

AN APOLOGY FOR ADS3

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- ADS3 IS A TOY MODEL THAT CAPTURES SOME OF THE MOST PUZZLING ASPECTS OF QUANTUM GRAVITY
- THERE ARE NO GRAVITATIONAL WAVES IN PURE ADS3 GRAVITY (AND THIS IS GOOD)
- A “SQUARE ROOT” OF PURE GRAVITY EXISTS
- ADS3 POSSESSES INFINITE-DIMENSIONAL ASYMPTOTICAL ALGEBRAS THAT IN PART REDUCE DYNAMICS TO KINEMATICS
- HIGH-SPIN FIELDS PROPAGATE ZERO DEGREES OF FREEDOM BUT THEY ARE NEITHER FREE NOR EQUIVALENT TO FREE FIELDS

THE EASY PROBLEM WITH QUANTUM GRAVITY: RENORMALIZABILITY

EINSTEIN ACTION HAS A COUPLING CONSTANT
WITH DIMENSION [square length] (NEWTON'S
CONSTANT)

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R(g) + L_{matter}]$$

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$$S_{div} \sim G^{n-1} \log(\Lambda/\mu) \int d^4x \sqrt{-g} R^{n+1}$$

SCALAR MADE OUT OF RIEMANN TENSORS

INFINITE SET OF COUNTERTERMS

WHAT IF THEORY IS FINITE (N=8 SUPERGRAVITY?)

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BEHAVES AT ENERGY

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$$A_{2 \rightarrow N} = \sum_{l=0}^{\infty} (GE^2)^{N/2+l} A_l$$


EXPANSION PARAMETER OF SERIES BECOMES LARGE AT
PLANCKIAN ENERGIES

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QUESTIONS SUCH AS UNITARITY IN BLACK-HOLE
EVAPORATION OR THE FATE OF SINGULARITIES
REMAIN AS UNKNOWN AS IN THE NON-
RENORMALIZABLE THEORY

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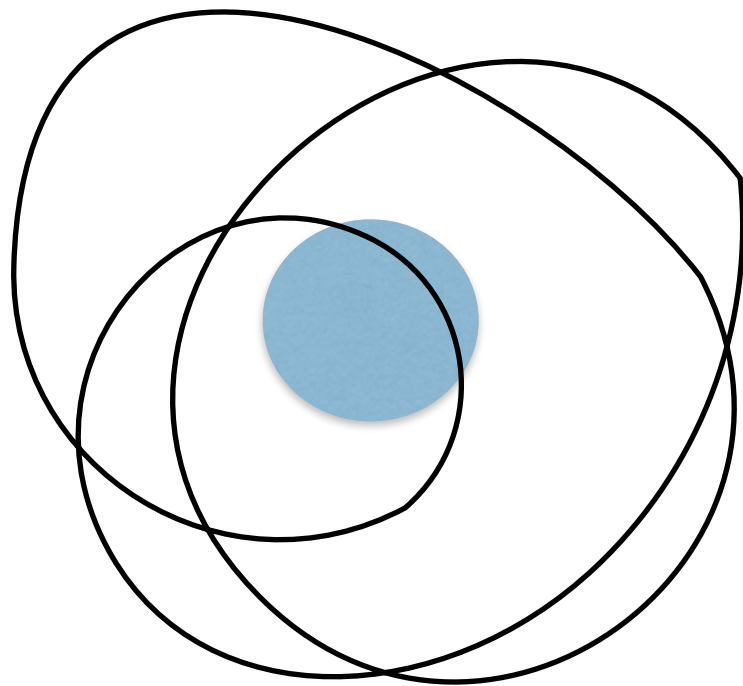
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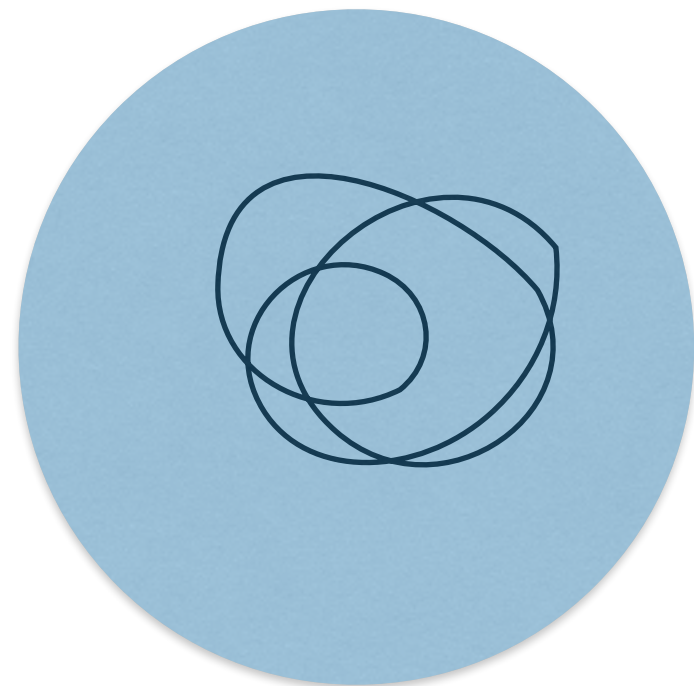
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$$R_S \ll L_{string}$$



$$R_S \gg L_{string}$$



WE STILL DON'T KNOW WHAT THE THEORY
DOES AT VERY HIGH ENERGY

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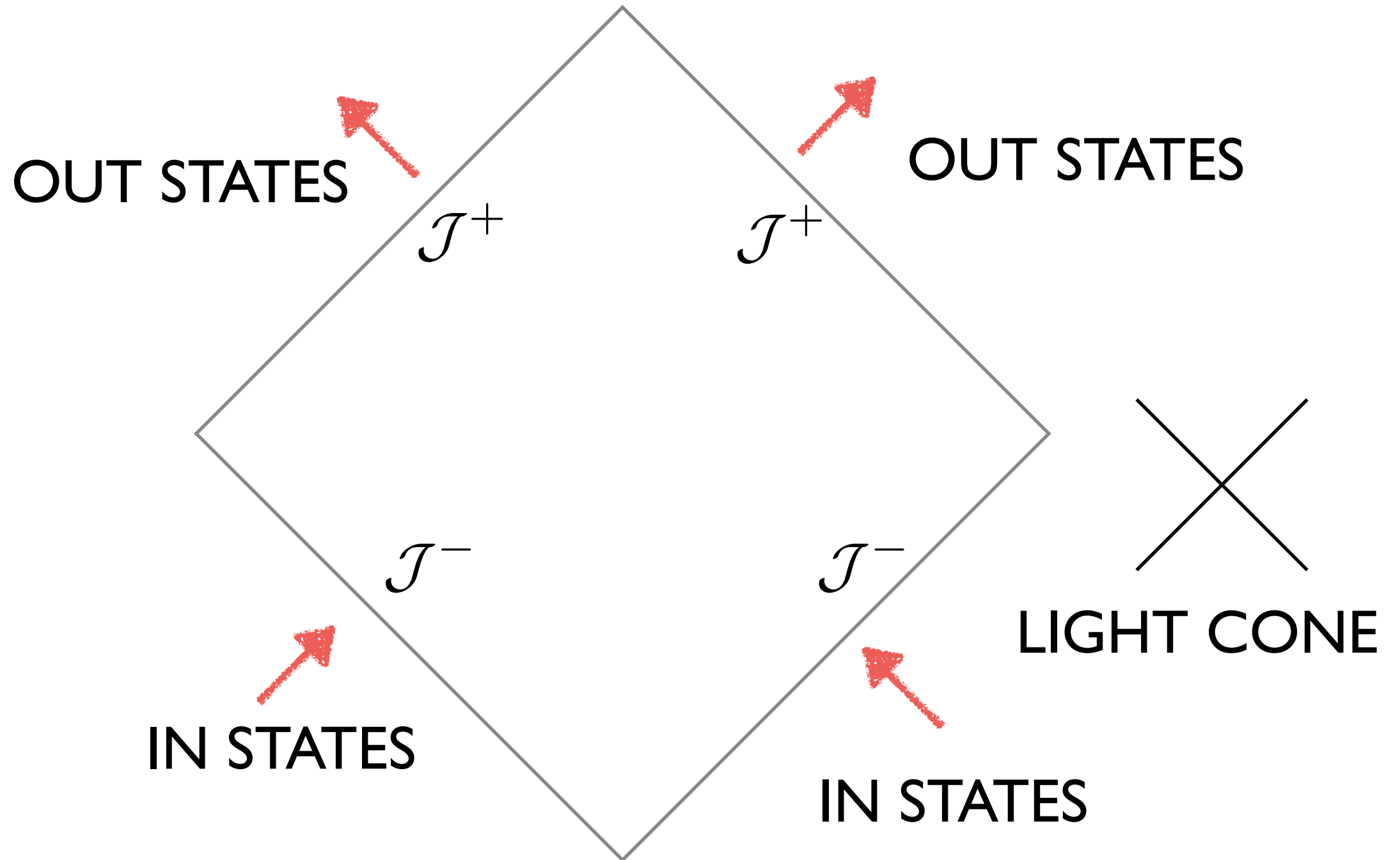
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NON-PERTURBATIVELY DEFINED
OBSERVABLES DEPEND ON THE ASYMPTOTIC
BEHAVIOR OF SPACETIME

IN ASYMPTOTICALLY FLAT SPACETIME THE ONLY
OBSERVABLE IS THE S-MATRIX



ANTI DE SITTER SPACE

SOLUTION OF EINSTEIN EQUATIONS WITH
NEGATIVE COSMOLOGICAL CONSTANT
(PROPAGATE SAME DEGREES OF FREEDOM AS
PURE GRAVITY)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}, \quad T_{\mu\nu} = g_{\mu\nu}(d-1)(d-2)/16\pi GL^2$$

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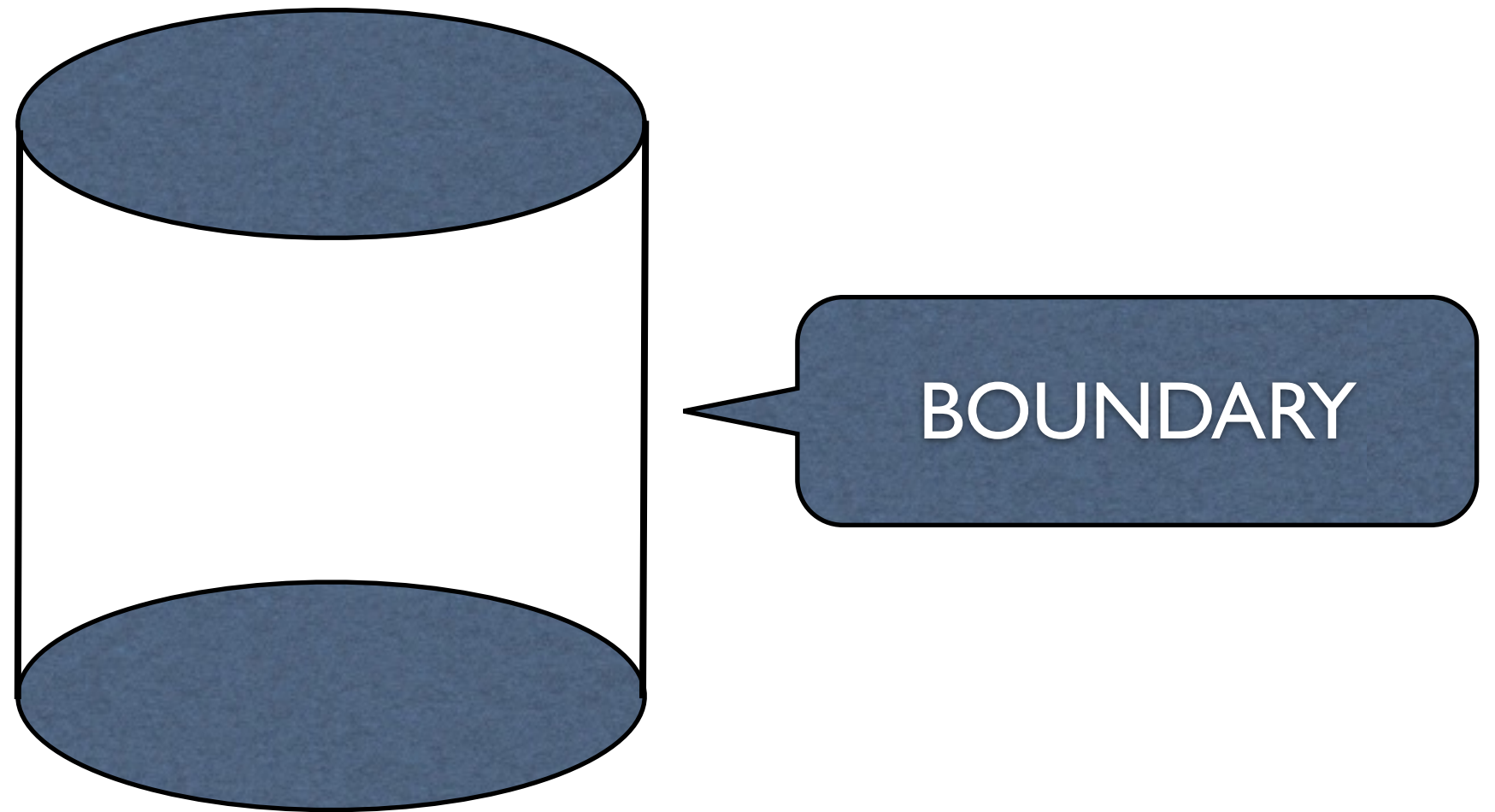
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GEOMETRICALLY IT IS A d-DIMENSIONAL
HYPERBOLOID IN A (2+d)-DIMENSIONAL SPACE
OF SIGNATURE (2,d)

$$-(x^0)^2 - (x^{d+1})^2 + \sum_{i=1}^d (x^i)^2 = -L^2$$

SIMPLEST CASE: $d=2$
ADS3 IS CONFORMAL TO A SOLID CYLINDER

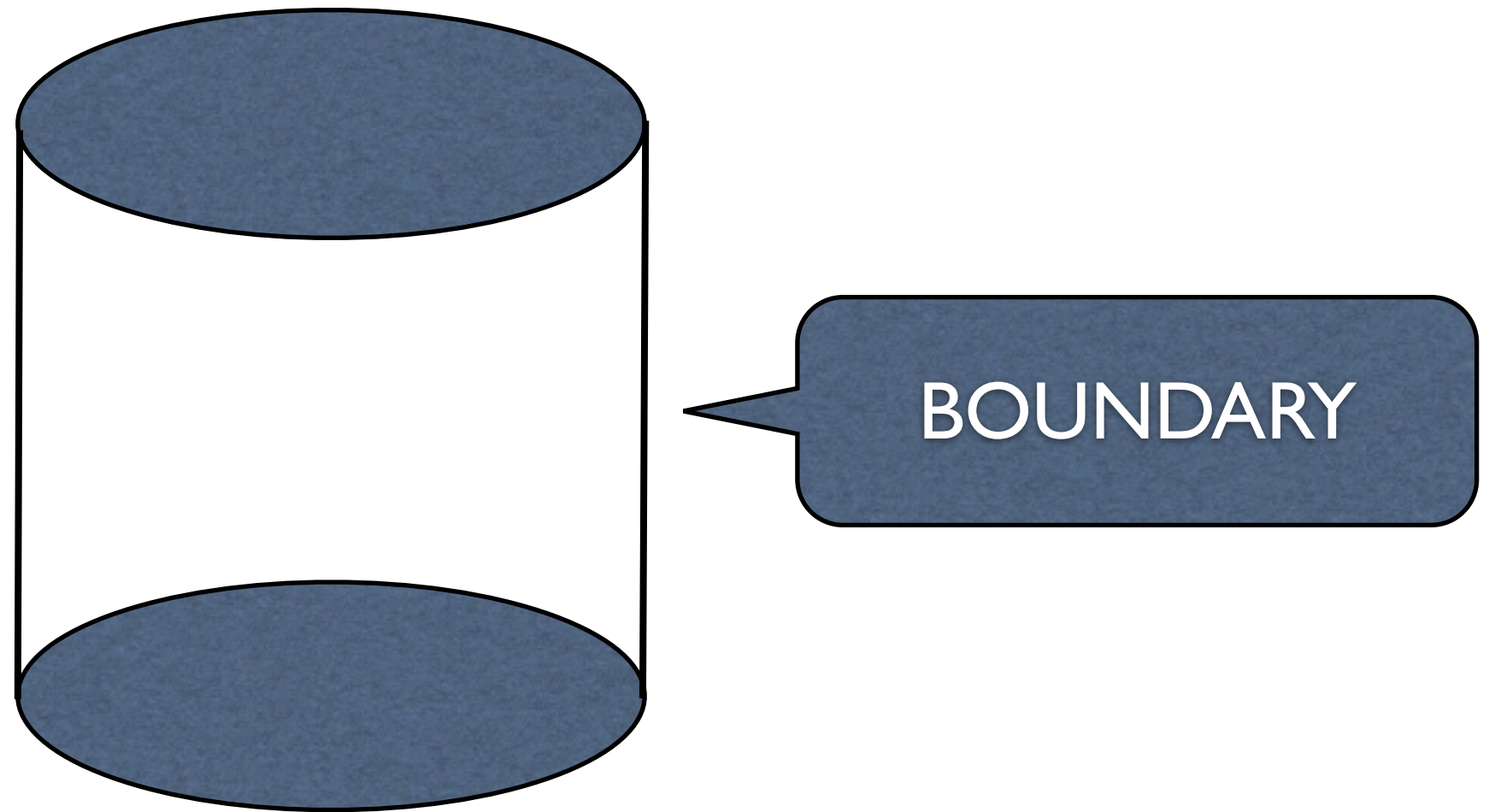


$$ds^2 = -\cosh^2 \rho dt^2 + \sinh^2 \rho d\phi^2 + d\rho^2$$



$$ds^2 = z^{-2}(-dw_+ dw_- + dz^2) \qquad w_{\pm} = t \pm \phi, \quad z = \exp(-\rho)$$

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OBSERVABLES: LOCAL OPERATORS DEFINED ON
THE BOUNDARY

THE ASYMPTOTICAL ADS3 METRIC IS PRESERVED
BY A LARGE GROUP OF COORDINATE
TRANSFORMATIONS THAT DEFINE A LARGE
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PRESERVED BY GENERAL COORDINATE TRANSFORMATION

$$w^\pm \rightarrow w^\pm + \epsilon^\pm(w^\pm) + \frac{1}{2} z^2 \epsilon''^\mp(w^\mp)$$

$$z \rightarrow z - \frac{1}{2} z [\epsilon'^+(w^+) + \epsilon'^-(w^-)]$$

THE ALGEBRA DEFINED BY THE CHARGES
ASSOCIATED TO ASYMPTOTIC DIFFEOMORPHISM
HAS A **CLASSICAL** CENTRAL CHARGE

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12}\delta_{n,-m}m(m^2 - 1)$$

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IN ANY THEORY OF QUANTUM GRAVITY ON
ADS3, STATES FIT IN UNITARY REPRESENTATIONS
OF THE VIRASORO ALGEBRA

ISN'T 3D GRAVITY NON-RENORMALIZABLE TOO?

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$R_{0\mu} - g_{0\mu}R + g_{0\mu}\Lambda = 0 \rightarrow -3$ non-dynamical equations = constraints

PURE ADS3 GRAVITY ADMITS BLACK HOLE SOLUTIONS, THE BTZ (BANADOS, TEITELBOIM, ZANELLI)

$$ds^2 = -\frac{(r^2 - R^2)^2}{r^2} dt^2 + \frac{L^2 r^2}{(r^2 - R^2)^2} dr^2 + r^2 d\phi^2, \quad R = \sqrt{8GM L}$$

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FOR ADS3, BOUNDARY CFT IS 2d. BEST KNOWN BECAUSE
OF INFINITE-DIMENSIONAL VIRASORO ALGEBRA

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$$\mathrm{Tr} e^{-\beta H} = Z(\beta) = Z(1/\beta)$$

THEN ASYMPTOTIC DENSITY OF STATES IS (CARDY)

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CLASSICAL GRAVITY AS CHERN-SIMONS

$$A = e/l - \omega, \quad \tilde{A} = e/l + \omega$$

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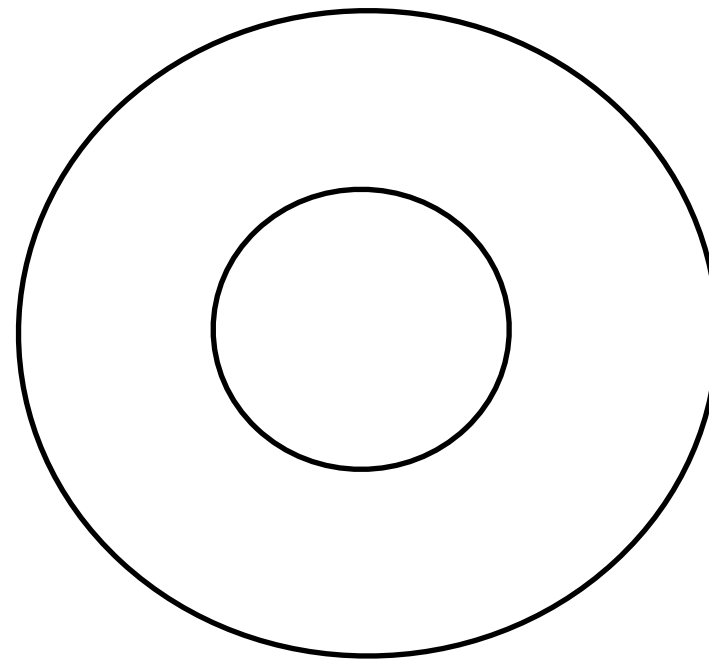
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CONSTRAINT EQUATION (GAUSS LAW)

$$F|_{\Sigma} = 0 \rightarrow A = dU U^{-1} \quad (\text{locally})$$

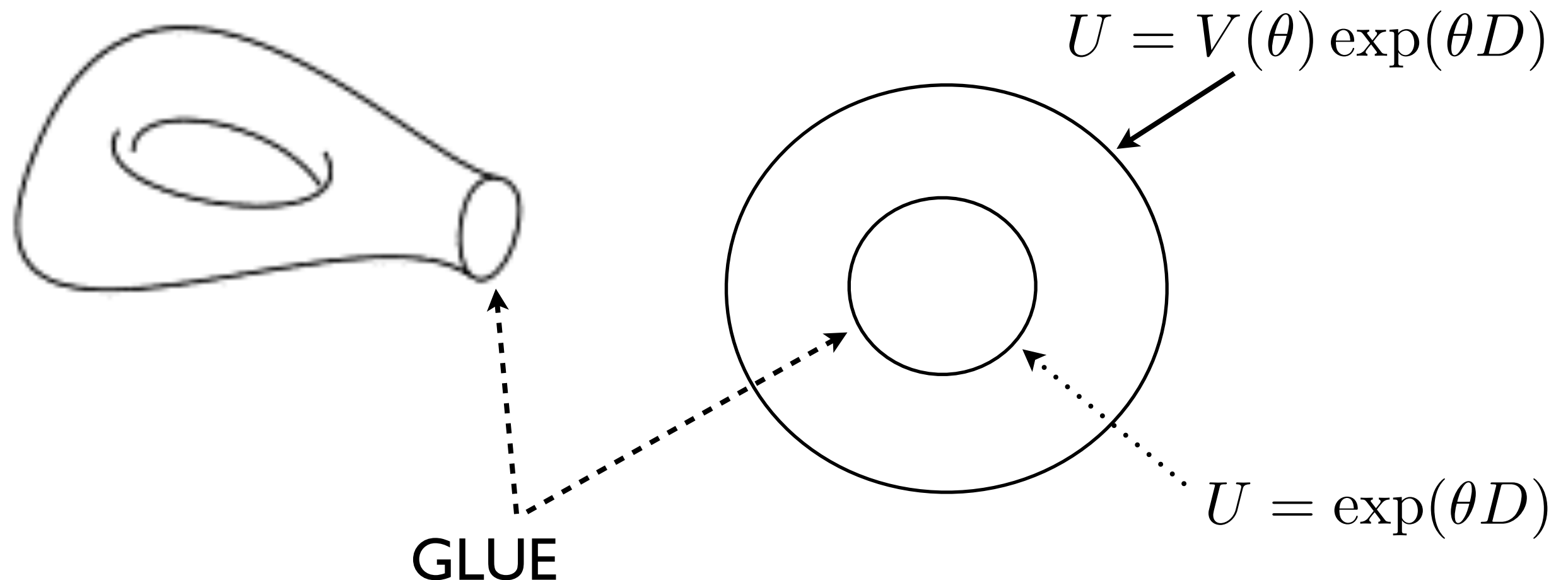
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THIS SPACE ADMITS A KÄHLER STRUCTURE AND A
KÄHLER FORM: WE CAN QUANTIZE USING COHERENT
STATES

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GUESS THE HOLOGRAPHIC DUAL OF PURE GRAVITY

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EXISTENCE OF THIS THEORY STILL DEBATED

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FOR GENERIC μ BAD THEORY WITH GHOSTS
AT $\mu L = 1$ GHOSTS DISAPPEAR FOR
BROWN-HENNEAUX BOUNDARY CONDITIONS

ASYMPTOTIC ALGEBRA IS ONLY ONE COPY OF VIRASORO

$$Q(\epsilon^+) = \frac{1}{4\pi G} \oint d\phi \epsilon^+ h_{++} \quad Q(\epsilon^-) = 0$$

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BOUNDARY CONDITIONS CAN BE RELAXED TO GIVE

$$Q(\epsilon^-) \neq 0$$

BUT THEORY HAS GHOST

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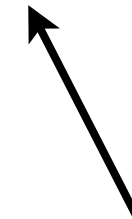
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CONFORMAL DIMENSION

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$$\Delta = s + (D-3)/2$$

ADS3 POSSESSES HIGH SPIN THEORIES DUAL TO
INTERACTING 2D HIGH SPIN ALGEBRAS

IN ANY DIMENSION $D > 2$ CONFORMAL GROUP IS

$$SO(D-1, 2) \supset SO(D-1) \times SO(2)$$

SPIN

s

CONFORMAL DIMENSION

$$\Delta \geq s + (D-3)/2$$

MASSLESS

$$\Delta = s + (D-3)/2$$

INTERACTING THEORIES OF MASSLESS
HIGH SPIN PARTICLES EXIST IN ADS
(VASILIEV)

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BUT NOT IN $D=3$!

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“ANALYTIC CONTINUATION” OF $SL(N, R)$

$$hs[\mu] = UEA[J^a] / \{ J_0^2 - \frac{1}{2}(J_- J_+ + J_+ J_-) - (\mu^2 - 1)/4 \}$$

$$hs[\mu] \times hs[\mu] \quad \mu \in R$$

CHERN-SIMONS POSSESS A B-H ASYMPTOTIC ALGEBRA

$$W_{\infty}(\mu) = \lim_{N \rightarrow \infty} W_{N,k} \quad \mu = \frac{N}{N+k} \quad k = l/4G$$

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GABERDIEL-GOPAKUMAR CONJECTURE: THIS IS CFT DUAL TO
ADS3 VASILIEV HIGH SPIN THEORY

TESTS: SPECTRUM OF LIGHT STATES, SYMMETRIES,
DEFORMATIONS OK

GRAVITATIONAL WAVES DO NOT PROPAGATE IN ADS3,
BUT IT IS NEVERTHELESS A PRECIOUS THEORETICAL
LABORATORY FOR QUANTUM GRAVITY
BECAUSE IT POSSESSES KEY FEATURES OF 4D GRAVITY,
LIKE BLACK HOLES,
TOGETHER WITH ESPECIALLY LARGE
ASYMPTOTIC ALGEBRAS THAT PARTIALLY REDUCE
DYNAMICS TO KINEMATICS