Charged Lepton Flavour Violation: Theory Introduction

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Outline

- Why LFV?
- Channels and BSM expectations
- Effective Field Theory for LFV (mainly $\mu \to e$)
- Conclusion

Neutrino masses imply Lepton Flavour Violation

The Standard Model Lagrangian (without right-handed neutrinos) is accidentally invariant under a phase rotation of each lepton flavor $U(1)_{L_\alpha}$

$$\ell_\alpha = \begin{pmatrix} \nu_\alpha \\ \alpha_L \end{pmatrix}, e_\alpha = \alpha_R \quad \text{with} \quad \alpha = e, \mu, \tau$$

$$U(1)_{L_\alpha} : \begin{cases} \ell_\alpha \to e^{i\chi_\alpha} \ell_\alpha \\ e_\alpha \to e^{i\chi_\alpha} e_\alpha \end{cases}$$

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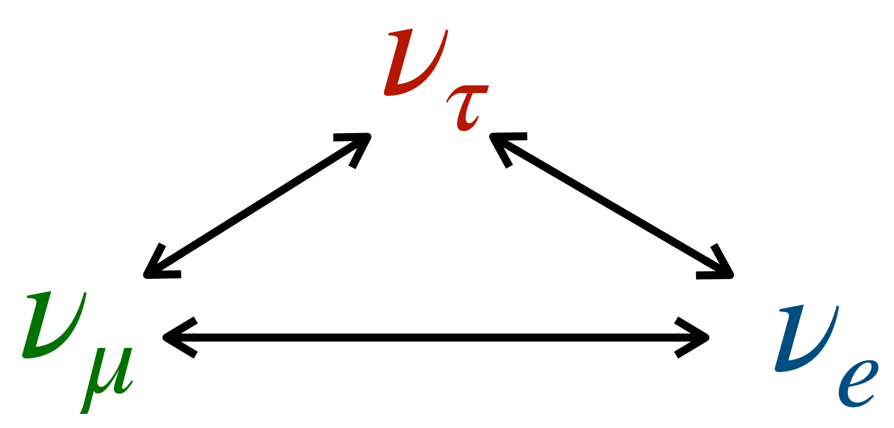
Neutrino oscillations break all symmetries



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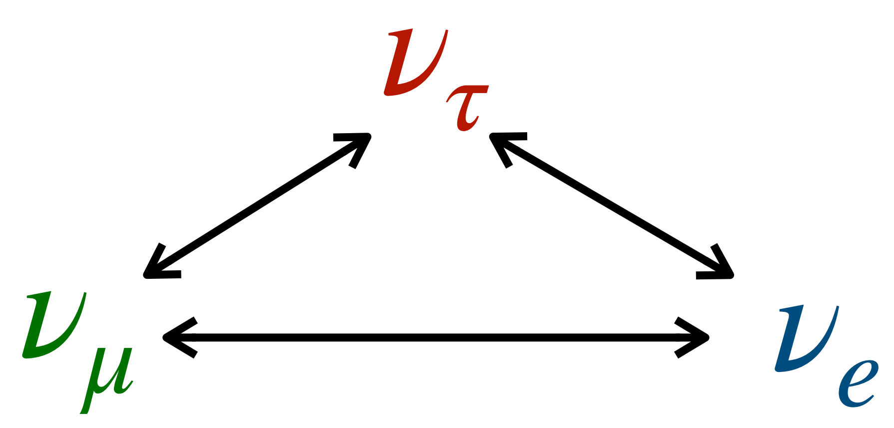
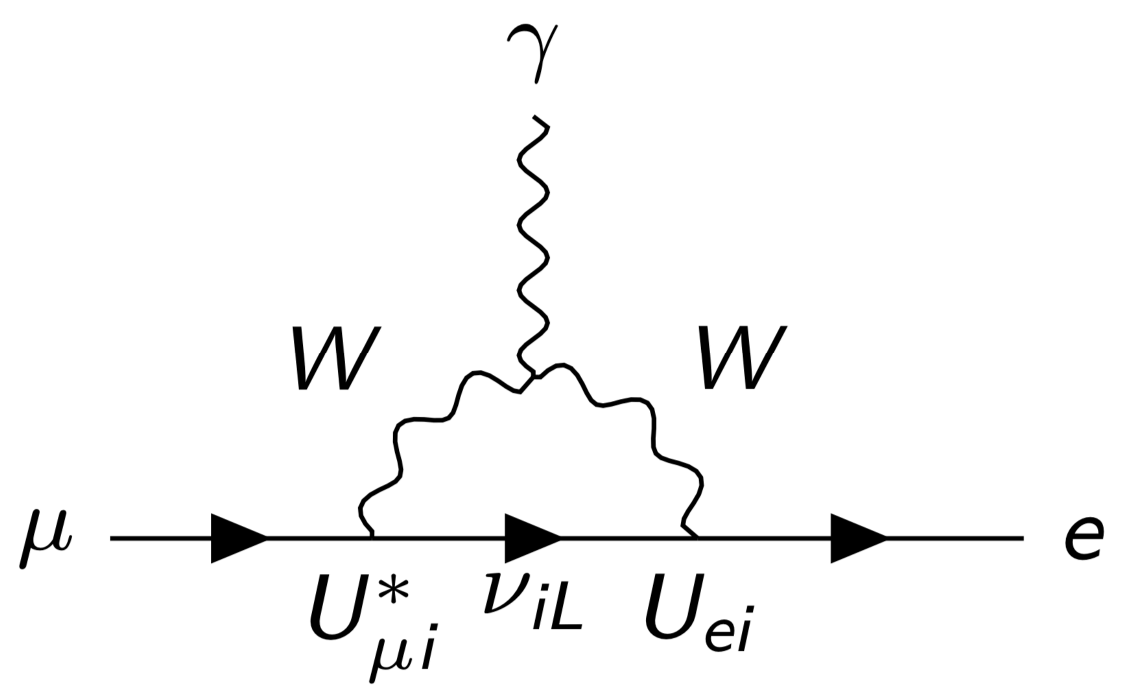


Since it is not a symmetry of nature, lepton flavour violation in the charged sector is inevitable:

$$\mu^\pm \to e^\pm \gamma \qquad \tau^\pm \to e^\pm e^+ e^- \qquad \ldots$$

but at what rates?

Charged Lepton Flavour Violation (LFV)



- SM+ν_R predicts small LFV

$$Br(\mu \to e\gamma) \simeq G_F^2(\Delta m_\nu^2)^2 \sim 10^{-50}$$

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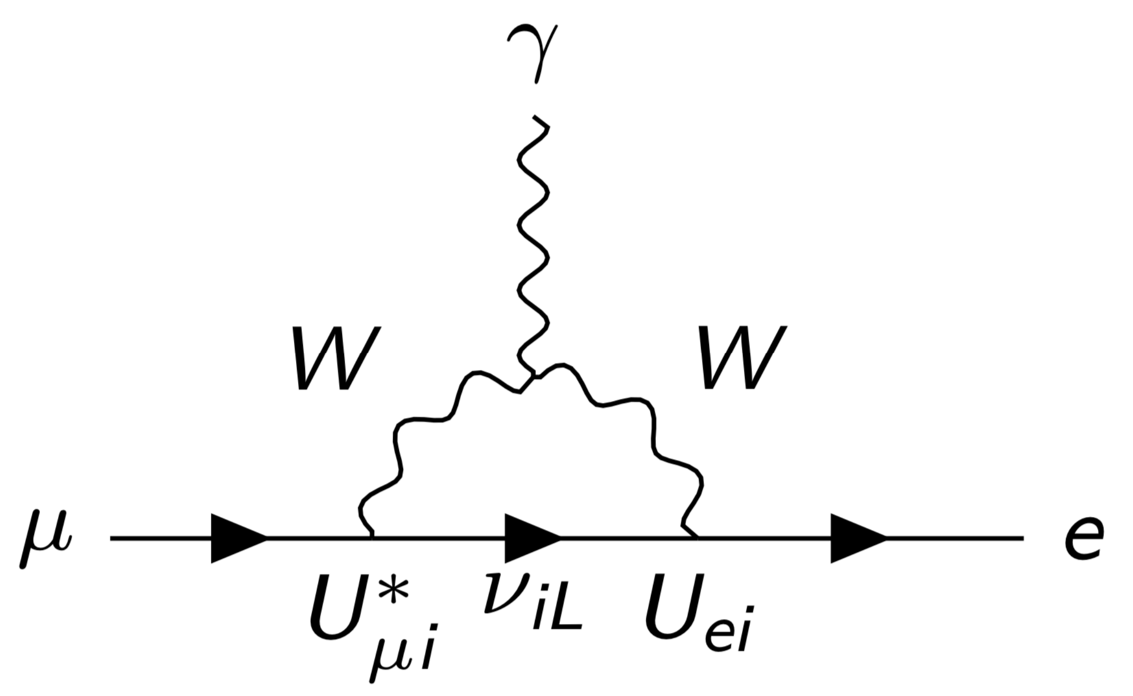

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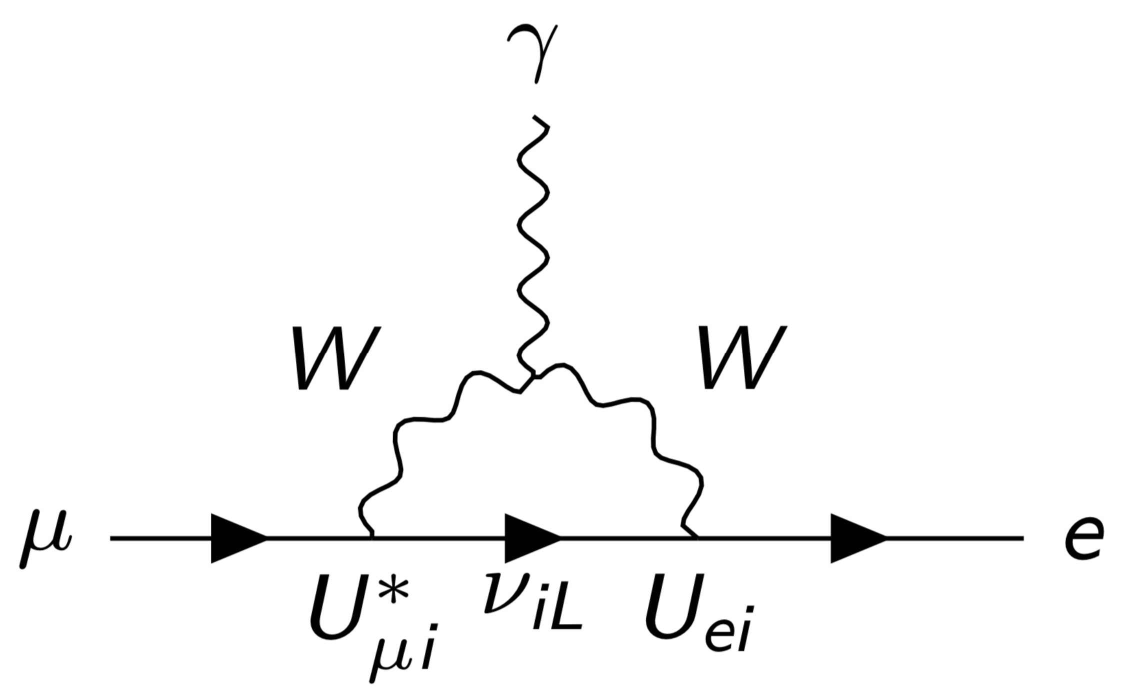

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- Many models that address unresolved puzzles (independently from neutrino masses) predict potentially observable LFV signals

- Some LFV reviews: Kuno+Okada hep-ph/9909265, Calibbi+Signorelli 1709.00294, Bernstein+Cooper 1307.5787, Cei+Donati 10.1155/2014/282915, Ardu+Pezzullo 2204.08220

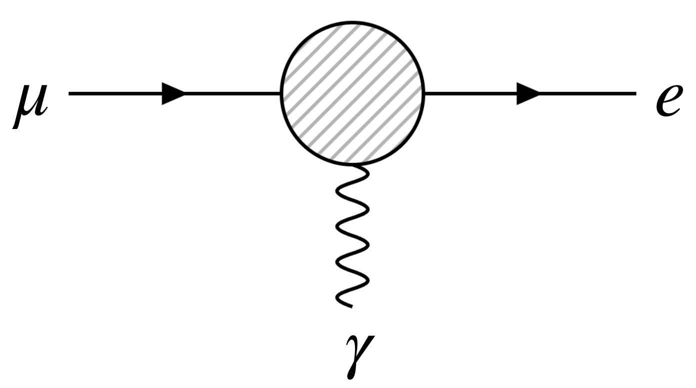
Experimental searches

Process	Current bound on BR	Future Sensitivity
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$ MEG	10^{-14} MEGII
$\mu \to \bar{e}ee$	$< 1.0 \times 10^{-12}$ SINDRUM	10^{-16} Mu3e
$\mu A \to eA$	$< 7 \times 10^{-13}$ SINDRUMII	$10^{-16} \to 10^{-18}$ COMET, Mu2e
$\tau \to l\gamma$	$< 3.3 \times 10^{-8}$	$3 \times 10^{-9}(e), 10^{-9}(\mu)$
$\tau \to e\bar{e}e$	$< 2.7 \times 10^{-8}$	5×10^{-9}
$\tau \to \mu\bar{\mu}\mu$	$< 2.1 \times 10^{-8}$	4×10^{-9}
$\tau \to \mu\bar{e}e, e\bar{\mu}\mu$	$< 1.8, 2.7 \times 10^{-8}$ Belle	$3, 5 \times 10^{-9}$ BelleII
...	...	...
$\tau \to l\pi^0$	$< 8.0 \times 10^{-8}$	4×10^{-9}
$\tau \to l\eta$	$< 6.5 \times 10^{-8}$	7×10^{-9}
$\tau \to l\rho$	$< 1.2 \times 10^{-8}$ Belle	10^{-9} BelleII
$K^0 \to \mu^\pm e^\mp$	$< 4.7 \times 10^{-12}$	
$B_d^0 \to \tau^\pm \mu^\mp$	$< 1.2 \times 10^{-5}$ LHCb	$\sim 10^{-6}$?
...	...	...
$h \to e^\pm \mu^\mp$	$< 6.1 \times 10^{-5}$ Atlas	2.1×10^{-5}
$h \to e^\pm \tau^\mp$	$< 2.2 \times 10^{-3}$ CMS	2.4×10^{-4}
$h \to \tau^\pm \mu^\mp$	$< 1.5 \times 10^{-3}$ CMS	2.3×10^{-4} ILC
$Z \to e^\pm \mu^\mp$	$< 7.5 \times 10^{-7}$ Atlas	
$Z \to l^\pm \tau^\mp$	$< 10^{-7}$ Atlas	

- $\mu \to e$ decays
- $\tau \to l$ decays
- Heavy particles decaying into LFV final states

$\mu \to e$ transitions

- $\mu \to e + \gamma$ $Br(\mu \to e\gamma) < 4 \times 10^{-13}$ (MEG) $\to$ $Br(\mu \to e\gamma) \sim 6 \times 10^{-14}$ (MEGII)



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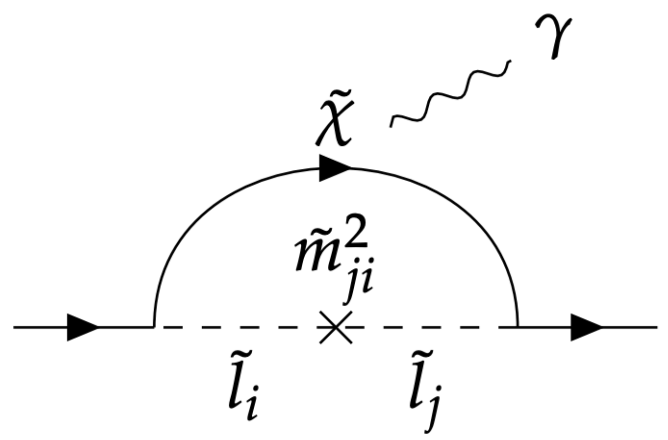
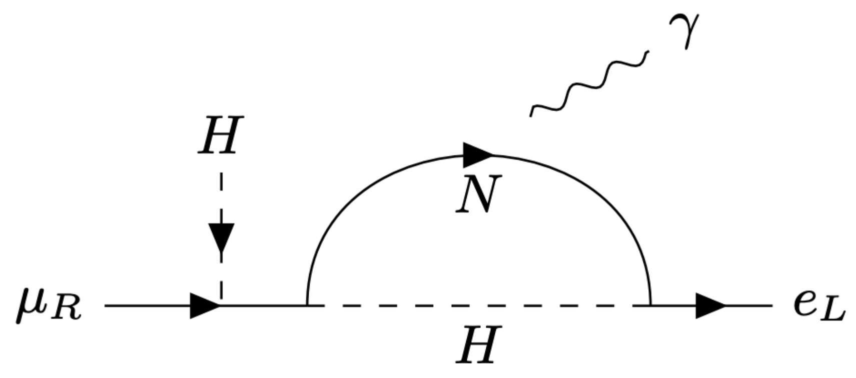
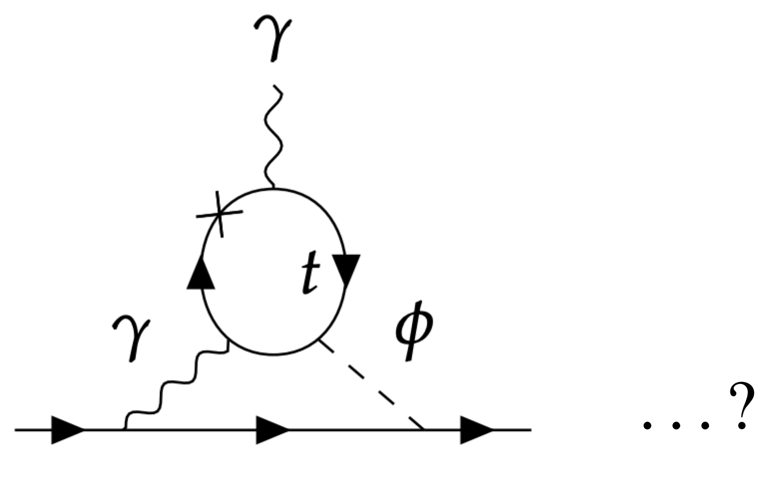


SUSY



Sterile Neutrinos



Scalar LFV (2HDM,…)

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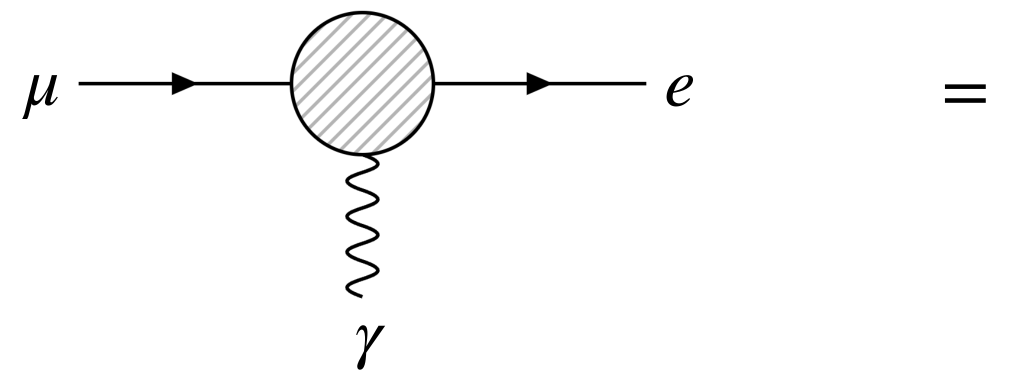

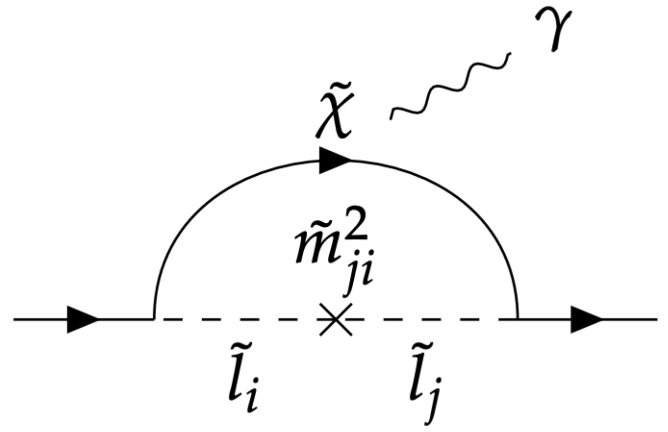

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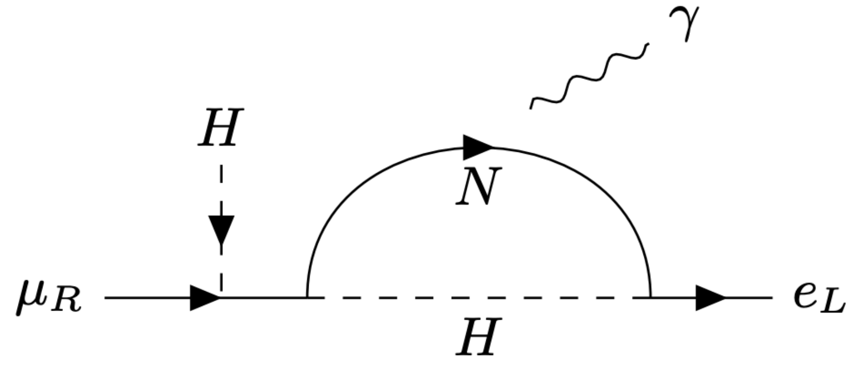

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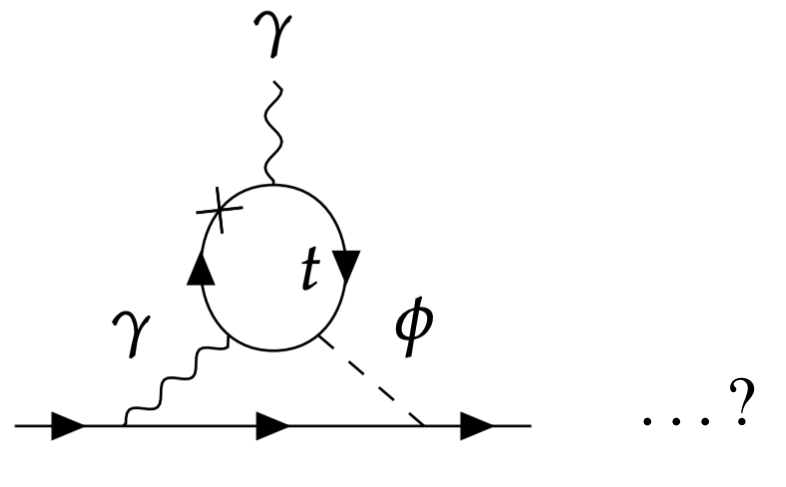
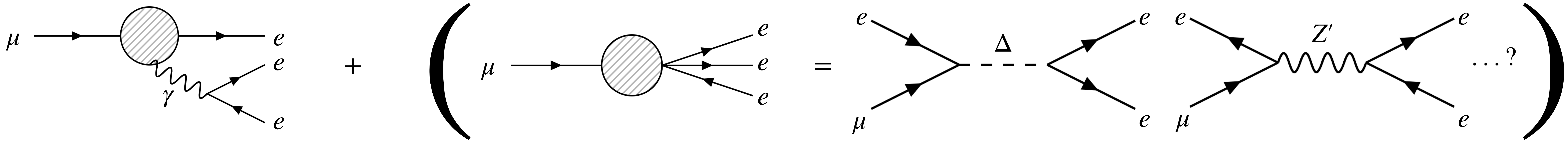


Scalar LFV (2HDM,…)

- $\mu \to e + \bar{e} + e$ $\quad Br(\mu \to e\bar{e}e) < 10^{-12}$ (SINDRUM) $\to$ $Br(\mu \to e\bar{e}e) \sim 10^{-16}$ (Mu3e)



Type-II seesaw

Extra gauge bosons

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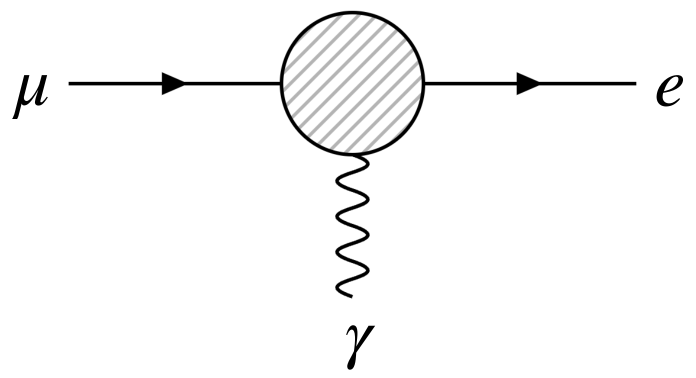

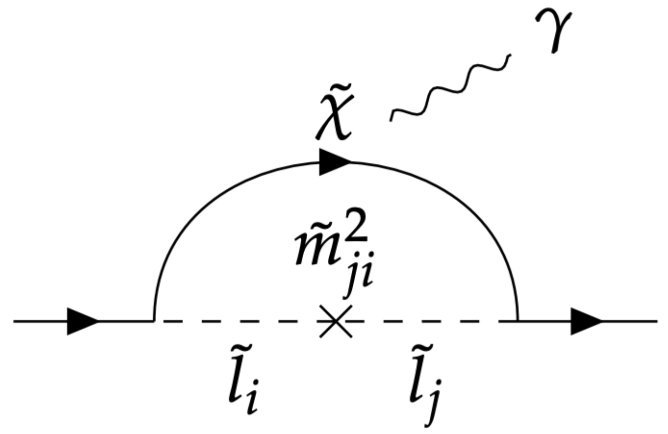

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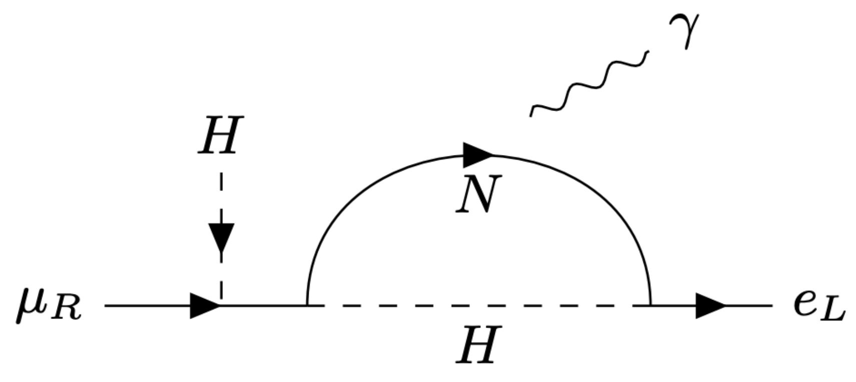

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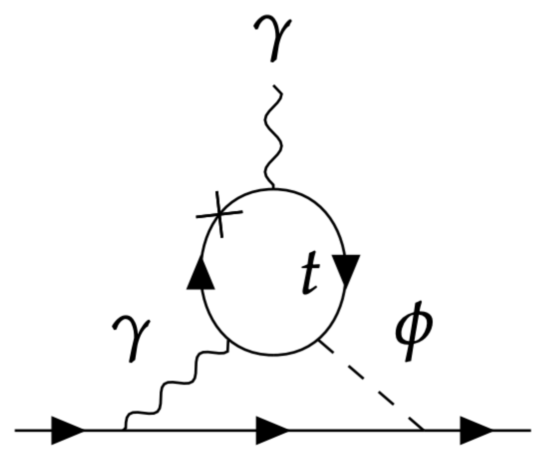

Scalar LFV (2HDM,…)

$\ldots ?$

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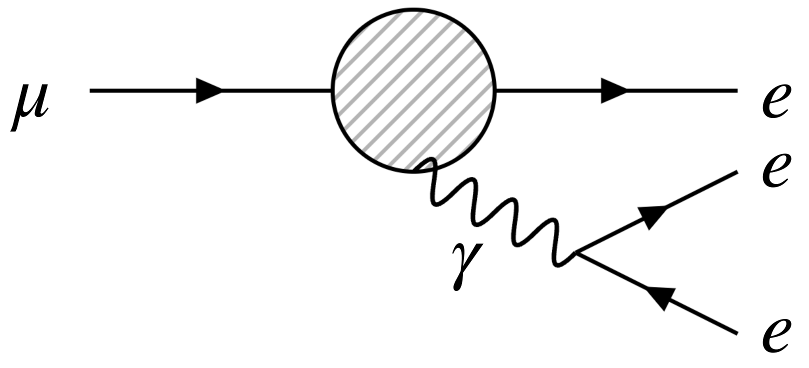

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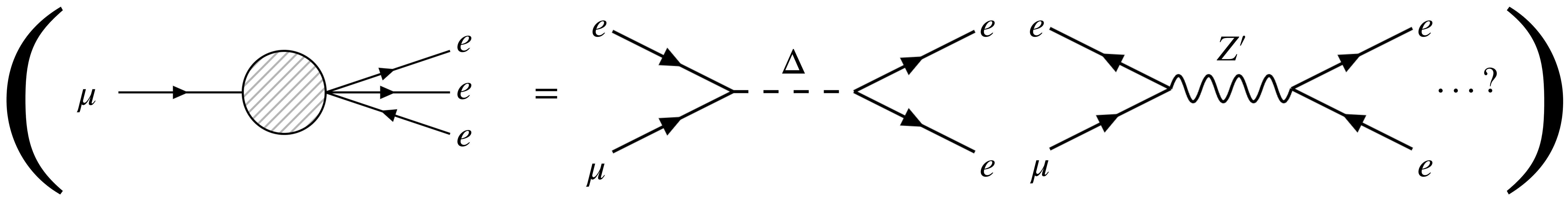
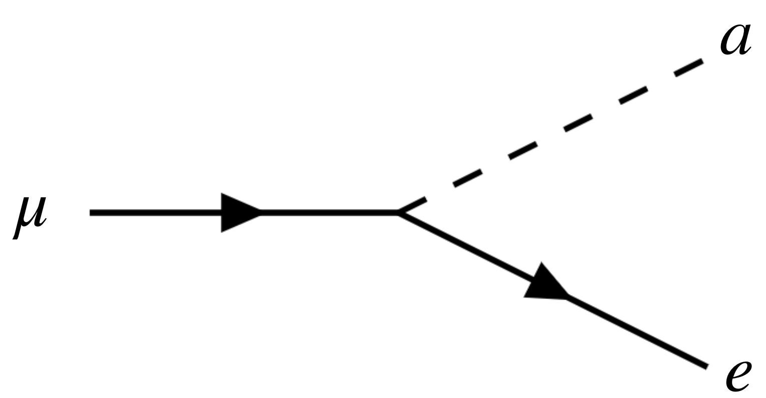


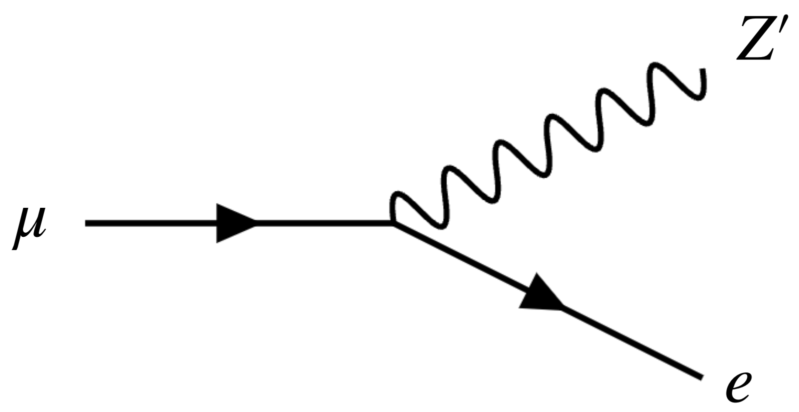
Type-II seesaw

Extra gauge bosons

- $\mu \to e + X$, where X is a light BSM particle $\quad Br(\mu \to eX) \lesssim 10^{-5}$ (TWIST) $\to$ MEG-II?



Axion (Like Particles)



Light Z prime

$\ldots ?$

$\mu \to e$ conversion in nuclei

Standard calculation in Kuno+Okada hep-ph/9909265

- The muon gets captured by the (Z,A) nucleus and tumbles down to the 1s state

- The SM processes that can happen are:

    A. $\mu + p \to \nu_\mu + n$ (capture)

    B. $\mu \to \nu_\mu + e + \overline{\nu_e}$ (Decay-In-Orbit)

- If there are LFV interactions with nucleons, an electron can be emitted without a neutrino (conversion)

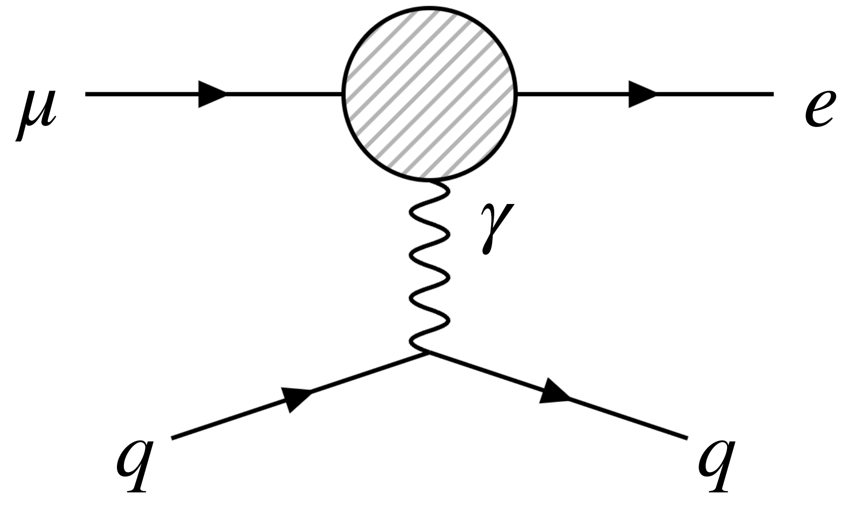
$$\mu + (Z, A) \to e + (Z, A)$$

- Spin-Independent rate is enhanced by $\propto A^2$ because the process is coherent (similar to WIMP scattering)

- The upcoming experiments (COMET, Mu2e) will deliver extremely intense muon beams allowing to probe $Br(\mu A \to eA) \sim 10^{-17}$

$\mu \to e$ conversion in nuclei

- Sensitivity to the dipole that could compete with $\mu \to e\gamma$ searches



$q = u, d$

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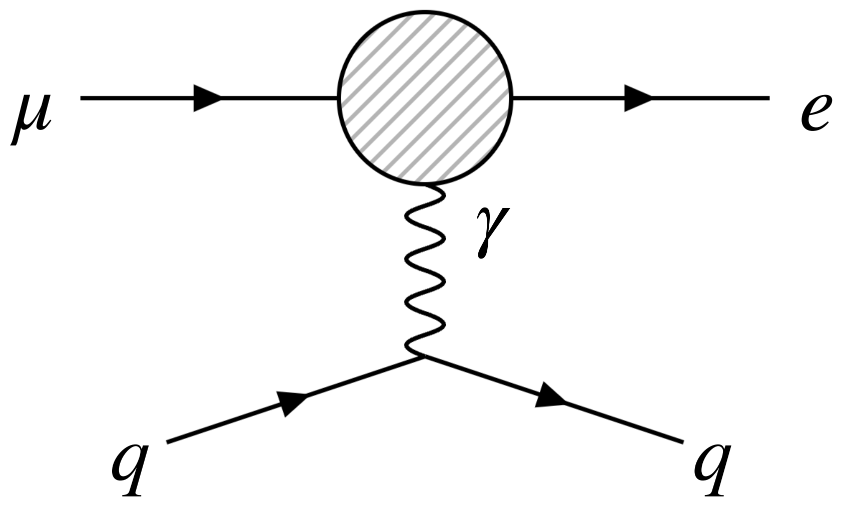
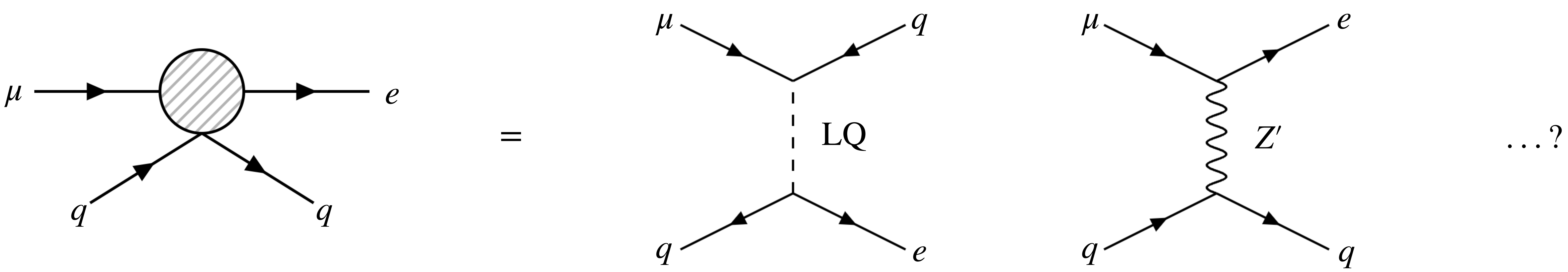


$q = u, d$

- But can also probe new interactions



Leptoquarks **Z prime**

$\tau \rightarrow l$ transitions

- One cannot make τ beams, so the sensitivity of $\tau \rightarrow l$ processes is $Br(\tau \rightarrow l) \sim 10^{-8} \rightarrow 10^{-10}$ (LHC(b), BaBar, Belle, Belle-II)

- Mostly insensitive to loops = if see $\tau \rightarrow l$ should be at tree-level if NP scale is above $\Lambda \gtrsim 4\ \mathrm{TeV}$

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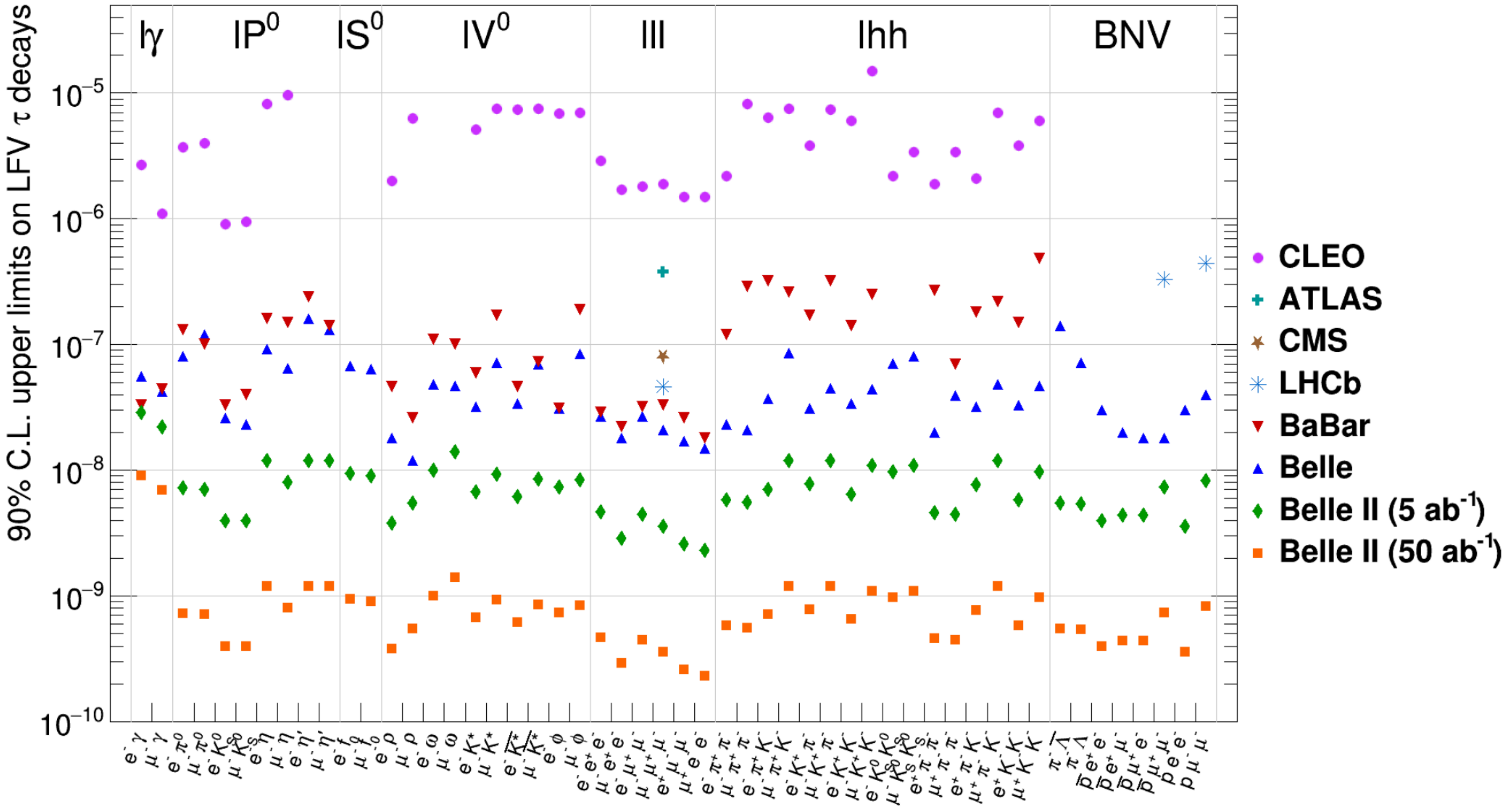
- Mostly insensitive to loops = if see $\tau \to l$ should be at tree-level if NP scale is above $\Lambda \gtrsim 4$ TeV
- The bigger phase available means there is a plethora of different channels (possible to overconstrain models = distinguish them)



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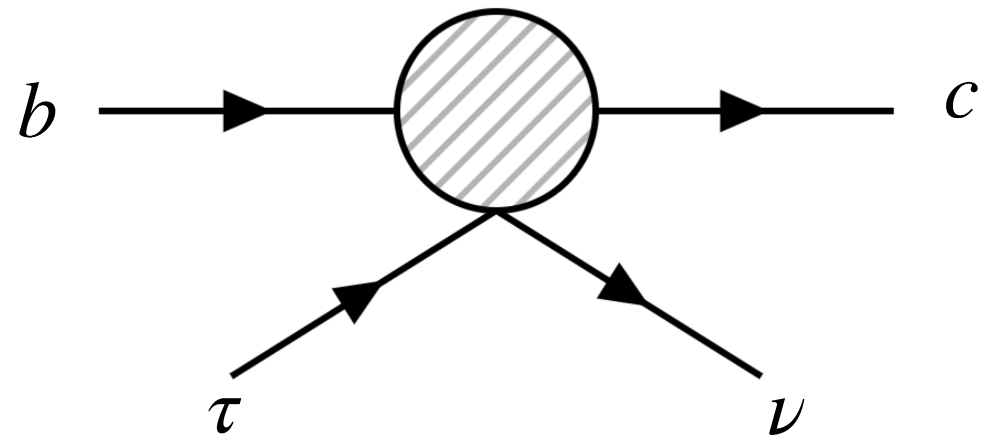
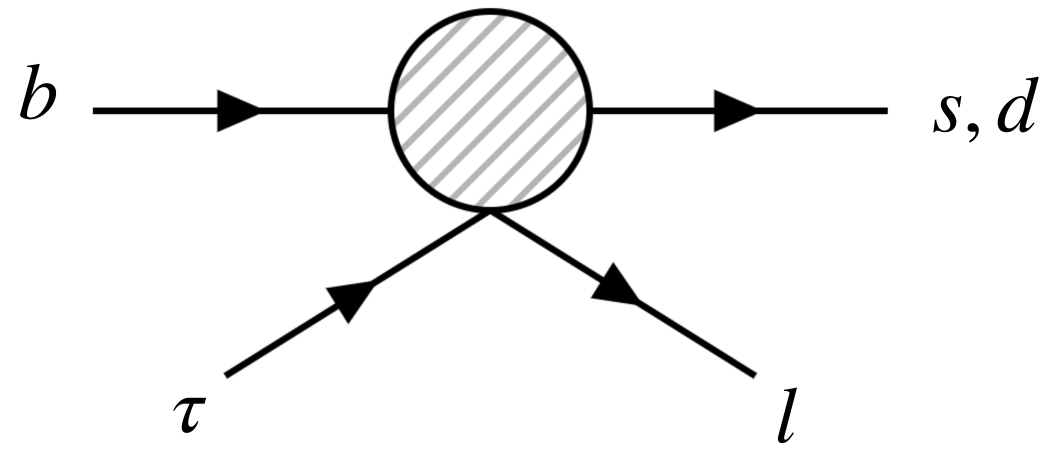
Hadron decays

- We can also have hadrons decaying into LFV final states

$$K \to e^{\pm}\mu^{\mp} \qquad B \to \tau^{\pm} l^{\mp} \qquad \cdots$$

- LFV decays of B meson with τ in the final states possibly related to $R_{D^{(*)}}$ anomaly?

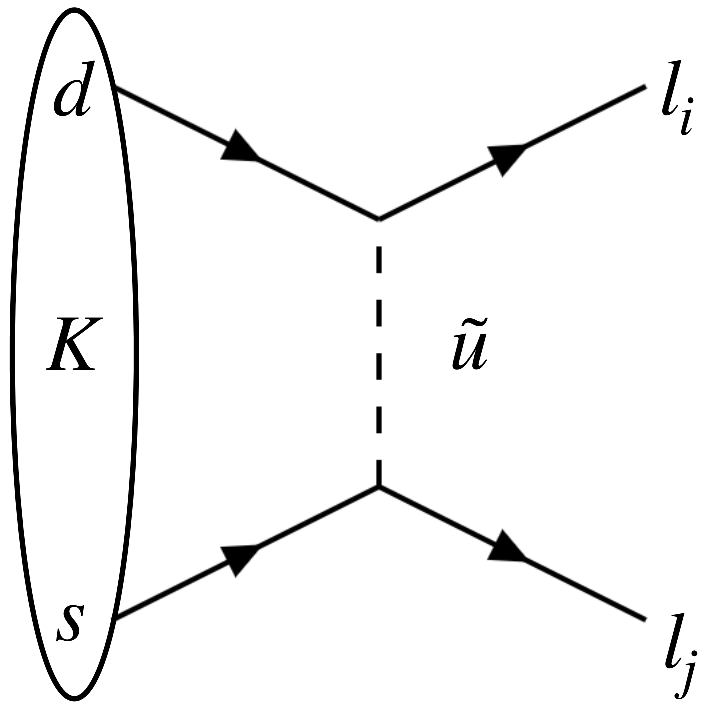
If 

also 

?

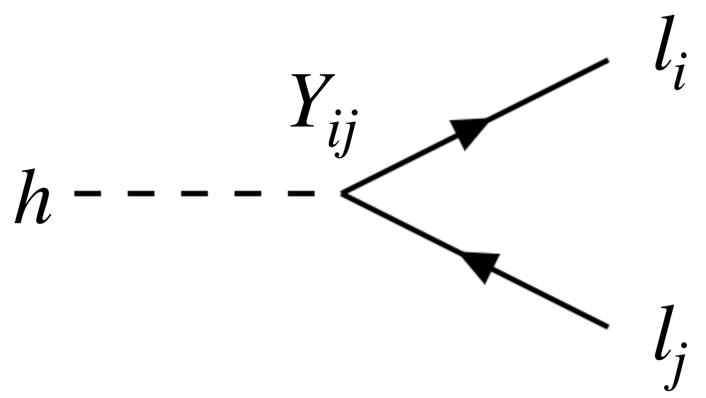
- SUSY with RPV?

$$W_{\rm RPV} \supset \lambda_{ijk} L_i Q_j \bar{E}_k$$



Heavy bosons decay

- LFV decays of heavy SM particles can be looked for at the LHC



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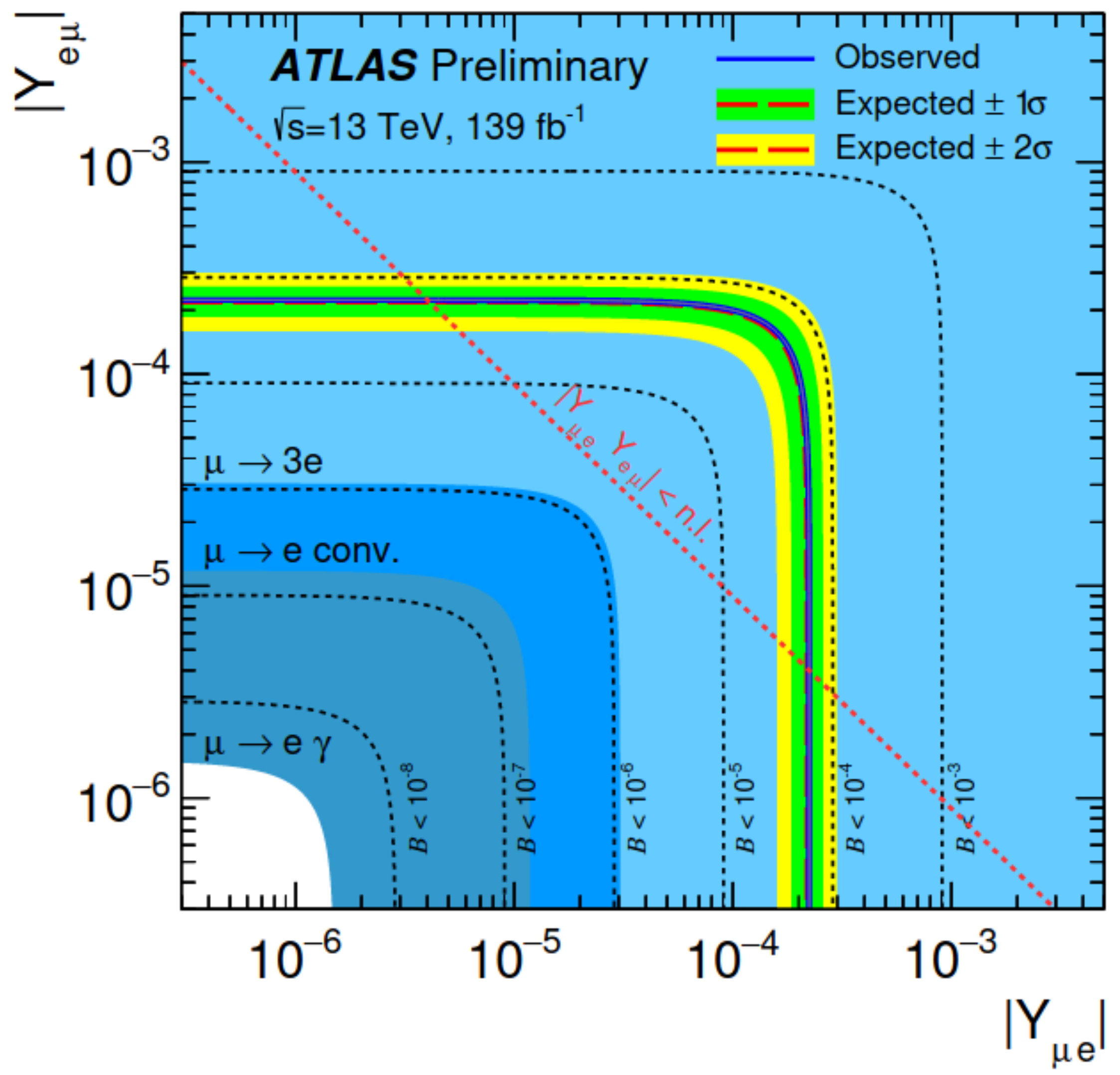

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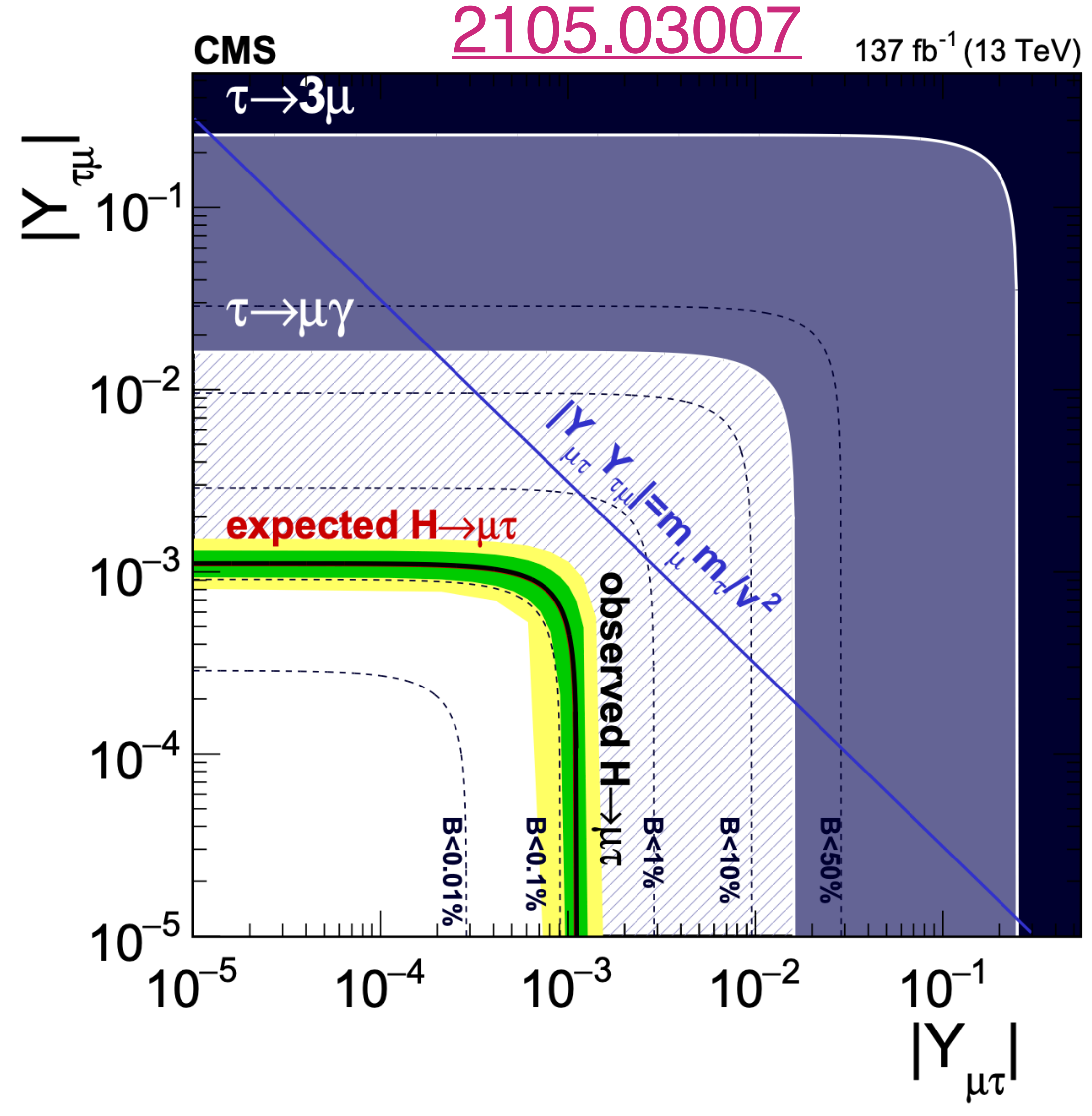
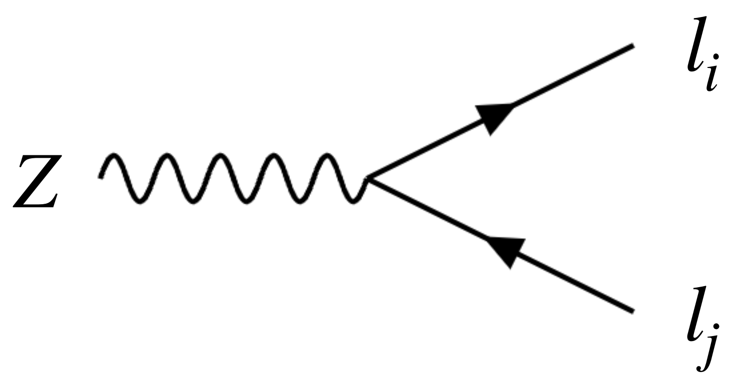
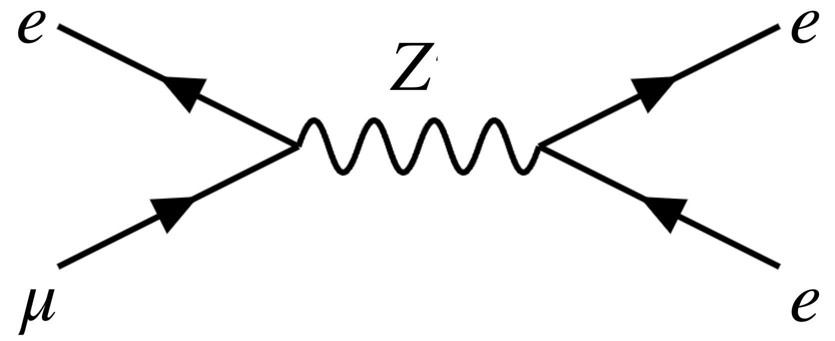
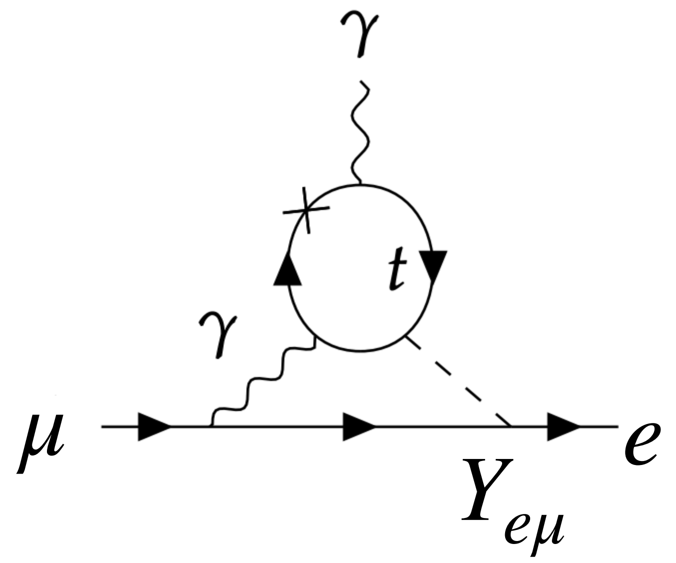




- In general with $e\mu$ final states the low-energy probes have a better sensitivity (but sensitivity $\neq$ constraint)





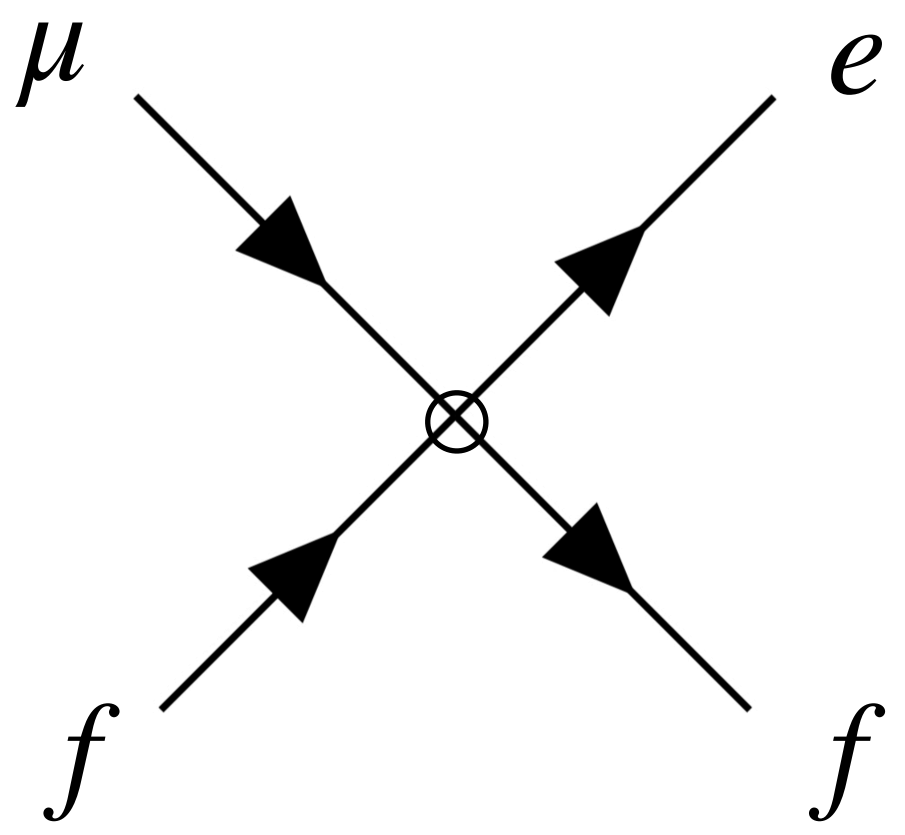
- Very competitive in the τ sector

Many channels, many more models… what to do?

Effective Field Theory for LFV

- Many models predict LFV = would be nice to know what experiments can tell us in a model-independent way

- If LFV New Physics is heavy $(\Lambda \gtrsim 4\ \mathrm{TeV})$ and it can be integrated out



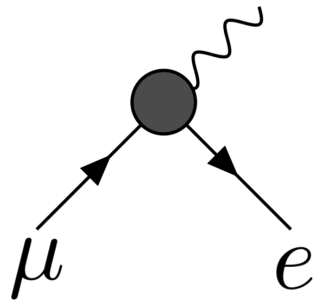
$$\sim \frac{C}{\Lambda^2}$$

- Add to the Lagrangian the contact interactions (non-renormalizable operators) compatible with the symmetries

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{d\leq 4} + \sum_{n>4} \frac{C_n \mathscr{O}_n}{\Lambda^{n-4}}$$

Effective Field Theory for LFV

- Observables are calculated in terms of the operator coefficients



$$\delta\mathscr{L}_{\mu\to e\gamma} = \frac{m_\mu}{\Lambda^2}(C^{e\mu}_{D,R}\bar{e}\sigma_{\alpha\beta}P_R\mu + C^{e\mu}_{D,L}\bar{e}\sigma_{\alpha\beta}P_L\mu)F^{\alpha\beta}$$

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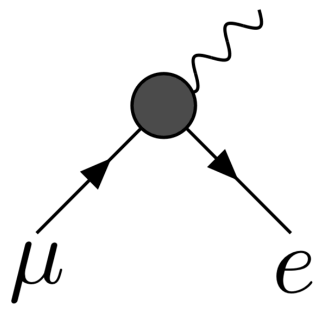

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$$Br(\mu \to e\gamma) = 384\pi^2\left(\frac{v}{\Lambda}\right)^4 (\; C^{e\mu}_{D,R}{}^{\;2} + \; C^{e\mu}_{D,L}{}^{\;2}) < 4.2 \times 10^{-13} \longrightarrow \left(\frac{v}{\Lambda}\right)^2 \; C^{e\mu}_{D,X} \; < 10^{-8}$$

$v^2 = (2\sqrt{2}G_F)^{-1} \sim (174\ \mathrm{GeV})^2$

$\Lambda \gtrsim 10^4 v$ (if $C_D \sim 1$)

Effective Field Theory for LFV

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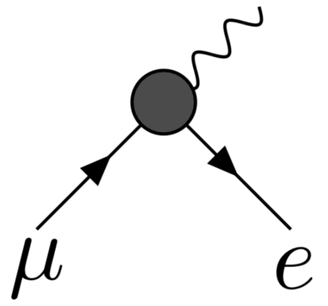

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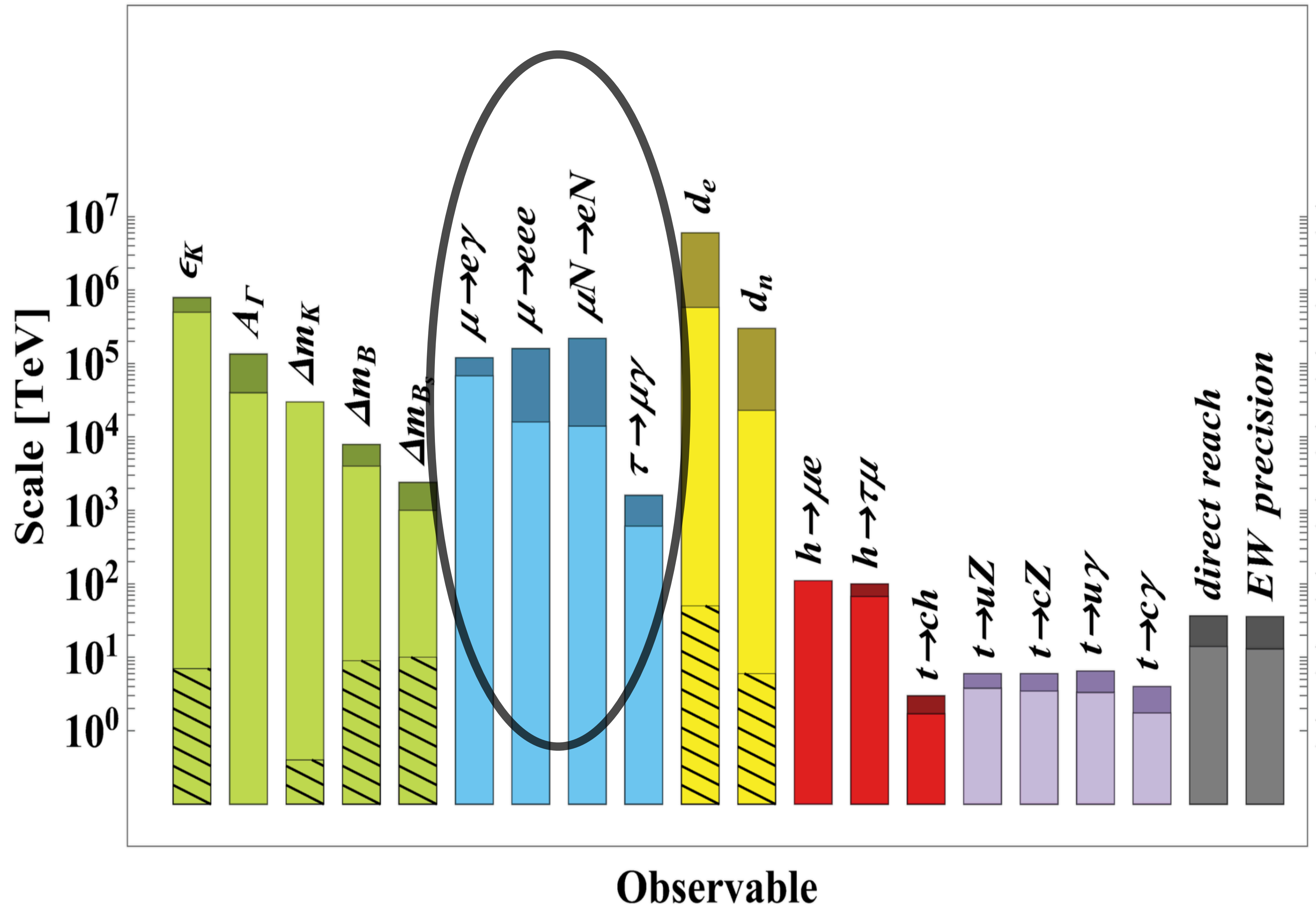
$$v^2 = (2\sqrt{2}G_F)^{-1} \sim (174\ \mathrm{GeV})^2 \qquad \Lambda \gtrsim 10^4 v\ (\text{if } C_D \sim 1)$$

- Translate branching ratios sensitivities/upper bound on New Physics scale (assuming $C \sim 1$; also depend on ops definition)



Including loops (RGEs)

- SM loops can decorate contact interactions, causing the coefficients to run with the energy scale (like any coupling does in QFT)

$$\frac{d\overrightarrow{C}_n(\mu)}{d\log\mu} = \overrightarrow{C_n}(\mu)\gamma + \dots$$

$n = \mathrm{op.dim.}$

γ anomalous dimension matrix

- The Renormalization Group Equations (RGEs) introduce operator mixing

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- SM loops can decorate contact interactions, causing the coefficients to run with the energy scale (like any coupling does in QFT)

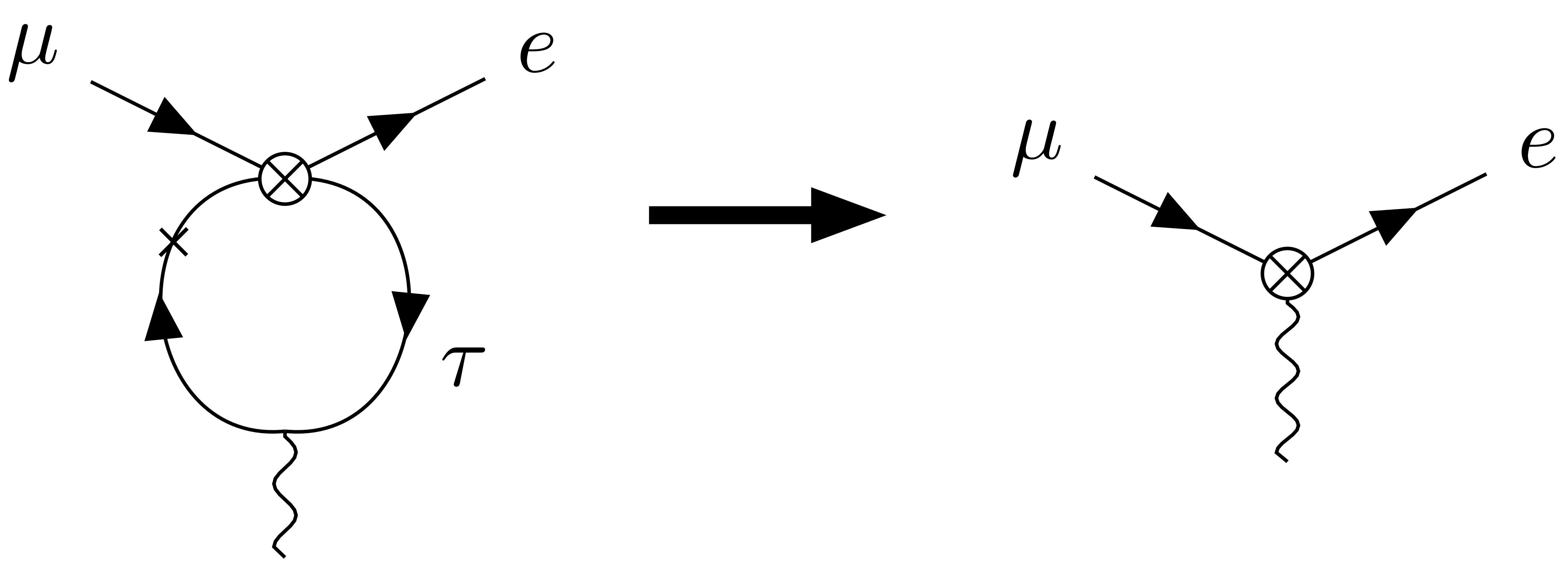
$$\frac{d\overrightarrow{C}_n(\mu)}{d\log\mu} = \overrightarrow{C_n}(\mu)\gamma + \dots$$

$n = \mathrm{op\,.\,dim\,.}$

γ anomalous dimension matrix

- The Renormalization Group Equations (RGEs) introduce operator mixing

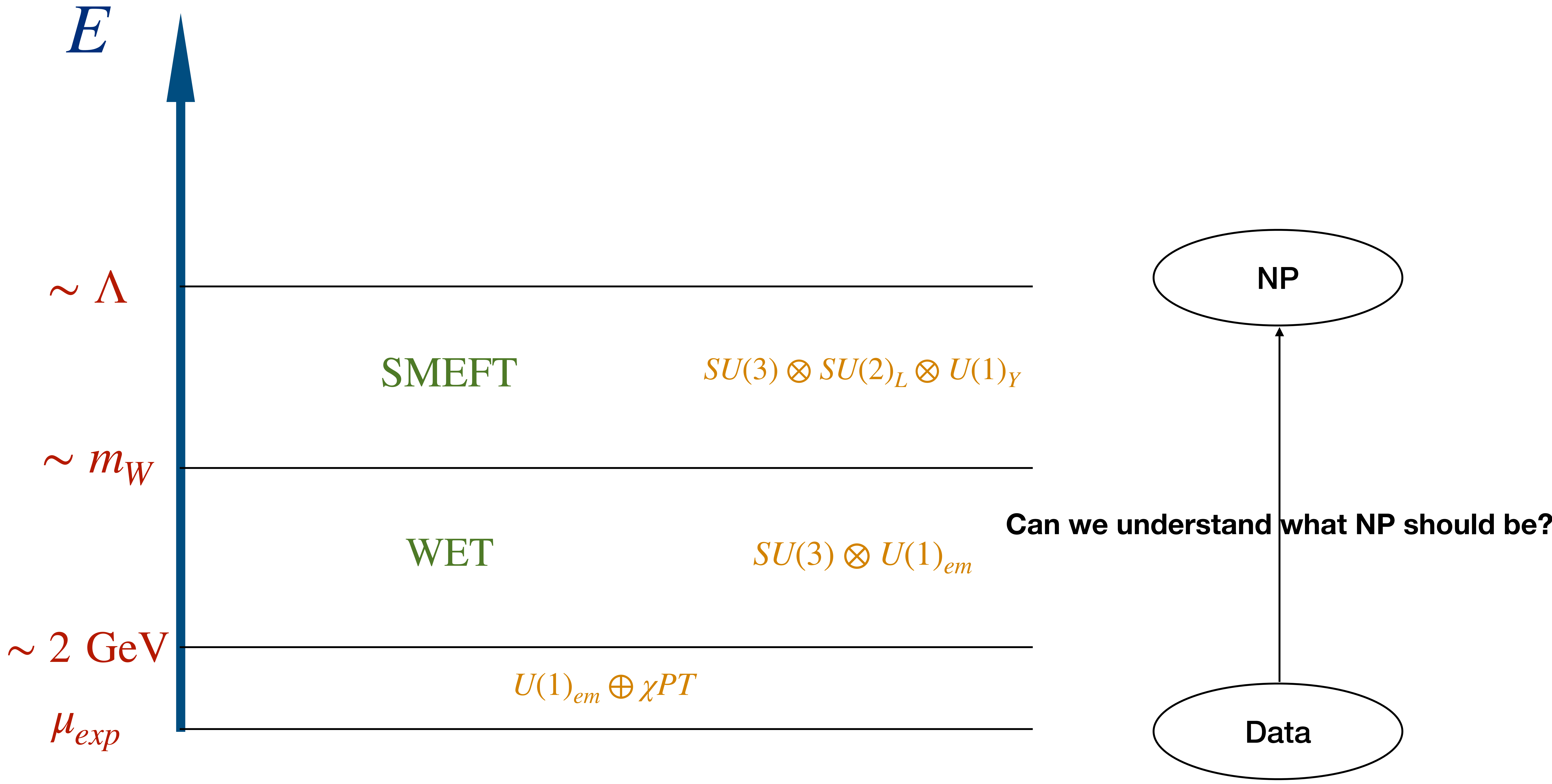
$C^{\tau}_{T,X}(\bar{e}\sigma P_X\mu)(\bar{\tau}\sigma P_X\tau)$



$$C_{D,X}(m_\mu) \sim \frac{m_\tau}{m_\mu}\frac{e}{16\pi^2}\log\left(\frac{\Lambda}{m_\mu}\right)C^{\tau}_{T,X}(\Lambda) \sim C^{\tau}_{T,X}(\Lambda)$$

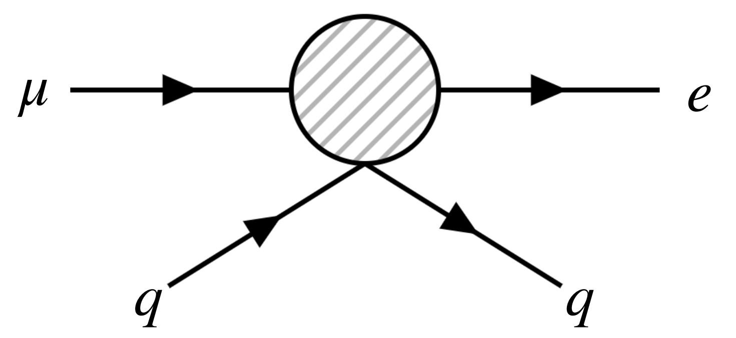
Loops are interesting because they allow to probe difficult-to-detect operators via operator mixing

Running from the bottom-up



Davidson+Echenard 22

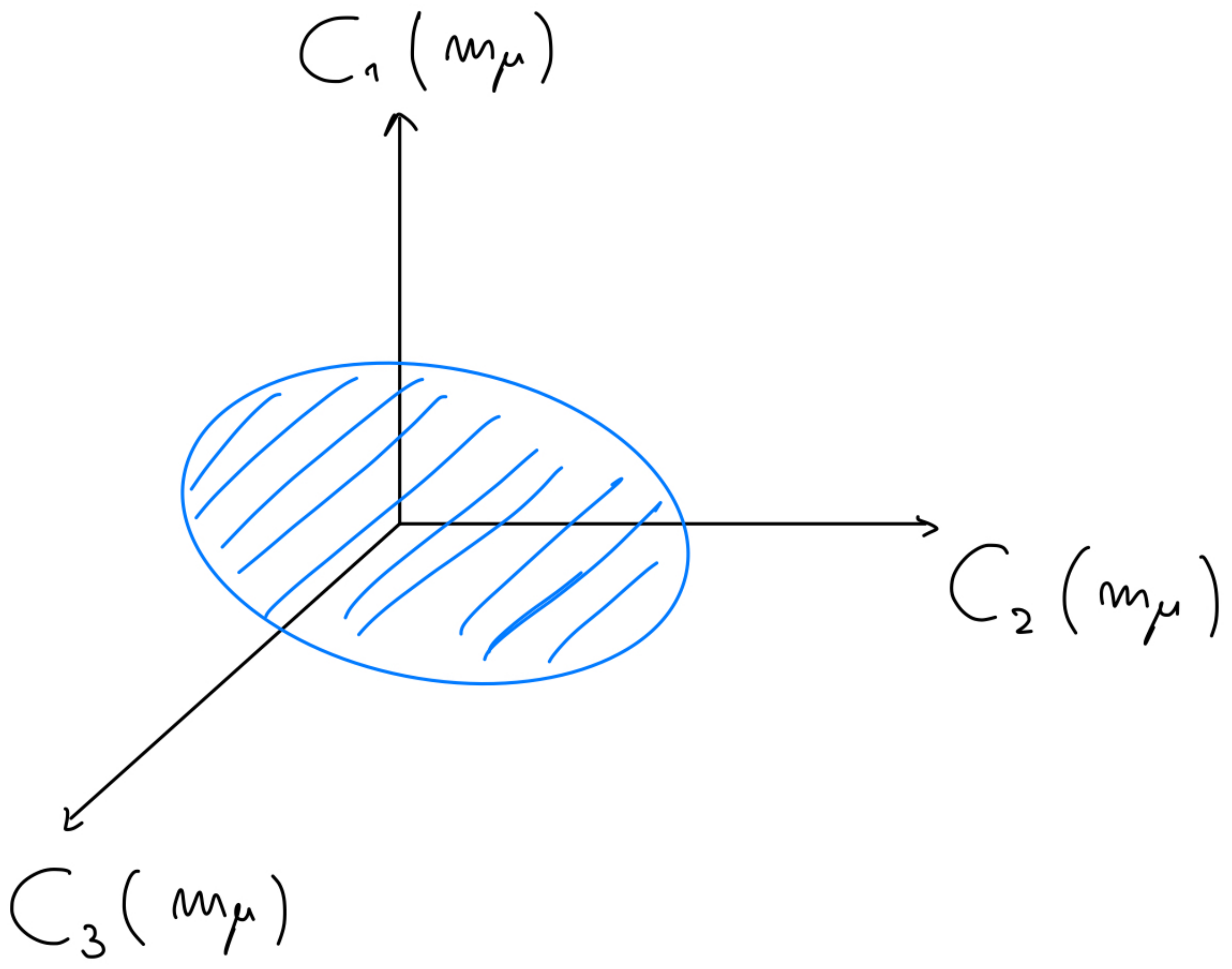
Bottom-up EFT for $\mu \to e$



- Focus on $\mu \to e$ because it has the best upcoming sensitivity

$$\mathcal{L}_{m_\mu} = \frac{1}{v^2} \sum_{X\in\{L,R\}} \left[C^{e\mu}_{D,X}(m_\mu \bar{e}\sigma^{\alpha\beta}P_X\mu)F_{\alpha\beta} + C^{e\mu ee}_{S,XX}(\bar{e}P_X\mu)(\bar{e}P_Xe) + C^{e\mu ee}_{V,LX}(\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma_\alpha P_X e) + C^{e\mu ee}_{V,RX}(\bar{e}\gamma^\alpha P_R\mu)(\bar{e}\gamma_\alpha P_X e) + C_{Alight,X}\mathcal{O}_{Alight,X} + C_{Aheavy\perp,X}\mathcal{O}_{Aheavy\perp,X} \right] + h.c.$$

- Data ($\mu \to e_X\gamma$, $\mu \to e_X\bar{e}_Y e_Z$, $\mu A \to e_X A \times 2$) constrain 12 operator coefficients at low energy to the interior of an ellipse in 12 dimensions



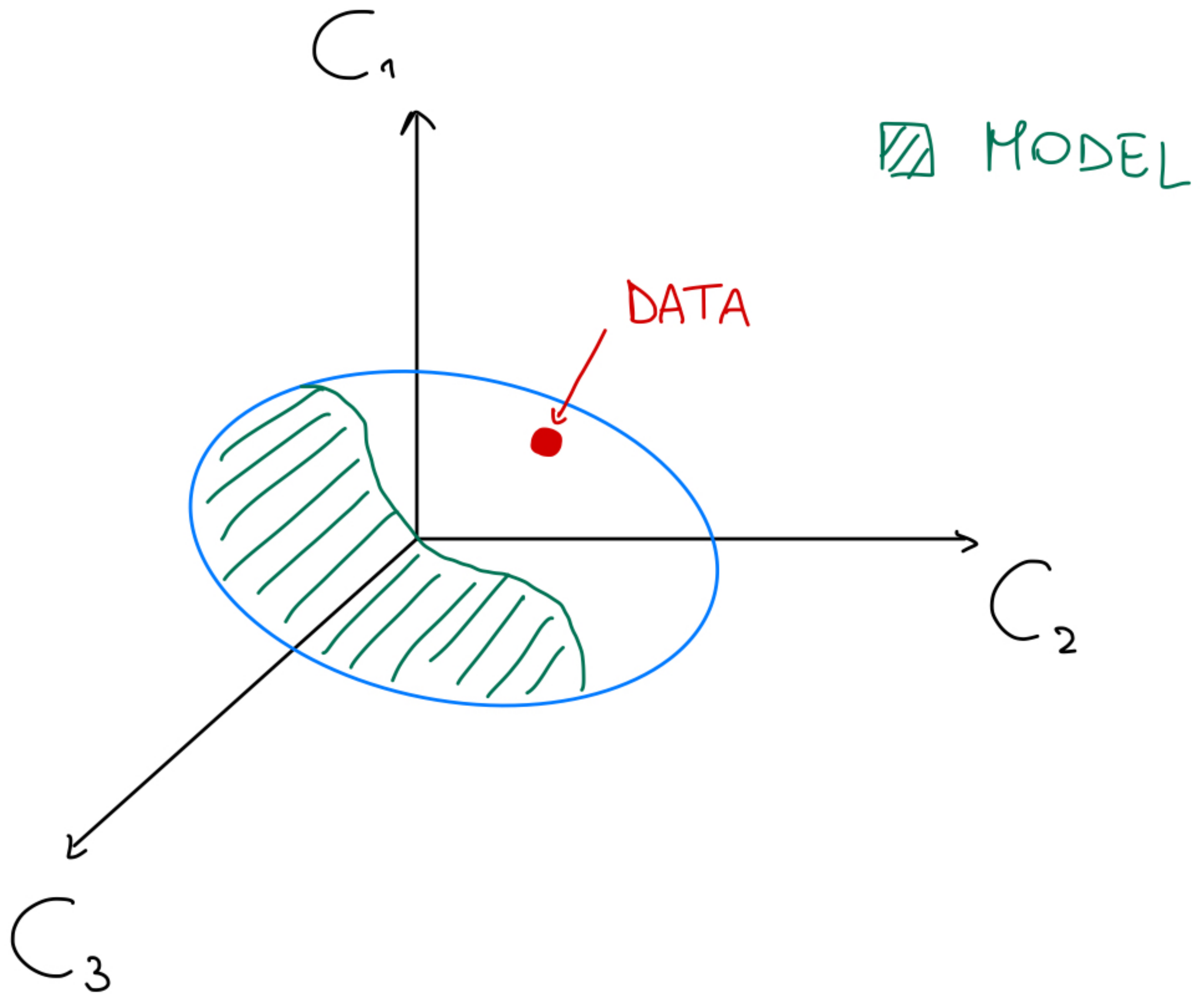
- The RGEs can tell us what these constrained directions are at the high scale Λ

$$\vec{C}(m_\mu) = \vec{C}(\Lambda) \cdot U(m_\mu, \Lambda)$$

A+Davidson+Lavignac 23

Distinguishing models?

- Suppose we observe $\mu \to e$ in the upcoming experiments (with theoretical optimism means a point in the 12-d ellipse)



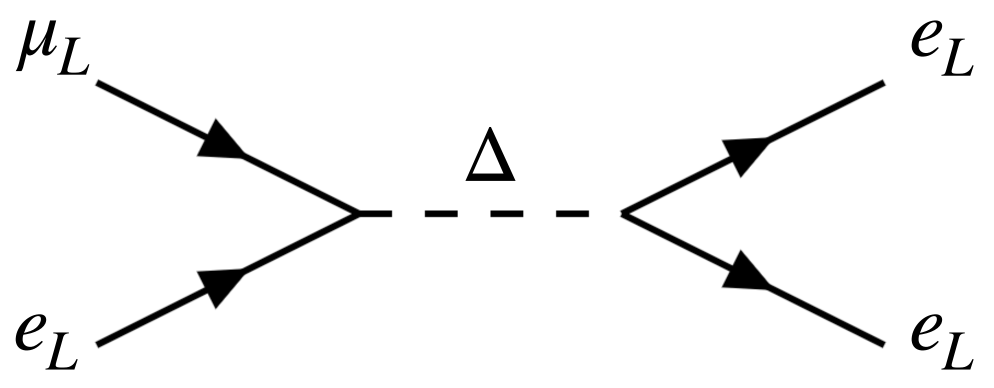
- And suppose I know regions where a model CAN NOT sit = If I see $\mu \to e$ there I can exclude it

- Instead of doing parameter scans like we usually do with top-down studies, this could be a complementary approach

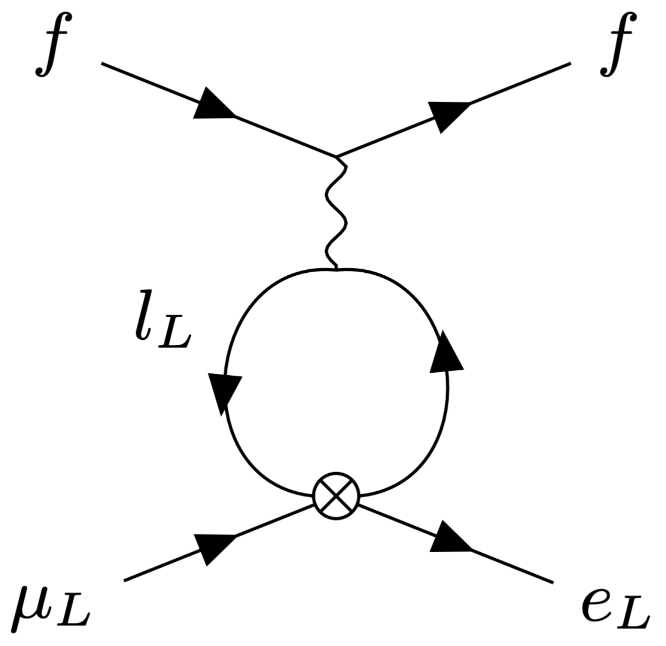
Ex: Type-II seesaw (SM + Triplet Δ)

$$\mathscr{L} \supset F_{\alpha\beta}\overline{\ell^c_\alpha}\epsilon\Delta\cdot\tau\ell_\beta + M_\Delta\lambda_H H^T\epsilon\Delta\cdot\tau H + \ldots \qquad [m_\nu]_{\alpha\beta} \sim 0.03\ \text{eV}\ F_{\alpha\beta}\frac{\lambda_H}{10^{-12}}\frac{\text{TeV}}{M_\Delta}$$

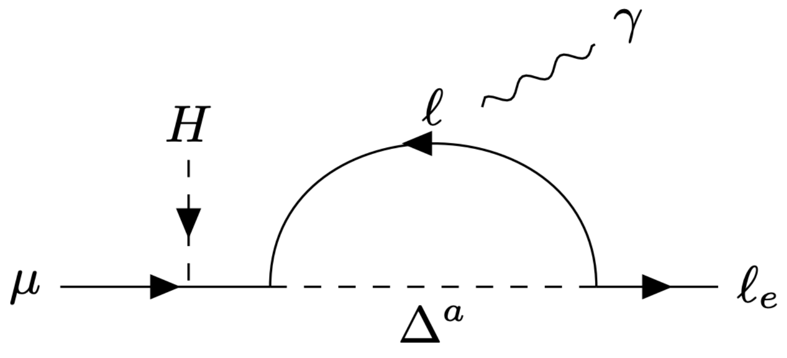
- Neutrino masses directly related to the Triplet Yukawas, but ordering, lightest mass and Majorana phases are unknown

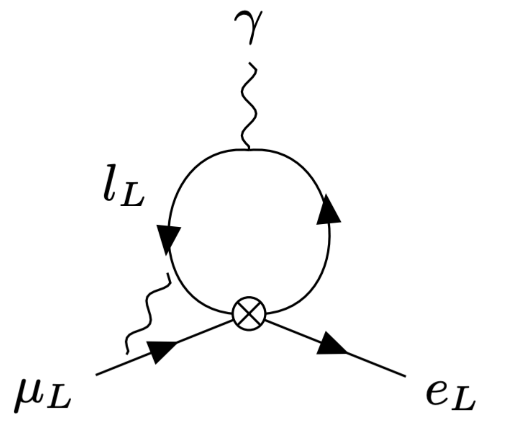


$C^{e\mu ee}_{V,LL} \quad (\mu \to e_L\bar{e}_L e_L)$



$C^{e\mu ee}_{V,LX} \quad (\mu \to e_L\bar{e}_X e_X) \qquad C^{e\mu}_{A,L} \quad (\mu A \to e_L A)$





$C^{e\mu}_{D,R} \quad (\mu \to e_L\gamma)$

Type-II: where does it live in the ellipse?

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- Cannot predict sizable $\mu \to e_R$ (because triplet couples to left-handed leptons and these are suppressed by m_e)

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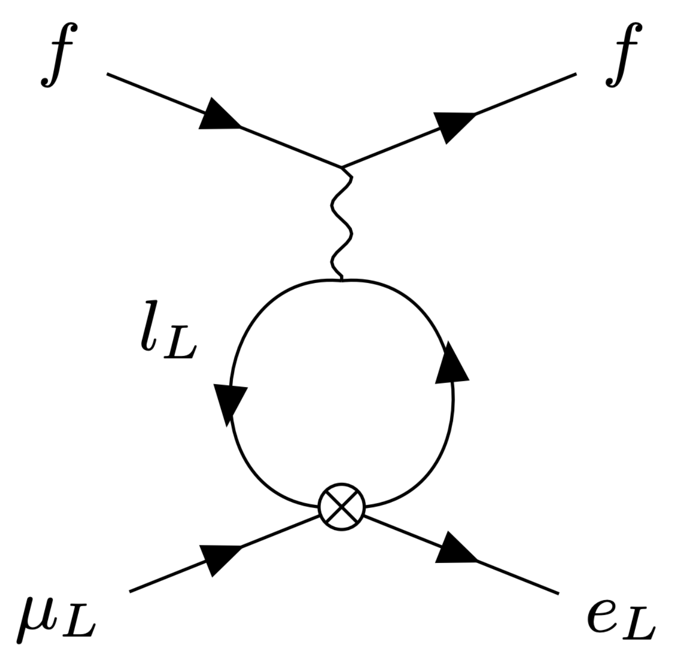

- $\mu A \to e_L A$ and $\mu \to e_L \bar{e}_R e_R$ vectors are proportional (because they come from the same diagram)

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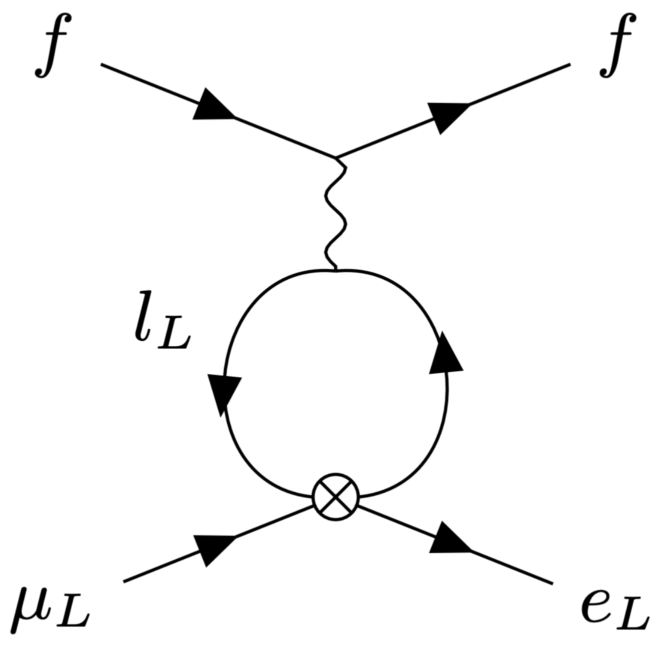
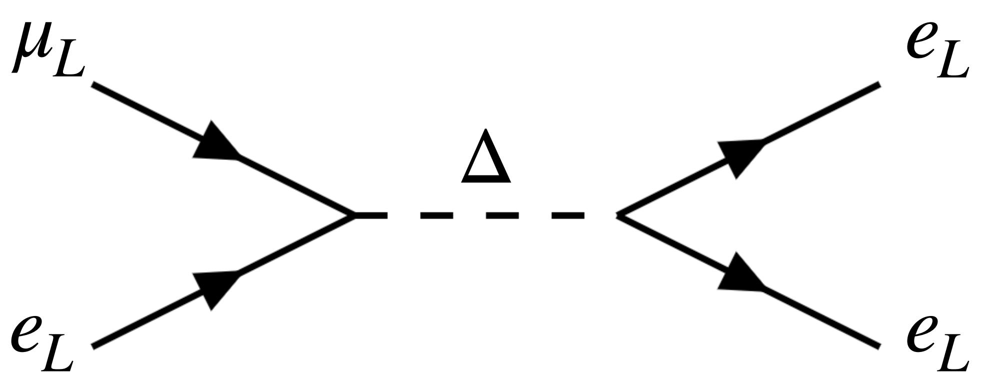


- $\mu A \to e_L A$ and $\mu \to e_L \bar{e}_R e_R$ vectors are proportional (because they come from the same diagram)

- The $C^{e\mu ee}_{V,LL}$ vector is expected to be large because is at tree-level but can also vanish (known for $0\nu2\beta$ that m_{ee} can vanish)



$\propto m_{ee} m^*_{\mu e}$

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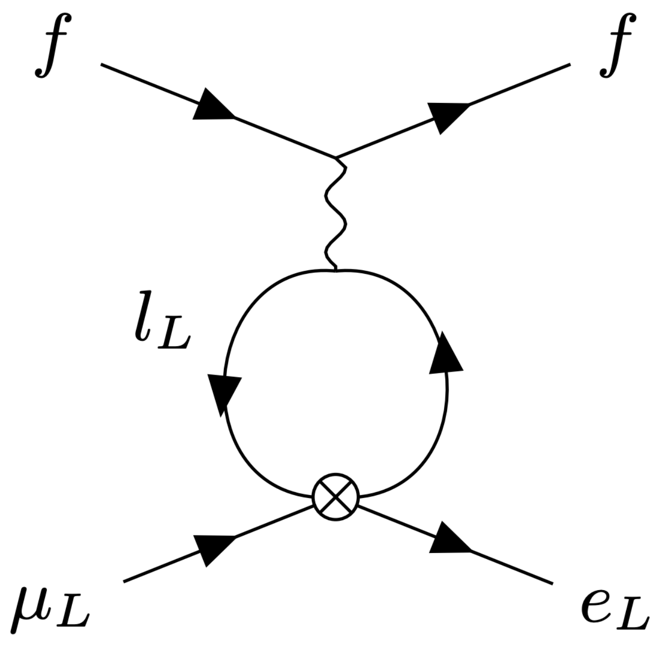

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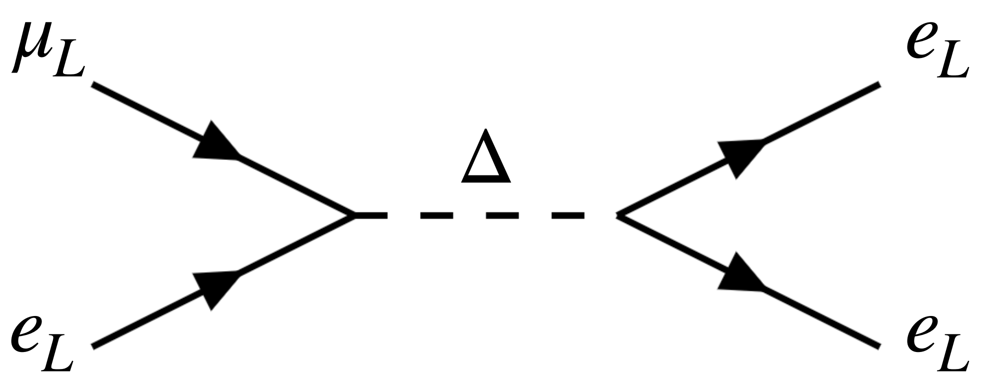
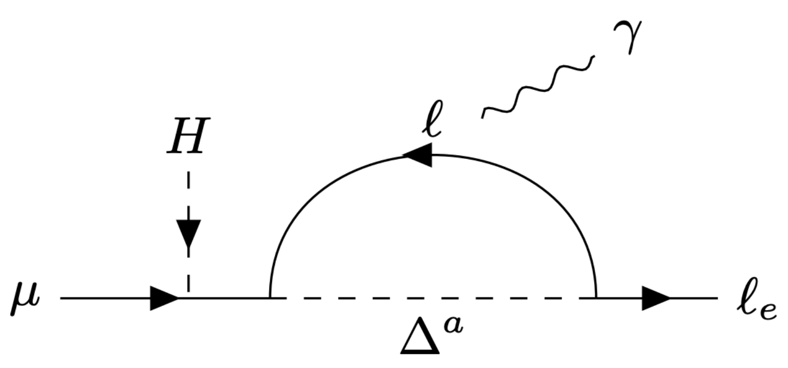


$\propto m_{ee} m_{\mu e}^*$

- Surprisingly also the dipole can be suppressed (although it requires some tuned cancellations and high m_ν — but we want to know what the model cannot do!)



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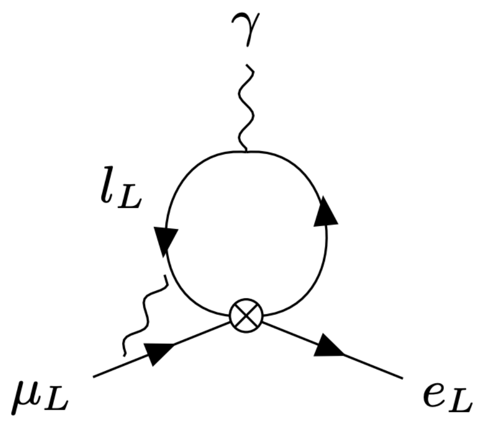

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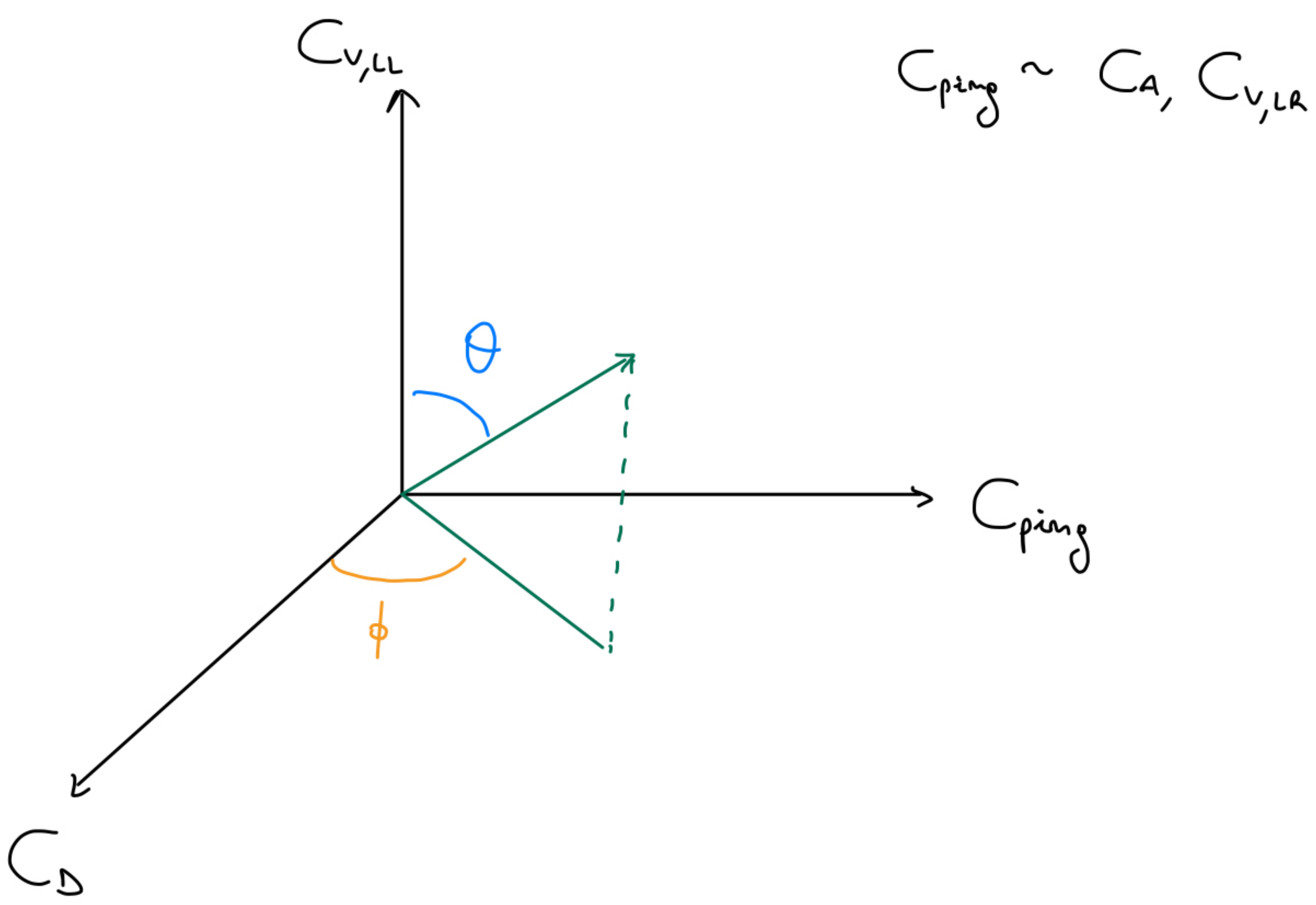
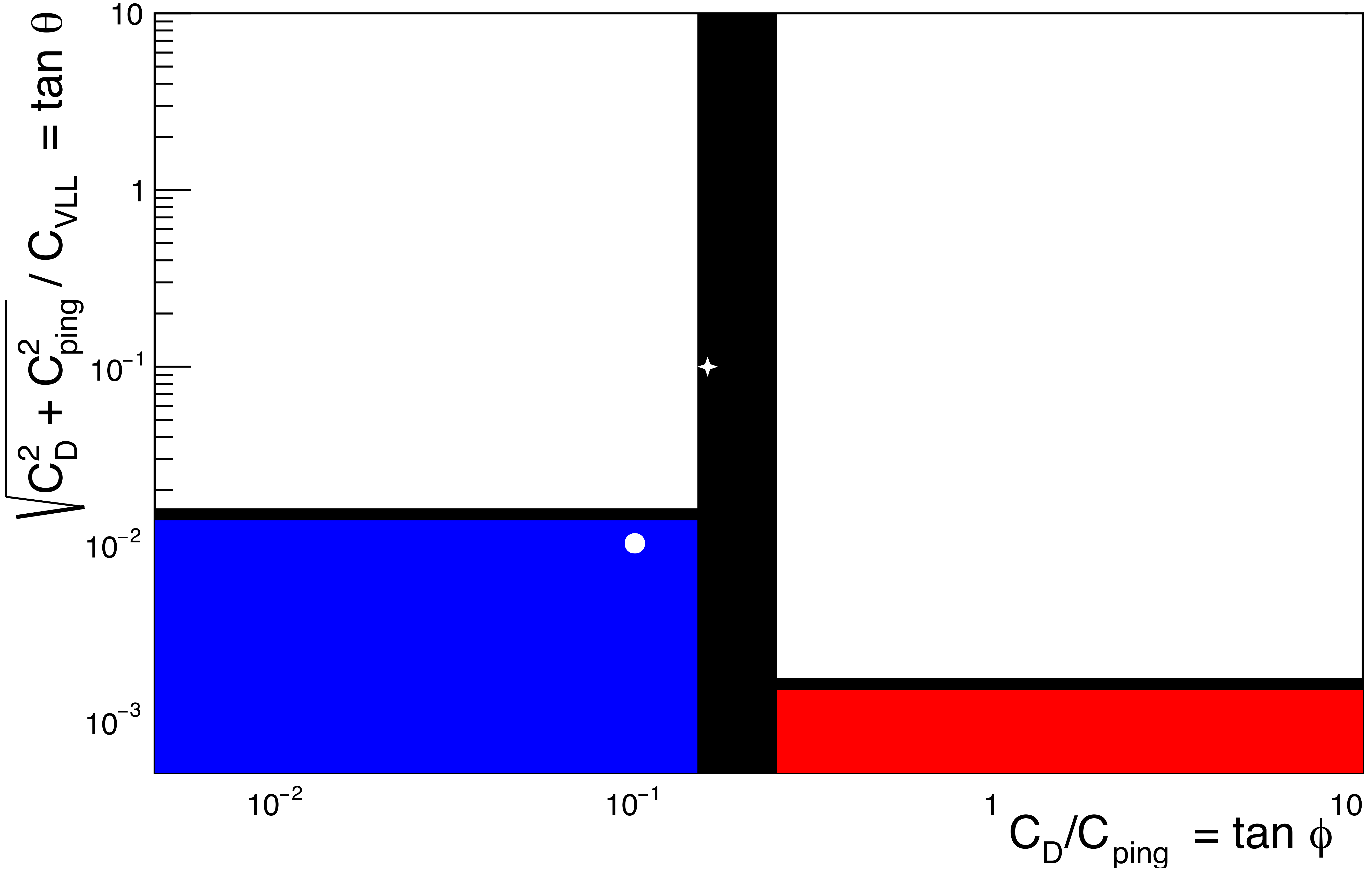
Type-II: where does it live in the ellipse?

- Luckily when coefficients are far from their "natural" values it means that we can say something about the other two





- Any observations outside the colored region can exclude the type-II seesaw!

Conclusion

- Neutrino masses imply leptonic New Physics that must introduce Lepton Flavour Violation
- Global symmetries of the SM are easily violated when new states and interactions are introduced = many models predict LFV
- Many different channels are available to probe a variety of BSM models
- New experiments are coming for LFV channels, and, especially in the $\mu \to e$ sector, they will reach impressive Branching ratio sensitivities
- By assuming heavy LFV physics, we can parametrise $\mu \to e$ data in terms of (in principle) observable EFT coefficients
- Running the EFT from the bottom-up, we identify the region where BSM theories should sit and explore how different models fill this space: if we find regions that are unaccessible we have a way to rule out the model with future experiments

Back-up

Low-energy basis

$$\mathcal{O}^{ll}_{V,YY} = (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{l}\gamma_{\alpha}P_Y l), \quad \mathcal{O}^{ll}_{V,YX} = (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{l}\gamma_{\alpha}P_X l)$$

$$\mathcal{O}^{ll}_{S,YY} = (\bar{e}P_Y\mu)(\bar{l}P_Y l) \qquad \mathcal{O}^{\tau\tau}_{S,YX} = (\bar{e}P_Y\mu)(\bar{\tau}P_X\tau)$$

$$\mathcal{O}^{\tau\tau}_{T,YY} = (\bar{e}\sigma^{\alpha\beta}P_Y\mu)(\bar{\tau}\sigma_{\alpha\beta}P_Y\tau)$$

$$\mathcal{O}^{qq}_{V,YY} = (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{q}\gamma_{\alpha}P_Y q) \quad , \quad \mathcal{O}^{qq}_{V,YX} = (\bar{e}\gamma^{\alpha}P_Y\mu)(\bar{q}\gamma_{\alpha}P_X q)$$

$$\mathcal{O}^{qq}_{S,YY} = (\bar{e}P_Y\mu)(\bar{q}P_Y q) \quad , \quad \mathcal{O}^{qq}_{S,YX} = (\bar{e}P_Y\mu)(\bar{q}P_X q)$$

$$\mathcal{O}^{qq}_{T,YY} = (\bar{e}\sigma^{\alpha\beta}P_Y\mu)(\bar{q}\sigma_{\alpha\beta}P_Y q)$$

$$\mathcal{O}_{D,L} = m_{\mu}\overline{e_R}\sigma^{\alpha\beta}\mu_L F_{\alpha\beta} \qquad m_{\mu}\overline{e_L}\sigma^{\alpha\beta}\mu_R F_{\alpha\beta}$$

$$\mathcal{O}_{GG,Y} = \frac{1}{v}(\bar{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \quad , \quad \mathcal{O}_{G\tilde{G},Y} = \frac{1}{v}(\bar{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta}$$

$$\mathcal{O}_{GGV,Y} = \frac{1}{v^2}(\bar{e}\gamma_{\sigma}P_Y\mu)G_{\alpha\beta}\partial_{\beta}G^{\alpha\sigma} \quad , \quad \mathcal{O}_{G\tilde{G}V,Y} = \frac{1}{v^2}(\bar{e}\gamma_{\sigma}P_Y\mu)G_{\alpha\beta}\partial_{\beta}\widetilde{G}^{\alpha\sigma}$$

$$\mathcal{O}_{FF,Y} = \frac{1}{v}(\bar{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \quad , \quad \mathcal{O}_{F\tilde{F},Y} = \frac{1}{v}(\bar{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

$$\mathcal{O}_{FFV,Y} = \frac{1}{v}(\bar{e}\gamma^{\sigma}P_Y\mu)F^{\alpha\beta}\partial_{\beta}F_{\alpha\sigma} \quad , \quad \mathcal{O}_{F\tilde{F}V,Y} = \frac{1}{v}(\bar{e}\gamma^{\sigma}P_Y\mu)F^{\alpha\beta}\partial_{\beta}\widetilde{F}_{\alpha\sigma}$$

where $l \in \{e, \mu\}, q \in \{u, d, s, c, b\}$

$\mu \to e$ Rates

$$BR(\mu \to e\gamma) = 384\pi^2(\; C_{DL}^{e\mu} \; ^2 + \; C_{DR}^{e\mu} \; ^2)$$

$$BR(\mu \to e\bar{e}e) = \frac{\; C_{S,LL}^{e\mu ee} \; ^2 + \; C_{S,RR}^{e\mu ee} \; ^2}{8} + 2 \; C_{V,RR}^{e\mu ee} + 4eC_{D,L}^{e\mu} \; ^2 + 2 \; C_{V,LL}^{e\mu ee} + 4eC_{D,R}^{e\mu} \; ^2$$

$$+(64 \ln \frac{m_\mu}{m_e} - 136)(\; eC_{D,R}^{e\mu} \; ^2 + \; eC_{D,L}^{e\mu} \; ^2) + \; C_{V,RL}^{e\mu ee} + 4eC_{D,L}^{e\mu} \; ^2 + \; C_{V,LR}^{e\mu ee} + 4eC_{D,R}^{e\mu} \; ^2$$

$$BR_{SI}(\mu A \to eA) = B_A(\; d_A C_{DR}^{e\mu} + C_{A,L} \; ^2 + \; d_A C_{DL}^{e\mu} + C_{A,R} \; ^2)$$

Type-II coefficients

- We list here the EFT coefficients in the type-II seesaw

$$C_{DR}^{e\mu} = \frac{3e}{128\pi^2}\Big[\frac{[m_\nu m_\nu^\dagger]_{e\mu}}{\lambda_H^2 v^2}\Big(1 + \frac{32}{27}\frac{\alpha_e}{4\pi}\ln\frac{M_\Delta}{m_\tau}\Big) + \frac{116\alpha_e}{27\pi}\ln\frac{m_\tau}{m_\mu}\sum_{\alpha\in e\mu}\frac{[m_\nu]_{\mu\alpha}[m_\nu^*]_{e\alpha}}{\lambda_H^2 v^2}\Big]$$

$$v = 174\ \mathrm{GeV}$$

$$C_{V,LL}^{e\mu ee} = \frac{[m_\nu^*]_{\mu e}[m_\nu]_{ee}}{2\lambda_H^2 v^2} + \frac{\alpha_e}{3\pi\lambda_H^2 v^2}\left[m_\nu^\dagger \ln\left(\frac{M_\Delta}{m_\alpha}\right) m_\nu\right]_{\mu e}$$

$$C_{V,LR}^{e\mu ee} = \frac{\alpha_e}{3\pi\lambda_H^2 v^2}\left[m_\nu^\dagger \ln\left(\frac{M_\Delta}{m_\alpha}\right) m_\nu\right]_{\mu e}$$