TMD phenomenology with the HSO approach

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GFI 2nd miniworkshop

Oasi di Cavoretto, Torino, Italy

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Jefferson Lab

OLD DOMINION UNIVERSITY

Based on

- The resolution to the problem of consistent large transverse momentum in TMDs (PhysRevD.107.094029)
  - (J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers)
- Combining nonperturbative transverse momentum dependence with TMD evolution (PhysRevD.106.034002)
  - (J. O. Gonzalez-Hernandez, T. C. Rogers, N. Sato)

Why TMDs?

Drell-Yan

SIDIS

Studying the role of intrinsic or **nonperturbative effects** in hadrons

$e^+ e^-$ --> H_a H_b

Predicting transverse momentum distributions in **cross sections** after evolution to **high energies**

Factorization theorems

Universality

Evolution equations

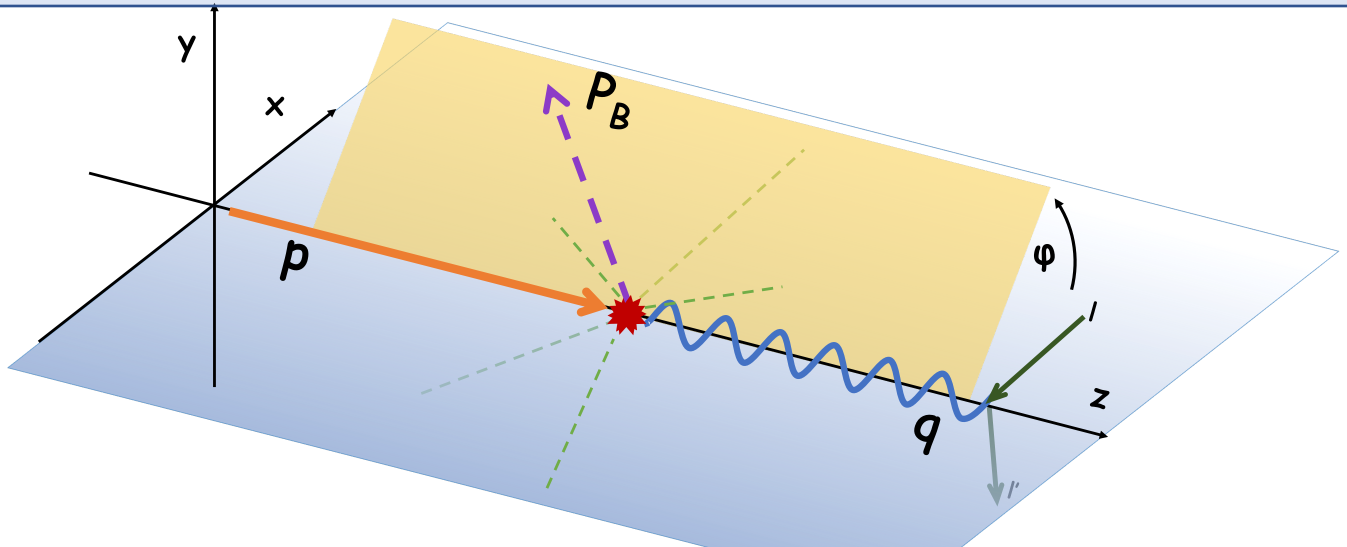
SIDIS

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}q_{\mathrm{T}}^2} = \underbrace{W_{\mathrm{SIDIS}}}_{q_{\mathrm{T}} \ll Q} + \underbrace{Y_{\mathrm{SIDIS}}}_{q_{\mathrm{T}} \sim Q} + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

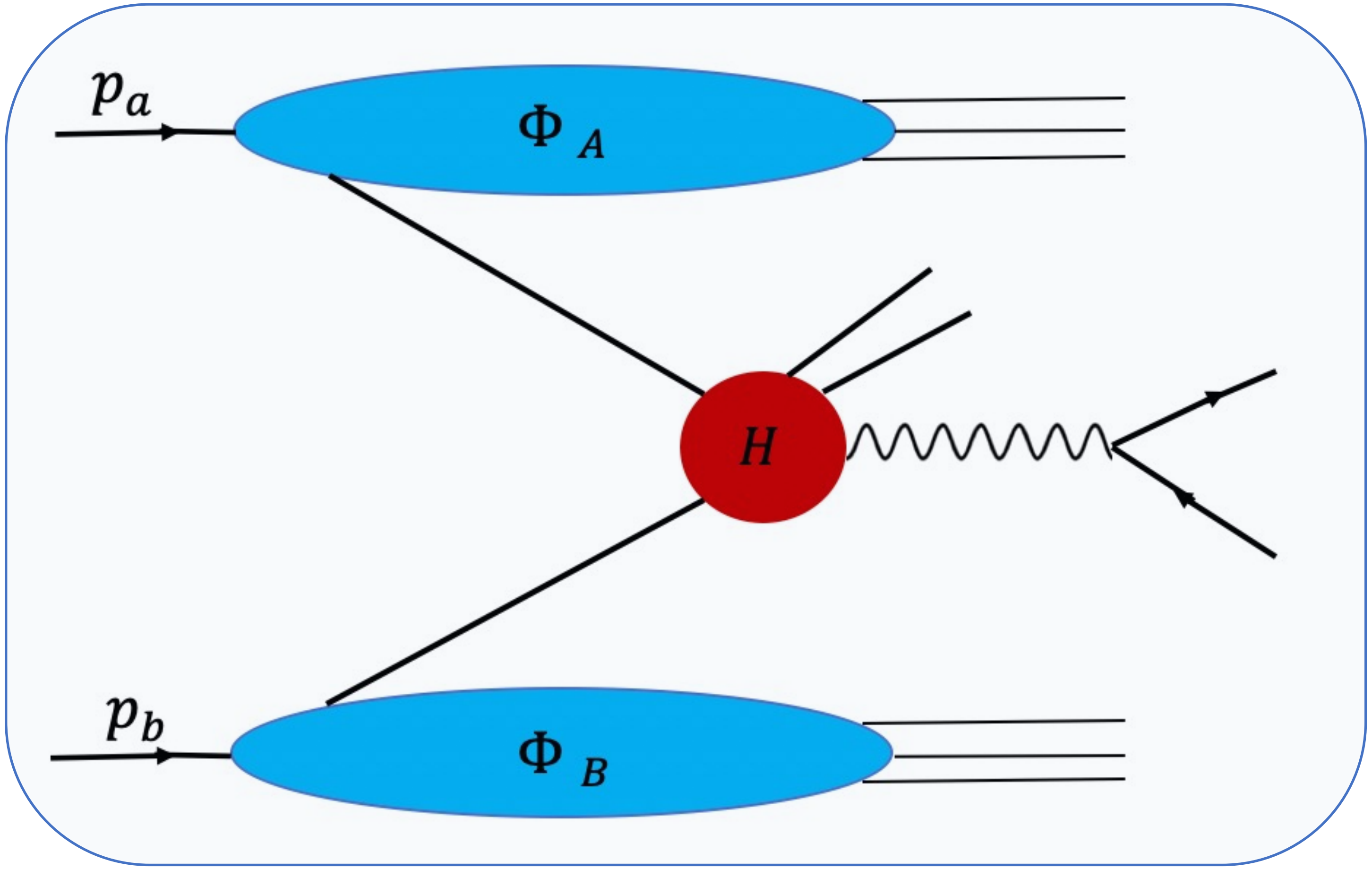
FO collinear factorization

$$\mathrm{FO}_{\mathrm{SIDIS}} - \mathrm{ASY}_{\mathrm{SIDIS}}$$

$$H \int \mathrm{d}^2\boldsymbol{k}_{1\mathrm{T}} \mathrm{d}^2\boldsymbol{k}_{2\mathrm{T}} f_{j/p}\left(x, \boldsymbol{k}_{1\mathrm{T}}; \mu, \sqrt{\zeta}\right) D_{h/j}\left(z, z\boldsymbol{k}_{2\mathrm{T}}; \mu, \sqrt{\zeta}\right) \delta^{(2)}\left(\boldsymbol{q}_{\mathrm{T}} + \boldsymbol{k}_{1\mathrm{T}} - \boldsymbol{k}_{2\mathrm{T}}\right)$$



Drell-Yan

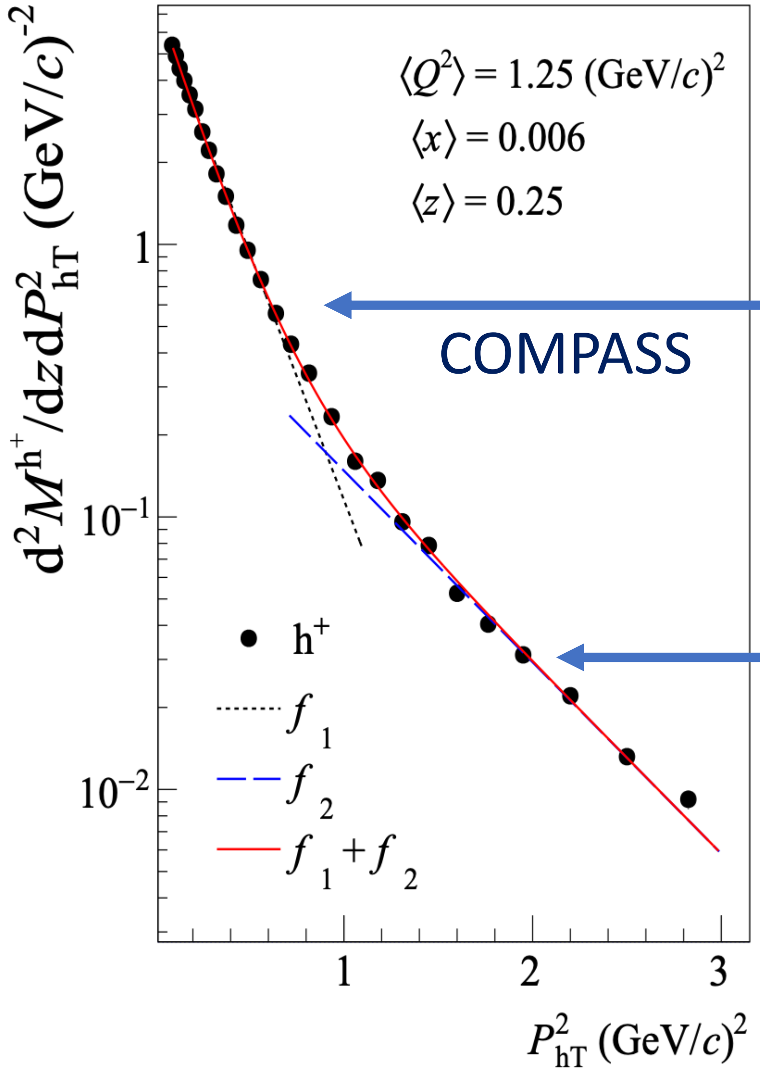


$$W_{\mathrm{DY}}^{\mu\nu}(x_a, x_b, Q, \boldsymbol{q}_T) =$$

$$= \sum_j H_{j,\mathrm{DY}}^{\mu\nu} \int \frac{\mathrm{d}^2 \boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T \cdot \boldsymbol{b}_T} \tilde{f}_{j/h_a}\left(x_a, \boldsymbol{b}_T; \mu_Q; Q^2\right) \tilde{f}_{\bar{j}/h_b}\left(x_b, \boldsymbol{b}_T; \mu_Q; Q^2\right)$$

$$+ (a \longleftrightarrow b) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{m}{Q}\right)$$

Pertubative and nonperturbative TM



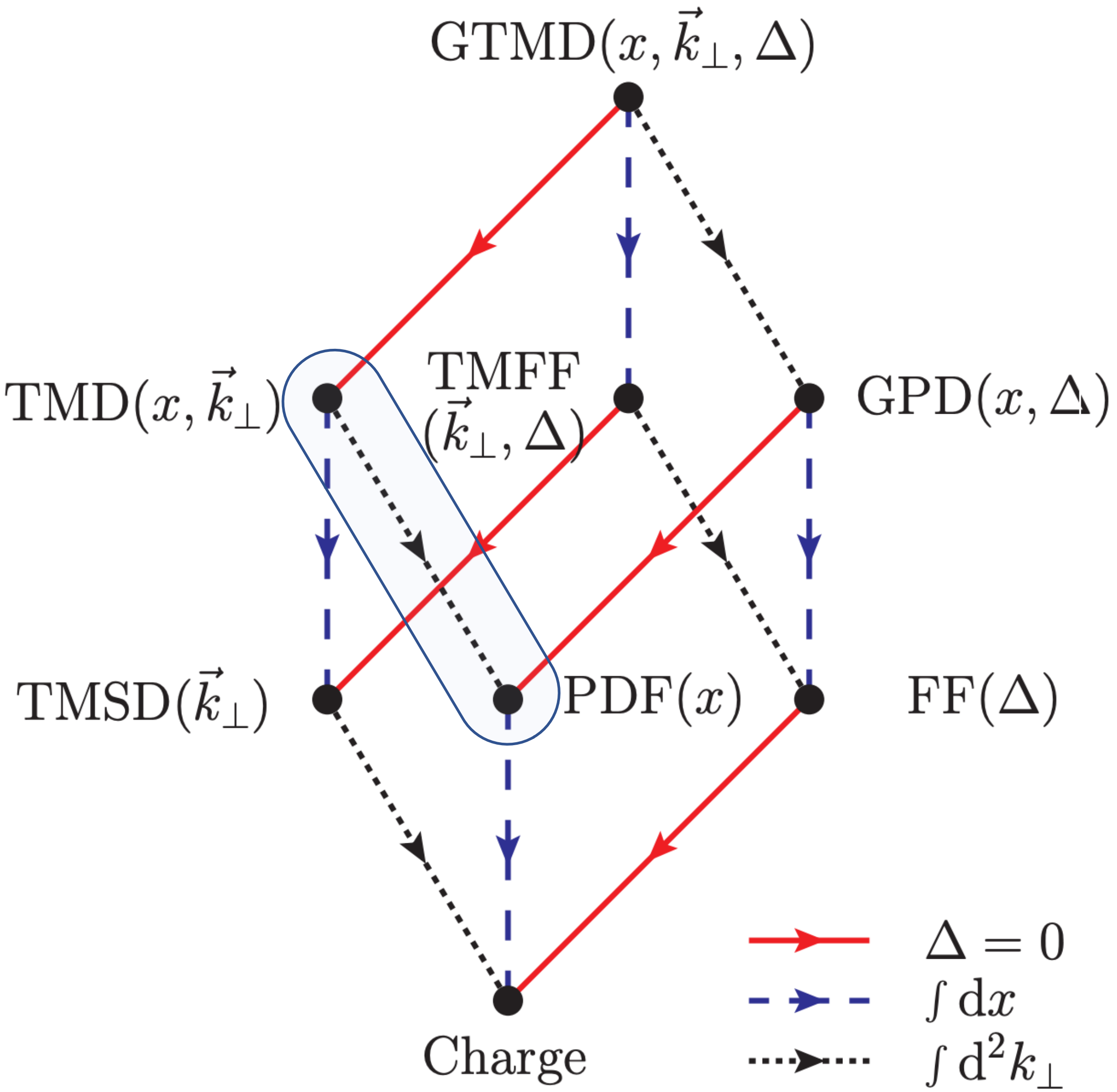
"the two exponential functions in our parameterisation F1 can be attributed to two completely different underlying physics mechanisms that overlap in the region $P_{hT}{}^2 \simeq 1\ (GeV/c)^2$ "

Intrinsic

Tail

Is there a way to be consistent and, at the same time, use all the information that we know about the TMDs?

The net of distribution functions



Most of these integrals are divergent.
A more careful treatment is necessary

Credits: Lorcé, Pasquini and Vanderhaeghen

Conventional approach :

①

$$H \int \mathrm{d}^2\boldsymbol{k}_{1\mathrm{T}} \mathrm{d}^2\boldsymbol{k}_{2\mathrm{T}} f_{j/p}\left(x, \boldsymbol{k}_{1\mathrm{T}}; \mu, \sqrt{\zeta}\right) D_{h/j}\left(z, z\boldsymbol{k}_{2\mathrm{T}}; \mu, \sqrt{\zeta}\right) \delta^{(2)}\left(\boldsymbol{q}_\mathrm{T} + \boldsymbol{k}_{1\mathrm{T}} - \boldsymbol{k}_{2\mathrm{T}}\right)$$

Fourier Transform

$$H \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{-i\boldsymbol{b}_\mathrm{T}\cdot\boldsymbol{q}_\mathrm{T}} \tilde{f}_{j/p}\left(x, \boldsymbol{b}_\mathrm{T}; \mu, \sqrt{\zeta}\right) \tilde{D}_{h/j}\left(z, \boldsymbol{b}_\mathrm{T}; \mu, \sqrt{\zeta}\right)$$

Solve evolution equations relating input scale with SIDIS scale

2

$$\frac{\partial \ln \tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \sqrt{\zeta}\right)}{\partial \ln \sqrt{\zeta}} = \tilde{K}\left(\boldsymbol{b}_{\mathrm{T}}; \mu\right)$$

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \sqrt{\zeta}\right)}{\mathrm{d} \ln \mu} = \gamma\left(\alpha_S(\mu); \mu/\sqrt{\zeta}\right)$$

$$\frac{\mathrm{d}\tilde{K}\left(\boldsymbol{b}_{\mathrm{T}}; \mu\right)}{\mathrm{d} \ln \mu} = -\gamma_K\left(\alpha_S(\mu)\right)$$

Same for FF

$$\mu = \sqrt{\zeta}$$
$$\mu_0 = \sqrt{\zeta_0}$$

$$\tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \sqrt{\zeta}\right) = \tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu_0, \sqrt{\zeta_0}\right) \times$$
$$\times \exp\left\{\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'}\left[\gamma\left(\alpha_S(\mu'); 1\right) - \ln\left(\frac{\sqrt{\zeta}}{\mu'}\right)\gamma_K\left(\alpha_S(\mu')\right)\right] + \ln\left(\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\tilde{K}\left(\boldsymbol{b}_{\mathrm{T}}; \mu_0\right)\right)\right\}$$

Separate $b_T < b_{max}$ & $b_T > b_{max}$ regions with a b_* prescription

3

$$\tilde{f}_{j/p}(x; \boldsymbol{b}_\mathrm{T}; \mu_Q, Q) = \tilde{f}_{j/p}(x; \boldsymbol{b}_*; \mu_Q, Q) \underbrace{\frac{\tilde{f}_{j/p}(x; \boldsymbol{b}_\mathrm{T}; \mu_Q, Q)}{\tilde{f}_{j/p}(x; \boldsymbol{b}_*; \mu_Q, Q)}}_{\exp\left\{-g_{j/p}(x, \boldsymbol{b}_\mathrm{T})\right\}}$$

Same for FF

Perturbatively calculable with fixed order collinear factorization

Nonperturbative

$$g_K(\boldsymbol{b}_\mathrm{T}) \equiv \tilde{K}(\boldsymbol{b}_*; \mu) - \tilde{K}(\boldsymbol{b}_\mathrm{T}; \mu)$$

Choose ansatzes for g functions

4

$$g_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}\right) = \frac{1}{4} M_F^2 b_{\mathrm{T}}^2$$

$$g_{h/j}\left(z, \boldsymbol{b}_{\mathrm{T}}\right) = \frac{1}{4z^2} M_D^2 b_{\mathrm{T}}^2$$

$$g_K\left(\boldsymbol{b}_{\mathrm{T}}\right) = \frac{g_2}{2M_K^2} \ln\left(1 + M_K^2 b_{\mathrm{T}}^2\right)$$

$$g_K\left(\boldsymbol{b}_{\mathrm{T}}\right) = \frac{1}{2} M_K^2 b_{\mathrm{T}}^2$$

Relate μ_{b_*} with input scale Q_0 and get OPE expansion

5

$$\tilde{f}_{j/p}\left(x;\boldsymbol{b}_\mathrm{T};\mu_Q,Q\right)=\tilde{f}^{\mathrm{OPE}}_{j/p}\left(x;\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)\times$$

$$\times\exp\left\{\int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[\gamma\left(\alpha_S(\mu');1\right)-\ln\left(\frac{Q}{\mu'}\right)\gamma_K\left(\alpha_S(\mu')\right)\right]+\ln\left(\frac{Q}{\mu_{b_*}}\right)\tilde{K}\left(\boldsymbol{b}_*;\mu_{b_*}\right)\right\}$$

$$\times\exp\left\{-g_{j/p}\left(x,\boldsymbol{b}_\mathrm{T}\right)-g_K\left(\boldsymbol{b}_\mathrm{T}\right)\ln\left(\frac{Q}{Q_0}\right)\right\}$$

Nonperturbative

Perturbatively calculable

Drop this

$$\tilde{f}^{\mathrm{OPE}}_{j/p}\left(x,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)=\widetilde{C}_{j/j'}\left(x/\xi,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)\otimes\tilde{f}_{j'/p}\left(\xi;\mu_{b_*}\right)+\mathcal{O}\left(m^2 b^2_{\mathrm{max}}\right)$$

Same for FF

Fixed order collinear factorization

Consistency checks

TMDs are uniquely determined by their operatorial definition

$$H \int \mathrm{d}^2\boldsymbol{k}_{1\mathrm{T}}\mathrm{d}^2\boldsymbol{k}_{2\mathrm{T}} f_{j/p}\left(x, \boldsymbol{k}_{1\mathrm{T}}; \mu, \sqrt{\zeta}\right) D_{h/j}\left(z, z\boldsymbol{k}_{2\mathrm{T}}; \mu, \sqrt{\zeta}\right) \delta^{(2)}\left(\boldsymbol{q}_{\mathrm{T}} + \boldsymbol{k}_{1\mathrm{T}} - \boldsymbol{k}_{2\mathrm{T}}\right)$$

At large q_T ~ Q the cross section is determined solely by fixed order collinear factorization

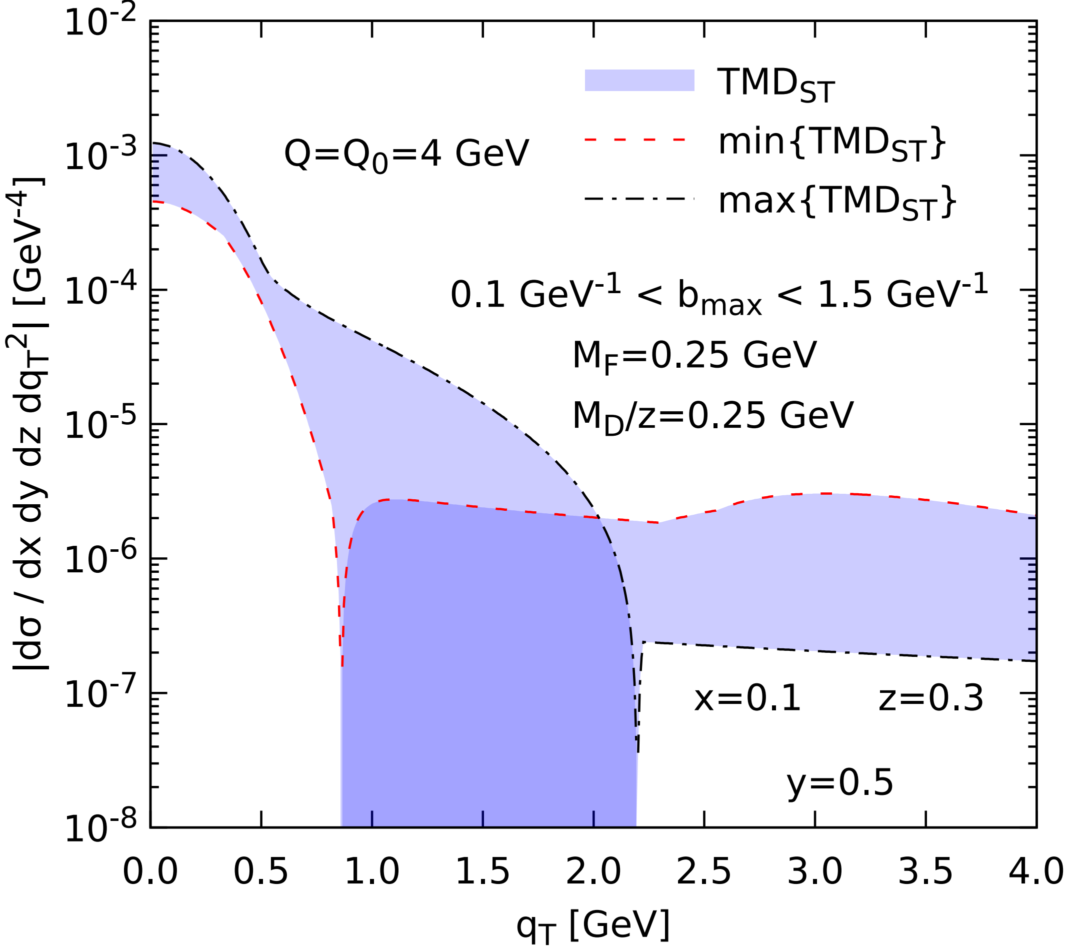
Similarly, at large TM (k_T) the TMDs are uniquely determined by an OPE expansion in terms of collinear PDFs/FFs

$$\frac{d}{db_{\max}} \frac{d\sigma}{dq_T^2 \ldots} = 0$$

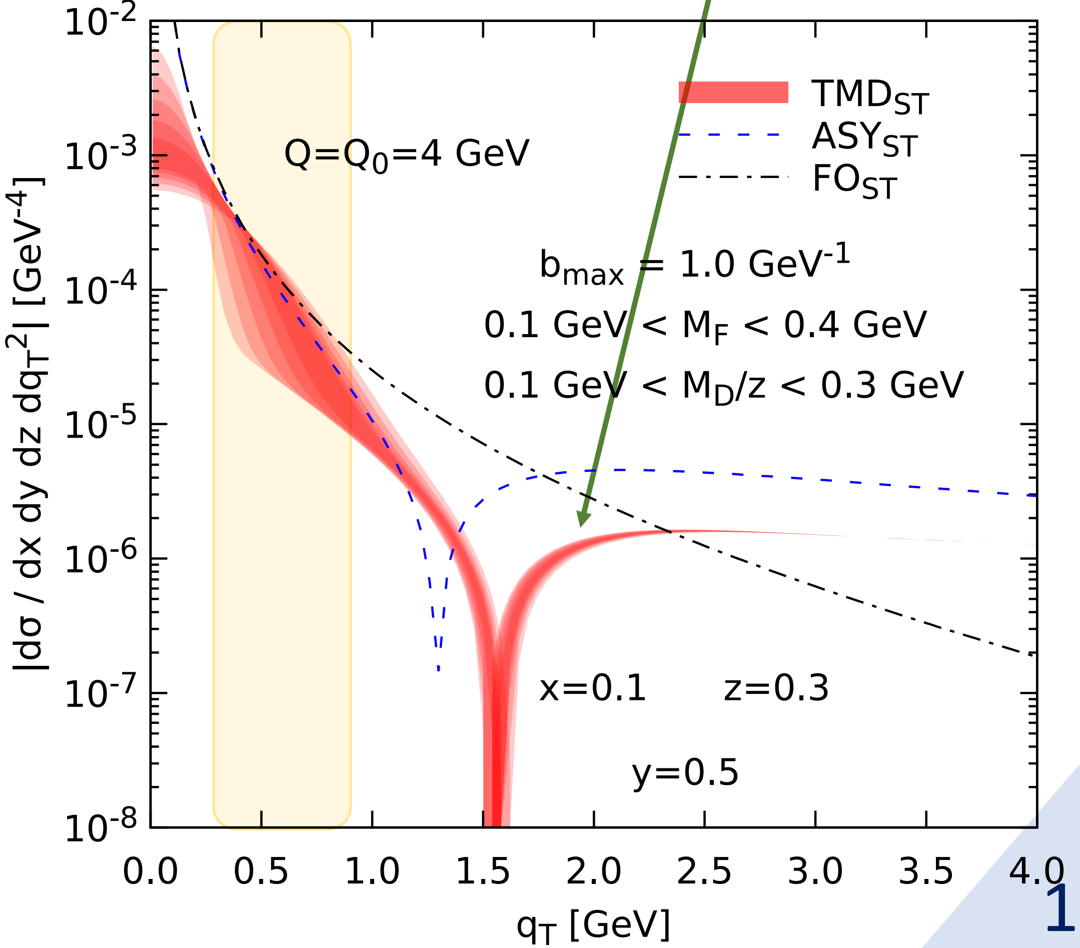
Any auxiliary parameter cannot change the Physics (b_{max} /b_{min})

(Some) Issues with conventional approach

Large b_{max} dependence

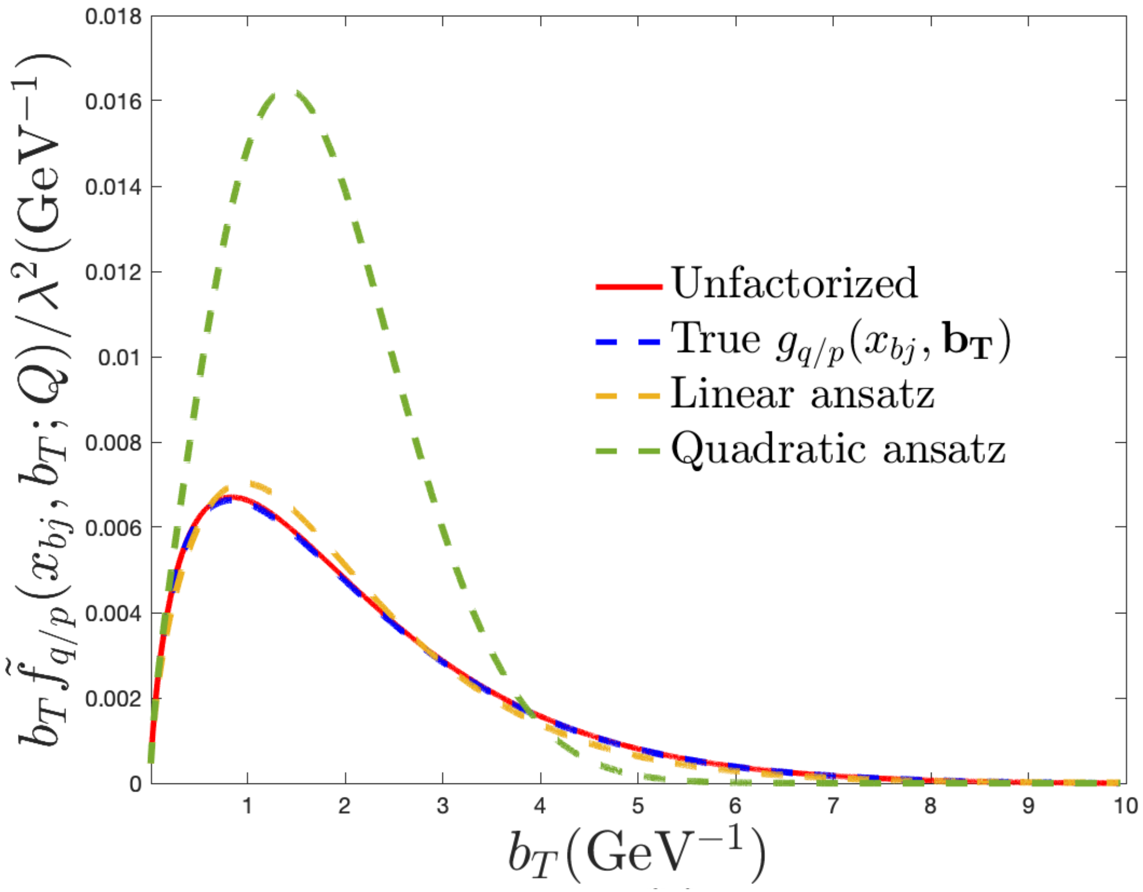


Large q_T inconsistency

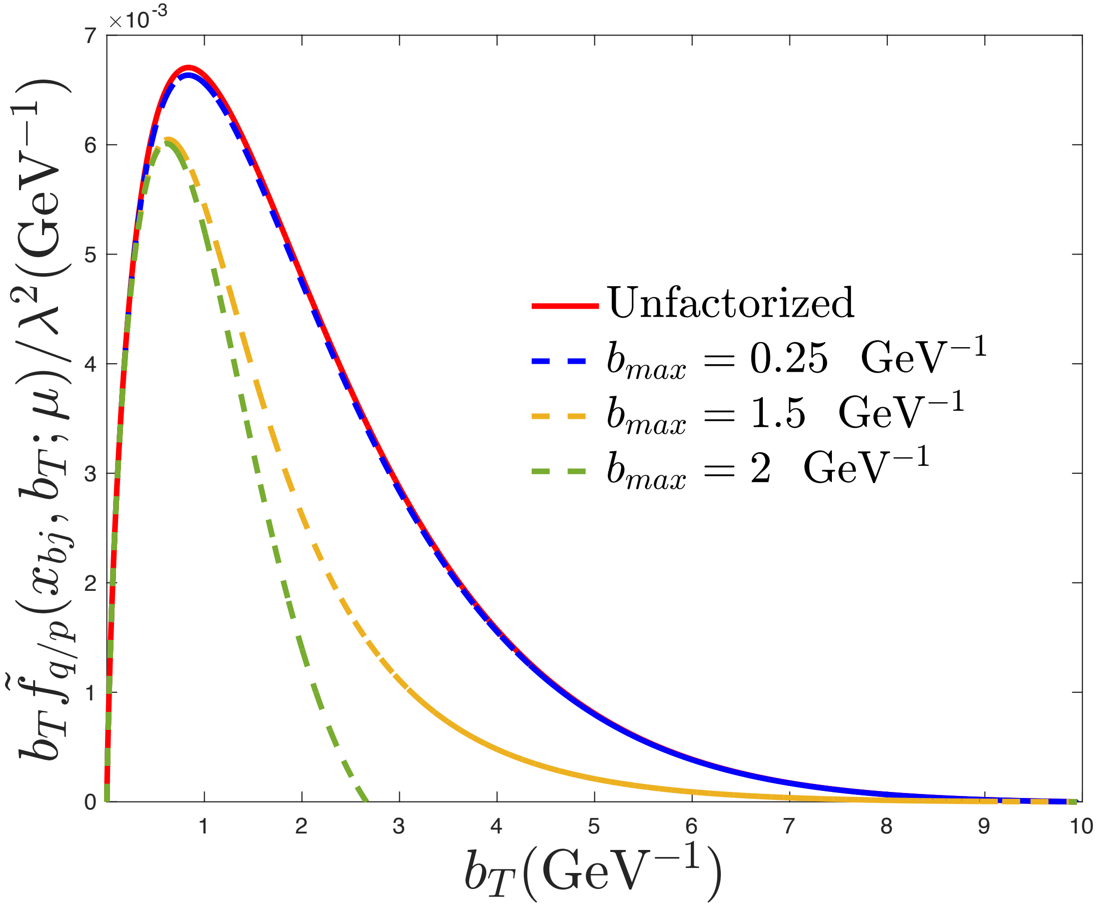


Common issues

Unconstrained g functions



Large b_{max} dependence (Yukawa)



Other issues

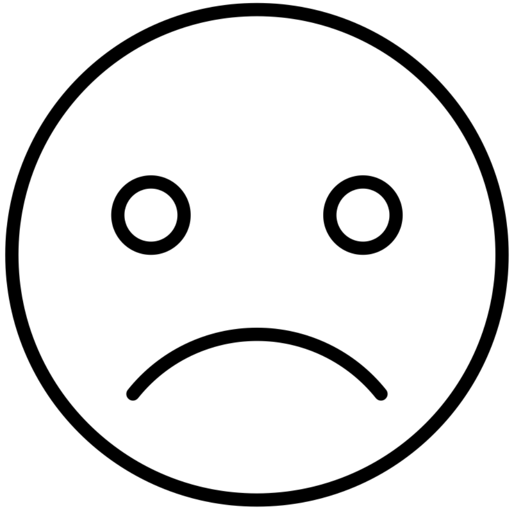
1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

2) Fixed order perturbation theory should work fine for $q_T \approx Q_0$, but evol. factors have a large effect. What is going on?

3) $\exists$ no region at input scale $Q = Q_0$ where $\Lambda_{QCD} \ll q_T \ll Q_0$

4) Backwards evolution…
No large, perturbative $\ln \frac{Q_0}{q_T}$.

5) $\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x, k_T; \mu_Q, Q^2) \approx f_{j/p}(x; \mu_Q)$
Very badly violated at moderate scales



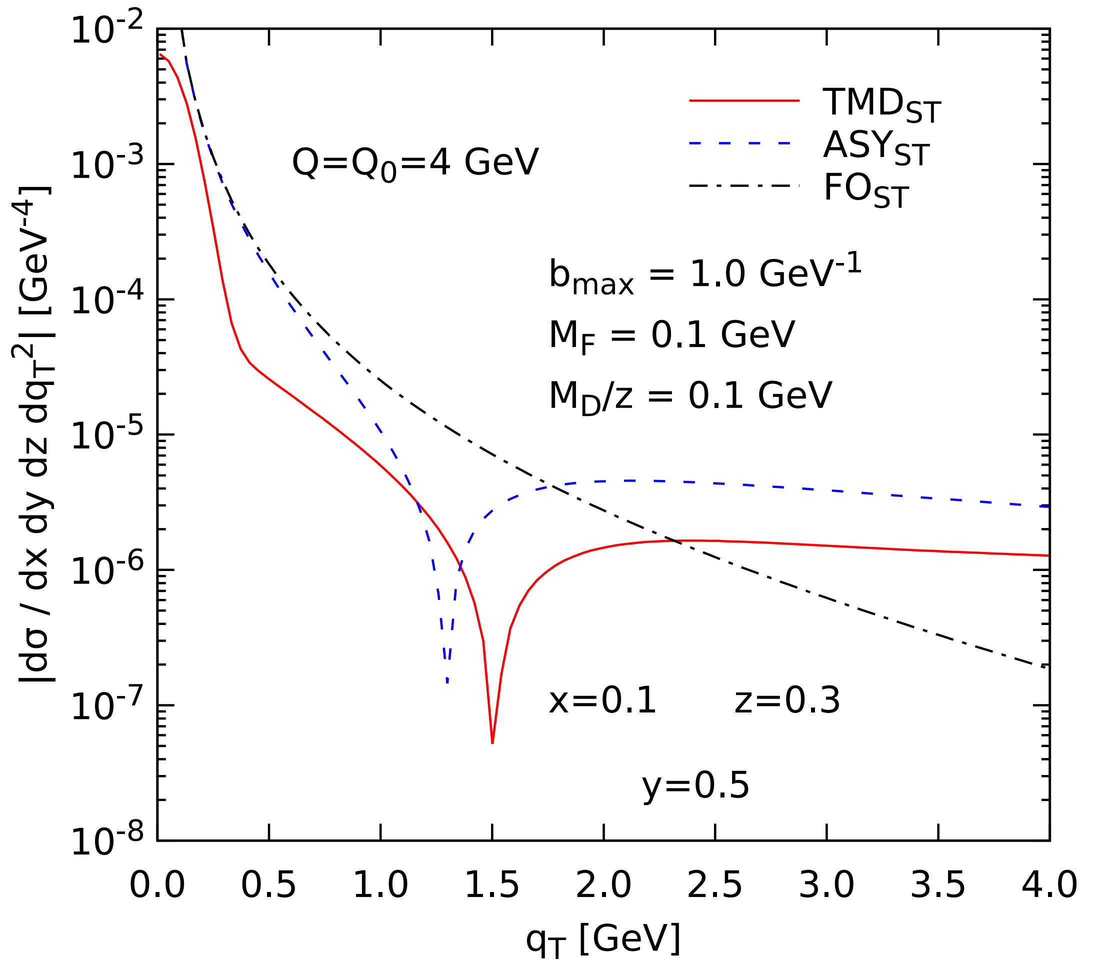
Hadron Structure Oriented approach (HSO)

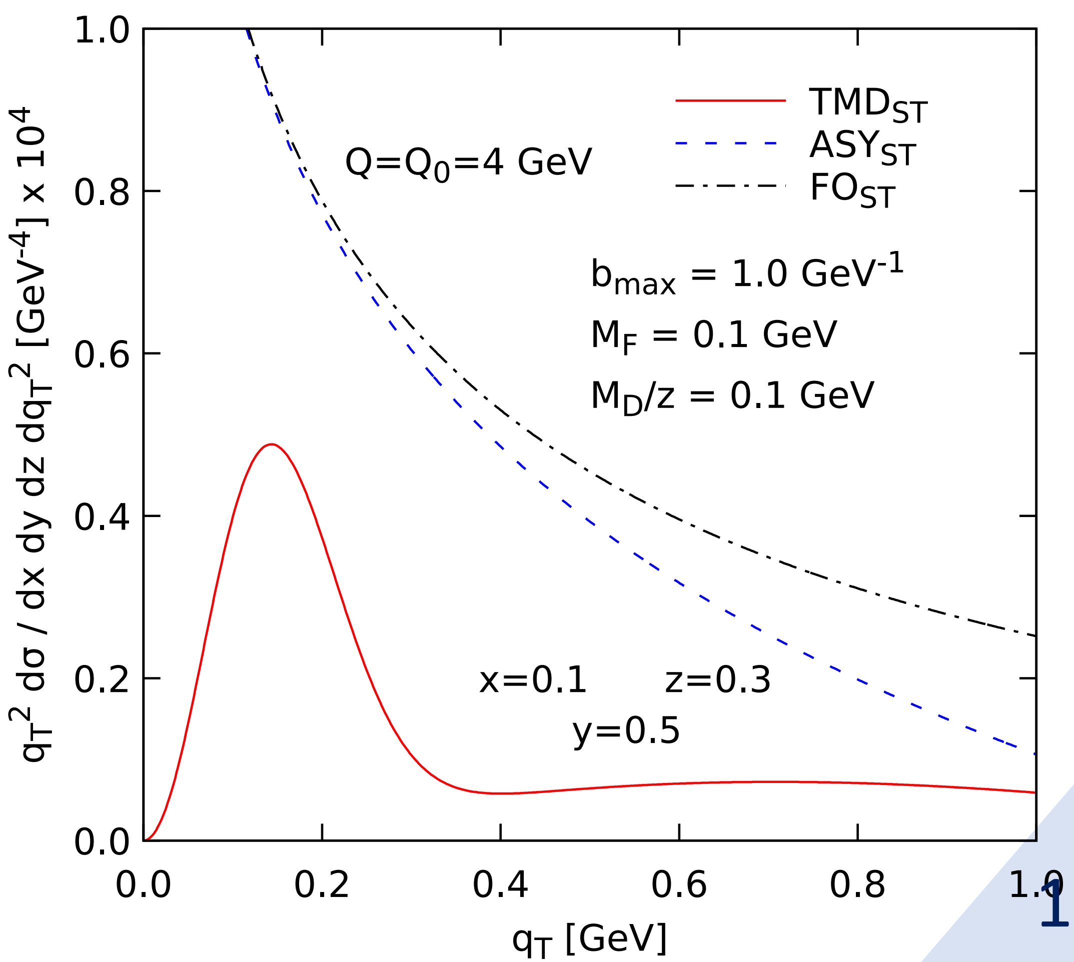
- Fixes TMDs parametrization at **input scale** Q_0
- Uses uniquely determined TMDs for **all transverse momenta**
- Interpolates perturbative (large k_T) and nonperturbative (small k_T) TM regions
- Can **swap NP models** easily
- Explicit (approximate) **probability interpretation**

No need for b_* prescription !

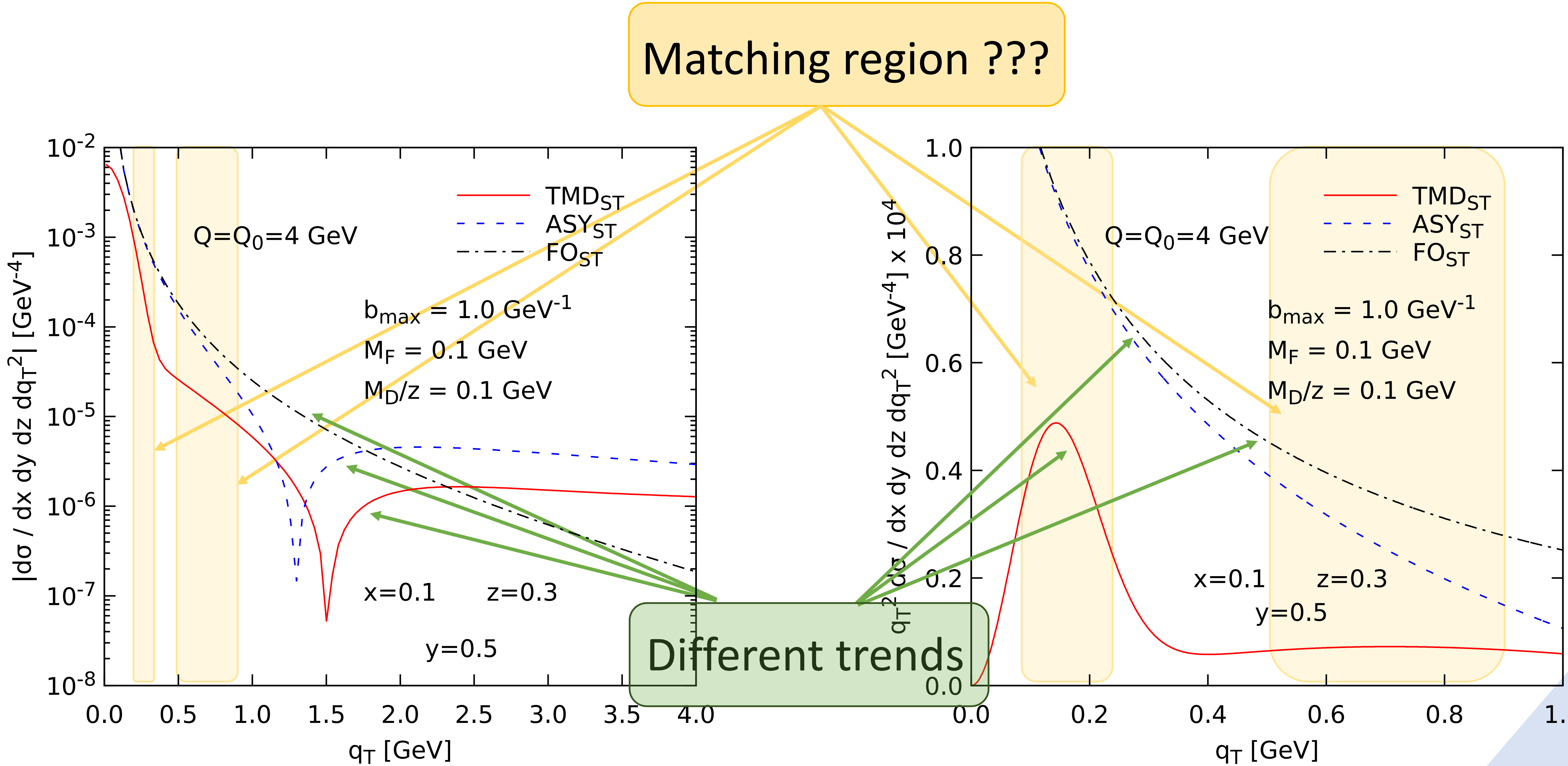
Conventional approach results for SIDIS

Matching region ? $\Lambda_{\rm QCD} \ll q_{\rm T} \ll Q_0$



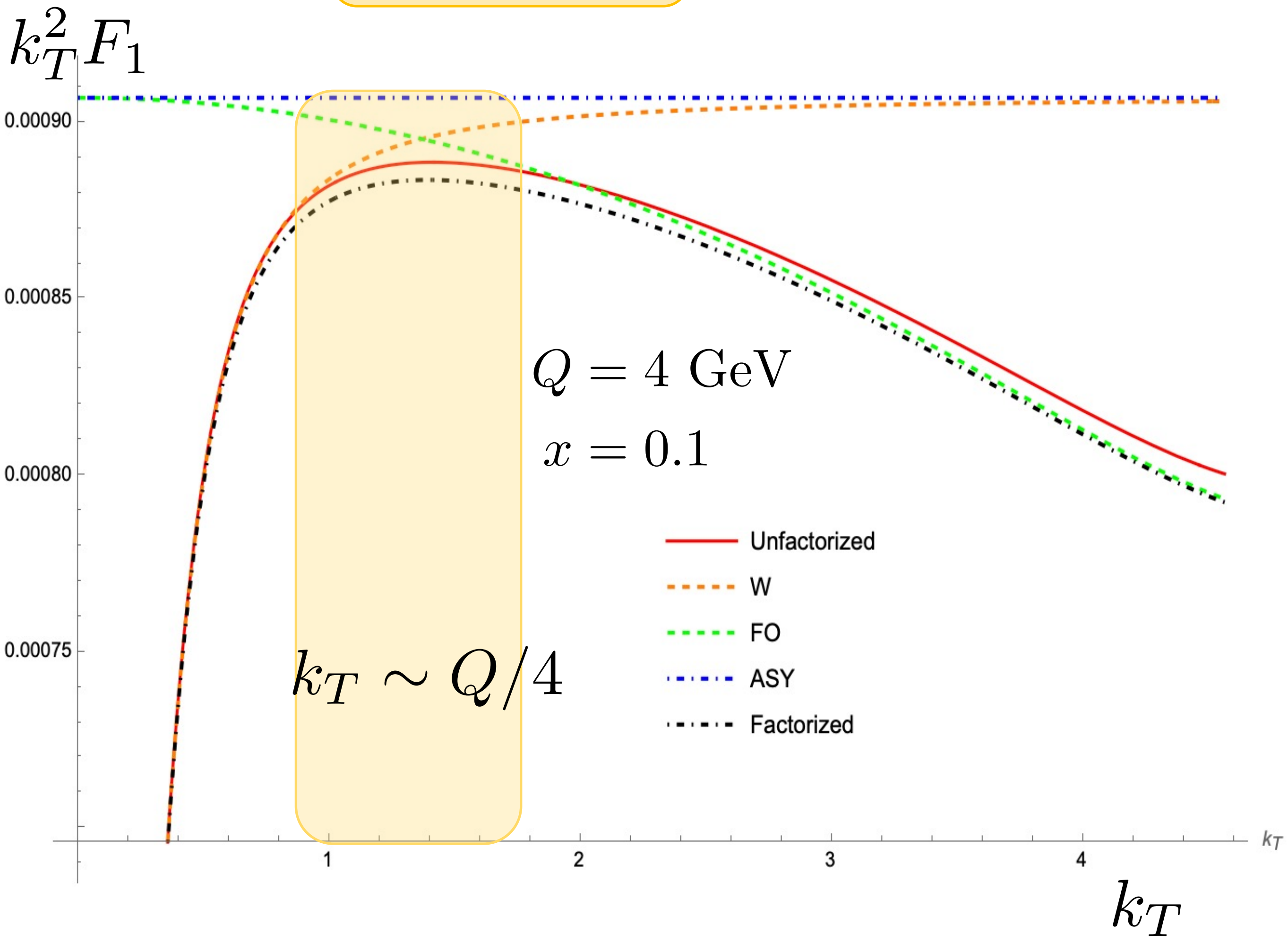


Conventional approach results for SIDIS

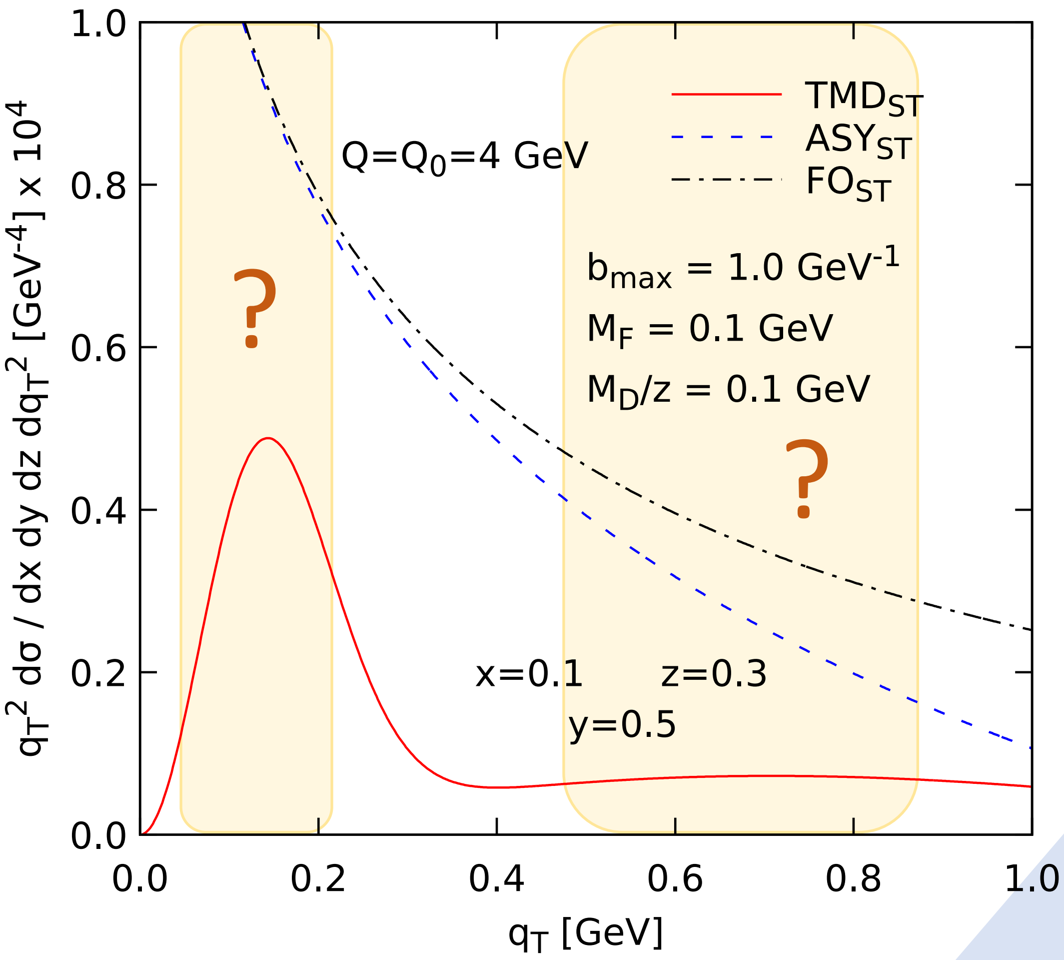


Matching region in SIDIS cross section?

Yukawa

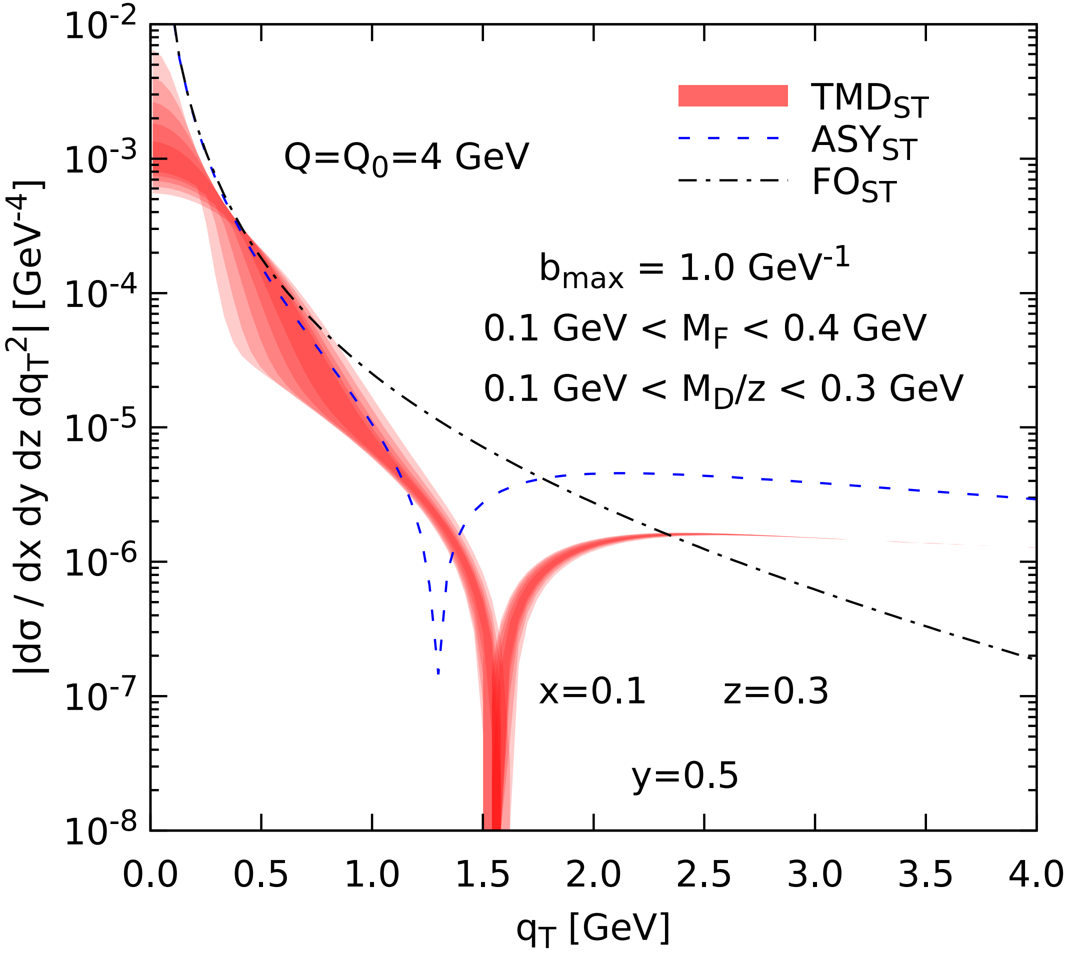


QCD (conventional)

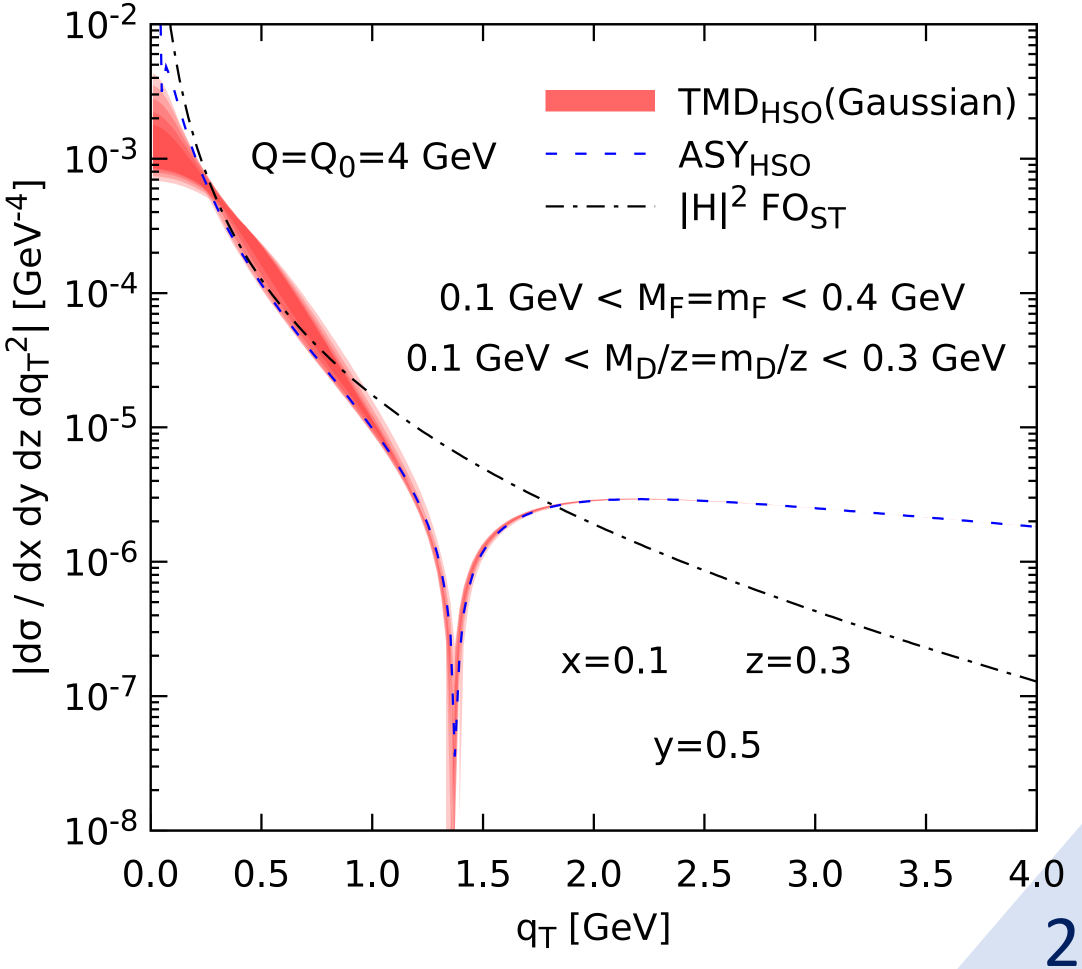


Conventional vs HSO - SIDIS cross section

Conventional

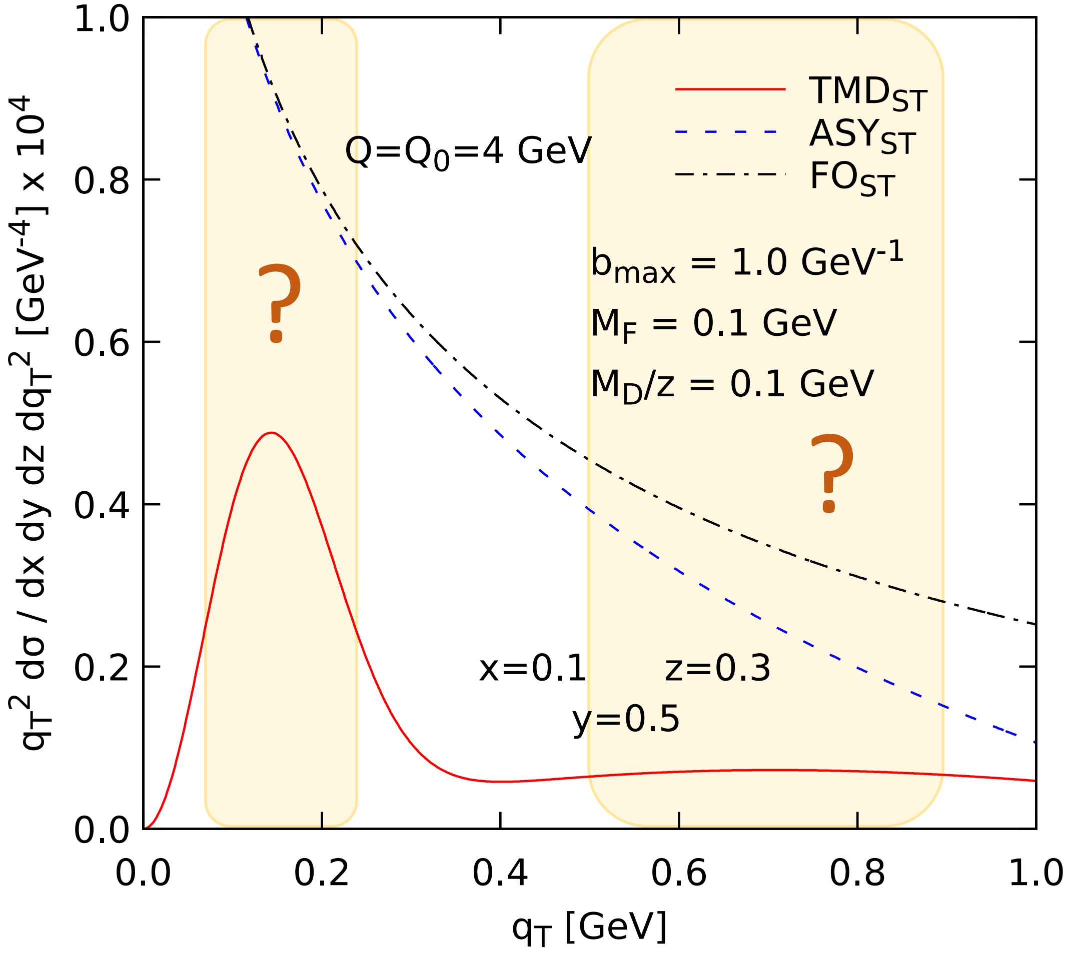


HSO (Gaussian)

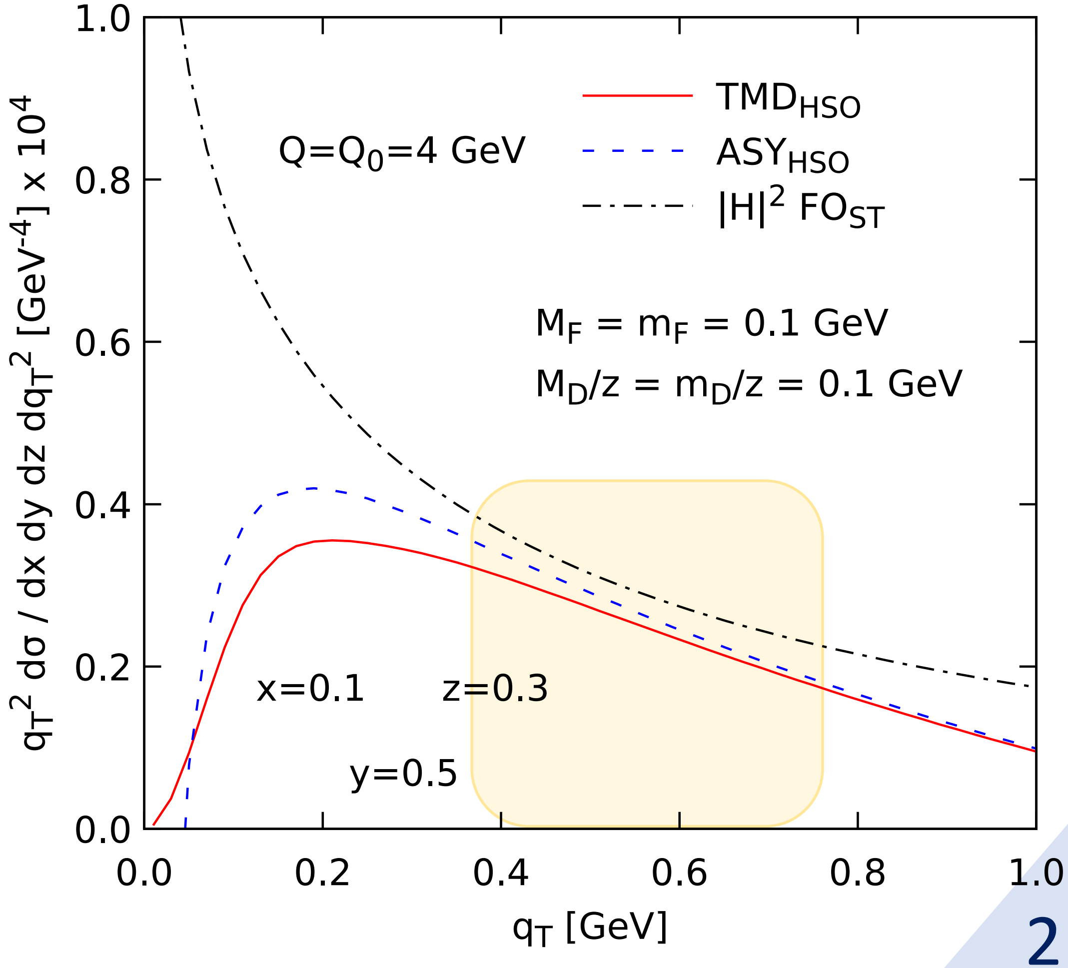


Conventional vs HSO - SIDIS cross section

Conventional



HSO (Gaussian)



The HSO setup

- Start with the perturbative expansion of the TMD and CS-kernel
- "Regularize" with mass (fit) parameter $k_T^2 \to k_T^2 + m^2$
- Add "core" model (small k_T / large b_T)
- Make sure **RG equations**, **OPE limits** and **integral relations** are satisfied
- RG improvement at the input scale via $\overline{Q_0}(b_T)$ prescription
- Evolution to higher scales

This is usual CSS

TMD from collinear factorization

$$\tilde{f}_i^{\mathrm{OPE}}(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \zeta = \mu^2)$$

$$= \sum_{n=0}^{\infty} \sum_{k=0}^{2n} a_S^n L_b^k \widetilde{C}_{ij}^{(n,k)}(x) \otimes f_j(x; \mu)$$

$\overline{\mathrm{MS}}$ OPE coefficients

$\overline{\mathrm{MS}}$ PDF

QCD running coupling

$$L_b \equiv \ln\left(\frac{\mu b_T}{2e^{-\gamma_E}}\right)$$

Map to k_T space

$$L_b^k \mapsto \mathrm{FT}\left\{L_b^k\right\}(k_T) \equiv \mathcal{F}^{(k)}(k_T)$$

$$\mathcal{F}^{(k)}(k_T) \to \overline{\mathcal{F}}^{(k)}(k_T, m) \equiv \mathcal{F}^{(k)}(k_T^2 + m^2)$$

$$\overline{\mathcal{F}}^{(k)} \equiv M_{kn}\Phi^{(n)} = M_{kn}\frac{1}{2\pi}\frac{\ln^n\left(\frac{\mu^2}{k_T^2+m^2}\right)}{k_T^2+m^2}$$

Map to k_T space

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & -\frac{3}{4} & 0 & 0 & 0 & 0 & 0 & \cdots \\
2\zeta(3) & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & \cdots \\
0 & 5\zeta(3) & 0 & 0 & -\frac{5}{16} & 0 & 0 & 0 & \cdots \\
9\zeta(5) & 0 & \frac{15}{2}\zeta(3) & 0 & 0 & -\frac{3}{16} & 0 & 0 & \cdots \\
-\frac{35}{2}\zeta(3)^2 & \frac{63}{2}\zeta(5) & 0 & \frac{35}{4}\zeta(3) & 0 & 0 & -\frac{7}{64} & 0 & \cdots \\
90\zeta(7) & -70\zeta(3)^2 & 63\zeta(5) & 0 & \frac{35}{4}\zeta(3) & 0 & 0 & -\frac{1}{16} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
$$

$$L_b^k \mapsto \mathrm{FT}\left\{L_b^k\right\}(k_T) \equiv \mathcal{F}^{(k)}(k_T)$$

$$\mathcal{F}^{(k)}(k_T) \to \mathcal{F}^{(k)}(k_T, m) \equiv \mathcal{F}^{(k)}(k_T^2 + m^2)$$

$$\overline{\mathcal{F}}^{(k)} \equiv M_{kn}\Phi^{(n)} = M_{kn}\frac{1}{2\pi}\frac{\ln^n\left(\frac{\mu^2}{k_T^2+m^2}\right)}{k_T^2+m^2}$$

Back to b_T space

$$\mathcal{L}^{(n)} \equiv \int \mathrm{d}^2\mathbf{k}_T e^{i\mathbf{k}_\mathrm{T}\cdot\mathbf{b}_\mathrm{T}} \Phi^{(n)}$$

$$\mu_b \equiv \frac{2e^{-\gamma_E}}{b_T}$$

Examples:

$$\mathcal{L}^{(0)} = K_0\left(mb_T\right) \overset{mb_T\to 0}{\to} -\ln\left(\frac{m}{\mu_b}\right)$$

$$\mathcal{L}^{(1)} = K_0\left(mb_T\right) \ln\left(\frac{\mu^2}{m\mu_b}\right) \overset{mb_T\to 0}{\to} -\ln\left(\frac{m}{\mu_b}\right)\ln\left(\frac{\mu^2}{m\mu_b}\right)$$

The input HSO TMD pdf

$$f_i^{\text{input}}(x, \boldsymbol{k}_T; \mu, \mu^2) = \sum_{l=1}^{\infty} \left(A_{ij}^{[l]} \otimes f_j \right) M_{lp} \Phi^{(p)} + C f_{i,core}$$

$$\tilde{f}_i^{\text{input}}(x, \boldsymbol{b}_\mathrm{T}; \mu, \mu^2) = \sum_{l=1}^{\infty} \left(A_{ij}^{[l]} \otimes f_j \right) M_{lp} \mathcal{L}^{(p)} + C \tilde{f}_{i,core}$$

Fixed power of log

$$A_{ij}^{[l]}(x, a_S(\mu)) \equiv \sum_{n=0}^{\infty} a_S^n \widetilde{C}_{ij}^{(n,l)}(x)$$

C is **not** just a normalization

At small b_T the OPE expansion is recovered

Cutoff Collinear PDF

Choose “core” models (examples)

$$f^{\text{Gauss}}_{\text{core},i/p}\left(x,\boldsymbol{k}_{\text{T}};Q_0^2\right) = \frac{e^{-k_{\text{T}}^2/M_F^2}}{\pi M_F^2}$$

$$D^{\text{Gauss}}_{\text{core},h/j}\left(z,z\boldsymbol{k}_{\text{T}};Q_0^2\right) = \frac{e^{-z^2k_{\text{T}}^2/M_D^2}}{\pi M_D^2}$$

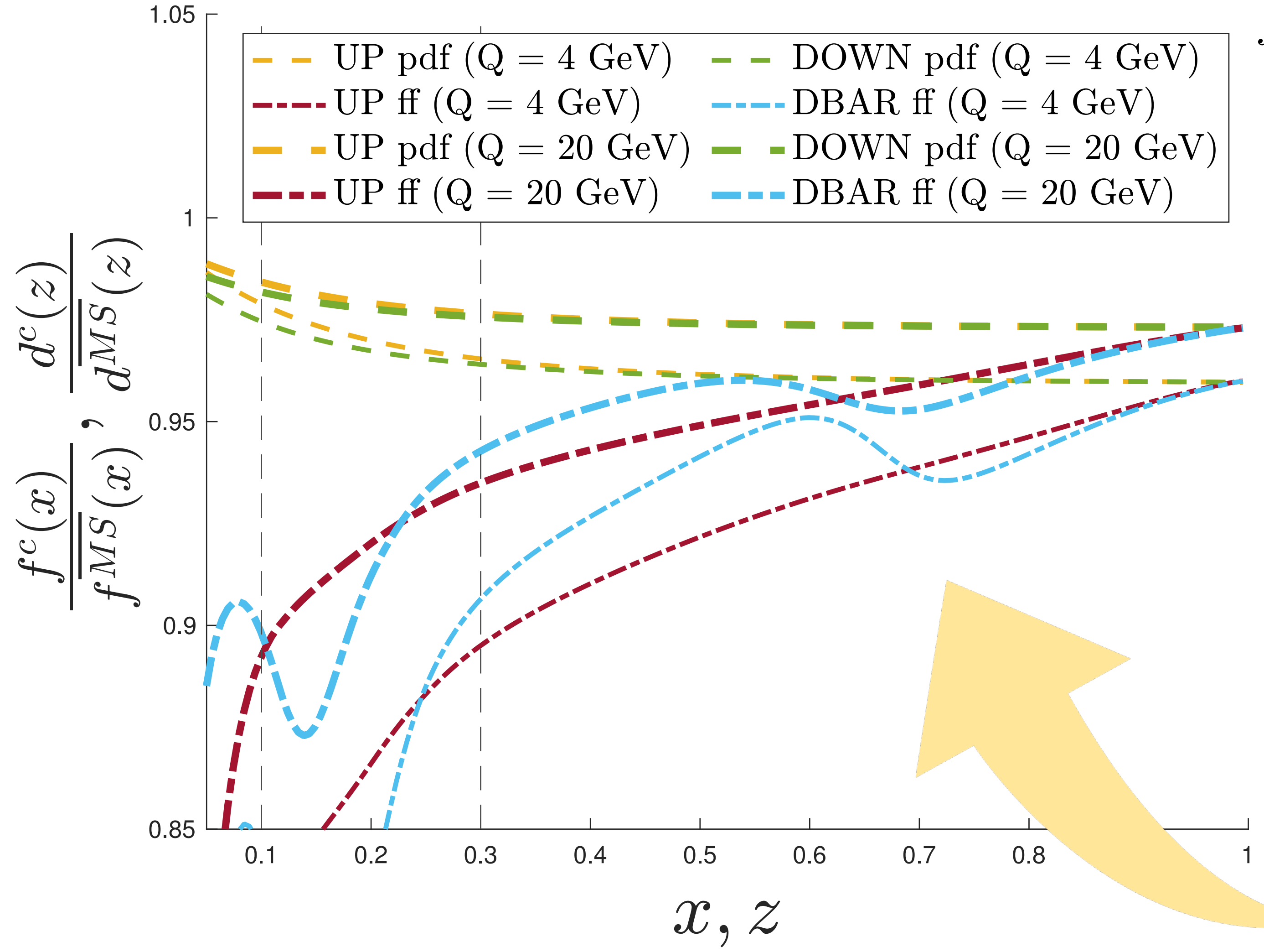
Gaussian “core” models

Spectator-like “core” models

$$f^{\text{Spect}}_{\text{core},j/p}\left(x,\boldsymbol{k}_{\text{T}};Q_0^2\right) = \frac{6M_{0F}^6}{\pi\left(2M_F^2+M_{0F}^2\right)}\frac{M_F^2+k_{\text{T}}^2}{\left(M_{0F}^2+k_{\text{T}}^2\right)^4}$$

$$D^{\text{Spect}}_{\text{core},h/j}\left(z,z\boldsymbol{k}_{\text{T}};Q_0^2\right) = \frac{2M_{0D}^4}{\pi\left(M_D^2+M_{0D}^2\right)}\frac{M_D^2+z^2k_{\text{T}}^2}{\left(M_{0D}^2+z^2k_{\text{T}}^2\right)^3}$$

Pseudo-probability distribution property saved



$$f_i^c(x;\mu,k_c) \equiv \pi \int_0^{k_c^2} \mathrm{d}k_T^2 f_{i/p,\mathrm{input}}(x,k_T;\mu,\zeta)$$

$$= f_i + C_{ij,\Delta}^c \otimes f_j + p.s.$$

$\overline{\mathrm{MS}}$ PDF

Completely determined by OPE expansion coefficients

It might make a BIG difference

Collinear Evolution

$$\text{Note}: \lim_{a_S \to 0} C_\Delta^c = 0$$

$$\frac{\mathrm{d} f_i^c}{\mathrm{d}\ln\mu} \equiv 2P_{ij}^c \otimes f_j^c + p.s.$$

$$= 2P_{ij} \otimes f_j + C_{\Delta,ij}^c \otimes 2P_{jk} \otimes f_k + \frac{\mathrm{d}C_{\Delta,ij}^c}{\mathrm{d}\ln\mu} \otimes f_j + \frac{\mathrm{d}p.s.}{\mathrm{d}\ln\mu}$$

Usual evolution

Additional term (scheme change)

Power suppressed

More on C

$$\Phi^{(p)}_{0,\mu} = \frac{1}{2(1+p)} \ln^{1+p}\left(\frac{\mu^2}{m^2}\right)$$

Important: C does **not** depend on the cutoff

$$C = f_i^{\overline{\mathrm{MS}}} + \underbrace{\left[A_{ij}^{[0]} \otimes f_j^{\overline{\mathrm{MS}}} - f_i^{\overline{\mathrm{MS}}} - \sum_{l=1}^{\infty} \left(A_{ij}^{[l]} \otimes f_j^{\overline{\mathrm{MS}}} \right) M_{lp} \overline{\mathcal{L}}_0^{(p)} \right]}_{C_{\Delta,ij} \otimes f_j^{\overline{\mathrm{MS}}}} - \sum_{l=1}^{\infty} \left(A_{ij}^{[l]} \otimes f_j^{\overline{\mathrm{MS}}} \right) M_{lp} \Phi_{0,\mu}^{(p)}$$

Cutoff Collinear PDF
(no p.s. and only μ dependence)

Necessary to satisfy
integral and OPE relations

Paper notation (LO)

A more familiar notation

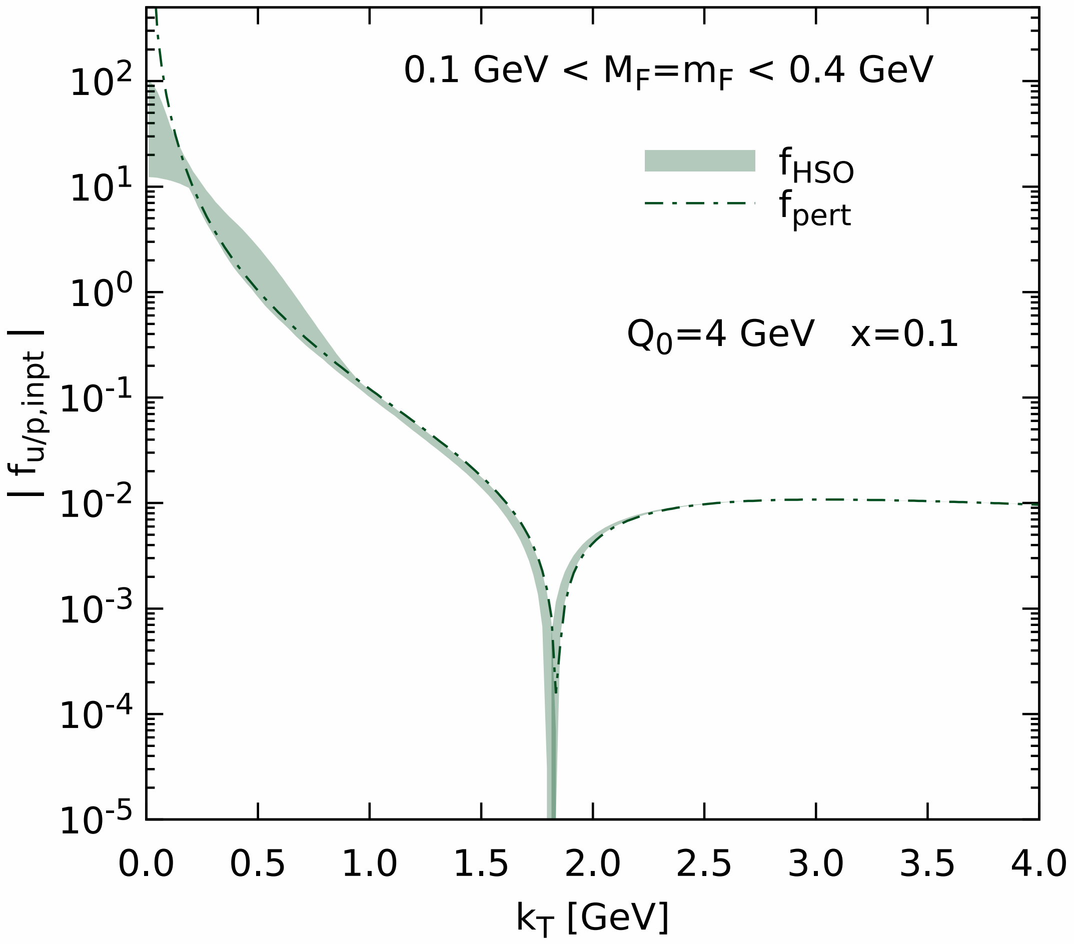
$$f_{\mathrm{inpt,i/p}}(x,\boldsymbol{k}_T;\mu_{Q_0};Q_0^2) = \frac{1}{2\pi}\frac{1}{k_T^2+m^2}\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_T^2+m^2}\right] + \frac{1}{2\pi}\frac{1}{k_T^2+m^2}A^{f,g}_{i/p}(x;\mu_{Q_0})$$
$$+ C^f_{i/p} f_{\mathrm{core,i/p}}(x,\boldsymbol{k_T};Q_0^2)$$

$$= \underbrace{\left(A^f_{i/p} + A^{f,g}_{i/p}\right)}_{M_{10}A^{[1]}\otimes f}\underbrace{\frac{1}{2\pi}\frac{1}{k_T^2+m^2}}_{\Phi^{(0)}} + \underbrace{B^f_{i/p}}_{M_{21}A^{[2]}\otimes f}\underbrace{\frac{1}{2\pi}\frac{\ln\left(\frac{Q_0^2}{k_T^2+m^2}\right)}{k_T^2+m^2}}_{\Phi^{(1)}} + C^f_{i/p} f_{\mathrm{core,i/p}}$$

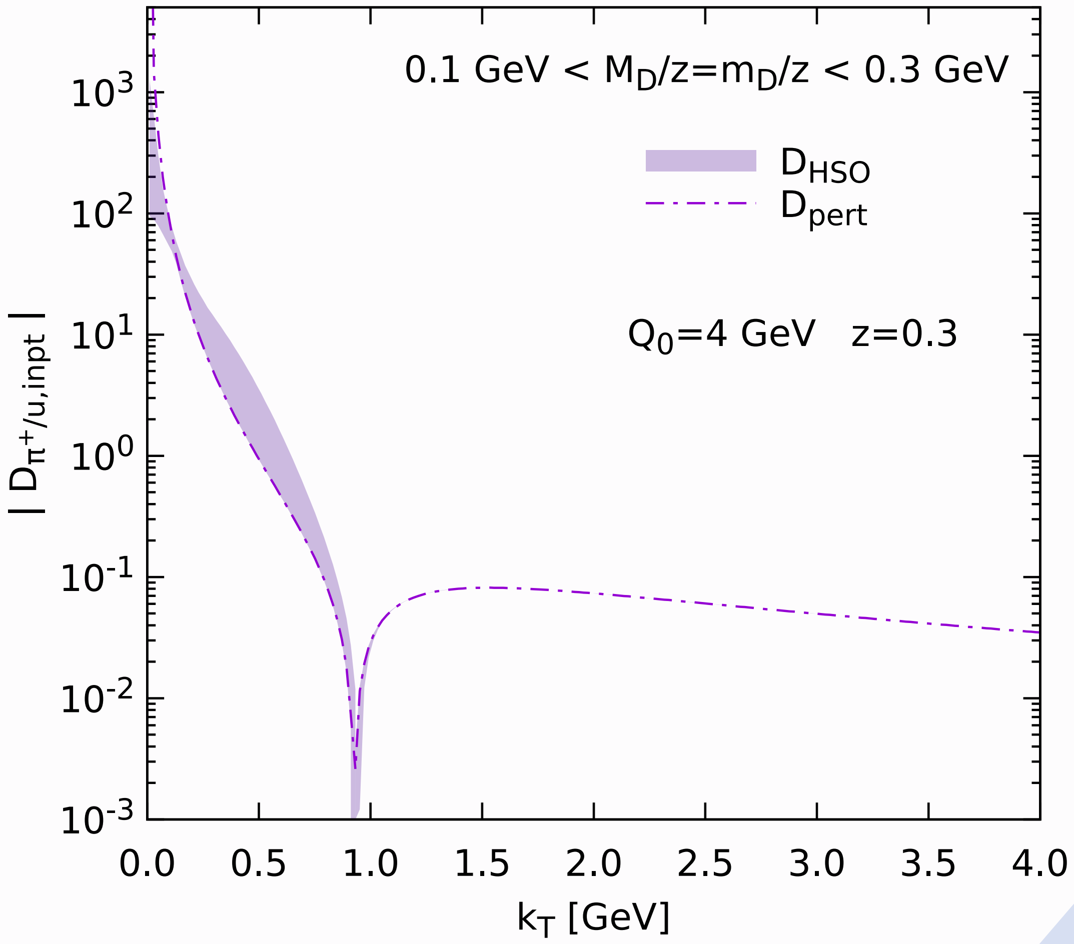
Leading Log terms (LL) at order a_s

Next-to-Leading Log terms (NLL) at order a_s

Up-quark from Proton TMD pdf



π^+ from Up-quark TMD ff



Higher orders

GRIDS of these coefficients are/will be available

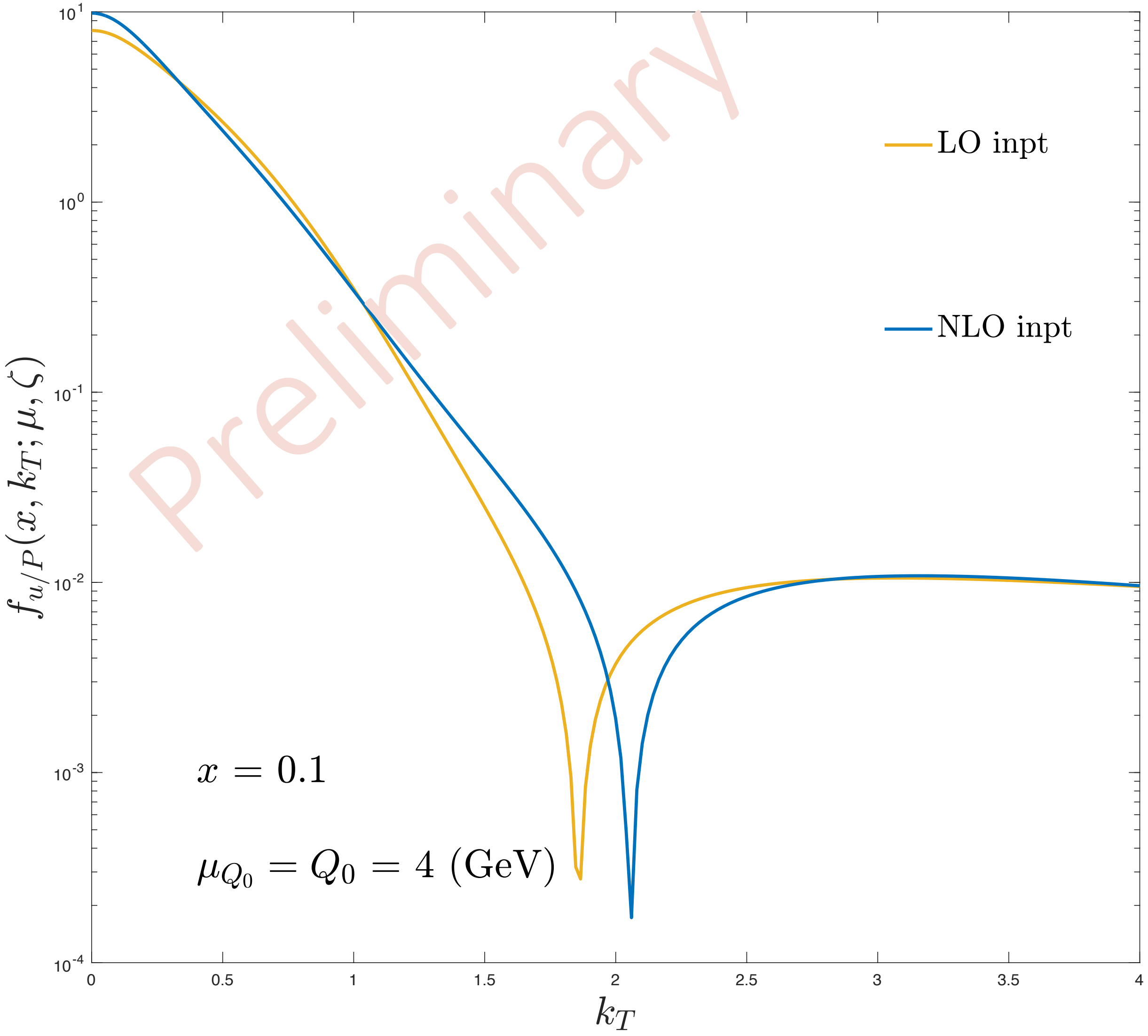
What do we need?

input	Const.	LL	NLL	N2LL	N3LL	...
LO	$A_{ij}^{[0]} \otimes f_j$	$A_{ij}^{[1]} \otimes f_j$	$A_{ij}^{[2]} \otimes f_j$	$\nexists$	$\nexists$	
NLO	$A_{ij}^{[0]} \otimes f_j$	$A_{ij}^{[1]} \otimes f_j$	$A_{ij}^{[2]} \otimes f_j$	$A_{ij}^{[3]} \otimes f_j$	$A_{ij}^{[4]} \otimes f_j$	...
$\vdots$						

HSO input CS - Kernel

HSO input TMD pdf/ff

LO vs NLO (example)



The shift of the TMD node seems consistent with what is extracted from fits, i.e.:

The true node is to the right of the "perturbative" one

However, a better investigation is necessary before drawing any rushed conclusions

HSO Collins-Soper Kernel

$$\widetilde{K}^{\rm OPE}(\boldsymbol{b}_{\rm T};\mu) = \sum_{n=1}^{\infty}\sum_{k=0}^{n} a_S^n L_b^k \hat{k}_{(n,k)}$$

$$\widetilde{K}_{\rm input}(b_T, a_S) = \sum_{n=1}^{\infty}\sum_{k=1}^{n} a_S^n M_{kl} \mathcal{L}^{(l)} \hat{k}_{(n,k)} + C_K + b_K \widetilde{K}_{core}$$

NP parameter

Large b_T model

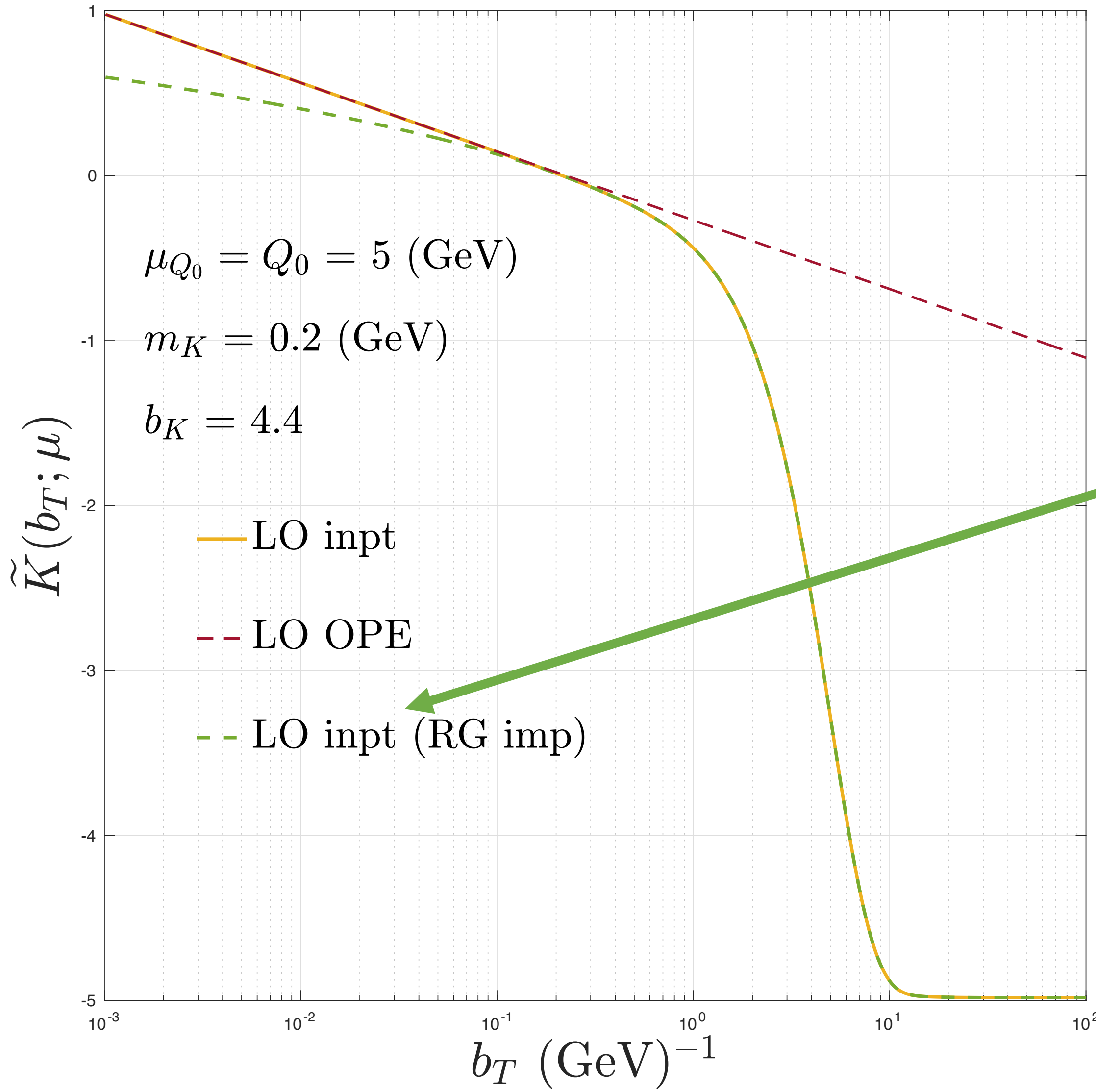
e.g.:

$$\sim e^{-b_T^2 m_K^2} - 1$$

NP parameter

Such that OPE limit and RG equations are satisfied

RG improvements (LO example)



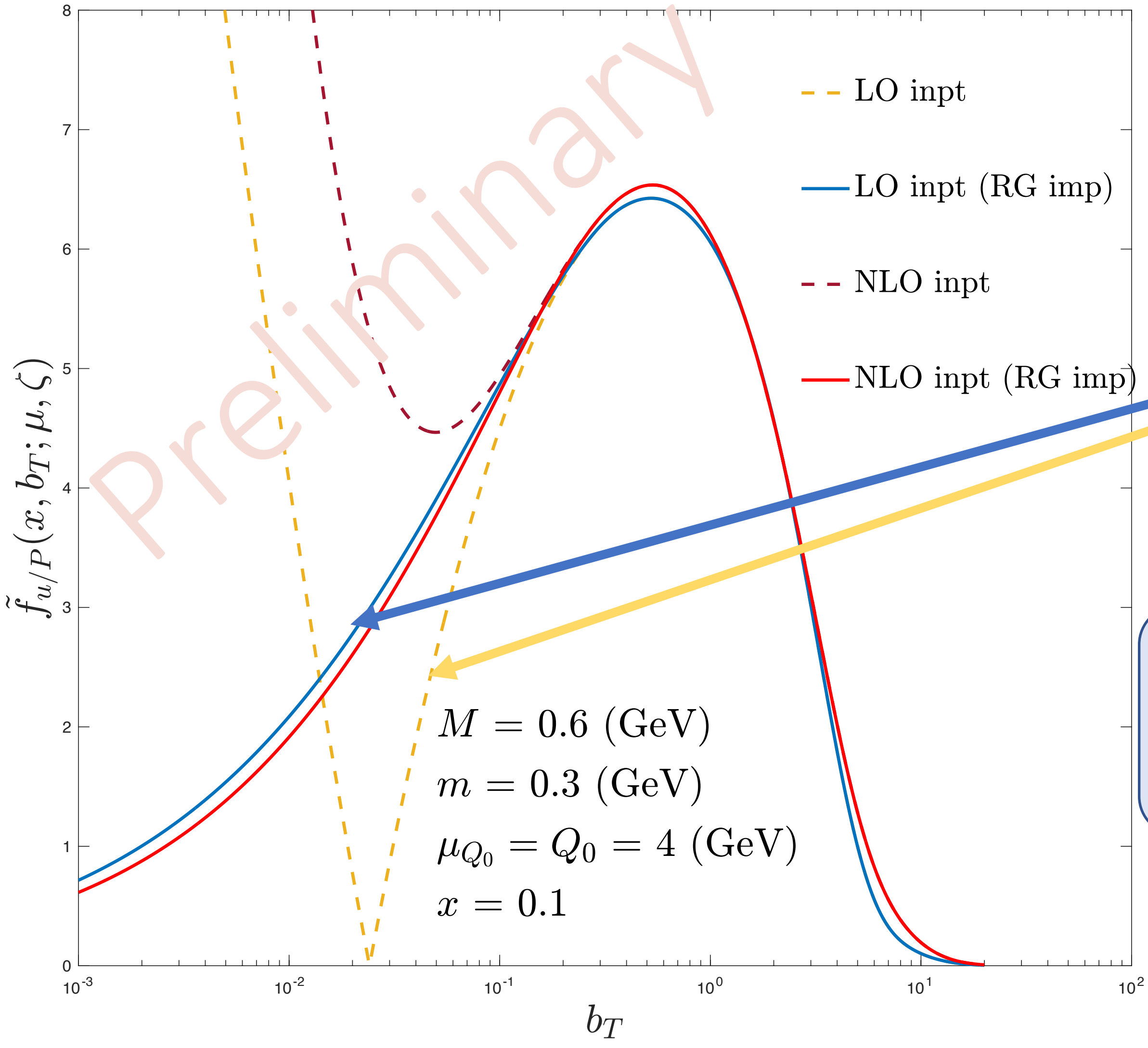
$$\overline{Q_0}\,(b_T, a) = Q_0 \left[1 - \left(1 - \frac{C_1}{Q_0 b_T}\right) e^{-a^2 b_T^2}\right]$$

$$\underline{\widetilde{K}}(b_T;\mu_{Q_0}) \equiv \widetilde{K}(b_T;\mu_{\overline{Q_0}}) - \int_{\mu_{\overline{Q_0}}}^{\mu_{Q_0}} \frac{\mathrm{d}\mu'}{\mu'} \gamma_K\left(a_S(\mu')\right)$$

A good approximation even for $b_T < 1/Q_0$

NO b_* and/or b_{max} / b_{min} necessary

Input scale RG improvement

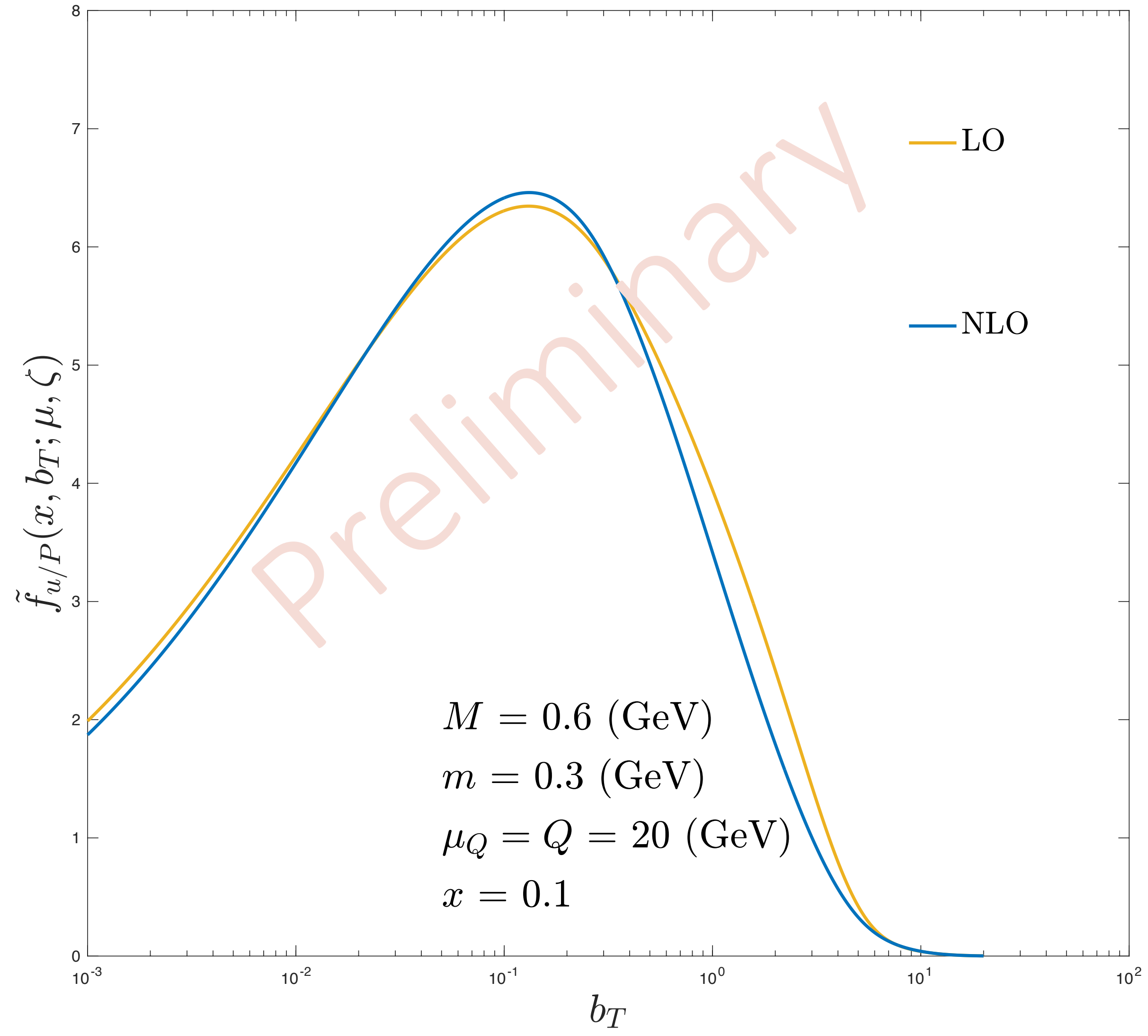


INPUT SCALE 4 GeV

$$\overline{Q_0}\,(b_T, a) = Q_0\left[1 - \left(1 - \frac{C_1}{Q_0 b_T}\right) e^{-a^2 b_T^2}\right]$$

$$\tilde{f}_i(x, b_T; \mu_{Q_0}, Q_0^2) = \tilde{f}_{i,\text{input}}(x, b_T; \mu_{\overline{Q_0}}, \overline{Q_0}^2)\times$$
$$\exp\left\{\int_{\mu_{\overline{Q_0}}}^{\mu_{Q_0}} \frac{\mathrm{d}\mu'}{\mu'}\left[\gamma\left(a_S(\mu')\right) - \ln\frac{Q_0}{\mu'}\gamma_K\left(a_S(\mu')\right)\right] + \ln\frac{Q_0}{\overline{Q_0}}\widetilde{K}_\text{input}\left(b_T; \mu_{\overline{Q_0}}\right)\right\}$$

Evolution to higher scales



Evolved to 20 GeV with HSO Collins-Soper Kernel

$$\underline{\widetilde{K}}\,(b_T;\mu) = \widetilde{K}_{\text{input}}\left(b_T;\mu_{\overline{Q_0}}\right) - \int_{\mu_{\overline{Q_0}}}^{\mu} \frac{\mathrm{d}\mu'}{\mu}\gamma_K\left(a_S(\mu')\right)$$

Check: the RG equations are satisfied

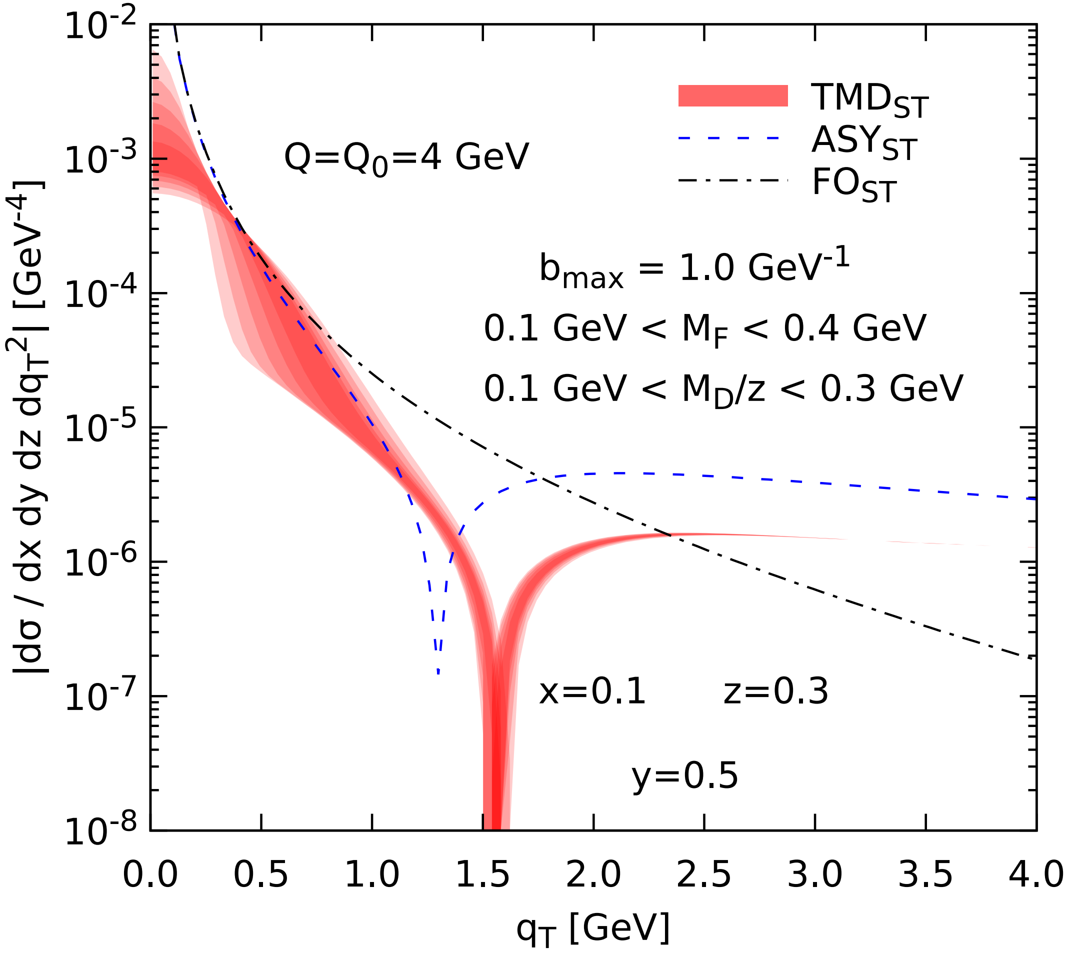
$$\frac{\partial \ln \tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \sqrt{\zeta}\right)}{\partial \ln \sqrt{\zeta}} = \tilde{K}\left(\boldsymbol{b}_{\mathrm{T}}; \mu\right)$$ ✓

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \sqrt{\zeta}\right)}{\mathrm{d} \ln \mu} = \gamma\left(\alpha_S(\mu); \mu/\sqrt{\zeta}\right)$$ ✓

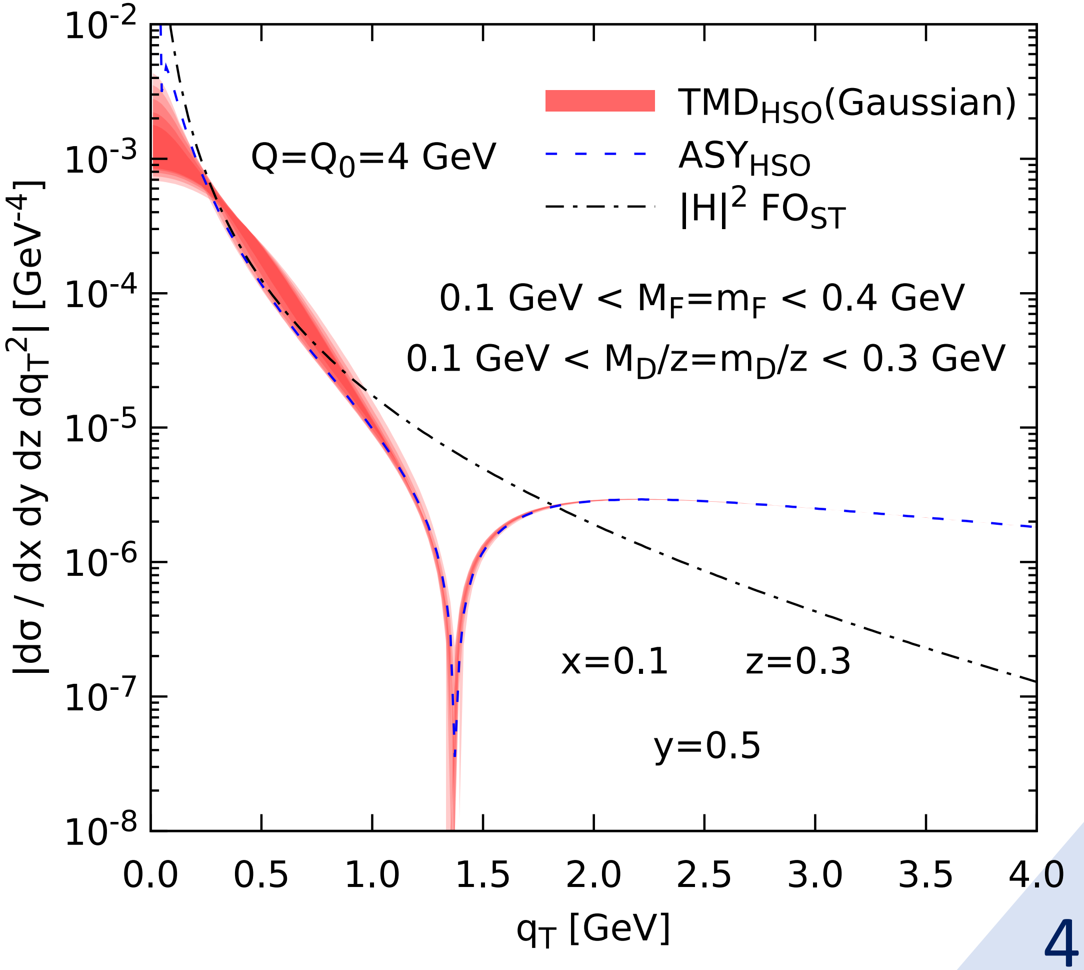
$$\frac{\mathrm{d}\tilde{K}\left(\boldsymbol{b}_{\mathrm{T}}; \mu\right)}{\mathrm{d} \ln \mu} = -\gamma_K\left(\alpha_S(\mu)\right)$$ ✓

Conventional vs HSO - SIDIS cross section

Conventional



HSO (Gaussian)



Summary

- Consistent TMD parametrization for large TM at input scale

- No need of b_{max}

- Improved TM behavior in matching region

NEXT/SOON:

- Check with data (SIDIS, DY, DIA, …)
- Add higher orders
- Incorporate NP calculations (lattice, EFT, …)

Thank you

Backup slides

Asymptotic term

HSO

$$\mathrm{ASY_{HSO}} = \lim_{\frac{q_T}{Q}\to\sim 1,\ \frac{m^2}{Q^2}\to 0} \mathrm{W_{HSO}}$$

Conventional

$$\mathrm{ASY_{ST}} = \lim_{\frac{q_T}{Q}\to 0,\ \frac{m^2}{Q^2}\to 0} \mathrm{FO_{ST}}$$

$$\begin{aligned}
[f, D]_{\mathrm{ASY}} = {} & D^{\mathrm{pert}}\left(z, z\boldsymbol{q}_{\mathrm{T}}; \mu_Q, Q^2\right) f^c\left(x; \mu_Q\right) + \frac{1}{z^2} f^{\mathrm{pert}}\left(x, -\boldsymbol{q}_{\mathrm{T}}; \mu_Q, Q^2\right) d^c\left(z; \mu_Q\right) \\
& + \int \mathrm{d}^2\boldsymbol{k}_{\mathrm{T}} \Big\{ f^{\mathrm{pert}}\left(x, \boldsymbol{k}_{\mathrm{T}} - \boldsymbol{q}_{\mathrm{T}}/2; \mu_Q, Q^2\right) D^{\mathrm{pert}}\left(z, z\left(\boldsymbol{k}_{\mathrm{T}} + \boldsymbol{q}_{\mathrm{T}}/2\right); \mu_Q, Q^2\right) \\
& - D^{\mathrm{pert}}\left(z, z\boldsymbol{q}_{\mathrm{T}}; \mu_Q, Q^2\right) f^{\mathrm{pert}}\left(x, \boldsymbol{k}_{\mathrm{T}} - \boldsymbol{q}_{\mathrm{T}}/2; \mu_Q, Q^2\right) \Theta\left(\mu_Q - |\boldsymbol{k}_{\mathrm{T}} - \boldsymbol{q}_{\mathrm{T}}/2|\right) \\
& - D^{\mathrm{pert}}\left(z, z\left(\boldsymbol{k}_{\mathrm{T}} + \boldsymbol{q}_{\mathrm{T}}/2\right); \mu_Q, Q^2\right) f^{\mathrm{pert}}\left(x, -\boldsymbol{q}_{\mathrm{T}}; \mu_Q, Q^2\right) \Theta\left(\mu_Q - |\boldsymbol{k}_{\mathrm{T}} + \boldsymbol{q}_{\mathrm{T}}/2|\right) \Big\}
\end{aligned}$$

NOTE: Collins-Soper kernel at large b_T

$$\tilde{K}(b_T;\mu) = \tilde{K}_P + \tilde{K}_{NP}$$

$$K(k_T;\mu) = \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{k_T}\cdot\boldsymbol{b}_T}\,\tilde{K}(b_T;\mu) = K_P + K_{NP}$$

$$\text{if} \quad \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{k_T}\cdot\boldsymbol{b}_T}\,\tilde{K}_{NP} \propto \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{k_T}\cdot\boldsymbol{b}_T}\, b_T^{2n} = 0$$

Two possibilities

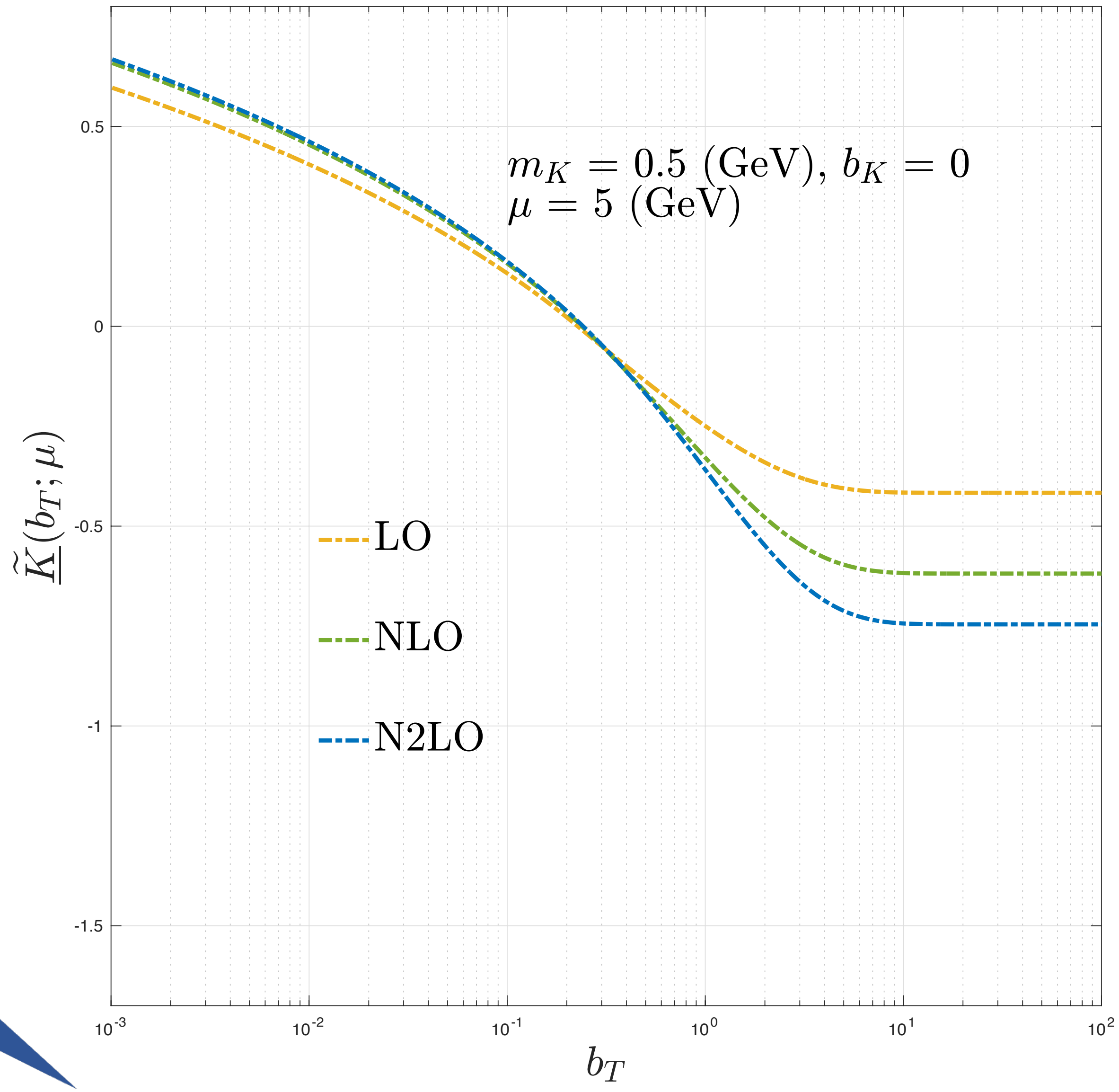
1 Do not choose even polynomials

- The Collins-Soper kernel has a meaning also in k_T space
- Perturbative and nonperturbative b_T/k_T parallel is preserved
- CS kernel and TMDs are physical

2 Choose even polynomials

- The Collins-Soper kernel is only meaningful in b_T space
- Perturbative and nonperturbative b_T/k_T parallel is **not** preserved
- Only TMDs are physical

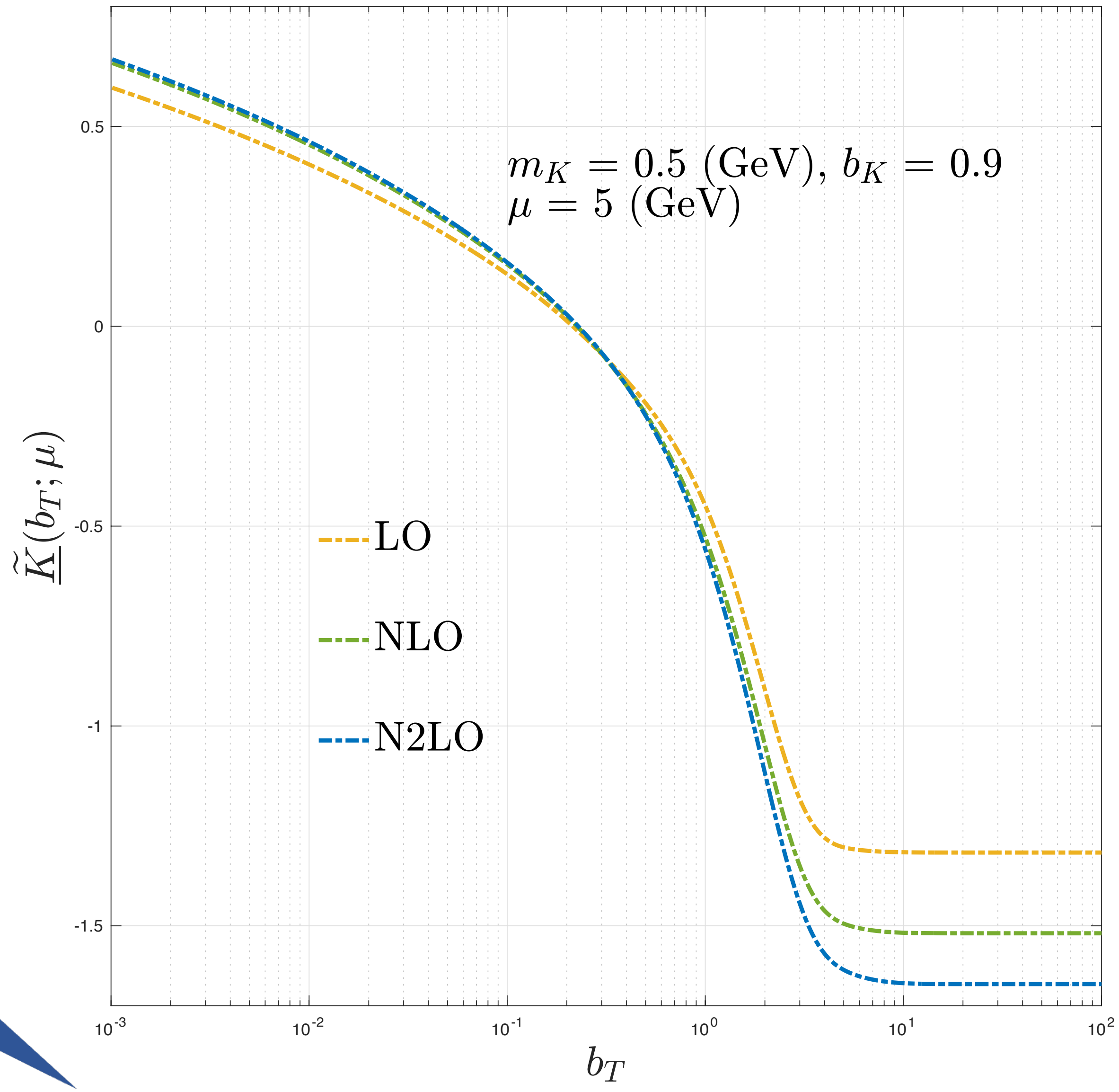
Some examples



Such that the OPE expansion is recovered at small b_T

$$\widetilde{K}(b_T, a_S) = \sum_{n=1}^{\infty} \sum_{k=1}^{n} a_S^n M_{kl} \mathcal{L}^{(l)} \hat{k}_{(n,k)} + C_K + b_K \widetilde{K}_{core}$$

Some examples



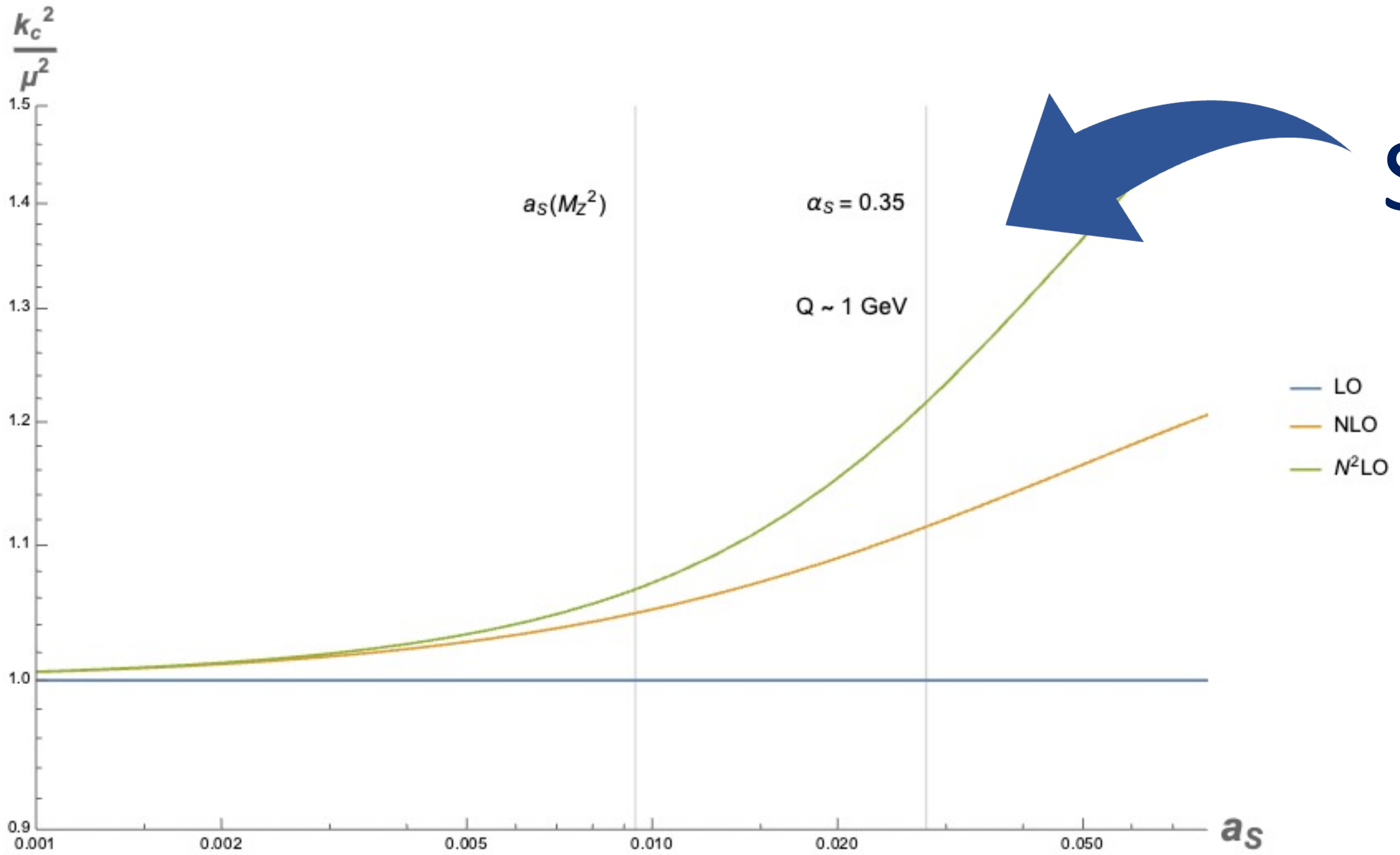
Such that the OPE expansion is recovered at small b_T

$$\widetilde{K}(b_T, a_S) = \sum_{n=1}^{\infty}\sum_{k=1}^{n} a_S^n M_{kl}\mathcal{L}^{(l)}\hat{k}_{(n,k)} + C_K + b_K\widetilde{K}_{core}$$

Vanishing Soft factor/CS Kernel?

$$\int_0^{k_c} \mathrm{d}^2\boldsymbol{k}_\mathrm{T} K_{\mathrm{input}}(k_T;\mu) \equiv \chi + p.s.$$

$$\chi = \sum_{n=1}^{\infty} a_S^n \left[\hat{k}_{(n,0)} - \sum_{k=1}^{n} M_{kl} \left(\overline{\mathcal{L}}_0^{(l)} - \Phi_{0,\mu c}^{(l)} \right) \hat{k}_{(n,k)} \right]$$



Solution to χ = 0

New numbers

$$\overline{\mathcal{L}}_0^{(n)} \equiv \lim_{x \tilde{\to} 0} \frac{\partial^n K_\alpha(x)}{\partial \alpha^n}\bigg|_{\alpha=0, x=2e^{-\gamma_E}} = \left[\frac{\partial^n}{\partial \alpha^n} \lim_{x \tilde{\to} 0} K_\alpha(x)\right]\bigg|_{\alpha=0, x=2e^{-\gamma_E}}$$

$$\overline{\mathcal{L}}_0^{(0)} = 0$$
$$\overline{\mathcal{L}}_0^{(1)} = 0$$
$$\overline{\mathcal{L}}_0^{(2)} = -\frac{2!}{3}\zeta(3)$$
$$\overline{\mathcal{L}}_0^{(3)} = 0$$
$$\overline{\mathcal{L}}_0^{(4)} = -\frac{2}{3}\pi^2\zeta(3) - \frac{4!}{5}\zeta(5)$$
$$\overline{\mathcal{L}}_0^{(5)} = 0$$
$$\overline{\mathcal{L}}_0^{(6)} = -\frac{3}{2}\pi^4\zeta(3) - 12\pi^2\zeta(5) - \frac{6!}{7}\zeta(7)$$
$$\overline{\mathcal{L}}_0^{(7)} = 0$$
$$\overline{\mathcal{L}}_0^{(8)} = -\frac{61}{9}\pi^6\zeta(3) - \frac{252}{5}\pi^4\zeta(5) - 480\pi^2\zeta(7) - \frac{1}{9}\left(2240\zeta(3)^3 + 8!\zeta(9)\right)$$

They come from here:

$$M_{lp}\Phi_{0,\mu}^{(p)} + L_b^l - M_{lp}\mathcal{L}_0^{(p)} = \begin{cases} 0, & p \text{ odd} \\ -M_{lp}\overline{\mathcal{L}}_0^{(p)}, & p \text{ even} \end{cases}$$

Examples: b_T space to k_T space

$$L_b \mapsto \mathcal{F}^{(1)} \equiv \frac{1}{2\pi k_T^2}(-1)$$

$$L_b^2 \mapsto \mathcal{F}^{(2)} \equiv \frac{1}{2\pi k_T^2}\left[-\ln\left(\frac{\mu^2}{k_T^2}\right)\right]$$

$$L_b^3 \mapsto \mathcal{F}^{(3)} \equiv \frac{1}{2\pi k_T^2}\left[-\frac{3}{4}\ln^2\left(\frac{\mu^2}{k_T^2}\right)\right]$$

$$L_b^4 \mapsto \mathcal{F}^{(4)} \equiv \frac{1}{2\pi k_T^2}\left[2\zeta(3)-\frac{1}{2}\ln^3\left(\frac{\mu^2}{k_T^2}\right)\right]$$