Heavy Quark Fragmentation in e+e- Collisions to NNLO+NNLL Accuracy in Perturbative QCD

BOOST Genova - 30/07/2024

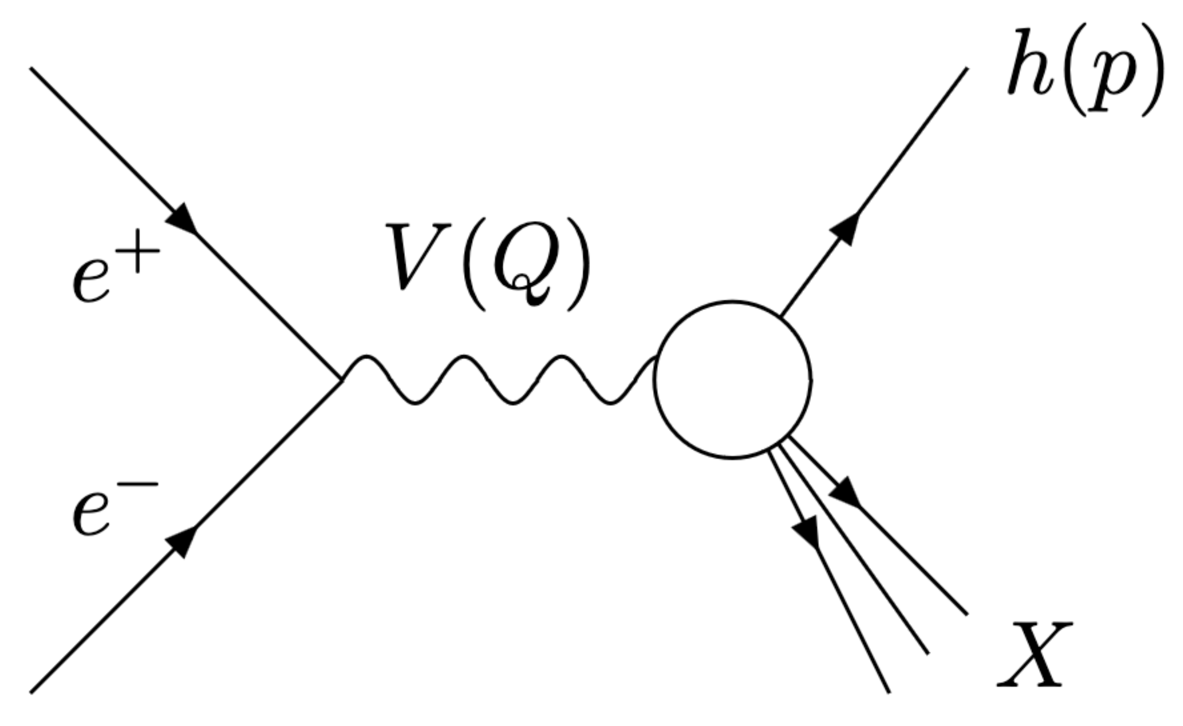
Universität Zürich UZH

Swiss National Science Foundation

Leonardo Bonino

Introduction

[2312.12519] with Matteo Cacciari (LPTHE) and Giovanni Stagnitto (UniMiB)



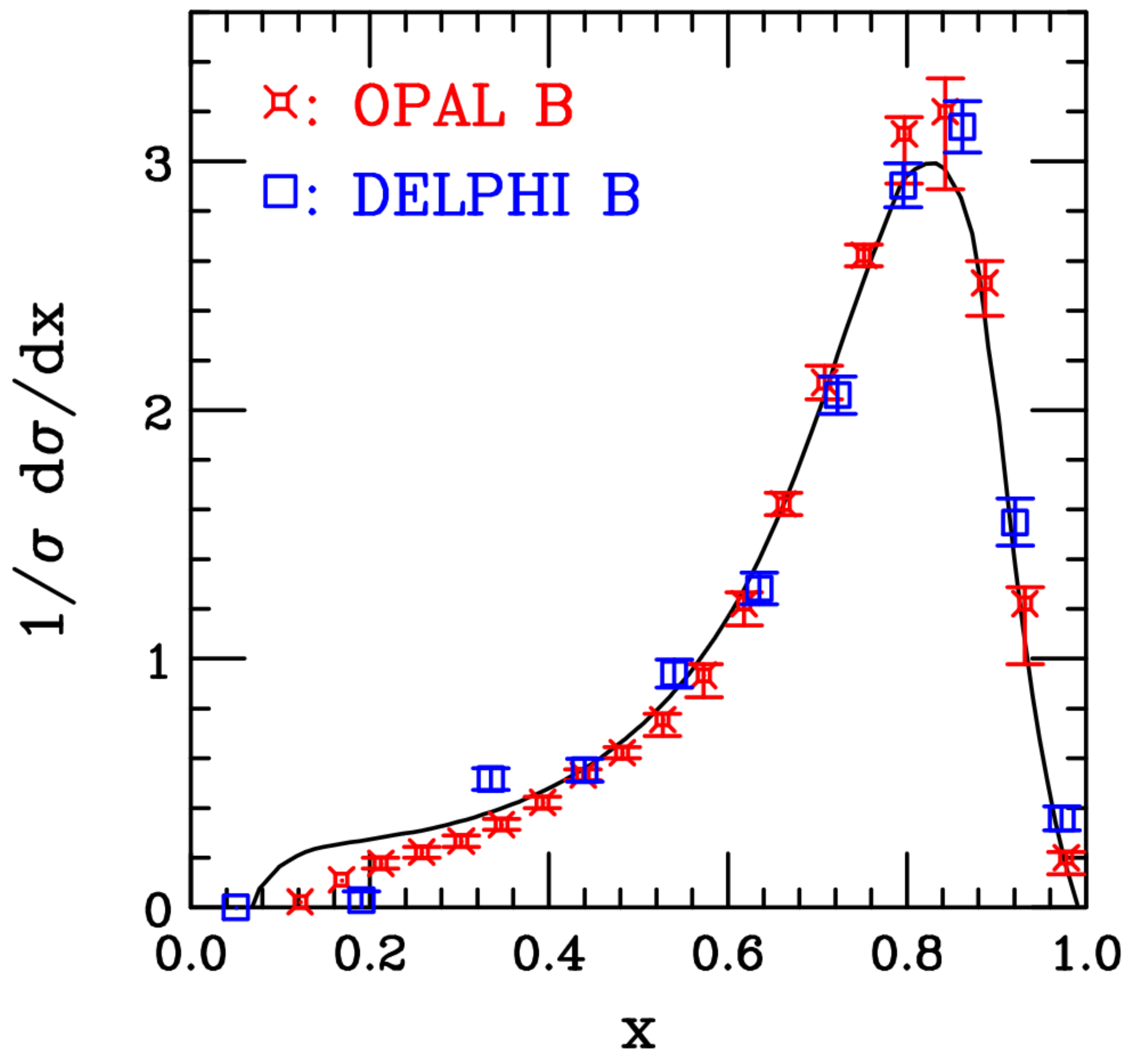
Fragmentation function formalism

- Goal: describe fragmentation of heavy quarks (c and b) into hadrons
- Process: $e^+e^- \to V(Q) \to h(p) + X$
- Tool: QCD perturbative fragmentation function formalism
  - Factorisation → hadron cross section $\sigma_h(x, Q)$

    $\sigma_h\left(x, Q\right) \simeq \sigma_{\mathcal{Q}}(x, Q, m) \otimes D^{np}_{\mathcal{Q}\to h}(x, \{\text{par}\})$
  - Fully perturbative cross section for heavy quark $\mathcal{Q}$ of mass m

    $\sigma_{\mathcal{Q}}(x, Q, m) = \hat{\sigma}_i(x, Q, \mu_F) \otimes D_{i\to\mathcal{Q}}(x, \mu_F, m) + \mathcal{O}\left((m/Q)^p\right)$ [Mele, Nason '91]
  - Fragmentation functions are universal (process independent)
- Mellin space: convolution → simple product

[Cacciari et al. '05]



$x = E_h/E_{\text{beam}}$
$Q = 2E_{\text{beam}}$
μ_F: factorisation scale

Introduction

Perturbative ingredients

- $$\frac{1}{\sigma_{\mathcal{Q}}^{tot}}\sigma_{\mathcal{Q}}\left(Q,m\right)=\frac{1}{\sigma_{\mathcal{Q}}^{tot}}\sigma^{(0)}\sum_{i,j}C_i\left(Q,\mu,\mu_F\right)E_{ij}\left(\mu_F,\mu_{0F}\right)D_{j\to\mathcal{Q}}\left(\mu_{0F},m\right)$$

2 factorisation scales: μ_F and μ_{0F}

  - Initial conditions $D_{j\to\mathcal{Q}}$ @ NNLO **[Melnikov, Mitov '04] [Mitov '04] [Maltoni et al. '22]**
  - DGLAP Evolution (ZM-VFNS @ NLO) E_{ij} with MELA **[Bertone et al. '15] [Ridolfi et al. '19]** MELA
  - Coefficient functions C_i @ NNLO **[Rijken, van Neerven '97] [Blümlein, Ravindran '06] [Mitov, Moch '06]**
- Poor behaviour in large-N $(x\to 1)$ region (Sudakov region)
  - Need resummation @ NNLL in initial conditions and coefficient functions **[Cacciari, Catani '01] [Aglietti et. al '06] [Maltoni et al. '22] [Czakon et al. '22]**

Theory overview

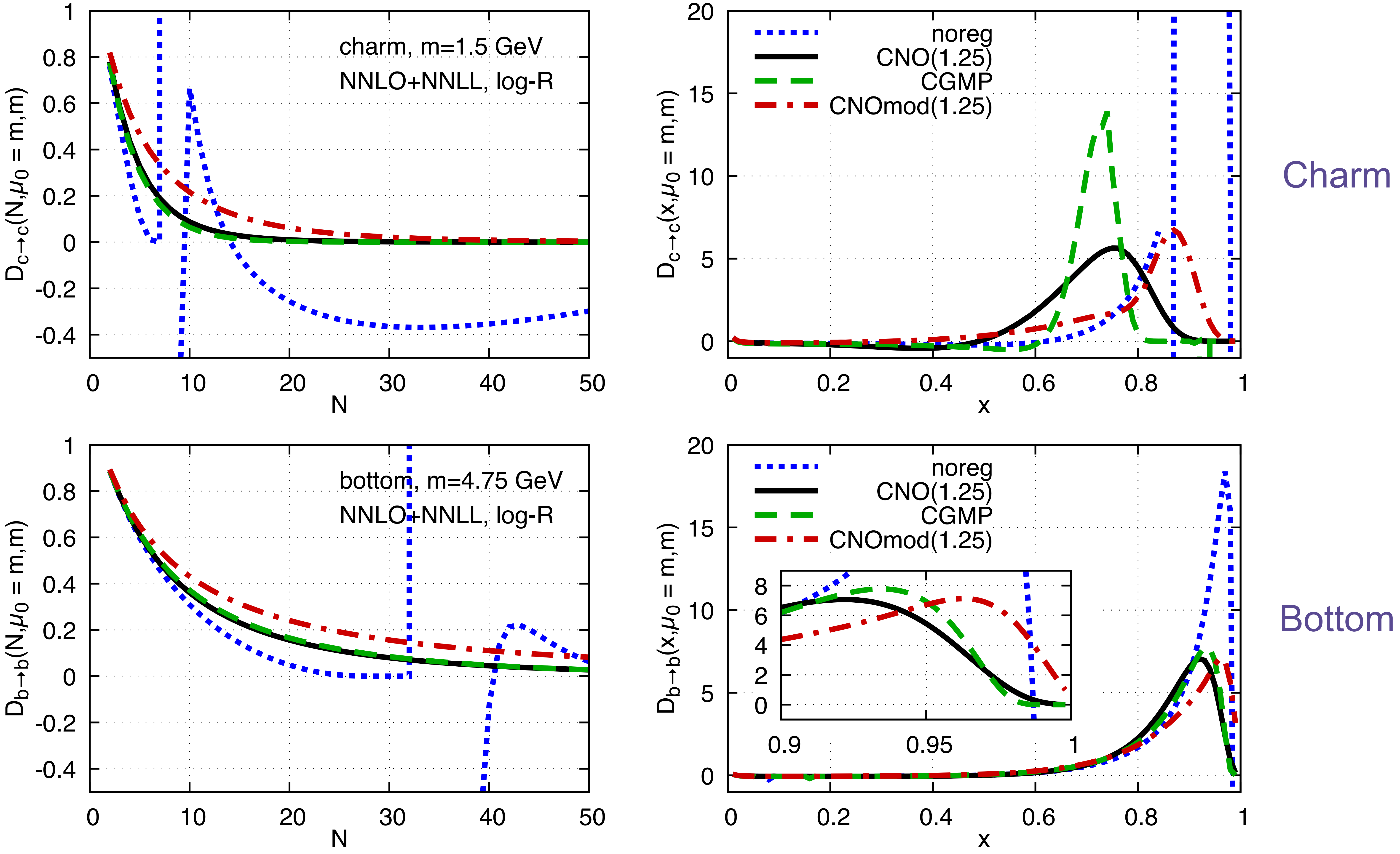
Soft-gluon resummation

- Sudakov-resummed initial conditions
  - Constant large-N limit of fixed order result & Sudakov factor
    $$D^{res}_{\mathcal{Q}\to\mathcal{Q}} = \left[D_{\mathcal{Q}\to\mathcal{Q}}\right]_c \exp\left[\ln N g^{(1)}_{\text{ini}}(\lambda_0) + g^{(2)}_{\text{ini}}(\lambda_0) + \alpha_S g^{(3)}_{\text{ini}}(\lambda_0)\right]$$
  - Different matchings to fixed order result (e.g. $\log \mathrm{R}$)
    $$\log D^{fo+res,\text{logR},reg}_{i\to\mathcal{Q}} = \log D^{fo}_{\mathcal{Q}\to\mathcal{Q}} + \log D^{res,reg}_{\mathcal{Q}\to\mathcal{Q}} - [\log D^{res(,reg)}_{\mathcal{Q}\to\mathcal{Q}}]_{\alpha_s^p}$$
- Landau pole in $g^{(i)}_{\text{ini}}(\lambda_0) : \lambda_0 = 1/2 \to N^L_0 = \exp\left(1/(2b_0\alpha_s)\right)$
  - $N^L_0 \sim 7$ for charm & $N^L_0 \sim 32$ for bottom → (many) prescriptions
    - “CNO” (Cacciari-Nason-Oleari): shift in N in $D^{res}_{\mathcal{Q}\to\mathcal{Q}}$ and $[D^{res}_{\mathcal{Q}\to\mathcal{Q}}]_{\alpha_s^p}$: parameter f **[Cacciari et al. ‘05]**
    - “CGMP” (Czakon-Generet-Mitov-Poncelet): truncation of Sudakov factor **[Czakon et al. ’22]**
    - “CNOmod” (Cacciari-Nason-Oleari + Czakon-Generet-Mitov-Poncelet): shift in N only in $D^{res}_{\mathcal{Q}\to\mathcal{Q}}$ **[Czakon et al. ’22]**

Numerical results

Heavy quark initial condition: NNLO + NNLL logR

- $D^{fo+res,match,reg}_{\mathcal{Q}\to\mathcal{Q}}(\,\cdot\,, Q, \mu_R, \mu_F, \mu_{0R}, \mu_{0F}, m)$
- Prescription chosen for Landau pole (reg) has huge impact
- Some shapes not suited for fits
  - NNLO+NNLL log-R CNO (default)
- Lower mass → more sensitivity to Landau
- Evolution and convolution do not help …

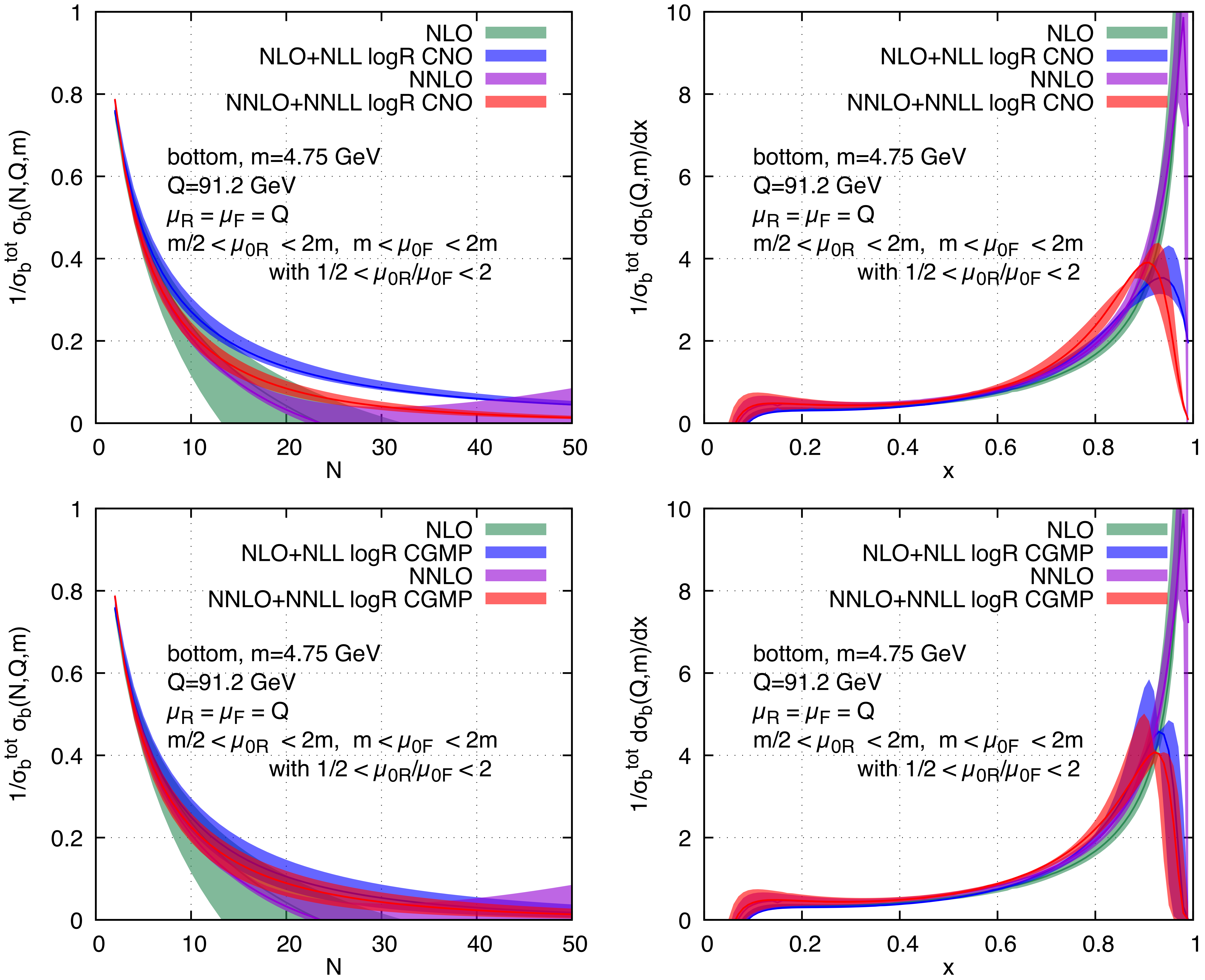


Numerical results

Full e^+e^- fragmentation function

- $\sigma_{\mathcal{Q}}^{fo+res,logR,reg}(\,\cdot\,, Q, \mu_R, \mu_F, \mu_{0R}, \mu_{0F}, m)$
- Uncertainty bands for μ_{0R} and μ_{0F} 5-point scale variation in initial conditions (around m)
  - NNLO+NNLL band (red) not much narrower then NLO+NLL one (blue)
  - Bands do not overlap → poor convergence of perturbative series (CNO)
- Drastic dependence on Landau pole regularisation
- Charm: worse

Bottom



CNO

CGMP

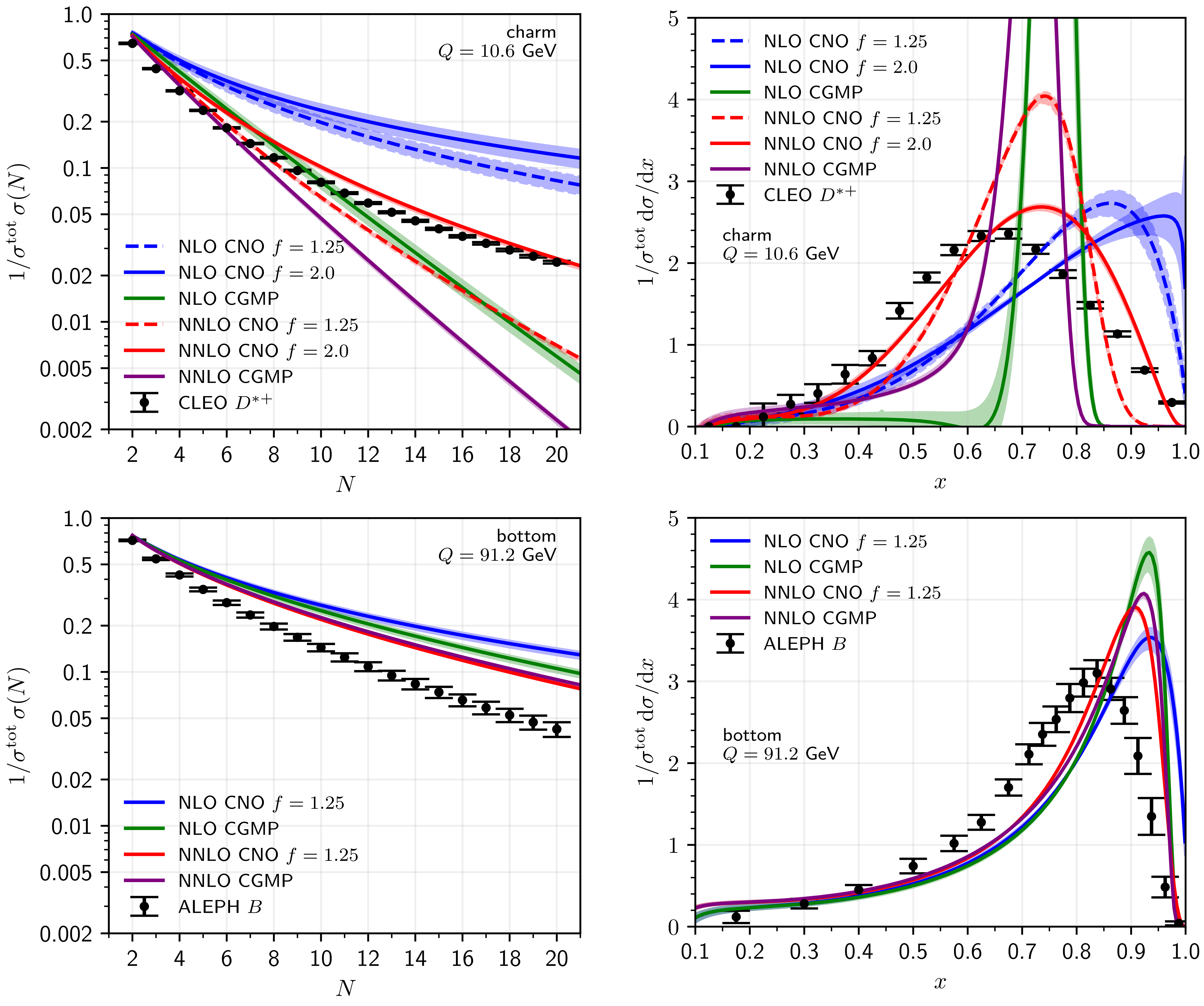
Towards phenomenology

pQCD at comparison with data

$$\sigma_h(N,Q) = \sigma_{\mathcal{Q}}(N,Q,m) D^{np}_{\mathcal{Q}\to h}(N,\{\text{par}\})$$

- Bottom: all prescriptions can be used for fits
- Charm: most NNLO+NNLL curves dip below data at $N \gtrsim 6$
- D^{np} can only “lower” theoretical prediction
  - Bottom: NNLO+NNLL logR CNO (f=1.25)
  - Charm: NNLO+NNLL logR CNO (f=2.0)
- Single-moment and single-parameter “fits”

f: CNO regularisation parameter



Charm

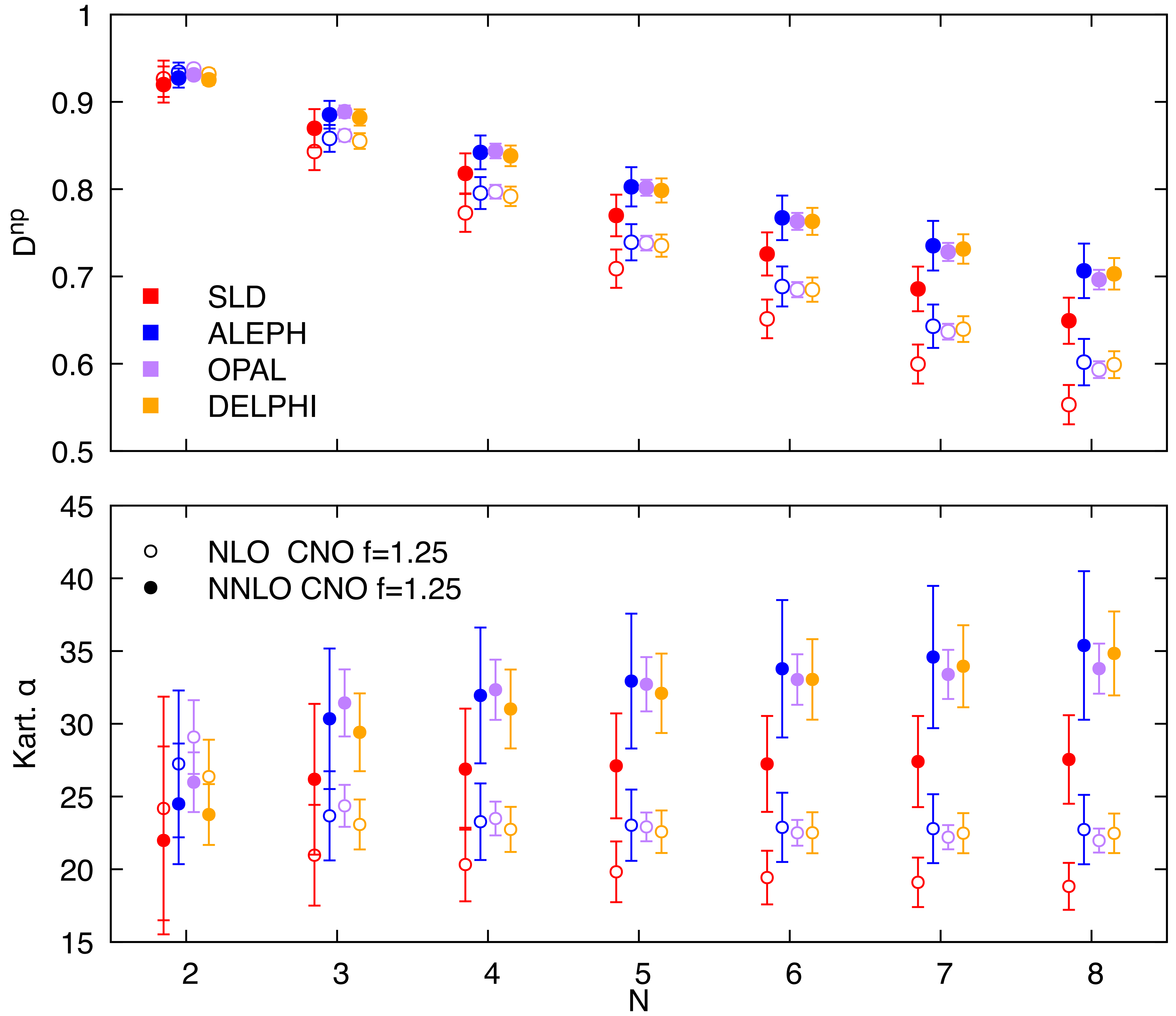
Bottom

Single-point fits

CNO bottom single-point fits up to NNLO + NNLL

Bottom

- Recall: non-perturbative FF factorised
  $\sigma_h(N,Q) = \sigma_{\mathcal{Q}}(N,Q,m) D^{np}_{\mathcal{Q}\to h}(N,\{\text{par}\})$
- Our take: as simple as possible
  - Single parameter non-perturbative function
    $D^{np}_K(x) = (\alpha+1)(\alpha+2)x^\alpha(1-x)$ **[Kartvelishvili et al. '78]**
- "Fit" moments between 2 and 8 (relevant for hadronic collisions)
- $D^{np} = \text{data}/(\text{pert . theo})$
  - α values stable in N
  - NNLO+NNLL results closer to data → smaller non-perturbative component

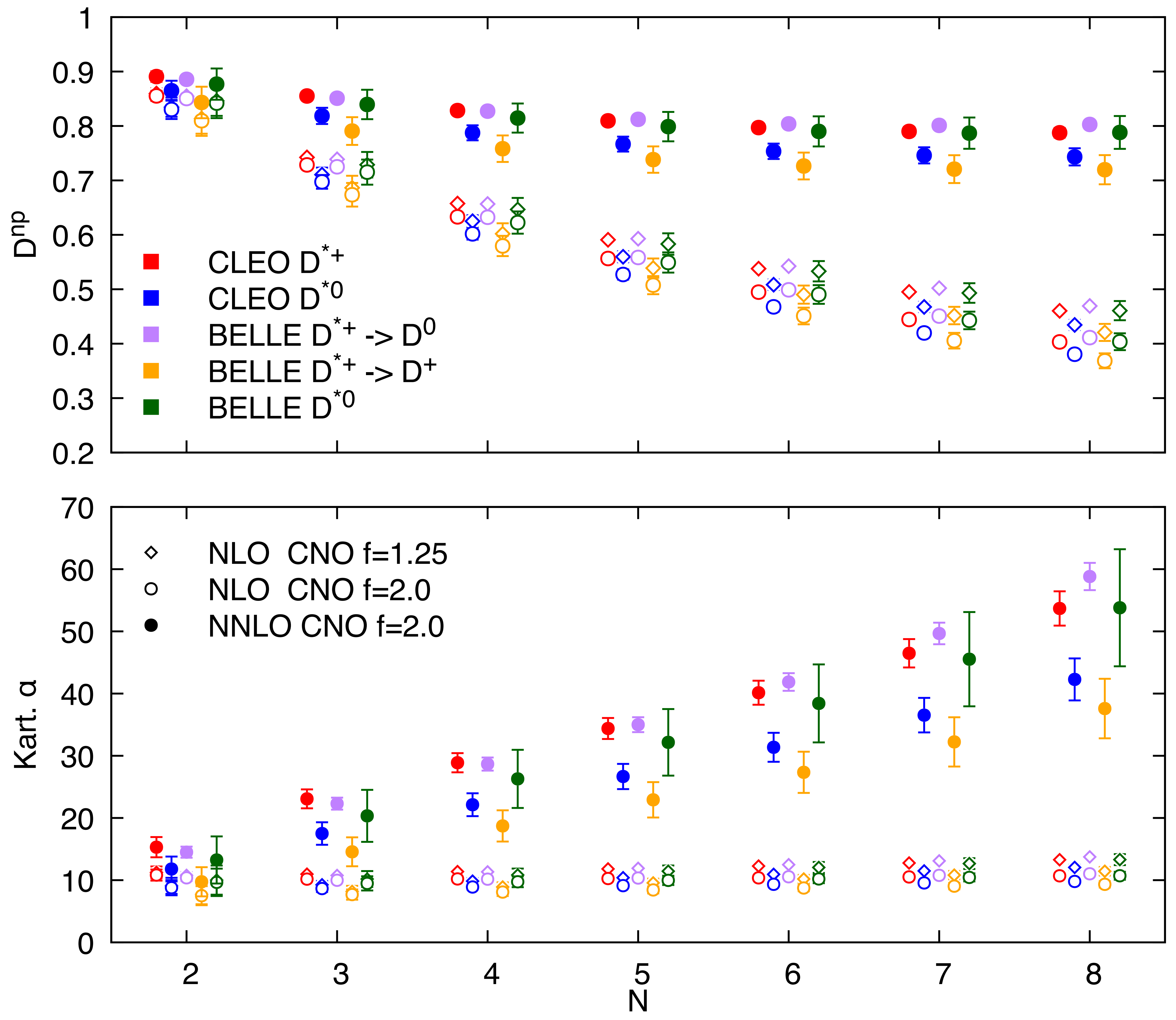


Single-point fits

CNO charm single-point fits up to NNLO + NNLL

Charm

- @ NLO+NLL
  - α values stable in N
  - Stable under variation of f
- @ NNLO+NNLL
  - large dependency of α on N
  - $f = 2.0$ mandatory choice

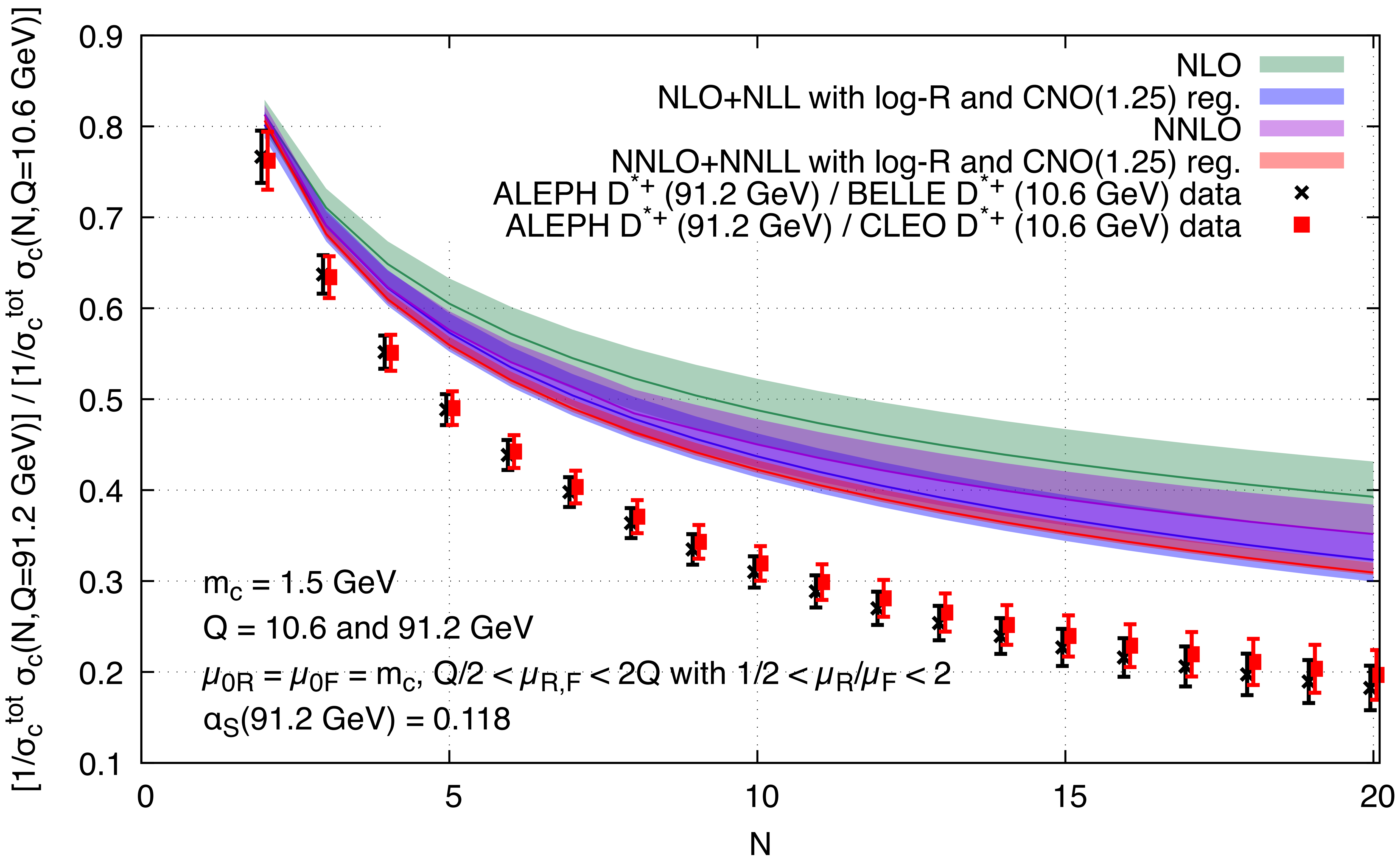


f: CNO regularisation parameter

Charm ratio

A perturbative observable

- Ratio of ALEPH ($91.2\,\mathrm{GeV}$) and BELLE/CLEO ($10.6\,\mathrm{GeV}$) moments for D^{*+}
- Non-perturbative and low-scales effects largely cancel in theory prediction → **entirely perturbative**
- Data undershoot pure QCD prediction
- Large power suppressed effects in coefficient functions?
- Discrepancy reduced if heavy-quark threshold effects also in resummed coefficient functions **[Cacciari et al. 2406.04173]**



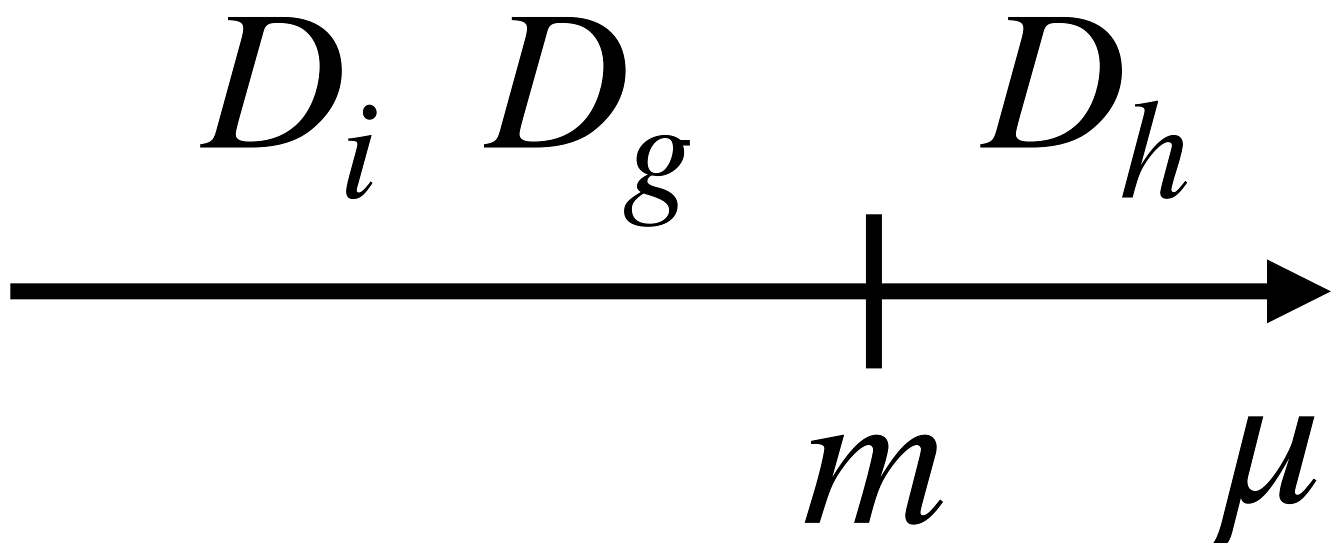
$$\frac{\sigma_{\mathbb{Q}}(M_Z, m)}{\sigma_{\mathbb{Q}}(M_\Upsilon, m)} \simeq \frac{C_q(M_Z,\mu_Z)}{\sigma_{\mathbb{Q}}^{tot}(M_Z)} E(\mu_Z,\mu_\Upsilon) \frac{\sigma_{\mathbb{Q}}^{tot}(M_\Upsilon)}{C_q(M_\Upsilon,\mu_\Upsilon)}$$

Matching conditions at NNLO

based on [2407.07623] in collaboration with Christian Biello (MPP)

Towards a ZM-VFNS at NNLO

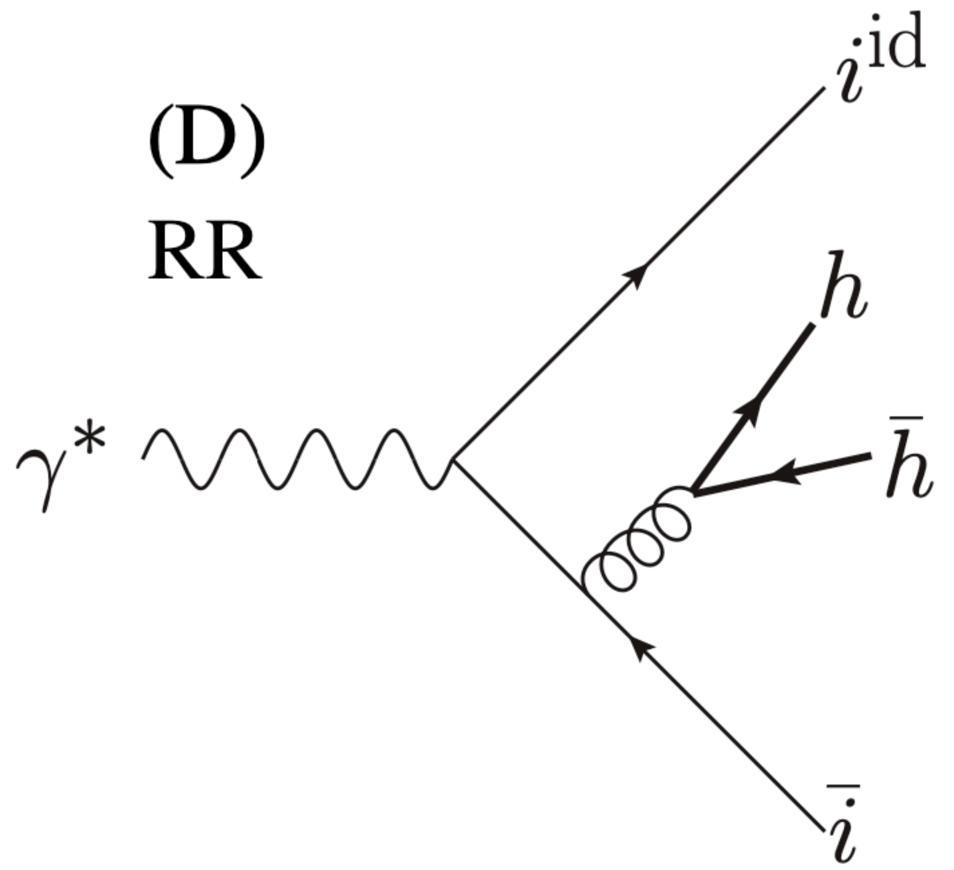
- $\frac{1}{\sigma_{\mathbb{Q}}^{tot}}\sigma_{\mathbb{Q}}(Q,m) = \frac{1}{\sigma_{\mathbb{Q}}^{tot}}\sigma^{(0)}\sum_{i,j} C_i(Q,\mu,\mu_F)\, E_{ij}(\mu_F,\mu_{0F})\, D_{j\to\mathbb{Q}}(\mu_{0F},m)$ → crossing heavy flavour thresholds ($\mu \sim m$)
  - need time-like thresholds matching conditions (ZM-VFNS)
  - NLO matching conditions for D_h and D_g **[Cacciari et al. '05]**
  - NNLO D_i (NNLO corrections to D_h D_g still missing) **[Biello, Bonino '24]**



- $\mu \ll m$ → decoupling scheme: only massless flavours FFs + mass effects in XS ($\mathrm{d}\sigma$)

$$\frac{\mathrm{d}\sigma^H}{\mathrm{d}x} = \int_x^1 \frac{\mathrm{d}z}{z}\left\{\sum_{j\in\mathbb{l}_n} D_j^{(n_L)}\frac{\mathrm{d}\sigma_j}{\mathrm{d}z} + D_g^{(n_L)}\frac{\mathrm{d}\sigma_{h\bar{h}g^{\mathrm{id.}}}}{\mathrm{d}z} + \sum_{i\in\mathbb{l}_{n_L}-g} D_i^{(n_L)}\frac{\mathrm{d}\sigma_{h\bar{h}i^{\mathrm{id.}}\bar{i}}}{\mathrm{d}z}\right\}$$

$\mu = \mu_F =$ factorization scale



- $\mu \gg m$ → full massless $\overline{MS}$ scheme $\frac{\mathrm{d}\sigma^H}{\mathrm{d}x} = \int_x^1 \frac{\mathrm{d}z}{z}\left\{\sum_{k\in\mathbb{l}_n} D_k^{(n)}\frac{\mathrm{d}\hat{\sigma}_k}{\mathrm{d}z}\right\}$

Matching conditions at NNLO

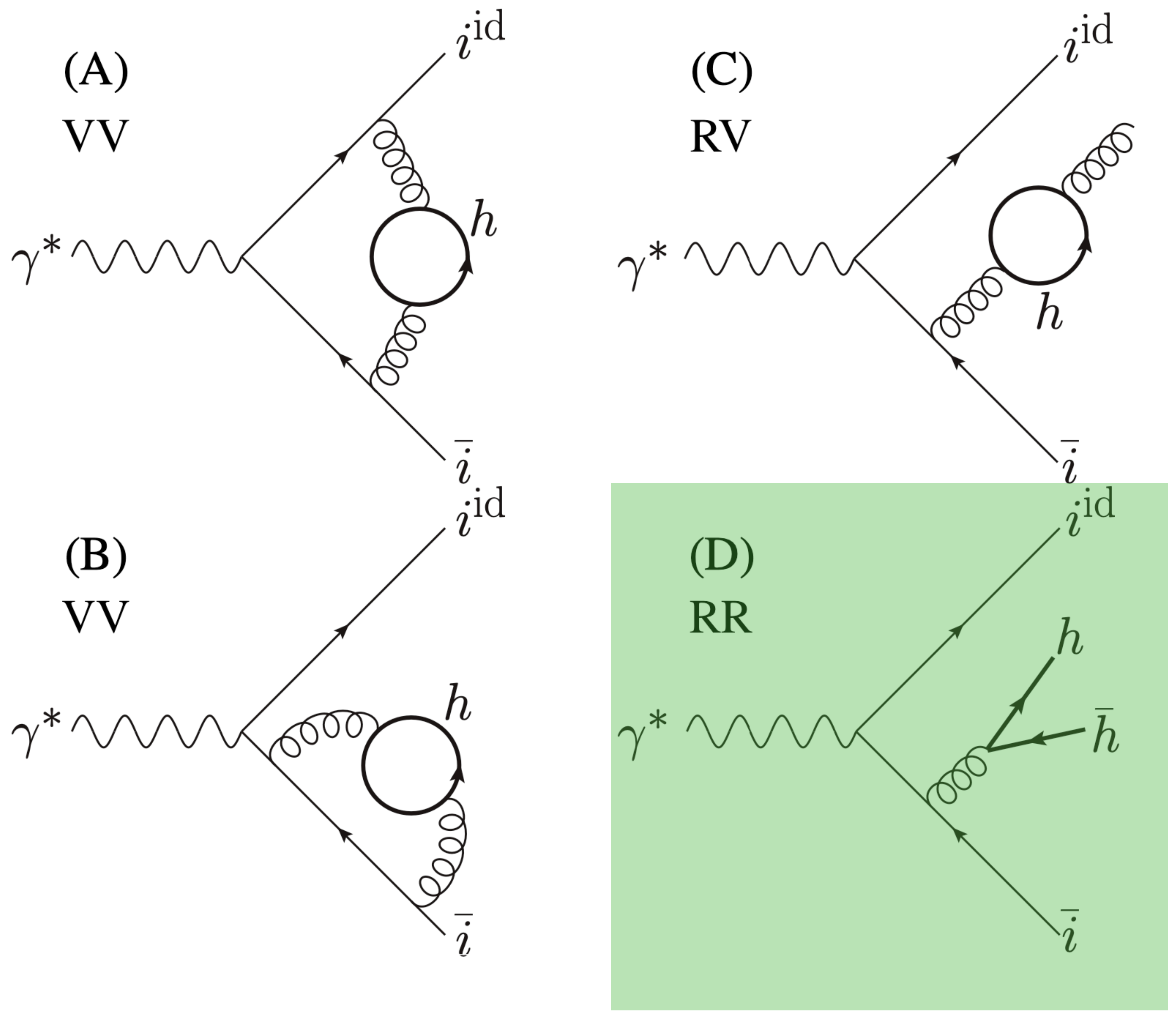
The light flavour matching condition

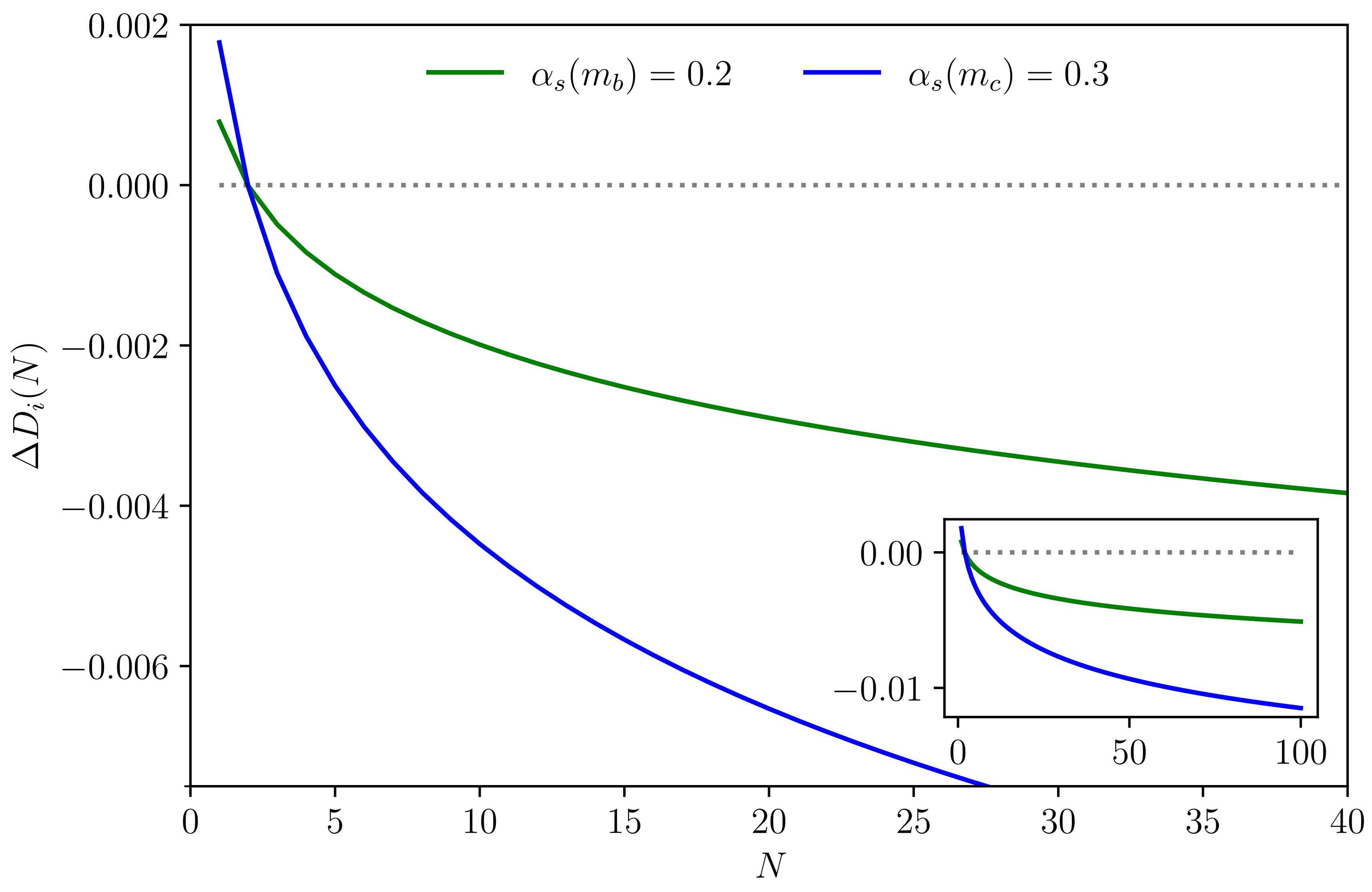
- NNLO matching conditions for light flavour (Mellin space)

$$D_i^{(n)}(N,\mu) = \left\{ 1 + \frac{1}{\sigma_{i\bar{i}}} \mathscr{M}_{(N,z)} \left[\delta^i_{D_i}(z) \right] \right\} D_i^{(n_L)}(N,\mu)$$

- Takes RR [Gehrmann Stagnitto '22] [Bonino et al. '24] RV and VV [Blümlein et al. '16] massive and massless corrections

$$\begin{aligned} D_i^{(n)}(N,\mu) = \Bigg\{ 1 + \left(\frac{\alpha_s}{2\pi}\right)^2 C_F \frac{1}{N^3(N+1)^3} \Bigg[& -\frac{2}{3}N^3(N+1)^3 S_{1,2}(N) - \frac{2}{3}N^3(N+1)^3 S_{2,1}(N) + \frac{1}{3}N^3(N+1)^3 S_3(N) \\ & + \frac{5}{9}N^3(N+1)^3 S_2(N) + S_1(N)\left(\frac{2}{3}N^3(N+1)^3 S_2(N) - \frac{28}{27}N^3(N+1)^3\right) - \frac{4}{3}N^3(N+1)^3\zeta_3 \\ & + \left(\frac{9307}{1296} - \frac{29}{108}\pi^2\right)N^6 + \left(\frac{9307}{432} - \frac{29}{36}\pi^2\right)N^5 + \left(\frac{3281}{144} - \frac{29}{36}\pi^2\right)N^4 + \left(\frac{10939}{1296} - \frac{29}{108}\pi^2\right)N^3 - \frac{5}{54}N^2 - \frac{1}{9}N + \frac{1}{6} \\ & - \frac{8}{9}N^3(N+1)^3\log^3 2 + \frac{29}{9}N^3(N+1)^3\log^2 2 + \frac{1}{54}\left(12\pi^2 - 359\right)N^3(N+1)^3\log 2 \\ & + \left(\frac{10}{9}N^3(N+1)^3 S_1(N) - \frac{2}{3}N^3(N+1)^3 S_2(N) - \frac{1}{36}N\left(3N^5 + 9N^4 + 53N^3 + 67N^2 + 8N - 12\right)\right)\log\left(\frac{\mu^2}{m^2}\right) \\ & + \left(\frac{1}{12}N^2(N+1)^2\left(3N^2+3N+2\right) - \frac{1}{3}N^3(N+1)^3 S_1(N)\right)\log^2\left(\frac{\mu^2}{m^2}\right)\Bigg] + \mathscr{O}(\alpha_s^3)\Bigg\} D_i^{(n_L)}(N,\mu) . \end{aligned}$$





Conclusions

- NNLO + NNLL reached for bottom & charm
  - Open question on Landau pole regularisation
    - Can we do better?
    - Current limit on theoretical precision
  - Interesting developments on charm-ratio
- 1 out of 3 of the NNLO matching conditions ✓
- Results important for $pp-$phenomelogy

Backup: Landau pole regularisations

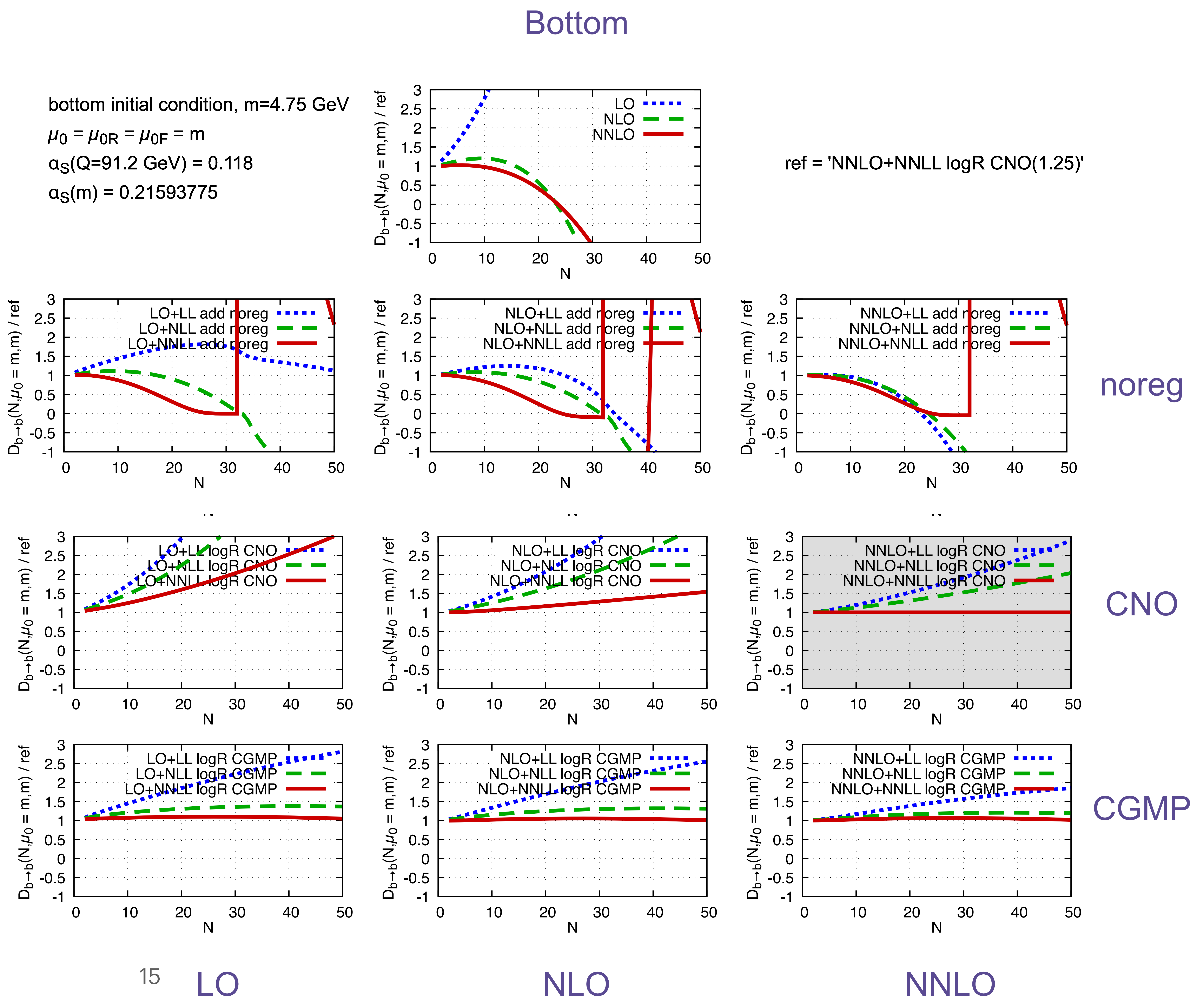
CNO & CGMP prescription

- "CNO" (Cacciari-Nason-Oleari): shift in N [Cacciari et al._0510032] $N \to N\frac{1+f/N^L}{1+fN/N^L}$
  - Consistent with all known perturbative results ✓
  - Yields physically acceptable results ✓
  - Does not introduce power corrections larger then generally expected for the process in question: $N\Lambda_{QCD}/m$ for IC and $N\Lambda^2_{QCD}/Q^2$ for CF ✓
- "CGMP" (Czakon-Generet-Mitov-Poncelet): truncation in the **exponent** of Sudakov factor [Czakon et al._2210.06078]
  - introduces power corrections larger then generally expected

Numerical results

Heavy quark initial condition

- Full range of perturbative orders
  - Ratio to NNLO+NNLL log-R CNO
- No obvious perturbative hierarchy NNLL < NLL < LL
- No systematic convergence
  - But for log-R CGMP
- NNLO+NNLL log-R CNO (default)

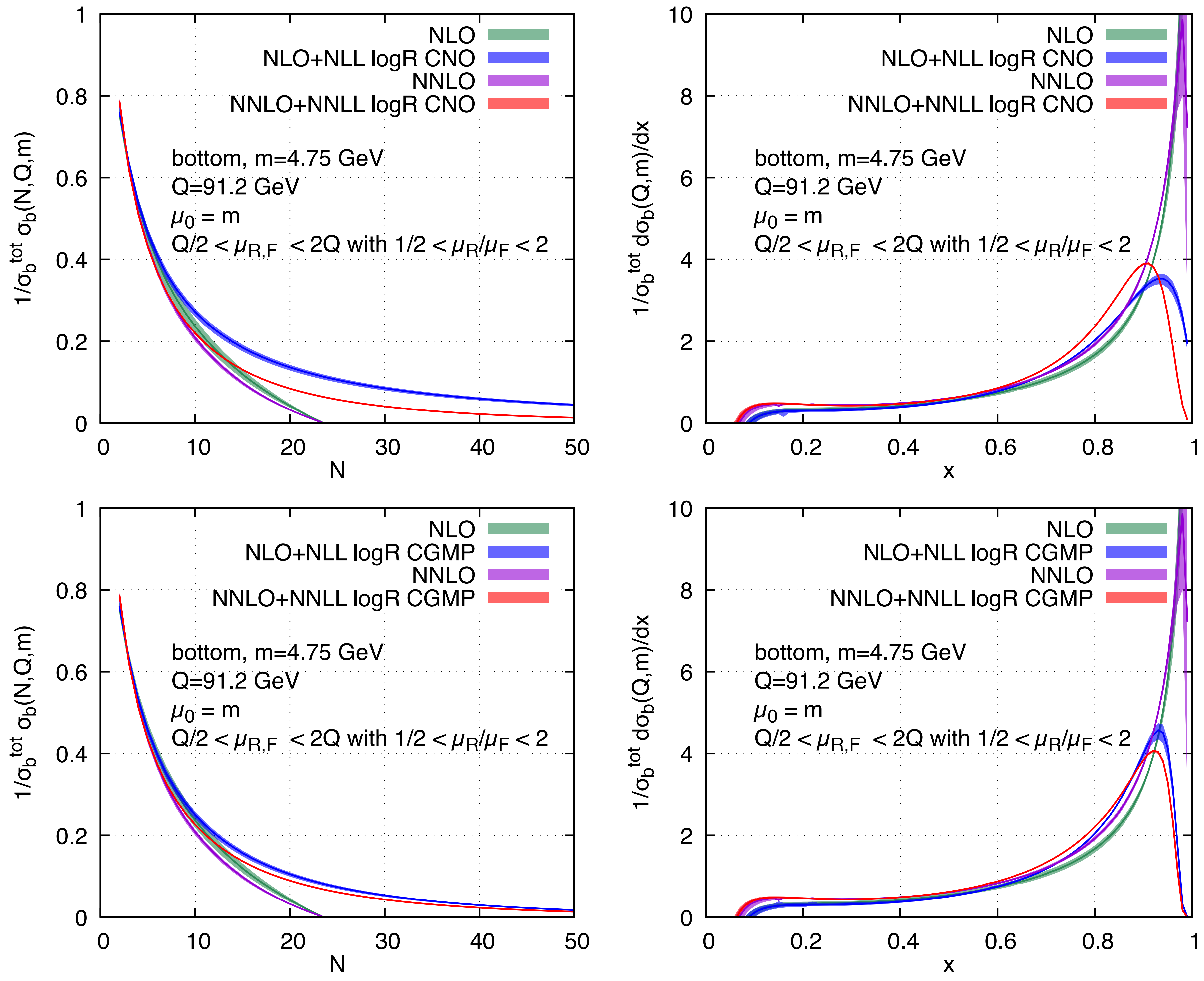


Backup: numerical results

Bottom

Full e^+e^- fragmentation function

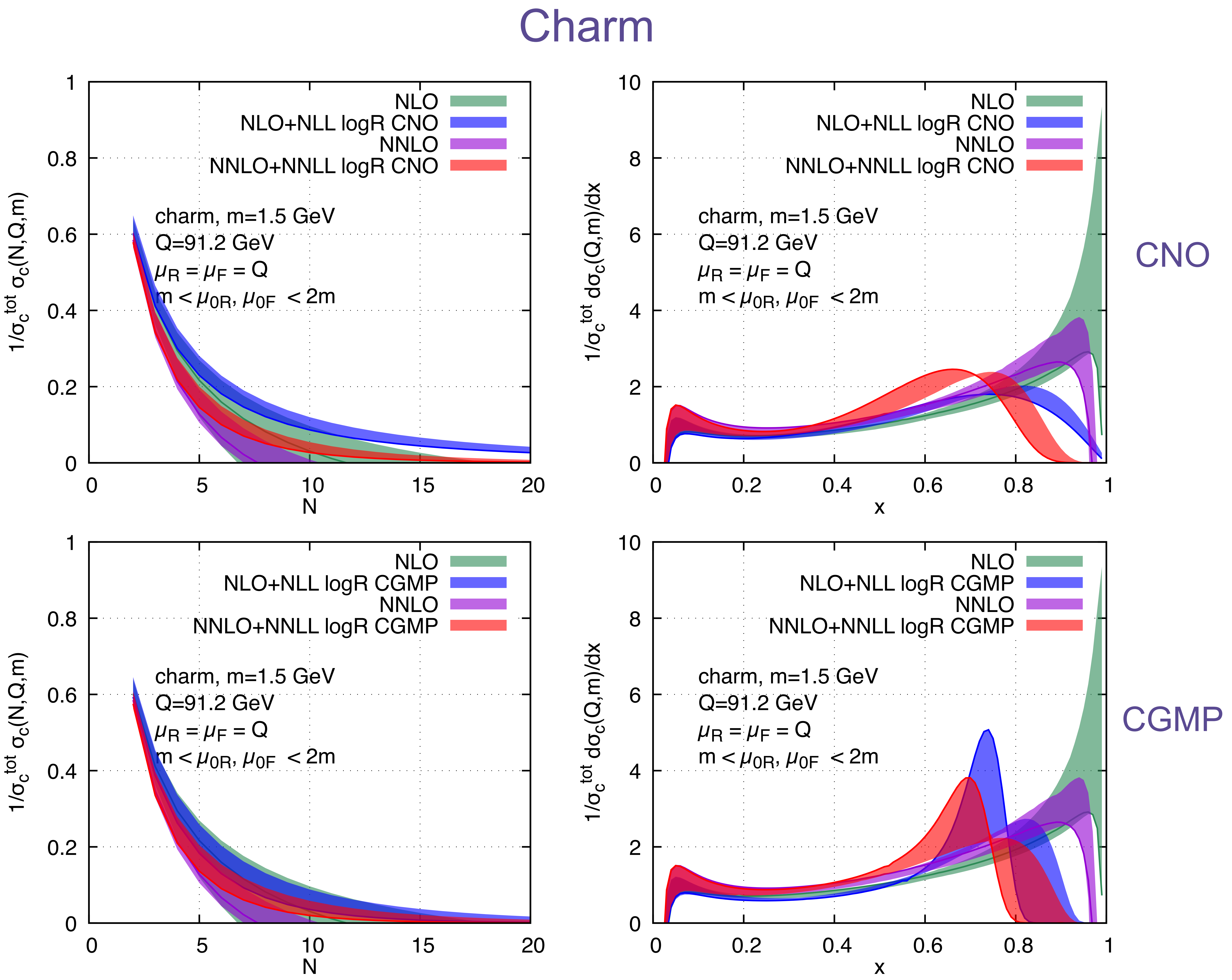
- Perturbative hierarchy better respected for μ_R and μ_F variations around Q



Numerical results

Full e^+e^- fragmentation function

- Scale variation around m
  - Wider bands than bottom case
  - Any perturbative hierarchy essentially lost
- Larger sensitivity to Landau pole regularisation than bottom
- Can we still compare with the data?

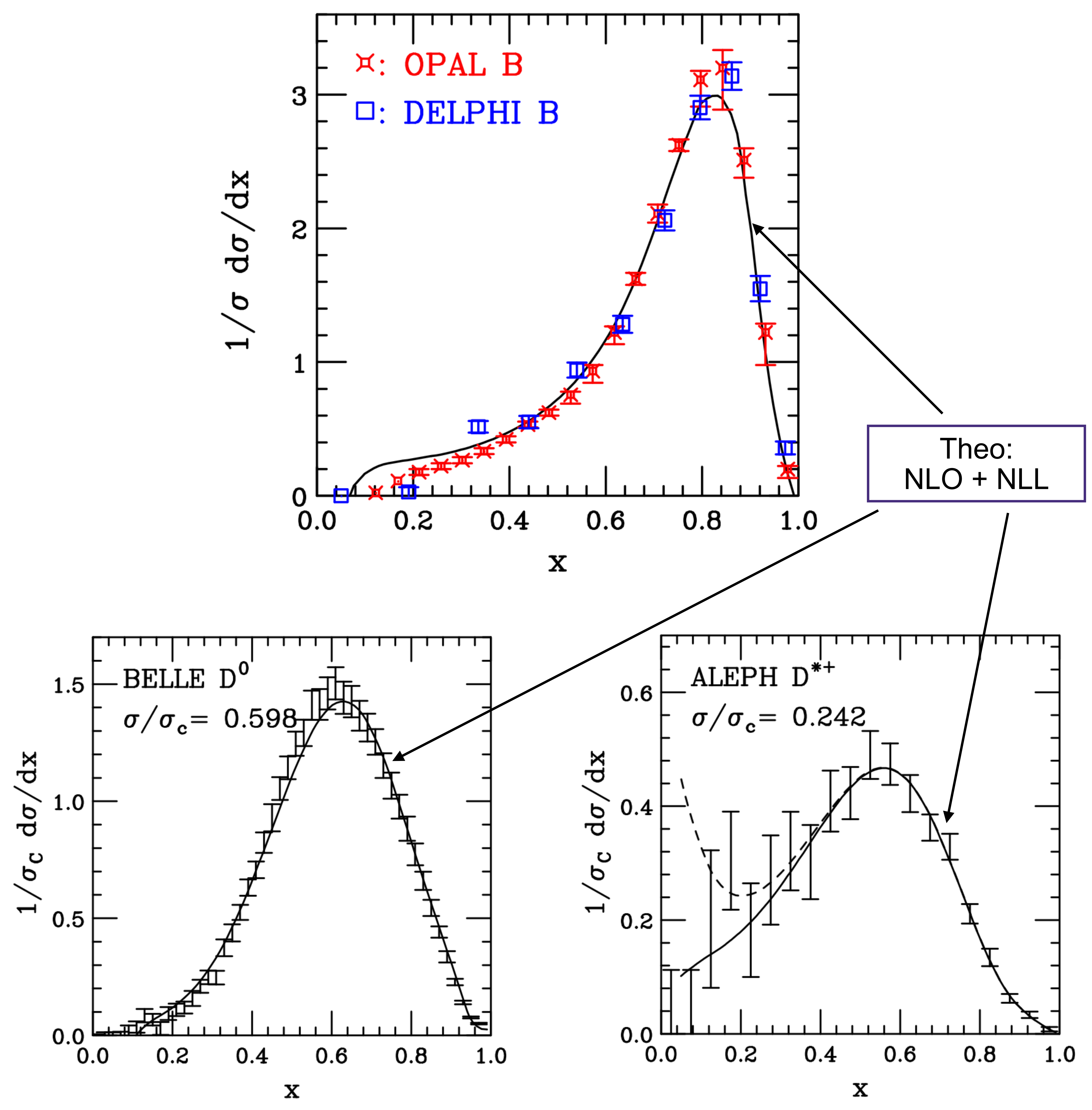


Backup: the data

At e^+e^- colliders

- Compare with the data, what do we have?
- Many data comprising bottom and charm
- B mesons @ $91.2\,\mathrm{GeV}$ (Z^0-peak):
  - ALEPH (mesons) [ALEPH_0106051], SLD [SLD_0202031], OPAL [OPAL_0210031], DELPHI (some baryons as well) [DELPHI_002-069 CONF 603]
- D mesons, 2 energies:
  - ALEPH @ $91.2\,\mathrm{GeV}$ [ALEPH_9909032], BELLE [Belle_0506068], CLEO @ $10.6\,\mathrm{GeV}$ (ISR-corrected) [CLEO_0402040] [CLEO_9707018]

From: [Cacciari et al._0510032]

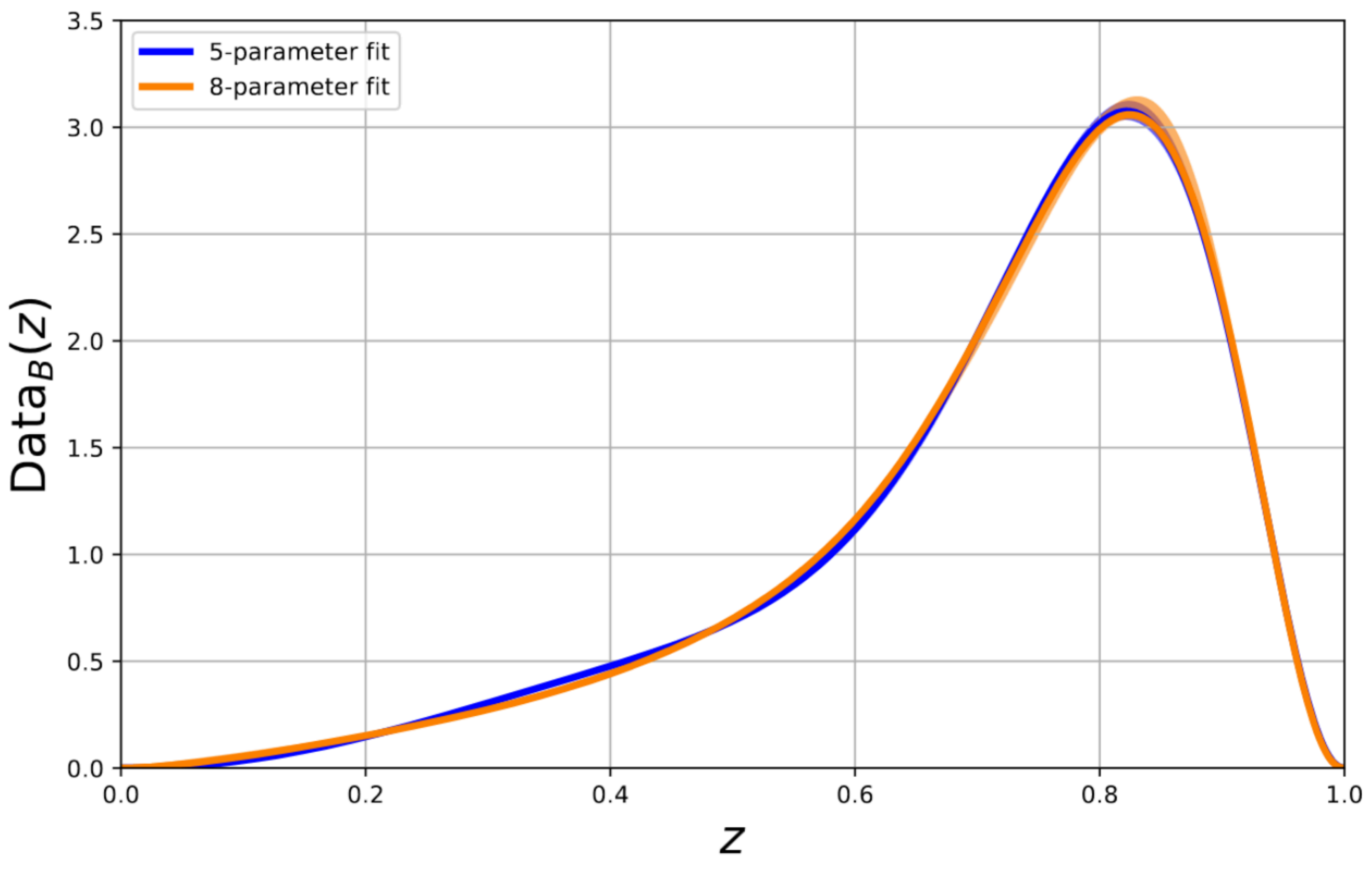


Backup:state of the art for fits

New results on bottom fragmentation

- Cacciari-Nason-Oleari 2005 → bottom & charm fits @ NLO + NLL
  - Good fits to D and B mesons fragmentation spectra
- Aglietti-Corcella-Ferrera 2007 → bottom & charm @ NLO + NNLL
  - Effective α_S: call for full NNLO analysis **[Aglietti et. al_0610035] [Corcella, Ferrera_0706.2357]**
- Czakon-Generet-Mitov-Poncelet 2022 → bottom @ NNLO + NNLL
  - Fits to D^{np} with 5-8 parameters
- Our contribution:
  - Charm @ NNLO + NNLL through b-threshold → fits to charm
  - Public code!

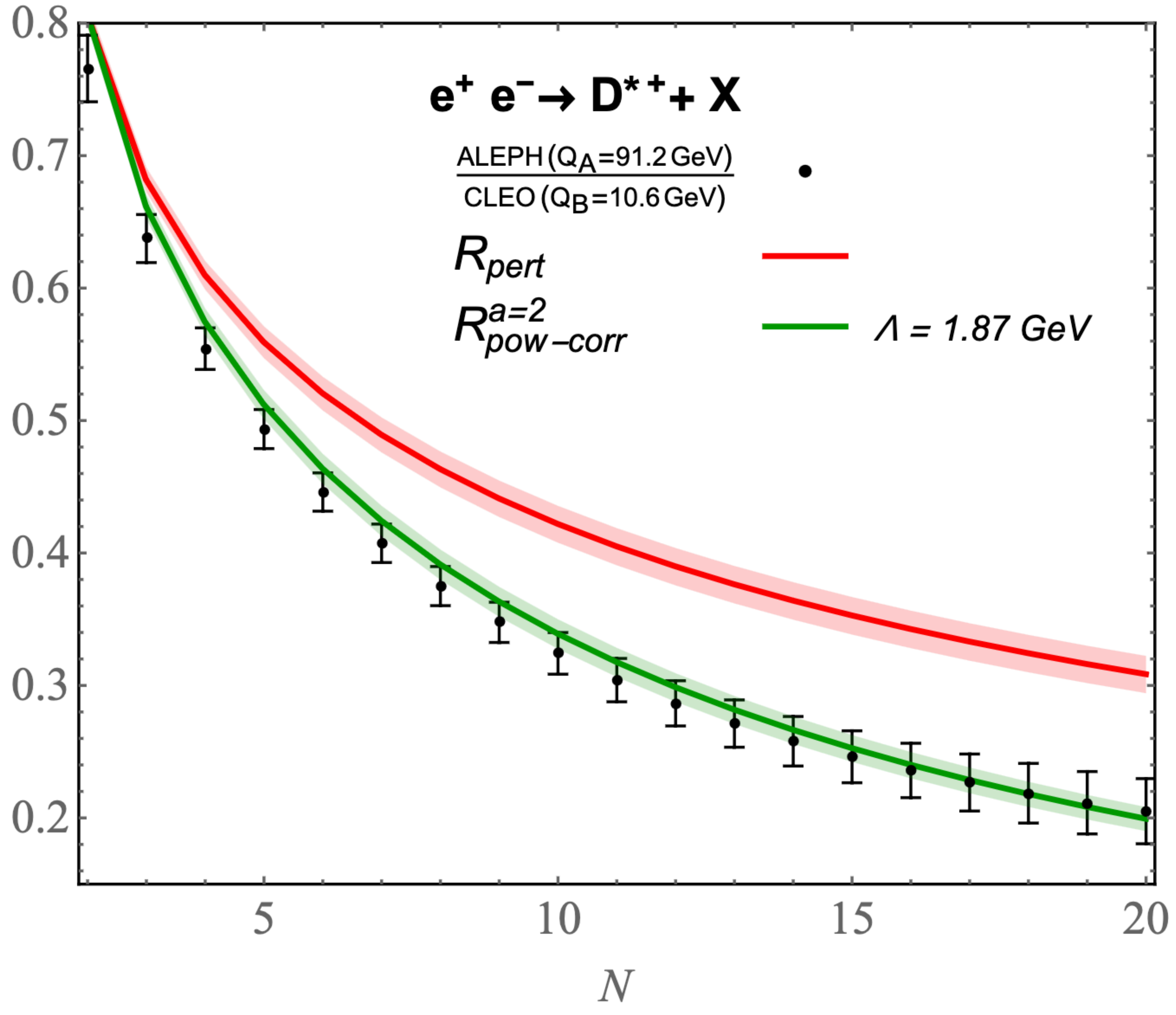
From: [Czakon et al._2210.06078]



Improved charm ratio

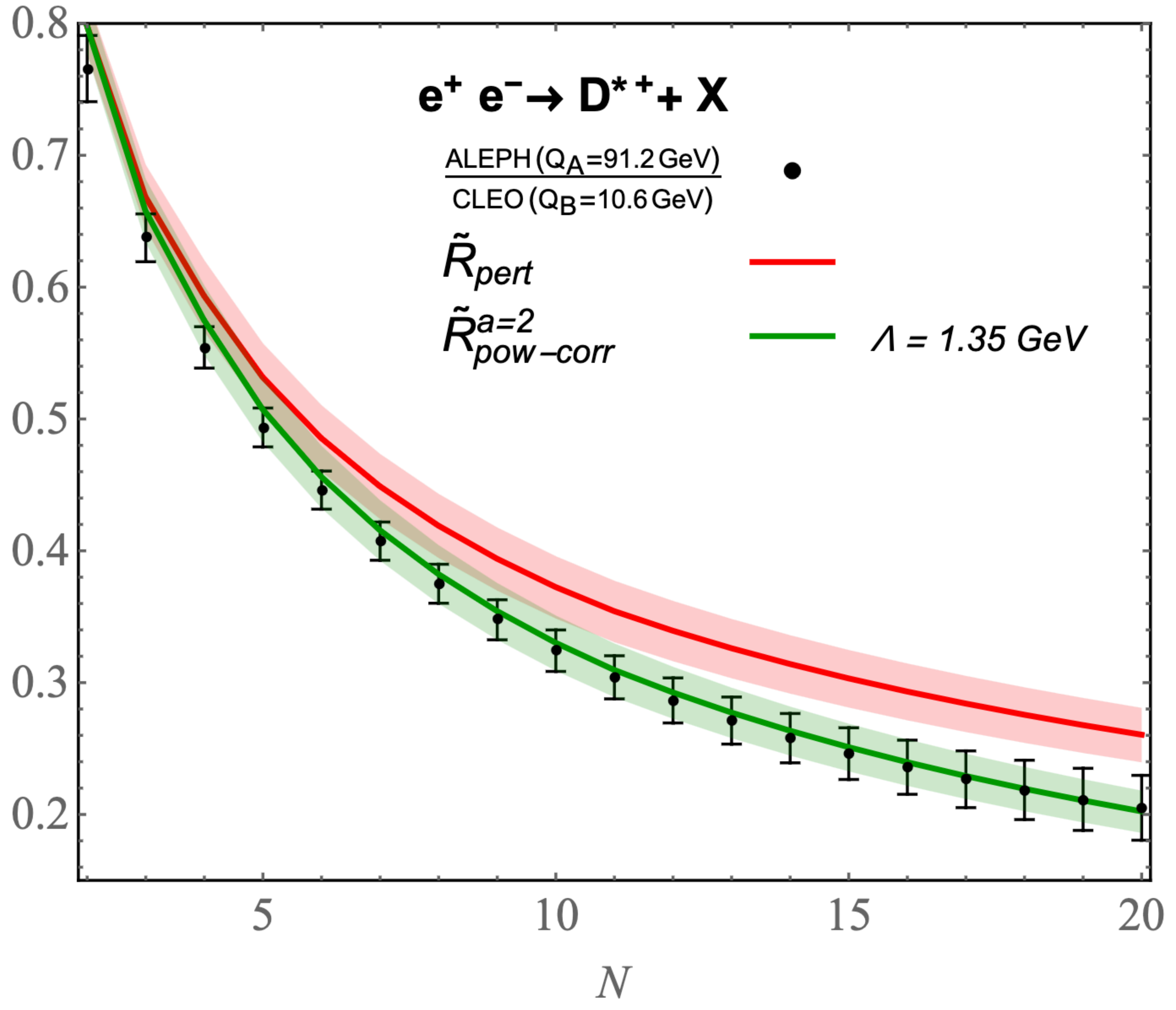
Power corrections and/or mass effects

NNLO+NNLL



$$R^{a}_{\text{pow}-\text{corr}} = R_{\text{per}} \times \frac{1 + \frac{\Lambda^a}{Q_A^a}\mathscr{C}(N)}{1 + \frac{\Lambda^a}{Q_B^a}\mathscr{C}(N)} \quad , \quad a = 2 \quad , \quad \mathscr{C}(N) = N - 1$$

K-factor at NLL



$$\tilde{R}_{\text{pert}} = R_{\text{pert}} \times K_{\text{th}}$$

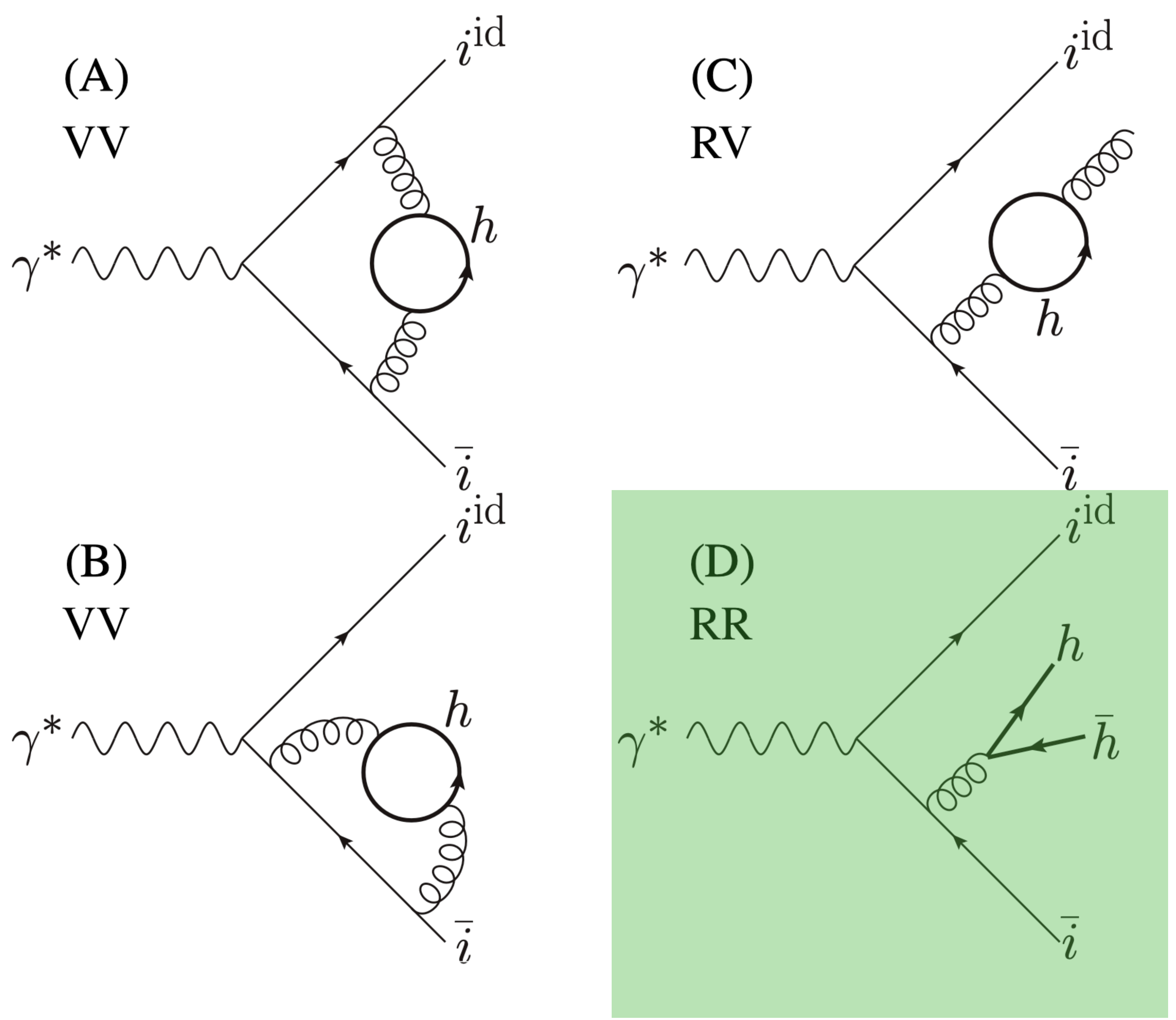
Matching conditions at NNLO

The light flavour matching condition

- NNLO matching conditions for light flavour (Mellin space)

$$D_i^{(n)}(N,\mu) = \left\{ 1 + \frac{1}{\sigma_{i\bar{i}}} \mathscr{M}_{(N,z)}\left[\delta^i_{D_i}(z)\right] \right\} D_i^{(n_L)}(N,\mu)$$

- Takes RR [Gehrmann Stagnitto '22] [Bonino et al. '24] RV and VV [Blümlein et al. '16] massive and massless corrections $\delta^i_{D_i} := \frac{\mathrm{d}\sigma^{Q_i}_{h\bar{h}i^{\mathrm{id}}\bar{i}}}{\mathrm{d}z} - \frac{\mathrm{d}\hat{\sigma}^{Q_i}_{h\bar{h}i^{\mathrm{id}}\bar{i}}}{\mathrm{d}z} + \frac{\mathrm{d}\sigma_{i,f}}{\mathrm{d}z} - \frac{\mathrm{d}\hat{\sigma}_{i,f}}{\mathrm{d}z} + \delta_{\alpha_s}$



$$\bar{\mathscr{B}}^{0,\mathrm{id}.i}_{ih\bar{h}\bar{i}}(z) = \frac{1}{216(z-1)}\Bigg[-18\left(z^2+1\right)\log^2\frac{m^2}{Q^2} + 6\left(-13z^2+6\left(z^2+1\right)\log(1-z)+6\left(z^2+1\right)\log(z)-16\right)\log\frac{m^2}{Q^2}$$
$$+36\left(z^2+1\right)\mathrm{Li}_2\left(\frac{z-1}{z}\right) + 6\pi^2\left(z^2+1\right) - 18\left(z^2+1\right)\log^2(1-z) + 6\log(1-z)\left(13z^2-6\left(z^2+1\right)\log(z)+16\right)$$
$$-z(115z+72)+12(z(8z+3)+8)\log(z)-172\Bigg] + \mathscr{O}\left(\frac{m}{Q}\right). \qquad \text{(C.25)}$$