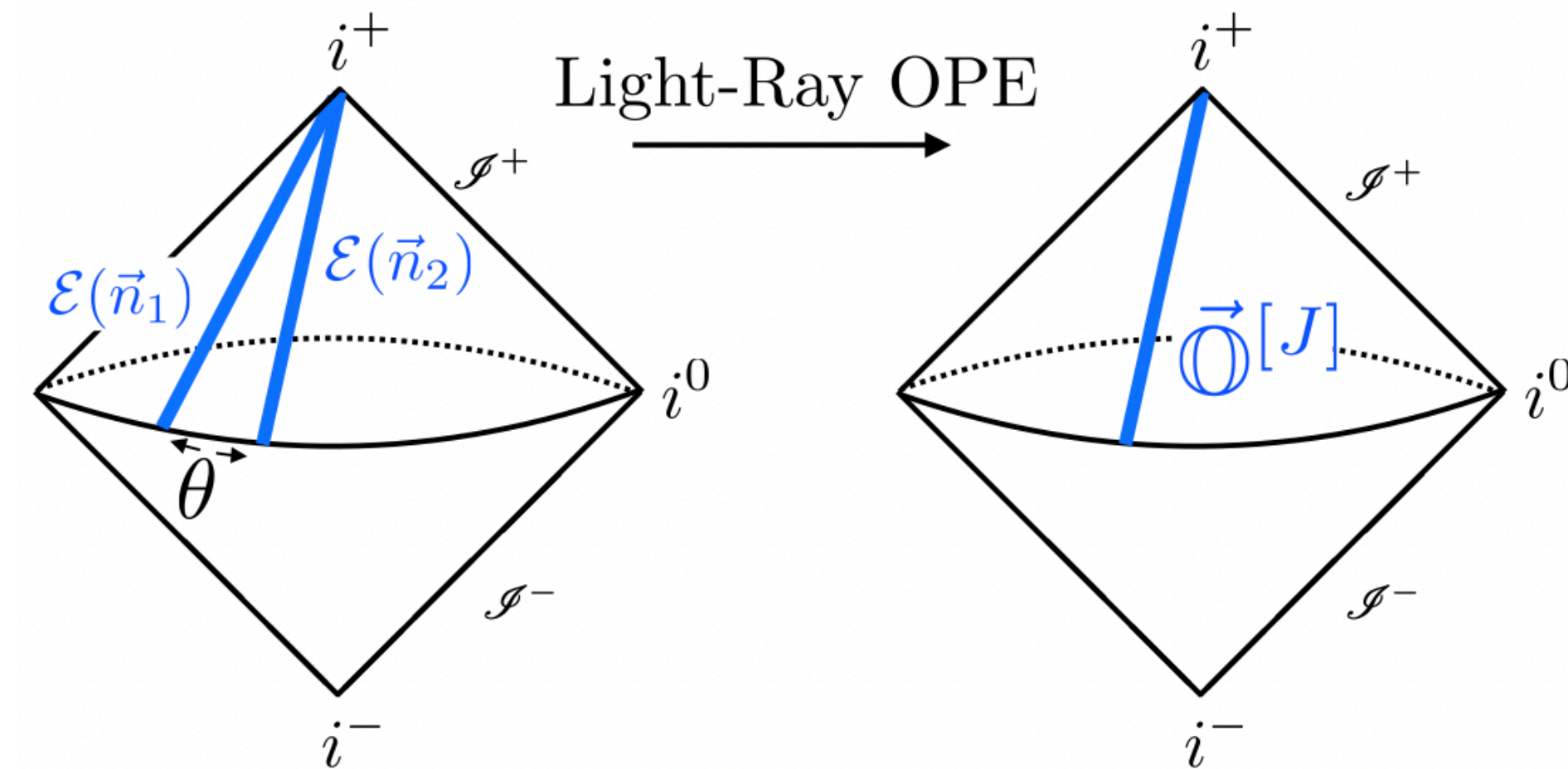
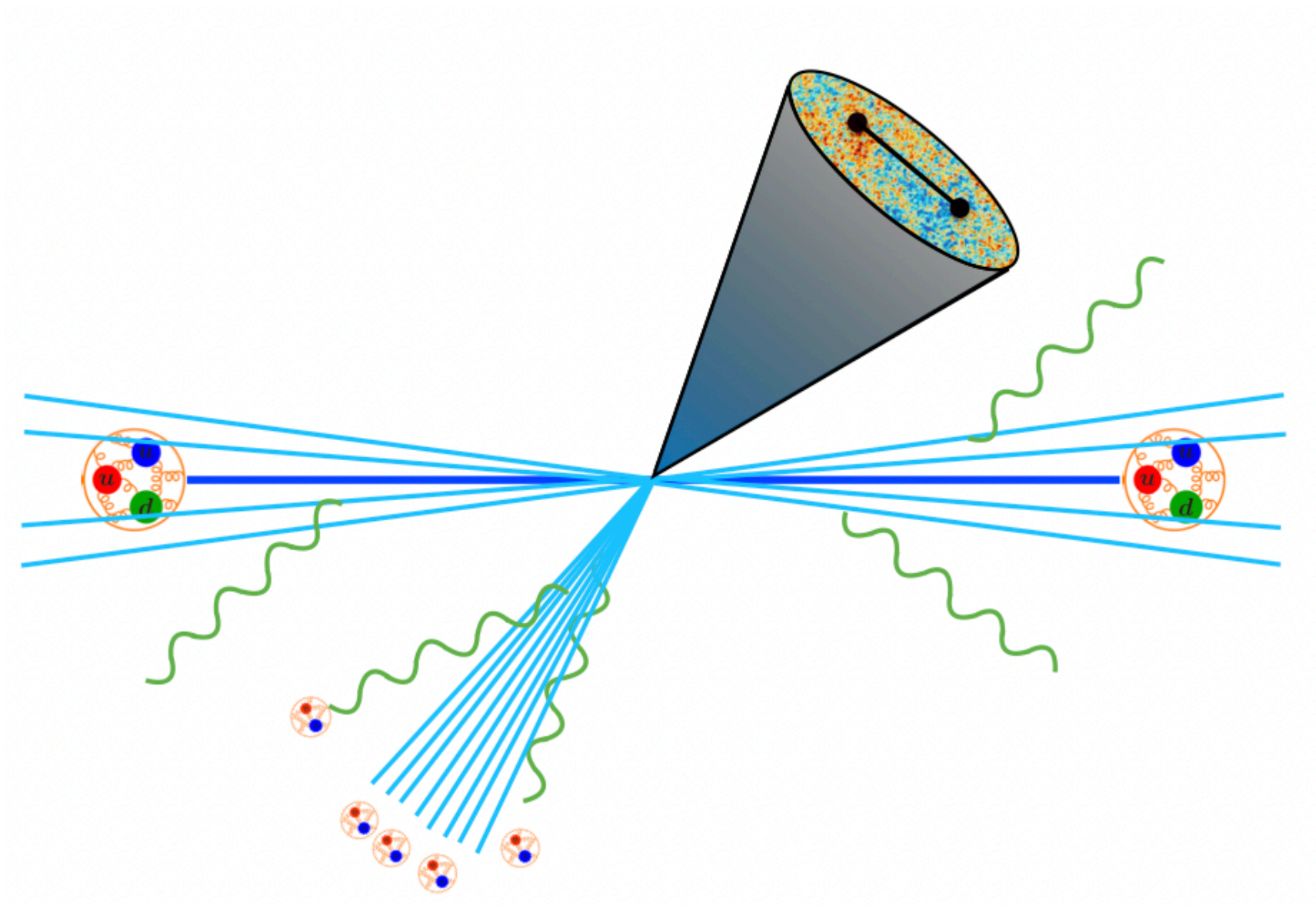
$$\langle \Psi | \mathcal{E}(\hat{n}_1) \dots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$

→ “Statistical Correlations”

These correlation functions measure the **flow** of energy at infinity.

Energy Flow Operators

Situations of interest at the LHC involve non-generic configurations of lightray operators: **interested in the small angle (OPE) limit.**



$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum_i \theta^{\tau_i-4} \mathbb{O}_i(\hat{n}_1)$$

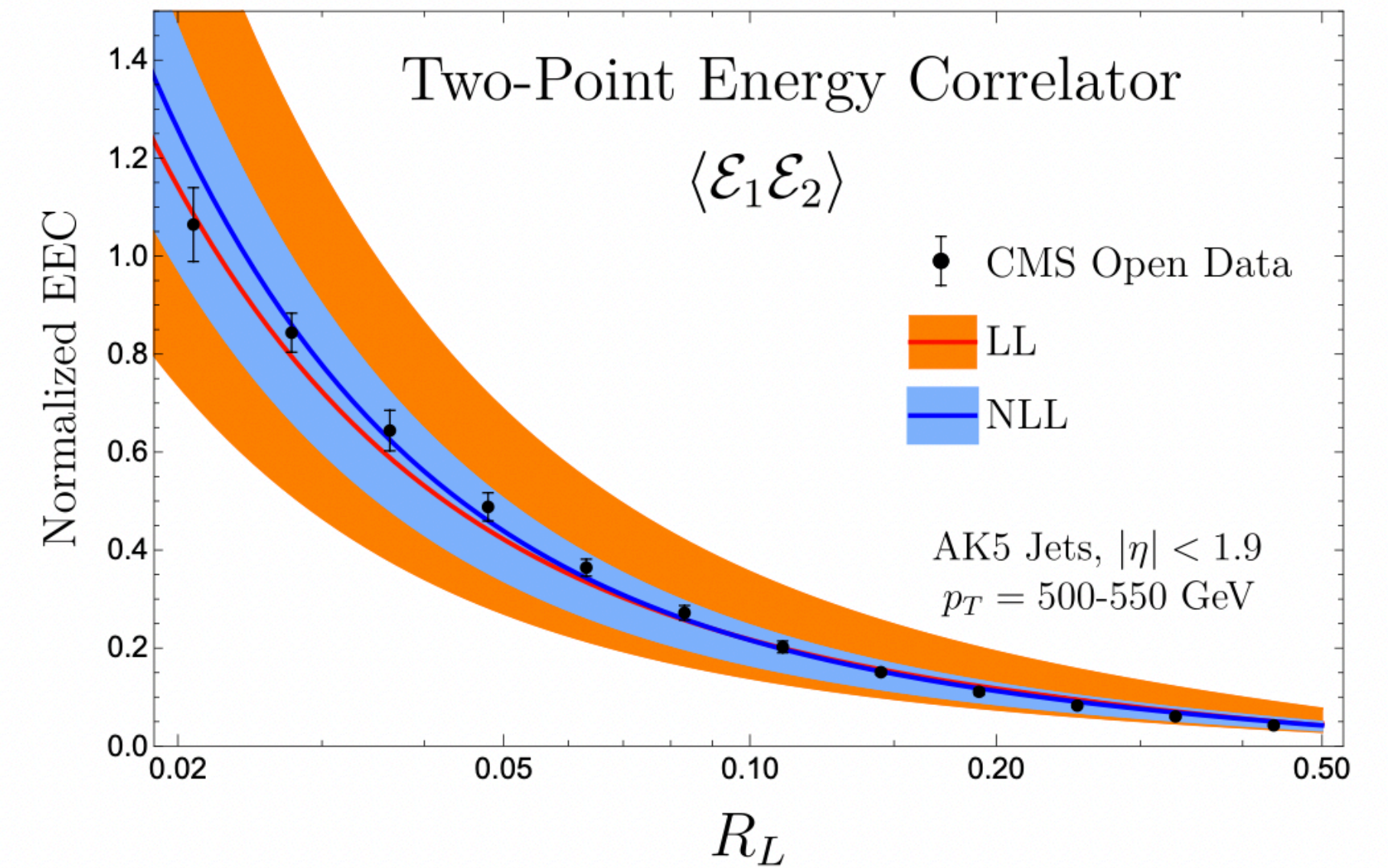
[Hofman, Maldacena]

In the small angle limit, these lightray operators should exhibit the **universal behavior of QCD**

Universal Behavior of QCD

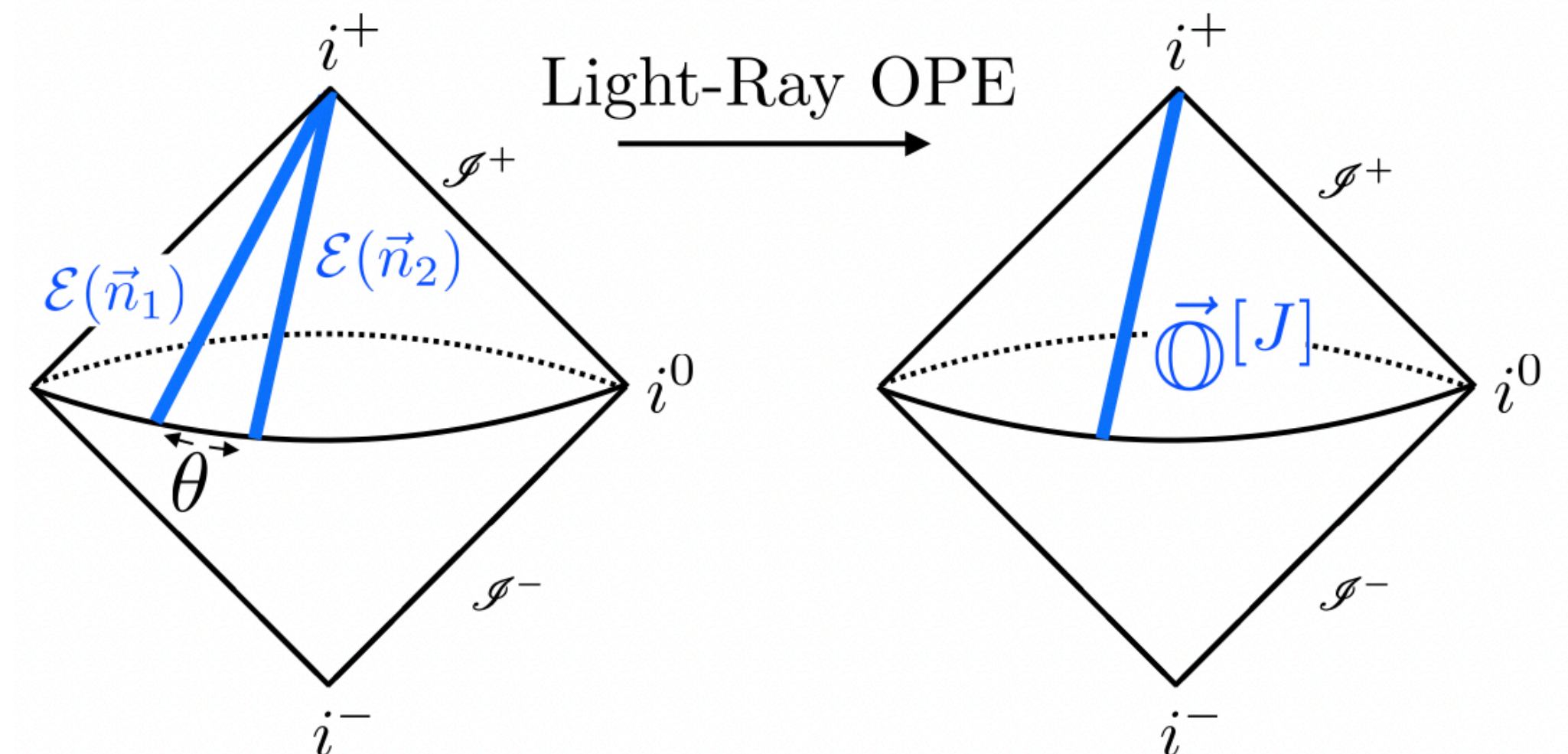
Energy flow operators exhibit universal scaling

→ The substructure of jets is completely determined by the OPE structure of **light-ray operators**



$$\mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \sim \sum_i \theta^{\tau_i - 4} \mathbb{O}_i(\hat{n}_1) \quad \begin{array}{l} \text{[Hofman, Maldacena]} \\ \text{[Moult, Zhu]} \end{array}$$

Reformulation of jet substructure using a **broader language** such that it can draw from diverse areas of physics



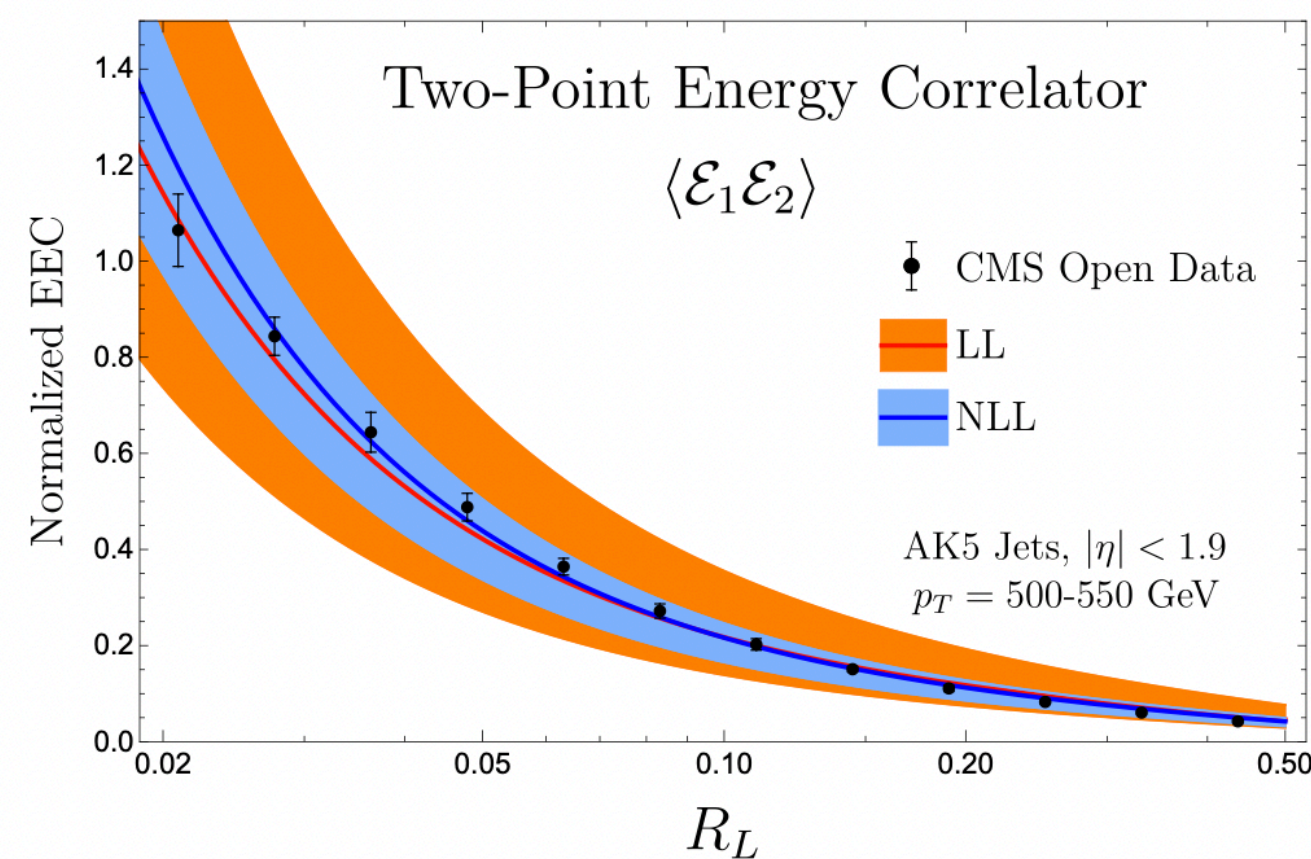
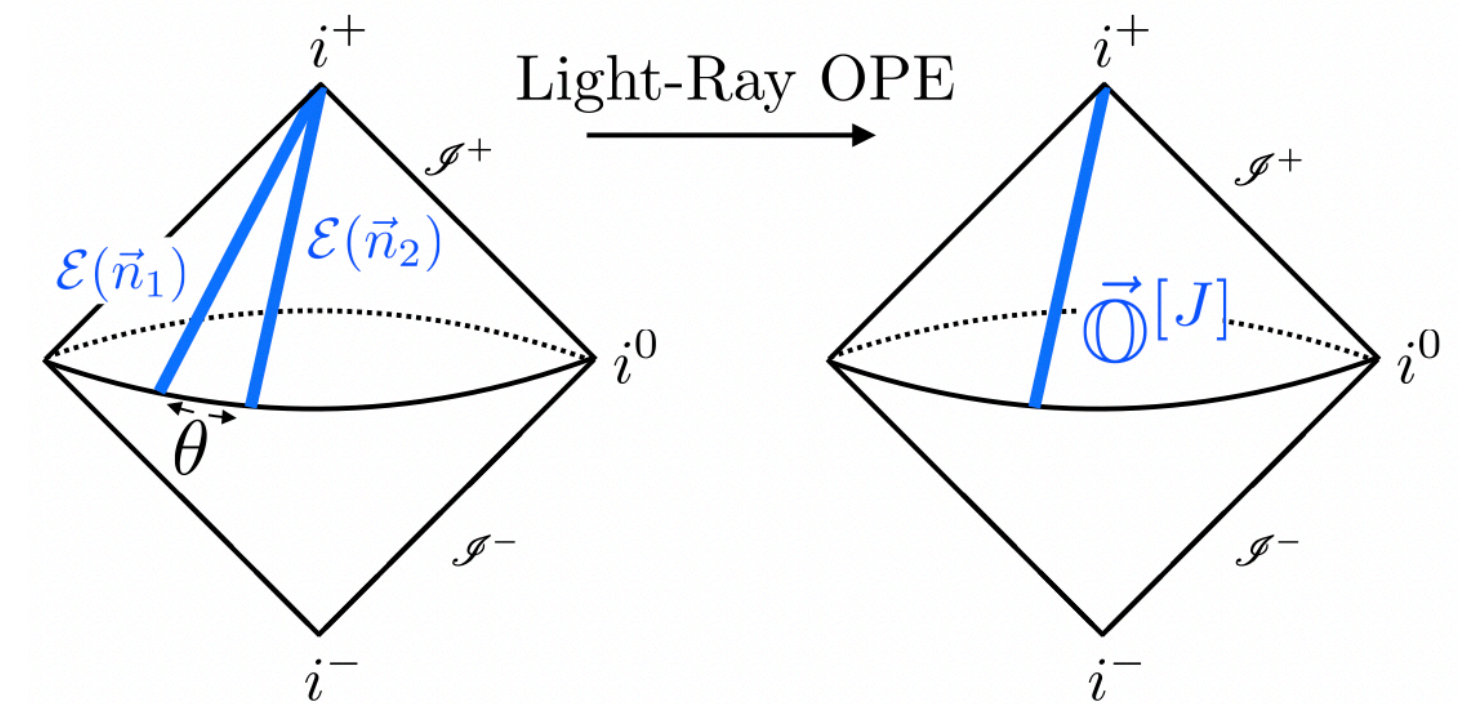
Universal Behavior of QCD

In the **UV regime**, we can explicitly do the OPE and obtain the exact scaling

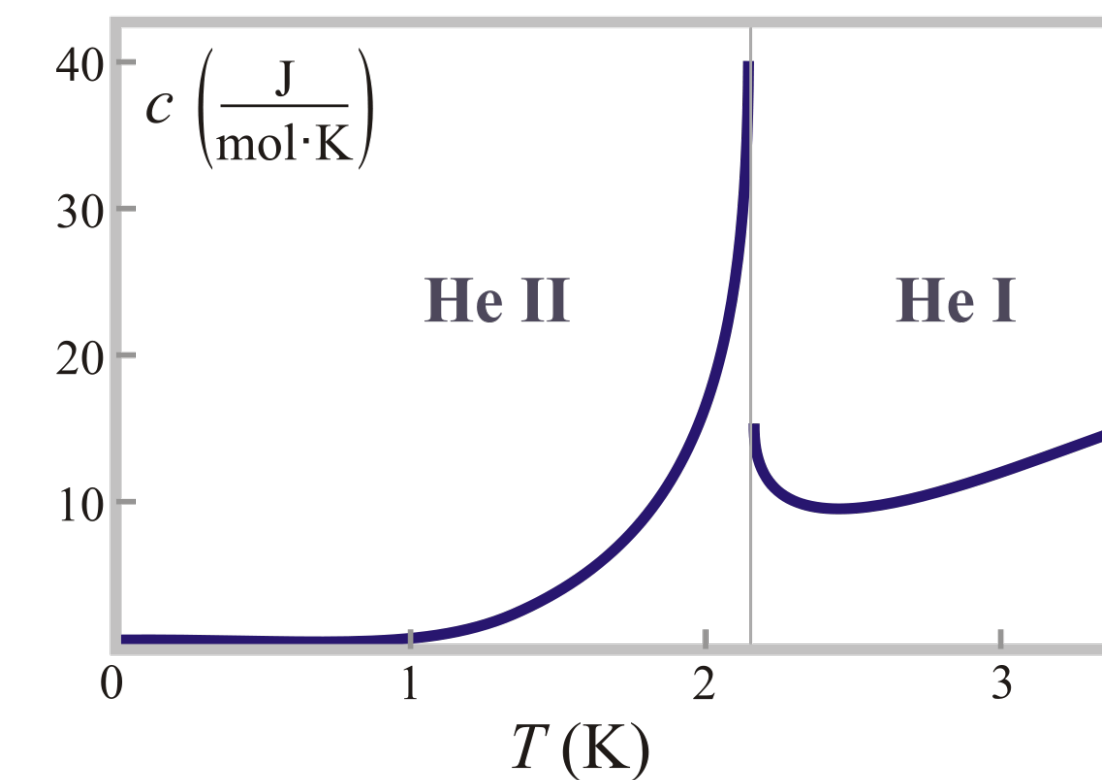
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi} \frac{2}{\theta_S^2} \hat{\mathcal{J}} [\hat{C}_{\phi_S}(2) - \hat{C}_{\phi_S}(3)] \overrightarrow{\mathbb{O}}^{[3]}(\hat{n}) + \dots \quad \text{[Moult, Zhu]}$$

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3) = -\frac{1}{2\pi} \frac{2}{\theta_S^2} \frac{2}{\theta_L^2} \hat{\mathcal{J}} [\hat{C}_{\phi_S}(2) - \hat{C}_{\phi_S}(3)] [\hat{C}_{\phi_L}(3) - \hat{C}_{\phi_L}(4)] \overrightarrow{\mathbb{O}}^{[4]}(\hat{n}) + \dots$$

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i-4} \mathbb{O}_i(\hat{n}_1)$$

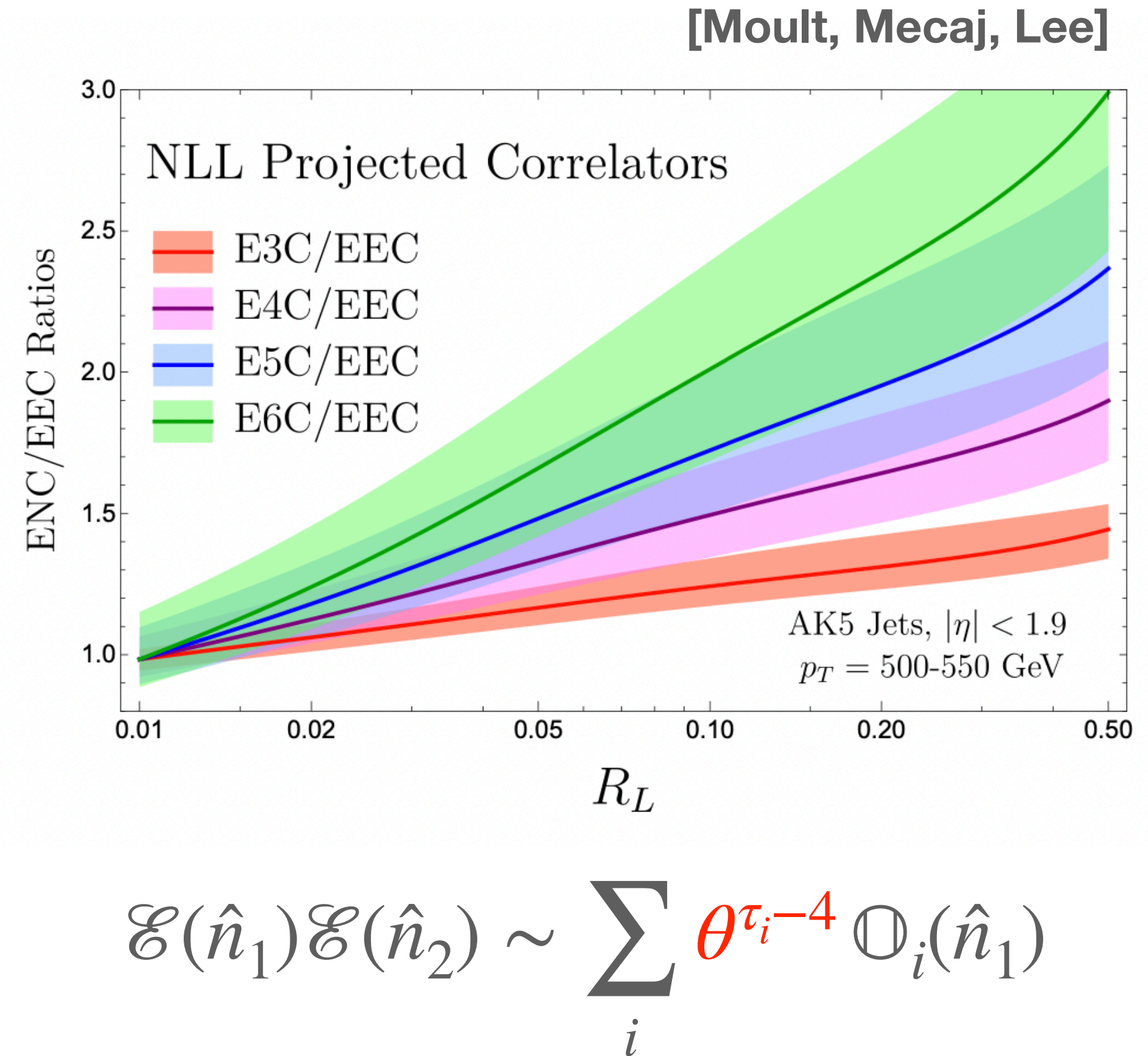
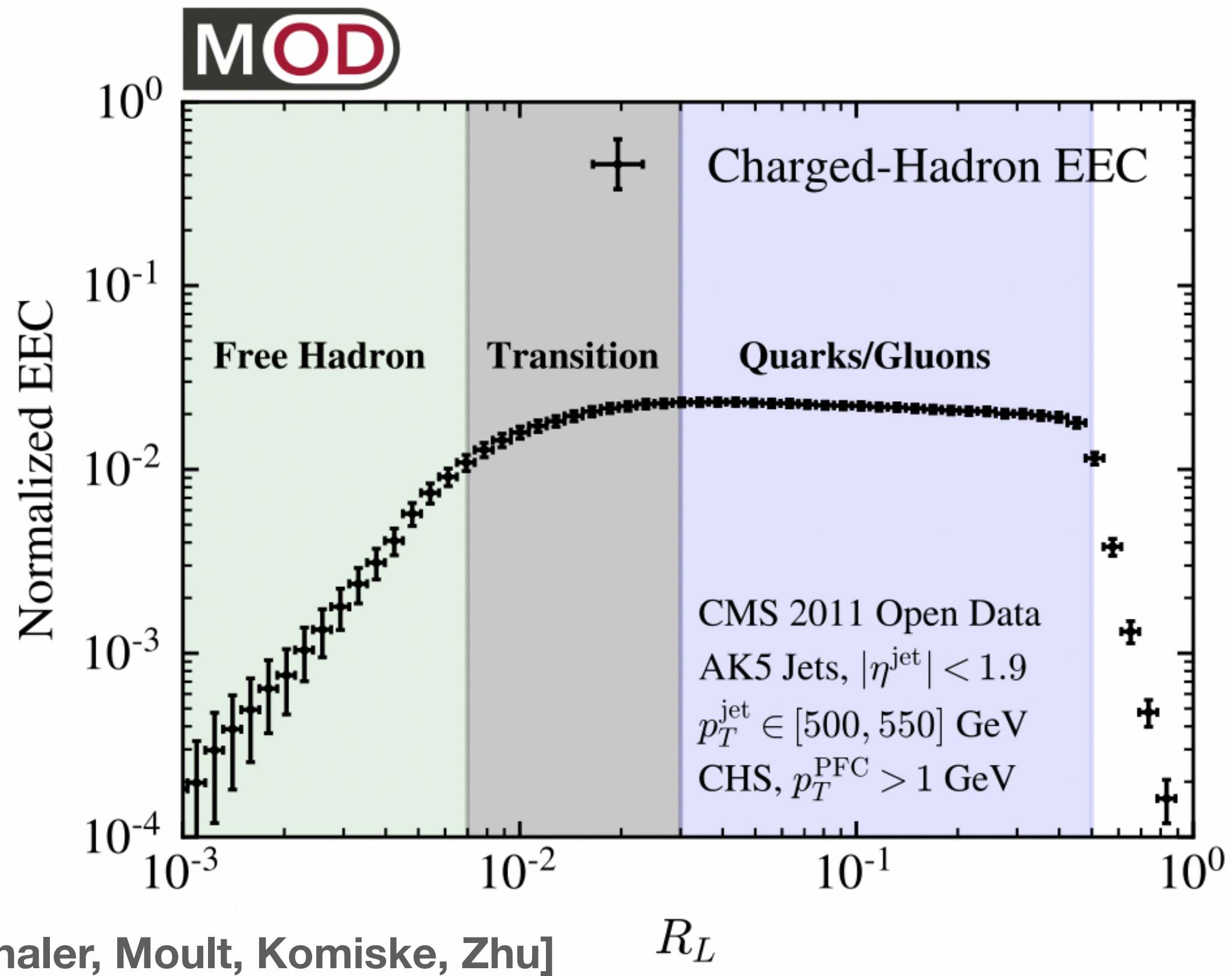


The scaling behavior of the twist-2 light ray operators completely control the leading behavior of **jet substructure**



In **superfluid helium**, most precise measurements suggest $C \sim |T - T_c|^{.009}$

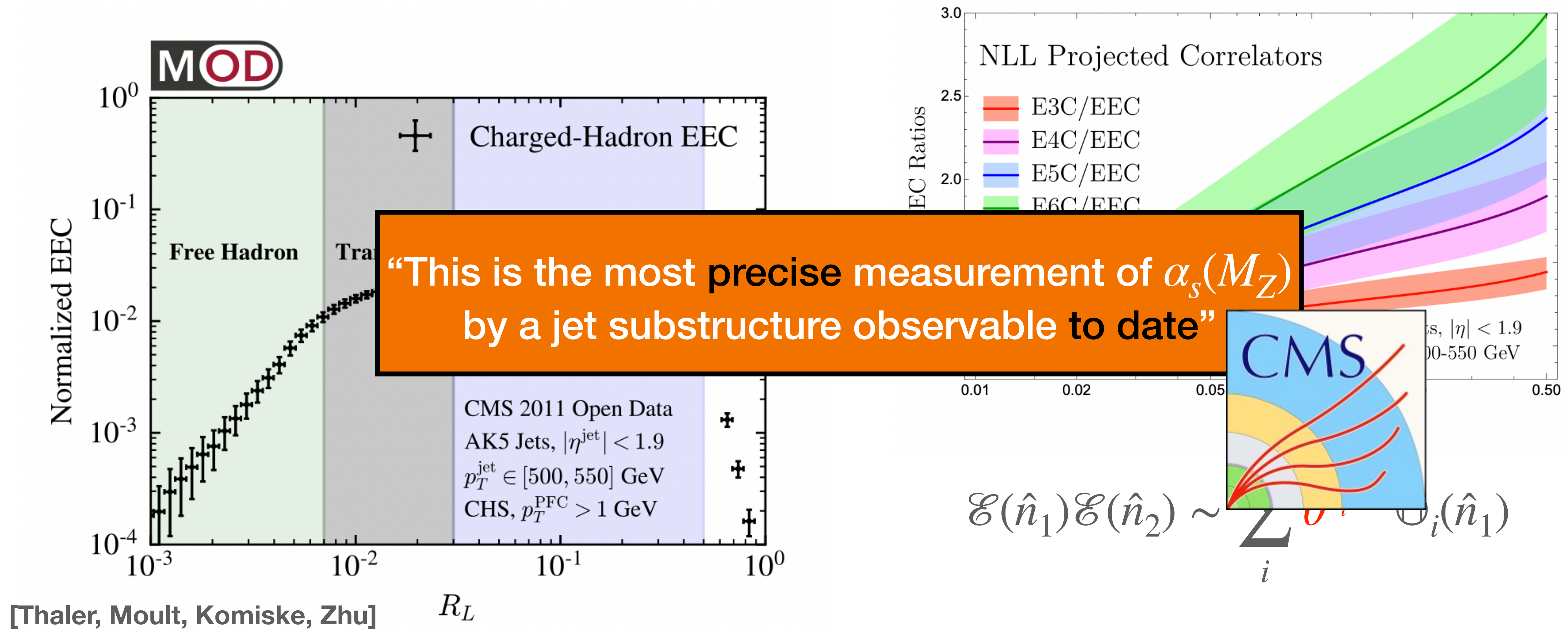
What can you do with this?



Access to **anomalous dimensions** of QCD twist-2 operators **directly** at the LHC

What can you do with this?

[Moult, Mecaj, Lee]

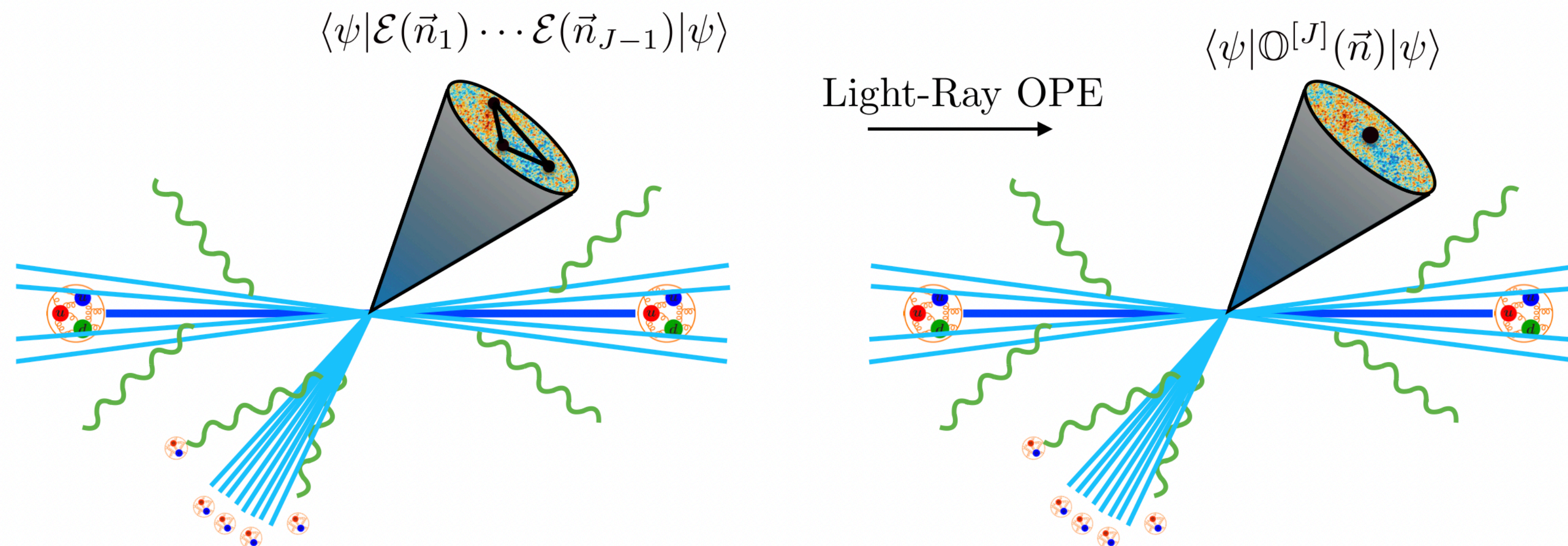


Access to **anomalous dimensions** of QCD twist-2 operators **directly** at the LHC

Can we ask other questions?

Allows us to **replace heuristic jet shapes** with **field theoretic objects** controlling the underlying theory

- Can directly relate **observations** to **field theoretic quantities**
- Able to exploit new, **formal theory developments** to understand collider experiments





Beautiful and Charming Energy Correlators

Evan Craft — Yale University
arXiv: 2210.09311



Based on work with K. Lee, B. Mecaj, I. Moulton

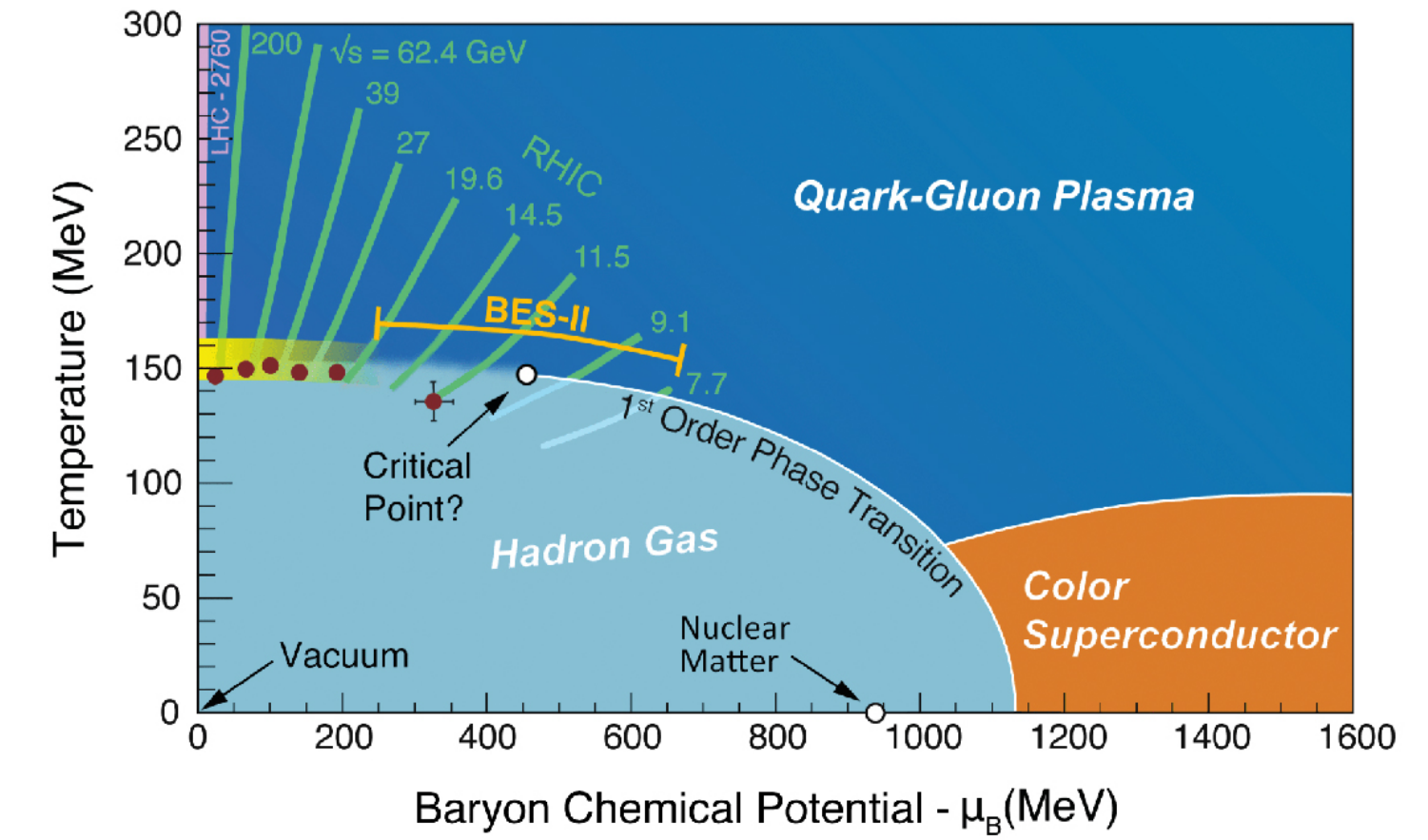
Yale University

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Many Puzzles in QCD

Several **open questions** remain across both Particle and Nuclear Physics

→ Many of these open problems are deeply connected to **Quantum Chromodynamics**



→ Why is color charge so complicated?

Hot QCD

- Quark Gluon Plasma
- Hadronization
- Quarkonia

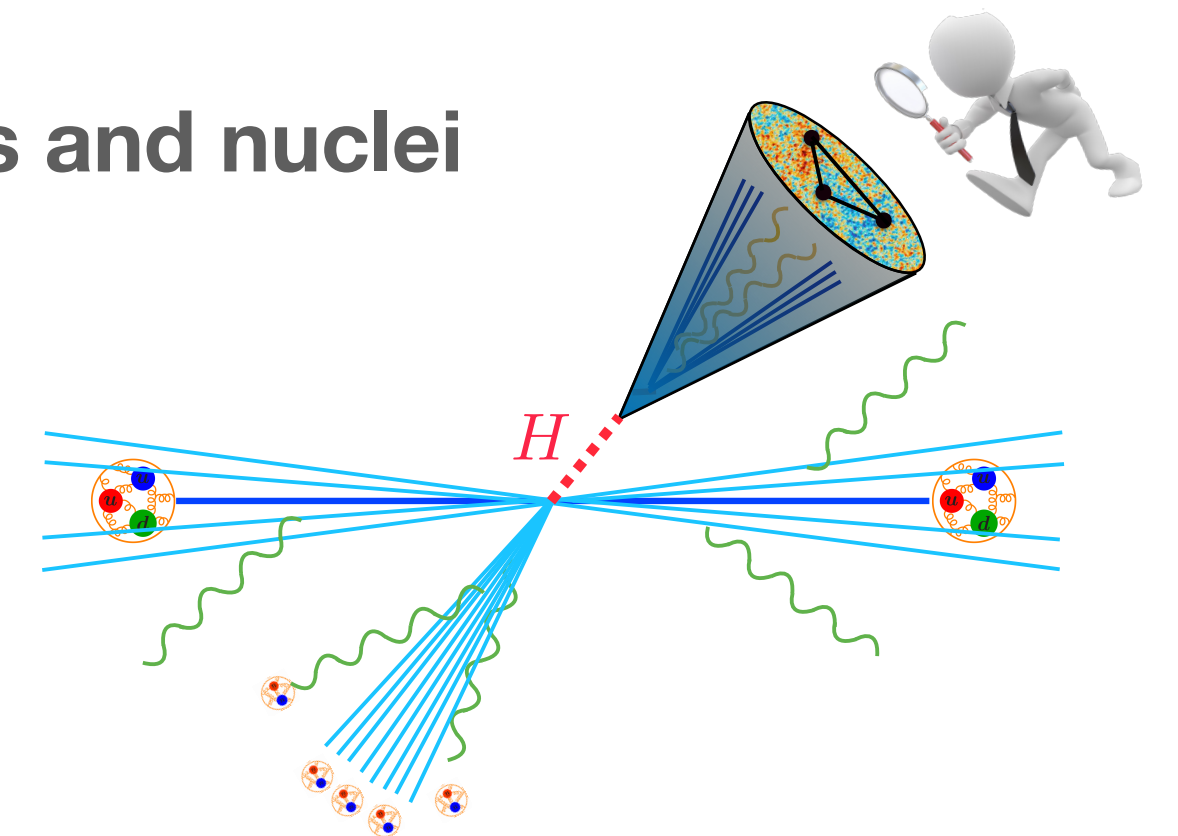
Medium QCD

- Strong CP
- Rare Higgs Decays
- Confinement

Cold QCD

- Gluon Saturation
- Proton Spin and Radius Puzzle
- 3D Structure of protons and nuclei

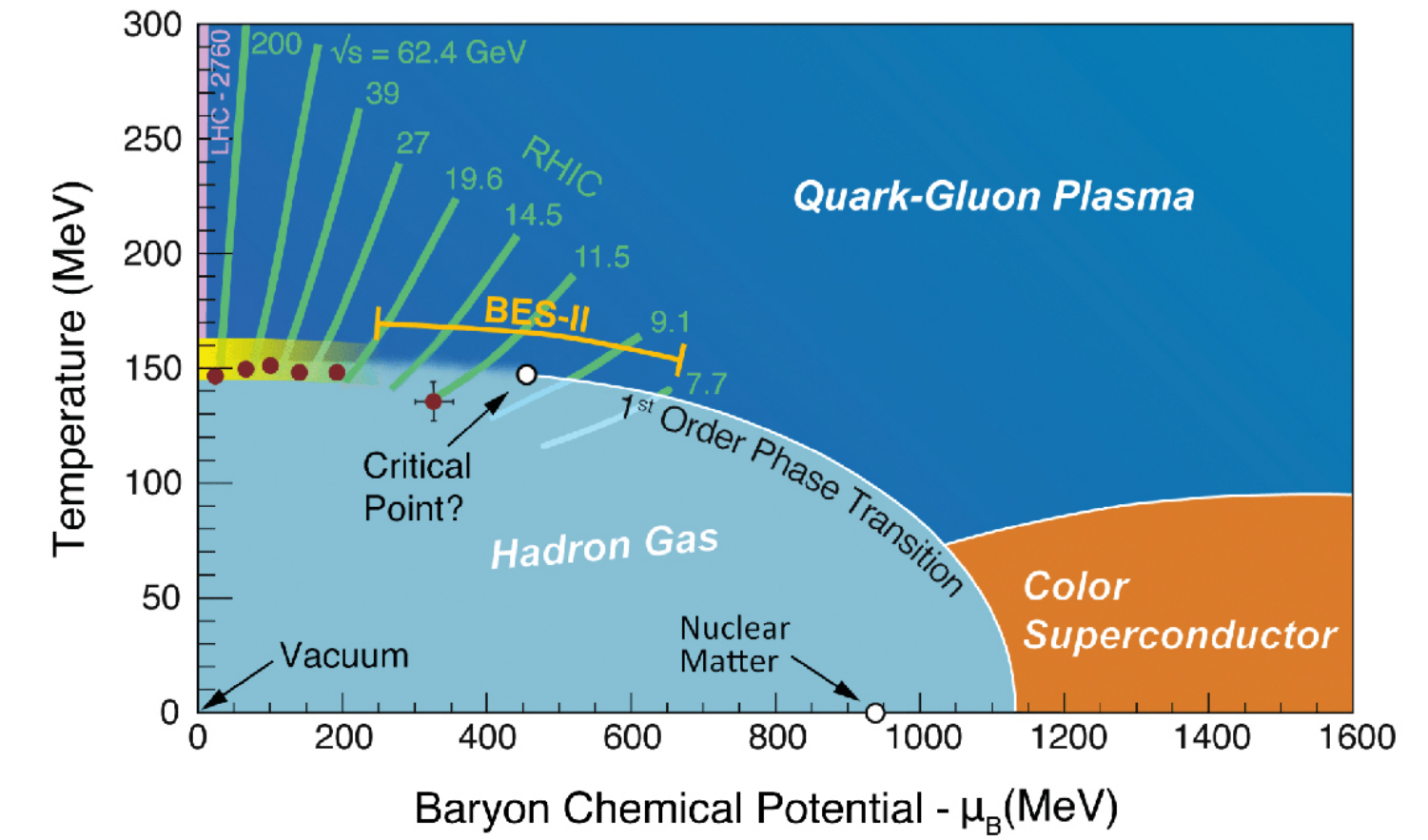
Numerous collider experiments spanning several continents working to resolve these **fundamental questions**



Many Puzzles in QCD

Several **open questions** remain across both Particle and Nuclear Physics

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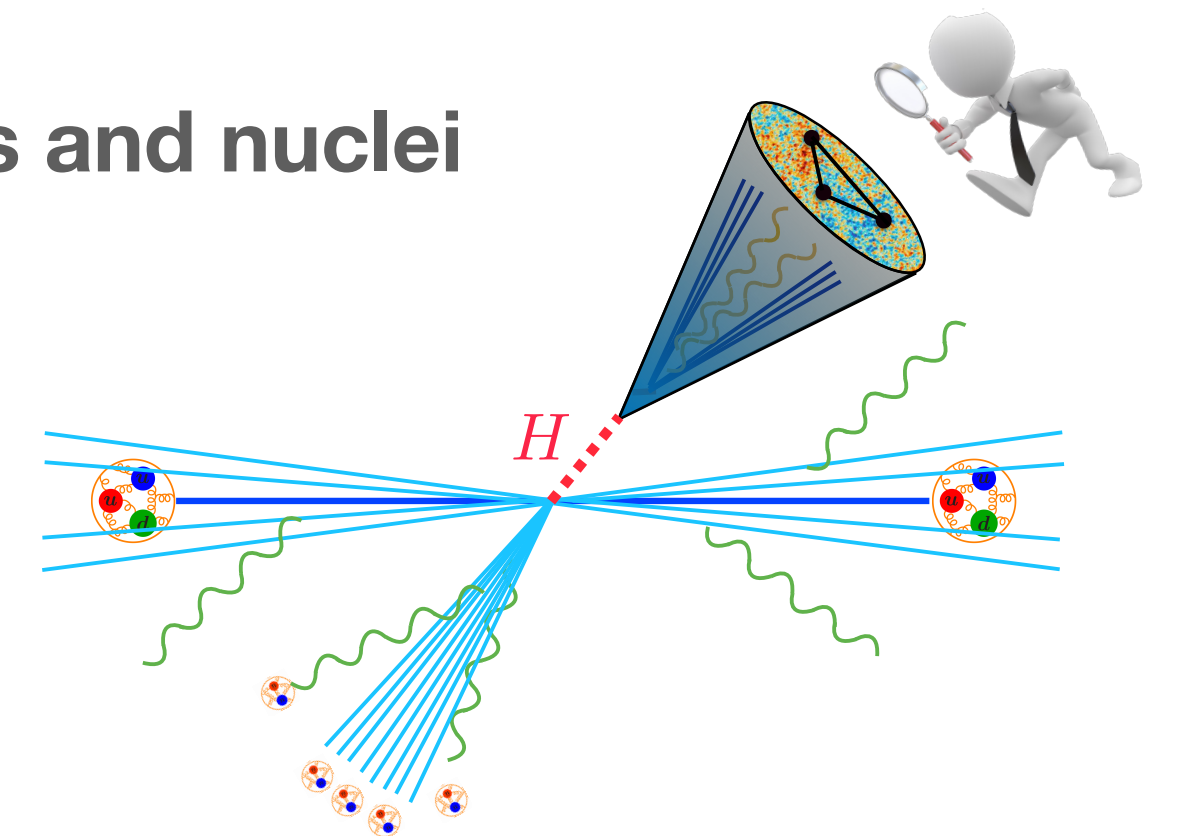
Flavor Dead Cone Effect

Cold QCD

- Gluon Saturation
- Proton Spin and Radius Puzzle
- 3D Structure of protons and nuclei

[\(Nature Physics, 2021\)](#)

Numerous collider experiments spanning several continents working to resolve these **fundamental questions**



Application: Confinement

[ALICE Collaboration, Nature Physics]

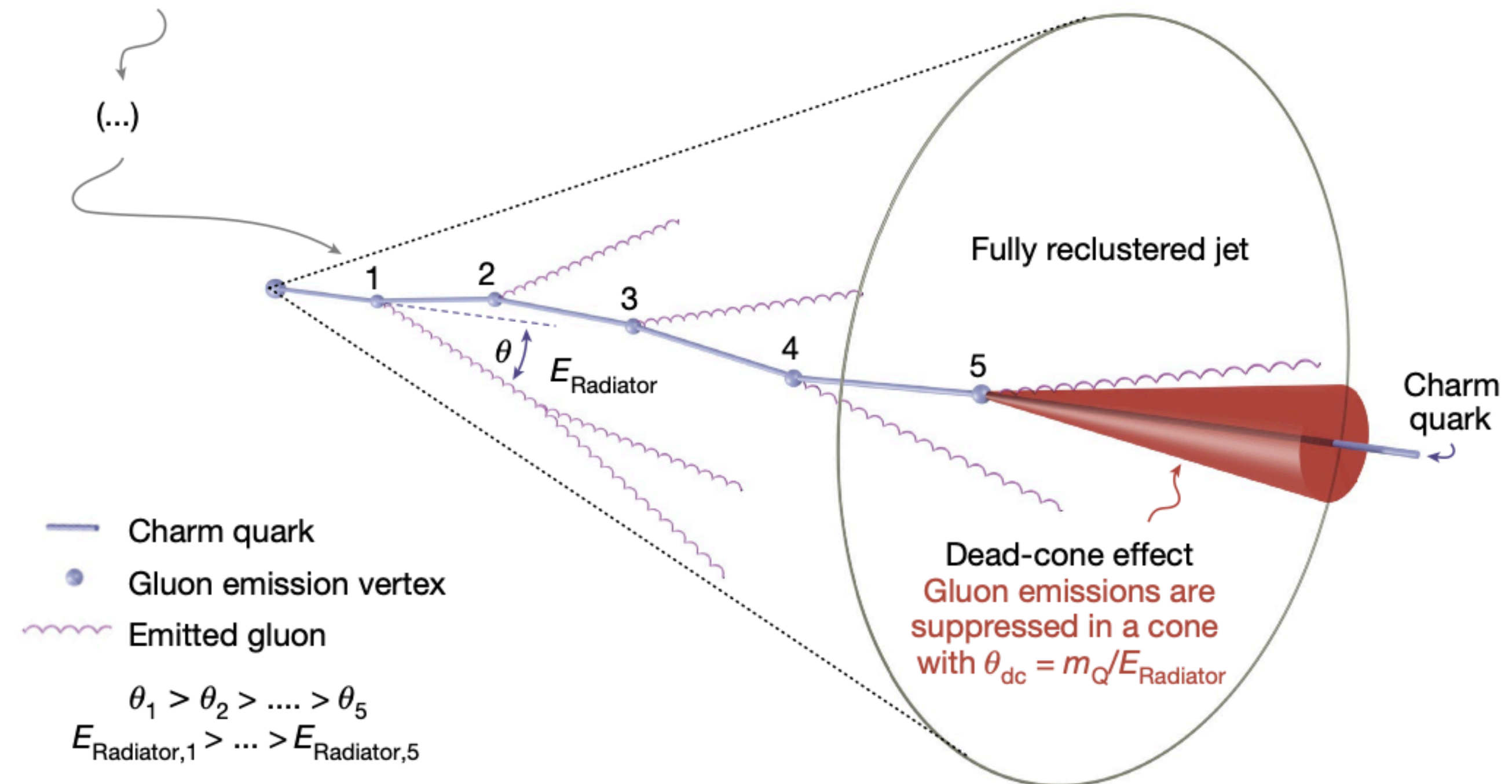
Dokshitzer, Khoze, Troyan (1991)

Heavy quark radiation of gluons must be **suppressed** within a cone of radius m_q/E_q around its center.

→ Fundamental property of all **gauge** field theories

→ Direct signature of intrinsic mass before **confinement**

We can access this effect simply with **statistical correlations (light-ray operators)** — providing a precise, **field theoretic** description of the dead cone.



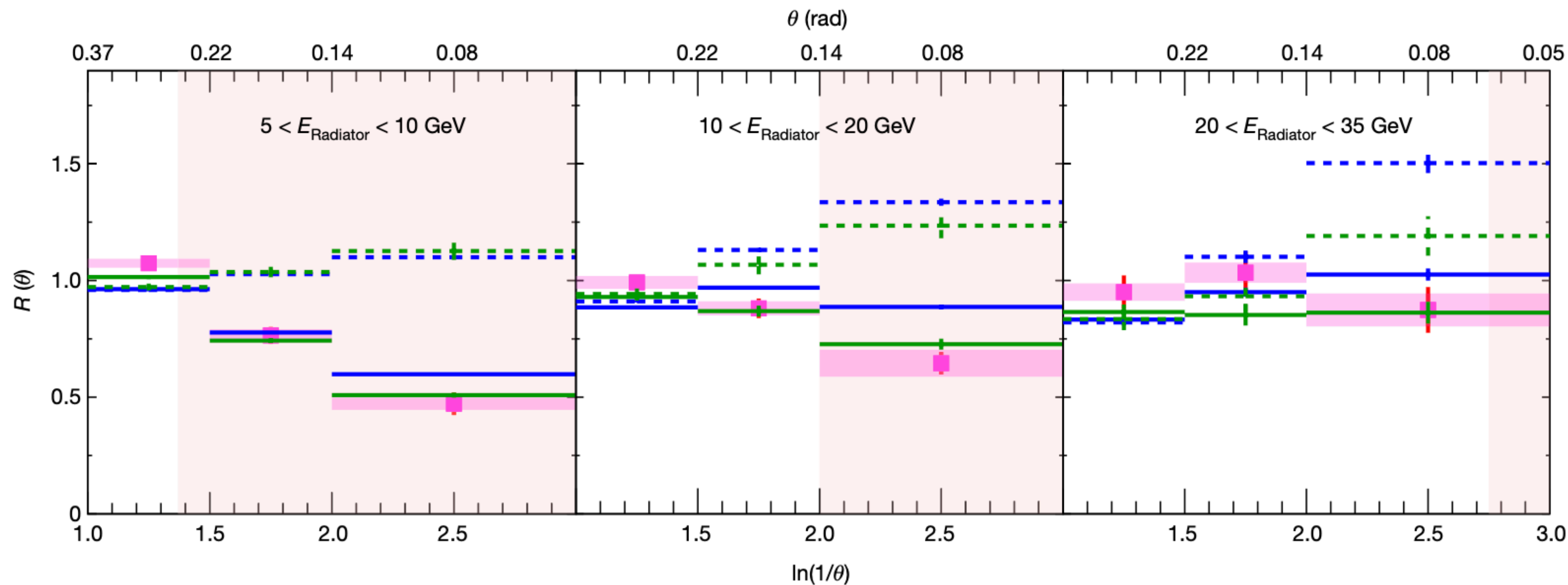
Measured this year by ALICE using a complex **iterative declustering** technique

→ Inferred all gluon emissions *directly*

→ State of the art analysis techniques

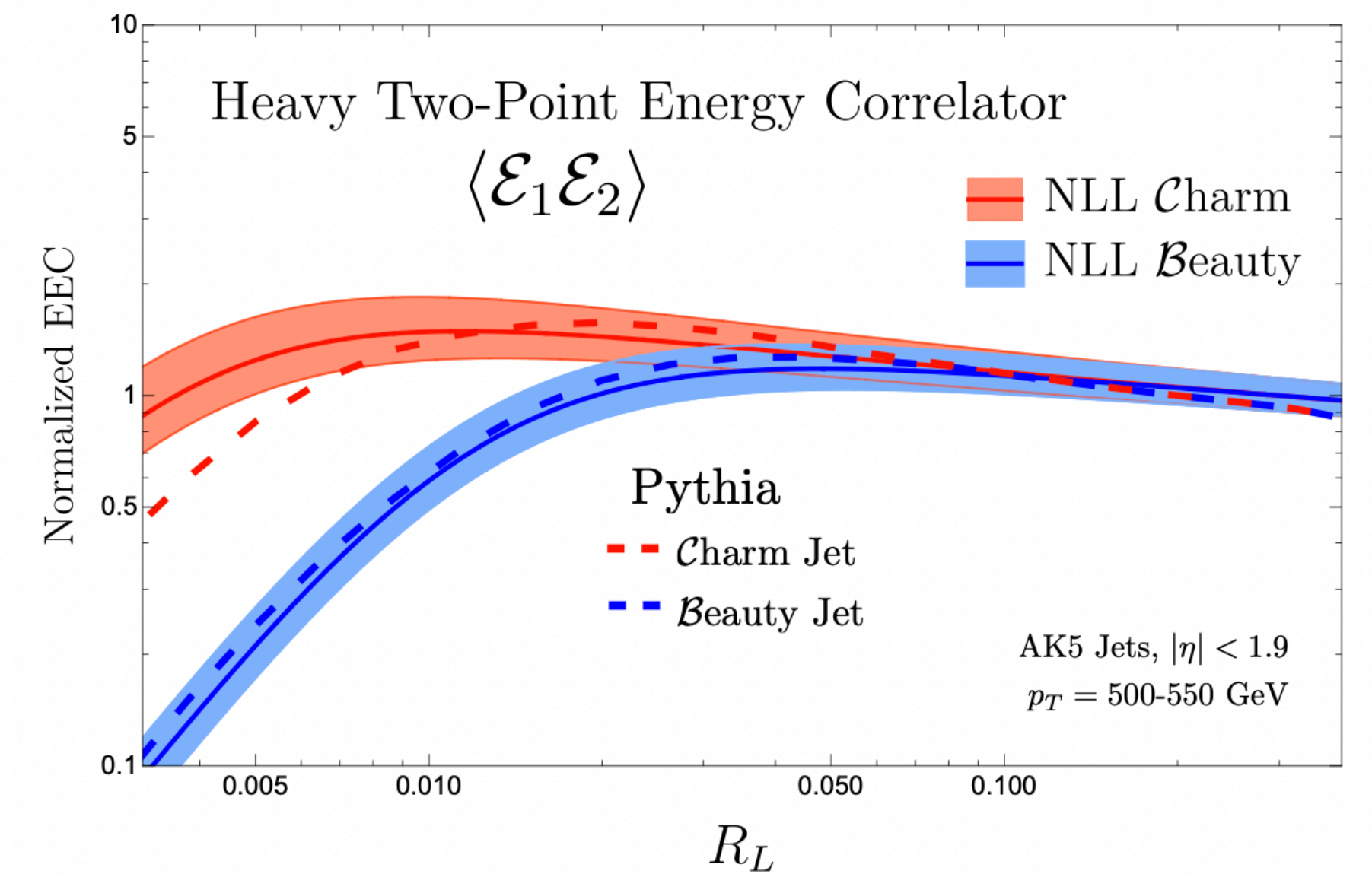
Application: Confinement

ALICE measured this effect using an iterative declustering algorithm and the **Lund Plane**



[ALICE Collaboration, Nature Physics]

[EC, Lee, Mecaj, Moult]



→ EECs can be **systematically computed** in perturbation theory

→ Dead cone effect is **visible by eye** using light-ray operators

→ Can easily be **extended** to other heavy flavor based analyses

Application: Confinement

In the **UV regime**, scaling should be independent of mass

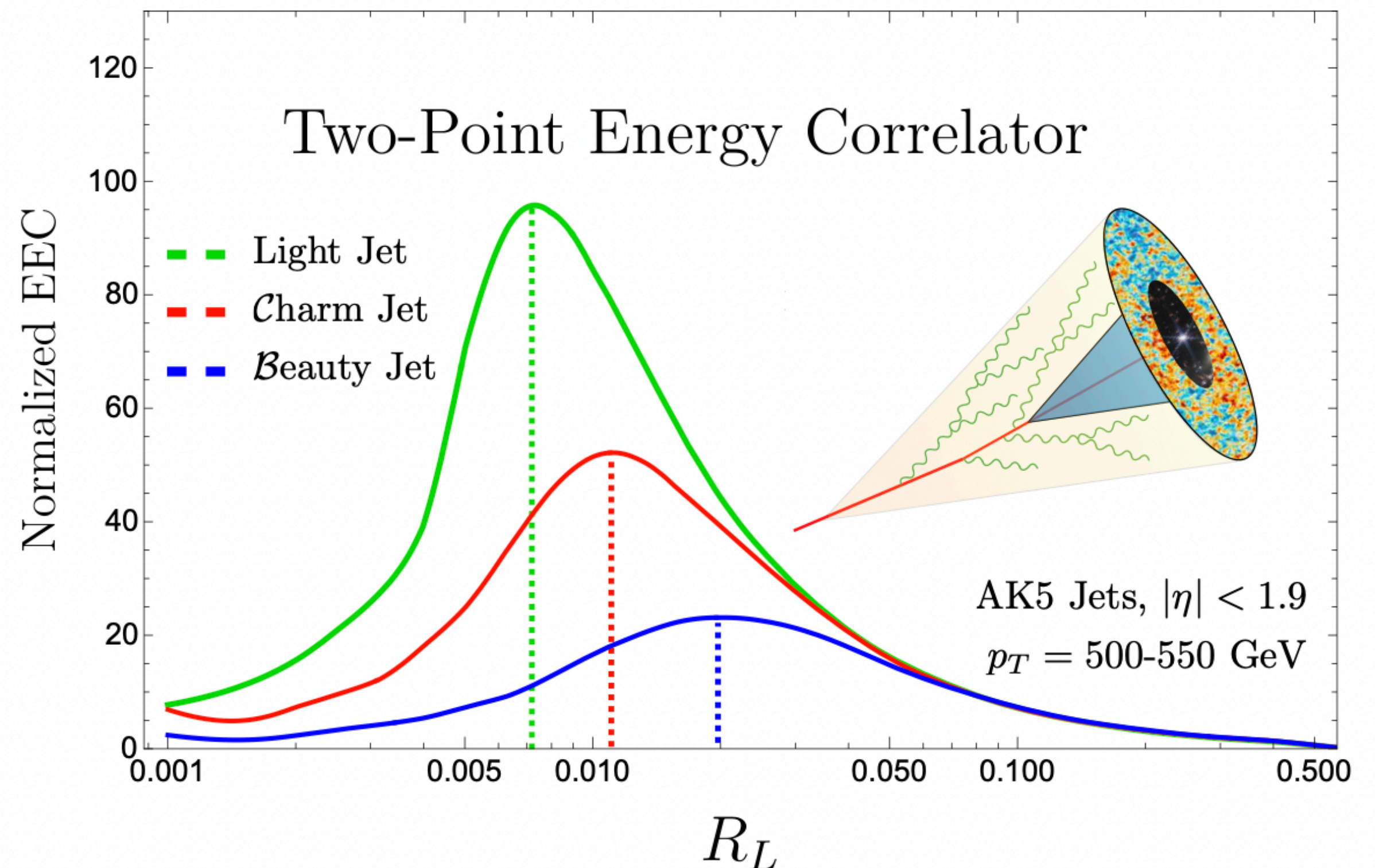
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum_i \theta^{\tau_i-4} \mathbb{O}_i(\hat{n}_1)$$

In the **IR regime**, mass is an intrinsic scale, and should be imprinted on the correlator

$$\langle \Psi | \mathcal{E}(\hat{n}_1) \dots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$

EECs provide a precise, **field-theoretic description** of the dead-cone effect

[EC, Lee, Mecaj, Moutl]



$$\text{Transition Scale} \sim \frac{m_q}{p_{T,jet}}$$

Application: Confinement

[EC, Lee, Mecaj, Moutl]

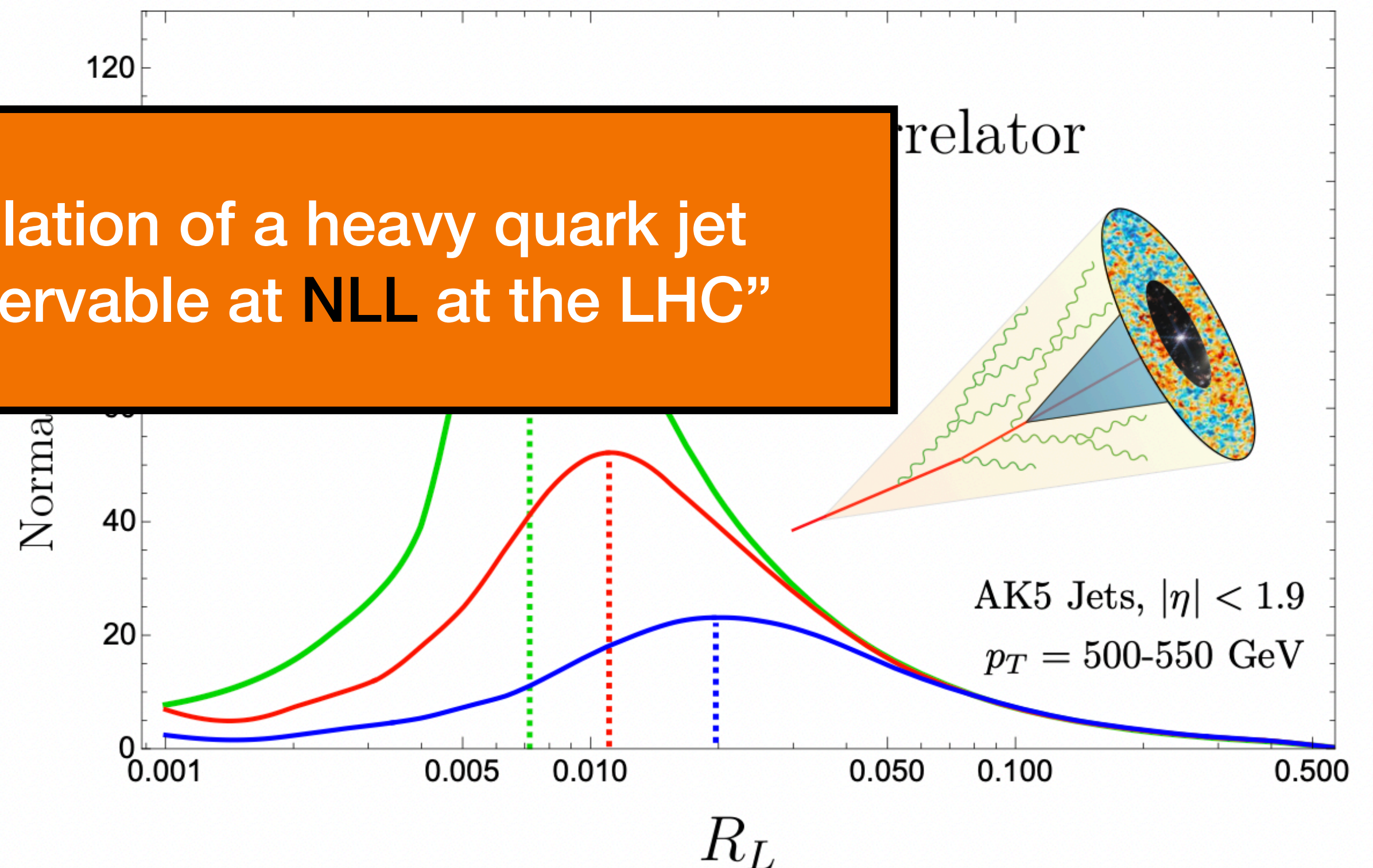
In the **UV regime**, scaling should be independent of mass

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim$$

“First ever calculation of a heavy quark jet substructure observable at **NLL** at the LHC”

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$$\langle \Psi | \mathcal{E}(\hat{n}_1) \dots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$



EECs provide a precise, **field-theoretic description** of the dead-cone effect

$$\text{Transition Scale} \sim \frac{m_q}{p_{T,jet}}$$



Pushing the Boundaries of Jet Substructure

Evan Craft — Yale University



Work **in prep.** with K. Lee, B. Mecaj, I. Moulton, & M. Gonzalez



Yale University

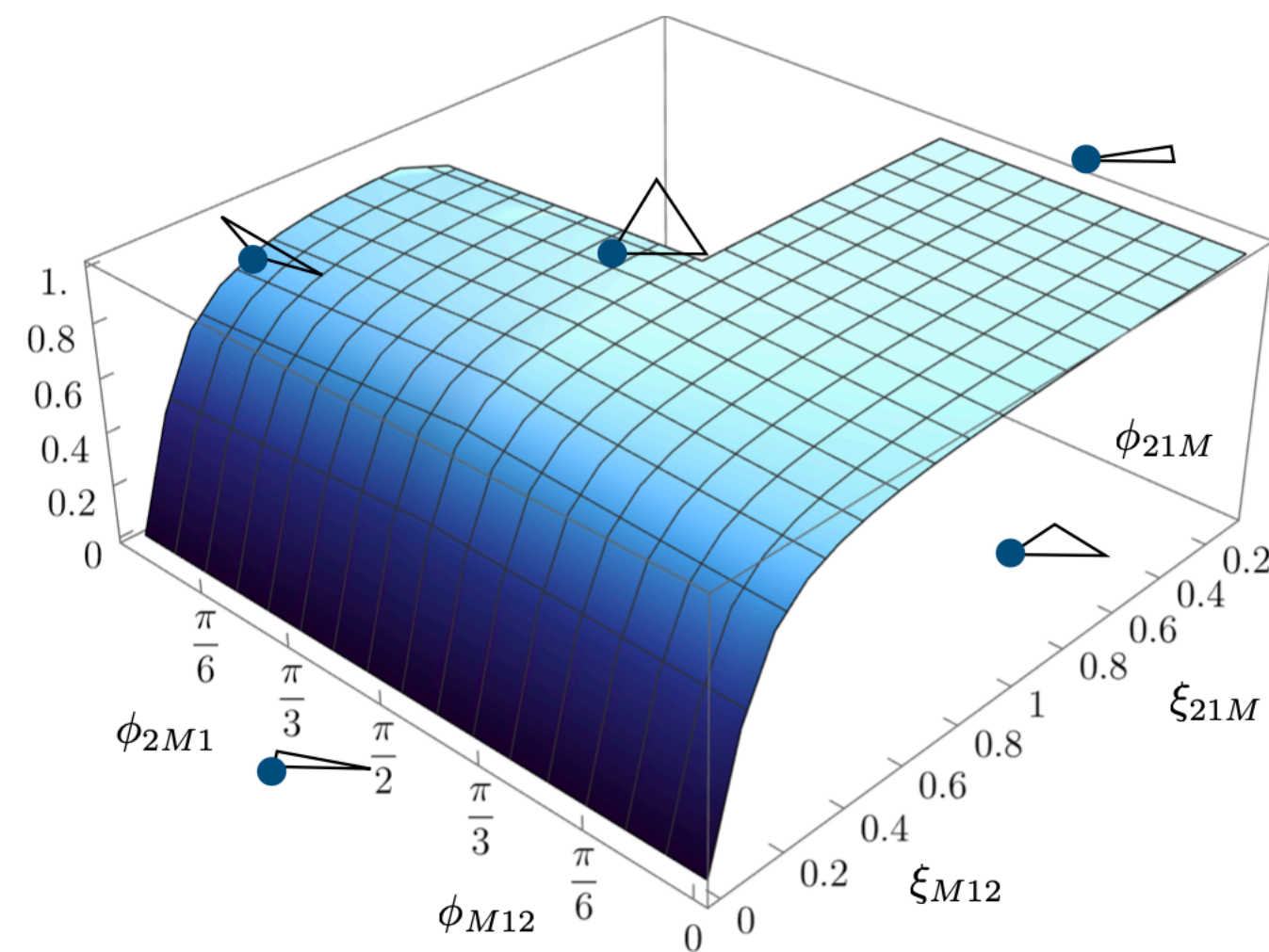


MIT

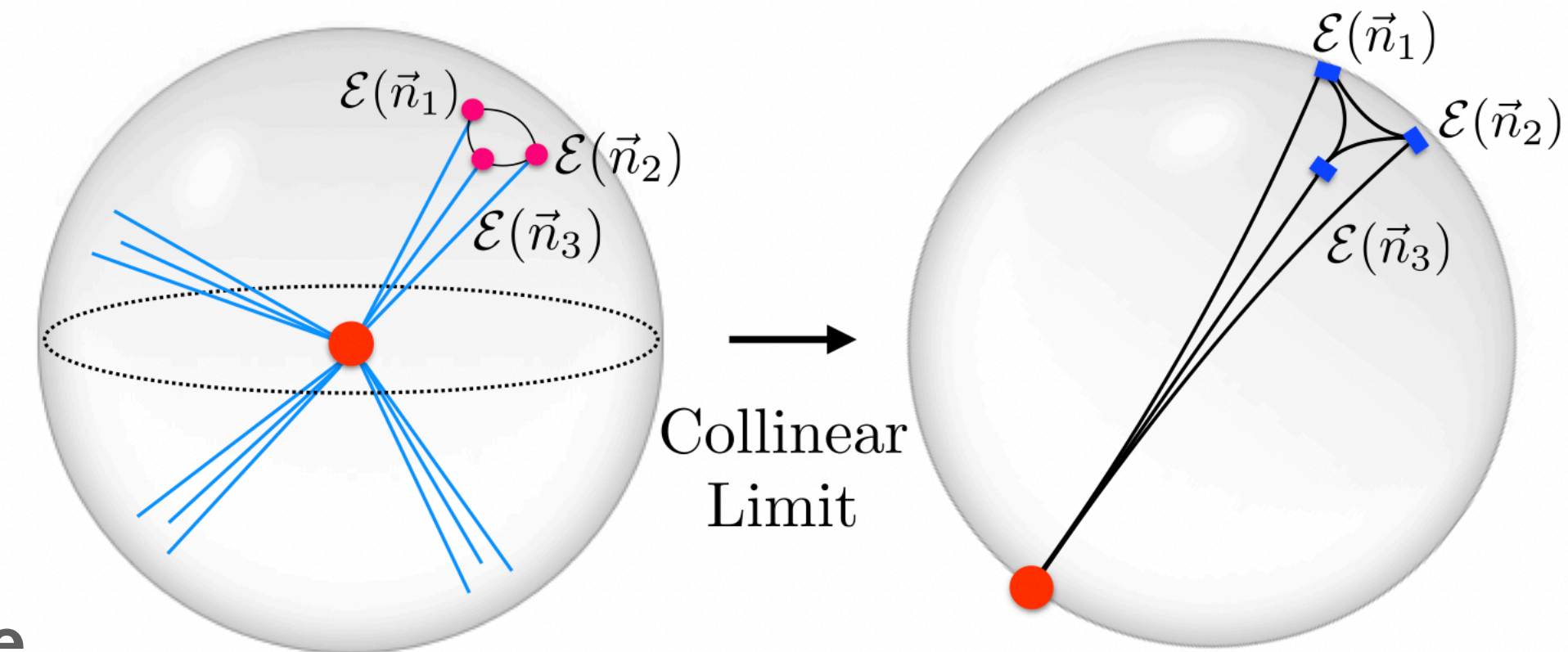
Extension: Higher Points

Natural to also consider **higher point** correlators

Experimental Side



3-point EEC allows access to the **shape** of the dead-cone!



Theoretical Side

transverse spin 0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J} \bar{\psi} \gamma^+ (iD^+)^{J-1} \psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J} F_a^{\mu+} \gamma^+ (iD^+)^{J-2} F_a^{\mu+}$$

excited by **2-point**

transverse spin 2

$$\mathcal{O}_{\tilde{g}\lambda}^{[J]} = -\frac{1}{2^J} F_a^{\mu+} \gamma^+ (iD^+)^{J-2} F_a^{\nu+} \epsilon_{\lambda\mu} \epsilon_{\lambda\nu}$$

↑
helicity ± 1

excited by **3-point**

→ Access to **non-Gaussianities**

→ Full **Shape** Dependence

$$\mathcal{E}(\hat{n}_1) \dots \mathcal{E}(\hat{n}_k) \sim \sum_i \theta^{\tau_i-4} \mathbb{O}_i(\hat{n}_1)$$

→ Probe fundamental operators of **QCD**

Elliptic Polylogarithms

In addition to elliptic integrals, one encounters further **complex structures** with the **EEEC**

→ Multiple Elliptic Polylogarithms (**MPLs**)

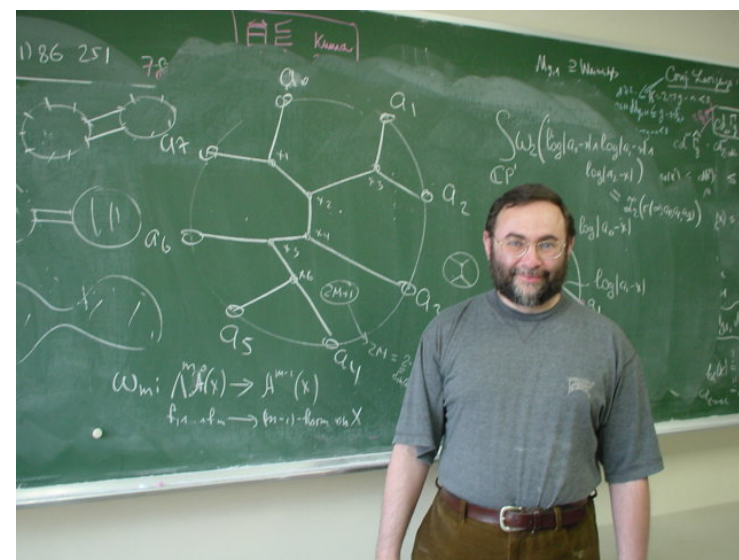
→ **Elliptic Curve** + Iterated **Logarithm**



C. Duhr

↓
Elliptic Polylogarithms

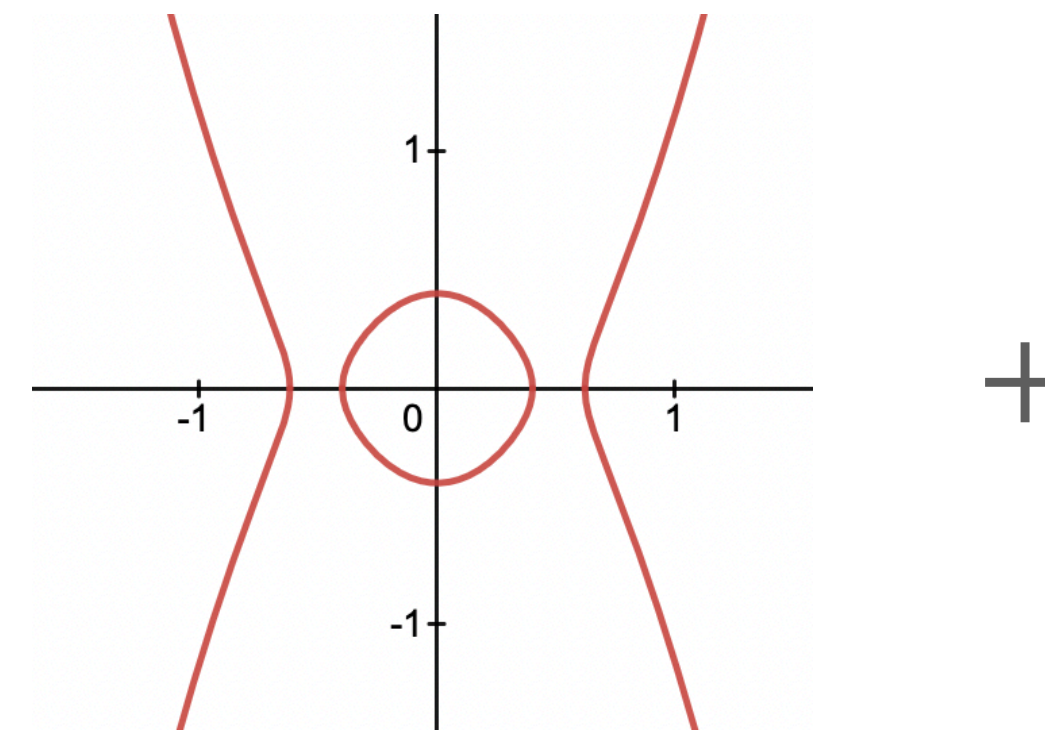
tools for **massive EEEC**



A. Goncharov

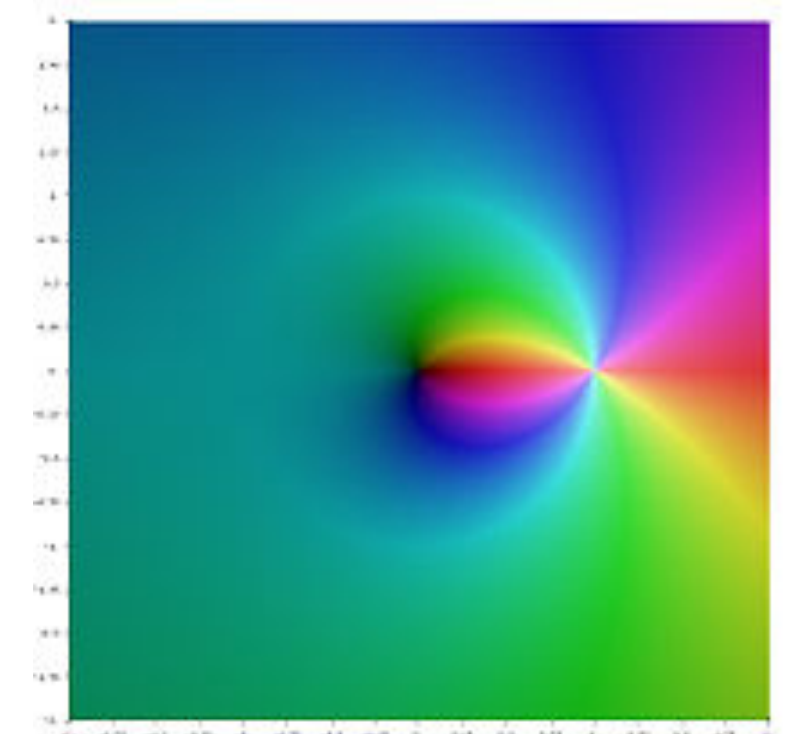
↓
Generalized Polylogarithms

tools for **massless EEEC**



elliptic structure

+



polylogarithmic structure

→ Only understood within the **last 5 years**

→ **EEC** at the **frontiers** of math and physics

Elliptic polylogarithms and iterated integrals on elliptic curves. Part I: general formalism

Johannes Broedel (Humboldt U., Berlin, Inst. Math. and Humboldt U., Berlin), Claude Duhr (CERN and Louvain U., CP3), Falko Dulat (SLAC), Lorenzo Tancredi (CERN)

Dec 19, 2017

54 pages

Published in: *JHEP* 05 (2018) 093

Published: May 15, 2018

e-Print: [1712.07089](https://arxiv.org/abs/1712.07089) [hep-th]

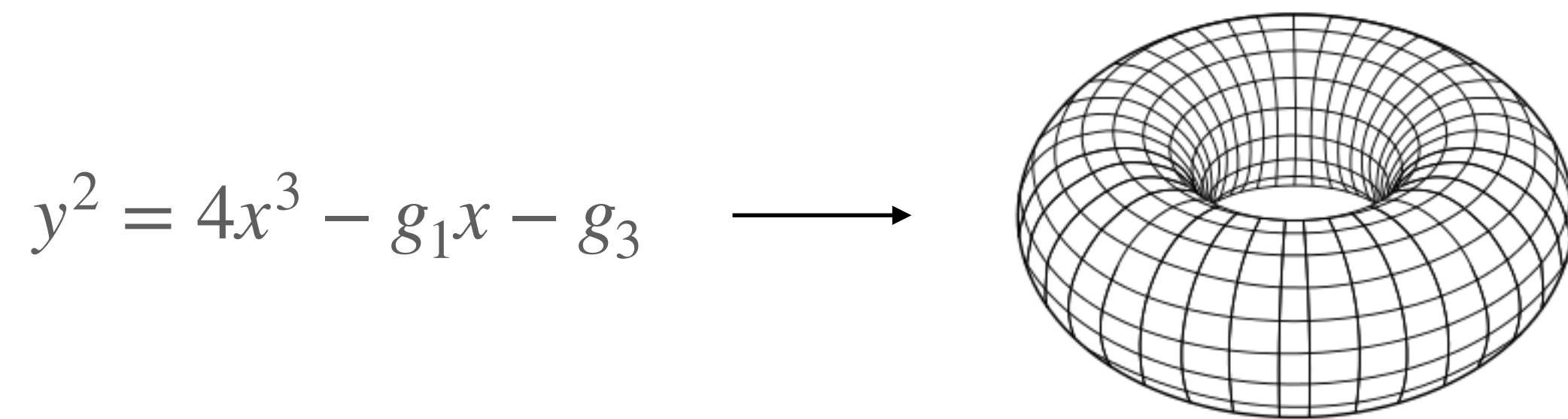
DOI: [10.1007/JHEP05\(2018\)093](https://doi.org/10.1007/JHEP05(2018)093)

Report number: CERN-TH-2017-273, CP3-17-57, HU-EP-17-29, HU-Mathematik-2017-09, SLAC-PUB-17194

View in: [ADS Abstract Service](#), [CERN Document Server](#)

Topological Aspects

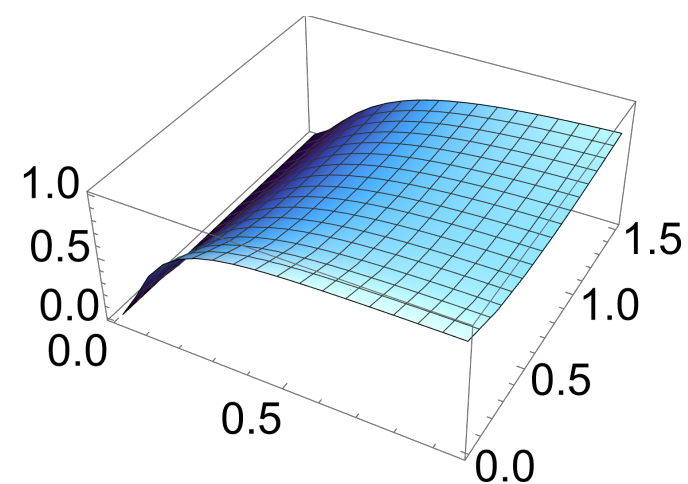
There is a direct mapping from the **kinematic configuration** of the **EEC**, to the torus



$$\omega_1 \sim {}_2F_1(1/2, 1/2, 1; \lambda)$$

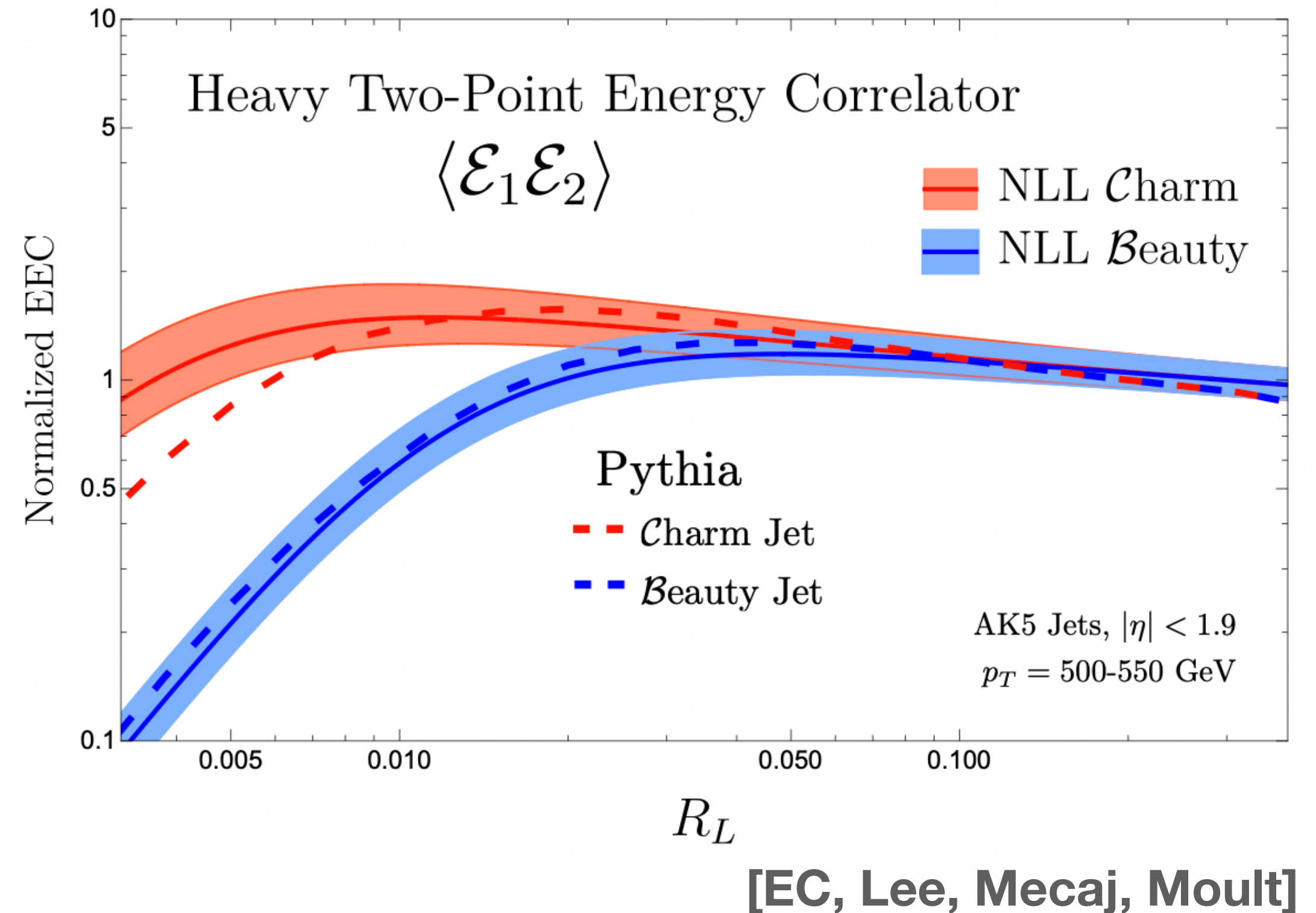
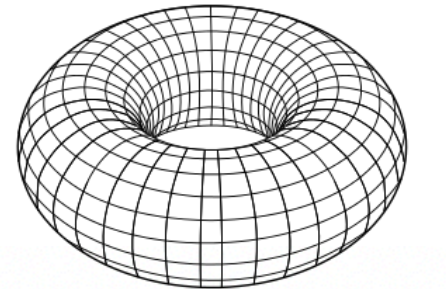
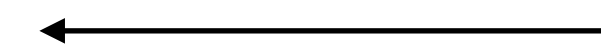
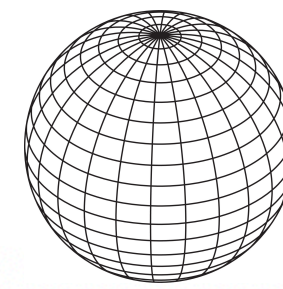
$$\omega_2 \sim {}_2F_1(1/2, 1/2, 1; 1 - \lambda)$$

periods deformed by kinematics



Similar degeneration for the three point!

topology **degenerates** in the collinear limit



Interesting to study the **topological** aspects of the observable

What can you do with this?

Formal

1. Understanding the **space of functions** generated by correlations of heavy energy flow

$$\int \frac{1}{y} \{E, F, \Pi\}, \int \frac{x^2}{y} \{E, F, \Pi\}, \dots \subset \text{E3C}$$

2. Playground for heavy **SCET factorization** with clear applications to collider physics

$$\frac{d\Sigma}{d\psi_1 d\psi_2 d\psi_3} \sim \vec{J} \otimes \vec{H} \otimes \vec{S}$$

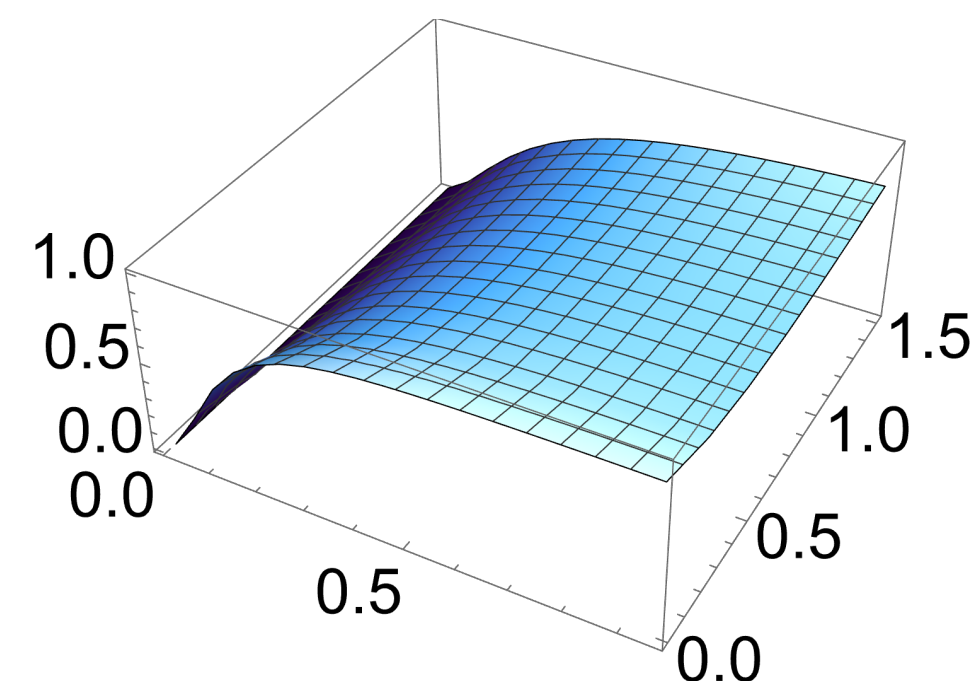
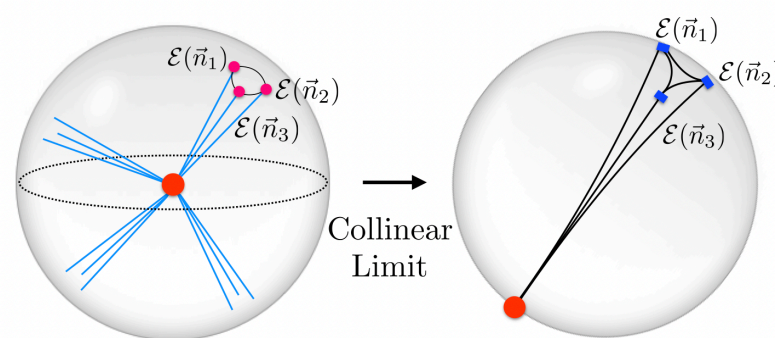
Phenomenological

1. Access to the shape of the deadcone with applications to **precision tests of QCD**

2. An example of a **3 parameter, heavy** event shape observable

→ Useful for tuning event generators

→ Playground for testing ML techniques



What can you do with this?

Formal

1. Understanding the **space of functions** generated by correlations of heavy energy flow

2. Playground for heavy **SCET factorization** with clear applications to collider physics

$$\int \frac{1}{y} \{E, F, \Pi\}, \quad \int \frac{x^2}{y} \{E, F, \Pi\}$$

Both halves of this slide are in *symbiosis*

$$\vec{J} \otimes \vec{H} \otimes \vec{S}$$

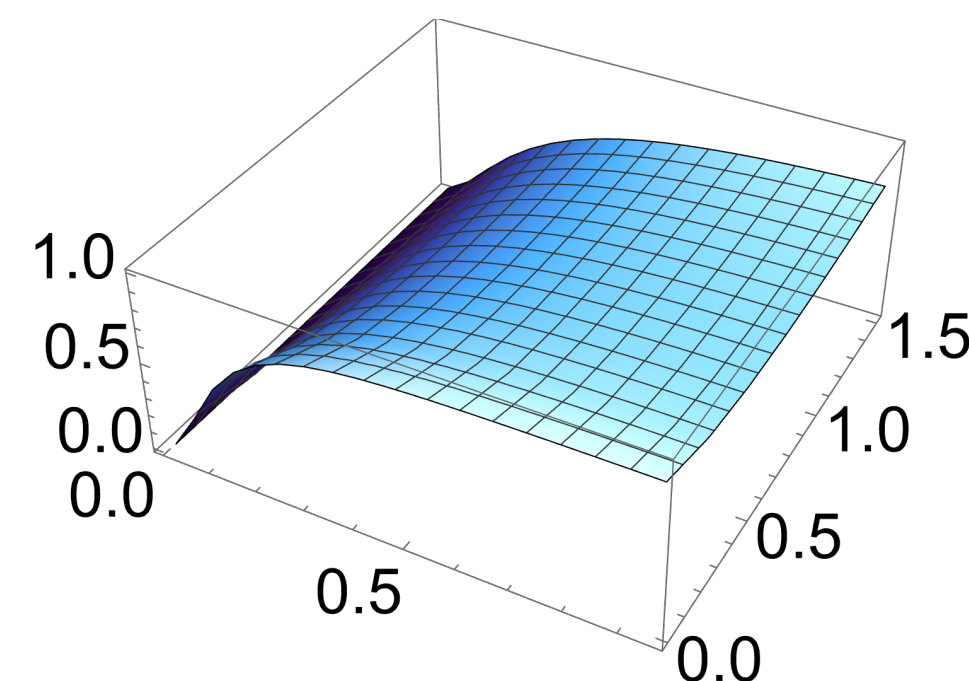
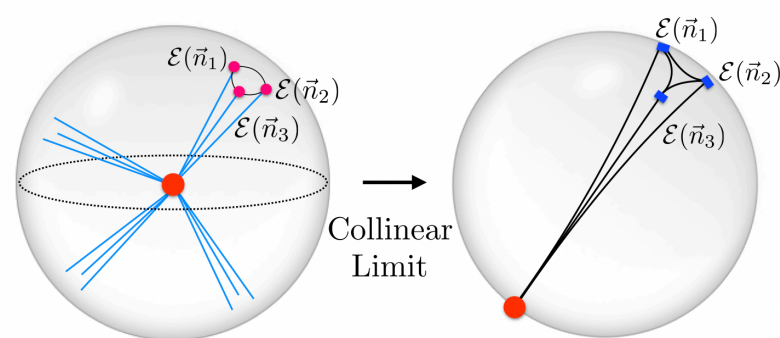
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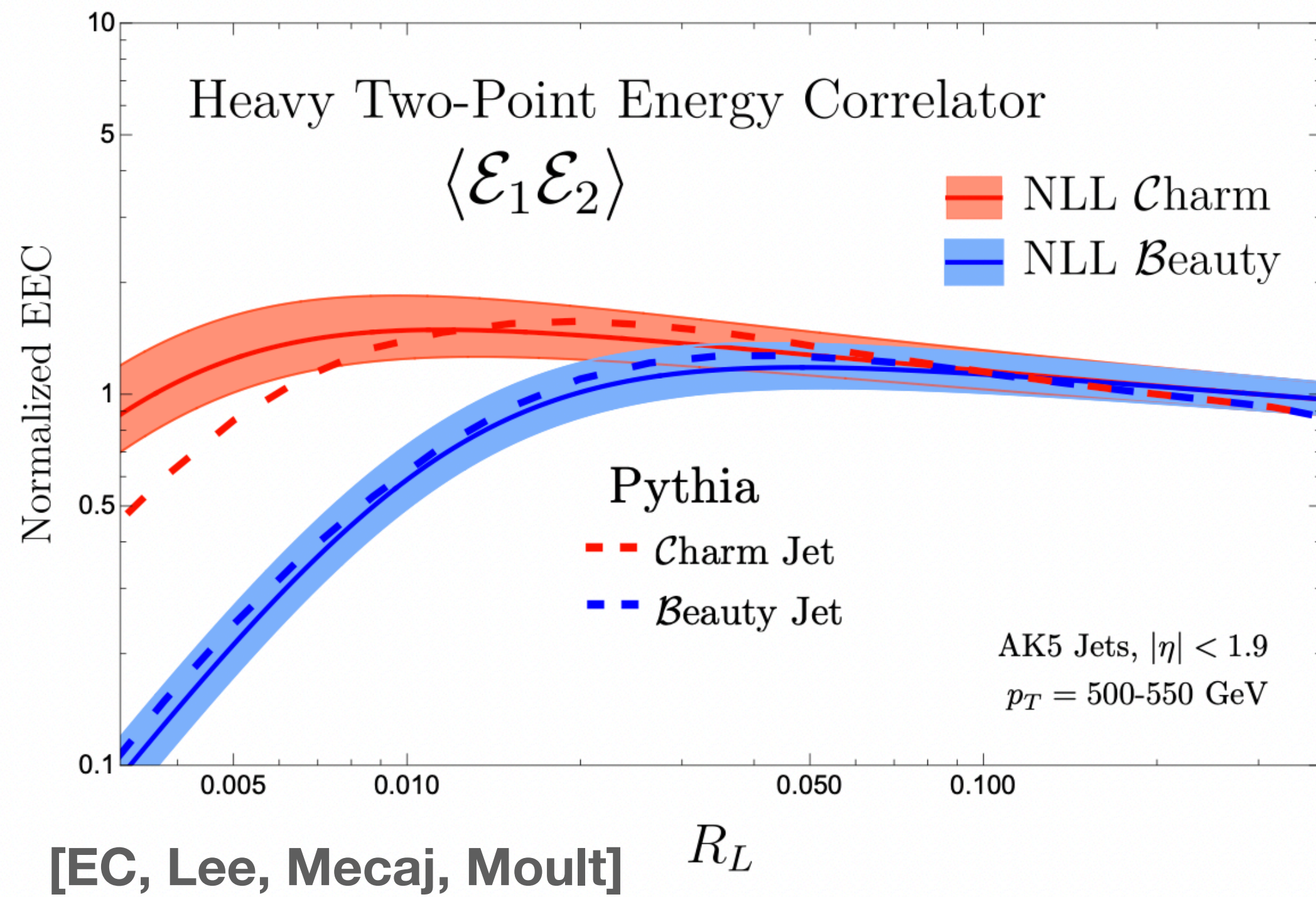


Concluding Remarks

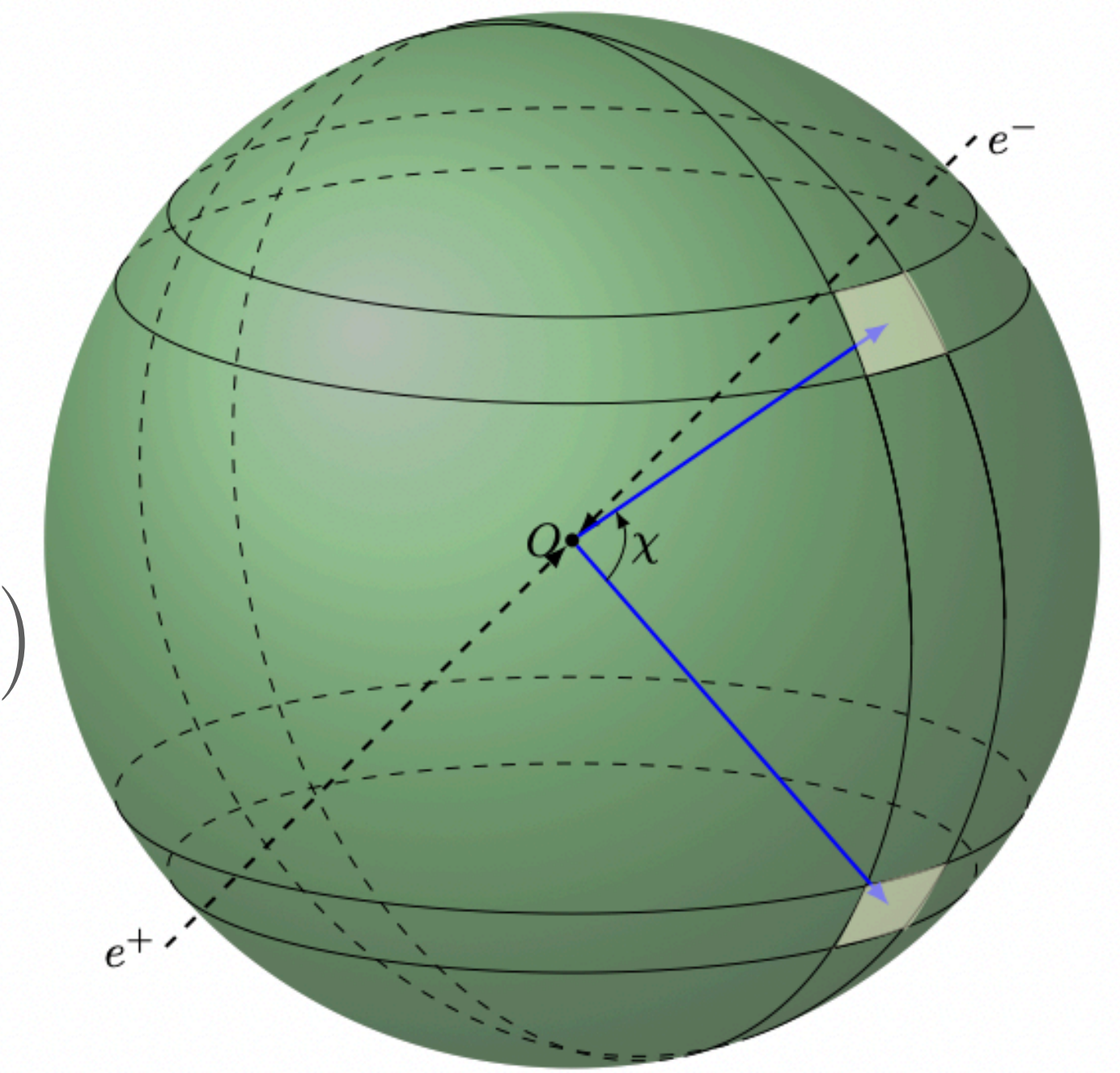
Unifying Theory and Experiment

Two Symbiotic Perspectives

Beautiful and Charming Interplay!



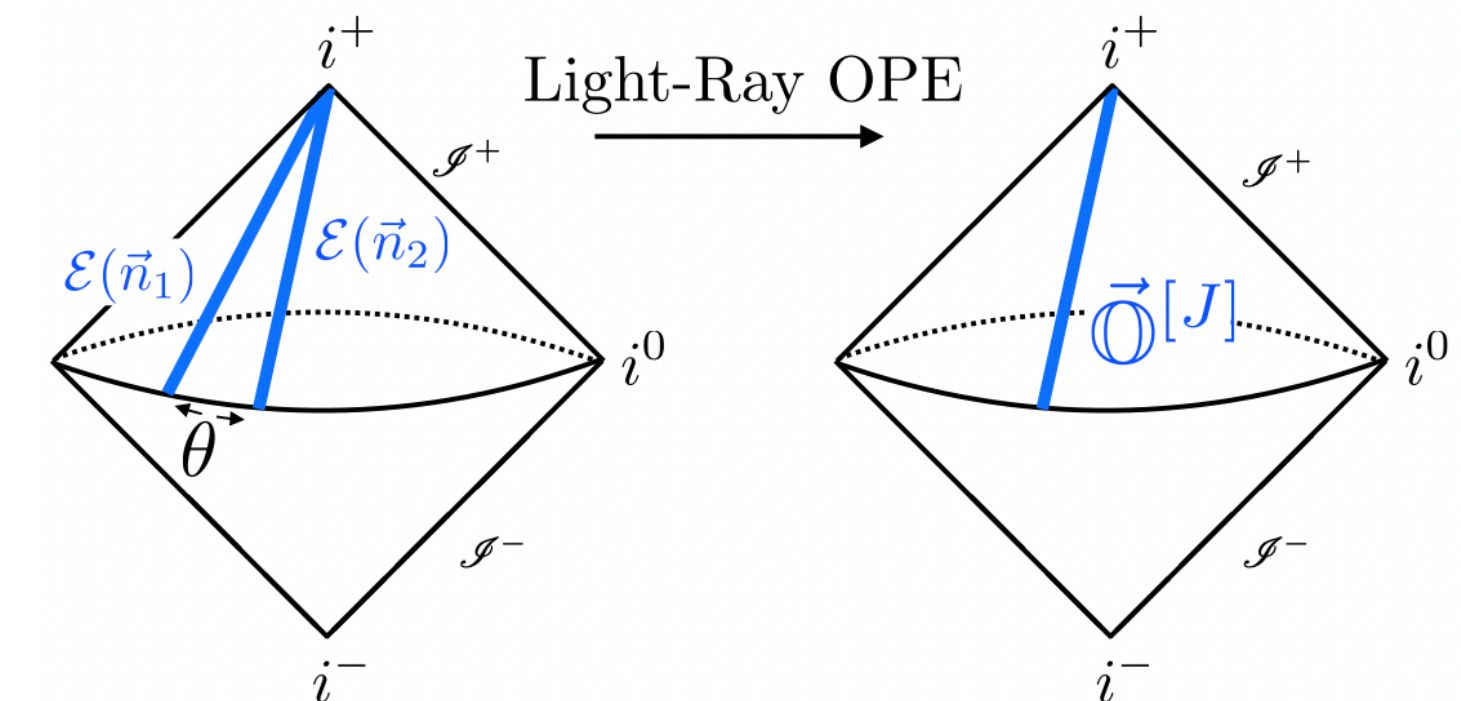
$$\frac{d\sigma}{d \cos \chi} = \sum_{i < j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos \chi)$$



Experiment

$$\mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \sim \sum_i \theta^{\tau_i - 4} \mathbb{O}_i(\hat{n}_1)$$

Theory



New Observables

This sort of collaboration is crucial for the success of future collider studies

Summary

Jet substructure provides a physical realization of the OPE limit of **light-ray operators**

→ Direct **bridge** between recent theoretical advancements and QCD Phenomenology

Creates an unprecedented symbiosis between **theory** and **experiment**

→ Allowing for sharp probes of interesting physics, new and old

