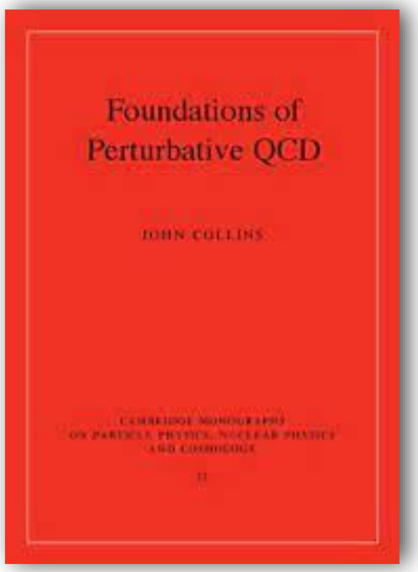
Andrea Simonelli

Hidden soft effects from TMD extractions

Old Dominion University

Jefferson Lab

TMD Factorization



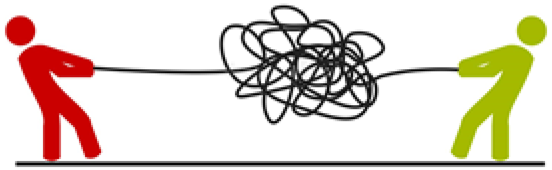
Standard processes:

- Drell-Yan $pp \to e^+e^- X$
- SIDIS $e^- p \to e^- h\, X$
- Double-Inclusive Annihilation (DIA) $e^+e^- \to h_1 h_2\, X$

TMD Factorization

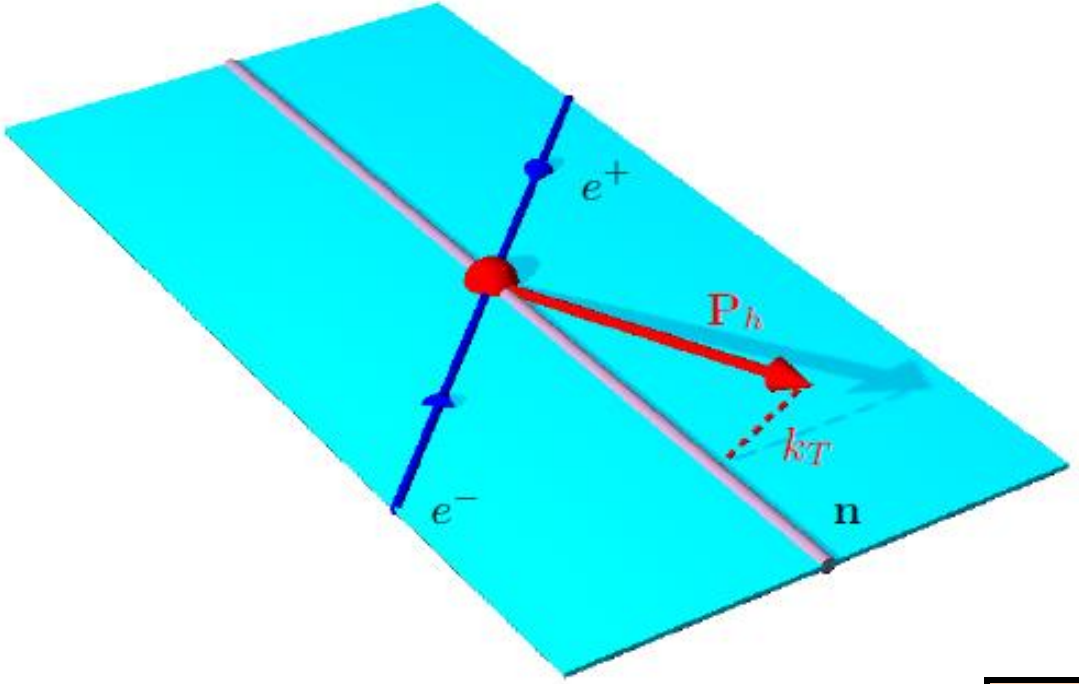
$$d\sigma = \mathcal{H}\int \frac{d^2\vec{b}_T}{(2\pi)^2}\mathcal{D}_A(z_A, b_T; y_A - y_n) \times \mathcal{D}_{\mathcal{B}}(z_B, b_T; y_n - y_B)$$

Always two TMDs that have to be extracted *simultaneously*



A process with a **single hadron** may offer a cleaner access to TMD (FFs)

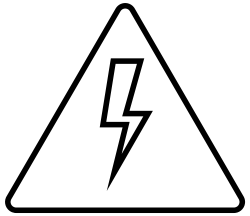
Single-Inclusive Annihilation (SIA)



The transverse momentum of the detected hadron is measured w.r.t. the thrust axis

$$z_h = \frac{E}{Q/2}, \quad T = \frac{\sum_i |\vec{P}_{(\mathrm{c.m.}),\, i} \cdot \hat{n}|}{\sum_i |\vec{P}_{(\mathrm{c.m.}),\, i}|}, \quad P_T \text{ w.r.t } \vec{n}$$

This process is **non**-standard!!



Soft Radiation Revealed

$$d\sigma = \mathcal{H} \int \frac{d^2\vec{b}_T}{(2\pi)^2} \mathcal{D}_A(z_A, b_T; y_A - y_n) \times \mathcal{D}_\mathcal{B}(z_B, b_T; y_n - y_B)$$

...it actually comes from the **re-arranging** of:

$$d\sigma = \mathcal{H} \int \frac{d^2\vec{b}_T}{(2\pi)^2} \mathcal{D}^\star_A(z_A, b_T; y_A - y_1) \times \mathcal{S}(b_T; y_1 - y_2) \times \mathcal{D}^\star_B(z_B, b_T; y_2 - y_B)$$

2-h Soft Factor correlating the two collinear groups

$$d\sigma = \mathcal{H} \int \frac{d^2\vec{b}_T}{(2\pi)^2} \mathcal{D}_A^\star(z_A, b_T; y_A - y_1) \times \mathcal{S}(b_T; y_1 - y_2) \times \mathcal{D}_B^\star(z_B, b_T; y_2 - y_B)$$

$$\frac{\mathcal{D}_A^{\text{uns.}}(z_A, b_T; y_A - (-\infty))}{\mathcal{S}(b_T; y_1 - (-\infty))} \times \mathcal{S}(b_T; y_1 - y_2) \times \frac{\mathcal{D}_B^{\text{uns.}}(z_B, b_T; \infty - y_B)}{\mathcal{S}(b_T; \infty - y_2)}$$

With:

$$\mathcal{D}^{\text{uns.}}(z, b_T; y_{\text{had}} - (-\infty)) = \frac{\text{Tr}_C}{N_C} \frac{\text{Tr}_D}{4} \sum_X \frac{1}{z} \int \frac{dx^-}{2\pi} e^{ik^+x^-}$$

$$\langle 0| \gamma^+ W_{(-))} (x/2 \to \infty) \, \psi_j (x/2) \, |P;\, X\rangle \langle P;\, X| \overline{\psi}_j (-x/2) \, W^\dagger_{(-))} (-x/2 \to \infty) \, |0\rangle$$

$$d\sigma = \mathcal{H} \int \frac{d^2\vec{b}_T}{(2\pi)^2} \mathcal{D}^\star_A(z_A, b_T; y_A - y_1) \times \mathcal{S}(b_T; y_1 - y_2) \times \mathcal{D}^\star_B(z_B, b_T; y_2 - y_B)$$

$$\frac{\mathcal{D}^{\text{uns.}}_A(z_A, b_T; y_A - (-\infty))}{\mathcal{S}(b_T; y_1 - (-\infty))} \times \mathcal{S}(b_T; y_1 - y_2) \times \frac{\mathcal{D}^{\text{uns.}}_B(z_B, b_T; \infty - y_B)}{\mathcal{S}(b_T; \infty - y_2)}$$

Same functional form!

TMD Parton Distribution and Fragmentation Functions with QCD Evolution

S.Mert Aybat (NIKHEF, Amsterdam and Vrije U., Amsterdam), Ted C. Rogers (Vrije U., Amsterdam) (Jan, 2011)

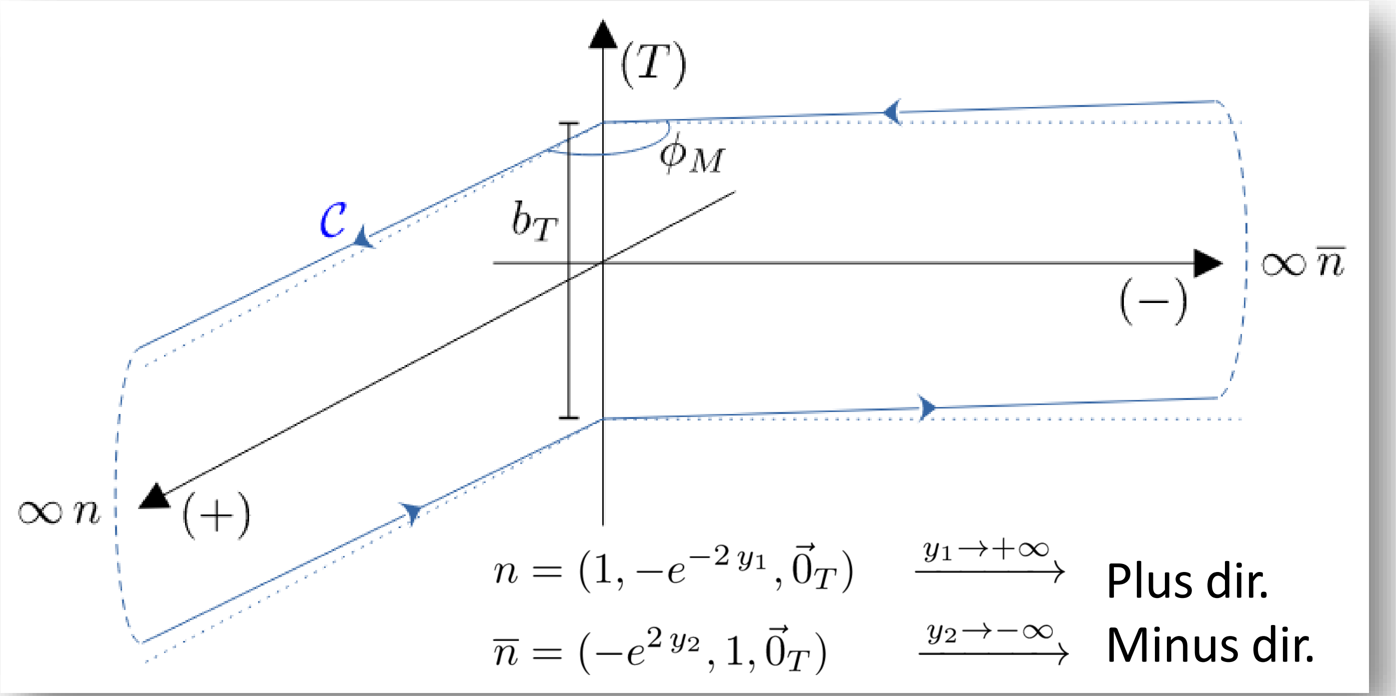
Published in: *Phys.Rev.D* 83 (2011) 114042 • e-Print: 1101.5057 [hep-ph]

pdf DOI cite claim reference search 394 citation

$$D(y_{\text{had}} - y_1) = D^{\text{uns.}}(y_{\text{had}} - (-\infty)) \sqrt{\frac{\mathcal{S}(\infty - y_1)}{\mathcal{S}(\infty - (-\infty))\, \mathcal{S}(y_1 - (-\infty))}}$$

- Light-cone limit $\begin{cases} y_1 \to \infty \\ y_2 \to -\infty \end{cases}$
- Evolution $\mathcal{S}(y_A, y_C) \propto \mathcal{S}(y_A, y_B)\mathcal{S}(y_B, y_C)$

Soft Factor



$$\cosh\phi_M = \frac{n \cdot \overline{n}}{\sqrt{n^2\,\overline{n}^2}} \equiv \cosh\left(y_1 - y_2\right)$$

$\phi_M = y_1 - y_2$ Minkowskian angle

$\phi_M \to \infty$ TMD Factorization

$\phi_M \to 0$ Bremsstrahlung function

$\phi_M \to i\pi$ quark-antiquark potential

$$\mathcal{S}(b_T, \phi_M) = \frac{\mathrm{Tr}}{N}\langle 0|W_{\mathcal{C}}(b_T, \phi_M)|0\rangle = \frac{\mathrm{Tr}}{N}\mathcal{P}\, Z_S\,\langle 0|e^{-ig_0 \oint_{\mathcal{C}} dx^\mu A_\mu^{(0),\,a}(x)\,t_a}|0\rangle$$

QCD cusp anomalous dimension: current status

Andrey Grozin (Novosibirsk, IYF) (Dec 10, 2022)

e-Print: 2212.05290 [hep-ph]

$$
\begin{aligned}
\Gamma = 4C_R\frac{\alpha_s}{4\pi}\Bigg\{&\varphi\coth\varphi - 1 + \frac{\alpha_s}{4\pi}\Bigg[C_A\Bigg[\frac{2}{3}\pi^2 - \frac{49}{9} + 2\varphi^2\\
&+ \coth\varphi\left(2\,\mathrm{Li}_2(e^{-2\varphi}) - 4\varphi\log(1-e^{-2\varphi}) - \frac{\pi^2}{3} - \frac{2}{3}\pi^2\varphi + \frac{67}{9}\varphi - 2\varphi^2 - \frac{2}{3}\varphi^3\right)\\
&+ \coth^2\varphi\left(2\,\mathrm{Li}_3(e^{-2\varphi}) + 2\varphi\,\mathrm{Li}_2(e^{-2\varphi}) - 2\zeta_3 + \frac{\pi^2}{3}\varphi + \frac{2}{3}\varphi^3\right)\Bigg]\\
&- \frac{20}{9}T_F n_f(\varphi\coth\varphi - 1)\Bigg] + \mathcal{O}(\alpha_s^2)\Bigg\}\\
= 4C_R\frac{\alpha_s}{4\pi}\Bigg\{&\varphi\coth\varphi - 1\\
&+ \frac{\alpha_s}{4\pi}\Bigg[C_A\Bigg[2\left(1+\frac{2}{3}\varphi^2\right) - \frac{1}{3}(\varphi\coth\varphi - 1)\left(2\pi^2 - \frac{67}{3} + 2\varphi^2\right)\\
&+ \coth\varphi(\varphi\coth\varphi + 1)\big(\mathrm{Li}_2(1-e^{2\varphi}) - \mathrm{Li}_2(1-e^{-2\varphi})\big)\\
&- 2\coth^2\varphi\big(\mathrm{Li}_3(1-e^{2\varphi}) + \mathrm{Li}_3(1-e^{-2\varphi})\big)\Bigg]\\
&- \frac{20}{9}T_F n_f(\varphi\coth\varphi - 1)\Bigg] + \mathcal{O}(\alpha_s^2)\Bigg\} \qquad (4.2)
\end{aligned}
$$

Non-Abelian Exponentiation Theorem

See works by E. Gardi, E. Leanen, L. Magnea, C. White etc...

Webs in multiparton scattering using the replica trick

Einan Gardi (Edinburgh U.), Eric Laenen (Amsterdam U. and Utrecht U. and NIKHEF, Amsterdam), Ge Stavenga (Fermilab), Chris D. White (Glasgow U. and Durham U., IPPP and Durham U.) (Aug, 2010)

Published in: *JHEP* 11 (2010) 155 • e-Print: 1008.0098 [hep-ph]

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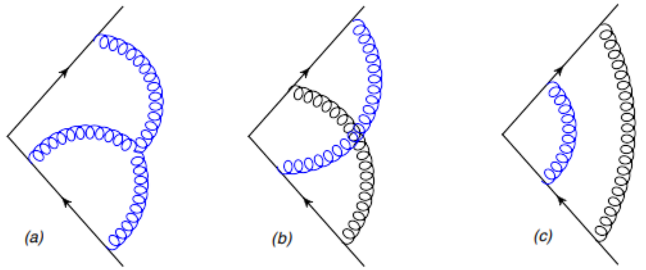
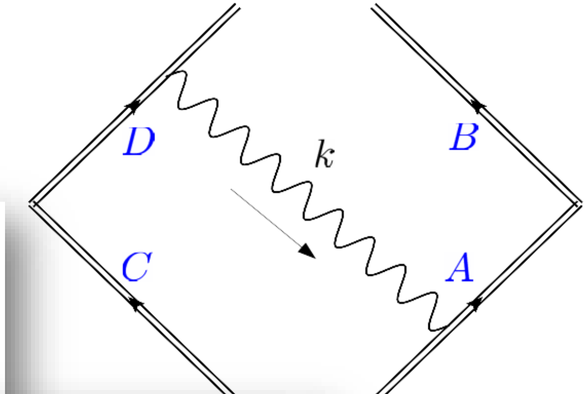


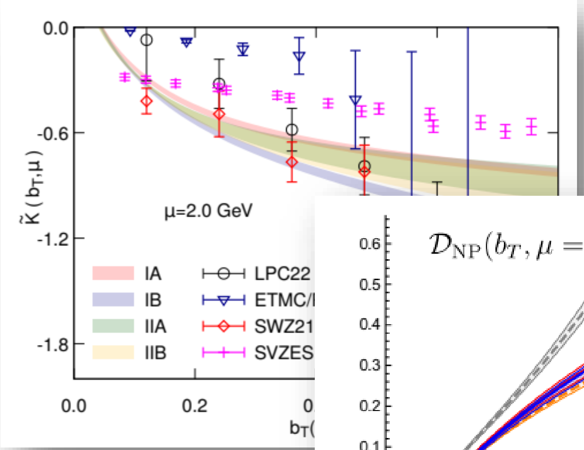
In the light-cone limit:

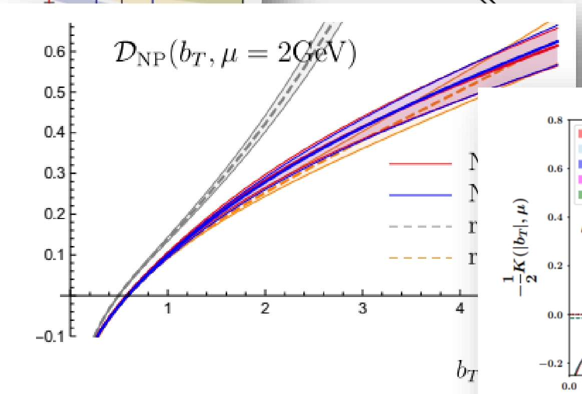
$$\mathcal{S}(b_T;\phi_M) = e^{\phi_M K + P}$$

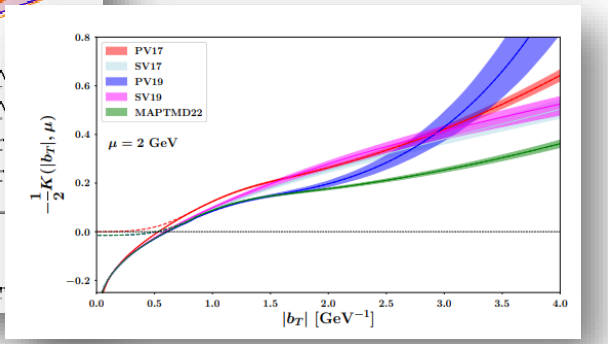
Studied a lot in TMD factorization...

Collins-Soper kernel

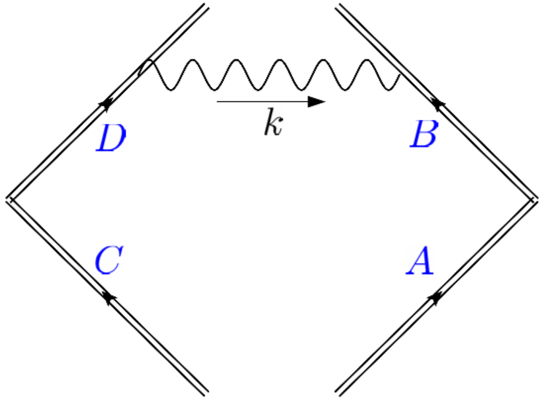








"Constant" P-term



The P-terms **disappear** in the standard TMD factorization...

$$\frac{\mathcal{D}_A^{\text{uns.}}(z_A, b_T; y_A - (-\infty))}{\mathcal{S}(b_T; y_1 - (-\infty))} \times \mathcal{S}(b_T; y_1 - y_2) \times \frac{\mathcal{D}_B^{\text{uns.}}(z_B, b_T; \infty - y_B)}{\mathcal{S}(b_T; \infty - y_2)}$$

$$= \frac{\mathcal{D}_A^{\text{uns.}}(z_A, b_T; y_A - (-\infty))}{e^{(y_1 - (-\infty))K + \frac{1}{2}\cancel{P}}} \times e^{(y_1 - y_2)K + \cancel{P}} \times \frac{\mathcal{D}_B^{\text{uns.}}(z_B, b_T; \infty - y_B)}{e^{(\infty - y_2)K + \frac{1}{2}\cancel{P}}}$$

...as well as in the standard TMD definition:

$$D(y_{\text{had}} - y_1) = D^{\text{uns.}}(y_{\text{had}} - (-\infty))\sqrt{\frac{\mathcal{S}(\infty - y_1)}{\mathcal{S}(\infty - (-\infty))\,\mathcal{S}(y_1 - (-\infty))}}$$

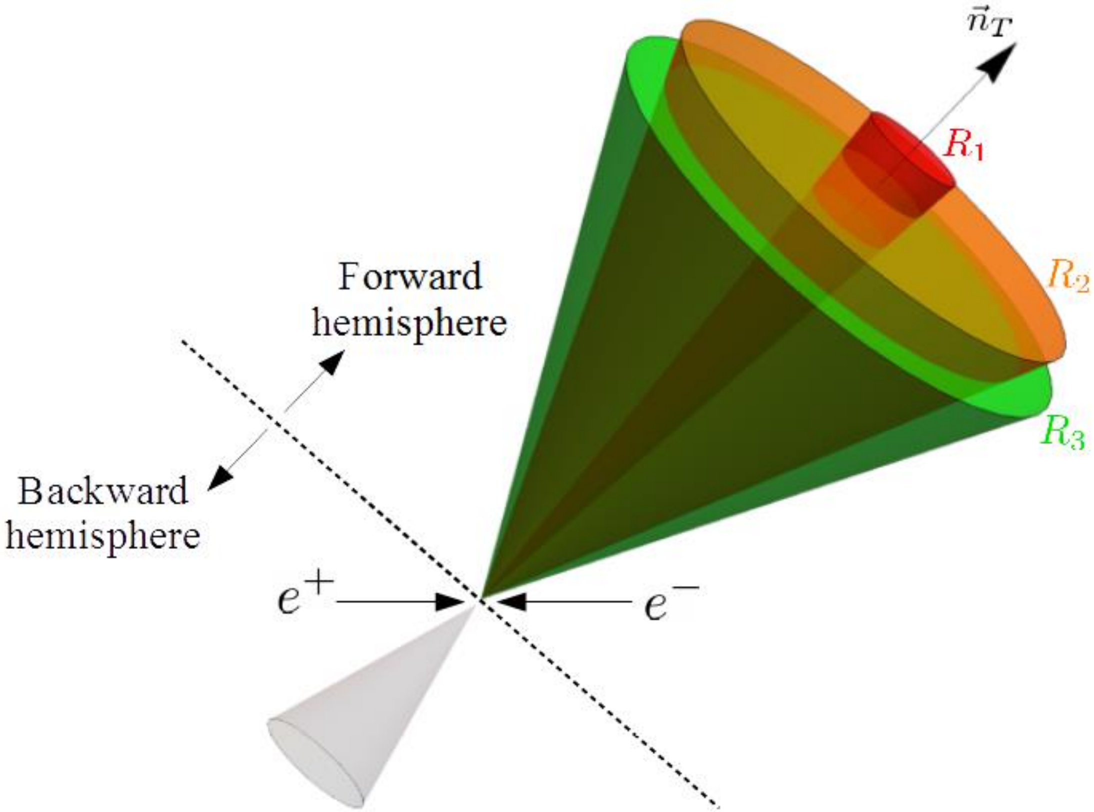
$$= D^{\text{uns.}}(y_{\text{had}} - (-\infty))\sqrt{\frac{e^{(\infty - y_1)K + \frac{1}{2}\cancel{P}}}{e^{(\infty - (-\infty))K}\, e^{(y_1 - (-\infty))K + \frac{1}{2}\cancel{P}}}}$$

The standard definition is optimal for standard TMD factorization.

We can forget about the existence of the P-term in the standard cases.

This is a naïve proof! Actual proof requires to modify also the unsubtracted TMDs

Soft Radiation in SIA (thrust)



The hadron is detected very close to the **axis** of the jet.
- ❑ Extremely small P_T
- ❑ Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

The hadron is detected in the **central region** of the jet.
- ❑ Most common scenario
- ❑ Majority of experimental data expected to fall into this case

The hadron is detected near the **boundary** of the jet.
- ❑ Moderately small P_T
- ❑ The hadron transverse momentum affects the topology of the final state directly

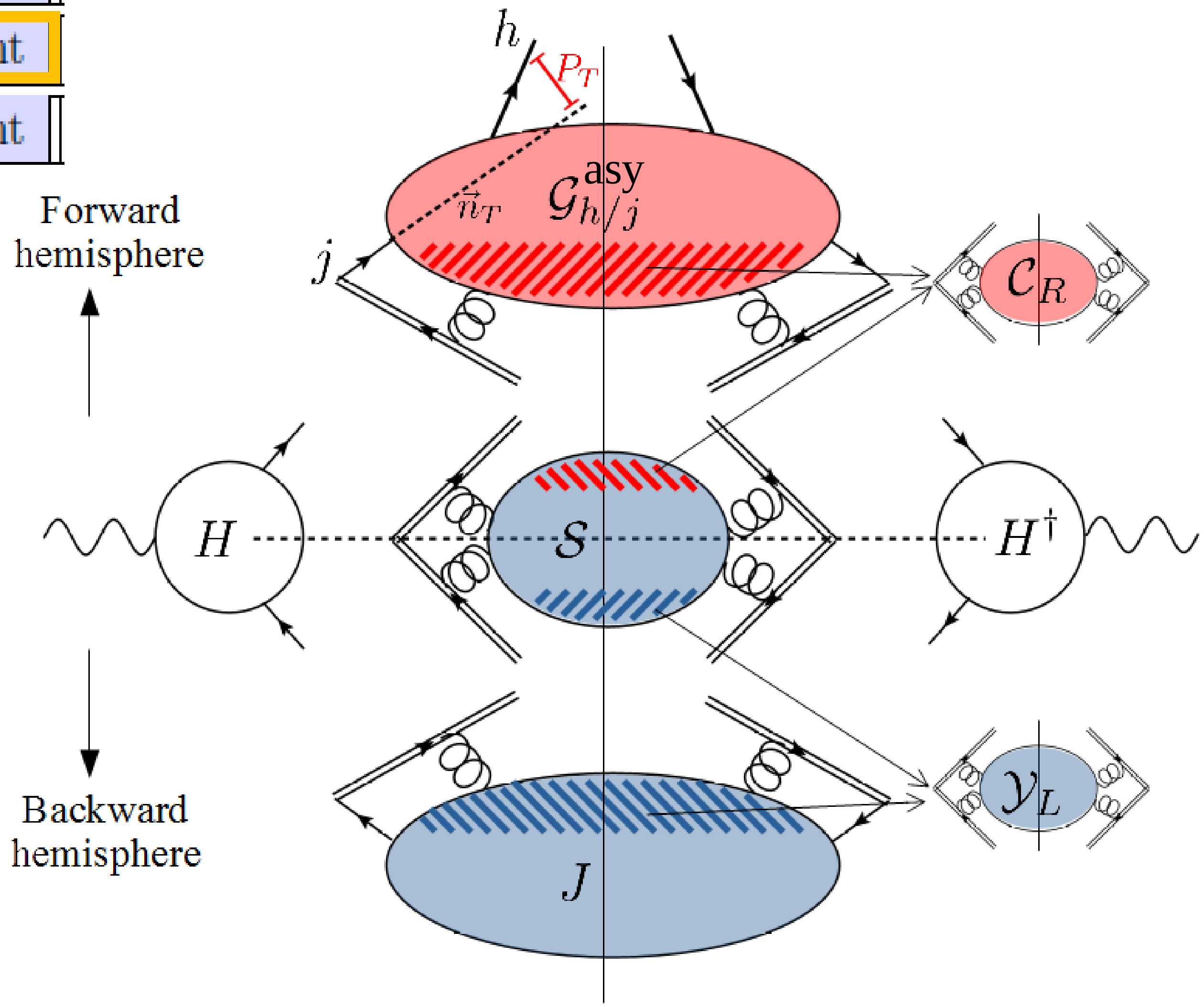
The three regions are uniquely determined by the specific role of **soft** and **soft-collinear** radiation:

	soft	soft-collinear	collinear
R_1	TMD-relevant	TMD-relevant	TMD-relevant
R_2	TMD-irrelevant	TMD-relevant	TMD-relevant
R_3	TMD-irrelevant	TMD-irrelevant	TMD-relevant

Red blobs are TMD-relevant

Blue blobs are TMD-irrelevant

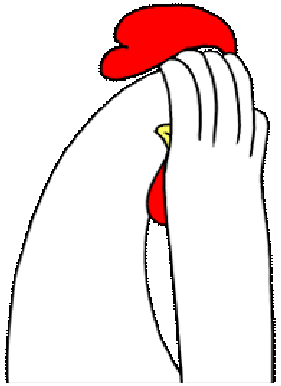
$$d\sigma_{R_2} \sim |H|^2 \, J \, \frac{\mathcal{S}}{\mathcal{Y}_L} \, \frac{\mathcal{G}^{\text{asy}}_{h/j}}{\mathcal{C}_R}$$



Different decompositions lead to different factorized cross sections

Factorization works in the same way for all the three regions, but it produces different results depending on the underlying kinematics

$$d\sigma = \mathcal{H} \int \frac{du}{2\pi i} e^{u\,\tau} \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, J(u;\infty - y_J)\, \frac{\widehat{\mathcal{S}}(u;y_1,y_2)}{\widehat{\mathcal{S}}(u;\infty,y_2)}\, D^{\star}(z, b_T; y_{\rm had} - y_1)$$



Not same functional form!
No magic this time...

$$\frac{\mathcal{D}^{\rm uns.}(z, b_T; y_{\rm had} - y_1)}{\mathcal{S}(b_T; y_1 - (-\infty))}$$

We are forced to use the definition* when we go beyond the standard processes

No extra soft contamination

- More universal
- Inclusion of P-term effects

Exploration of new physics!

P-term from definition comparison

From comparing **operators**:

$$\mathcal{D}^{\star}(z, b_T; y_{\text{had}} - y_1) = \mathcal{D}(z, b_T; y_{\text{had}} - y_1) e^{-\frac{1}{2} P(b_T)}$$

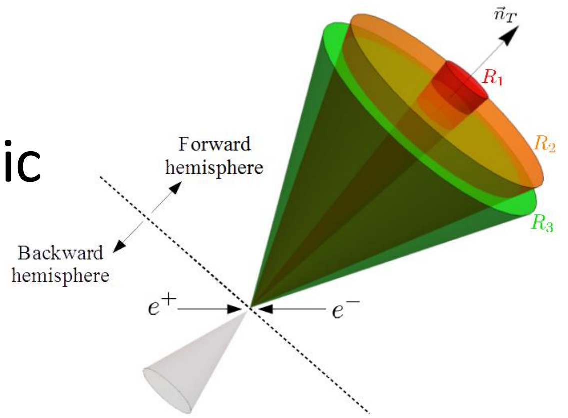
$$\frac{\partial \log D^{\star}(z, b_T; \mu, y_1)}{\partial y_1} = -K(b_T, \mu)$$

$$\frac{\partial \log D^{\star}(z, b_T; \mu, y_1)}{\partial \log \mu} = \gamma_d(a_S(\mu)) + \boxed{\frac{1}{2}\gamma_P(a_S(\mu))} - \gamma_K(a_S(\mu)) \log \frac{Q e^{-y_1}}{\mu}$$

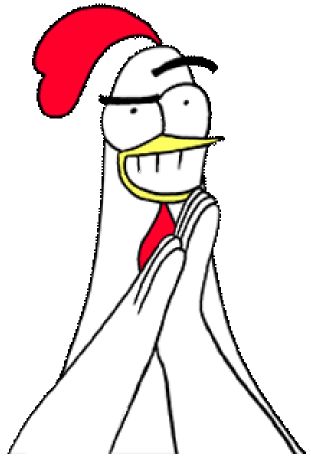
Extra (new!) term

P-term from extraction comparison

Central kinematic region of SIA



$$R = \frac{\text{TMD extraction from } \textbf{beyond} \text{ standard process}}{\text{TMD extraction from standard process}} = e^{-\frac{1}{2}P(b_T,\mu)}$$



Strategy for accessing the hidden soft effects

P-terms from SIA cross sections are:

$$d\sigma_{\mathrm{R}_2} \propto e^{I_R(u,y_1)-\frac{1}{2}P(b_T)} = e^{I_R|_{\mu_R}-\frac{1}{2}P|_{\mu_b}-\frac{1}{2}\int_{\mu_R}^{\mu_b}\frac{d\mu'}{\mu'}\gamma_P(a_S(\mu'))}$$

(right) counterpart of P in the thrust sector

$$\mu_R = \frac{Q}{u_E}e^{y_1}$$

Non-Perturbative behavior of the P-term

CSS approach: $$P(b_T,\mu) = P(a_S(\mu_b^\star)) - \int_{\mu_b^\star}^{\mu}\frac{d\mu'}{\mu'}\gamma_P(a_S(\mu')) - g_P(b_T)$$

However, **within the approximations** of

$$d\sigma_{\mathrm{R}_2} \propto e^{\frac{1}{2}g_P(b_T)}$$

Full treatment of the thrust distribution in single inclusive $e^+e^- \to h\,X$ processes #1

M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Old Dominion U. and Jefferson Lab) (Jun 5, 2023)

Published in: *JHEP* 09 (2023) 006 • e-Print: 2306.02937 [hep-ph]

pdf DOI cite claim reference search 1 citatior

$$d\sigma_{\mathrm{R}_2} \propto e^{\frac{1}{2} g_P(b_T)}$$

Effectively:

$$\mathcal{D}^{\star}_{h/j}(z, b_T, \mu, y_1) = \mathcal{D}\left(z,\, a_S(\mu_b^\star)\right) \times$$
$$\times\ \exp\left\{\frac{1}{2}\,\widetilde{K}_\star\left(a_S(\mu_b^\star)\right)\,\log\frac{\sqrt{\zeta}}{\mu_b^\star} + \int_{\mu_b^\star}^{\mu}\frac{d\mu'}{\mu'}\,\gamma_D\left(a_S(\mu'),\,\log\left(\frac{\sqrt{\zeta}}{\mu'}\right)\right)\right\}\times$$
$$\times\ M_D(z, b_T;\, j, h)\, e^{\frac{1}{2}g_P(b_T)}\exp\left\{-\frac{1}{2}\,g_K(b_T)\,\log\frac{\sqrt{\zeta}}{M_h}\right\}$$

This is the combination effectively extracted in

Full treatment of the thrust distribution in single inclusive $e^+e^- \to h\,X$ processes

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Making "predictions"

Full treatment of the thrust distribution in single inclusive $e^+e^- \to h X$ processes
M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Old Dominion U. and Jefferson Lab) (Jun 5, 2023)
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$$R = \frac{\quad}{\text{TMD FF model from standard process}} = e^{\frac{1}{2} g_P(b_T)}$$

→ Independent of z (collinear physics)

SV19

Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan scattering at small transverse momentum
Ignazio Scimemi (Madrid U.), Alexey Vladimirov (Regensburg U.) (Dec 13, 2019)
Published in: *JHEP* 06 (2020) 137 • e-Print: 1912.06532 [hep-ph]

MAP22

Unpolarized transverse momentum distributions from a global fit of Drell-Yan and semi-inclusive deep-inelastic scattering data
MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration • Alessandro Bacchetta (Pavia U. and INFN, Pavia) et al. (Jun 15, 2022)
Published in: *JHEP* 10 (2022) 127 • e-Print: 2206.07598 [hep-ph]

Beware! This would be an extraction of an extraction!

Comparison with SV19

Full treatment of the thrust distribution in single inclusive $e^+e^- \to h X$ processes #1

M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Old Dominion U. and Jefferson Lab) (Jun 5, 2023)

Published in: *JHEP* 09 (2023) 006 • e-Print: 2306.02937 [hep-ph]

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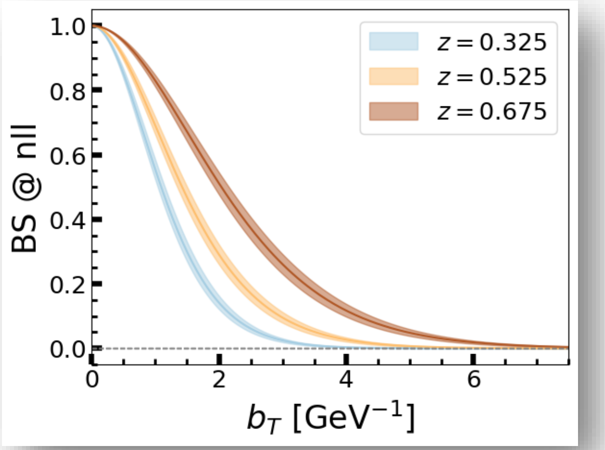
2 free parameters

$$M_D(z, b_T) = \frac{2}{\Gamma(p(z)-1)} \left(\frac{b_T\, m(z)}{2}\right)^{p(z)-1} K_{p(z)-1}\left(b_T\, m(z)\right)$$



Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan scattering at small transverse momentum #

Ignazio Scimemi (Madrid U.), Alexey Vladimirov (Regensburg U.) (Dec 13, 2019)

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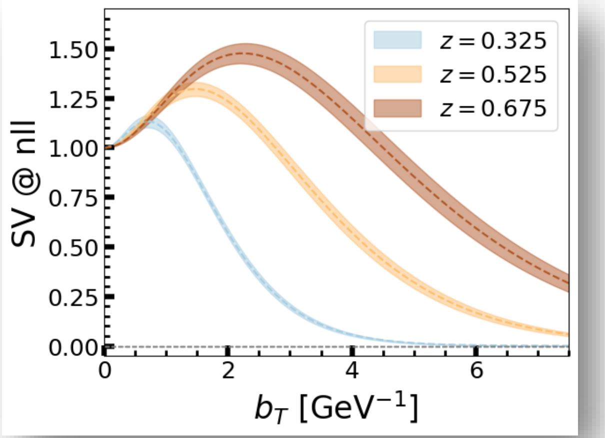
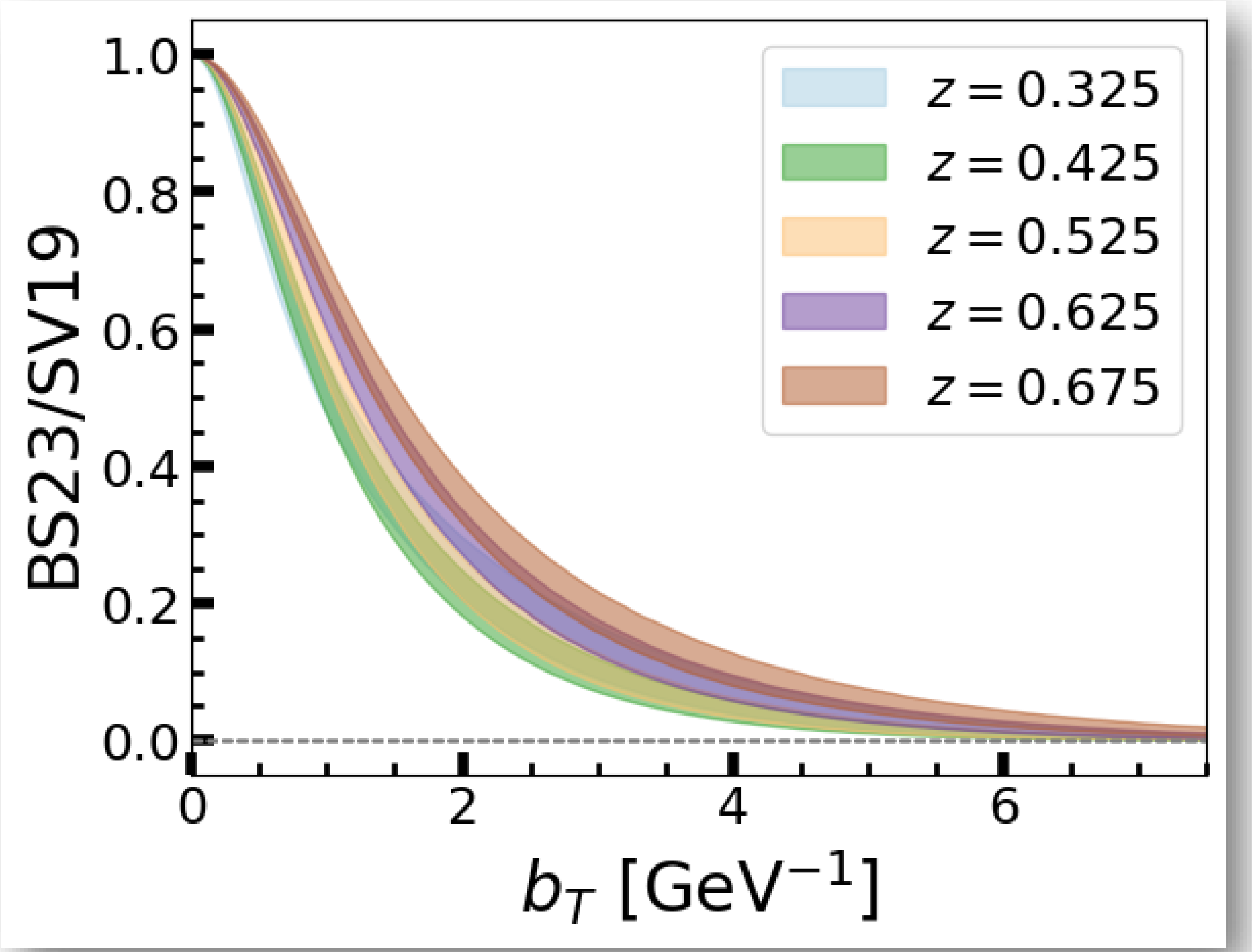
pdf DOI cite claim reference search 138 citation

4 free parameters

$$D_{NP}(x, b) = \exp\left(-\frac{\eta_1 z + \eta_2 (1-z)}{\sqrt{1+\eta_3 (\boldsymbol{b}/z)^2}} \frac{\boldsymbol{b}^2}{z^2}\right)\left(1 + \eta_4 \frac{\boldsymbol{b}^2}{z^2}\right)$$



Comparison with SV19



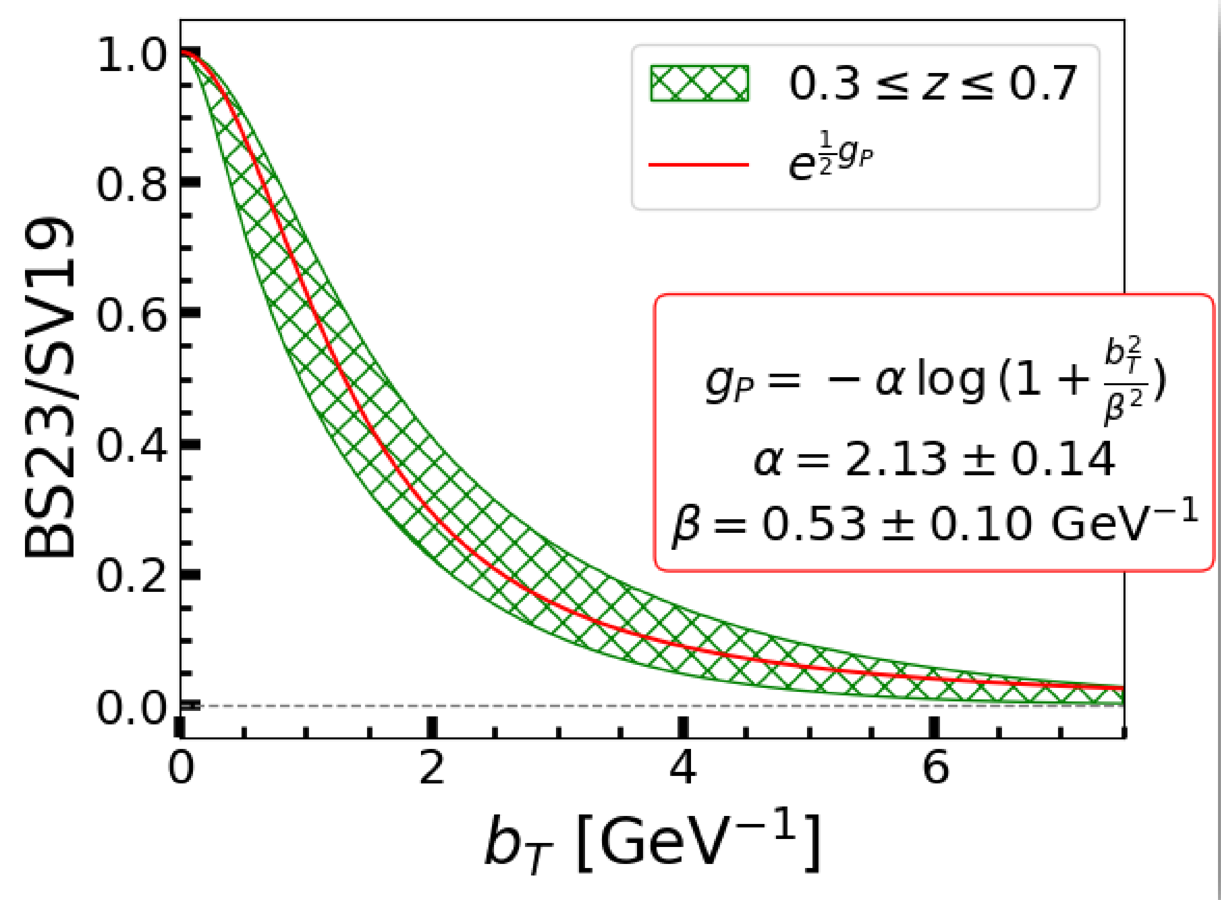
z-independence ✓

Keep in mind that:

- Very different functional forms at numerator and denominator
- Extractions from completely different data sets (BELLE vs SIDIS/DY)

Comparison with SV19

Spread of error bands can be used to constraint the "extraction" of g_P



Comparison with MAP22

Full treatment of the thrust distribution in single inclusive $e^+e^- \to h\,X$ processes #1

M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Old Dominion U. and Jefferson Lab) (Jun 5, 2023)

Published in: *JHEP* 09 (2023) 006 • e-Print: 2306.02937 [hep-ph]

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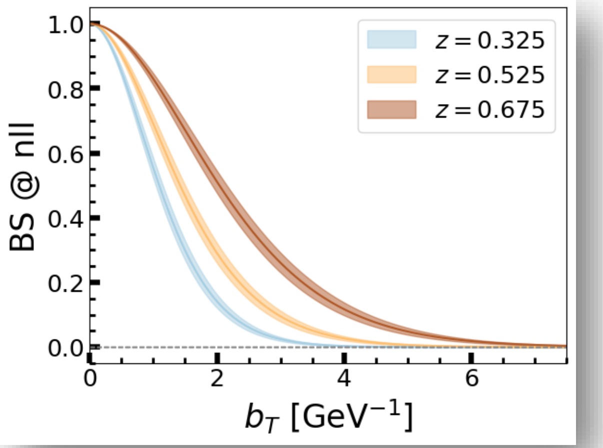
2 free parameters

$$M_D(z,b_T) = \frac{2}{\Gamma(p(z)-1)} \left(\frac{b_T\, m(z)}{2}\right)^{p(z)-1} K_{p(z)-1}\left(b_T\, m(z)\right)$$



Unpolarized transverse momentum distributions from a global fit of Drell-Yan and semi-inclusive deep-inelastic scattering data #4

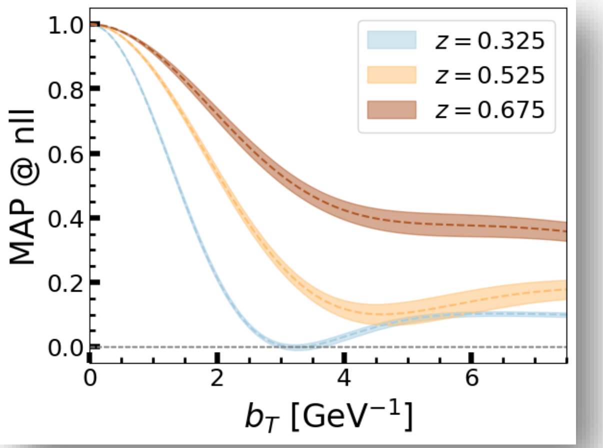
MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration • Alessandro Bacchetta (Pavia U. and INFN, Pavia) et al. (Jun 15, 2022)

Published in: *JHEP* 10 (2022) 127 • e-Print: 2206.07598 [hep-ph]

pdf DOI cite claim reference search 50 citations

9 free parameters

$$D_{1\,NP}(z, b_T^2; \zeta, Q_0) = \frac{g_3(z)\, e^{-g_3(z)\frac{b_T^2}{4z^2}} + \frac{\lambda_F}{z^2}\, g_{3B}^2(z) \left[1 - g_{3B}(z)\frac{b_T^2}{4z^2}\right] e^{-g_{3B}(z)\frac{b_T^2}{4z^2}}}{g_3(z) + \frac{\lambda_F}{z^2}\, g_{3B}^2(z)}$$

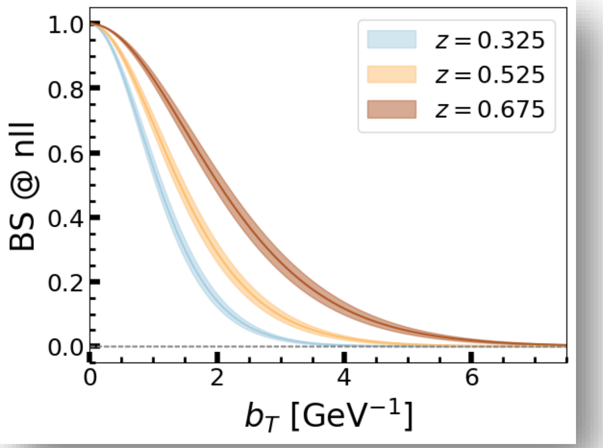


Comparison with MAP22

Full treatment of the thrust distribution in single inclusive $e^+e^- \to h\,X$ processes #1
M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Old Dominion U. and Jefferson Lab) (Jun 5, 2023)
Published in: *JHEP* 09 (2023) 006 • e-Print: 2306.02937 [hep-ph]
pdf DOI cite claim reference search 1 citation

2 free parameters

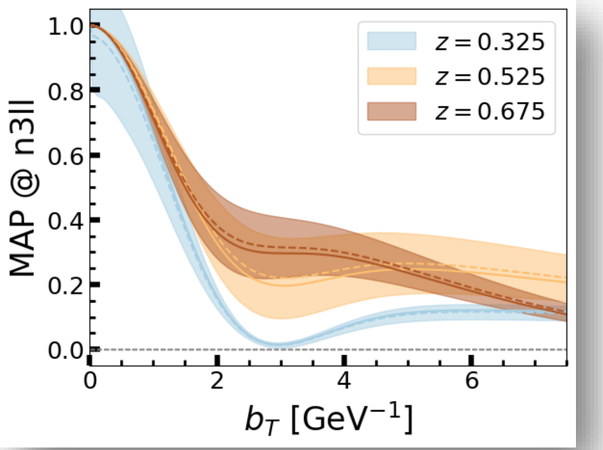
$$M_D(z,b_T) = \frac{2}{\Gamma(p(z)-1)} \left(\frac{b_T\, m(z)}{2}\right)^{p(z)-1} K_{p(z)-1}\left(b_T\, m(z)\right)$$



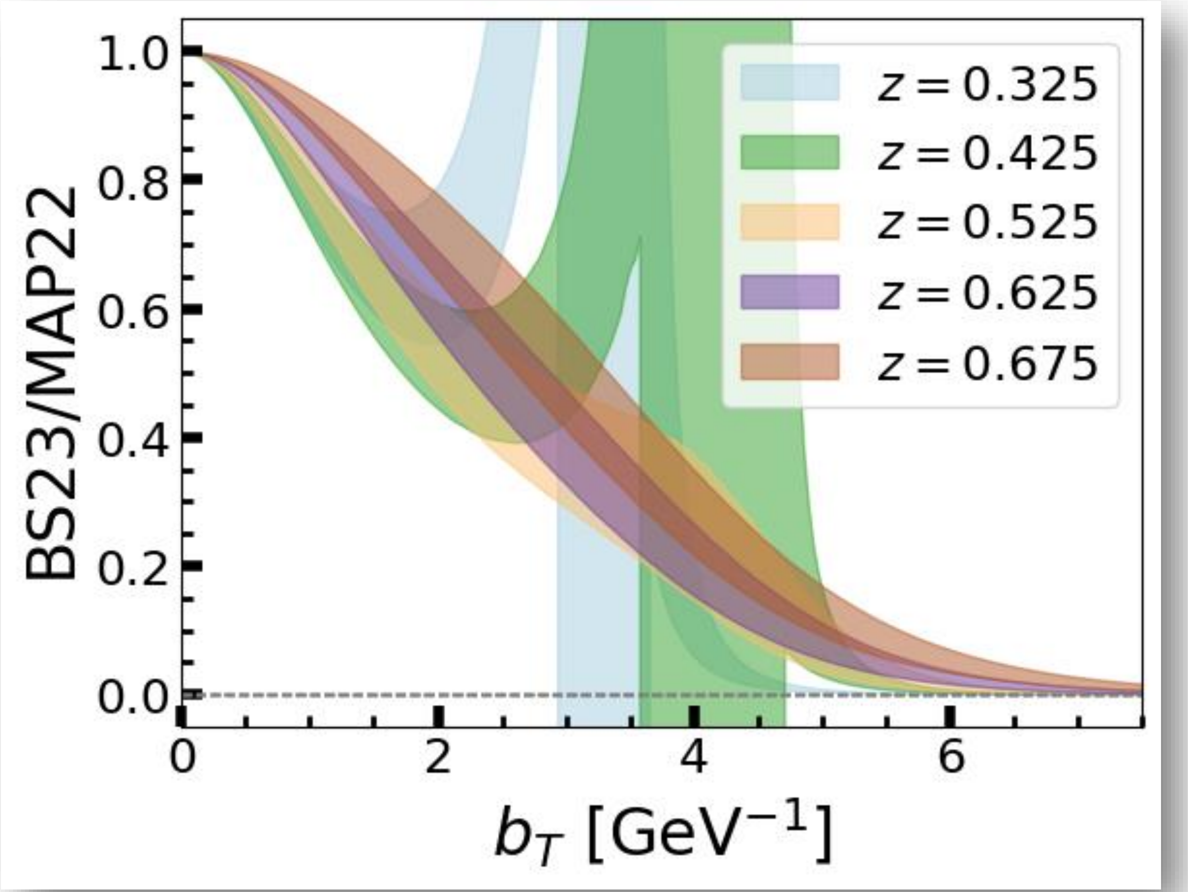
Unpolarized transverse momentum distributions from a global fit of Drell-Yan and semi-inclusive deep-inelastic scattering data #4
MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration • Alessandro Bacchetta (Pavia U. and INFN, Pavia) et al. (Jun 15, 2022)
Published in: *JHEP* 10 (2022) 127 • e-Print: 2206.07598 [hep-ph]
pdf DOI cite claim reference search 50 citations

9 free parameters

$$D_{1\,NP}(z,b_T^2;\zeta,Q_0) = \frac{g_3(z)\, e^{-g_3(z)\frac{b_T^2}{4z^2}} + \frac{\lambda_F}{z^2}\, g_{3B}^2(z)\left[1 - g_{3B}(z)\frac{b_T^2}{4z^2}\right] e^{-g_{3B}(z)\frac{b_T^2}{4z^2}}}{g_3(z) + \frac{\lambda_F}{z^2}\, g_{3B}^2(z)}$$

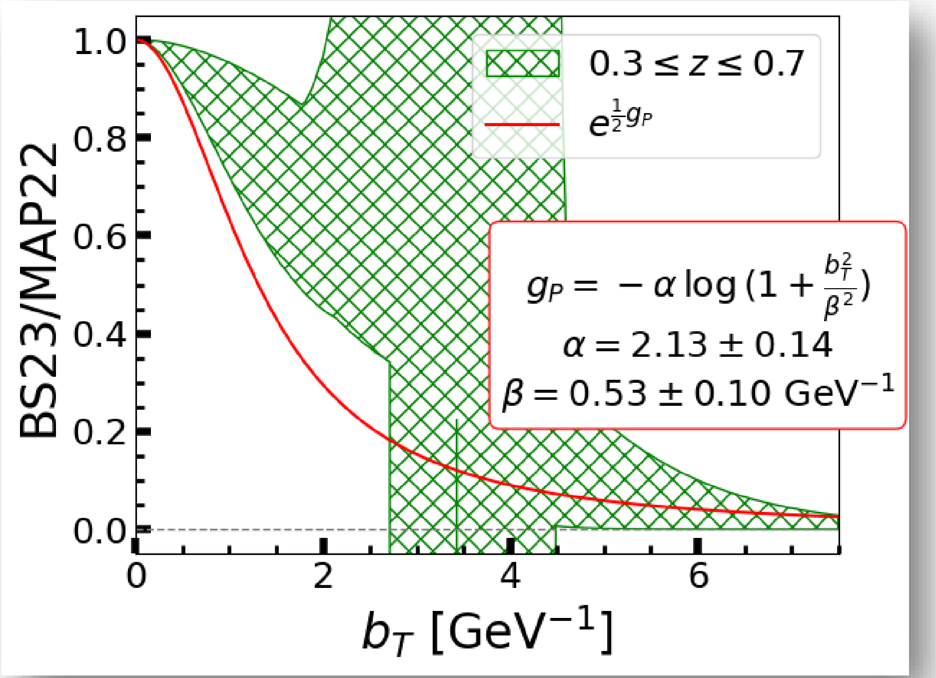


Comparison with MAP22



z-independence ?

Not really constrained



Thank You!