A hadron structure oriented (HSO) approach to TMD phenomenology

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Based on:

- J.O. Gonzalez, TCR, N. Sato, Phys.Rev.D 106 (2022) 3, 034002
- J.O. Gonzalez, T. Rainaldi, TCR (2023), Phys.Rev.D 107 (2023) 9, 094029

&

- Work in progress with F. Aslan, M. Boglione, J.O. Gonzalez, T. Rainaldi, A. Simonelli

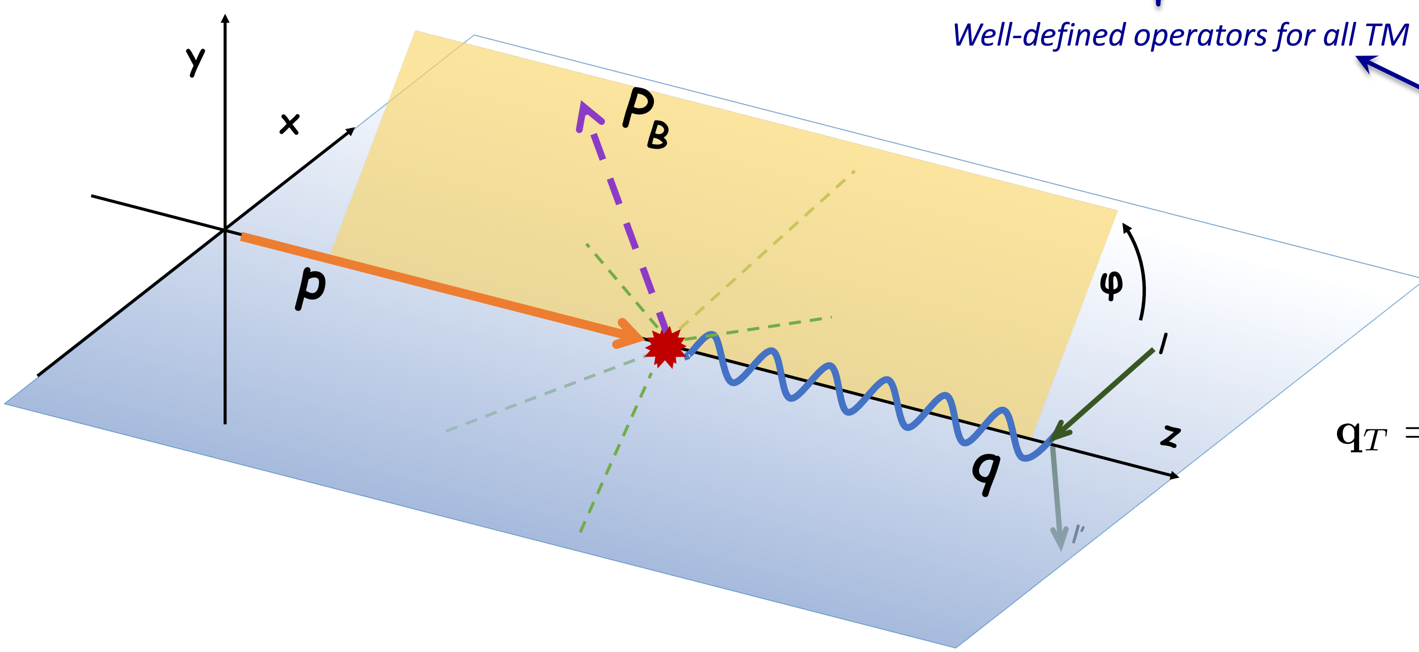
GFI 2nd miniworkshop
Oasi di Cavoretto, Torino, September 20, 2023

TMD factorization & SIDIS $\left(for\ \frac{q_T}{Q} \ll 1\right)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T}\underbrace{f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2)D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)}_{\text{Well-defined operators for all TM}}\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T})$$

+ power suppressed

The "W-term"



$$\mathbf{q}_T = -\frac{\mathbf{P}_{BT}}{z}$$

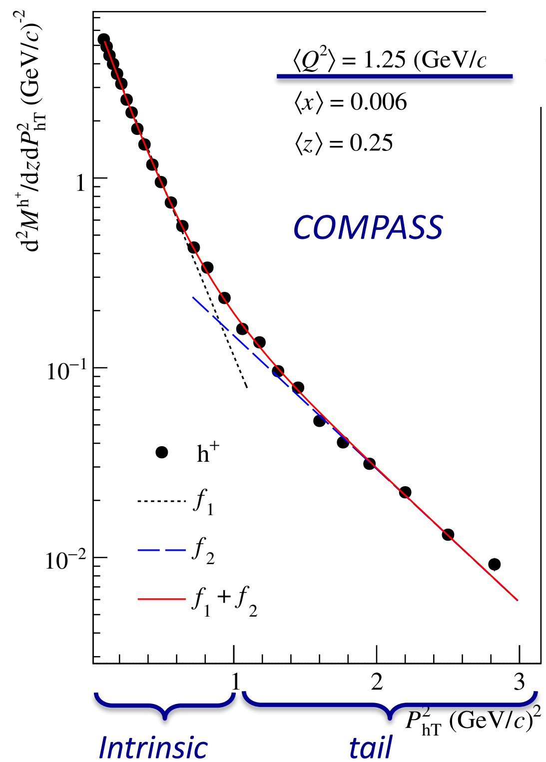
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\int \frac{\mathrm{d}^2\mathbf{b}_T}{(2\pi)^2}e^{-i\mathbf{b}_T\cdot\mathbf{q}_T}\tilde{f}_{j/p}(x,\mathbf{b}_T;\mu_Q,Q^2)\tilde{D}_{h/j}(z,\mathbf{b}_T;\mu_Q,Q^2)$$

+ power suppressed

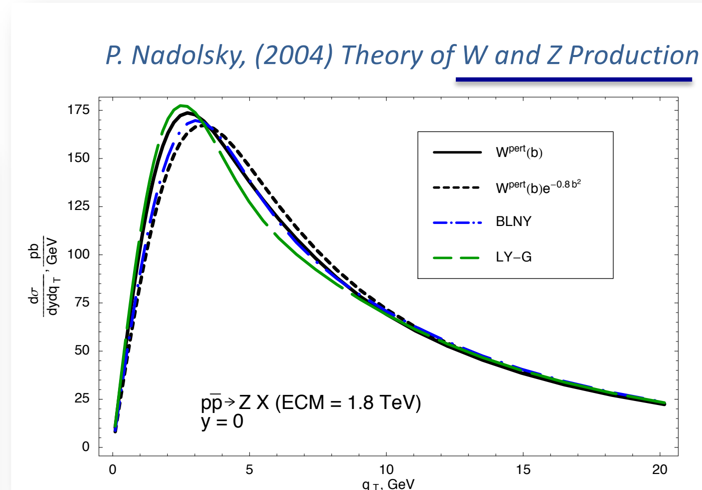
Nonperturbative structures in pheno

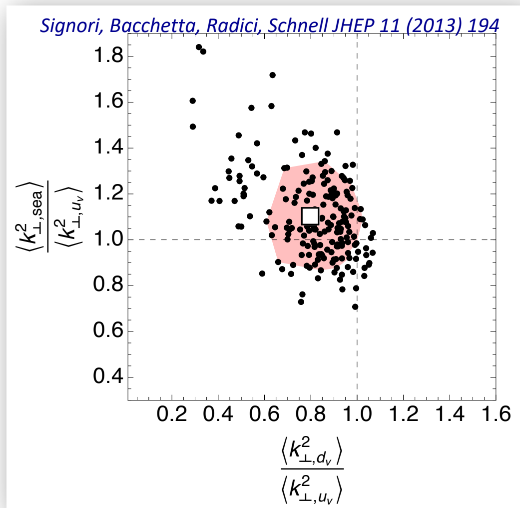
PHYSICAL REVIEW D **97,** 032006 (2018)

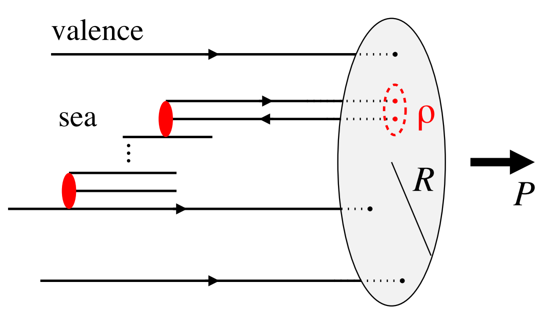
Transverse-momentum-dependent multiplicities of charged hadrons in muon-deuteron deep inelastic scattering

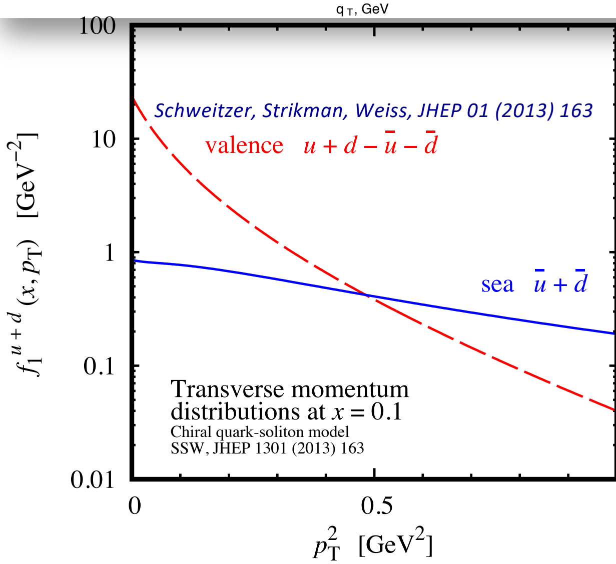


"While significant effort has been put into the study of W(b) ***at large b*** *[36, 42, 43, 44], none … adequately describe the observed Z boson distribution without introducing free parameters."*





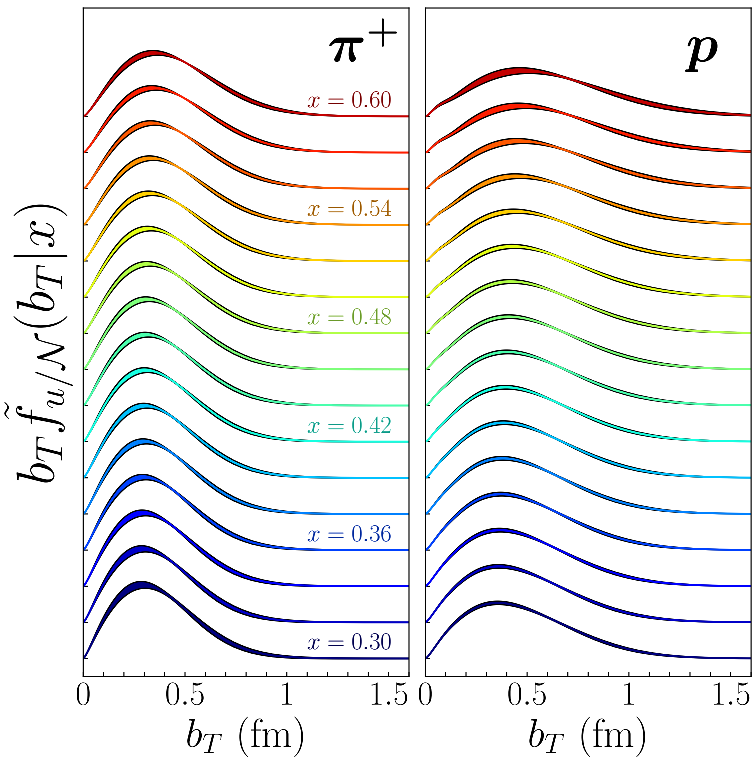




"… the two exponential functions in our parametrizations F_1 can be attributed to two completely different underlying physics mechanisms that overlap in the region $P_{hT} \simeq 1\ \mathrm{GeV}^2$."

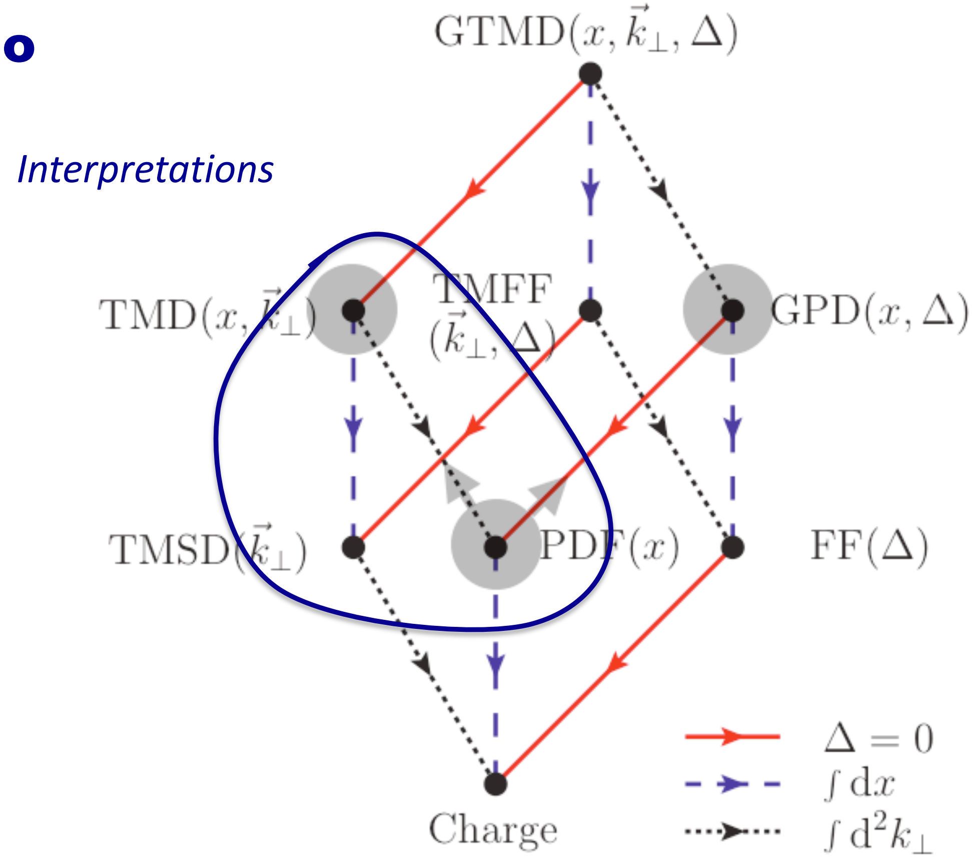
Nonperturbative structures in pheno

JAM: Barry, et al (2302.01192)



??

"Importantly, we have checked that the differences between the proton and pion ⟨bT |x⟩ are completely due to the nonperturbative TMD structure, independent of the collinear PDFs." ??



These integrals are often divergent

The conventional organization

1) Solve evolution equations to relate overall SIDIS hard scale ($\mu_Q = Q$) to input scale ($\mu_{Q_0} = Q_0$)

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\mathrm{d} \ln \mu} = \gamma(\alpha_s(\mu); \zeta/\mu^2)$$

$$\frac{\mathrm{d}\tilde{K}(b_T; \mu)}{\mathrm{d} \ln \mu} = -\gamma_K(\alpha_s(\mu))$$

The conventional organization

1) Solve evolution equations to relate overall SIDIS hard scale ($\mu_Q = Q$) to input scale ($\mu_{Q_0} = Q_0$)
2) How to use small $b_T \ll 1/\Lambda_{QCD}$ collinear factorization?
   - Partition small ($b_T < b_{\max}$) & large ($b_T > b_{\max}$) regions with a b_*
   - Define hard scale $\mu_{b_*} \sim 1/b_*$
3) Evolve again to relate Q_0 to μ_{b_*}

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\mathrm{d} \ln \mu} = \gamma(\alpha_s(\mu); \zeta/\mu^2)$$

$$\frac{\mathrm{d}\tilde{K}(b_T; \mu)}{\mathrm{d} \ln \mu} = -\gamma_K(\alpha_s(\mu))$$

$$\tilde{f}_{j/p}(x, b_T; \mu, \zeta) = \int_x^1 \frac{\mathrm{d}\xi}{\xi} \tilde{C}_{j/k}(x/\xi, b_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu) + O(b_T \Lambda_{\mathrm{QCD}})$$

Or

$$f_{j/p}(x, k_T; \mu, \zeta) = \frac{1}{k_T^2} \left[\int_x^1 \frac{\mathrm{d}\xi}{\xi} C_{j/k}(x/\xi, k_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}}{k_T}\right) \right]$$

$$\tilde{K}(b_T; \mu) = \tilde{K}_{\mathrm{pert}}(b_T \mu, \alpha_s(\mu)) + O(b_T \Lambda_{\mathrm{QCD}})$$

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4) **Place remaining NP parts in an exponent:**

$$\tilde{f}_{j/p}(x, \boldsymbol{b}_\mathrm{T}; \mu_Q, \zeta) = \tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_Q, \zeta) \underbrace{\frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_\mathrm{T}; \mu_Q, \zeta)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_Q, \zeta)}}_{??}$$

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\mathrm{d} \ln \mu} = \gamma(\alpha_s(\mu); \zeta/\mu^2)$$

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Or

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$$\tilde{K}(b_T; \mu) = \tilde{K}_\mathrm{pert}(b_T \mu, \alpha_s(\mu)) + O(b_T \Lambda_\mathrm{QCD})$$

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4) **Place remaining NP parts in an exponent:**

$$-g_{j/p}(x, b_{\mathrm{T}}) \equiv \ln \left(\frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)} \right)$$

$$g_K(b_{\mathrm{T}}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_{\mathrm{T}}; \mu)$$

5) **Perform small-$\boldsymbol{b_T}$ expansions & drop $\boldsymbol{O(\Lambda_{QCD}\, b_{max})}$ errors**
6) **Ansatz for g-functions**

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

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<u>*Or*</u>

$$f_{j/p}(x, k_T; \mu, \zeta) = \frac{1}{k_T^2} \left[\int_x^1 \frac{\mathrm{d}\xi}{\xi} C_{j/k}(x/\xi, k_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}}{k_T}\right) \right]$$

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$$\int \mathrm{d}^2\mathbf{k}_{1T} \mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x, \mathbf{k}_{1T}; \mu_Q, Q^2) D_{h/j}(z, z\mathbf{k}_{2T}; \mu_Q, Q^2) \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$

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$$\Downarrow$$

$$\int \frac{d^2\boldsymbol{b}_{\rm T}}{(2\pi)^2} e^{-i\boldsymbol{q}_{\rm T}\cdot\boldsymbol{b}_{\rm T}} \tilde{f}^{\rm OPE}_{j/p}(x_{\rm bj}, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2) \tilde{D}^{\rm OPE}_{h/j}(z_{\rm h}, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2)$$
$$\times \exp\left\{ 2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu')) \right] + \ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*; \mu_{b_*}) \right\}$$
$$\times \exp\left\{ -g_{j/p}(x_{\rm bj}, b_{\rm T}) - g_{h/j}(z_{\rm h}, b_{\rm T}) - g_K(b_{\rm T}) \ln\left(\frac{Q^2}{Q_0^2}\right) \right\} + O(b_{\max}\Lambda_{\rm QCD})$$

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$$\int \frac{d^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{-i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \tilde{f}_{j/p}^{\mathrm{OPE}}(x_{\mathrm{bj}}, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2) \tilde{D}_{h/j}^{\mathrm{OPE}}(z_{\mathrm{h}}, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2)$$
$$\times \exp\left\{ 2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'} \gamma_K(\alpha_s(\mu')) \right] + \ln\frac{Q^2}{\mu_{b_*}^2} \tilde{K}(b_*; \mu_{b_*}) \right\}$$
$$\times \exp\left\{ -g_{j/p}(x_{\mathrm{bj}}, b_{\mathrm{T}}) - g_{h/j}(z_{\mathrm{h}}, b_{\mathrm{T}}) - g_K(b_{\mathrm{T}}) \ln\left(\frac{Q^2}{Q_0^2}\right) \right\} + O\left(b_{\max}\Lambda_{\mathrm{QCD}}\right)$$

$$\tilde{f}_{j/p}(x, b_T; \mu, \zeta) = \underbrace{\int_x^1 \frac{d\xi}{\xi} \tilde{C}_{j/k}(x/\xi, b_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu)}_{\tilde{f}_{j/p}^{\mathrm{OPE}}(x, b_T; \mu, \zeta)} + O\left(b_T \Lambda_{\mathrm{QCD}}\right)$$

How to test consistency

- $$\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T})$$
  is uniquely determined by its operator definition

- At $q_T \approx Q$

$$= \underbrace{\frac{1}{q_{\mathrm{T}}^2}\left[\mathcal{C}\left(q_{\mathrm{T}}/\mu_Q,\alpha_s\left(\mu_Q\right)\right)\otimes f_{j/p}(x;\mu_Q)\otimes d_{h/j}(z;\mu_Q)\right.}_{q_T\sim Q,\, Q\to\infty \text{ asymptote}} \left. + O\left(\frac{\Lambda_{\mathrm{QCD}}}{q_{\mathrm{T}}}\right)\right]$$

- For TMD pdfs & ffs

$$f_{j/p}(x,k_T\approx Q;\mu_Q,Q^2)=\frac{1}{k_T^2}\left[\int_x^1\frac{\mathrm{d}\xi}{\xi}C_{j/k}(x/\xi,k_T/Q,\alpha_s(Q))f_{k/p}(\xi;Q)+O\left(\frac{\Lambda_{\mathrm{QCD}}}{k_T}\right)\right]$$

&

$$\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x,k_T;\mu_Q,Q^2)\approx f_{j/p}(x;\mu_Q)$$

- $$\frac{\mathrm{d}}{\mathrm{d}b_{\max}}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_{\mathrm{T}}\ldots}\right)=0$$ *for* $b_{\max}\ll 1/\Lambda_{QCD}$

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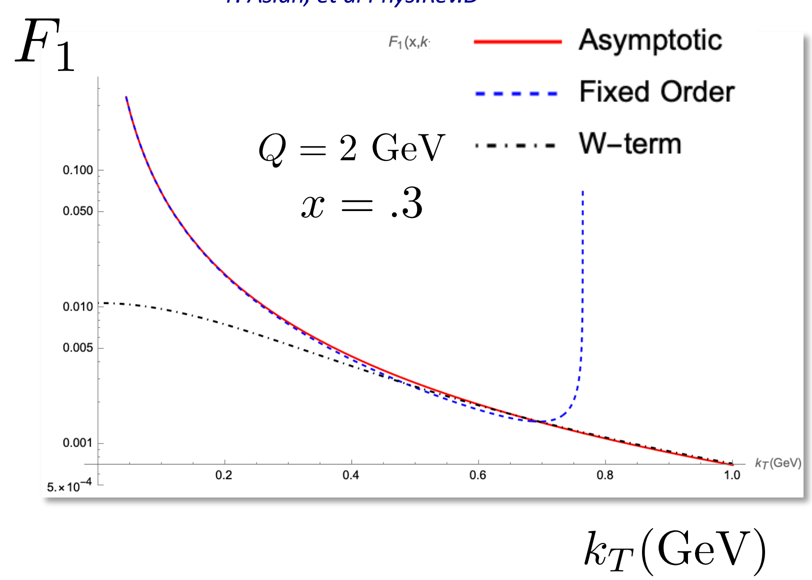

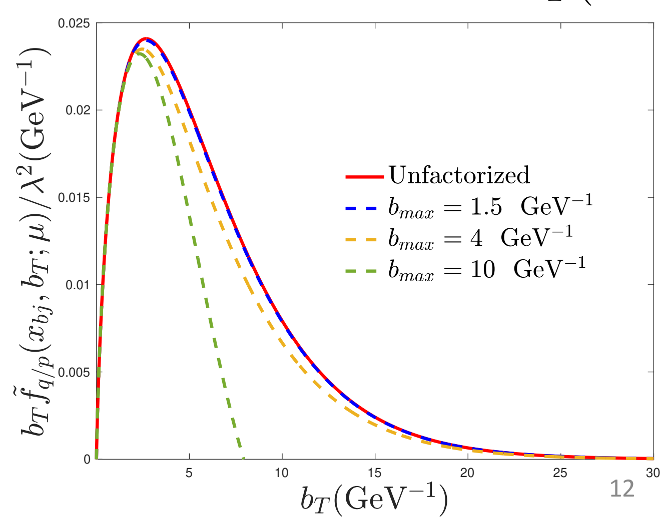

How to test consistency

e+e- annihilation

- What does $q_T \approx Q$ mean?
  - No sensitivity to parameters related nonperturbative transverse momentum
  - $\Lambda_{QCD} \ll q_T \ll Q$? Look for matching between fixed order x-section and asymptotic term
    - Is there a region where both

      $$\frac{\Lambda_{QCD}}{q_T} \;\&\; \frac{q_T}{Q}$$

      powers are simultaneously negligible?
  - No large logarithms: Look for node in asymptotic term

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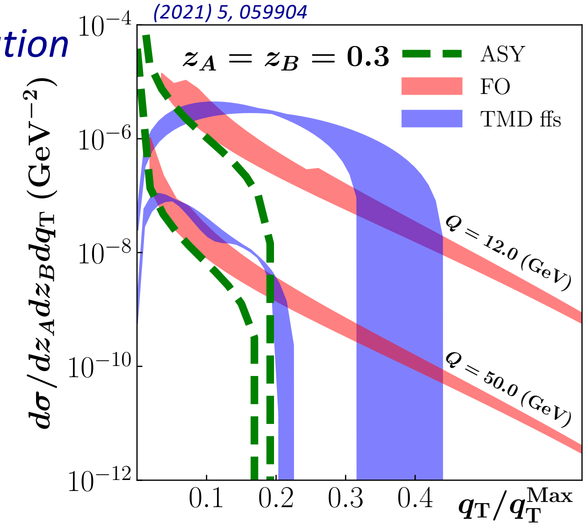
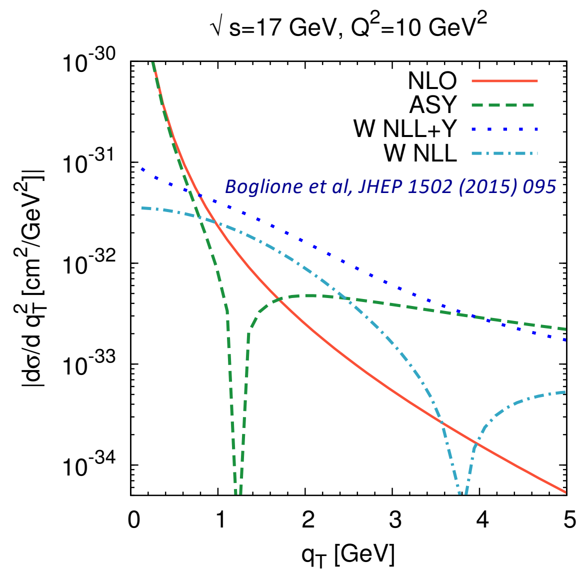


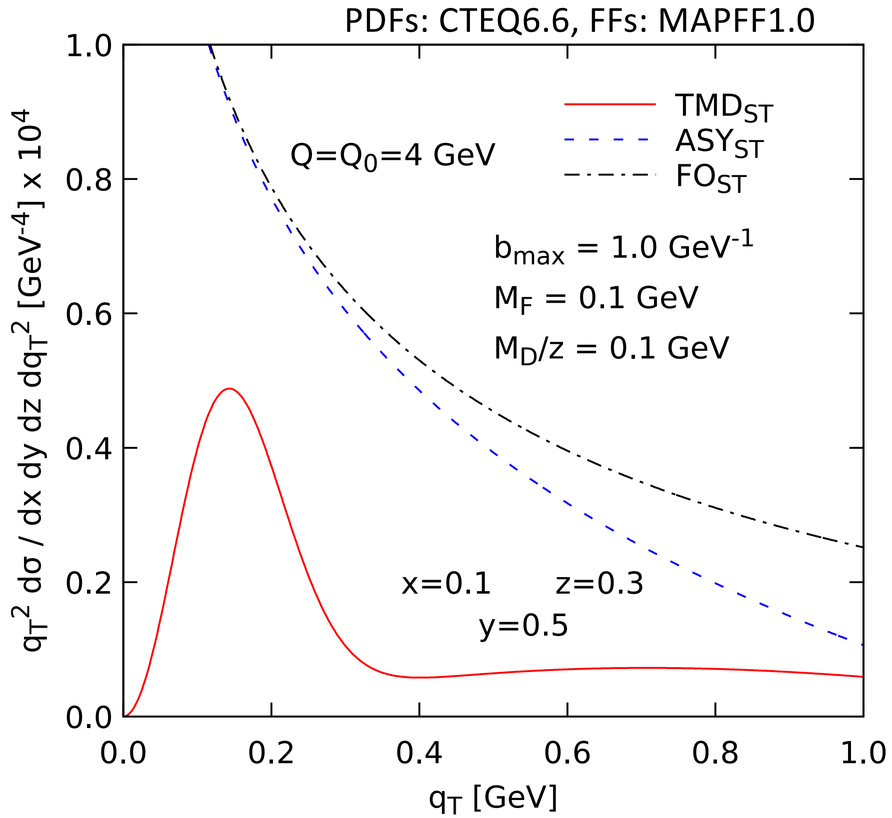


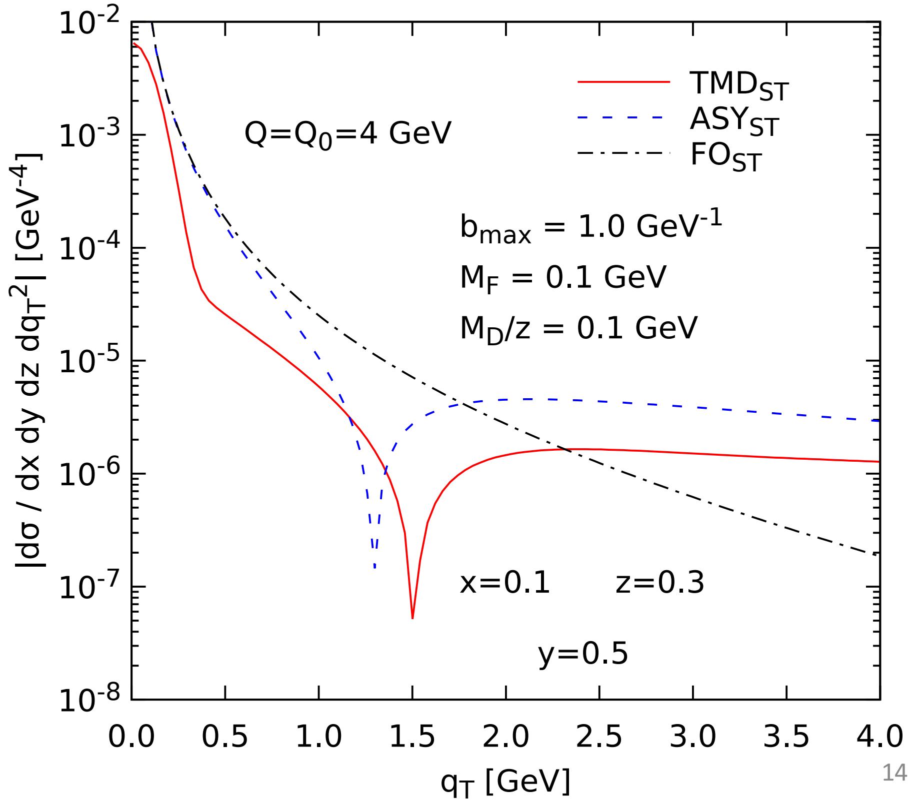
Conventional organization & complications

(An example typical of conventional approach)

$$g_{j/p}(x, b_{\rm T}) = \frac{1}{4} M_F^2 \, b_{\rm T}^2$$

$$g_{h/j}(z, b_{\rm T}) = \frac{1}{4\,z^2} M_D^2 \, b_{\rm T}^2$$





Conventional organization & complications

$$g_{j/p}(x, b_{\mathrm{T}}) = \frac{1}{4} M_F^2 \, b_{\mathrm{T}}^2$$

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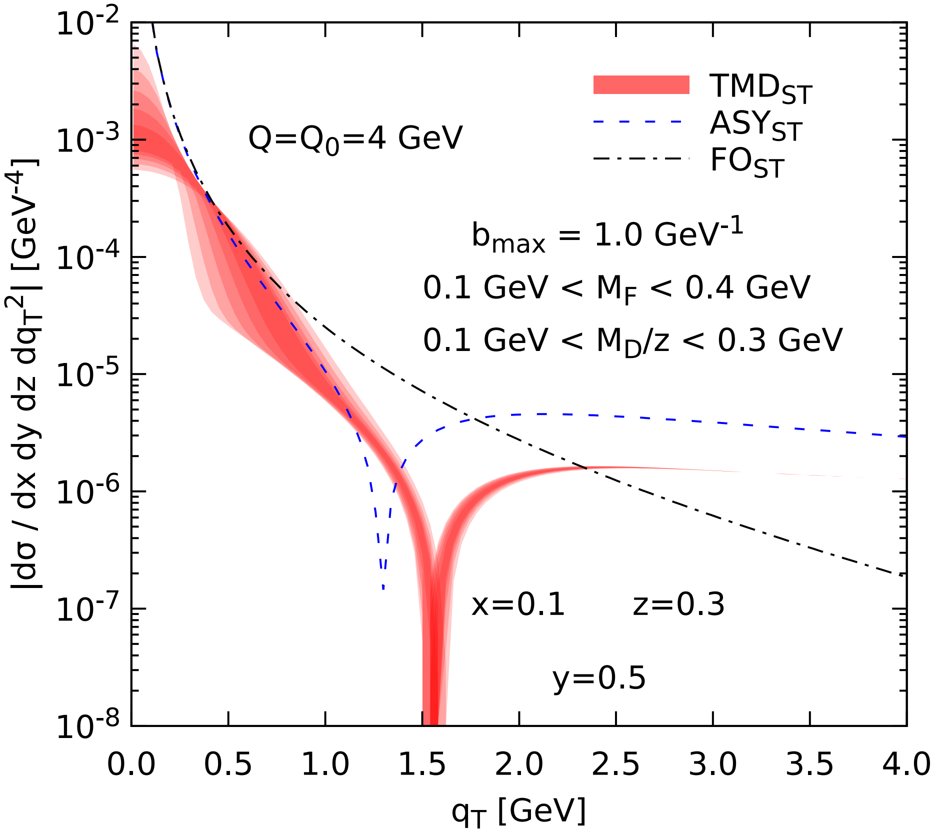

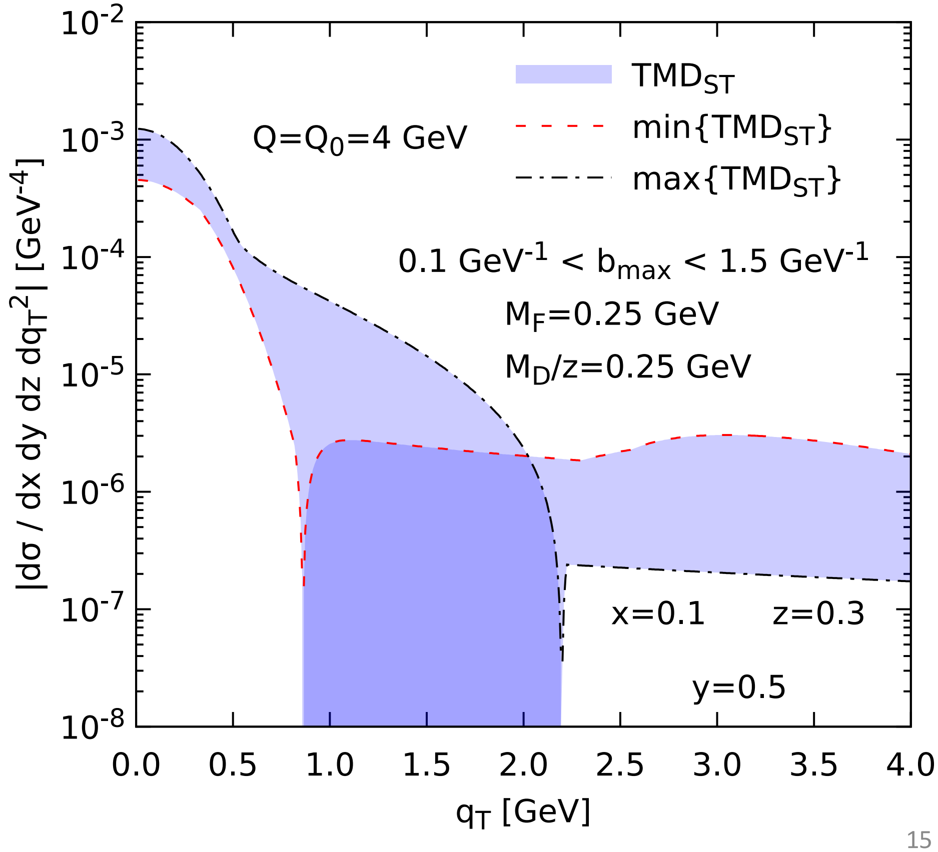

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\underbrace{\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)}_{\textit{Well-defined operators for all TM}}\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T}) + \text{power suppressed}$$

Diagnosis

1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

2) Fixed order perturbation theory should work fine for $q_T \approx Q_0$, but evol. factors have a large effect. What is going on?

3) $\exists$ no region at input scale $Q = Q_0$ where $\Lambda_{QCD} \ll q_T \ll Q_0$

4) Backwards evolution… No large, perturbative $\ln \frac{Q_0}{q_T}$.

5) $\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x,k_T;\mu_Q,Q^2) \approx f_{j/p}(x;\mu_Q)$

Very badly violated at moderate scales

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\underbrace{\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)}_{\textit{Well-defined operators for all TM}}\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T}) + \text{power suppressed}$$

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1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} =$$

$$H(Q/\mu_Q)\int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\ \tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)\ \tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)$$

$$\times \exp\left\{\tilde{K}(b_\mathrm{T};\mu_{Q_0})\ln\left(\frac{Q^2}{Q_0^2}\right) + \int_{\mu_{Q_0}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1) - \ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]\right\}$$

2) Fixed order perturbation theory should work fine for $q_T \approx Q_0$, but evol. factors have a large effect. What is going on?

3) $\exists$ no region at input scale $Q = Q_0$ where $\Lambda_{QCD} \ll q_T \ll Q_0$

4) Backwards evolution…
No large, perturbative $\ln\frac{Q_0}{q_T}$.

5) $\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x,k_T;\mu_Q,Q^2) \approx f_{j/p}(x;\mu_Q)$
Very badly violated at moderate scales

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q) \underbrace{\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)}_{\textit{Well-defined operators for all TM}} \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T}) + \text{power suppressed}$$

Diagnosis

1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

2) Fixed order perturbation theory should work fine for $q_T \approx Q_0$, but evol. factors have a large effect. What is going on?

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$$\tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) = \tilde{D}_{\mathrm{inpt},h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{\overline{Q}_0},\overline{Q}_0^2)E(\overline{Q}_0/Q_0,b_\mathrm{T})$$

$$\tilde{f}_{i/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) = \tilde{f}_{\mathrm{inpt},i/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{\overline{Q}_0},\overline{Q}_0^2)E(\overline{Q}_0/Q_0,b_\mathrm{T})$$

$$\lim_{b_\mathrm{T}\to 0}\overline{Q}_0 \sim 1/b_\mathrm{T}$$

Instead, characterize the full range of k_T behavior of TMD functions at the input scale

A hadron structure oriented (HSO) reorganization

1) Use the uniquely determined TMDs for all k_T

2) Smoothly interpolate between nonperturbative TM dependence at small TM ($k_T \approx \Lambda_{QCD}$) & perturbative (collinear) TM at large TM ($k_T \approx Q$)

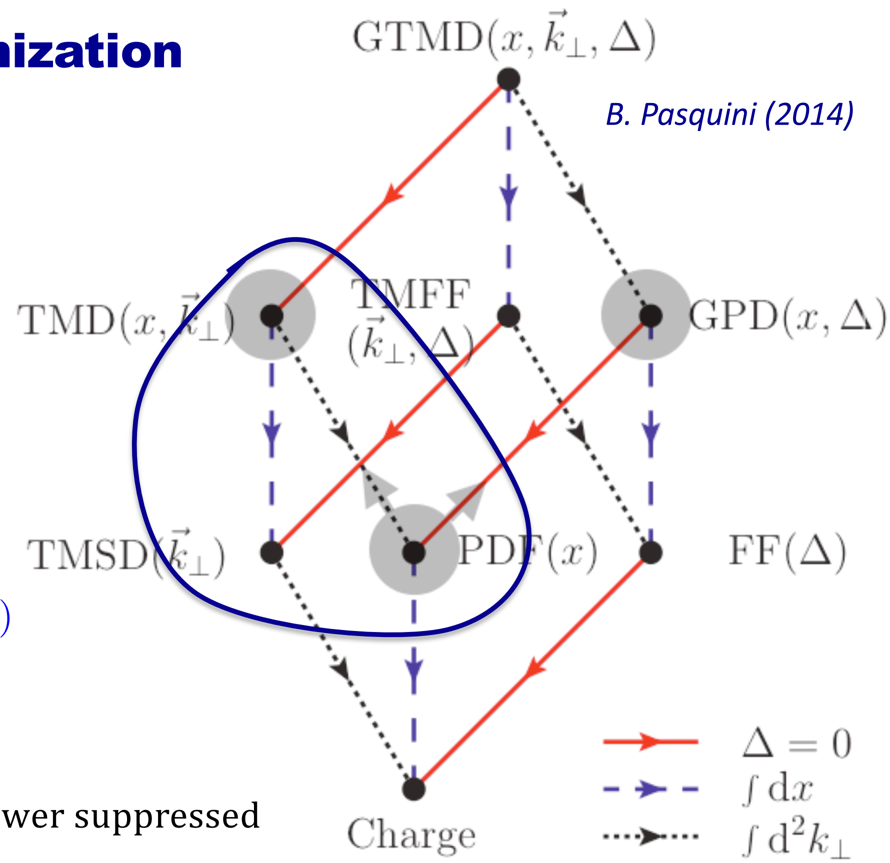
3) (Approximate) probability interpretation

- Parton model: $\int d^2\mathbf{k}_T f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q)$

- QCD:

$$\pi \int^{\mu_Q^2} dk_T^2 f_{j/p}(x, k_T; \mu_Q, Q^2) = \underset{\overline{\text{MS}}}{f_{j/p}(x; \mu_Q)} + \underset{\underbrace{\sum_{j'} \mathcal{C}^{\Delta}_{j/j'} \otimes f_{j'/p}}_{O(\alpha_s)}}{\Delta_{j/p}} + \text{power suppressed}$$

The "W-term"



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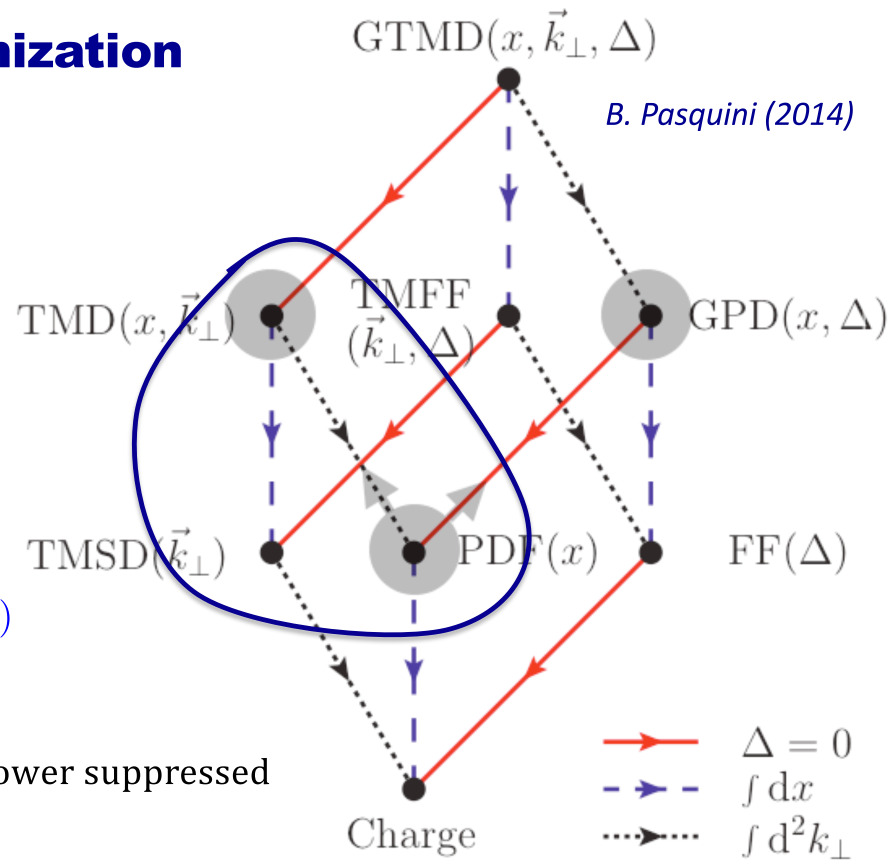
  + power suppressed

4) All should apply at an input scale, Q_0

5) Pheno requirement: Must be simple to swap one model/parametrization for another while still satisfying 1-4



A hadron structure oriented (HSO) reorganization

- We provide a recipe to transform a NP TMD parametrization into an evolved parametrization at other scales:
  - *Sec. VI of Phys.Rev.D 106 (2022) 3, 034002*

Called "bottom up" approach here

- No b_{max} or b_* necessary

- HSO approach is equivalent to standard TMD factorization, CSS, etc, just with additional consistency constraints on the g-functions

- It is straightforward to translate between standard treatment and HSO

  - *Sec. IX of Phys.Rev.D 106 (2022) 3, 034002*

An $O(\alpha_s)$ example with $\overline{\text{MS}}$ pdfs and ffs

$$f_{\text{inpt},i/p}(x, \boldsymbol{k}_{\text{T}}; \mu_{Q_0}, Q_0^2) = \frac{1}{2\pi} \frac{1}{k_{\text{T}}^2 + m_{f_{i,p}}^2} \left[A_{i/p}^f(x; \mu_{Q_0}) + B_{i/p}^f(x; \mu_{Q_0}) \ln \frac{Q_0^2}{k_{\text{T}}^2 + m_{f_{i,p}}^2} \right] + \frac{1}{2\pi} \frac{1}{k_{\text{T}}^2 + m_{f_{g,p}}^2} A_{i/p}^{f,g}(x; \mu_{Q_0})$$
$$+ C_{i/p}^f \, f_{\text{core},i/p}(x, \boldsymbol{k}_{\text{T}}; Q_0^2)$$

$$D_{\text{inpt},h/j}(z, z\boldsymbol{k}_{\text{T}}; \mu_{Q_0}, Q_0^2) = \frac{1}{2\pi z^2} \frac{1}{k_{\text{T}}^2 + m_{D_{h,j}}^2} \left[A_{h/j}^D(z; \mu_{Q_0}) + B_{h/j}^D(z; \mu_{Q_0}) \ln \frac{Q_0^2}{k_{\text{T}}^2 + m_{D_{h,j}}^2} \right] + \frac{1}{2\pi z^2} \frac{1}{k_{\text{T}}^2 + m_{D_{h,g}}^2} A_{h/j}^{D,g}(z; \mu_{Q_0})$$
$$+ C_{h/j}^D \, D_{\text{core},h/j}(z, z\boldsymbol{k}_{\text{T}}; Q_0^2)$$

- C^f & C^D constrained by:

$$f_{i/p}^c(x; \mu_{Q_0}) \equiv 2\pi \int_0^{\mu_{Q_0}} \mathrm{d}k_{\text{T}} \, k_{\text{T}} f_{i/p}(x, \boldsymbol{k}_{\text{T}}; \mu_{Q_0}, Q_0^2) = f_{i/p}(x; \mu_{Q_0}) + \mathcal{C}_{i/i'}^{\Delta^f} \otimes f_{i'/p} + \text{p.s.}$$
$$d_{h/j}^c(z; \mu_{Q_0}) \equiv 2\pi z^2 \int_0^{\mu_{Q_0}} \mathrm{d}k_{\text{T}} \, k_{\text{T}} D_{h/j}(z, z\boldsymbol{k}_{\text{T}}; \mu_{Q_0}, Q_0^2) = d_{h/j}(z; \mu_{Q_0}) + \mathcal{C}_{j'/j}^{\Delta^d} \otimes d_{h/j'} + \text{p.s.}$$

An $O(\alpha_s)$ example with $\overline{\mathrm{MS}}$ pdfs and ffs

- Parametrizing the very small transverse momentum

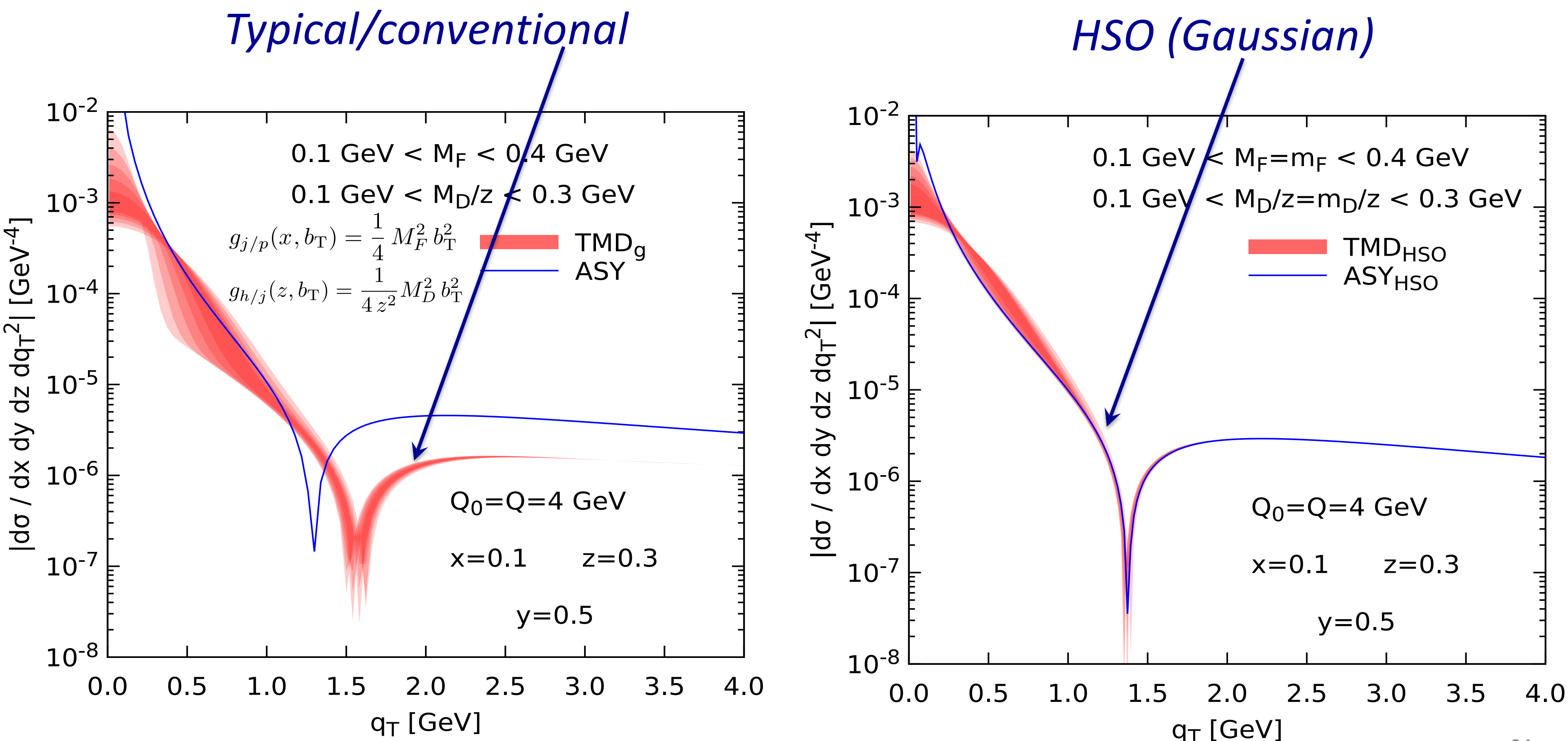
A. Gaussian model (very commonly used)

$$f^{\mathrm{Gauss}}_{\mathrm{core},i/p}(x,\boldsymbol{k}_\mathrm{T};Q_0^2) = \frac{e^{-k_\mathrm{T}^2/M_\mathrm{F}^2}}{\pi M_\mathrm{F}^2}\,, \qquad D^{\mathrm{Gauss}}_{\mathrm{core},h/j}(z,z\boldsymbol{k}_\mathrm{T};Q_0^2) = \frac{e^{-z^2\,k_\mathrm{T}^2/M_\mathrm{D}^2}}{\pi M_\mathrm{D}^2}$$

B. Spectator model

$$f^{\mathrm{Spect}}_{\mathrm{core},i/p}(x,\boldsymbol{k}_\mathrm{T};Q_0^2) = \frac{6M_{0\mathrm{F}}^6}{\pi\,(2M_\mathrm{F}^2+M_{0\mathrm{F}}^2)}\,\frac{M_\mathrm{F}^2+k_\mathrm{T}^2}{(M_{0\mathrm{F}}^2+k_\mathrm{T}^2)^{\,4}}\,, \quad D^{\mathrm{Spect}}_{\mathrm{core},h/j}(z,z\boldsymbol{k}_\mathrm{T};Q_0^2) = \frac{2M_{0\mathrm{D}}^4}{\pi\,(M_\mathrm{D}^2+M_{0\mathrm{D}}^2)}\,\frac{M_\mathrm{D}^2+k_\mathrm{T}^2 z^2}{(M_{0\mathrm{D}}^2+k_\mathrm{T}^2 z^2)^{\,3}}$$

Compare standard/unconstrained with HSO ($O(\alpha_s)$)



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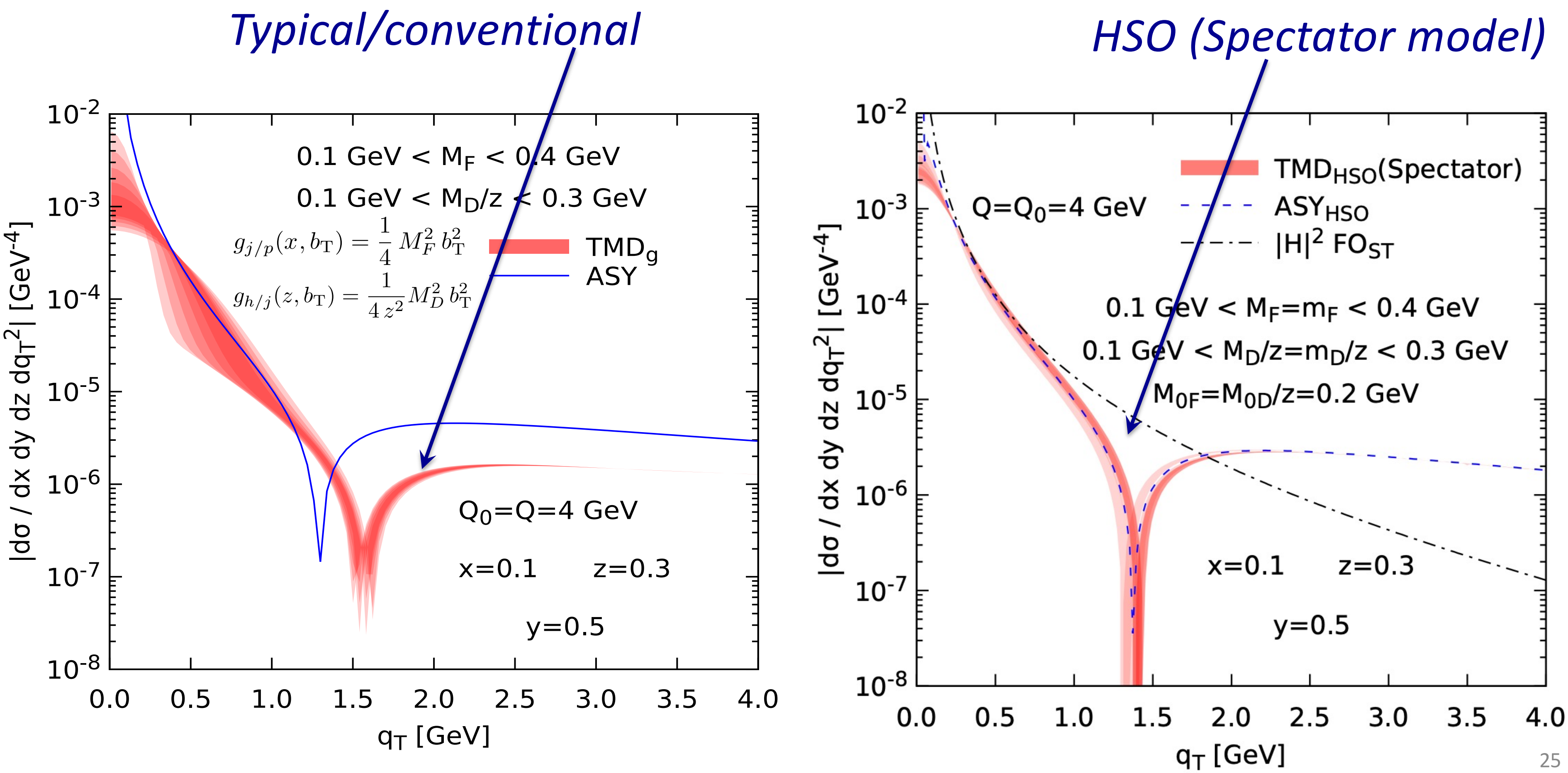
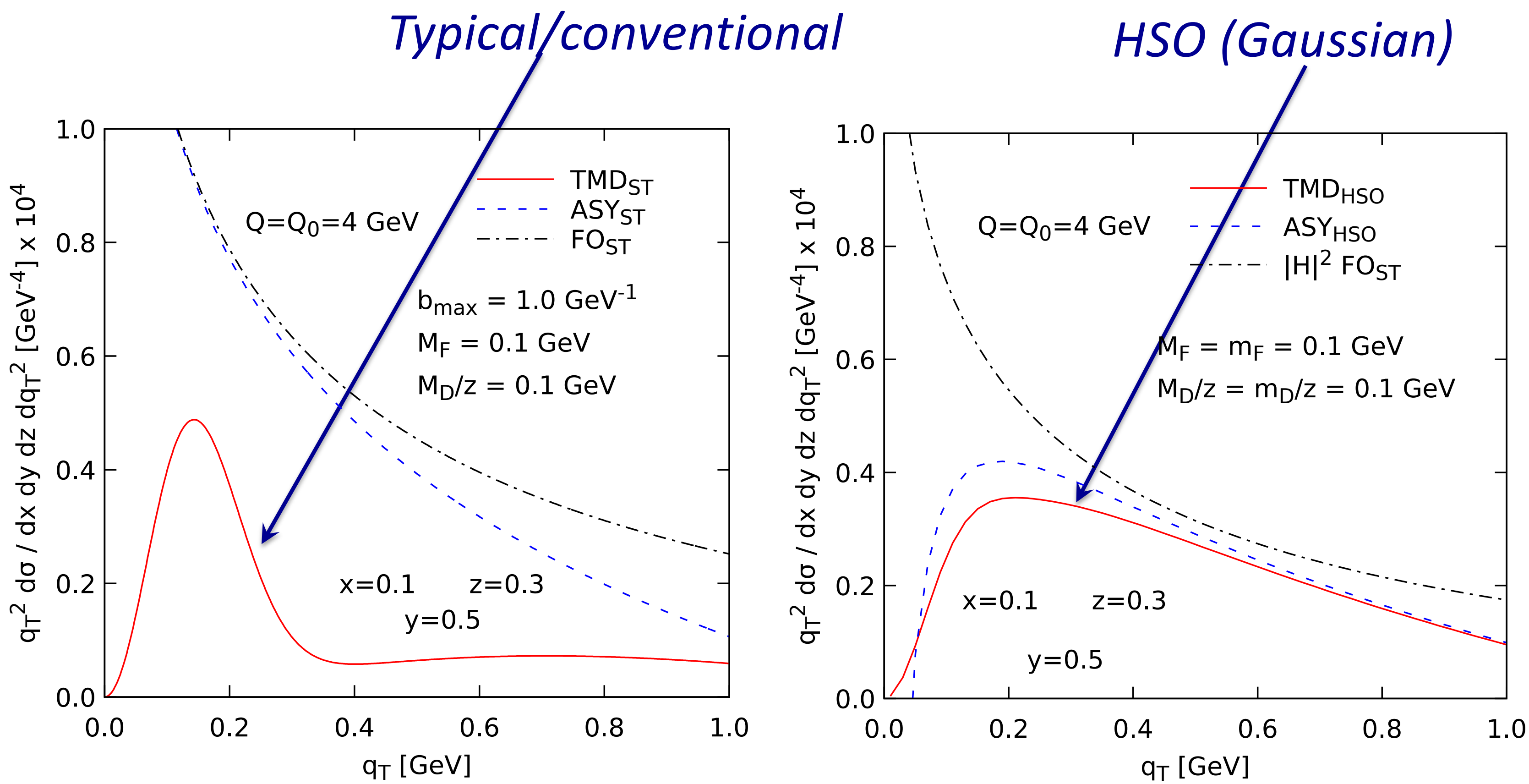


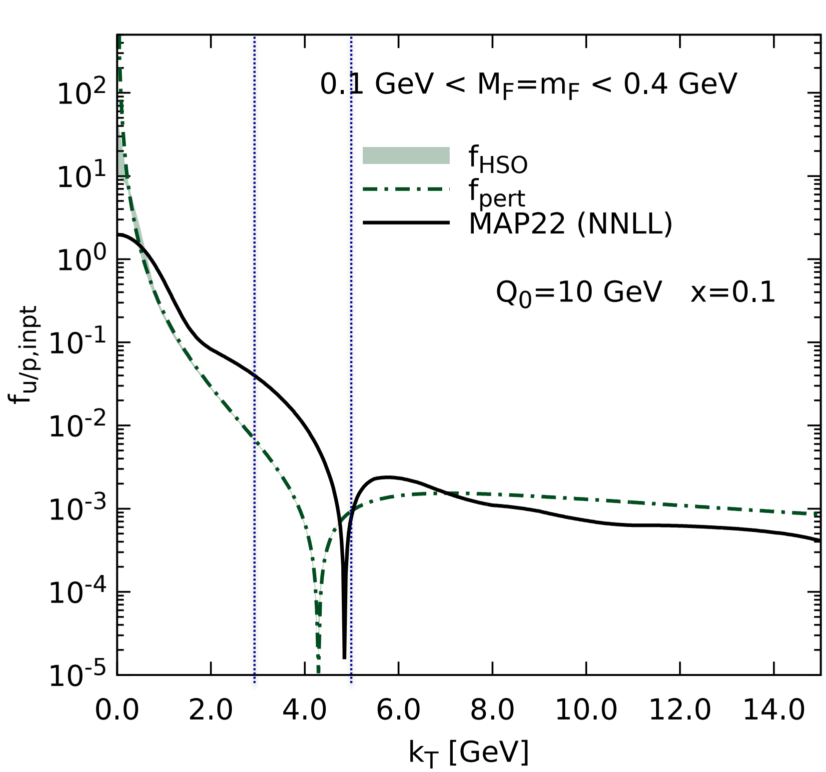
Compare standard/unconstrained with HSO ($O(\alpha_s)$)

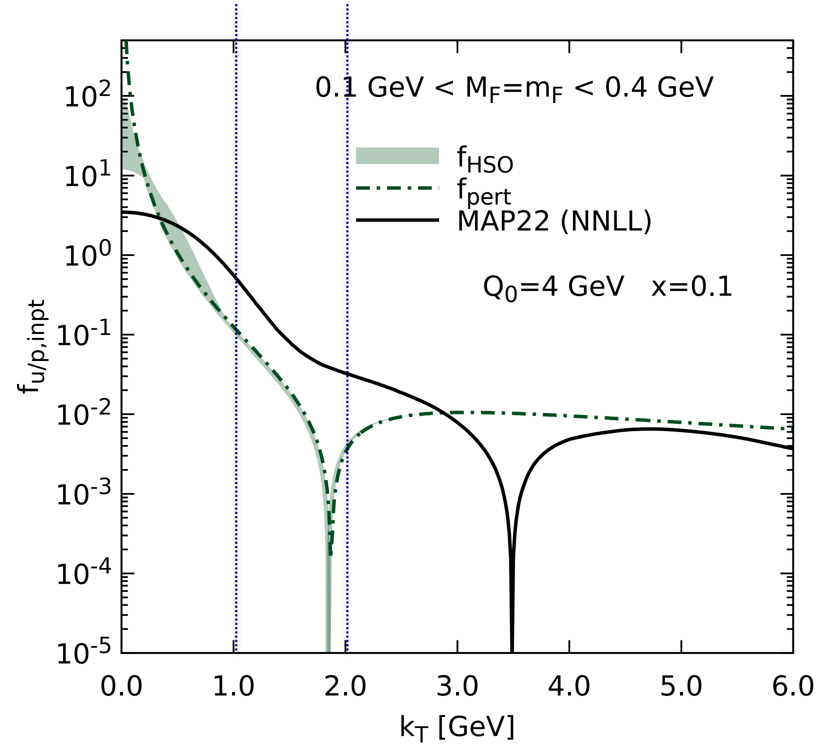


What about the individual TMD pdfs?

Proton pdfs MAP (2023)

Bacchetta et al, JHEP 10 (2022) 127, 2206.07598 [hep-ph]





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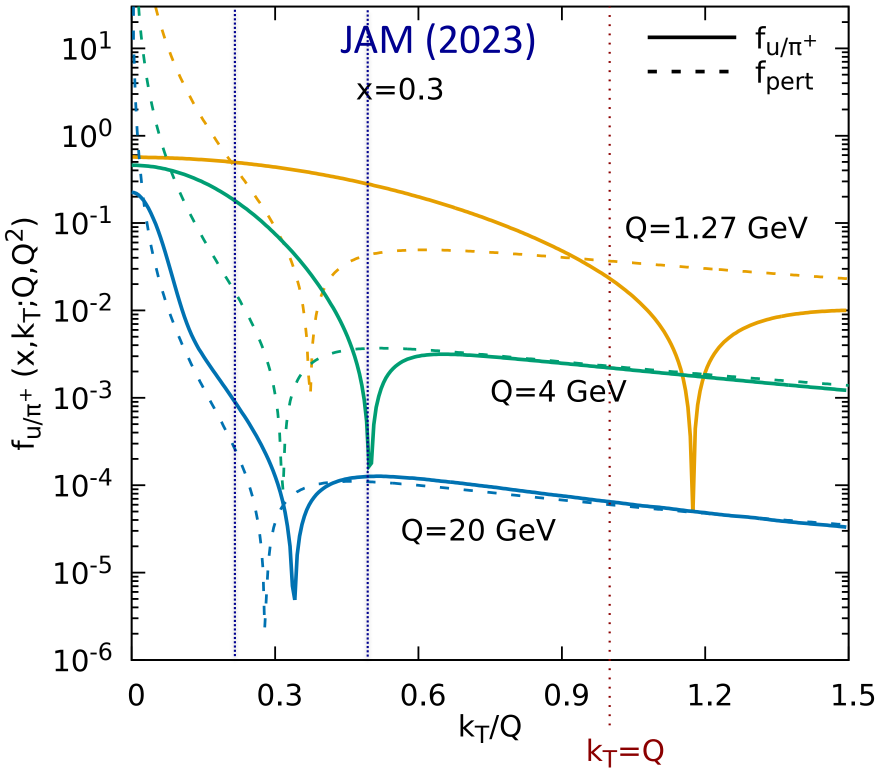
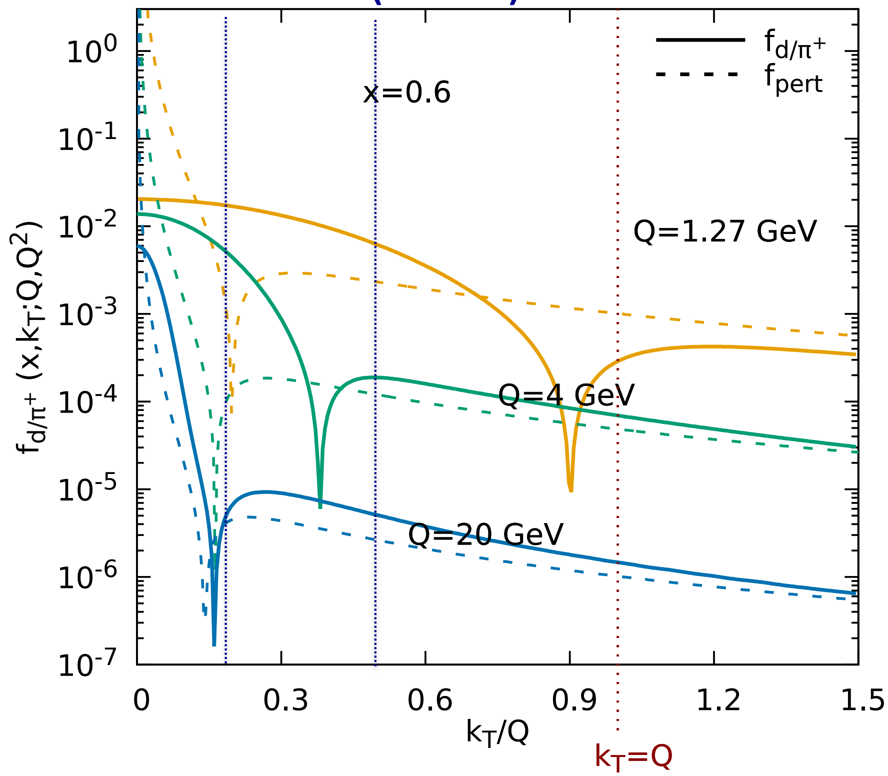
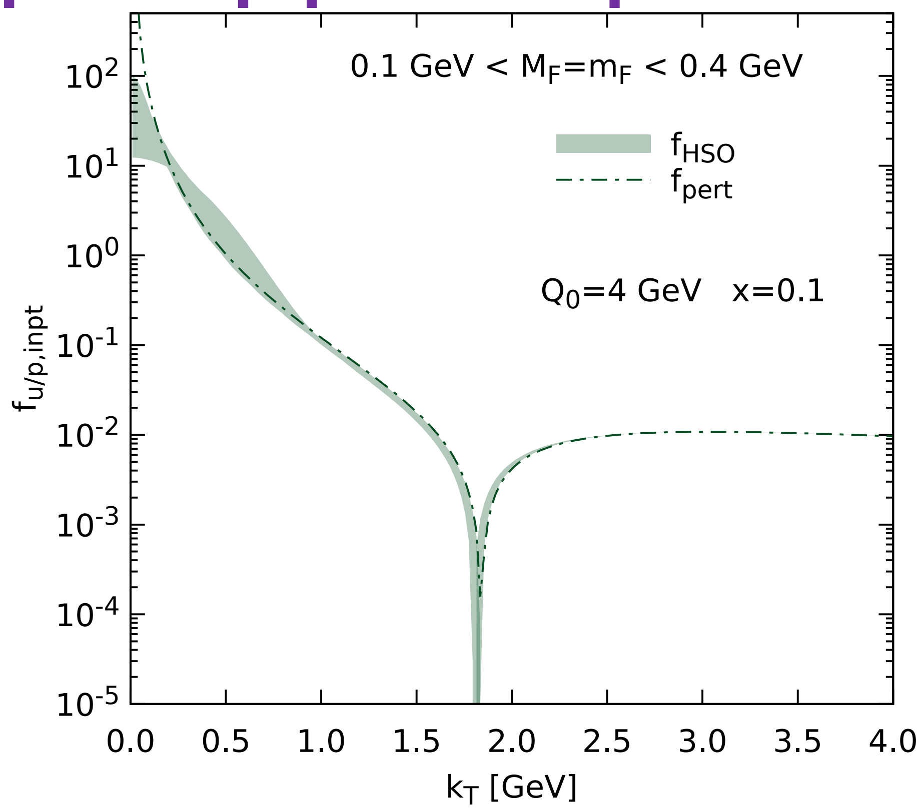


JAM (2023) 2302.01192 [hep-ph]

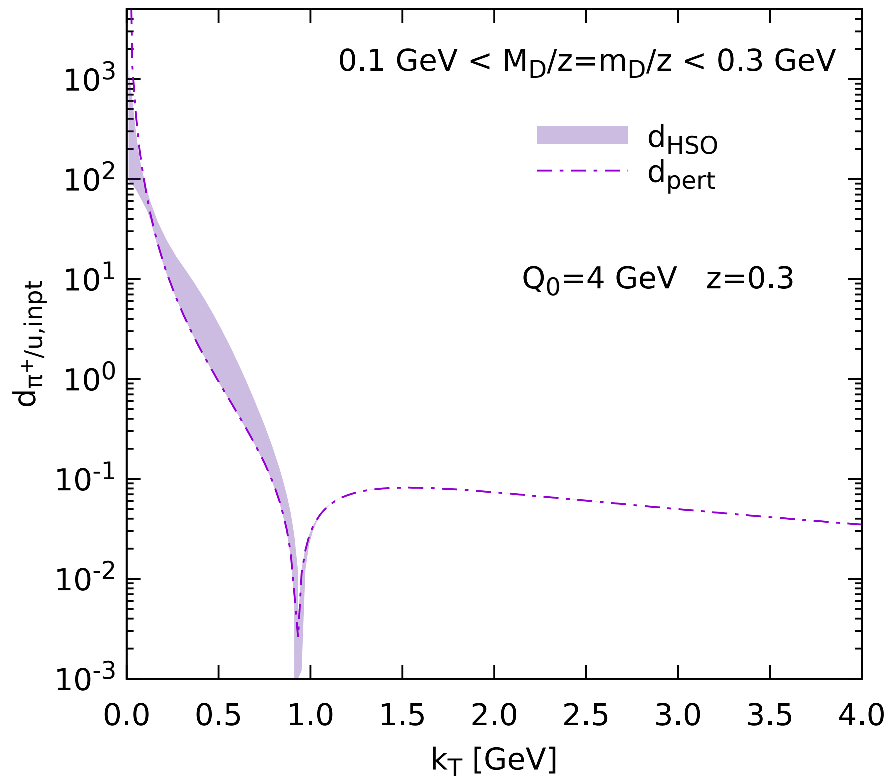


What about the individual TMD pdfs?

proton up-quark TMD pdfs



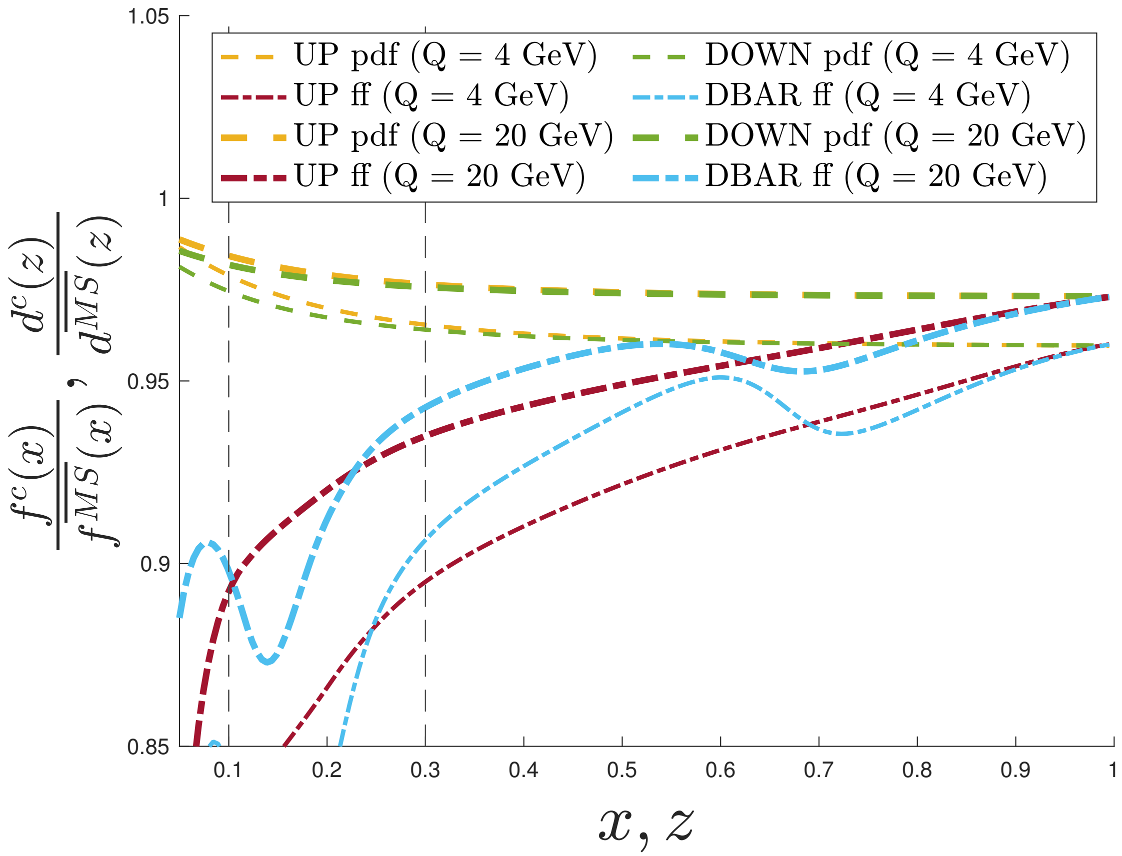
π^+ from up-quark TMD ff



HSO (Gaussian)

Improvements beyond the TMD pdfs

- Cutoff TMD versus renormalized collinear pdfs?



Summary

- Switching to a hadron-structure-oriented approach to pheno with TMD factorization improves consistency in the large transverse behavior of TMD correlation functions
- Necessary for understanding the shapes of nonperturbative distributions, separating perturbative and nonpeturbative parts, etc
- Necessary for transforming claims about nonperturbative TM physics into testable/falsifiable hypotheses
- HSO is not a new formalism; **HSO = “standard CSS”!**
- Next:
  - Applications
  - Higher orders
  - Incorporating NP calculations (lattice, EFTs, models etc)
  - Spin dependent TMDs

Backup

$$A^{f}_{i/p}(x;\mu_{Q_0}) \equiv \sum_{ii'} \delta_{i'i} \frac{\alpha_s(\mu_{Q_0})}{\pi} \left\{ \left[(P_{i'i} \otimes f_{i'/p})(x;\mu_{Q_0}) \right] - \frac{3C_F}{2} f_{i'/p}(x;\mu_{Q_0}) \right\} ,$$

$$B^{f}_{i/p}(x;\mu_{Q_0}) \equiv \sum_{i'i} \delta_{i'i} \frac{\alpha_s(\mu_{Q_0}) C_F}{\pi} f_{i'/p}(x;\mu_{Q_0}) \, ,$$

$$A^{f,g}_{i/p}(x;\mu_{Q_0}) \equiv \frac{\alpha_s(\mu_{Q_0})}{\pi} \left[(P_{ig} \otimes f_{g/p})(x;\mu_{Q_0}) \right] ,$$

$$C^{f}_{i/p} \equiv \frac{1}{N^{f}_{i/p}} \left[f_{i/p}(x;\mu_{Q_0}) - A^{f}_{i/p}(x;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right) - B^{f}_{i/p}(x;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right) \ln\left(\frac{Q_0^2}{\mu_{Q_0} m_{f_{i,p}}}\right) ,\right.$$

$$\left. - A^{f,g}_{i/p}(x;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{f_{g,p}}}\right) + \frac{\alpha_s(\mu_{Q_0})}{2\pi} \left\{ \sum_{ii'} \delta_{i'i} [\mathcal{C}^{i/i'}_{\Delta} \otimes f_{i'/p}](x;\mu_{Q_0}) + [\mathcal{C}^{i/g}_{\Delta} \otimes f_{g/p}](x;\mu_{Q_0}) \right\} \right] .$$

$$P_{ig}(x) = T_F \left[x^2 + (1-x)^2 \right] ,$$

$$\mathcal{C}^{i/i}_{\Delta}(x) = C_F(1-x) - C_F \frac{\pi^2}{12} \delta(1-x) \, ,$$

$$\mathcal{C}^{g/p}_{\Delta}(x) = 2T_F x(1-x) \, ,$$

$$N^{f}_{i/p} \equiv \, 2\pi \int_0^\infty dk_{\mathrm{T}} k_{\mathrm{T}} \, f_{\mathrm{core},i/p}(x, \boldsymbol{k}_{\mathrm{T}}; Q_0^2)$$

Backup

$$A^D_{h/j}(z;\mu_{Q_0}) \equiv \sum_{jj'} \delta_{j'j} \frac{\alpha_s(\mu_{Q_0})}{\pi} \left\{ [(P_{jj'} \otimes d_{h/j'})(z;\mu_{Q_0})] - \frac{3C_F}{2} d_{h/j'}(z;\mu_{Q_0}) \right\} ,$$

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$$A^{D,g}_{h/j}(z;\mu_{Q_0}) \equiv \frac{\alpha_s(\mu_{Q_0})}{\pi} [(P_{gj} \otimes d_{h/g})(z;\mu_{Q_0})] ,$$

$$C^D_{h/j} \equiv \frac{1}{N^D_{h/j}} \Bigg[d_{h/j}(z;\mu_{Q_0}) - A^D_{h/j}(z;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{D_{h,j}}}\right) - B^D_{h/j}(z;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{D_{h,j}}}\right) \ln\left(\frac{Q_0^2}{\mu_{Q_0} m_{D_{h,j}}}\right) ,$$

$$- A^{D,g}_{h/j}(z;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{D_{h,g}}}\right) + \frac{\alpha_s(\mu_{Q_0})}{2\pi} \left\{ \sum_{jj'} \delta_{j'j} [\mathcal{C}^{j'/j}_{\Delta} \otimes d_{h/j'}](z;\mu_{Q_0}) + [\mathcal{C}^{g/j}_{\Delta} \otimes d_{h/g}](z;\mu_{Q_0}) \right\} \Bigg] .$$

$$P_{qq}(z) = P_{\bar{q}\bar{q}}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right] ,$$

$$P_{gq}(z) = C_F \frac{1+(1-z)^2}{z} ,$$

$$\mathcal{C}^{q/q}_{\Delta}(z) = 2P_{qq}(z)\ln z + C_F(1-z) - C_F\frac{\pi^2}{12}\delta(1-z) ,$$

$$\mathcal{C}^{g/q}_{\Delta}(z) = 2P_{gq}(z)\ln z + C_F z ,$$

$$N^D_{h/j} \equiv 2\pi\, z^2 \int_0^\infty dk_{\mathrm{T}} k_{\mathrm{T}}\, D_{\mathrm{core},h/j}(z, z\boldsymbol{k}_{\mathrm{T}}; Q_0^2) .$$

Backup

$$
\begin{aligned}
f^c_{\mathrm{inpt},i/p}(x;\mu_{Q_0}) =& 2\pi \int_0^{\mu_{Q_0}} \mathrm{d}k_\mathrm{T}\, k_\mathrm{T} f_{\mathrm{inpt},i/p}(x,\boldsymbol{k}_\mathrm{T};\mu_{Q_0},Q_0^2) = \\
& C^f_{i/p}\, f^c_{\mathrm{core},i/p}(x;\mu_{Q_0}) + \frac{1}{2}A^{f,g}_{i/p}(x;\mu_{Q_0}) \ln\left(1+\frac{\mu^2_{Q_0}}{m^2_{f_{g,p}}}\right) \\
& + \frac{1}{2}A^f_{i/p}(x;\mu_{Q_0}) \ln\left(1+\frac{\mu^2_{Q_0}}{m^2_{f_{i,p}}}\right) + \frac{1}{4}B^f_{i/p}(x;\mu_{Q_0}) \left[\ln^2\left(\frac{m^2_{f_{i,p}}}{Q_0^2}\right) - \ln^2\left(\frac{\mu^2_{Q_0}+m^2_{f_{i,p}}}{Q_0^2}\right)\right] \\
& = f_{i/p}(x;\mu_{Q_0}) + O\left(\alpha_s(\mu_0), \frac{m^2}{Q_0^2}\right) ,
\end{aligned}
$$

$$
\begin{aligned}
d^c_{\mathrm{inpt},h/j}(z;\mu_{Q_0}) =& 2\pi z^2 \int_0^{\mu_{Q_0}} \mathrm{d}k_\mathrm{T}\, k_\mathrm{T} D_{\mathrm{inpt},h/j}(z,z\boldsymbol{k}_\mathrm{T};\mu_{Q_0},Q_0^2) = \\
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$$