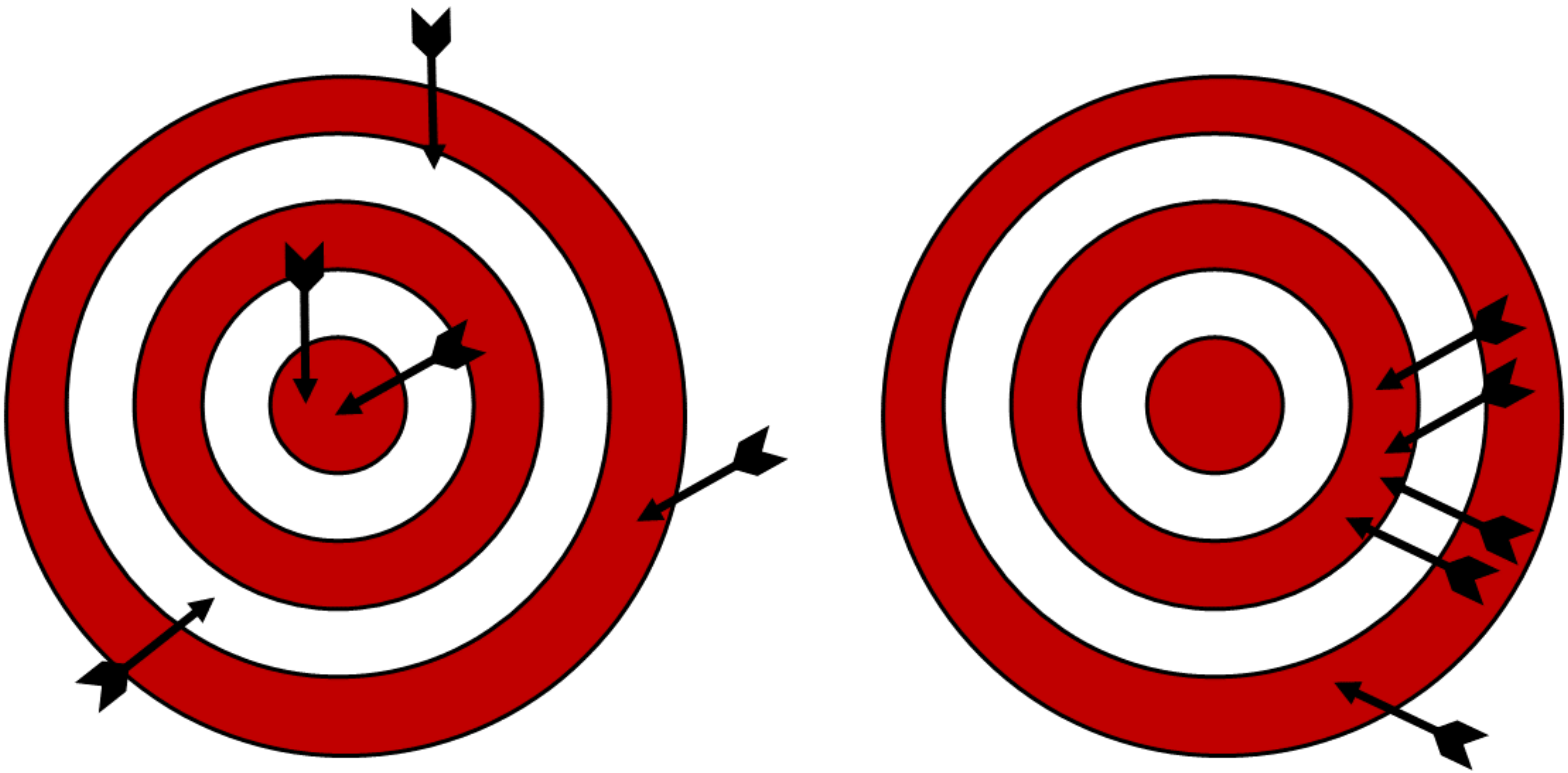

Systematics Uncertainties in Small-scale Experiments

Francesco Renga
INFN Roma

Disclaimer

- Small-scale experiments —> Single-purpose experiments
  - e.g. Muon g-2 experiment, ~ 200 authors
- Technical aspects already covered in previous lectures, I will concentrate on practical applications
  - A very biased selection of experimental examples, mostly from muon and QED precision physics
- Not intended as a review of physics results
  - you could find incomplete references, out-of-date results, etc.

Outline

- General aspects of systematic uncertainties in single-purpose experiments
  - precision measurements
  - rare event searches
- Inclusion of systematics in confidence interval computations
- (Very biased) collection of relevant examples

Generalities

Single-purpose experiments

- Most single-purpose experiments can be classified into two categories:

    1) highly accurate measurements of particles properties

    2) searches for rare processes (rare decays, interactions of elusive particles)

- Besides accumulating statistics, high sensitivity is achieved through **high precision** (extremely good resolutions, extremely high background rejection) and/or **high accuracy** (no bias in measurements and background estimates)

High Accuracy vs. High precision

PHYSICAL REVIEW LETTERS **131,** 161802 (2023)

Editors' Suggestion

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm

Measurement based on the rate of e^+ from μ^+ decays, above a given energy threshold (**2.5%** e^+ energy resolution)

High Accuracy vs. High precision

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Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm

Measurement based on the rate of e⁺ from μ⁺ decays, above a given energy threshold (**2.5%** e⁺ energy resolution)

Quantity	Correction (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	$\cdots$	201
ω_a^m (systematic)	$\cdots$	25
C_e	451	32
C_p	170	10
C_{pa}	−27	13
C_{dd}	−15	17
C_{ml}	0	3
$f_{\text{calib}} \cdot \langle \omega'_p(\vec{r}) \times M(\vec{r}) \rangle$	$\cdots$	46
B_k	−21	13
B_q	−21	20
$\mu'_p(34.7°)/\mu_e$	$\cdots$	11
m_μ/m_e	$\cdots$	22
$g_e/2$	$\cdots$	0
Total systematic for $\mathcal{R}'_\mu$	$\cdots$	70
Total external parameters	$\cdots$	25
Total for a_μ	622	215

High Accuracy vs. High precision

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https://doi.org/10.1140/epjc/s10052-024-12416-2

THE EUROPEAN PHYSICAL JOURNAL C

Check for updates

Regular Article - Experimental Physics

A search for $\mu^+ \to e^+\gamma$ with the first dataset of the MEG II experiment

MEG II Collaboration

Upper Limit based on the discrimination of 2-body vs. 3-body kinematics with extremely precise measurements (e.g. **0.2%** e^+ energy resolution)

High Accuracy vs. High precision

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Total external parameters	⋯	25
Total for a_μ	622	215

Measurement based on the rate of e^+ from μ^+ decays, above a given energy threshold (**2.5%** e^+ energy resolution)

Parameter	Impact on limit
$\phi_{e\gamma}$ uncertainty	1.1 %
E_γ uncertainty	0.9 %
$\theta_{e\gamma}$ uncertainty	0.7 %
Normalization uncertainty	0.6 %
$t_{e\gamma}$ uncertainty	0.1 %
E_e uncertainty	0.1 %
RDC uncertainty	< 0.1 %

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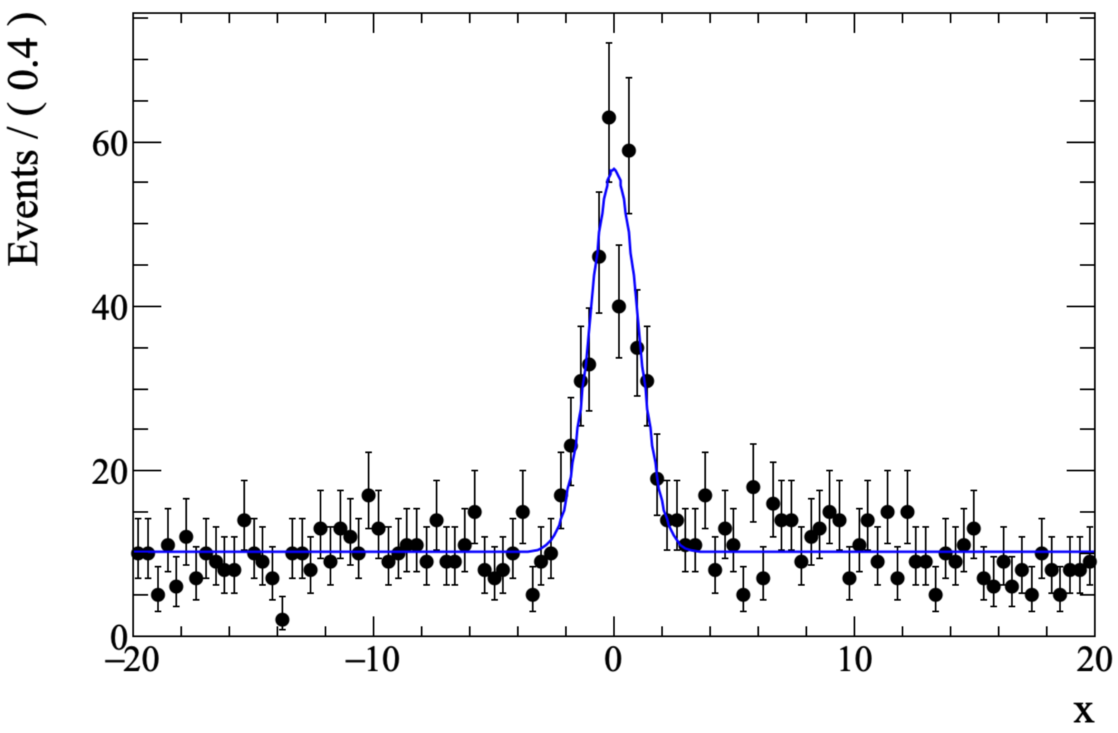
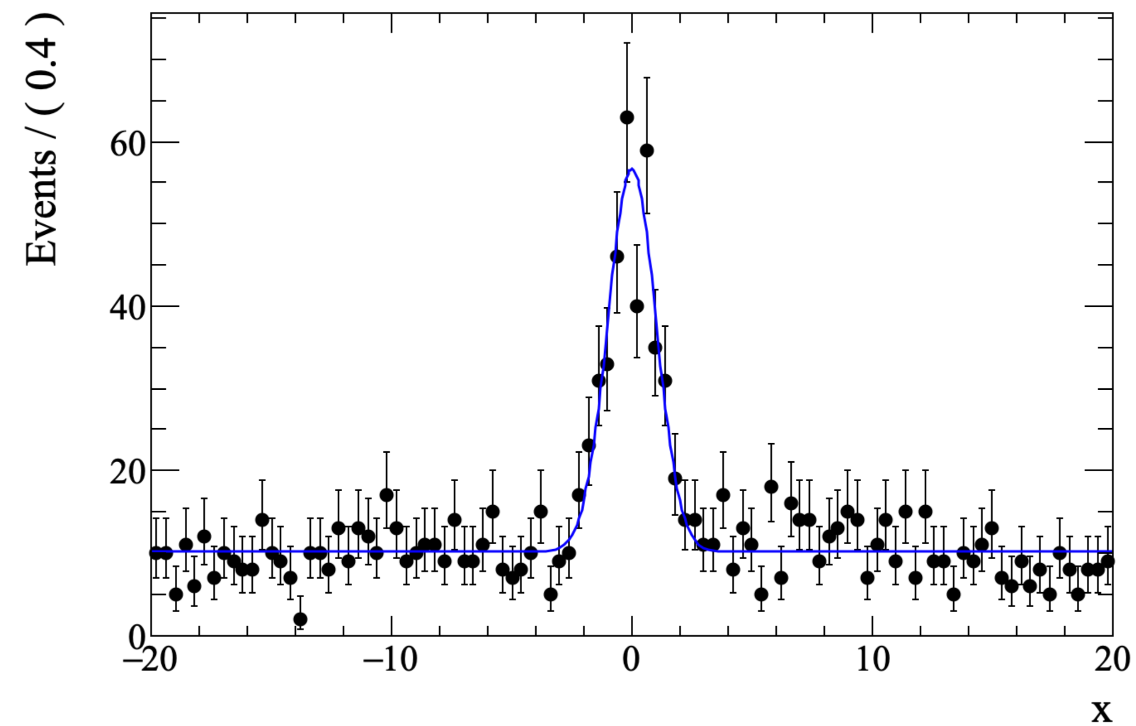
Rare events with high-precision

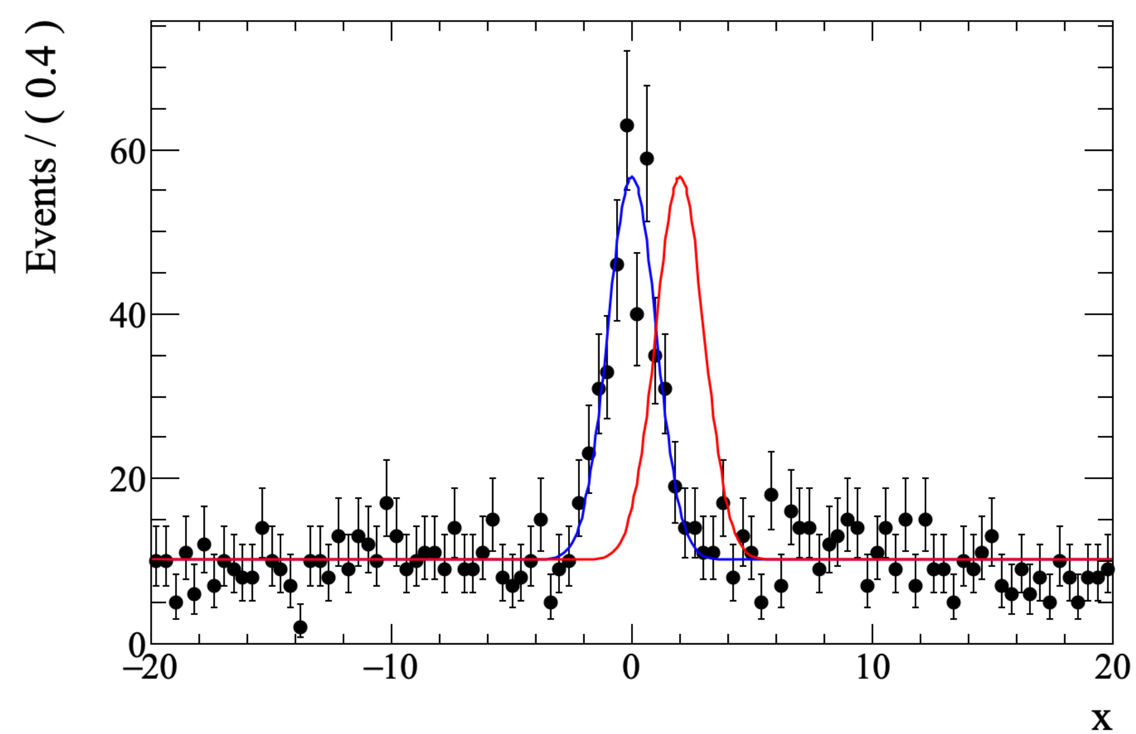
- Let's suppose to search for a rare process, looking for a peak over the background in a known position of a distribution
  - high sensitivity through high precision



Rare events with high-precision

- Let's suppose to search for a rare process, looking for a peak over the background in a known position of a distribution
    - high sensitivity through high precision
    - the measurement has to be also highly accurate (no relevant bias)

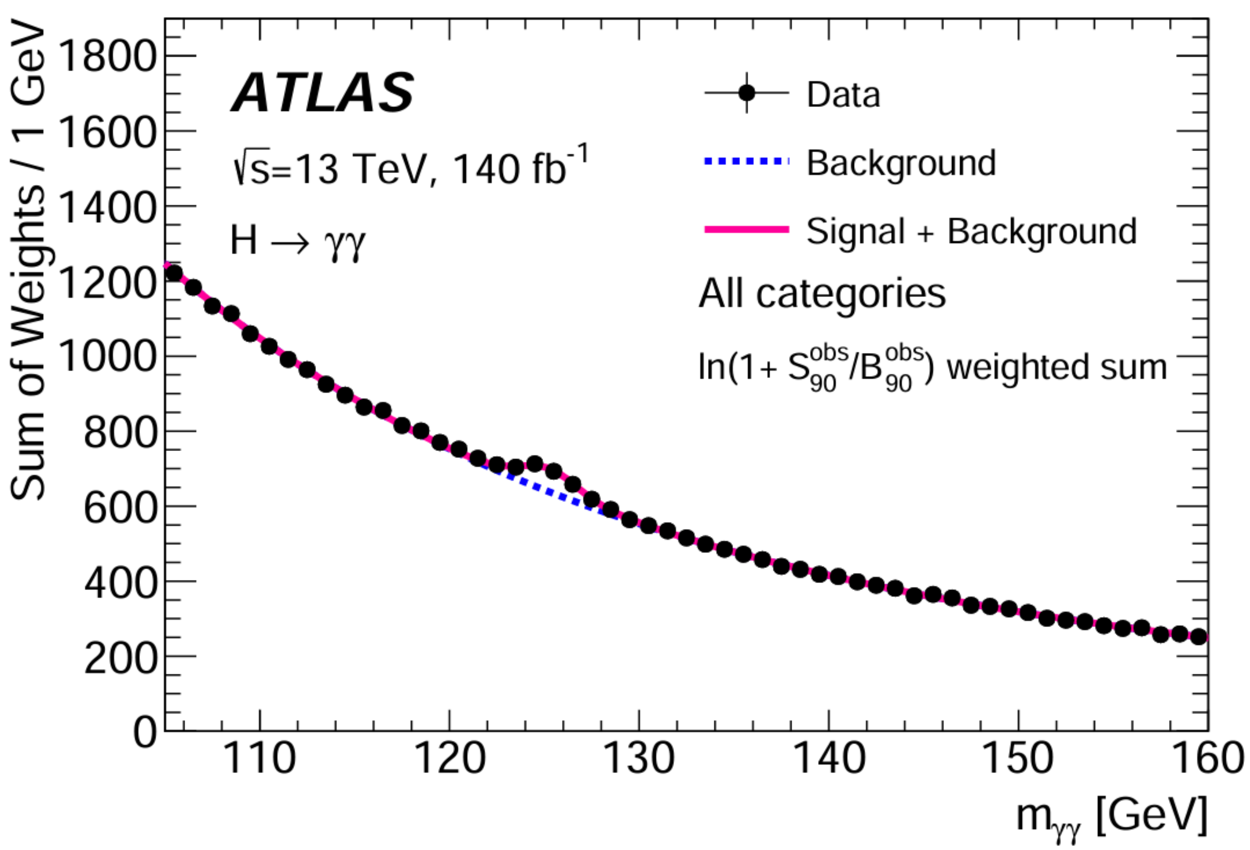


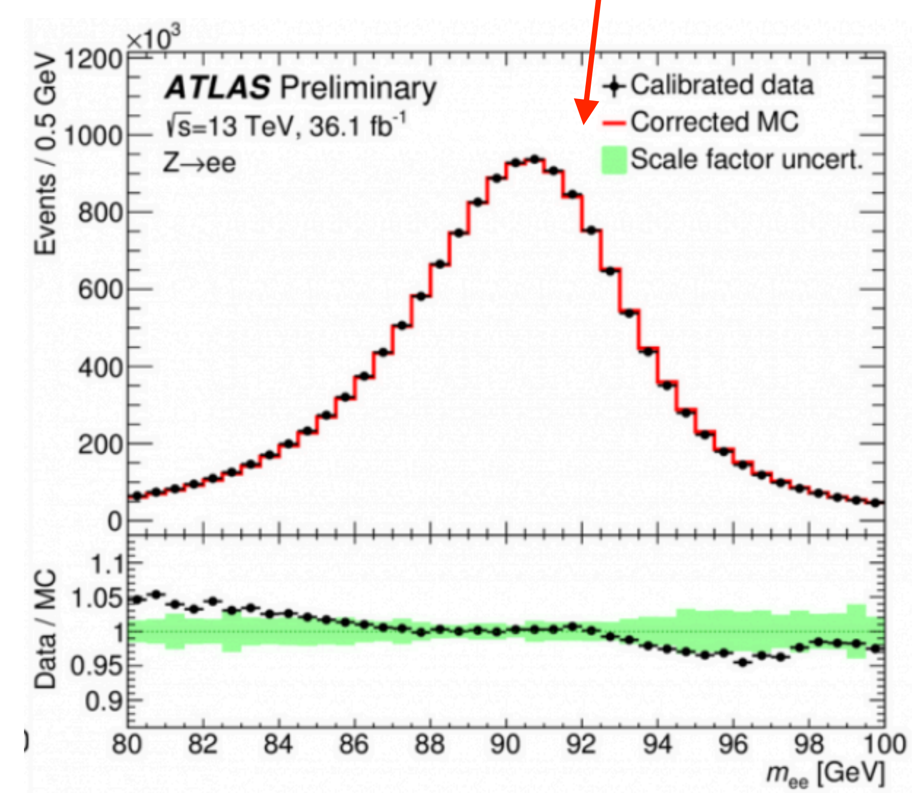


Rare events with high-precision

- In general-purpose experiments, many non-rare physics processes can be used to calibrate the measurements and remove biases

Source	Impact [MeV]
Photon energy scale	83
$Z \to e^+e^-$ calibration	59
E_T-dependent electron energy scale	44
$e^{\pm} \to \gamma$ extrapolation	30
Conversion modelling	24
Signal–background interference	26
Resolution	15
Background model	14
Selection of the diphoton production vertex	5
Signal model	1
Total	90





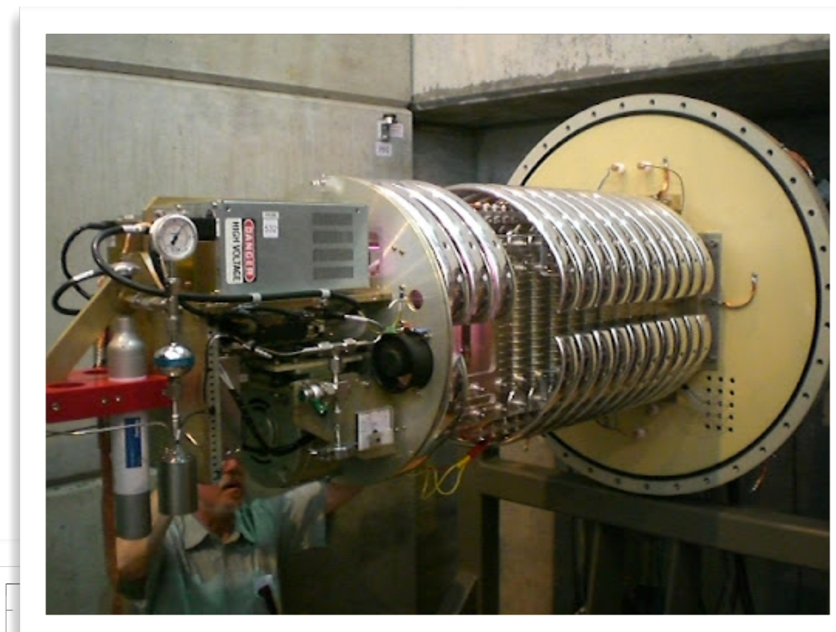
Rare events with high-precision

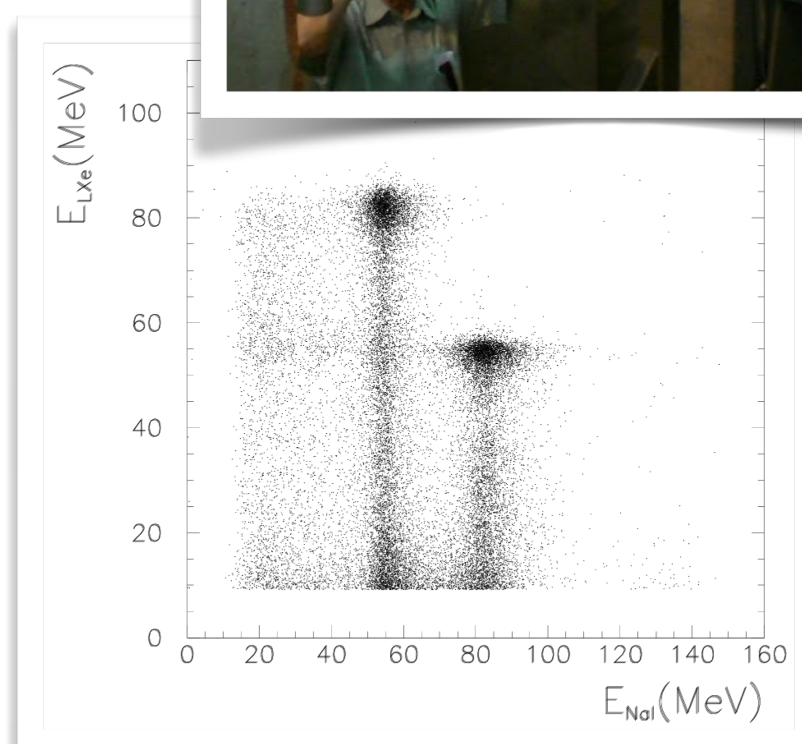
- Single-purpose experiments typically have extreme resolution (i.e. need for extreme accuracy) and a scarcity of physics processes to be used for calibrations

    - **dedicated tools need to be developed**

Rare events with high-precision - the MEG case

- Single-purpose experiments typically have extreme resolution (i.e. need for extreme accuracy) and a scarcity of physics processes to be used for calibrations
  - **dedicated tools need to be developed**
- In the MEG experiment, where a muon beam is used to search for $\mu \to e\gamma$, a profusion of calibration tools has been developed, including a **dedicated Cockroft-Walton accelerator** and a $\pi^-(p,n)\,\pi^0$ **experiment** with the only purpose of calibrating the photon energy reconstruction.





Systematic Uncertainties for High Accuracy

- **High Accuracy for zero tests**

e.g. particles EDMs

$$\Delta \stackrel{?}{=} 0$$

$$\Delta_{meas} = k\Delta + \delta \quad \text{with } k = 1 \pm \sigma_k\,,\ \delta = 0 \pm \sigma_\delta$$

$$\Rightarrow \ \sigma_\Delta^{syst} = \frac{\sigma_k}{k}\Delta + \sigma_\delta \sim \sigma_\delta$$

- Additive uncertainties dominate over multiplicative uncertainties

Systematic Uncertainties for High Accuracy

- **High Accuracy for non-zero measurements**

e.g. particles MDMs,
coupling constants

  - multiplicative uncertainties are also important

  - comparison with SI units is critical —> **a metrology problem**

precession frequency

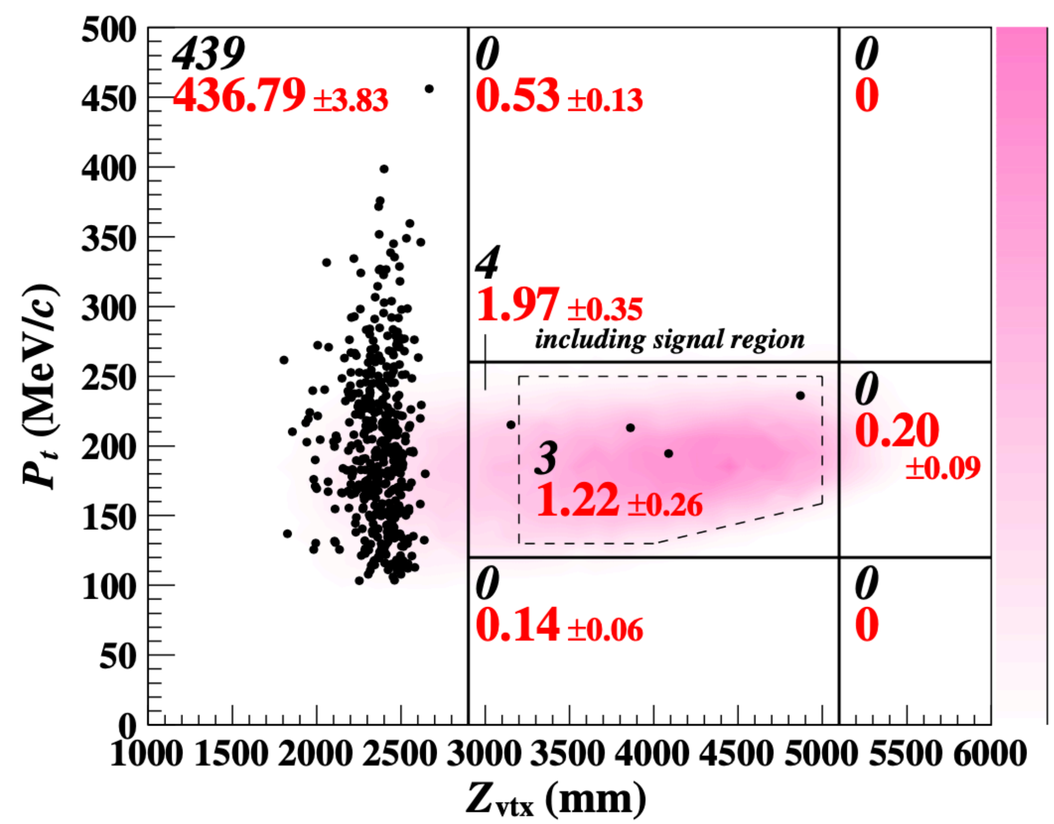
$$\frac{g_\mu - 2}{2} \sim \frac{m\omega}{qB}$$

magnetic field

Measured dimensional quantities need to be calibrated against SI standards with << ppm accuracy

Systematic Uncertainties in Rare Event Searches

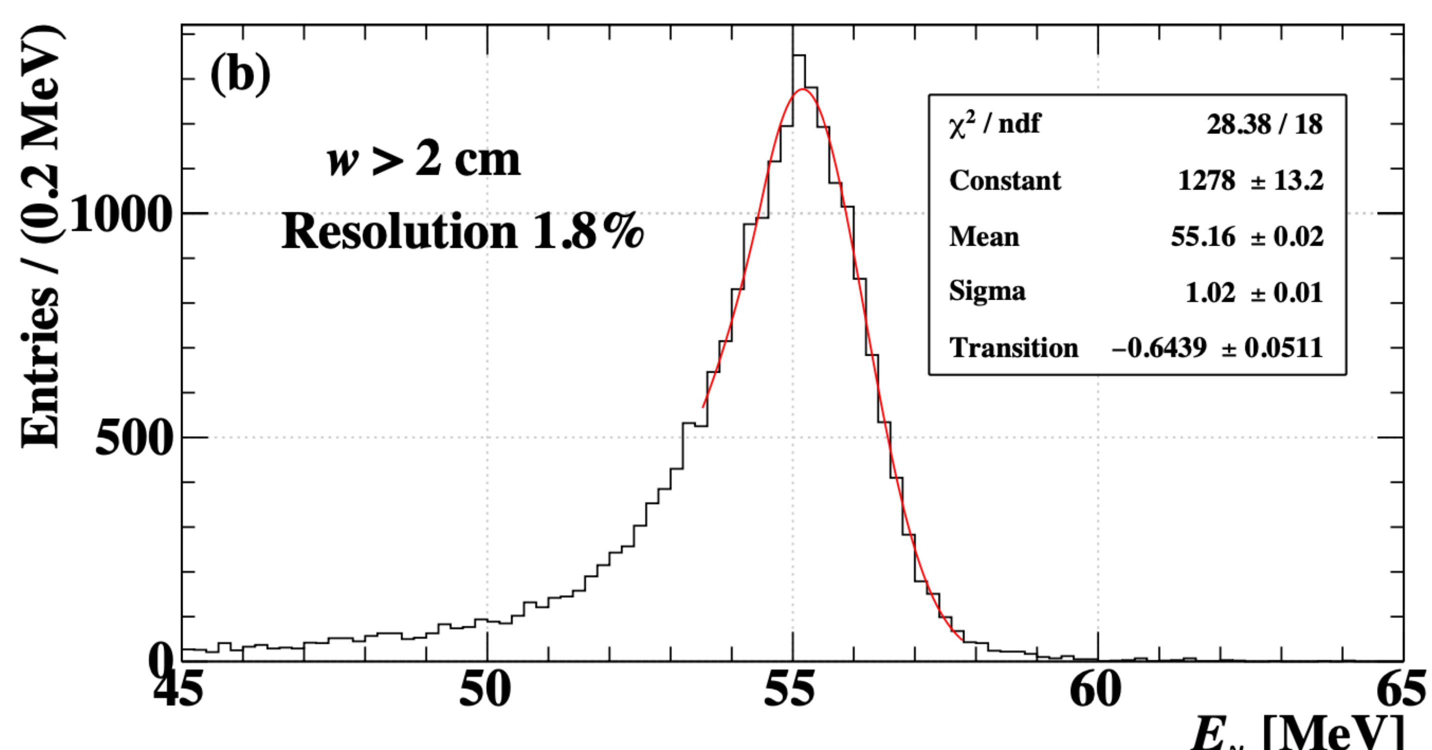
- **Rare event searches through event patterns**
  - rare event searches (rare decays, dark matter, etc.) where background rejection is mostly achieved through **particle identification, vetos, event topology**, etc.
  - dominant systematic uncertainties typically from the control of the background rejection efficiency



Systematic Uncertainties in Rare Event Searches

- **Rare event searches through precision**
  - rare event searches (e.g. rare decays) where the precise measurement of some observable (e.g. kinematics) is required to discriminate signal and background
  - high precision requires also high accuracy, which can be only achieved with dedicated tools

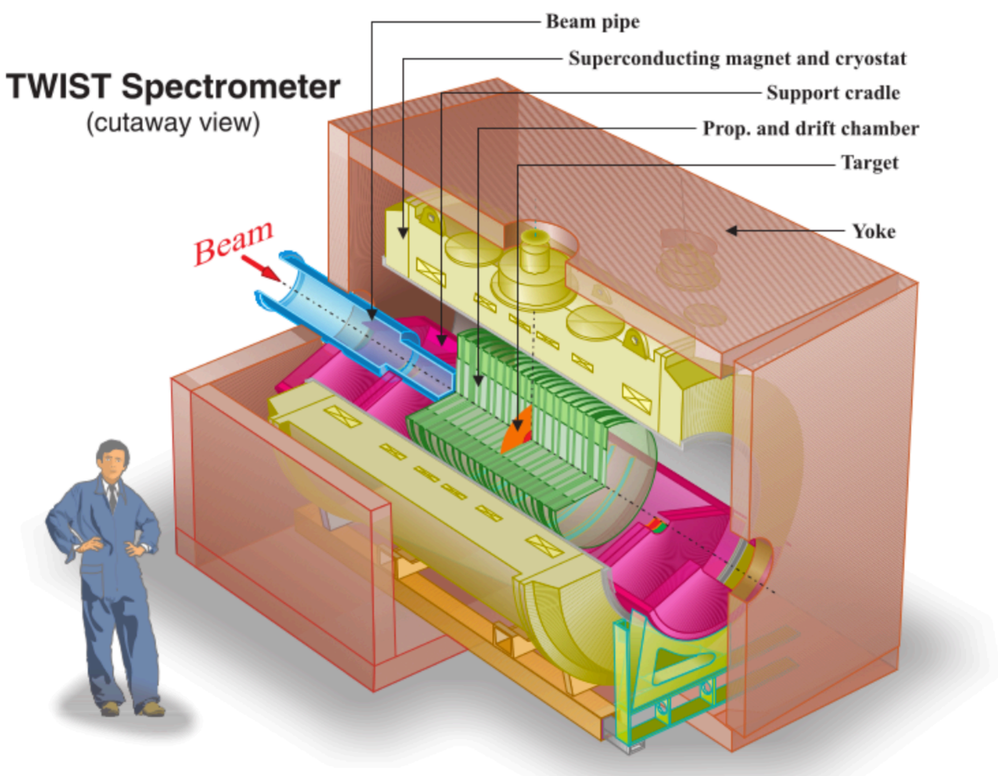
Photon energy calibration in MEG with a pion beam, $\pi^- + p \to \pi^0(\gamma\gamma) + n$



A ognuno il suo *(Each to their own)*

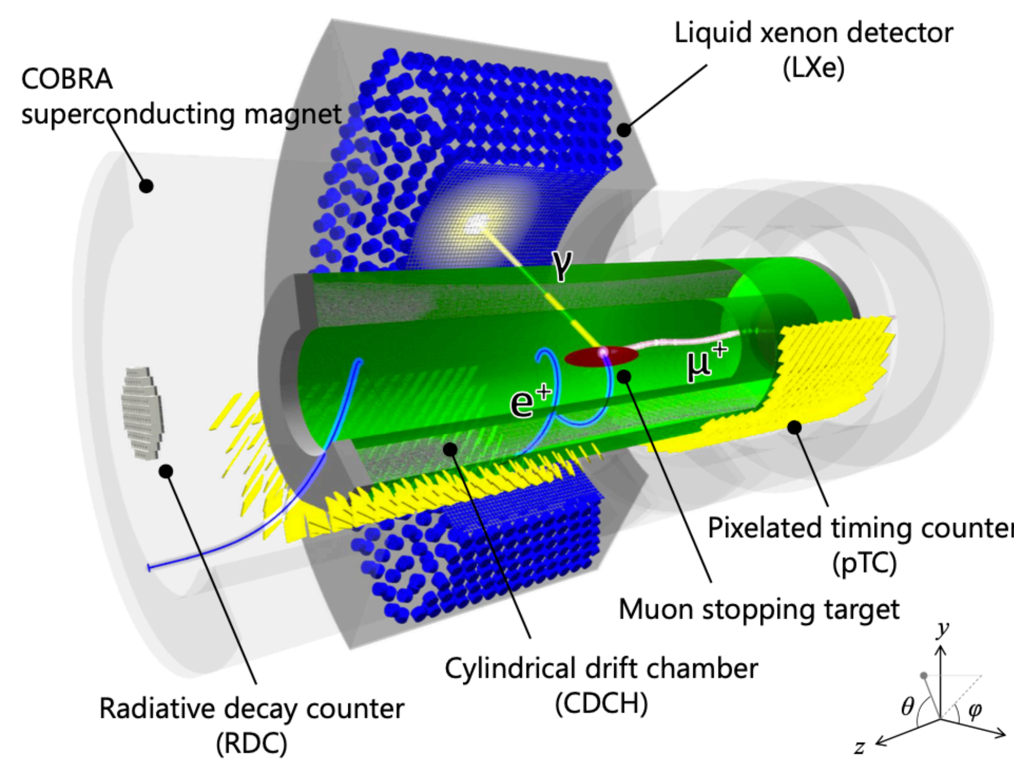
- MEG vs. TWIST

Measurement of parameters of the e+ energy spectrum in μ⁺ decays



Very accurate knowledge of the magnetic field is necessary to measure the spectrum parameters

Measurement of e⁺ energy spectrum for the search of $\mu^+ \to e^+\gamma$



Magnetic field adjusted from data, exploiting the theoretical knowledge of the Michel spectrum

The role of Monte Carlo simulations

- The extremely high resolutions and accuracies of single-purpose experiments pose strong challenges to Monte Carlo simulations

- Additionally, for rare event searches, MC productions resulting in sufficient statistics of reconstructed background events are computationally unachievable or unreliable

    - e.g. MEG II 2021 data

Potential background events	$\sim 10^{12}$
After the trigger	2×10^{7}
In the analysis region	66
Within 1σ to the signal	< 1

The role of Monte Carlo simulations

- The extremely high resolutions and accuracies of single-purpose experin ... lations
- Additi ... ting in sufficie ... e compu
  - e.g.

The use of Monte Carlo simulations is typically very limited and, when MC inputs are unavoidable, the related systematic uncertainties are large

Potential background events	$\sim 10^{12}$
After the trigger	2×10^{7}
In the analysis region	66
Within 1σ to the signal	< 1

Confidence intervals and systematics

The Feldman-Cousins approach

- In FC, the confidence belt is built using the likelihood ratio test statistics, defined as:

$$\mathscr{R}(\mathbf{p}) = \frac{\mathscr{L}(\mathbf{p})}{\mathscr{L}(\hat{\mathbf{p}})}$$

$\mathbf{p}$: set of parameters

$\hat{\mathbf{p}}$: set of parameters maximizing the likelihood (i.e. fitted value)

- Given a set of values of the parameters **p**, the expected distribution of $\mathscr{R}$ is computed
- When the experiment is performed, the value of $\mathscr{R}$ is computed for different hypothetical sets of **p**:
  - a set of **p** is included in a confidence interval at C.L. = 1 - α if more than a fraction α of the experiments is expected to give a larger $\mathscr{R}(\mathbf{p})$ than data

Dealing with physical limits on a parameter

- For rare event searches ($p = N_{\text{sig}}$), the so-called "conditioning" is also included to properly treat the physical constraint $N_{\text{sig}} > 0$:

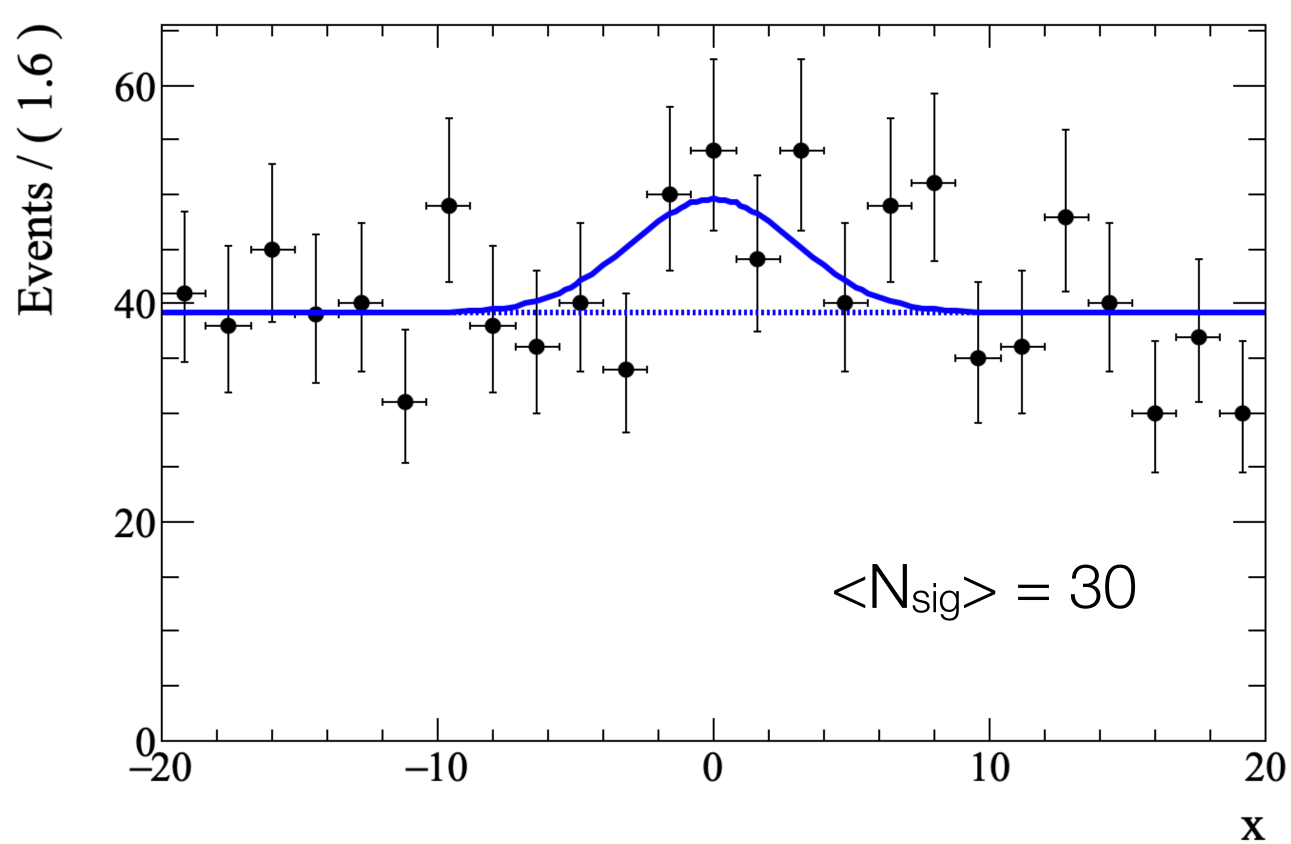
$$\mathcal{R} = \begin{cases} \frac{\mathcal{L}(N_{\text{sig}})}{\mathcal{L}(N_{\text{sig,best}})} & \text{if} \, N_{\text{sib,best}} \geq 0 \\ \\ \frac{\mathcal{L}(N_{\text{sig}})}{\mathcal{L}(0)} & \text{if} \, N_{\text{sib,best}} < 0 \end{cases}$$

The Feldman-Cousins approach

- The FC approach requires to evaluate the expected distribution of $\mathscr{R}(\mathbf{p})$
- Typically obtained by generating pseudo-experiments (toy Monte Carlo exp.) according to the expected PDFs

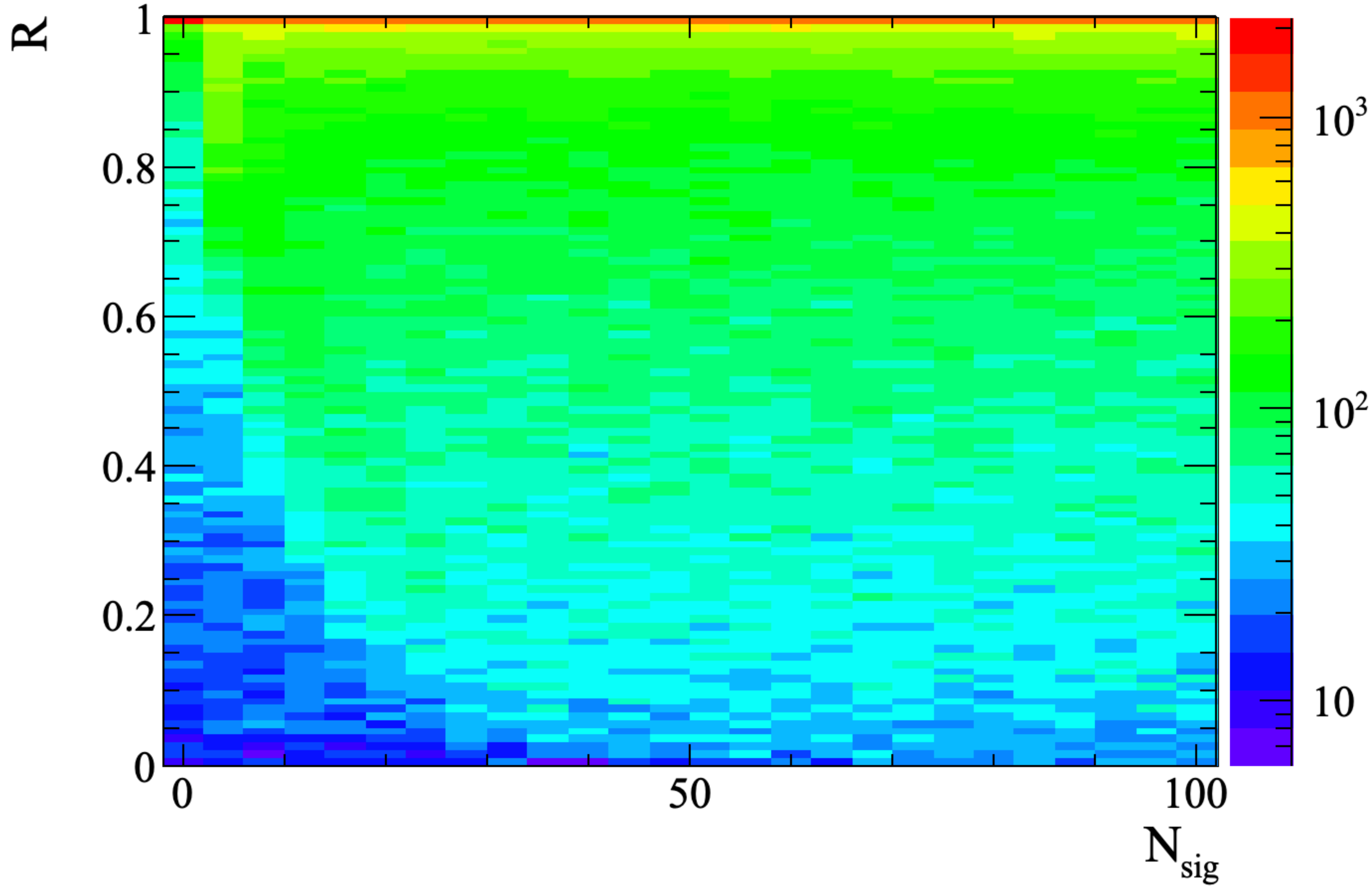
The Feldman-Cousins approach

- Let's consider the case of:
  - a single variable x
  - a gaussian signal (μ = 0, σ = 3)
  - a flat background of 1000 events in x ϵ [-20,20]



The Feldman-Cousins approach

Distribution of R from toy MCs generated with different <N_{sig}>



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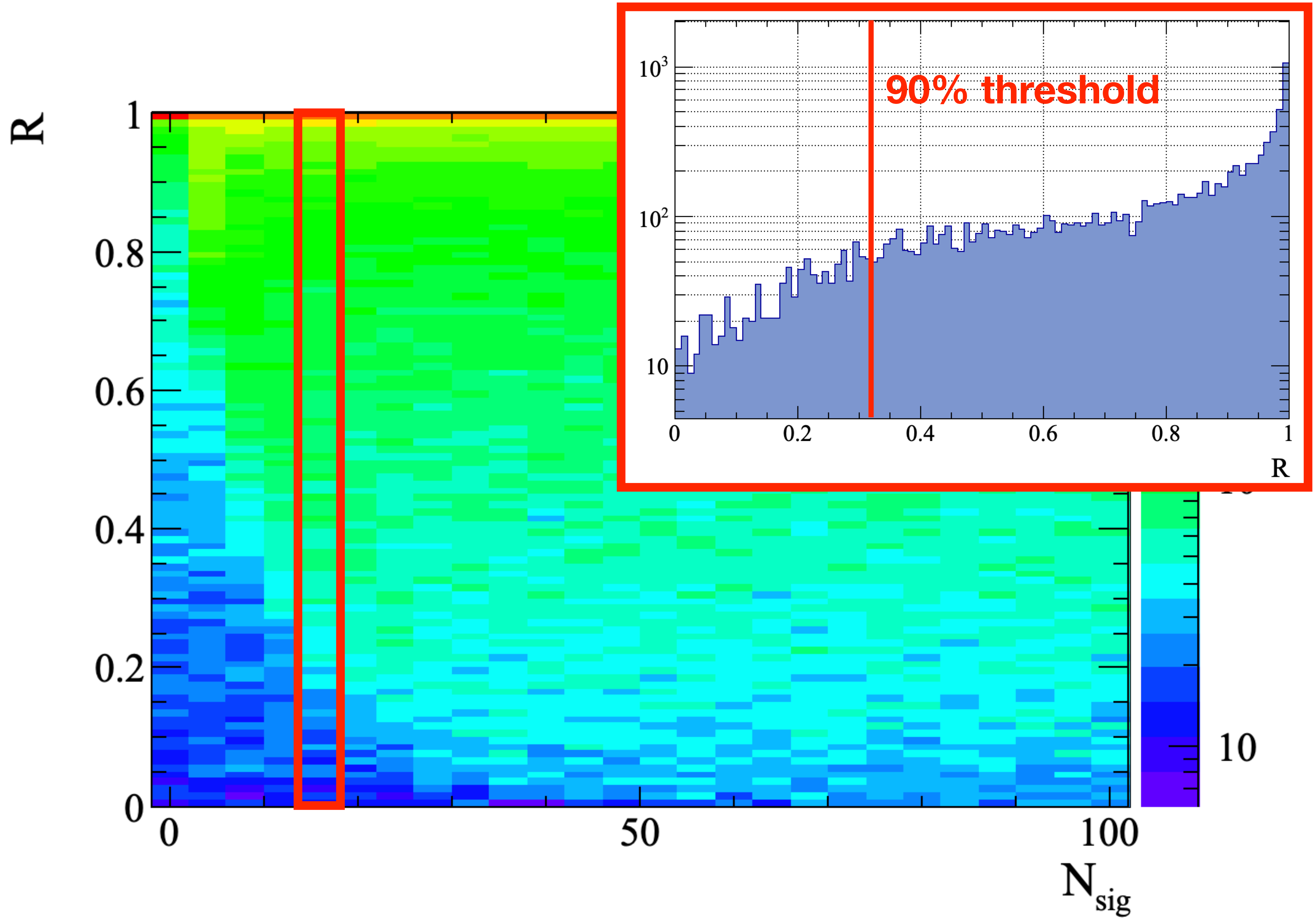

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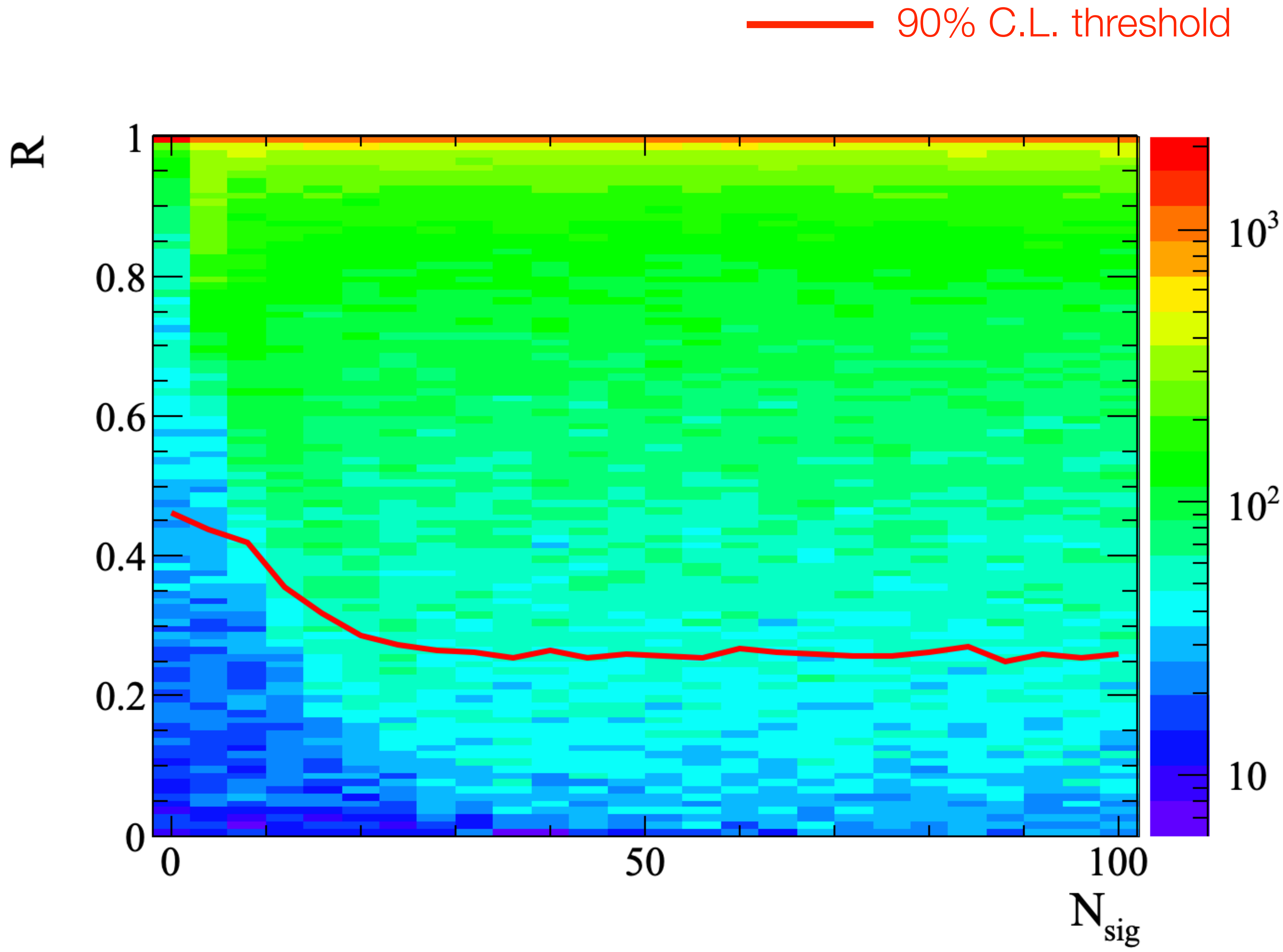

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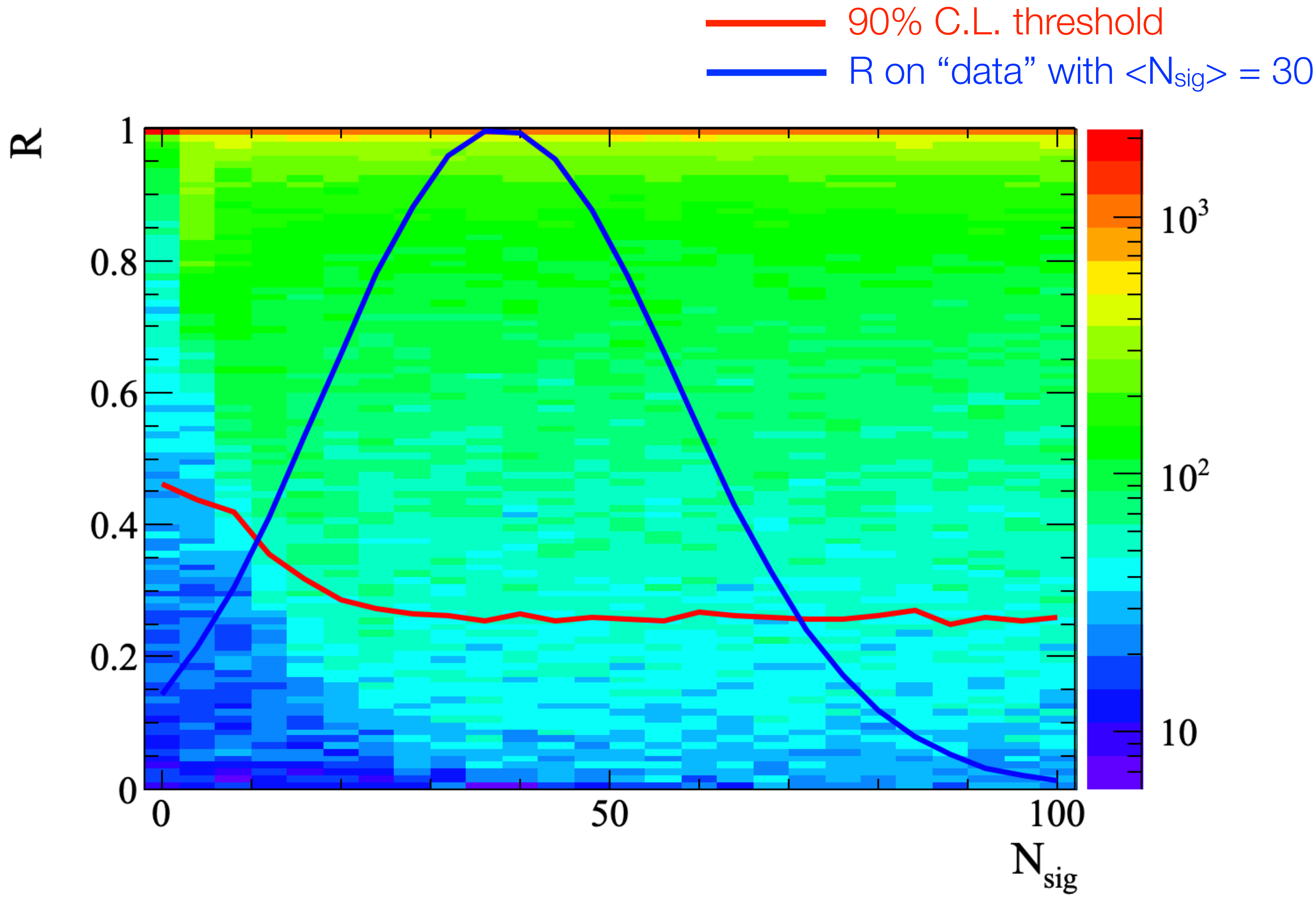

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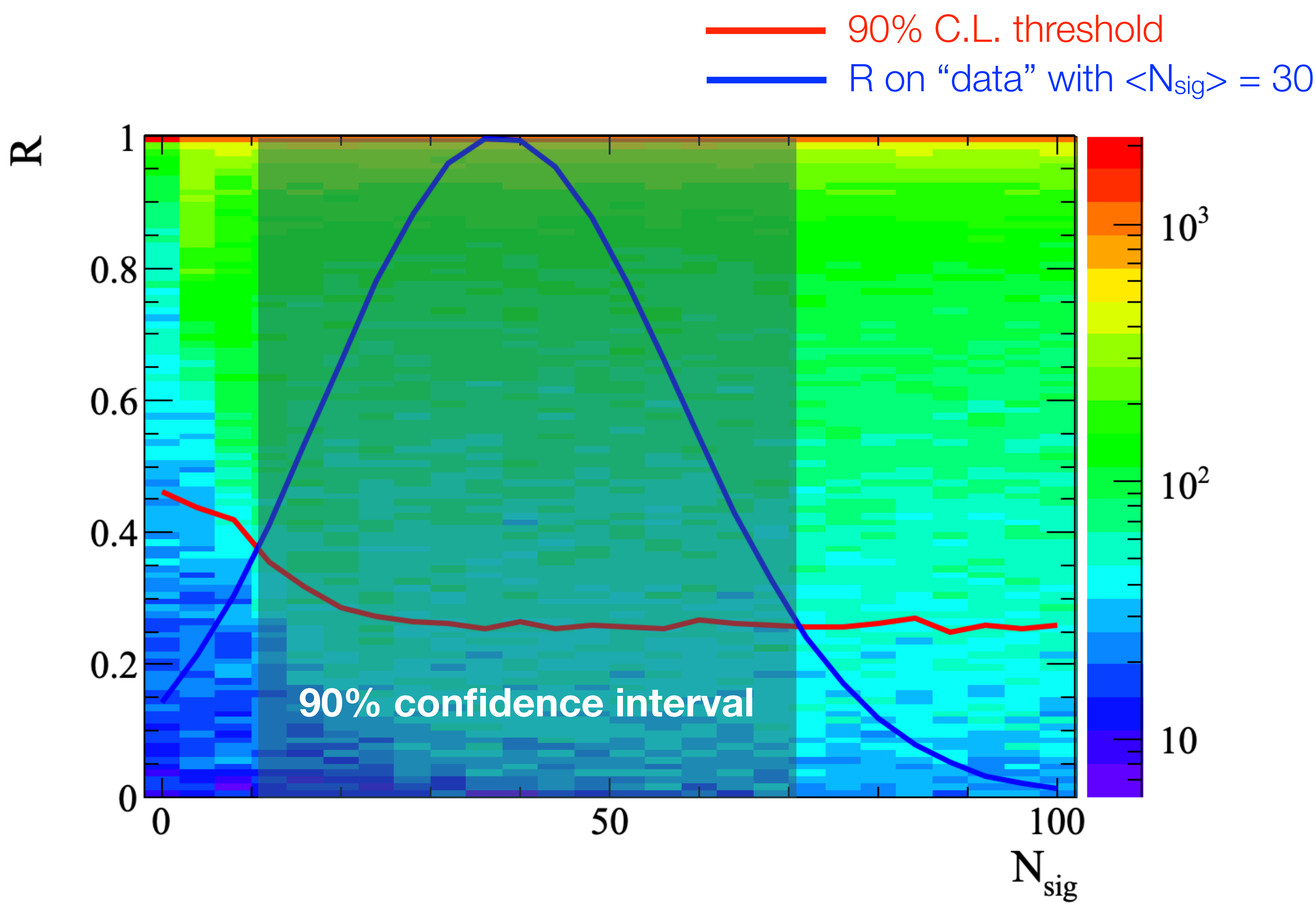
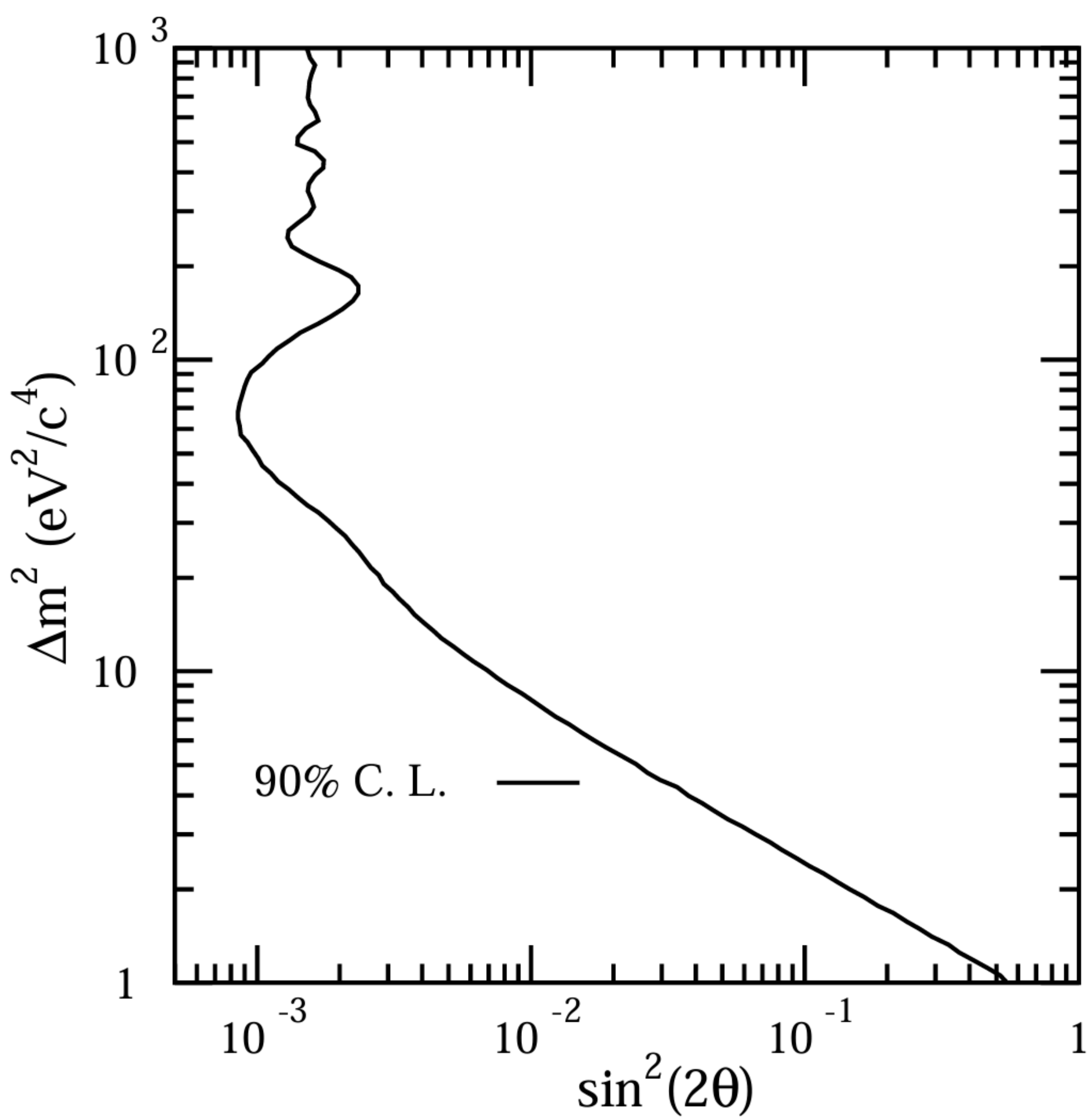


Multi-dimensional case

- The original FC problem (neutrino oscillation parameters)
- Counting experiment:
  - the observable is the number of countings n_{obs} (Poisson PDF)
  - there is an expected number of background events, n_{bkg}
  - for each point in the 2D space, there is an expected number of signal events, n_{sig}
  - for each point, the R distribution is derived from toy MCs with Poisson distribution
  - R from data is compared to the R distribution in toy MCs

$$\mathscr{R}(\Delta m^2, \sin^2(2\theta)) = \frac{\text{Poisson}(n_{\text{obs}}; n_{\text{sig}} + n_{\text{bkg}})}{\text{Poisson}(n_{\text{obs}}; n_{\text{sig,best}} + n_{\text{bkg}})}$$

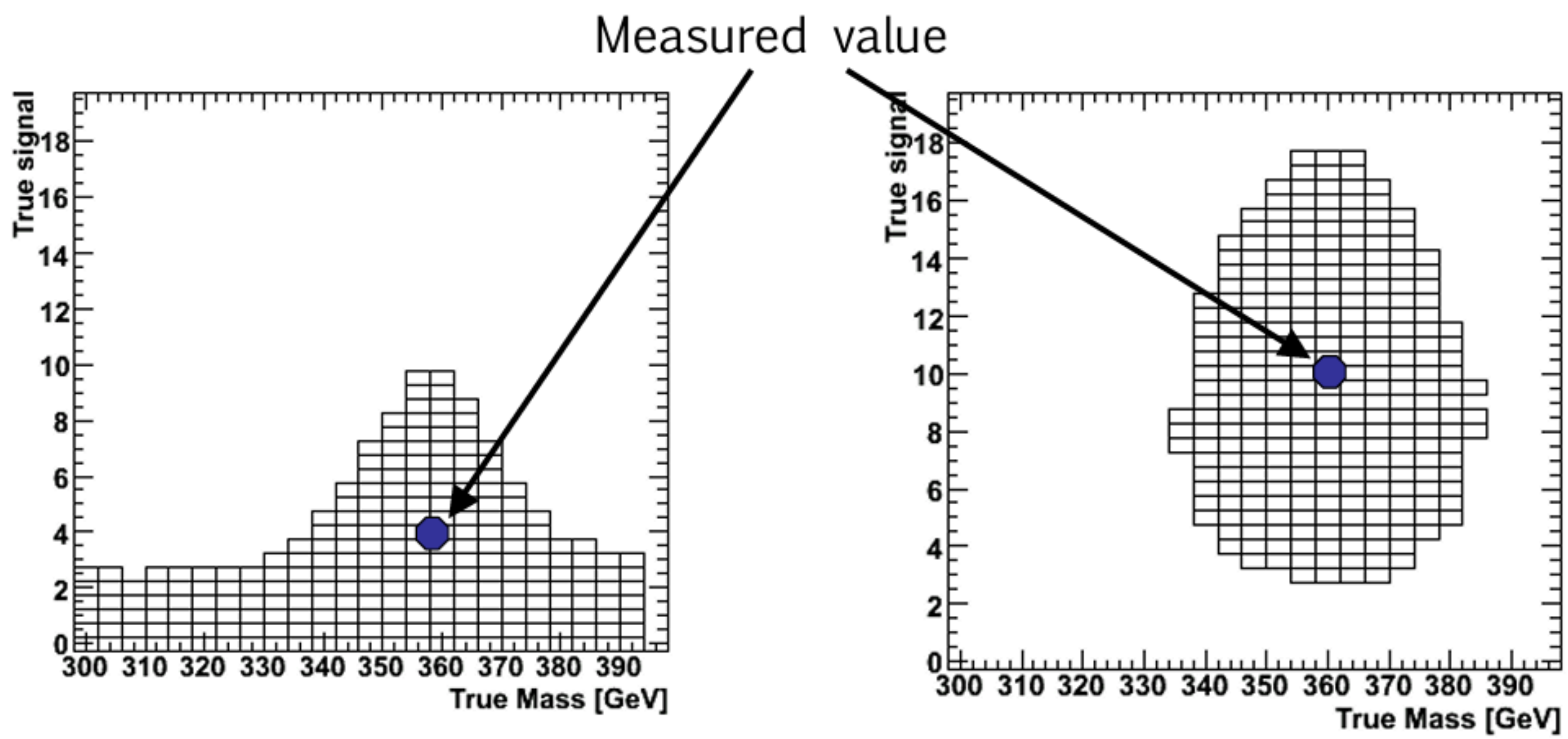
with $n_{\text{sig,best}} > 0$



Multi-dimensional case

$$\mathscr{R} = \frac{\mathscr{L}(N_{\text{sig}})}{\mathscr{L}(\hat{N}_{\text{sig}})}$$

$$\mathscr{R} = \frac{\mathscr{L}(N_{\text{sig}}, M)}{\mathscr{L}(\hat{N}_{\text{sig}}, \hat{M})}$$



- Coverage guaranteed for each pair of true (N_{sig}, M)
  - no need of look-elsewhere corrections

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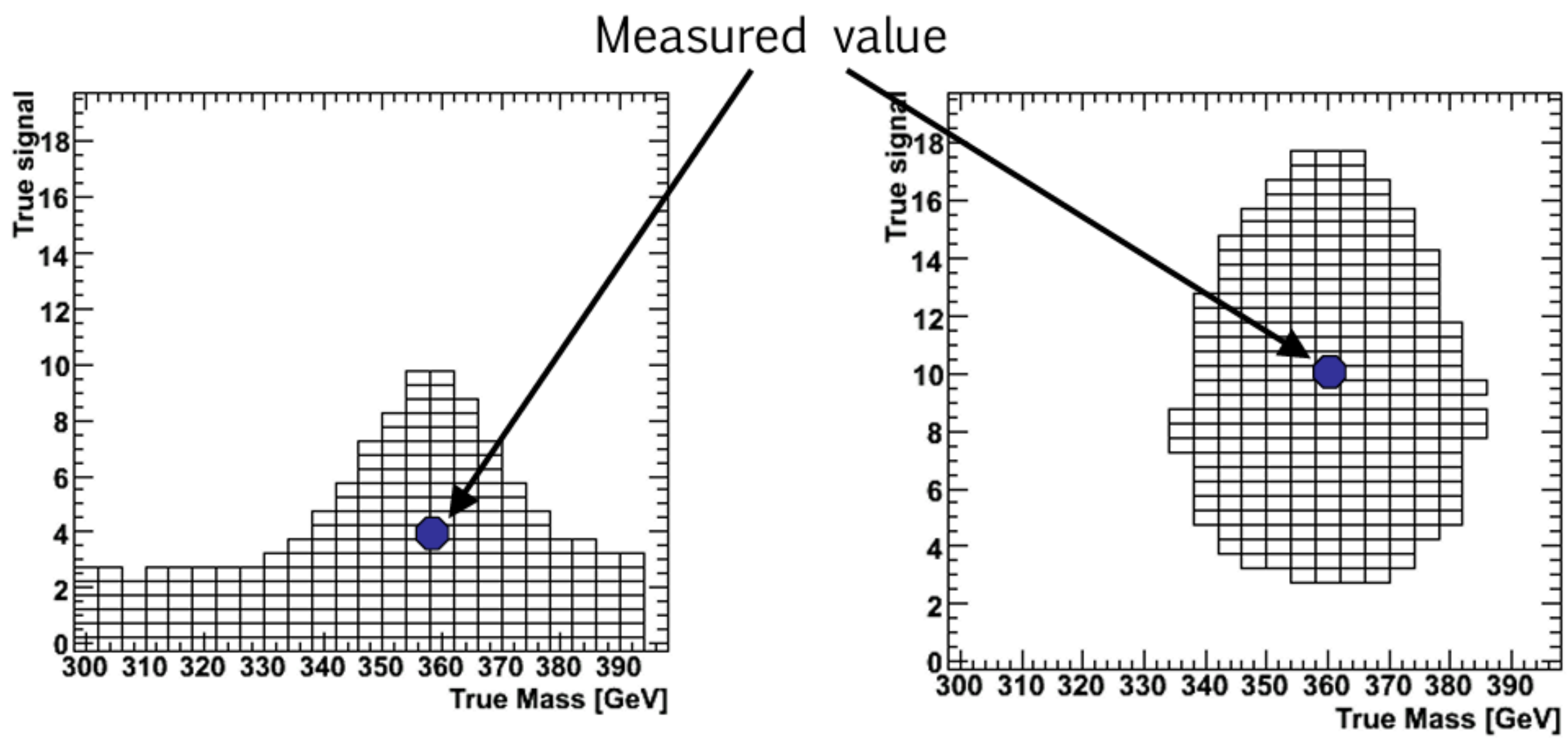

- Coverage guaranteed for each pair of true (N_{sig}, M)
  - no need of look-elsewhere corrections

The Feldman-Cousins approach

- The FC approach requires to evaluate the expected distribution of $\mathscr{R}(\mathbf{p})$
- Typically obtained by generating pseudo-experiments (toy Monte Carlo exp.) according to the expected PDFs
- CAVEAT: can be computationally heavy for multi-dimensional parameter space, complex likelihoods and very small p-values:
  - for a 5σ test, need to generate ~ 10^9 toy MC experiments

Inclusion of systematics

- For the inclusion of systematics, the most popular approaches are:

  - semi-bayesian approach (Highland-Cousins): the likelihood is integrated over the nuisance parameters before applying the desired statistical approach

e.g. likelihood for poisson-distributed yields, integrated over a gaussian uncertainty on the expected background *b*

$$q(n)_{s+b} = \frac{1}{\sqrt{2\pi}\sigma_b} \int_0^\infty p(n)_{s+b'} e^{-(b-b')^2/2\sigma_b^2} db'$$

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e.g. likelihood for poisson-distributed yields, integrated over gaussian uncertainties on the expected background *b* and signal efficiency ϵ

$$q(n)_{s+b} = \frac{1}{2\pi\sigma_b\sigma_\epsilon}\int_0^\infty\int_0^\infty p(n)_{b'+\epsilon' s}$$

$$\times e^{-(b-b')^2/2\sigma_b^2} e^{-(1-\epsilon')^2/2\sigma_\epsilon^2} db'\, d\epsilon'$$

Inclusion of systematics

- For the inclusion of systematics, the most popular approaches are:

  - profile likelihood ratio: the likelihood is maximized with respect to the nuisance parameters when building the likelihood ratio:

$$\mathscr{L}(\mathbf{p}, \mathbf{q}) = P(\mathrm{data} \,|\, \mathbf{p}, \mathbf{q}) P(\mathbf{q})$$

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External constraint: PDF (e.g. gaussian) representing the uncertainty on the nuisance parameters

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$$\mathscr{R}(\mathbf{p}) = \frac{\mathscr{L}(\mathbf{p}, \hat{\hat{\mathbf{q}}})}{\mathscr{L}(\hat{\mathbf{p}}, \hat{\mathbf{q}})}$$

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$$\mathscr{L}(\mathbf{p}, \mathbf{q}) = P(\mathrm{data} \,|\, \mathbf{p}, \mathbf{q}) P(\mathbf{q})$$

Likelihood maximized over **q** for fixed **p**

$$\mathscr{R}(\mathbf{p}) = \frac{\mathscr{L}(\mathbf{p}, \hat{\mathbf{q}})}{\mathscr{L}(\hat{\mathbf{p}}, \hat{\mathbf{q}})}$$

Likelihood maximized over **p** and **q**

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$$\mathscr{R} = \frac{\text{Poisson}(n_{\rm obs}; n_{\rm sig} + n_{\rm bkg}) P(n_{\rm bkg})}{\text{Poisson}(n_{\rm obs}; n_{\rm sig,best} + n_{\rm bkg,best}) P(n_{\rm bkg,best})}$$

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$$\mathcal{R} = \begin{cases} \dfrac{\mathcal{L}(N_{\rm sig}, \hat{\hat{\mathbf{p}}}(N_{sig}))}{\mathcal{L}(\hat{N}_{\rm sig}, \hat{\mathbf{p}})} & \text{if } N_{\rm sig,best} \geq 0 \\ \\ \dfrac{\mathcal{L}(N_{\rm sig}, \hat{\hat{\mathbf{p}}}(N_{sig}))}{\mathcal{L}(0, \hat{\hat{\mathbf{p}}}(0))} & \text{if } N_{\rm sig,best} < 0 \end{cases}$$

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$$\mathscr{L}(N_{\text{sig}}, \mathbf{p}) = P(\text{data} \,|\, N_{\text{sig}}, \mathbf{p}) P(\mathbf{p})$$

Set of nuisance parameters which maximizes the likelihood for a given N_{sig}

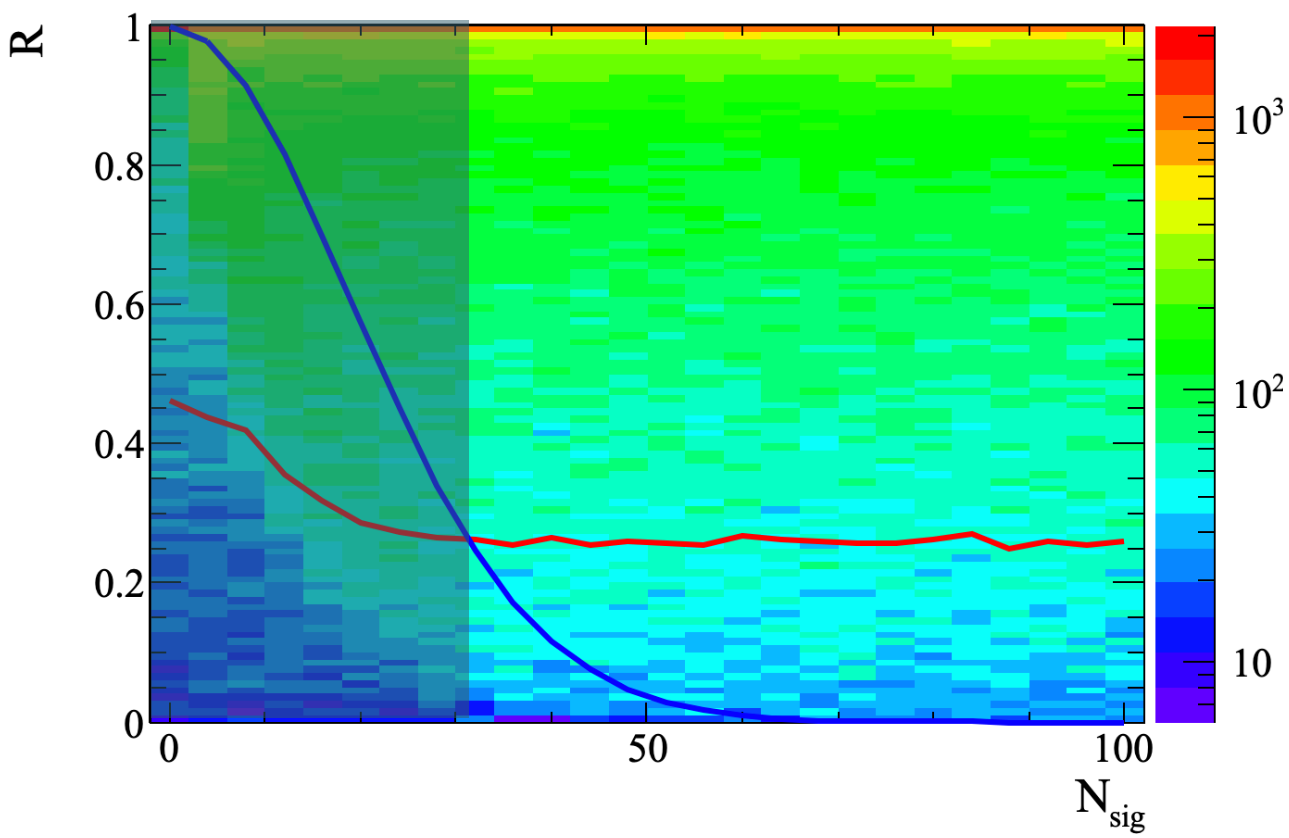
$$\mathcal{R} = \begin{cases} \dfrac{\mathcal{L}(N_{\text{sig}}, \hat{\hat{\mathbf{p}}}(N_{sig}))}{\mathcal{L}(N_{\text{sig}}, \hat{\mathbf{p}})} & \text{if}\, N_{\text{sig,best}} \geq 0 \\[2ex] \dfrac{\mathcal{L}(N_{\text{sig}}, \hat{\hat{\mathbf{p}}}(N_{sig}))}{\mathcal{L}(0, \hat{\hat{\mathbf{p}}}(0))} & \text{if}\, N_{\text{sig,best}} < 0 \end{cases}$$

Inclusion of systematics

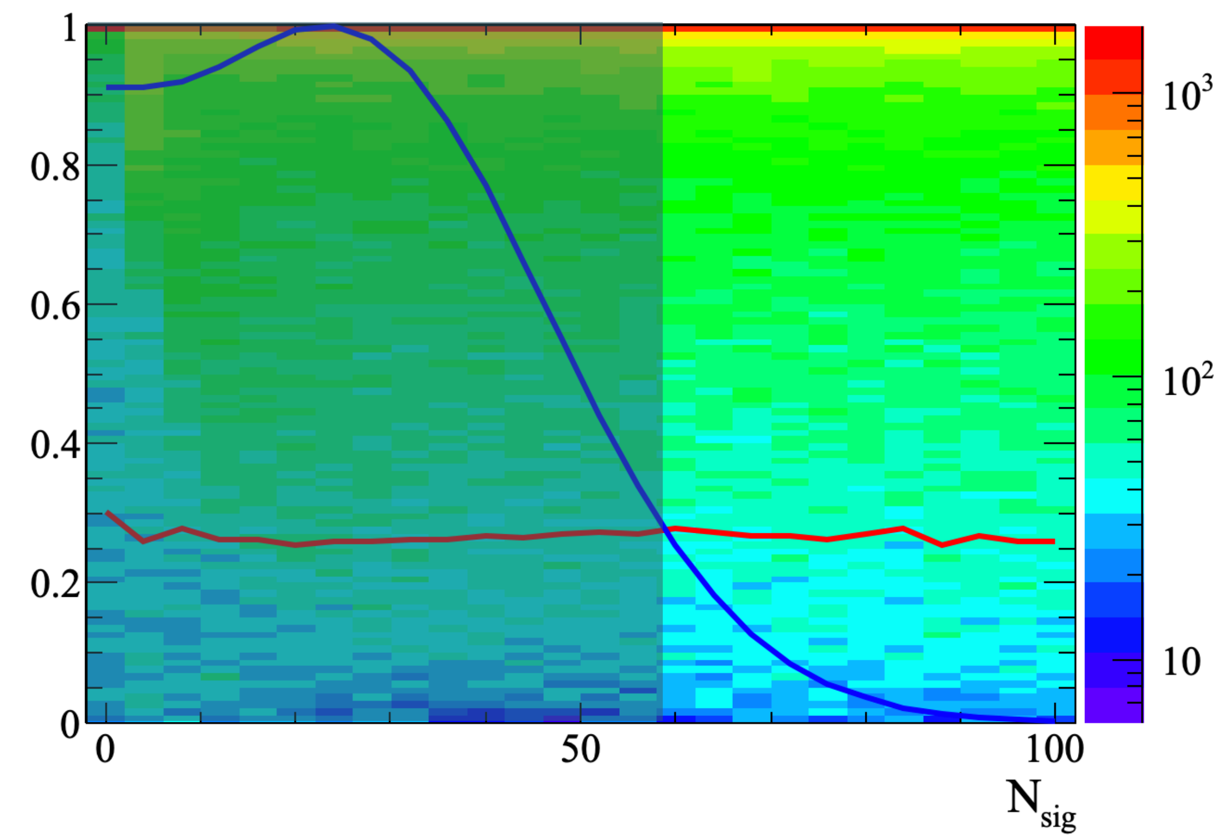
90% C.L. threshold

R on “data” with <N_{sig}> = 0

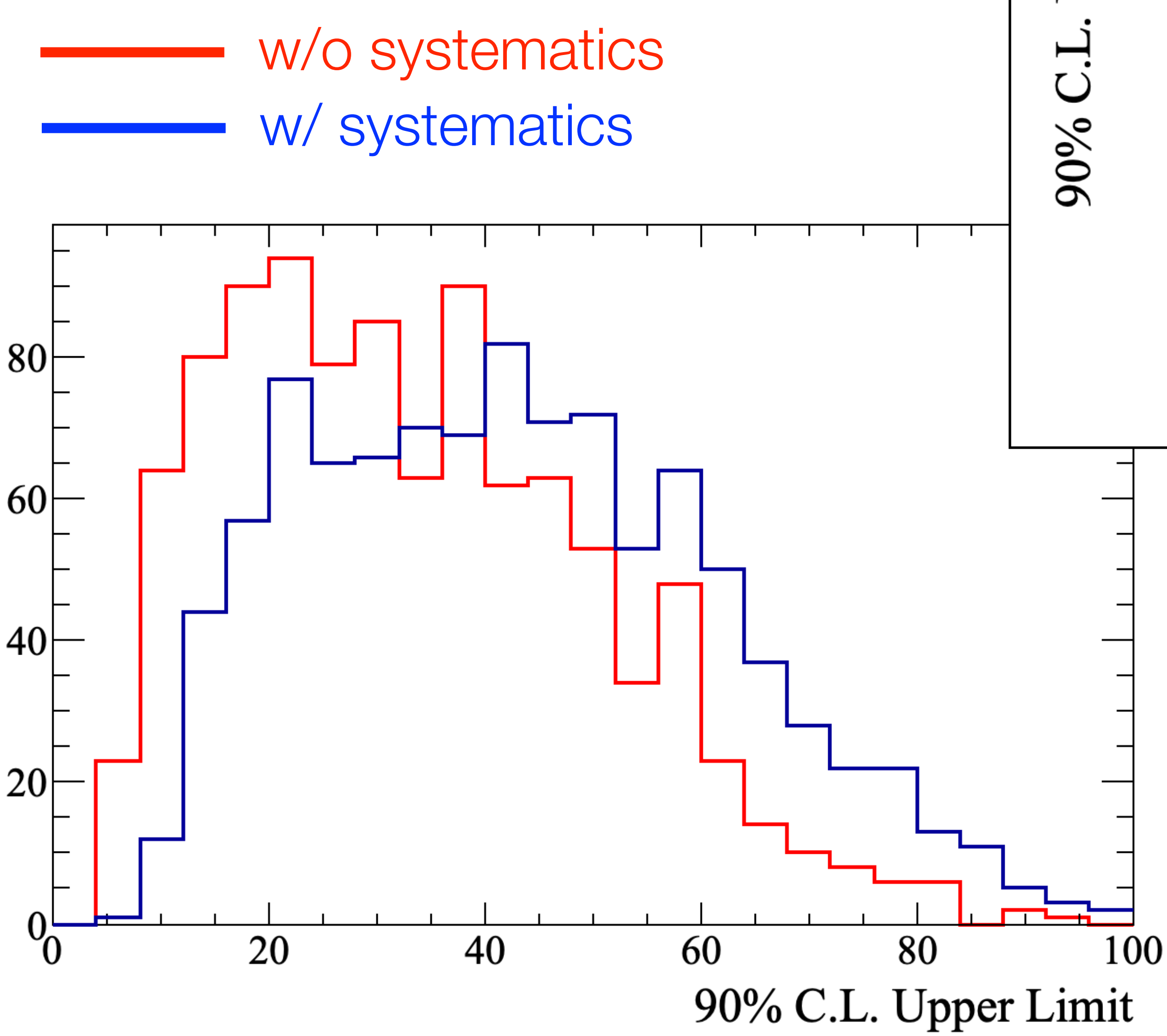
w/o systematics

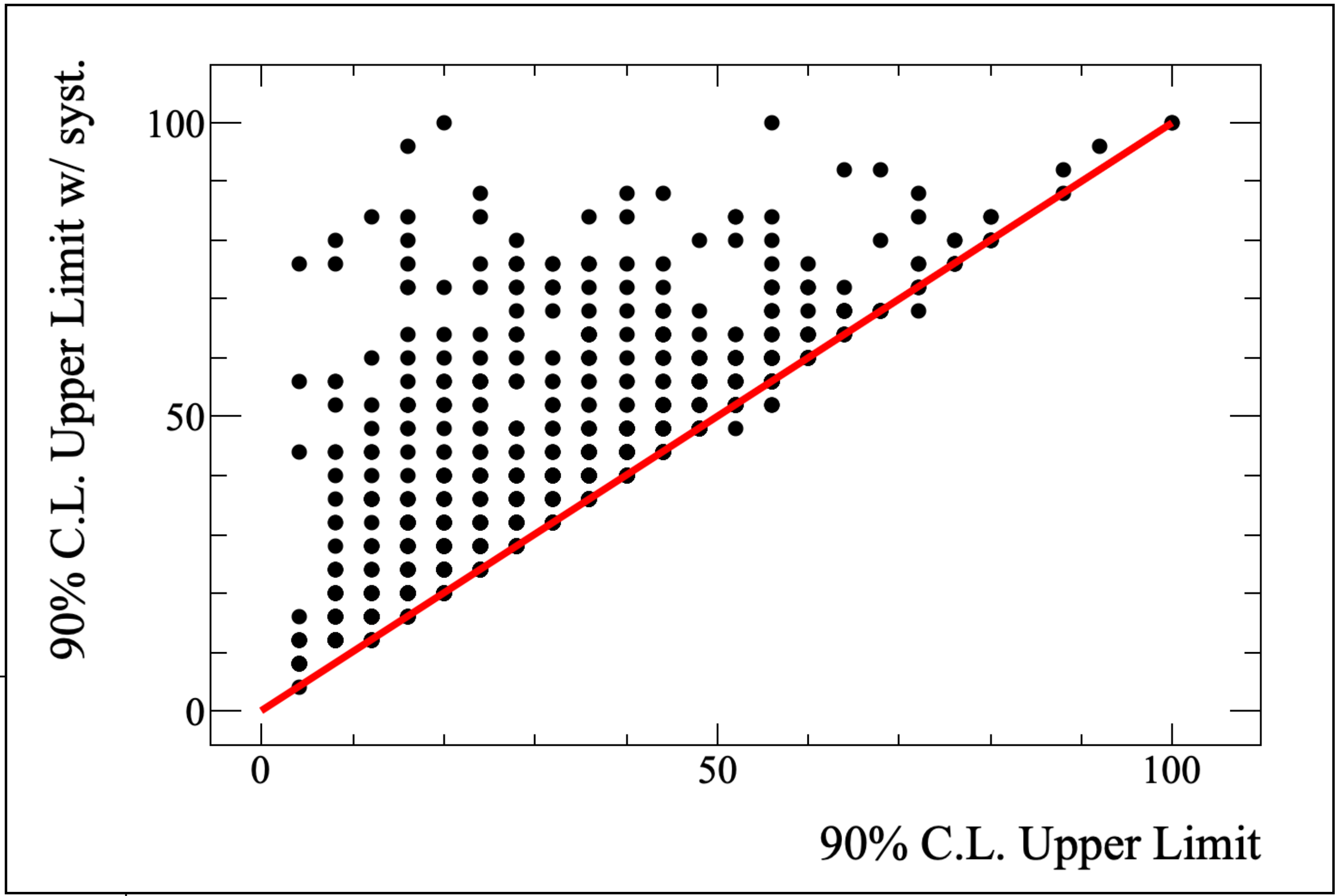


w/ ±3 uncertainty on μ



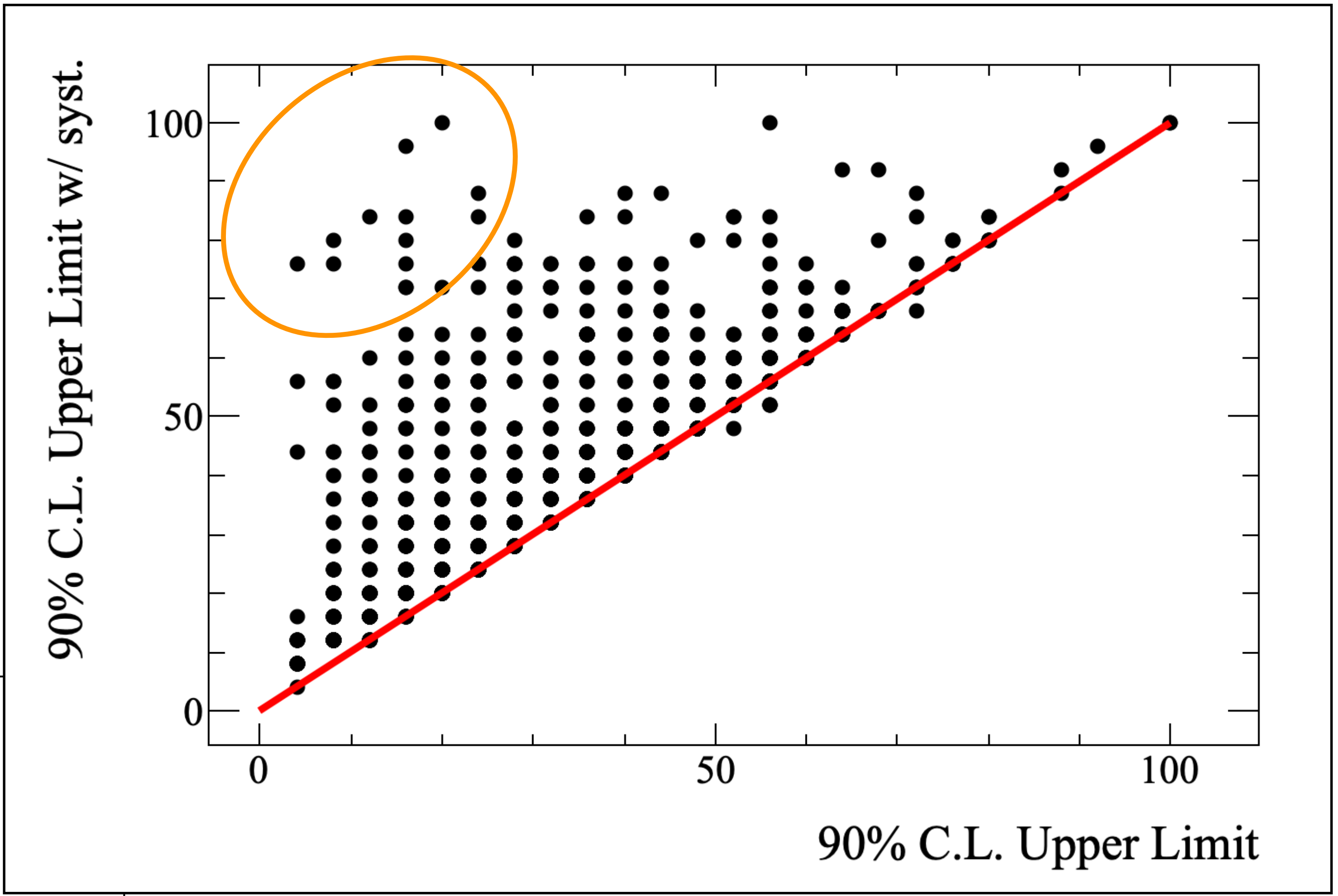
Inclusion of systematics





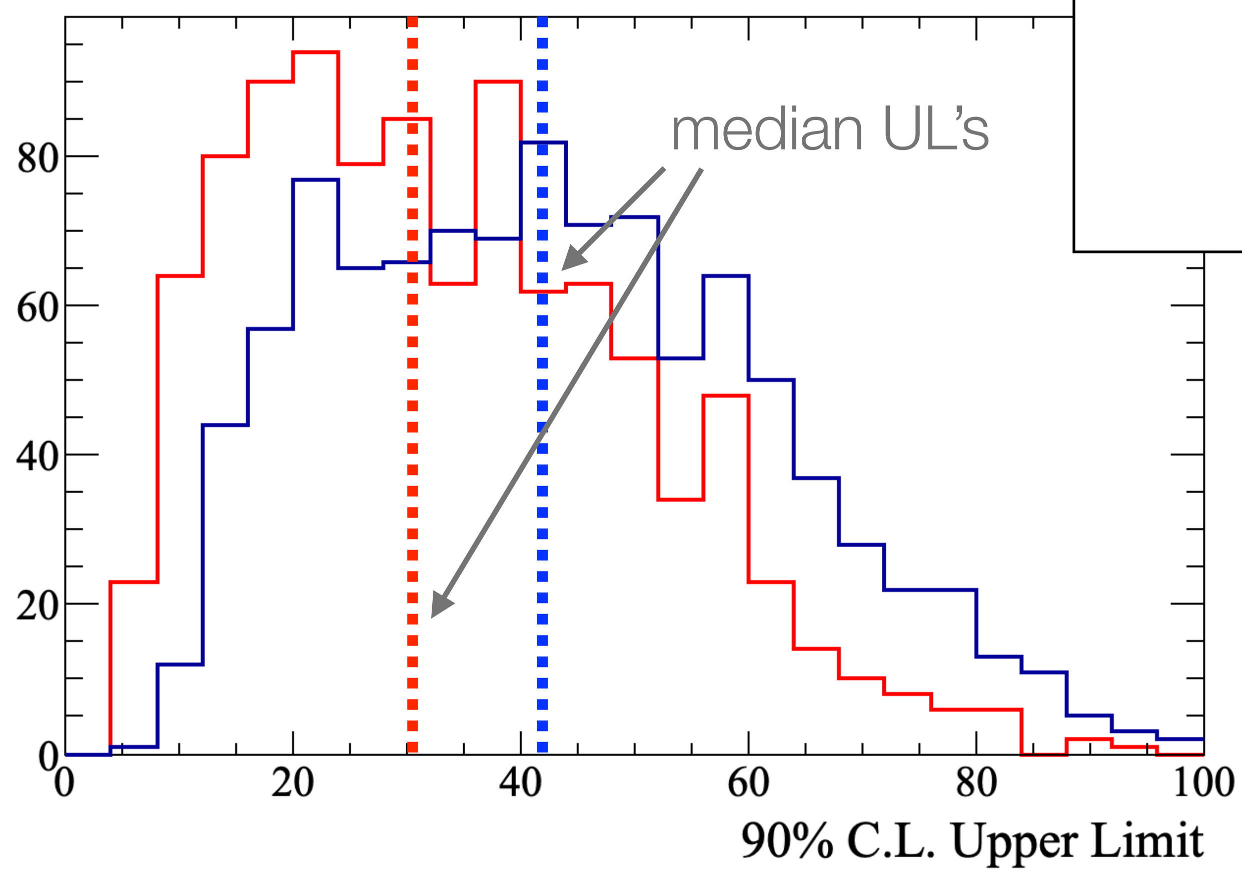
Inclusion of systematics

With an uncertainty in the mean equal to the gaussian width, the impact in some experiments can be huge (up to a factor 10!!!)



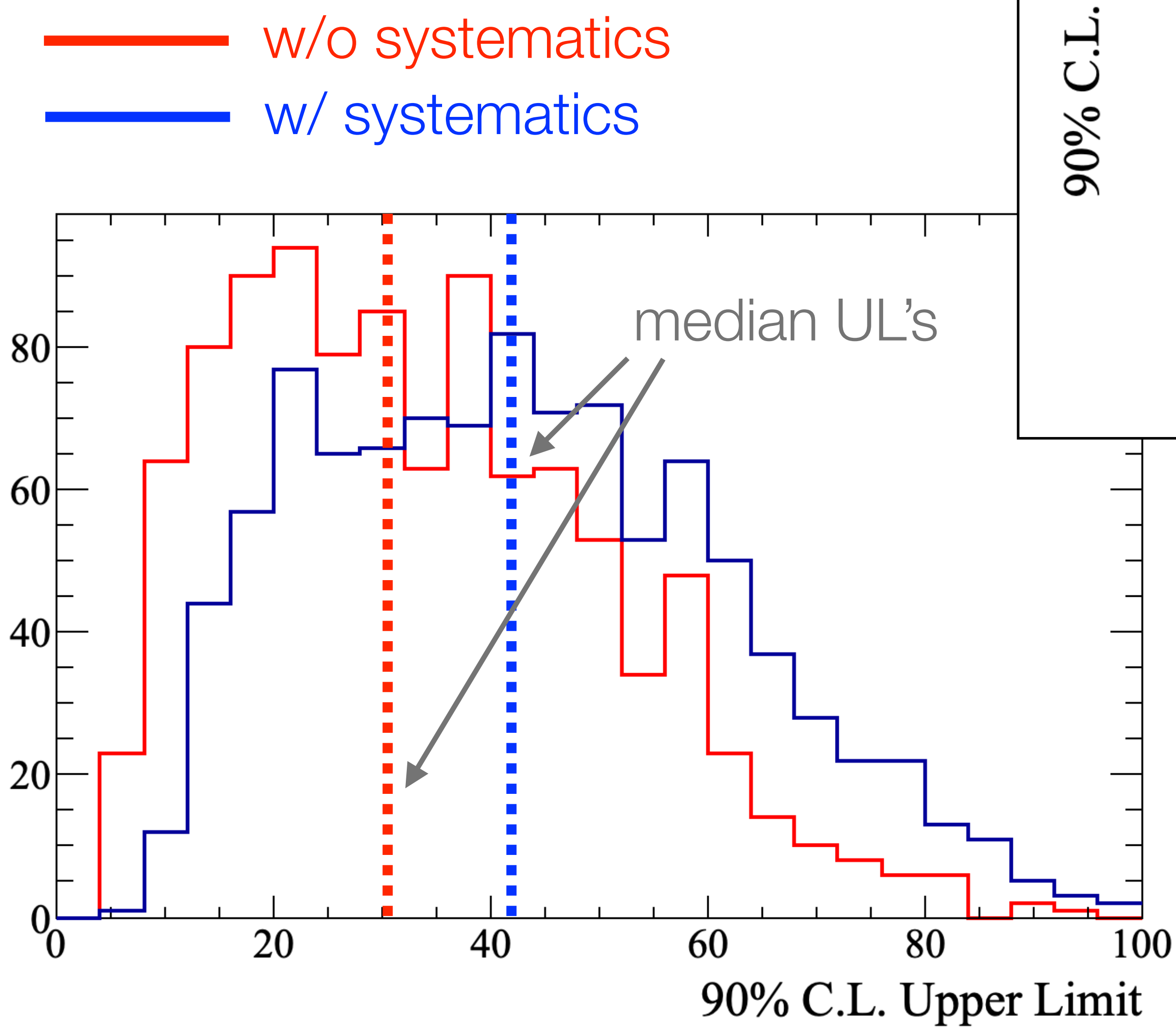
w/o systematics

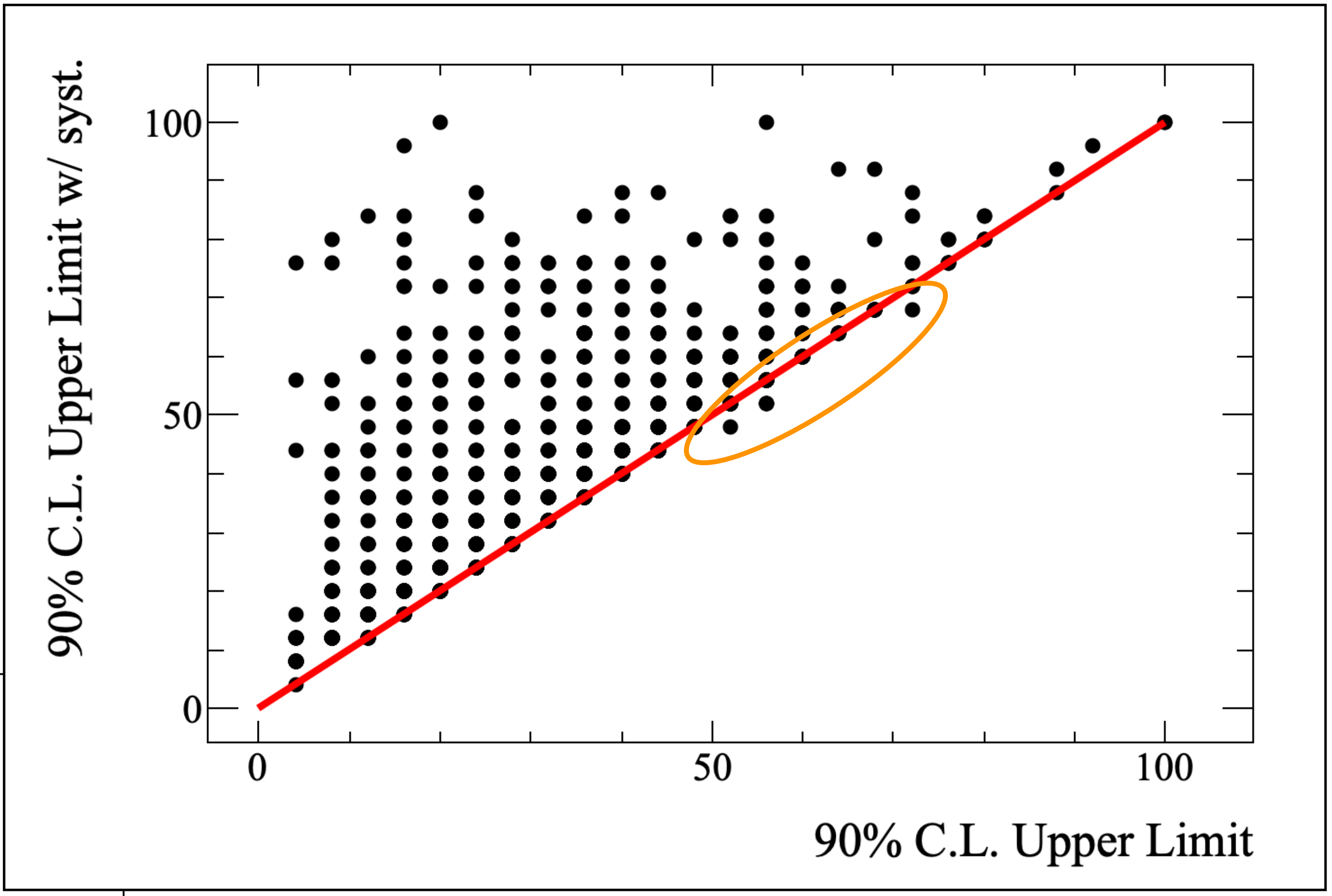
w/ systematics



Inclusion of systematics

With an uncertainty in the mean equal to the gaussian width, the impact in some experiments can be huge (up to a factor 10!!!)





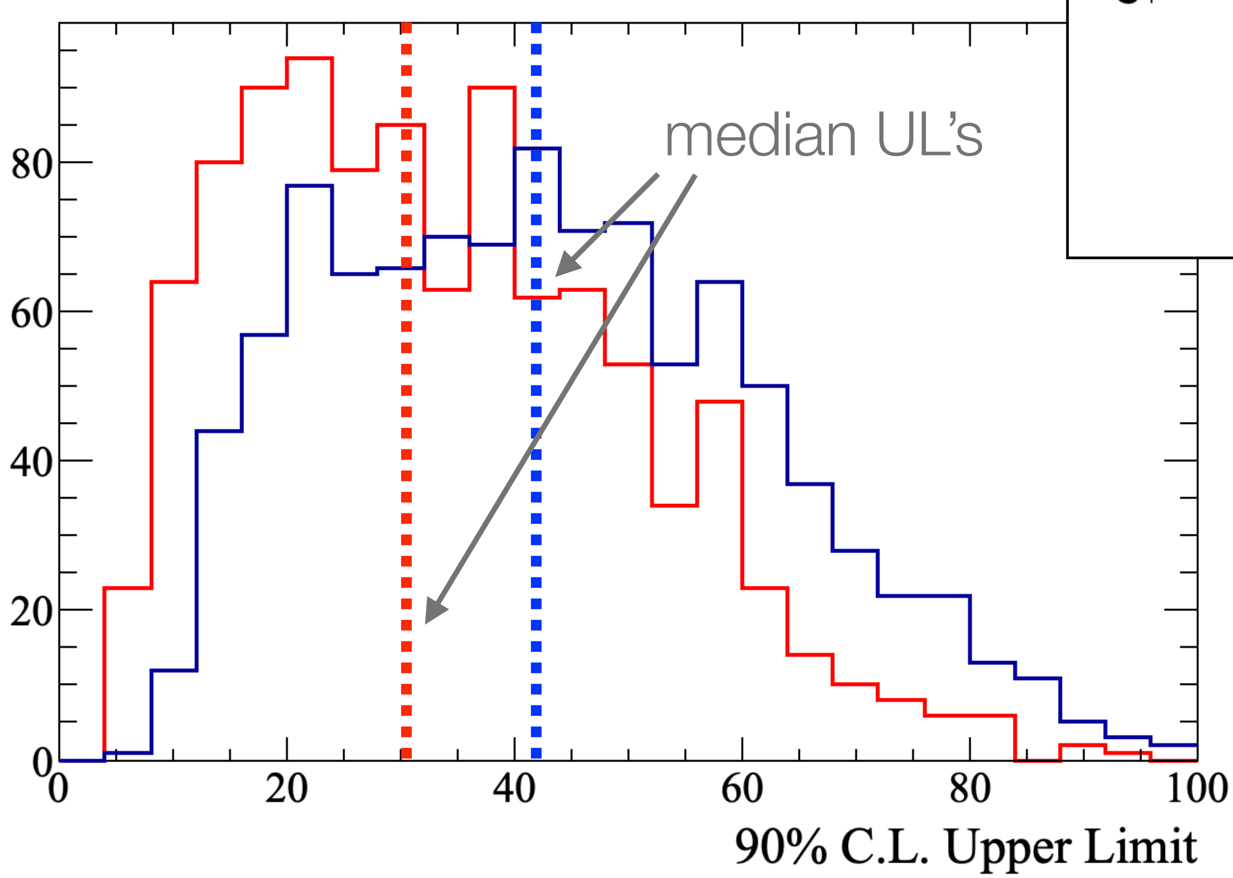
N.B. due to stat. fluctuations, in some experiments the UL w/ systematics can be lower than w/o

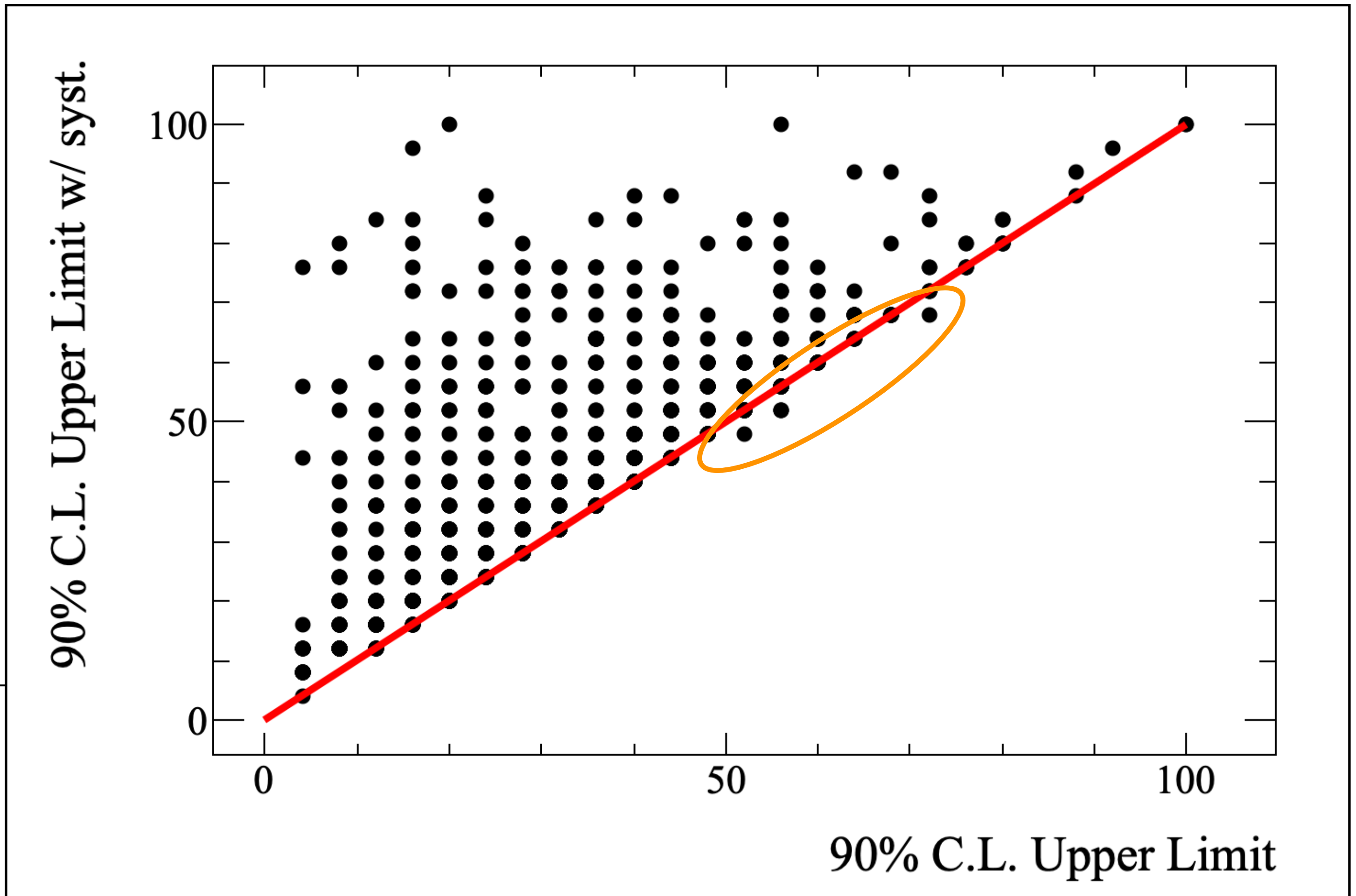
Is it a problem?

Inclusion of systematics

With an uncertainty in the mean equal to the gaussian width, the impact in some experiments can be huge (up to a factor 10!!!)

w/o systematics
w/ systematics





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Is it a problem? *No, systematics are treated here in a frequentistic way, what matters is the coverage*

Inclusion of systematics

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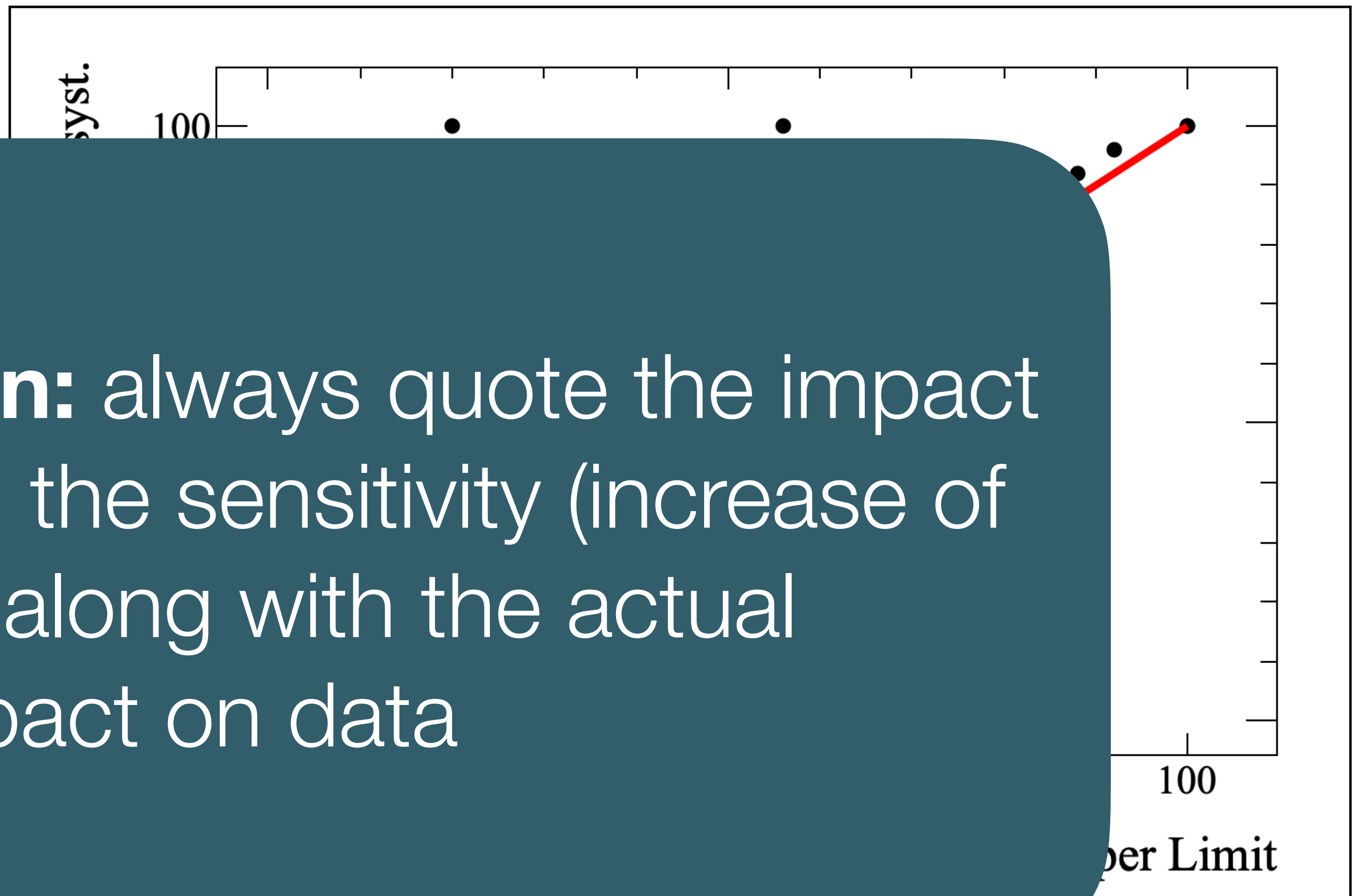
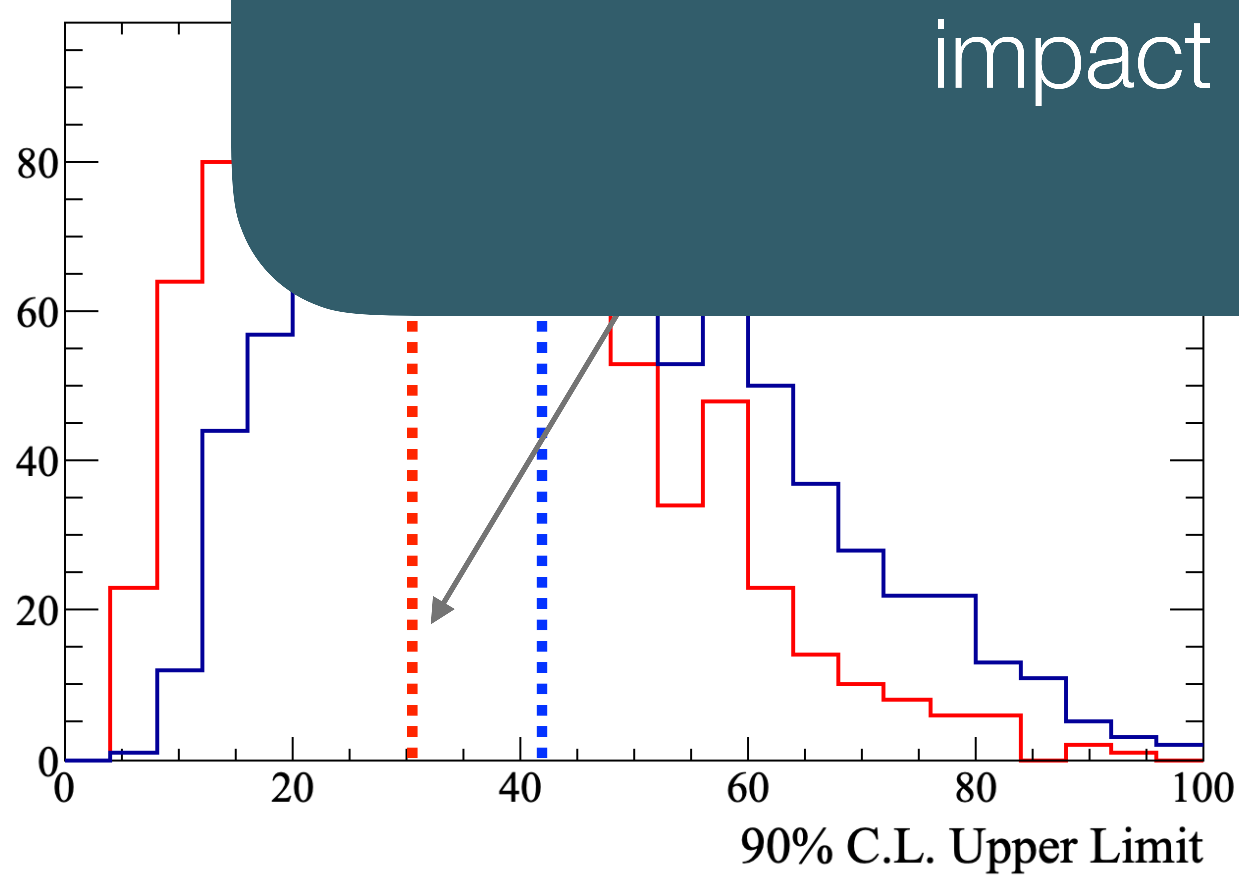


Recommendation: always quote the impact of systematics on the sensitivity (increase of median UL) along with the actual impact on data



N.B. due to stat. fluctuations, in some experiments the UL w/ systematics can be lower than w/o

Is it a problem? *No, systematics are treated here in a frequentistic way, what matters is the coverage*

Inclusion of systematics

- For the inclusion of systematics, the most popular approaches are:

    - simplified approach: in the toy MCs used to build the likelihood ratio distributions, nuisance parameters are randomly fluctuated in the generation or fit of the toy MC experiments (conceptually similar to the semi-bayesian approach)

The generation of the toy MC experiments

- What values of the nuisance parameters should be used when generating the toy MC samples?

- Several options:

    - **a priori estimate:** fixed values decided a priori

    - can have significant under- or over-coverage

The generation of the toy MC experiments

- What values of the nuisance parameters should be used when generating the toy MC samples?

- Several options:

  - **conservative:** generate with the values giving the worst upper limit

  - can have very large over-coverage

arXiv:2207.14353

The generation of the toy MC experiments

- What values of the nuisance parameters should be used when generating the toy MC samples?

- Several options:

  - **Highland-Cousins:** extract a random value of the nuisance parameters for each toy MC experiment, according to an a-priori distribution

  - can have some over-coverage when the nuisance parameter have a true single value (not varying from experiment to experiment)

The generation of the toy MC experiments

- What values of the nuisance parameters should be used when generating the toy MC samples?

- Several options:

  - **a-posteriori Highland-Cousins:** extract a random value of the nuisance parameters for each toy MC experiment, according to an a-posteriori distribution derived from data

  - can still have some over-coverage, but less than the a-priori method

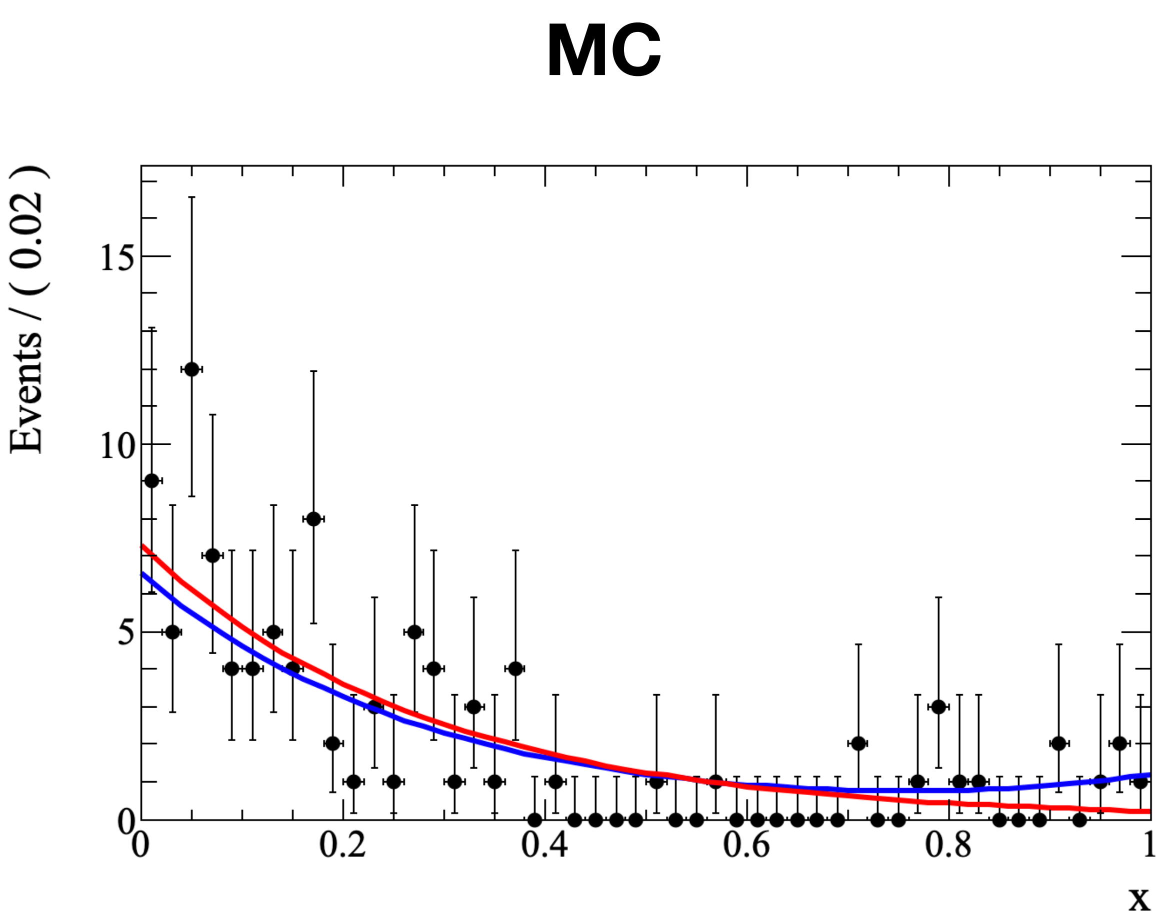
PDFs from small Monte Carlo samples

- We have seen that the generation of MC samples for extremely rare events can be problematic
- Nonetheless, sometimes the use of MC to extract PDFs is unavoidable
  - if the PDF shape is not known a priori, and the MC sample is too small to infer a reliable parameterization, there is a strong risk of overestimating or underestimating the systematic uncertainties, due to the inclusion of unnecessary shape uncertainties

PDFs from small Monte Carlo samples

True distribution

Exponential estimated from the MC



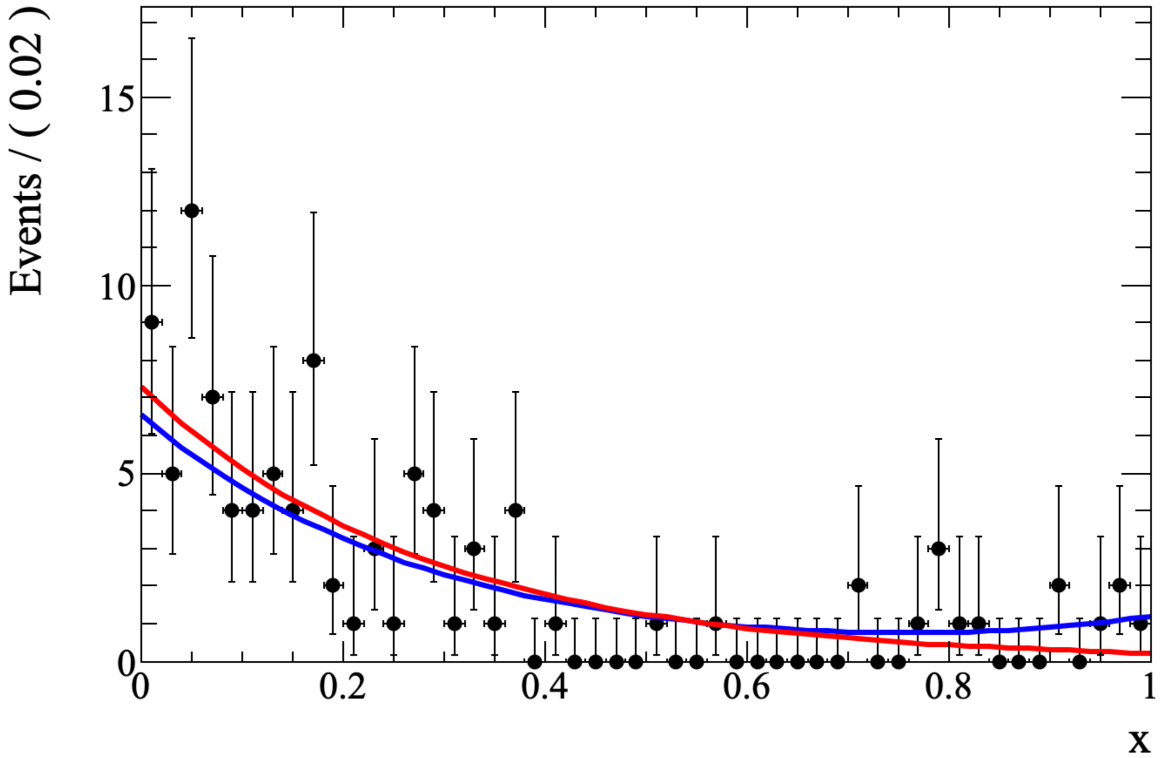
PDFs from small Monte Carlo samples

True distribution

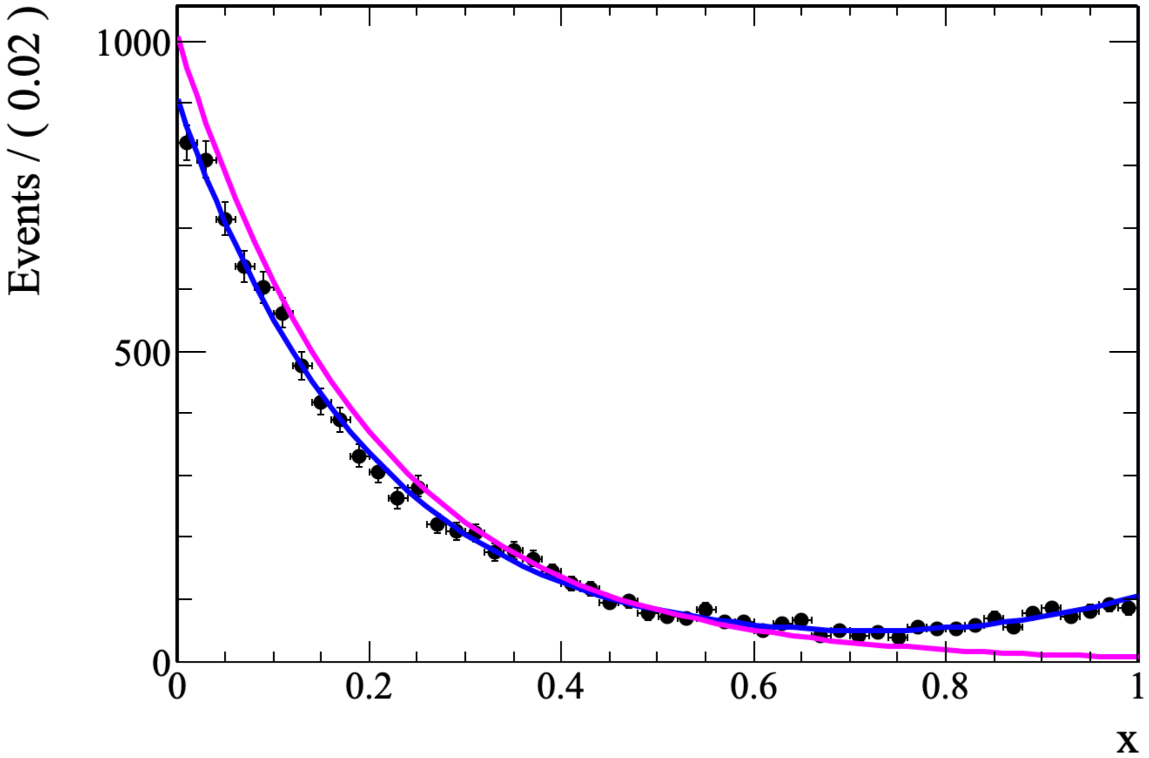
Exponential estimated from the MC

Exponential estimated on data with constraint from the MC

MC



Data



PDFs from small Monte Carlo samples

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- Nonetheless, sometimes the use of MC to extract PDFs is unavoidable
  - if the PDF shape is not known a priori, and the MC sample is too small to infer a reliable parameterization, there is a strong risk of overestimating or underestimating the systematic uncertainties, due to the inclusion of unnecessary shape uncertainties
  - using MC histograms to represent the PDFs, with a proper treatment of uncertainties, could be the solution

Comput.Phys.Commun. 77 (1993), 219-228

The Beeston-Barlow approach

- Binned fit with different populations (e.g. signal, backgrounds)
- The expected MC content of each bin i for each population j (A_{ji}) is treated as a nuisance parameter, constrained from the actual MC

$$f_i = \sum_{j=0}^{m} p_j A_{ji}$$

f_i: expected yield in bin i of data
p_j: data/MC scale factor for population j (to be estimated)

$$\ln \mathscr{L} = \sum_{i=1}^{n} d_i \ln f_i - f_i + \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ji} \ln A_{ji} - A_{ji}$$

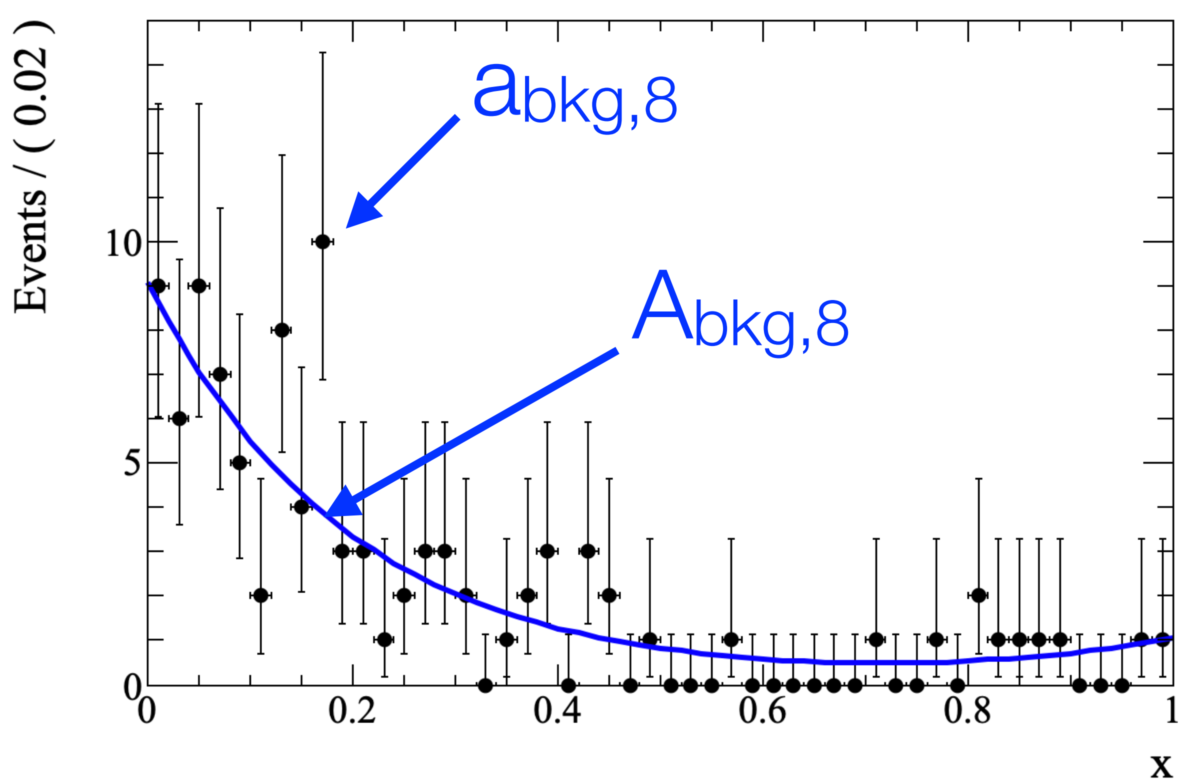
Likelihood for data (Poisson)
d_i: observed yield in bin i of data

Likelihood for MC (Poisson)
a_{ij}: observed yield in bin i of MC for population j
A_{ji}: expected yield in bin i of MC for population j (nuisance parameters)

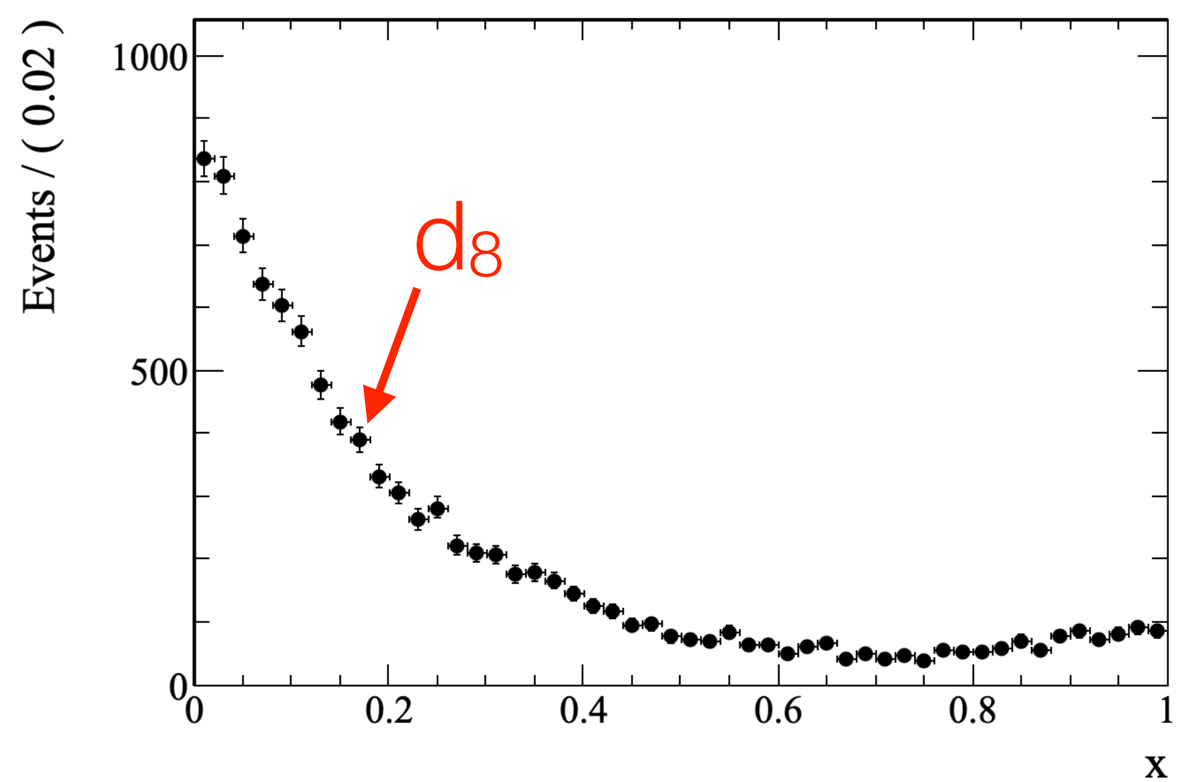
Comput.Phys.Commun. 77 (1993), 219-228

The Beeston-Barlow approach

MC



Data



$$f_i = \sum_{j=0}^{m} p_j A_{ji}$$

Can be computationally unaffordable, lite versions exist

$$\ln \mathscr{L} = \sum_{i=1}^{n} d_i \ln f_i - f_i + \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ji} \ln A_{ji} - A_{ji}$$

Practical Examples

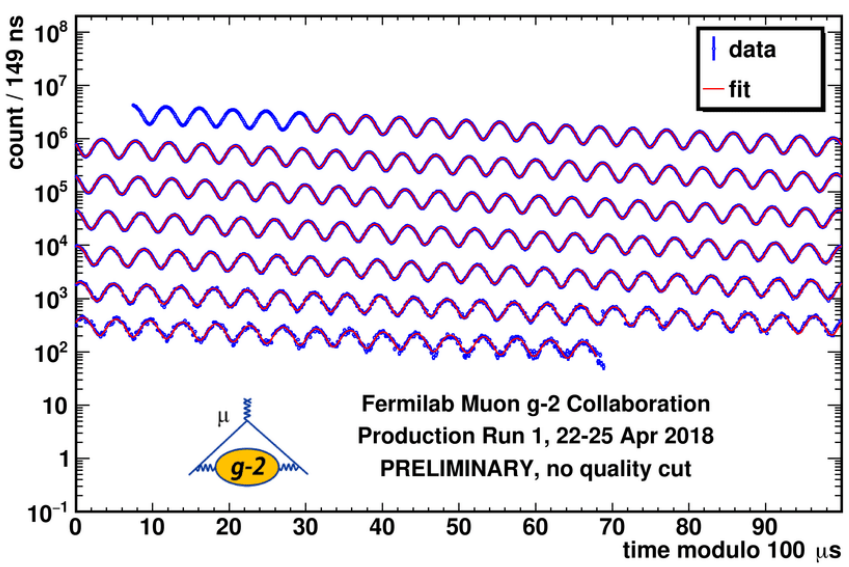
Phys. Rev. Lett. 131, 161802 (2023)

2021 JINST 16 P12041

Muon g-2 experiment at FNAL

OBJECTIVE AND EXPERIMENTAL APPROACH

- Objective: measurement of the muon g-2
- Approach: measurement of the spin precession frequency of muons in orbit in a magnetic field



$$\frac{g_\mu - 2}{2} \sim \frac{m\omega}{qB}$$

- Experimental observable: rate of positrons with E > E_{thr} emitted in forward direction w.r.t. the muon momentum

SYSTEMATICS

- Main criticality: accuracy of the magnetic field

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

- Solution: instead of taking absolute field and frequency measurements, a **standard metrologic process** (*precession frequency of shielded protons in a spherical sample, ω_p*) is measured in the same field, and the ratio is used
- Residual uncertainties from beam dynamics, temperature stability, external inputs

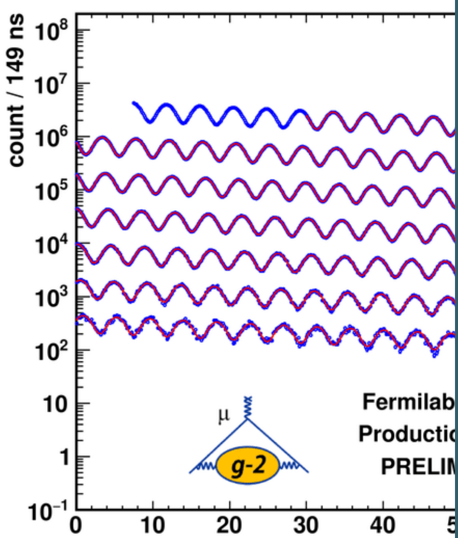
Phys. Rev. Lett. 131, 161802 (2023)

2021 JINST 16 P12041

Muon g-2 experiment at FNAL

OBJECTIVE AND E

- Objective: r
- Approach: r
precession
in a magnet



- Experiment
positrons with E > E_{thr} emitted in forward direction w.r.t. the muon momentum

Quantity	Correction (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	⋯	201
ω_a^m (systematic)	⋯	25
C_e	451	32
C_p	170	10
C_{pa}	−27	13
C_{dd}	−15	17
C_{ml}	0	3
$f_{\text{calib}} \cdot \langle \omega'_p(\vec{r}) \times M(\vec{r}) \rangle$	⋯	46
B_k	−21	13
B_q	−21	20
$\mu'_p(34.7°)/\mu_e$	⋯	11
m_μ/m_e	⋯	22
$g_e/2$	⋯	0
Total systematic for $\mathcal{R}'_\mu$	⋯	70
Total external parameters	⋯	25
Total for a_μ	622	215

accuracy of the

$$T_r) \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$
$$H)$$

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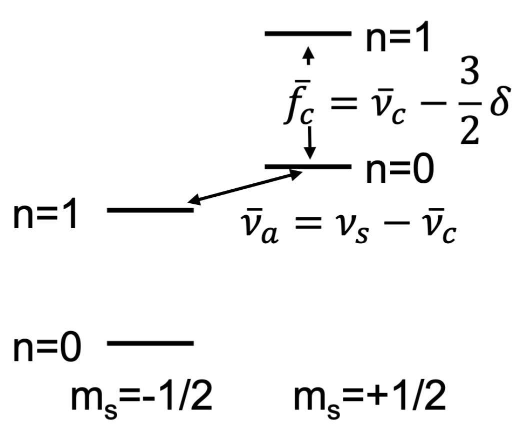
s in a spherical
measured in the
the ratio is used
tainties from
beam dynamics, temperature
stability, external inputs

Phys. Rev. Lett. 130, 071801 (2023)

Electron g-2

OBJECTIVE AND EXPERIMENTAL APPROACH

- Objective: measurement of the electron g-2
- Approach: measurement of energy transitions for a single electron in a magnetic field inside a Penning trap

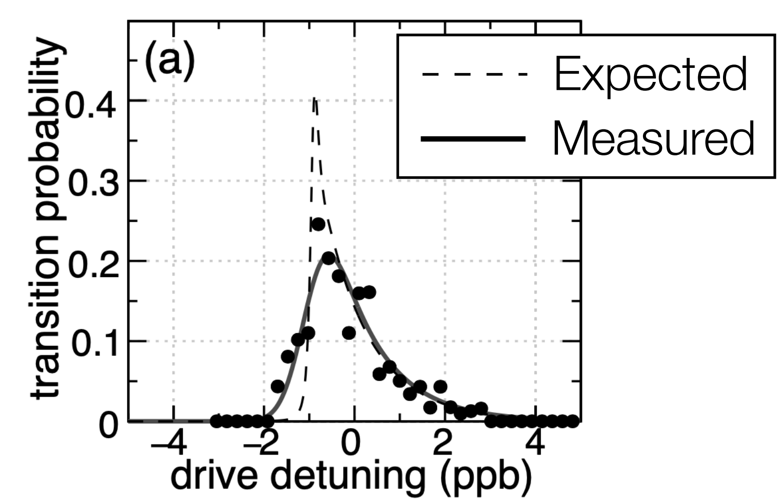


$$-\frac{\mu}{\mu_B} = \frac{g}{2} = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_a}{\nu_c}$$

- Experimental observable: quantum jumps hν, excited with electrodes, induce currents in the electrodes themselves, with a resonance if the frequency of the excitation matches ν

SYSTEMATICS

- The necessary quantities ν_a , ν_c are measured in situ with the same approach—> no metrology issue
- Dominant uncertainty is expected from the correction between frequencies for free and trapped electrons
- Indeed, B field fluctuations are observed and induce additional systematics

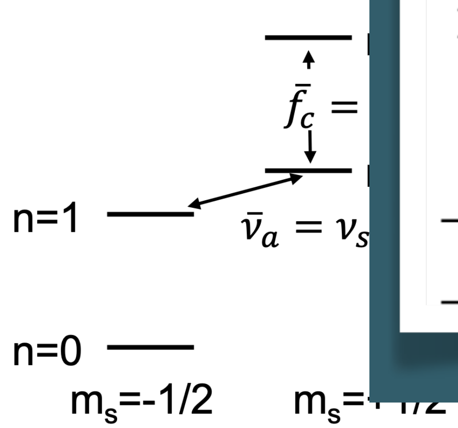


Phys. Rev. Lett. 130, 071801 (2023)

Electron g-2

OBJECTIVE AND EXPERIMENTAL APPROACH

- Objective: r
  g-2
- Approach:
  transitions f
  magnetic fie



Source	Uncertainty×10^{13}
statistical	0.29
cyclotron broadening	0.94
cavity correction	0.90
nuclear paramagnetism	0.12
anomaly power shift	0.10
magnetic field drift	0.09
total	1.3

- Experimental observable: quantum jumps hν, excited with electrodes, induce currents in the electrodes themselves, with a resonance if the frequency of the excitation matches ν

SYSTEMATICS

quantities ν_a , ν_c
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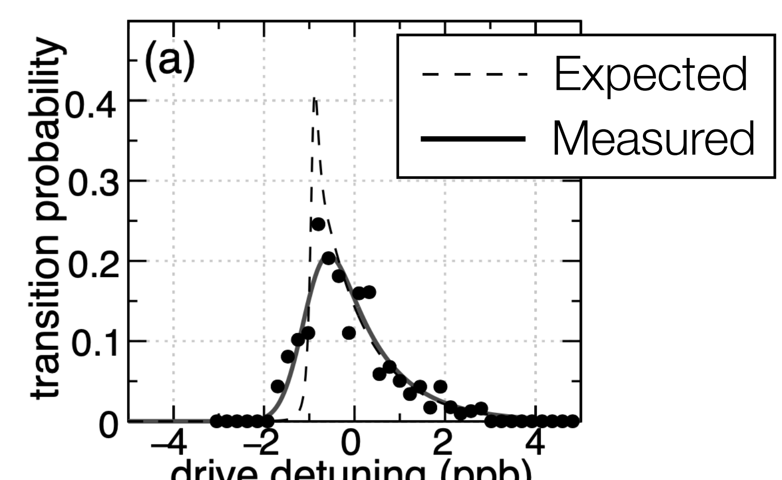
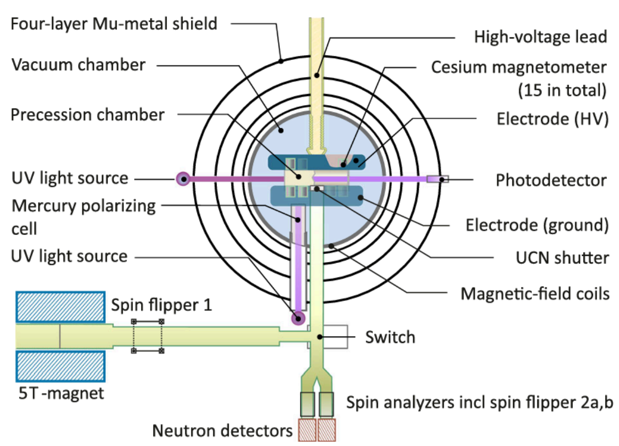


Phys. Rev. Lett. 124, 081803 (2020)

nEDM

OBJECTIVE AND EXPERIMENTAL APPROACH

- Objective: search for a neutron EDM
- Approach: measurement of spin precession frequency in E + B field



$$f_n = \frac{1}{\pi\hbar}\left|\mu_n \vec{B}_0 + d_n \vec{E}\right|$$

- Experimental technique: Ramsey spectroscopy of polarized ultra-cold neutrons within a shielded chamber:
  - π/2 spin flip by oscillating megnetic field
  - find the frequency f_n that maximizes the asymmetry btw. spin up and down

SYSTEMATICS

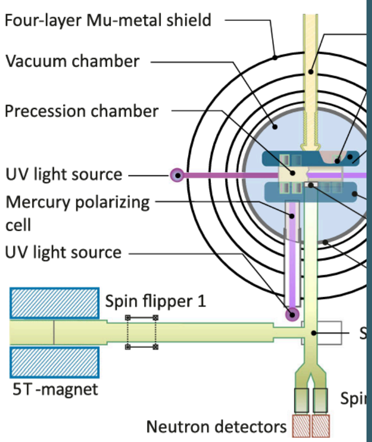
- Main criticality: this is not a genuine measurement of zero, because $\mu_n B_0$ has to be subtracted from a non-zero measurement:
  - absolute value of B_0 need to be known
  - comagnetometry against a metrologic standard (^{199}Hg)
- Several residual systematics:
  - systematics in the ^{199}Hg measurement
  - magnetic non-uniformities
  - asymmetries in the distribution of neutrons w.r.t. the magnetic field in the chamber
  - ….

Phys. Rev. Lett. 124, 081803 (2020)

nEDM

OBJECTIVE AND EXPERIMENTAL APPROACH

- Objective:
- Approach
  precession



- Experimen
  spectrosc
  neutrons v
  - π/2 spi
    field
  - find the frequency f_n that maximizes the asymmetry btw. spin up and down

Effect	Shift	Error
Error on $\langle z \rangle$	...	7
Higher-order gradients $\hat{G}$	69	10
Transverse field correction $\langle B_T^2 \rangle$	0	5
Hg EDM [8]	–0.1	0.1
Local dipole fields	...	4
$v \times E$ UCN net motion	...	2
Quadratic $v \times E$	...	0.1
Uncompensated G drift	...	7.5
Mercury light shift	...	0.4
Inc. scattering ^{199}Hg	...	7
TOTAL	69	18

SYSTEMATICS

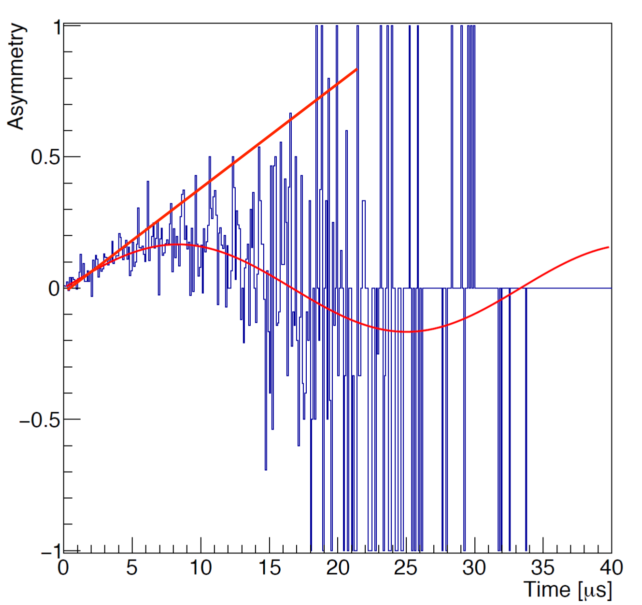
- Main criticality: this is not a
  ment of zero,
  to be
  non-zero
- e of B_0 need to
- etry against a
  ndard (^{199}Hg)
- ystematics:
  the ^{199}Hg
  - -uniformities
  - n the distribution
    r.t. the magnetic
    field in the chamber
  - ....

Eur. Phys. J. C 84 (2024) 3, 262

muEDM

OBJECTIVE AND EXPERIMENTAL APPROACH

- Objective: search for a muon EDM
- Approach: detection of non-zero spin precession in a magnetic field, with MDM precession canceled by a suitable combination of E and B fields



$$\vec{d} = \frac{\eta e}{2mc}\vec{s}$$

$$\omega = \frac{\eta q}{2m}\vec{\beta} \times \overrightarrow{B}$$

$$A(t) \propto \frac{2P_0 E_f \alpha |d_\mu|}{a\hbar\gamma^2} t$$

- Experimental observable: time-dependent asymmetry of positrons emitted along and opposite to the B field

SYSTEMATICS

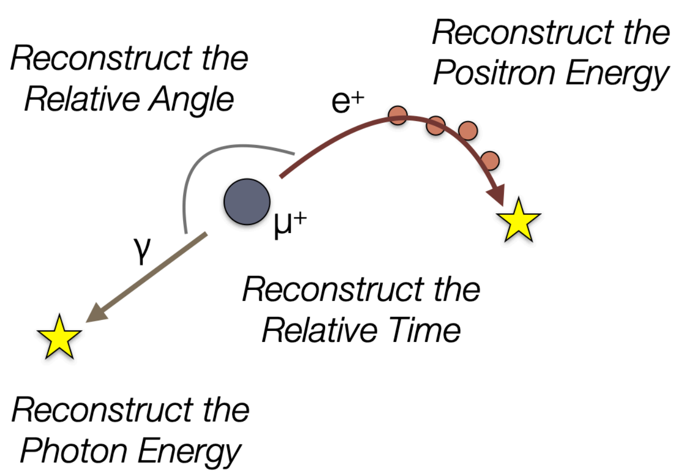
- Main criticality:
  - it is not necessary to know extremely well the main components of the field, but fake EDM can arise from fringe fields
- EDM is CP-violating, standard electrodynamics is CP-conserving:
  - systematics can be canceled by inverting B and injection direction
  - indeed, it moves the systematics from electrodynamics to the symmetry between the two injection modes
- Detector asymmetries to be kept under control

Eur. Phys. J. C 84 (2024) 3, 216

MEG & MEG II

OBJECTIVE AND EXPERIMENTAL APPROACH

- Objective: search for $\mu \rightarrow e\gamma$
- Approach: search for e^+ and γ in μ^+ decays at rest with 2-body kinematics (monochromatic e^+ and γ at 180º)



$Ee = E\gamma = 52.8$ MeV

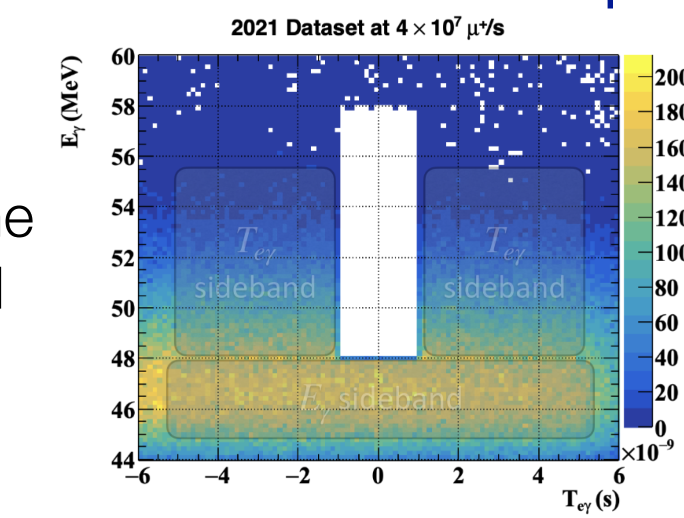
$\Theta e\gamma = 180°$

$Te\gamma = 0$

- Experimental technique: photon reconstruction in a LXe calorimeter, positron reconstruction in a magnetic spectrometer

SYSTEMATICS

- Robust control of the background from sidebands



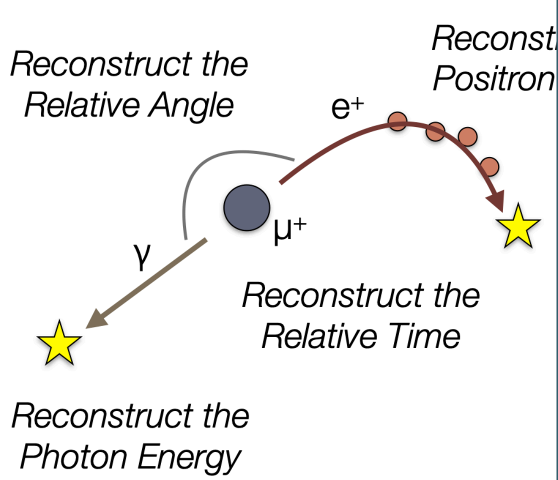
- Main criticalities:
  - calibrating the photon enery scale requires a dedicated $\pi^-(p,n)\pi^0$ experiment
  - **no physics process to calibrate the relative angle —> rely on detector alignments**
- Dominant systematics from target alignment:
  - tolerable in MEG, required a dedicated target monitoring system with photo cameras in MEG II due to better resolutions

Eur. Phys. J. C 84 (2024) 3, 216

MEG & MEG II

OBJECTIVE AND EXPERIMENTAL APPROACH

- Objective: search for $\mu \rightarrow e\gamma$
- Approach: search for e^+ and γ in μ^+ decays at rest w
(monochromatic

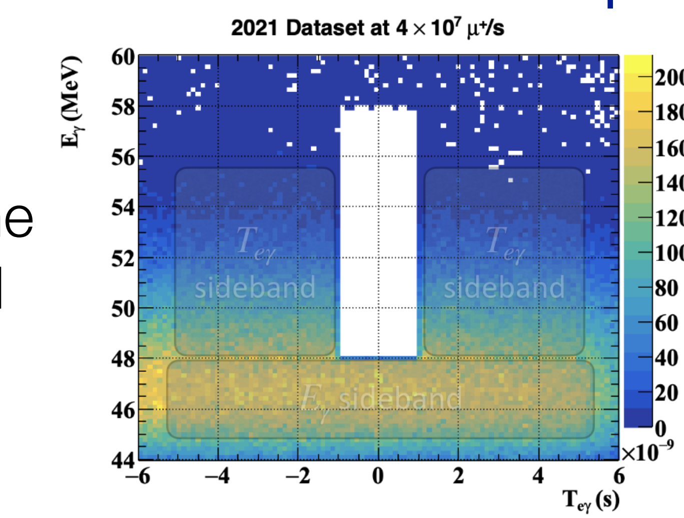


Parameter	Impact on limit
$\phi_{e\gamma}$ uncertainty	1.1 %
E_γ uncertainty	0.9 %
$\theta_{e\gamma}$ uncertainty	0.7 %
Normalization uncertainty	0.6 %
$t_{e\gamma}$ uncertainty	0.1 %
E_e uncertainty	0.1 %
RDC uncertainty	< 0.1 %

- Experimental technique: photon reconstruction in a LXe calorimeter, positron reconstruction in a magnetic spectrometer

SYSTEMATICS

- Robust control of the background



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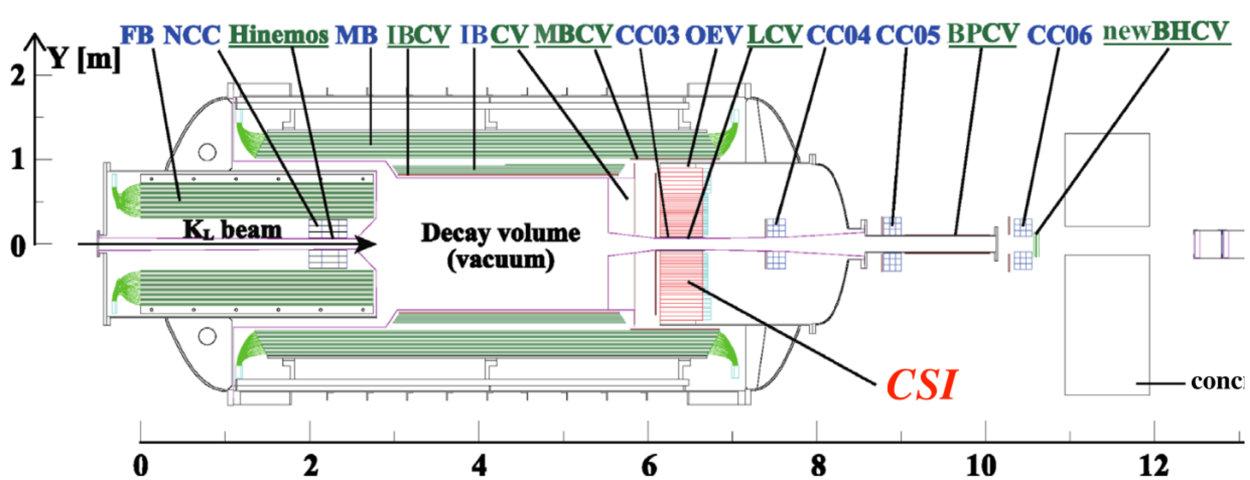
- tolerable in MEG, required a dedicated target monitoring system with photo cameras in MEG II due to better resolutions

Phys. Rev. Lett. 126, 121801 (2021)

KOTO

OBJECTIVE AND EXPERIMENTAL APPROACH

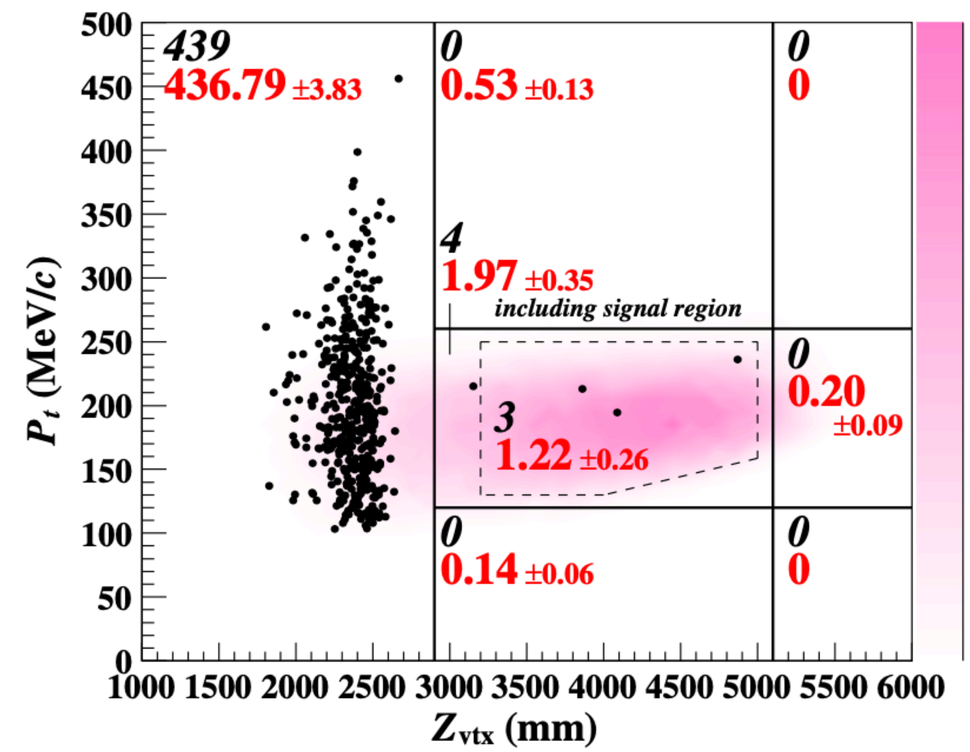
- Objective: search for $K_L^0 \to \pi^0 \nu \bar{\nu}$
- Approach: search for kaon decays with only 2 photons and nothing else



- Experimental technique: kaons decaying in a volume surrounded by hermetic neutral and charged particle detectors, used to veto background decays

SYSTEMATICS

- Main criticality: hermeticity and PID to be precisely controlled to get rid of the Standard Model and beam-halo backgrounds up 10^7 rejection factor:
  - dominant systematics from the expected background rates



Phys. Rev. Lett. 126, 121801 (2021)

KOTO

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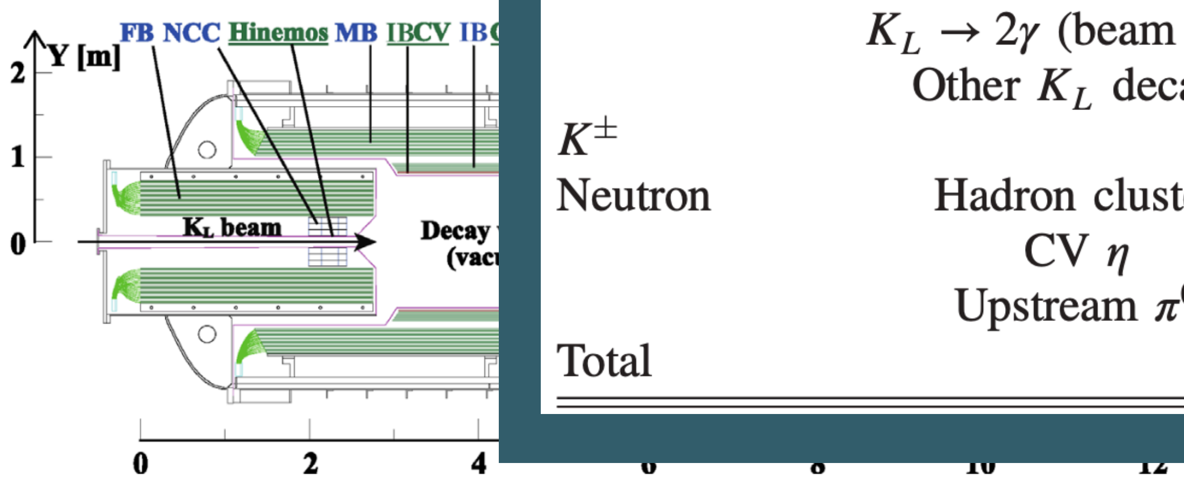
- Approach: search for kaon decays with only 2 photons

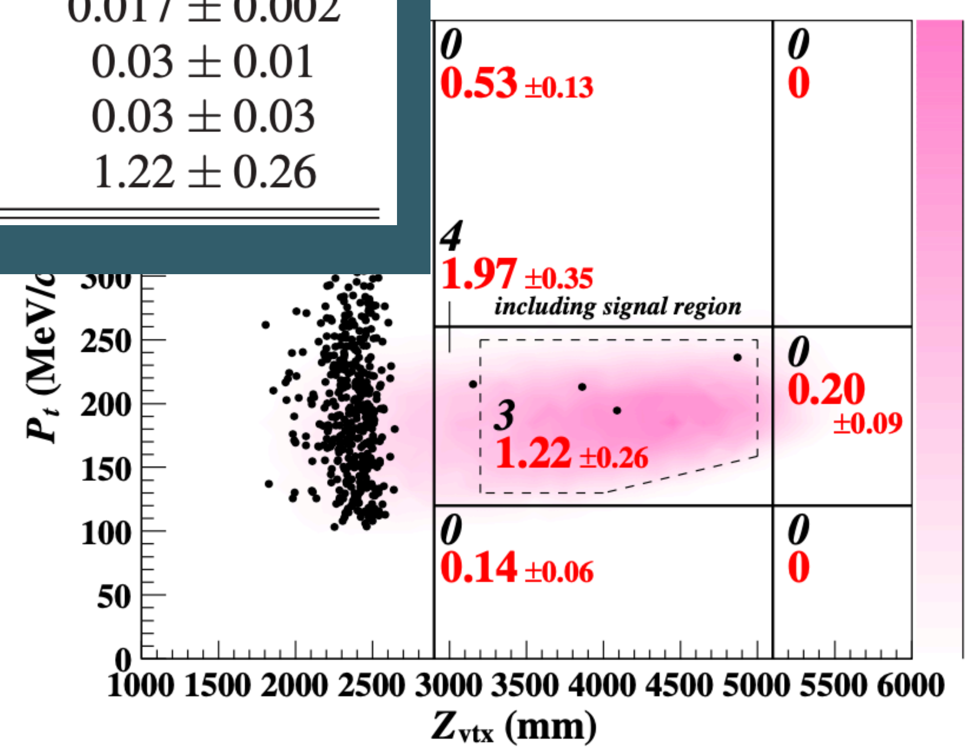


Source		Number of events
K_L	$K_L \to 3\pi^0$	0.01 ± 0.01
	$K_L \to 2\gamma$ (beam halo)	0.26 ± 0.07[a]
	Other K_L decays	0.005 ± 0.005
$K^\pm$		0.87 ± 0.25[a]
Neutron	Hadron cluster	0.017 ± 0.002
	CV η	0.03 ± 0.01
	Upstream π^0	0.03 ± 0.03
Total		1.22 ± 0.26

- Experimental technique: kaons decaying in a volume surrounded by hermetic neutral and charged particle detectors, used to veto background decays

SYSTEMATICS

- Main criticality: hermeticity and PID to be precisely controlled to get rid of the Standard Model … halo backgrounds up … factor:
- … systematics from the background rates

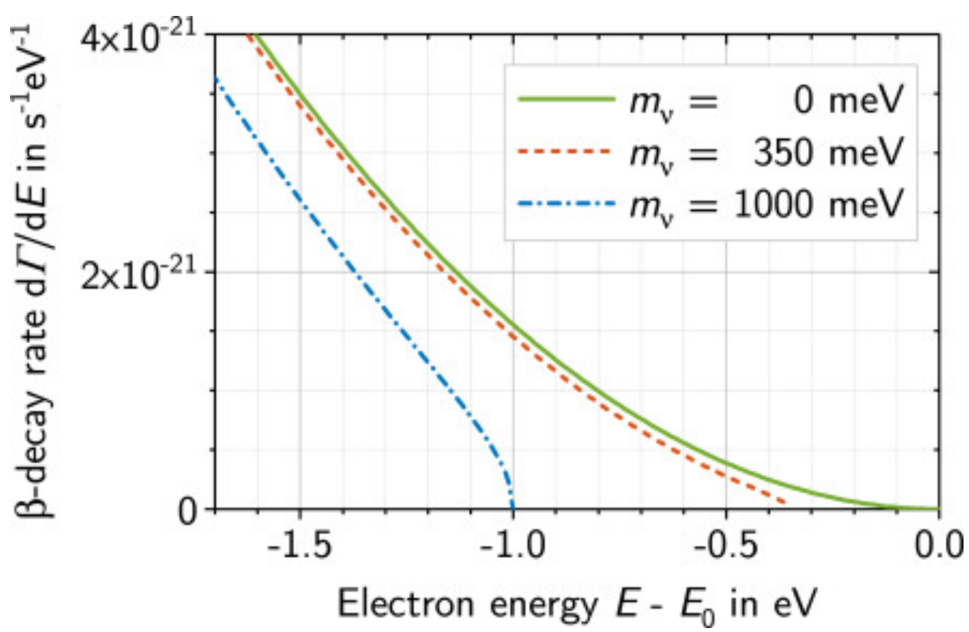


Nature Physics 18, 160 (2022)

KATRIN

OBJECTIVE AND EXPERIMENTAL APPROACH

- Objective: measurement of the electron anti-neutrino mass
- Approach: measurement of the beta decay end-point



- Experimental technique: electromagnetic filter to count events above a certain energy threshold

SYSTEMATICS

- Dominant systematics:
  - electric and magnetic field (accuracy & stability)
  - non-Poisson background
- Origin of non-Poisson background:
  - nuclear decays from contaminants produce keV electrons
  - they ionize the residual gas, producing secondaries
  - many secondaries from a very small number of primaries (i.e. correlated background)—> non-Poisson fluctuations

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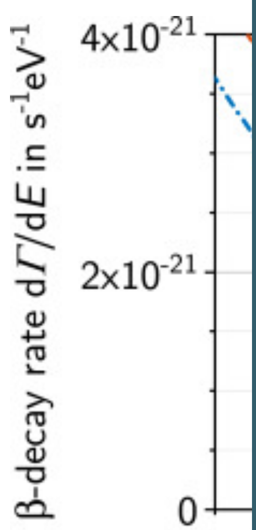
OBJECTIVE AND E

- Objective: r
anti-neutrin
- Approach:
decay end-



- Experimental technique: electromagnetic filter to count events above a certain energy threshold

Effect	68.2% CL uncertainty on m_ν^2 (eV²)
Statistical	0.29
Non-Poissonian background	0.11
Source-potential variations	0.09
Scan-step-duration-dependent background	0.07
qU-dependent background	0.06
Magnetic fields	0.04
Molecular final-state distribution	0.02
Column density × inelastic scattering cross section	0.01
Activity fluctuations	0.01
Energy-loss function	<0.01
Detector efficiency	<0.01
Theoretical corrections	<0.01
High-voltage stability and reproducibility	<0.01
Total uncertainty	0.34

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background) —> non-
Poisson fluctuations

Conclusions

- Compared to general-purpose experiments, systematics in single-purpose experiments pose some special challenges:
  - metrology issues
  - need of dedicated (hardware) tools for the control of systematics
- Control of systematic uncertainties is a critical aspect in the design of single-purpose experiments, often requiring special expertise from outside the HEP field