

Dark sector searches at NA64

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Outline

- 1 Introduction
- 2 The NA64-*e* experiment at CERN
- 3 The POKER project
- 4 NA64 future initiatives
- 5 Conclusions

Light dark matter

The light dark matter hypothesis predicts that DM is made by sub-GeV particles interacting with SM via a new force.¹

Simplest possibility: “vector-portal”.

DM-SM interaction through a new U(1) gauge-boson (“dark-photon”) coupling to electric charge

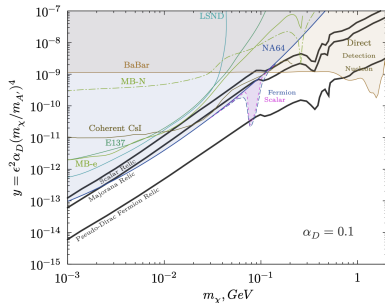
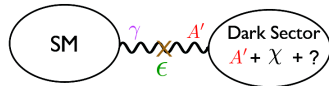
Model parameters:

- Dark-photon mass, $M_{A'}$ and coupling to electric charge ϵ
- Dark matter mass, M_χ and coupling to dark photon, g_D ($\alpha_D \equiv g_D^2/4\pi$)

Annihilation cross section reads:

$$\langle\sigma v\rangle \propto \frac{\epsilon^2 \alpha_D m_\chi^2}{m_{A'}^4} = \frac{\epsilon^2 \alpha_D m_\chi^4}{m_{A'}^4} \frac{1}{m_\chi^2} \equiv \frac{y}{m_\chi^2}$$

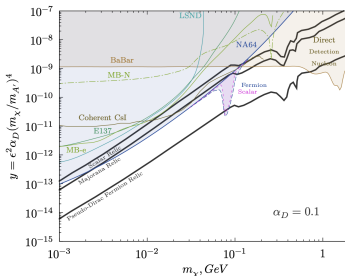
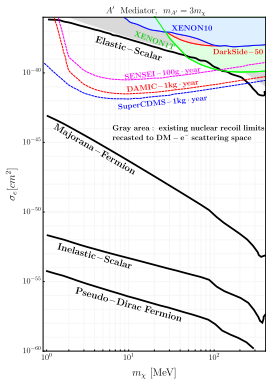
For a fixed m_χ value, the thermal origin hypothesis (DM relic density) imposes a unique value of y



¹For a comprehensive review: 1707.04591, 2005.01515, 2011.02157

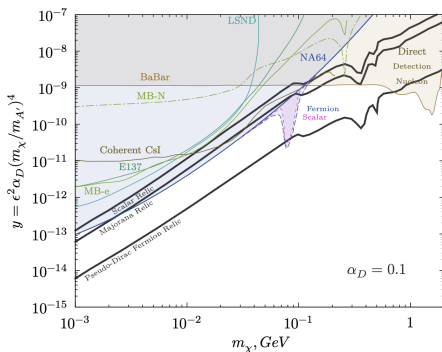
Light dark matter searches at the intensity frontier

- Dark Matter direct detection experiments, typically optimized for $M_\chi \geq 1$ GeV, have a limited sensitivity in the sub-GeV range
 - $E_R \propto v^2 M_\chi^2 / M_N$, $v \simeq 220$ km/s $\sim 7 \cdot 10^{-4} c$
 - Many ongoing efforts to overcome this kinematic effect
- LDM-SM interaction cross section at low energy has a sizable dependence on the impinging particle velocity, with a drastic reduction for specific models



Light dark matter searches at the intensity frontier

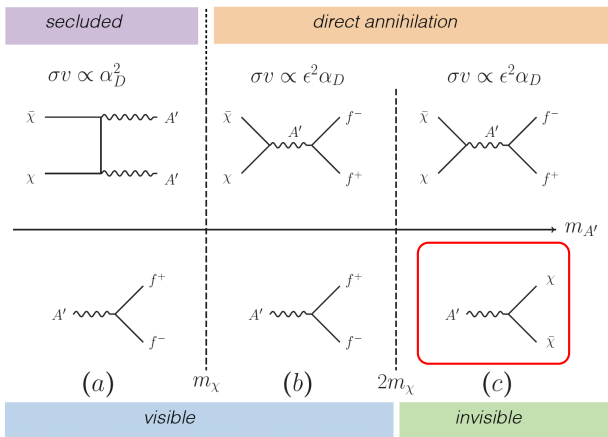
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LDM at accelerators

Accelerator-based experiments at the *intensity frontier* are uniquely suited to explore the light dark matter hypothesis

Light dark matter signatures



- Ⓐ $m_{A'} < m_\chi$: **secluded** scenario. Provides no thermal target for accelerator-based experiments: **any** ϵ value is allowed
- Ⓑ $m_\chi < m_{A'} < 2m_\chi$: **visible decay** scenario (although off-shell $\chi - \bar{\chi}$ production is allowed!)
- Ⓒ $m_{A'} > 2m_\chi$: **invisible decay** scenario

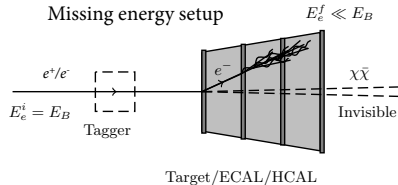
The NA64- e approach to Dark Sector search

NA64- e approach: missing energy measurement, high energy e^-/e^+ beam impinging on an active thick target.

Number of signal events scales as: $N_S \propto \epsilon^2$

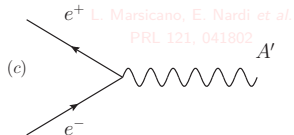
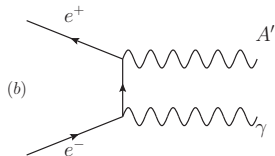
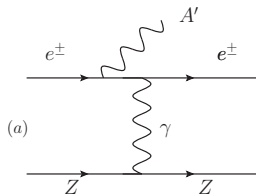
Missing Energy Experiment

- Specific beam structure: impinging particles impinging “one at time” on the active target
- Deposited energy E_{dep} measured event-by-event
- Signal: events with large $E_{miss} = E_B - E_{dep}$
- Backgrounds: events with ν / long-lived (K_L) / highly penetrating (μ) escaping the detector / eventual beam contaminants

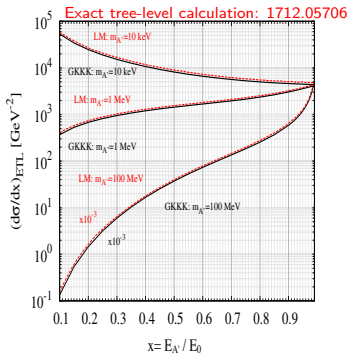


LDM production mechanisms with lepton beams

Three main LDM production mechanisms in fixed-target, lepton-beam experiments

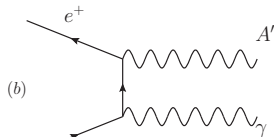
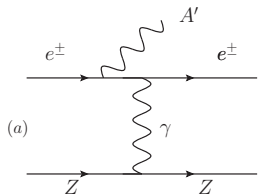
a) A' -strahlung

- Radiative A' emission (nucleus EM field) followed by $A' \rightarrow \chi\bar{\chi}$
- Scales as $Z^2\alpha_{EM}^3$
- Forward-boosted, high-energy A' emission



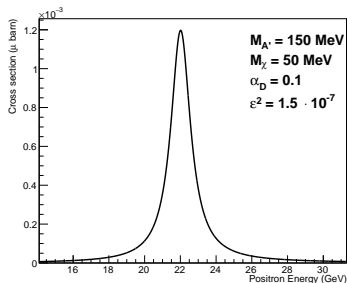
LDM production mechanisms with lepton beams

Three main LDM production mechanisms in fixed-target, lepton-beam experiments



c) Resonant e^+e^- annihilation

- $e^+e^- \rightarrow A' \rightarrow \chi\bar{\chi}$
- Scales as $Z\alpha_{EM}$
- Closed kinematics:
 $P_\chi + P_{\bar{\chi}} = P_{e^+} + P_{e^-}$
- Resonant, Breit-Wigner like cross section with $M_{A'} = \sqrt{2}m_e\bar{E}_{e^+}$

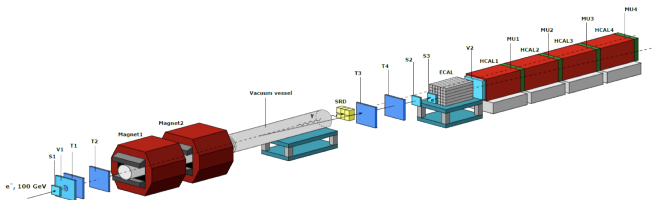
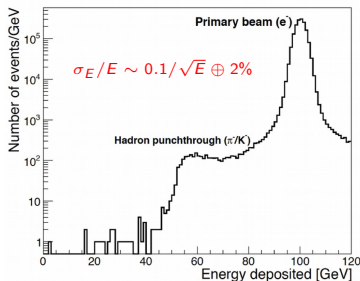


The NA64-e experiment at CERN North Area

NA64-e: missing energy experiment at CERN North Area, 100 GeV e^- beam²
 H4 line: $\simeq 10^7 e^-/\text{spill}$ (γ conversion). $\sigma_E < 1\%$, hadron contamination $\sim 0.5\%$

Experiment Setup

- EM-Calorimeter: $40X_0$, Pb/Sc Shashlik
- Hadron calorimeter: 4 m, $30 \lambda_I$
- Beam identification system: SRD + MM trackers
- Plastic scintillator based scintillator counters for VETO



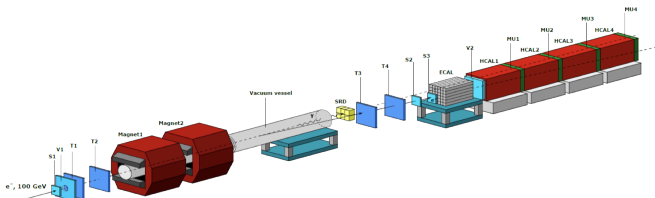
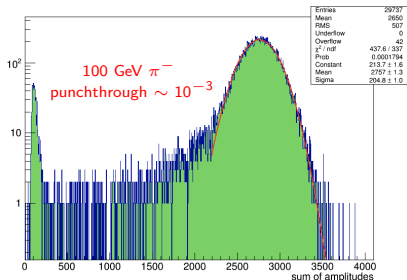
²Phys. Rev. Lett. 131, 161801 (2023)

The NA64-*e* experiment at CERN North Area

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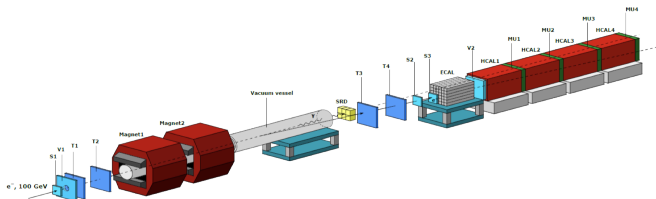
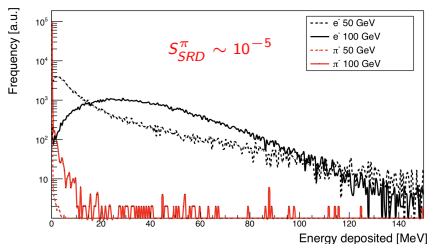
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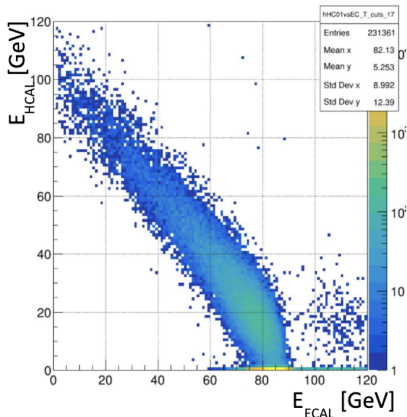
NA64-*e*: backgrounds

Possible background sources: production of long-lived and highly penetrating neutral SM particles upstream / within ECAL (neutrons, kaons)

- (i) Di-muon events: $eZ \rightarrow eZ\mu^+\mu^-$, with one or both muons decaying or escaping without being detected by VETO/HCAL.
- (ii) Decay of mis-identified contaminating hadrons to $e\nu_e$ final state.
- (iii) Electro- / photo-nuclear interactions with upstream beamline materials.
Critical contribution, yield estimated directly from data by side-band extrapolation.
- (iv) Hadron productions in the ECAL undetected by VETO/HCAL.

Background yield for 2021-2022 runs

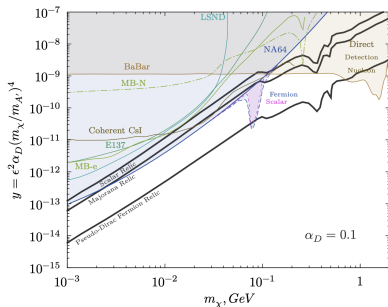
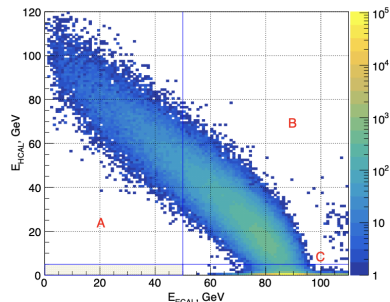
Background source	Background, n_b
(i) dimuons losses or decays in the target	0.04 ± 0.01
(ii) $\mu, \pi, K \rightarrow e + \dots$ decays in the beam line	0.3 ± 0.05
(iii) lost γ, n, K^0 from upstream interactions	0.16 ± 0.12
(iv) Punch-through leading n, K_L^0	< 0.01
Total n_b (conservatively)	0.51 ± 0.13



NA64-*e*: results

Latest results:

- Accumulated statistics $\simeq 10^{12}$ EOT (2016-2022)
- After applying all selection cuts, no events are observed in the signal region $E_{ECAL} < 50$ GeV, $E_{HCAL} < 1$ GeV
- Expected number of background events ~ 0.5 compatible with null observation
- **Today, the most competitive exclusion limits** in large portion of the LDM parameters space. New ideas are being explored to cover the large-mass region, $m_{A'} \gtrsim 100$ MeV.



POKER: POsitrion resonant annihilation into darK mattER

An optimized light dark matter search with positrons in the NA64 framework

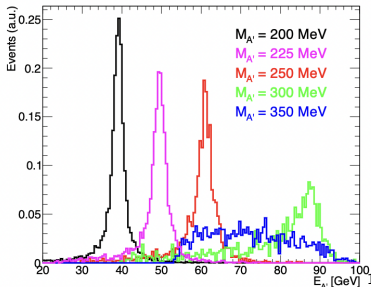
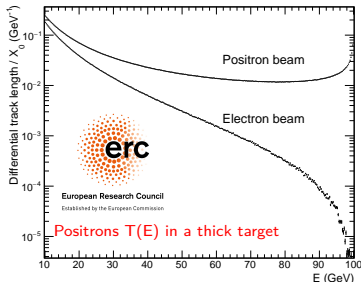
Why positrons?

Signal production reaction: $e^+e^- \rightarrow A' \rightarrow \chi\bar{\chi}$

- Large event yield:
 $N_s^{annihil} \propto Z\alpha_{EM}$ vs $N_s^{brem} \propto Z^2\alpha_{EM}^3$
- Missing energy distribution shows a **peak** around $E_R = \frac{M_{A'}^2}{2m_e}$

Project goal

- Perform a preliminary missing energy measurement with a positron beam, using a new high resolution / high segmentation detector replacing the current NA64 ECAL.
- Demonstrate the technique and set the basis of the first **optimized** light dark matter search at a positron-beam facility



POKER (NA64- e^+): current status

A first proof-of-concept measurement was completed in 2022 (10^{10} E^+OT at 100 GeV)

Goals: backgrounds study (higher beam hadronic contamination), resonant production characterization via the $e^+e^- \rightarrow \mu^+\mu^-$ reaction, extraction of the first upper limit with an analysis optimized for resonant LDM production.

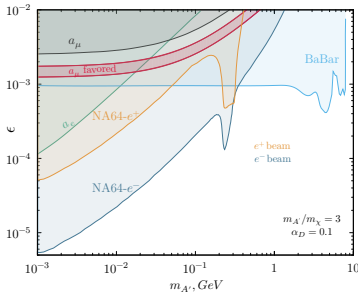
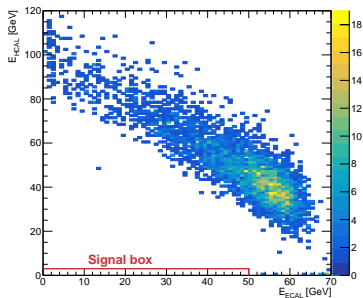
Experimental setup: as in NA64- e^- .

Analysis strategy: blind-analysis (signal-like region $E_{ECAL} < 50$ GeV, $E_{HCAL} < 3$ GeV).

Main backgrounds:

- $\pi^+ \rightarrow e^+\nu_e + \text{fake-SRD tag}$
- Upstream hadrons electro-production, soft e^+ in ECAL
- Overall expected background yield: < 0.1 events.

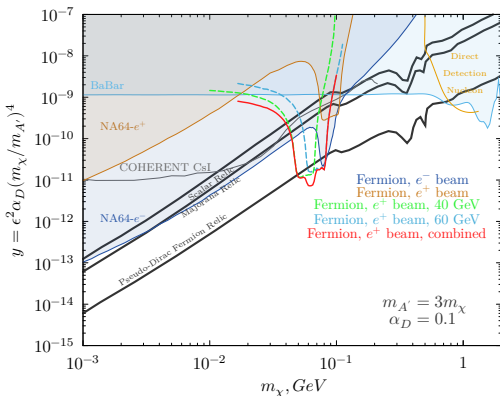
Results: no events observed in the signal region after unblinding, new upper limits set to the LDM parameters space.



POKER (NA64- e^+): future plans

The successful 100 GeV e^+ beam run promoted a multi-measurement program at NA64 at lower energies.

- Two measurements at 60 GeV and 40 GeV will allow to probe the LDM parameters space down to the thermal targets.
- 10^{11} E⁺OT requested, 2x 1-week data-taking runs.
- Modest detector upgrades: new SRD detector, new active target



POKER (NA64- e^+): new LYSO-based SRD detector

At low beam energy, the SR emission drops significantly $E_{SR} \propto E_{beam}^4$: an optimized SRD detector is necessary.

Possible option: LYSO-based homogeneous detector.

LYSO key properties:

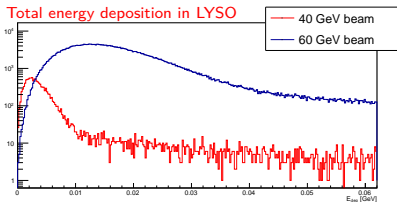
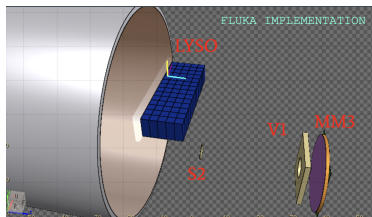
- Large density: $\simeq 7 \text{ g/cm}^3$
- High light yield: $\simeq 3 \cdot 10^4 \gamma/\text{MeV}$
- Very fast response: $\tau \simeq 40 \text{ ns}$

Possible critical items for false-positives:

- Lu intrinsic radioactivity
- Afterglow effects

Detector design currently being optimized through MC simulations.

Preliminary results demonstrate that e^+ detection efficiencies up to 99% (60 GeV) / 89% (40 GeV) can be reached, with minimal rate of false-positives.



POKER (NA64- e^+): new active target

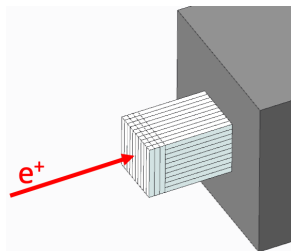
To properly measure the A' signal resonant line shape, a new active target with optimized energy resolution is required: **new PKR-CAL detector**.

Baseline design: $33.5 X_0$ PbWO₄ calorimeter with SiPM readout

- 9x9 matrix of $20 \times 20 \times 220$ mm³ crystals + 4 layers in front (pre-shower): $31.5 X_0$
 - Fully absorb EM shower with minimal longitudinal/transverse energy leakage
- Required $\sigma_E/E \sim 2.5\%/\sqrt{E} \oplus 0.5\%$
 - $LY \sim 2.5$ phe/MeV
 - Use four 6×6 mm² SiPMs, $10 \mu\text{m}$ cell coupled to each crystal

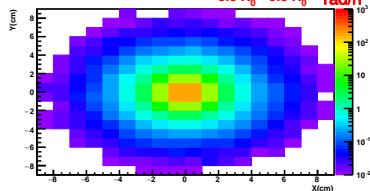
Radiation levels are critical

- EM dose up to 200 rad/h (CMS ECAL max: 500 rad/h)
 - Light-induced annealing
 - Beam-spot rastering: factor $\simeq 5$ reduction
- $\phi_n \leq 10^3$ n_{eq} cm⁻²s⁻¹: no significant effects expected



100 kHz 100 GeV e^+ : radiation dose

$8.5 X_0 - 9.5 X_0$ rad/h



POKER (NA64- e^+): POKERINO prototype

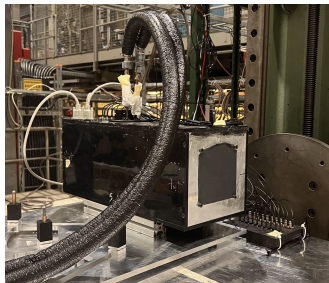
POKERINO: 3x3 PbWO₄ matrix with SiPM readout.

Goal: characterize the prototype response to cosmic rays / beam to validate the technical choices and measure the detector performances

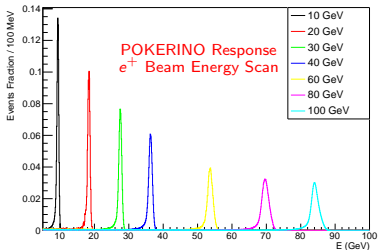
Measurements:

- Cosmic-ray measurements with EEE setup in Genova: commissioning / LY measurement and uniformity studies.
- August 2023: characterization with high-energy e^+ (10...120 GeV) at CERN, H8 beamline.

Data analysis currently in progress, first results are promising.



Energy Scan



NA64 program

Electron Beam:

- NA64-e: 10×10^{12} EOT already measured
- Additional $\sim 6 \times 10^{11}$ EOT collected in 2023 (data analysis ongoing), after detector upgrade (electronics, straw detectors and veto hadron calorimeter)
- **Already probed significant part of the A' invisible parameter space up to the thermal relic targets**

Positron Beam:

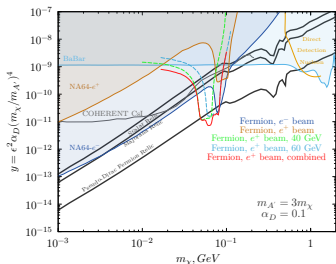
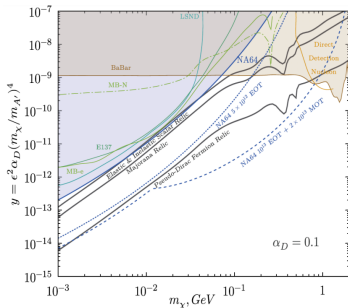
- Primary e^+ beam allows to exploit the enhanced resonant annihilation cross section \rightarrow **high sensitivity to large A' masses**
- **Dedicated short e^+ run in 2022:** $10^{10} e^+ \text{OT}$ accumulated
- Multi-measurement program at lower energies, first **POKER run** foreseen in 2024-2025.

Muon Beam:

- NA64- μ : missing momentum and energy experiment with a muon beam
- Ongoing parallel effort of the NA64 collaboration, data-taking at M2 beamline

Hadron Beam

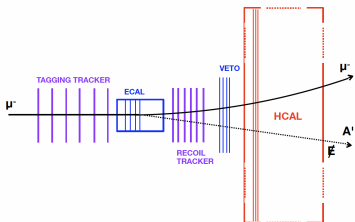
- Explore light mesons fully invisible decay modes
- First ideas are currently being developed



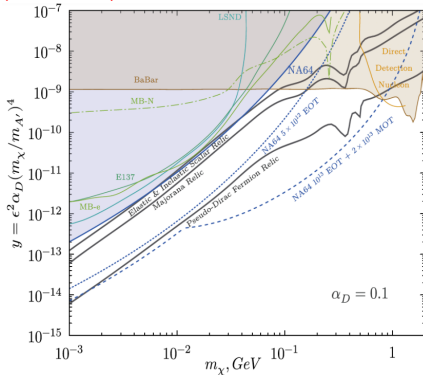
NA64 short-term future: NA64- μ : experimental technique

LDM search with a **muon beam** impinging on a fixed target, complementary to e^\pm searches in the high-mass region.

- **Signal production:** A' radiative emission by beam muons impinging on an active target (ECAL).
- **Signal signature: *missing momentum*.** Well-identified impinging beam track and final-state low-energy deflected track. No additional activity in downstream detectors (VETO / hadronic calorimeters).



Combined sensitivity of NA64- e (10^{13} EOT) and NA64- μ (2×10^{13} MOT).



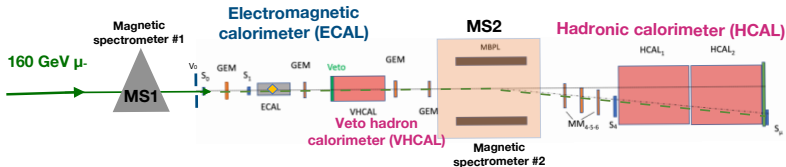
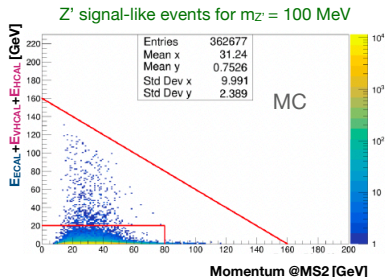
NA64 short-term future: NA64- μ . Experimental setupNA64- μ experiment: muon beam missing energy + momentum search

Beam: M2 beamline at CERN SPS, 160 GeV μ^- , $10^5 - 10^7$ μ/s .

Detector:

- Two magnetic spectrometers, MS1 (impinging μ) / MS2 (scattered μ)
- Three calorimeters: ECAL (active target), VHCAL, HCAL
- Beam-defining plastic scintillator counters

Signal signature: $P_1 \simeq 160$ GeV,
 $P_2 < 80$ GeV, $E_{CAL} \simeq \text{MIP}$.



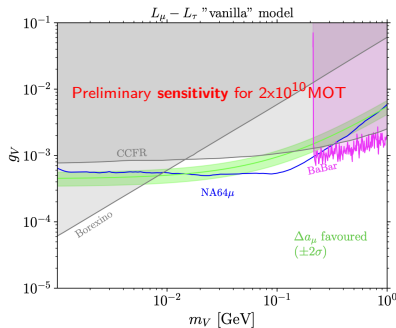
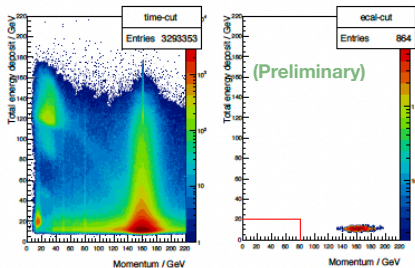
NA64- μ : status

Three test runs have already been performed in 2021 (5×10^9 MOT), 2022 (5×10^{10} MOT), and 2023 (10^{11} MOT)

- Very successful runs, *efficient use of the M2 beam thanks to the positive coordination with the Compass / AMBER experiment.*
- First results from 2021 data analysis: less than 10^{-11} background events expected per MOT. No events observed in the signal region.
- 2022/2023 data analysis in progress.

Possibility for a long physics run in 2024

- Detector optimization in progress.
- Final goal: 10^{12} MOT.



Conclusions

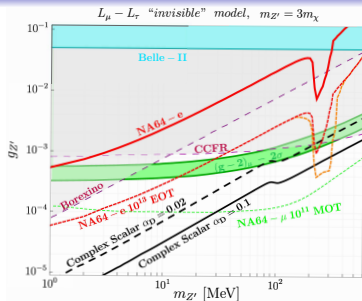
- Light dark matter scenario (MeV-to-GeV range) is largely unexplored
 - Can efficiently explain DM relic density.
 - Theoretically founded as the WIMP DM paradigm, assuming a **new** DM-SM interaction mechanics exists.
 - Accelerator-based experiments are uniquely suited to explore it.
- The NA64 experiment at CERN aims to investigate the Dark Sector through missing-energy/momentum measurements, exploiting different probes:
 - Electron beam: 1.6×10^{12} EOT already accumulated, most stringent limits in the low-mass LDM region.
 - Positron beam: exploit resonant LDM production (high signal yield, clear kinematic signature) - POKER project
 - Muon beam: focus on the high mass region. Three test runs completed, results expected by early next year.
 - Hadron beam: future ideas being developed.

Backup slides

NA64-e at CERN - further recent results

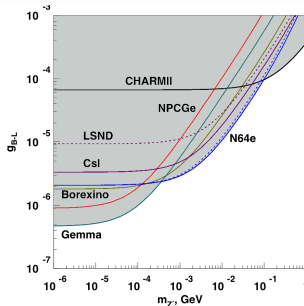
Light Z' search in the $L_\mu - L_\tau$ scenario

- Re-analysis of the 2016-2018 dataset searching for a new gauge boson coupling predominantly to muons and taus
- Ad-hoc calculation of the loop-induced coupling to photons (kinetic mixing)
- Results are more stringent than Belle-II limits, and compatible with exclusion contours from the re-analysis of neutrino experiments



Search for a new B-L Z' gauge boson

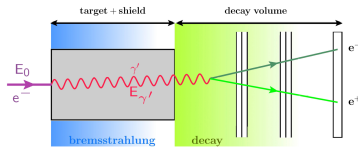
- First NA64 analysis using the 2021 dataset
- New constraints on the Z' parameters space, more stringent compared to those obtained from the neutrino-electron scattering data for the 0.3-100 MeV mass range
- Results can be re-casted to A' space via $g_{B-L} \leftrightarrow \epsilon e$.



A' production and visible decay detection in a fixed thick-target setup

Reaction topology:

- A' production: radiative A' emission
 $e^- N \rightarrow e^- NA'$
- A' propagation: for low ε values ($\lesssim 10^{-5}$) the A' is long-lived, resulting to a detached decay vertex.
- A' detection: measurement of the $e^+ e^-$ decay pair in a downstream detector.

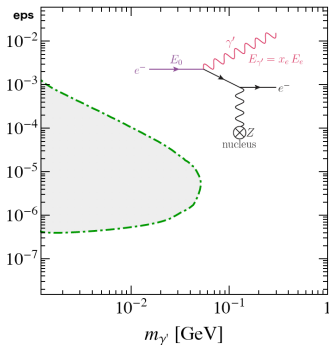


Number of events:

Dependence on main parameters³:

$$N \sim N_{eot} n_{sh} \int dE' dE_e dt l_e(E_e, t) \frac{d\sigma}{dE'} e^{-L_{sh}/\lambda} (1 - e^{-L_d/\lambda})$$

- Upper bound:
 $N_{evt} \propto \varepsilon^2 e^{-L_{sh}/l_{A'}} , I_{A'} \propto E_0/\varepsilon^2$
- Lower bound:
 $N_{evt} \propto \varepsilon^2 L_d/l_{A'} \propto \varepsilon^4$



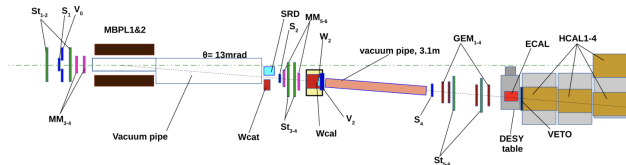
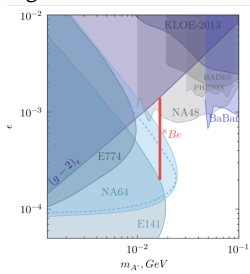
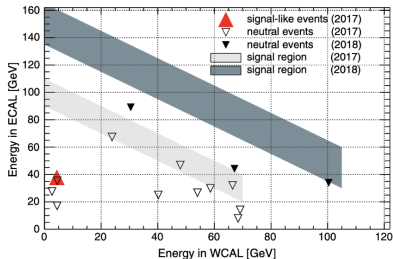
³For a review: S. Andreas, Phys.Rev. D86 (2012) 095019

Visible A' search at NA64

A' production by radiative emission in compact WCAL active target and decay to e^+e^- in vacuum pipe

- Clean impinging e^- with $P \simeq P_0$
- No activity in W2 and V2 veto counters
- MIP-like signature in S4, signals in trackers
- No activity in VETO and HCAL
- Energy deposition in WCAL and ECAL, with $E_{WCAL} + E_{ECAL} = E_0$

Combined analysis of 2017 ($5.4 \cdot 10^{10}$ EOT, 100 GeV/c) and 2018 ($3 \cdot 10^{10}$ EOT, 150 GeV/c) data:
no signal-like events observed in the signal region



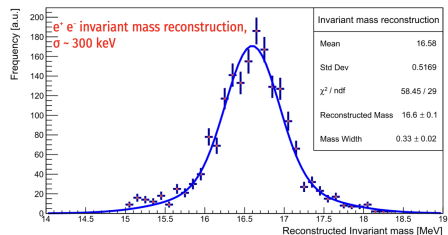
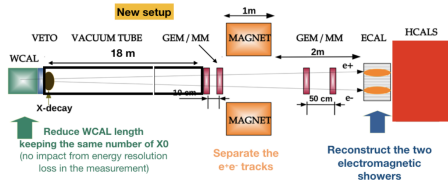
Visible A' search at NA64

Future prospects for post-2023 run: improved setup with e^+e^- invariant mass reconstruction.

Detector upgrades:

- New compact tungsten calorimeter (same X_0) to measure short-lived A' and improve sensitivity to large ε
- New SRD optimized for 150 GeV beam energy
- New larger transverse size MM
- New ECAL with larger transverse dimensions

With these modifications, 20 days of beam time at 150 GeV/c will be required to fully scrutinize the still-unexplored X17 parameters space.



POKER (NA64- e^+): PbWO_4 R&D

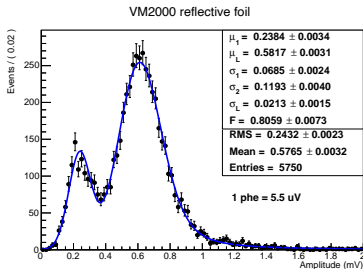
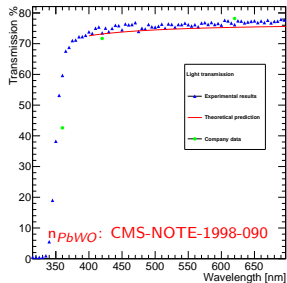
Massive characterization of PbWO_4 main properties (producer: CRYTUR)

Longitudinal light transmission vs λ :

- Spectrophotometer-based setup
- Measurements reproducibility better than 1%.
- $T > 70\%$ at $\lambda = 450$ nm for all samples.

Absolute light yield

- Setup:
 - Cosmic-ray telescope with plastic-scintillator counters
 - Crystal temperature stabilized at $+18$ °C
 - Readout: 4x S14160-6010 SiPM (6×6 mm², 10 μ m pixel size)
- Single-phe amplitude obtained from a pulsed-laser measurement
- LY from energy distribution fit $\simeq 5$ phe/MeV for all samples



POKER (NA64- e^+): new FADC digitizer

Development of a new FADC, possibly based on the Waveboard platform (INFN-Roma1), to replace the existing NA64 digitizers (MSADC from Compass).

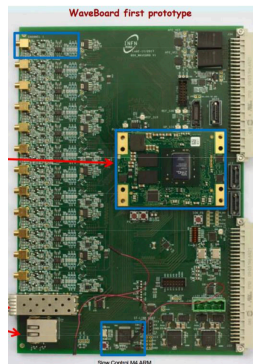
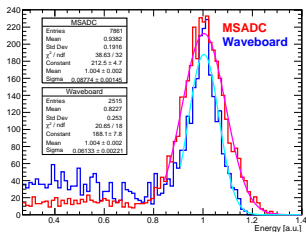
Requirements:

- At least 250 MHz sampling frequency
- Support for digital trigger logic

The use of the Waveboard in NA64 has already been validated with standalone measurements:

- Single-channel response
- Multiple-channels response

Ongoing R&D: Waveboard integration in NA64 (UCF protocol)

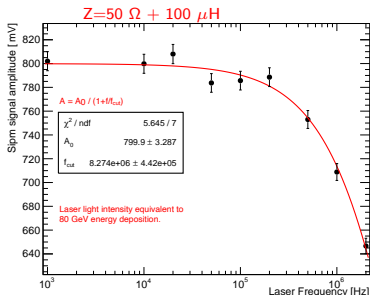
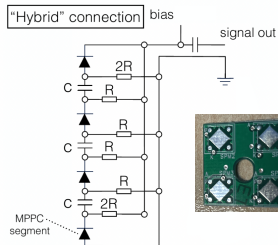


F. Ameli et al., NIMA 936

POKER (NA64- e^+): SiPM and readout R&D

Current readout scheme:

- 4x 6x6 mm² SiPMS coupled to each crystal to maximize light collection. Pixel size: 10 μm to minimize saturation effect (Hamamatsu S14160-6010)
- Hybrid-connection to minimize capacitance toward readout amplifier while keeping low bias voltage.
- Ad-hoc biasing scheme (LC network) to minimize bias current-induced gain variations.
- Ad-hoc connection circuit with embedded thermal-dissipation planes.



NA64 long-term future: NA64-*h*. Invisible K^0 decay.

- The Standard Model prediction for $S \rightarrow$ invisible is extremely small, $\lesssim 10^{-16}$
 - $S \rightarrow \nu\bar{\nu}$ is rigorously forbidden by helicity conservation if neutrino is massless.
 - $BR \simeq (M_\nu/M_S)^2$ due to finite neutrino mass.
 - $S \rightarrow 4\nu$ suppressed by $(G_F M_S^2)^4$.
- Any observation of these processes would be an unambiguous signal for the presence of new BSM physics.
- No upper limit exist today for $K_{S,L}^0 \rightarrow$ invisible
- This challenging search is complementary to existing and planned efforts for $K_{S,L}^0 \rightarrow \pi^0 +$ invisible (KOTO, Klever) and $K^\pm \rightarrow \pi^\pm +$ invisible (NA62)



S	$BR(\text{invisible})$	
π^0	$< 4.4 \cdot 10^{-9}$	NA62 ⁴
η	$< 1.0 \cdot 10^{-4}$	BESIII ⁵
η'	$< 6 \cdot 10^{-4}$	BESIII
$K_{S,L}^0$	NO	

⁴ JHEP 02 (2021) 201

⁵ PRL 97 (2006) 202002

NA64 long-term future: NA64-*h*. Experimental setup

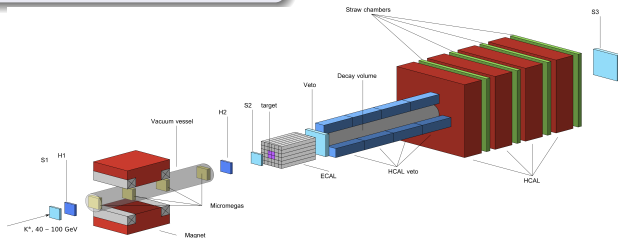
NA64-*h* approach: exploit the charge-exchange reaction $K^- p \rightarrow K^0 n$ as a source of a secondary neutral kaons, and search for the $K_{S,L}^0$ invisible decay.
Unique experimental signature: complete disappearance of the beam energy.

Experiment setup

- Low-Z active target surrounded by an hermetic EM calorimeter
- High-efficiency veto counters
- Massive HCAL around and downstream a decay vessel
- Upstream beam tracking/tagging

Signal signature

- Well-defined impinging track
- MIP-like energy deposition in the active target
- No activity in ECAL or VETO
- No activity in the HCAL

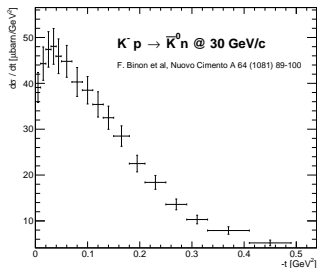


NA64 long-term future: NA64-*h*. Sensitivity

The number of signal events is proportional to:

- The number of K^- OT N_{K^-}
- The p numerical density in the active target n
- The charge-exchange cross section $\frac{d\sigma_{CE}}{dt}$
- The decay probability in the decay vessel length L
- The signal efficiency ε
- **The invisible channel branching fraction BF_I**

$$N_S = N_{K^-} \cdot n \cdot \int dt \frac{d\sigma_{CE}}{dt} \cdot \varepsilon(t) \left[1 - \exp\left(-\frac{L}{\gamma(t)c\tau}\right) \right] BF_I$$



Charge-exchange cross section:

- Data for 30 GeV/c K^- beam available from IHEP-70 measurement
- Regge-based model, $\rho + a_2$ t -channel exchange: energy dependence $\sim s^{\alpha(0)-1}$.
 Z -dependence $\sim Z^{2/3 \div 1}$.

NA64-*h* sensitivity for 10^{12} K^- OT

Assuming zero background events, from $N_S = N_{K_L} + N_{K_S} < N_{up}^{90\%CL}$:

- $Br(K_S \rightarrow \text{invisible}) \lesssim 10^{-8}$
- $Br(K_L \rightarrow \text{invisible}) \lesssim 10^{-6}$