## J. OSVALDO GONZÁLEZ-HERNÁNDEZ

WHY HSO?
(AND OTHER QUESTIONS)

## 1) TOO MANY CHOICES IN PHENOMENOLOGY

MODEL<br>FOR<br>TMD PDF/FF

INTERPLAY BETWEEN PERTURBATIVE
AND
NONPERTURBATIVE "INGREDIENTS"

MODEL FOR COLLINS-SOPER
KERNEL (EVOLUTION)

CHOICE OF
COLLINEAR
FUNCTIONS

TREATMENT
OF
THEORETICAL ERRORS

## 1) TOO MANY CHOICES IN PHENOMENOLOGY

| $\begin{gathered} \text { MODEL } \\ \text { FOR } \\ \text { TMD PDF/FF } \end{gathered}$ | MODEL FOR COLLINS-SOPER KERNEL (EVOLUTION) |
| :---: | :---: |
|  |  |
| INTERPLAY BETWEEN PERTURBATIVE AND | CHOICE OF COLLINEAR |
| NONPERTURBATIVE "INGREDIENTS" | FUNCTIONS |
| TREATMENT | TREATMENT |
| EXPERIMENTAL ERRORS | THEORETICAL ERRORS |



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## CAN WE RELY ON RESULT?

## STATISTICAL METHODS HELP

THE LARGE-SAMPLE DISTRIBUTION OF THE LIKELIHOOD RATIO FOR TESTING COMPOSITE HYPOTHESES ${ }^{1}$

By S. S. Wilks

Theorem: If a population with a variate $x$ is distributed according to the probability function $f\left(x, \theta_{1}, \theta_{2} \ldots \theta_{h}\right)$, such that optimum estimates $\bar{\theta}_{i}$ of the $\theta_{i}$ exist which are distributed in large samples according to (3). then when the hypothesis $H$ is true that $\theta_{i}=\theta_{0 i}, i=m+1, m+2, \cdots h$, the distribution of $-2 \log \lambda$, where $\lambda$ is given by (2) is, except for terms of order $1 / \sqrt{n}$, distributed like $\chi^{2}$ with $h-m$ degrees of freedom.

$$
\begin{equation*}
\frac{\left|c_{i j}\right|^{\frac{1}{1}}}{(2 \pi)^{h / 2}} e^{-\frac{1}{i, i=1} \sum_{i j}^{h} c_{i z_{i} \dot{k}_{j}}}(1+\phi) d z_{1} \cdots d z_{h} \tag{3}
\end{equation*}
$$

where $z_{i}=\left(\bar{\theta}_{i}-\theta_{i}\right) \sqrt{n}, c_{i j}=-E\left(\frac{\partial^{2} \log f}{\partial \theta_{i} \partial \theta_{j}}\right), E$ denoting mathematical expectation, and $\phi$ is of order $1 / \sqrt{ } \bar{n}$ and $\left\|c_{i j}\right\|$ is positive definite. Denoting (3) by
${ }^{3}$ For conditions under which the $\bar{\theta}$ 's exist which are distributed according to (3), see J. L. Doob, Probability and Statistics, Trans. Amer. Math. Soc. Vol. 36, p. 759-775.

## BUT NOTE: ALL THIS WORKS ONLY IF MODEL IS CORRECT

EPJ Web of Conferences


## SIMPLE GENERALIZED PARTON MODEL (NO CS KERNEL, ETC.)

CAN WE RELY ON RESULT?

## CSS USUAL APPROACH

$$
\begin{aligned}
W\left(q_{\mathrm{T}}, Q\right) & =H\left(\mu_{Q} ; C_{2}\right) \int \frac{\mathrm{d}^{2} \boldsymbol{b}_{\mathrm{T}}}{(2 \pi)^{2}} e^{-i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \tilde{D}_{A}\left(z_{A}, \boldsymbol{b}_{*} ; \mu_{b_{*}}, \mu_{b_{*}}^{2}\right) \tilde{D}_{B}\left(z_{B}, \boldsymbol{b}_{*} ; \mu_{b_{*}}, \mu_{b_{*}}^{2}\right) \\
& \times \exp \left\{2 \int_{\mu_{b_{*}}}^{\mu_{Q}} \frac{d \mu^{\prime}}{\mu^{\prime}}\left[\gamma\left(\alpha_{s}\left(\mu^{\prime}\right) ; 1\right)-\ln \frac{Q}{\mu^{\prime}} \gamma_{K}\left(\alpha_{s}\left(\mu^{\prime}\right)\right)\right]+\ln \frac{Q^{2}}{\mu_{b_{*}}^{2}} \tilde{K}\left(b_{*} ; \mu_{b_{*}}\right)\right\} \\
& \times \exp \left\{-g_{A}\left(z_{A}, b_{\mathrm{T}}\right)-g_{B}\left(z_{B}, b_{\mathrm{T}}\right)-g_{K}\left(b_{\mathrm{T}}\right) \ln \left(\frac{Q^{2}}{Q_{0}^{2}}\right)\right\} .
\end{aligned}
$$

CONSIDER A SET OF MODELS THAT SATISFY
$g_{A}\left(b_{T}=0\right)=0$


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& \times \exp \left\{-g_{A}\left(z_{A}, b_{\mathrm{T}}\right)-g_{B}\left(z_{B}, b_{\mathrm{T}}\right)-g_{K}\left(b_{\mathrm{T}}\right) \ln \left(\frac{Q^{2}}{Q_{0}^{2}}\right)\right\} .
\end{aligned}
$$

CONSIDER A SET OF MODELS THAT SATISFY
$g_{A}\left(b_{T}=0\right)=0$


## CSS IN HSO

$$
\begin{aligned}
W^{(n)}\left(q_{\mathrm{T}}, Q\right) & \equiv H^{(n)}\left(\alpha_{s}\left(\mu_{Q}\right) ; C_{2}\right) \int \frac{\mathrm{d}^{2} \boldsymbol{b}_{\mathrm{T}}}{(2 \pi)^{2}} e^{-i \boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \underline{\tilde{D}}_{A}^{\left(n, d_{r}\right)}\left(z_{A}, \boldsymbol{b}_{\mathrm{T}} ; \mu_{Q_{0}}, Q_{0}^{2}\right) \underline{\tilde{D}}_{B}^{\left(n, d_{r}\right)}\left(z_{B}, \boldsymbol{b}_{\mathrm{T}} ; \mu_{Q_{0}}, Q_{0}^{2}\right) \\
& \times \exp \left\{\underline{\tilde{K}}^{(n)}\left(b_{\mathrm{T}} ; \mu_{Q_{0}}\right) \ln \left(\frac{Q^{2}}{Q_{0}^{2}}\right)+\int_{\mu_{Q_{0}}}^{\mu_{Q}} \frac{\mathrm{~d} \mu^{\prime}}{\mu^{\prime}}\left[2 \gamma^{(n)}\left(\alpha_{s}\left(\mu^{\prime}\right) ; 1\right)-\ln \frac{Q^{2}}{{\mu^{\prime 2}}^{2}} \gamma_{K}^{(n)}\left(\alpha_{s}\left(\mu^{\prime}\right)\right)\right]\right\}
\end{aligned}
$$

$\tilde{D}_{h / j}\left(z, \boldsymbol{b}_{\mathrm{T}} ; \mu_{Q_{0}}, Q_{0}^{2}\right)=\tilde{D}_{\mathrm{inpt}, h / j}\left(z, \boldsymbol{b}_{\mathrm{T}} ; \mu_{\bar{Q}_{0}} \bar{Q}_{0}^{2}\right) E\left(\bar{Q}_{0} / Q_{0}, b_{\mathrm{T}}\right)$. RG IMPROVEMENTS

$$
\begin{aligned}
D_{\text {inpt }, h / j}\left(z, z \boldsymbol{k}_{\mathrm{T}} ; \mu_{Q_{0}}, Q_{0}^{2}\right) & =\frac{1}{2 \pi z^{2}} \frac{1}{k_{\mathrm{T}}^{2}+m_{D_{h, j}}^{2}}\left[A_{h / j}^{D}\left(z ; \mu_{Q_{0}}\right)+B_{h / j}^{D}\left(z ; \mu_{Q_{0}}\right) \ln \frac{Q_{0}^{2}}{k_{\mathrm{T}}^{2}+m_{D_{h, j}}^{2}}\right] \\
& +\frac{1}{2 \pi z^{2}} \frac{1}{k_{\mathrm{T}}^{2}+m_{D_{h, g}}^{2}} A_{h / j}^{D, g}\left(z ; \mu_{Q_{0}}\right) \\
& +C_{h / j}^{D} D_{\text {core }, h / j}\left(z, z \boldsymbol{k}_{\mathrm{T}} ; Q_{0}^{2}\right), \quad \text { MODEL }
\end{aligned}
$$

## CSS IN HSO

$$
\begin{aligned}
& \text { ( } 0.1 \mathrm{GeV}<\mathrm{M}_{\mathrm{D} / \mathrm{z}=\mathrm{m}_{\mathrm{D}} / \mathrm{z}<0.3 \mathrm{GeV}} \\
& D_{\text {inpt }, h / j}\left(z, z \boldsymbol{k}_{\mathrm{T}} ; \mu_{Q_{0}}, Q_{0}^{2}\right)=\frac{1}{2 \pi z^{2}} \frac{1}{k_{\mathrm{T}}^{2}+m_{D_{h, j}}^{2}}\left[A_{h / j}^{D}\left(z ; \mu_{Q_{0}}\right)+B_{h / j}^{D}\left(z ; \mu_{Q_{0}}\right) \ln \frac{Q_{0}^{2}}{k_{\mathrm{T}}^{2}+m_{D_{h, j}}^{2}}\right] \\
& +\frac{1}{2 \pi z^{2}} \frac{1}{k_{\mathrm{T}}^{2}+m_{D_{h, g}}^{2}} A_{h / j}^{D, g}\left(z ; \mu_{Q_{0}}\right) \\
& +C_{h / j}^{D} D_{\text {core }, h / j}\left(z, z \boldsymbol{k}_{\mathrm{T}} ; Q_{0}^{2}\right),
\end{aligned}
$$

## 2) TMDS HAVE CONCRETE DEFINITIONS IN QCD

IMPROVEMENT IN DESCRIBING/PREDICTING DATA BY IMPOSING THE CORRECT paCD TAIL
(TO BE TESTED)



## 2) TMDS HAVE CONCRETE DEFINITIONS IN QCD

## ANOTHER SCENARIO:

- EXTRACTION "A" WITH CORRECT PQCD TAIL.
- EXTRACTION "B" WITH INCONSISTENT LARGE-K BEHAVIOR


## BUT OTHERWISE EQUIVALENT

(E.G. SAME $\chi^{2}$ )



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## BUT OTHERWISE EQUIVALENT

(E.G. SAME $\chi^{2}$ )
"A" IS A STRONGER CANDIDATE FOR THE TRUE BEHAVIOR OF TMDS



## Q: DO WE TRUST OUR FRAMEWORK?

MODEL
FOR TMD PDF/FF
INTERPLAY BETWEEN PERTURBATIVE AND
NONPERTURBATIVE "INGREDIENTS"

TREATMENT<br>OF<br>EXPERIMENTAL ERRORS

D

A
TA

MODEL FOR COLLINS-SOPER
KERNEL (EVOLUTION)

CHOICE OF COLLINEAR FUNCTIONS

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THEORETICAL ERRORS

## STATISTICAL THEOREMS + ADVANCED TOOLS/FRAMEWORKS

## Q: DO WE TRUST OUR FRAMEWORK?



## Q: POSTDICTIONS = PREDICTIONS?

## Q: POSTDICTIONS = PREDICTIONS?

## POSTDICTIONS = PREDICTIONS

## FITS $=$ PREDICTIONS

## $0<$ <br> $\begin{gathered}\text { INTERPOLATING } \\ \text { LINES }\end{gathered}<\underset{\text { SIMPLE }}{\text { FIT }}<\stackrel{\text { ROBUST }}{\text { MODEL FIT }} \cdots$ <br> THEORY <br> < COMPLIANT FIT < POSTDICTION < PREDICTION

## Q: THE FUTURE EIC DATA WILL

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## The case for an EIC Theory Alliance: Theoretical Challenges of the EIC

Guiding and understanding the future experimental measurements will require a laborious and meticulous analysis of the data, new approaches and new methods in the theoretical treatment and in the phenomenological extraction of TMDs. The EIC Theory Alliance will provide an essential framework for guiding and organizing the broad theoretical

- Theoretical and phenomenological exploration of QCD factorization theorems and expanding the region of their applicability, for instance by inclusion of power corrections in $q_{T} / Q$. A crucial ingredient will be matching collinear factorization ( $\Lambda_{\mathrm{QCD}} \ll q_{T} \sim Q$ ) and TMD factorization ( $\Lambda_{\mathrm{QCD}} \lesssim q_{T} \ll Q$ ) in the overlap region $\Lambda_{\mathrm{QCD}} \ll q_{T} \ll Q$ in a stable and efficient way. Such a matching is needed for our ability to describe the measured quantities, differential in transverse momentum, in the widest possible region of phase space. In turn, this will lead to a much more reliable understanding of both collinear and TMD related functions and uncertainties in their determinations.


## Q: ARE WE DOING PHENO USING THE HSO?

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Plot from (MAP collaboration):
JHEP 10 (2022) 127

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## Q: ARE WE DOING PHENO USING THE HSO?

E288: test. $\mathrm{E}=400 \mathrm{GeV}$


## FINAL (PERSONAL) REMARK

## ( HOPEFULLY, EVENTUALLY)

A: THE FUTURE EIC DATA WAS SUCCESSFULLY PREDICTED

