TECHNISCHE UNIVERSITÄT MÜNCHEN

XYZ Exotics Theory

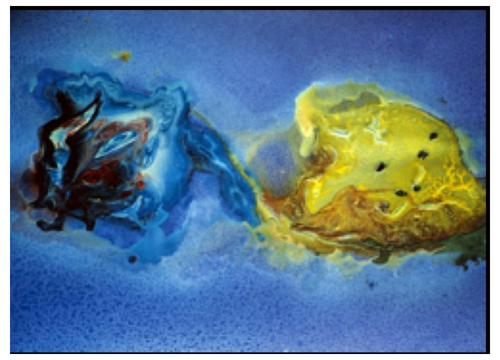
Nora Brambilla

T30f

TUMQCD LATTICE Collaboration

STRONG 2020

TUM-IAS Focus Group Physics with Effective Field Theories



Quark Confinement and the Hadron Spectrum since 1994

MCQST

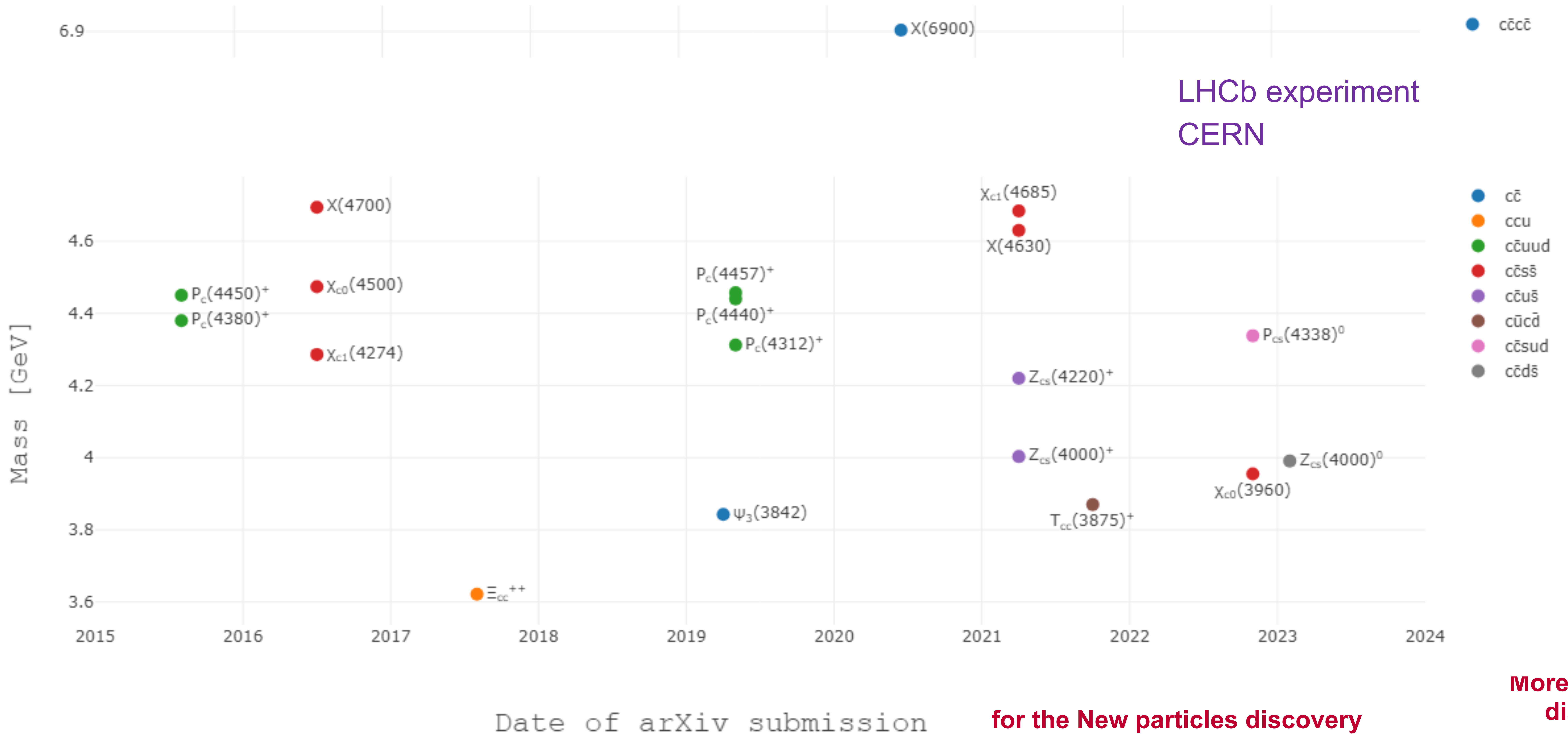
MDSI

Munich Data Science Institute

ORIGINS Excellence Cluster

We are currently through a new revolution in particle physics

The present revolution: new particles discoveries beyond the Quark Model



LHCb experiment
CERN

for the New particles discovery

More new particles discovered at

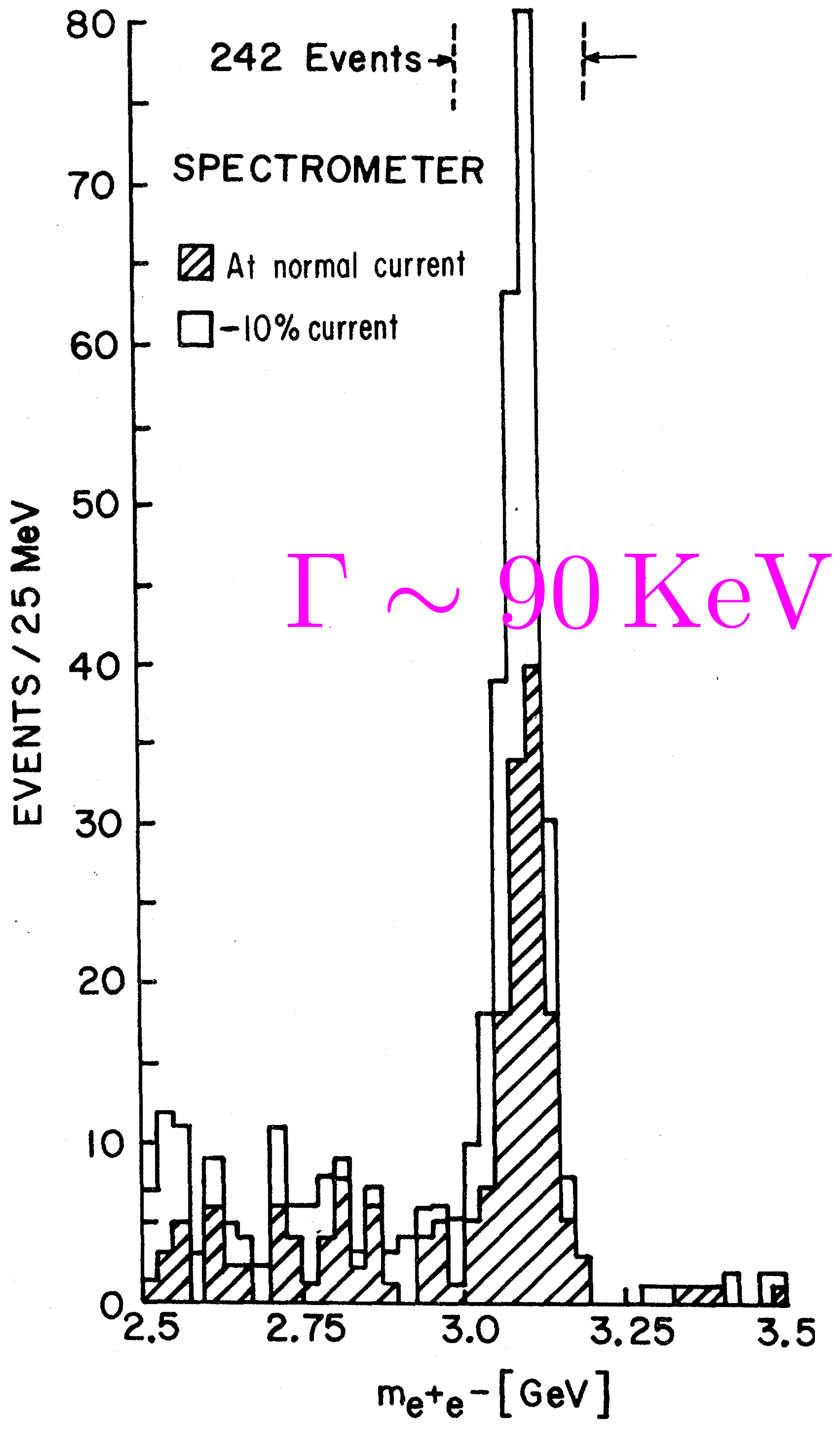
Plots for each experiment available at our

https://qwg.ph.nat.tum.de/exoticshub/

Past Revolution

The discovery of the J/psi (ccbar lowest state) is at the origin of the November revolution in 1974

Samuel Ting: It was like to stumble in a village where people were living 70000 years



Aubert et al. BNL 74

- Discovery of the first quark of heavy type Q (m_c > Lambda_QCD)
- Confirmation of the quark model and QCD

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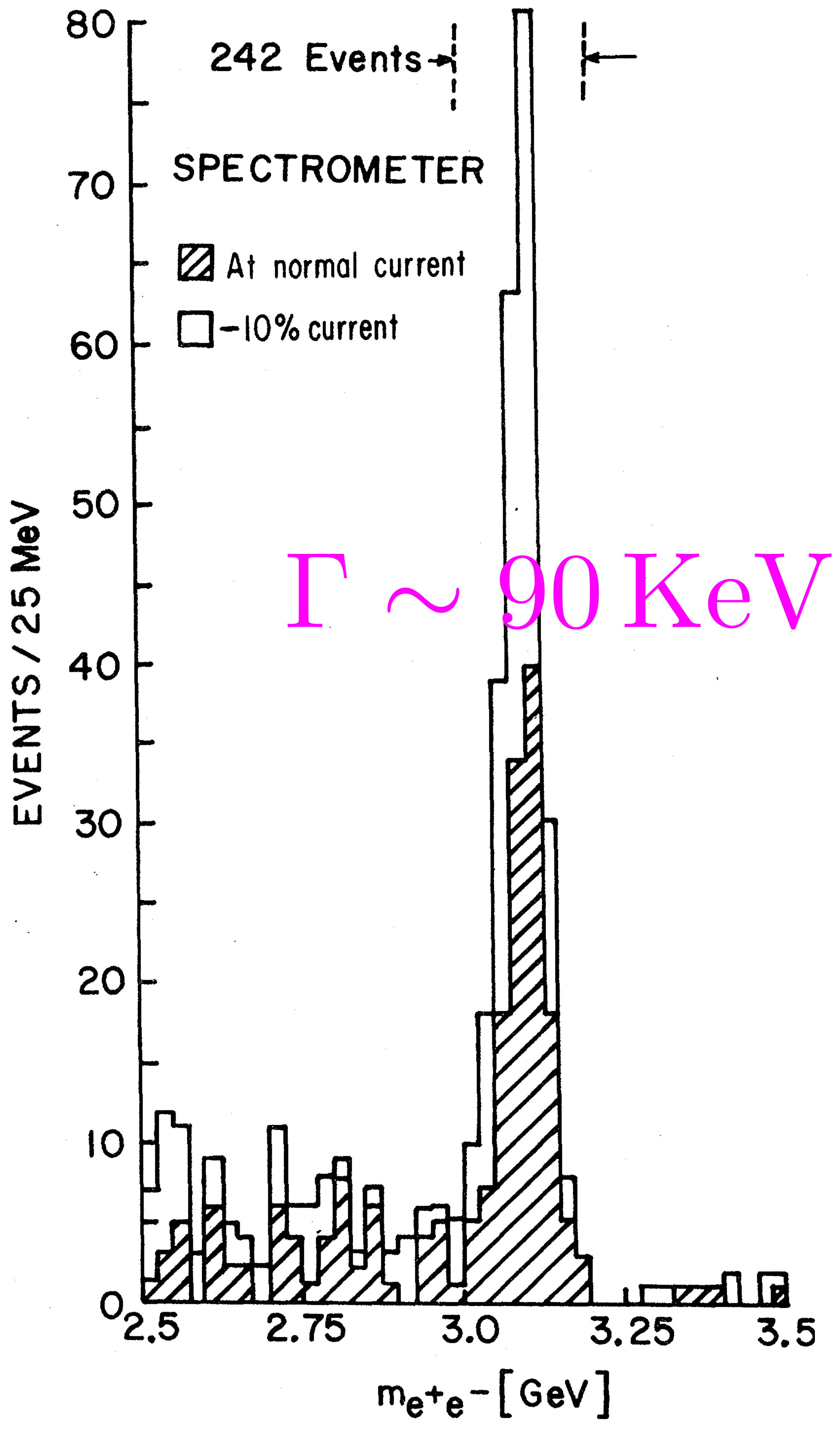

$\Gamma \sim 90\,\mathrm{KeV}$

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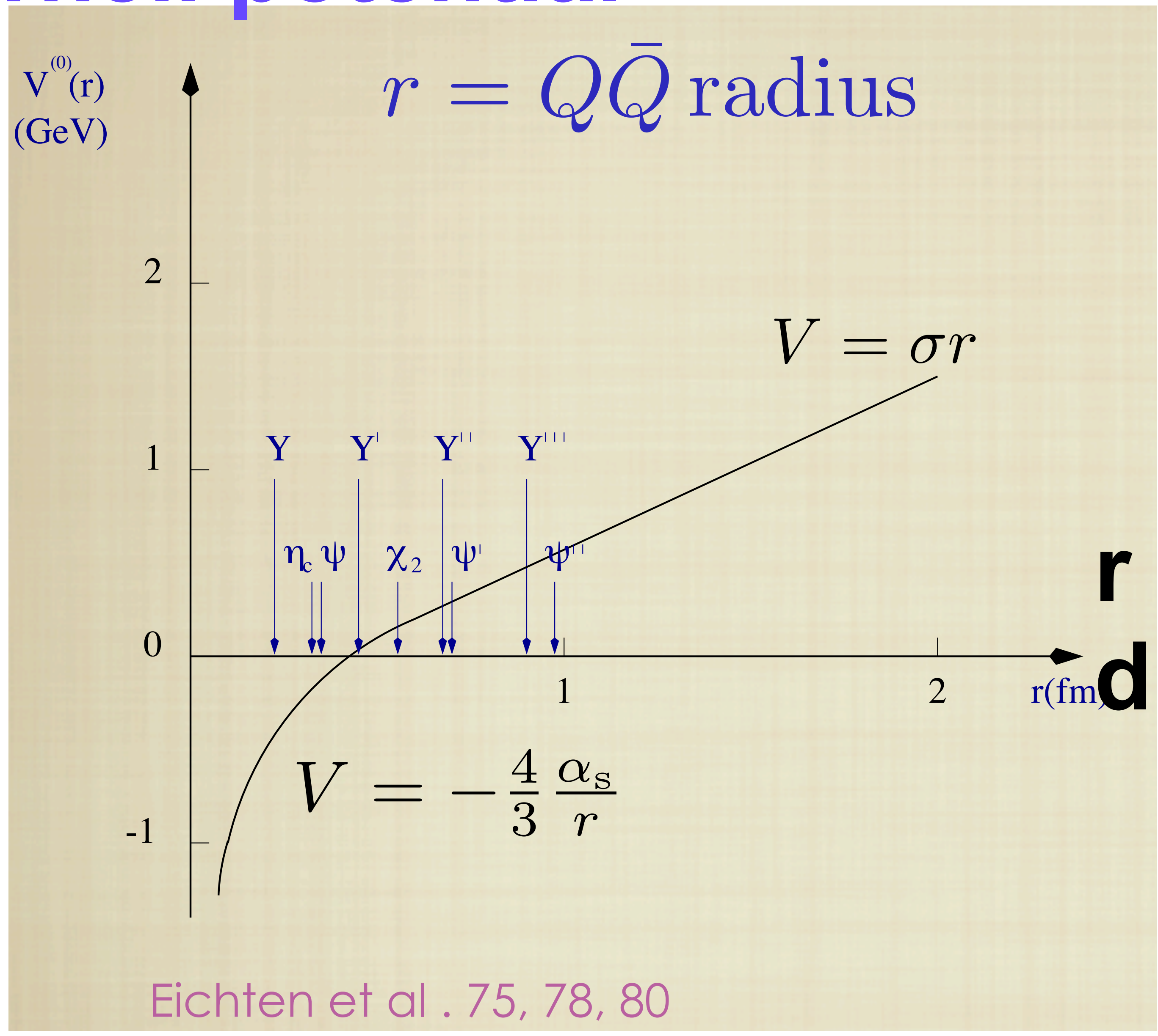
—>**narrow width and asymptotic freedom:** annihilation at large scale controlled by a small coupling constant $\alpha_s(2m_c) \ll 1$

—>**energy levels and confinement:** structure of levels controlled by a Cornell potential in a Schroedinger eq.

Cornell potential



r QQbar distance

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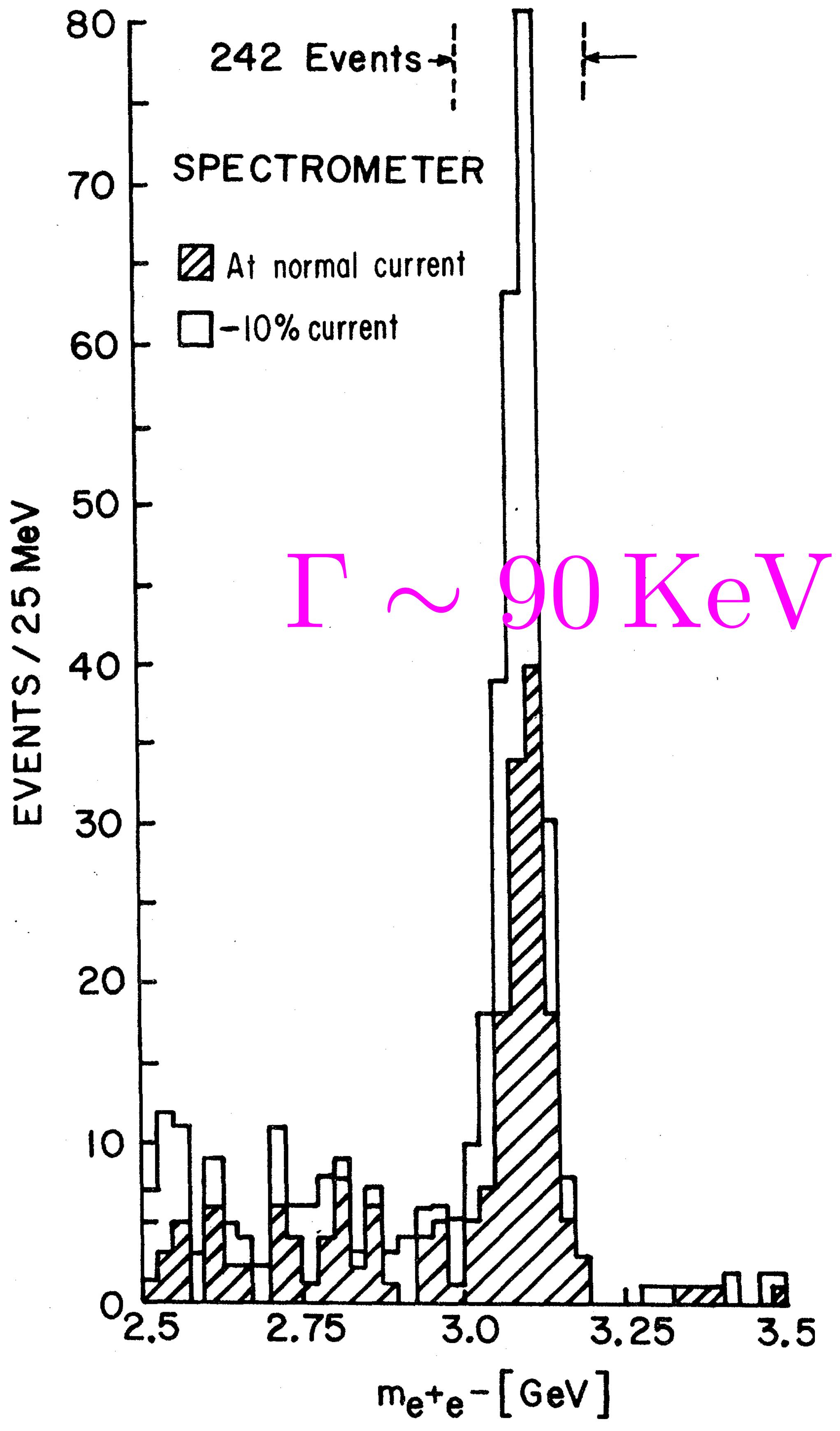

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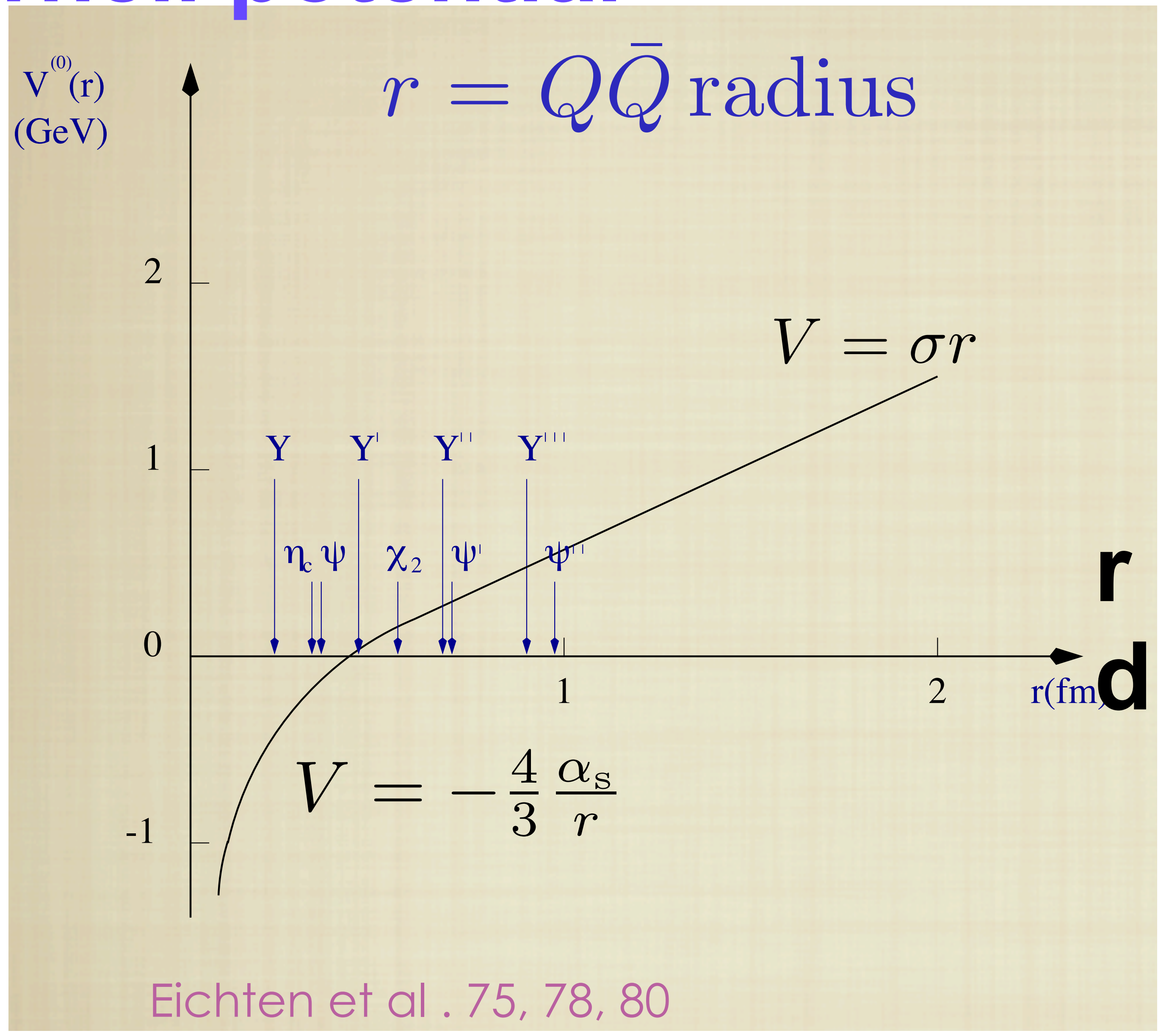

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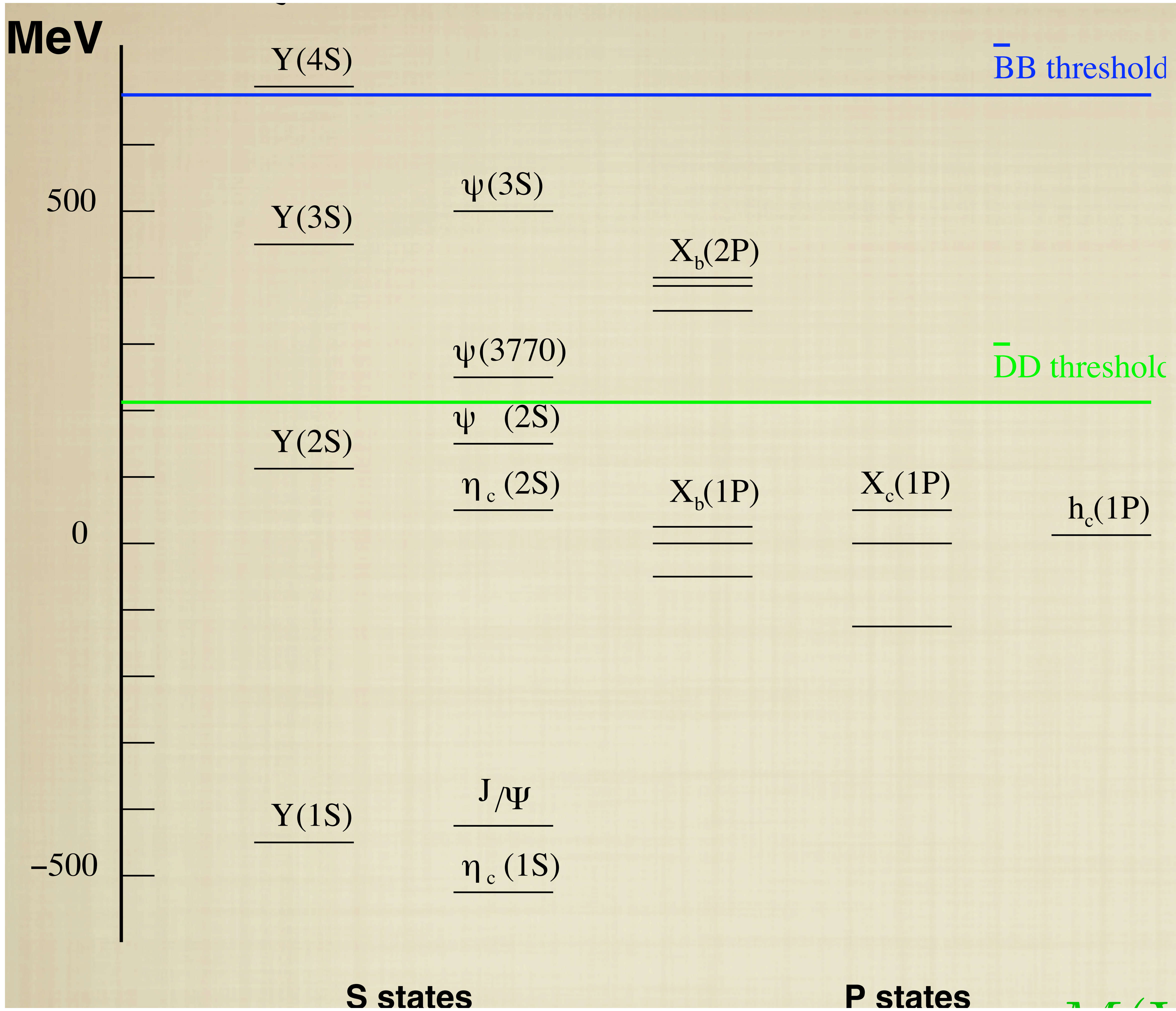
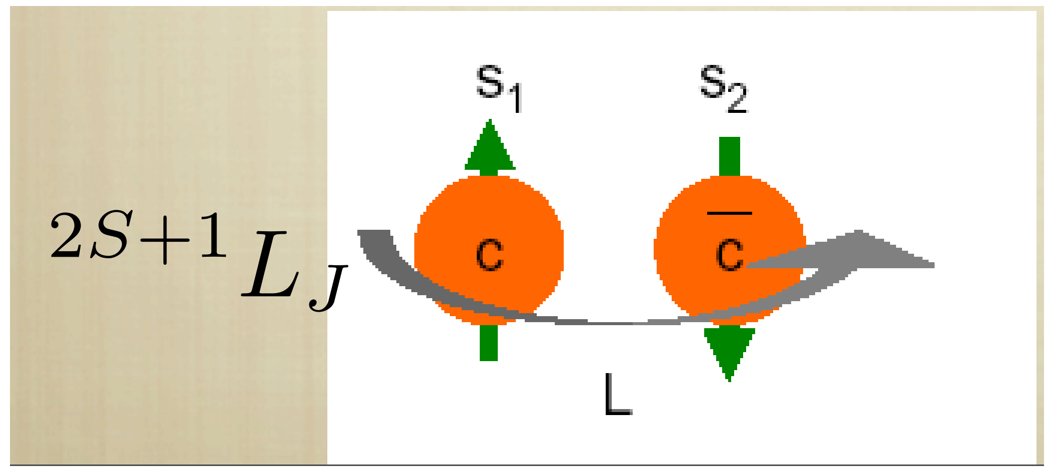
Heavy quarkonia are nonrelativistic systems:multiscale systems

Many scales: a challenge and an opportunity

Quarkonium scales



Levels normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$



$M(Y(1S)) = 9460\,\mathrm{GeV}$

$M(J/\Psi) = 3097\,\mathrm{GeV}$

The mass scale is perturbative

$$m_Q \gg \Lambda_{\mathrm{QCD}}$$

$$m_b \simeq 5\,\mathrm{GeV};\, m_c \simeq 1.5\,\mathrm{GeV}$$

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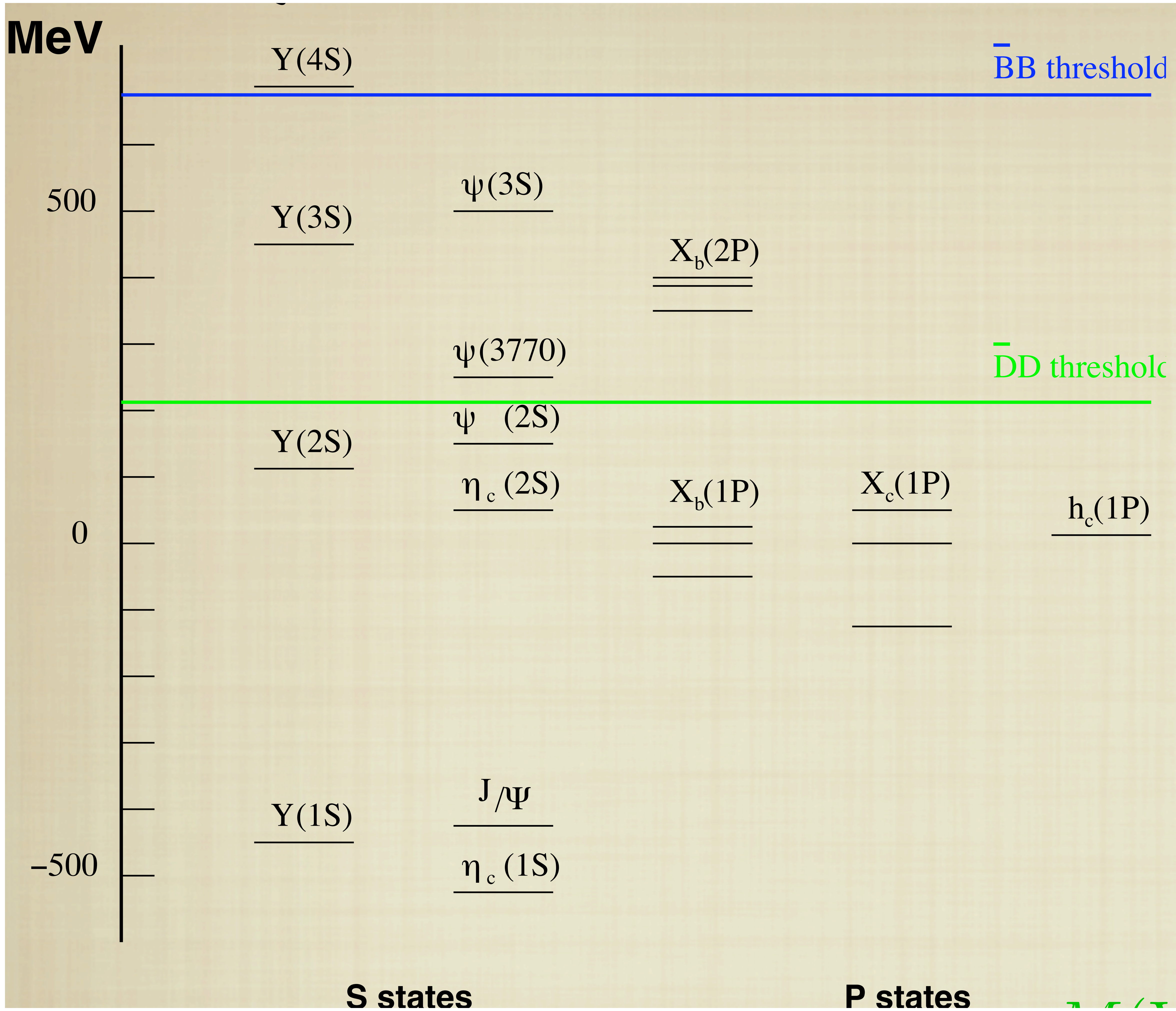

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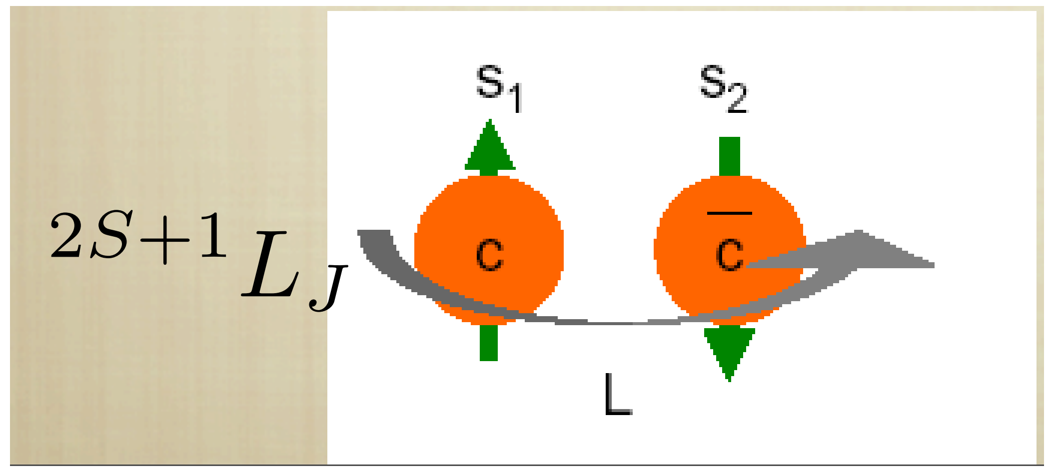

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$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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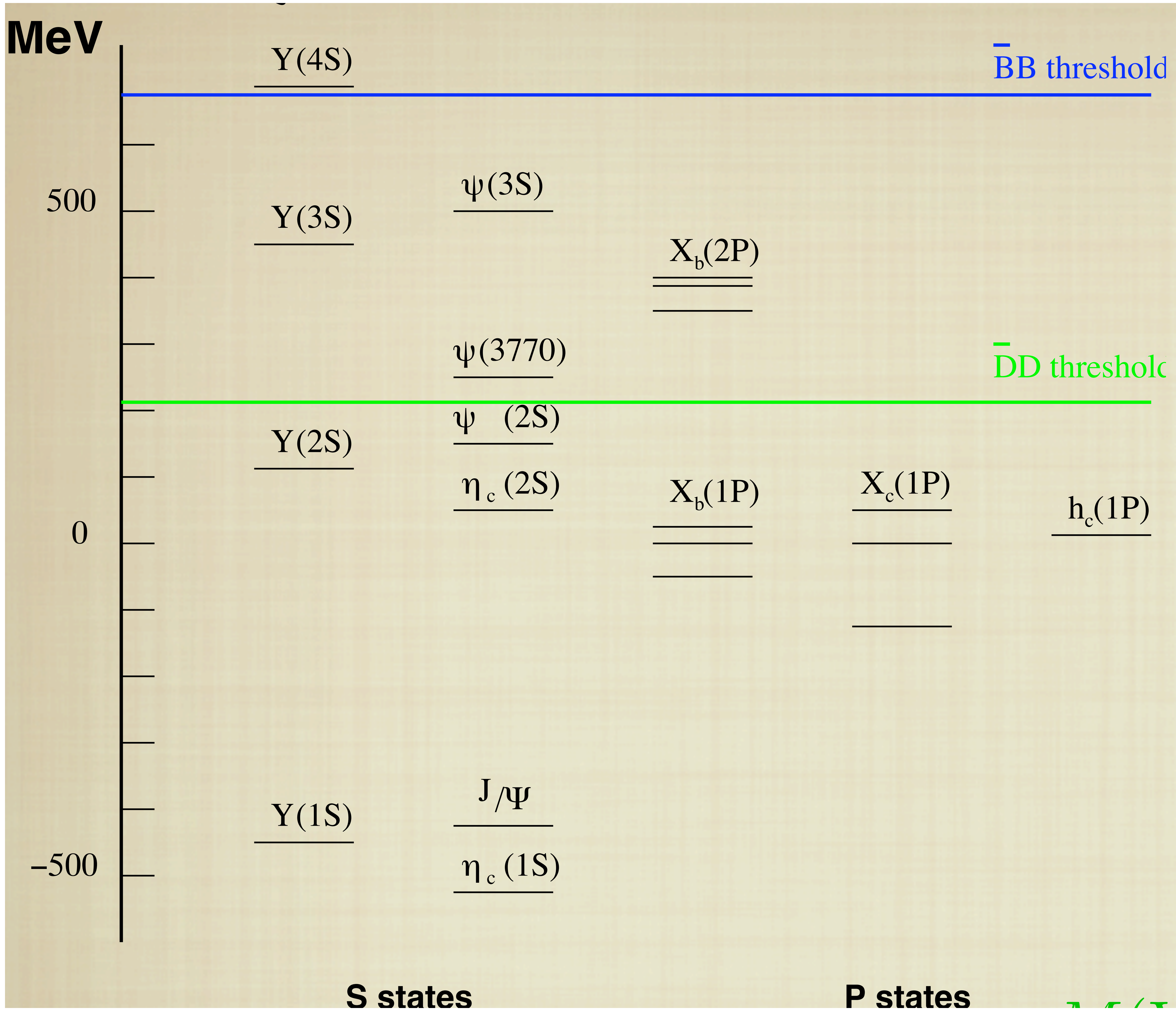

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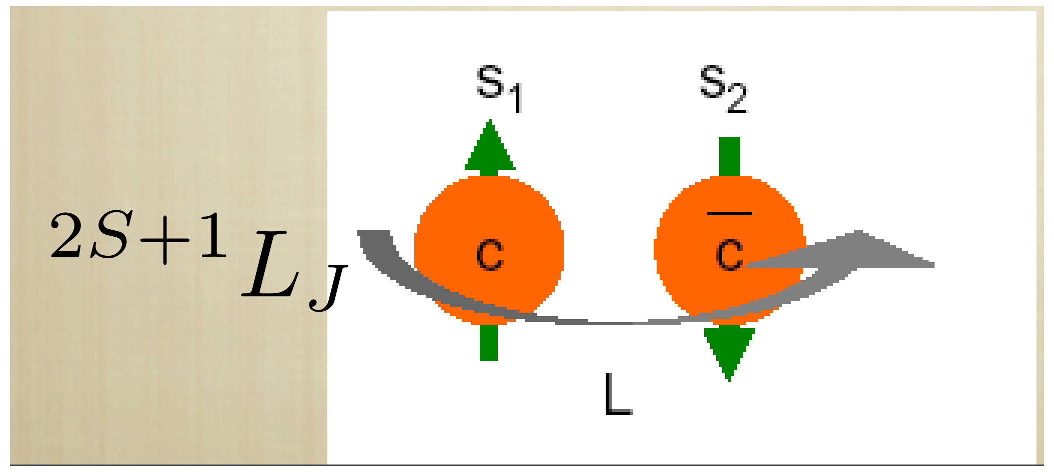

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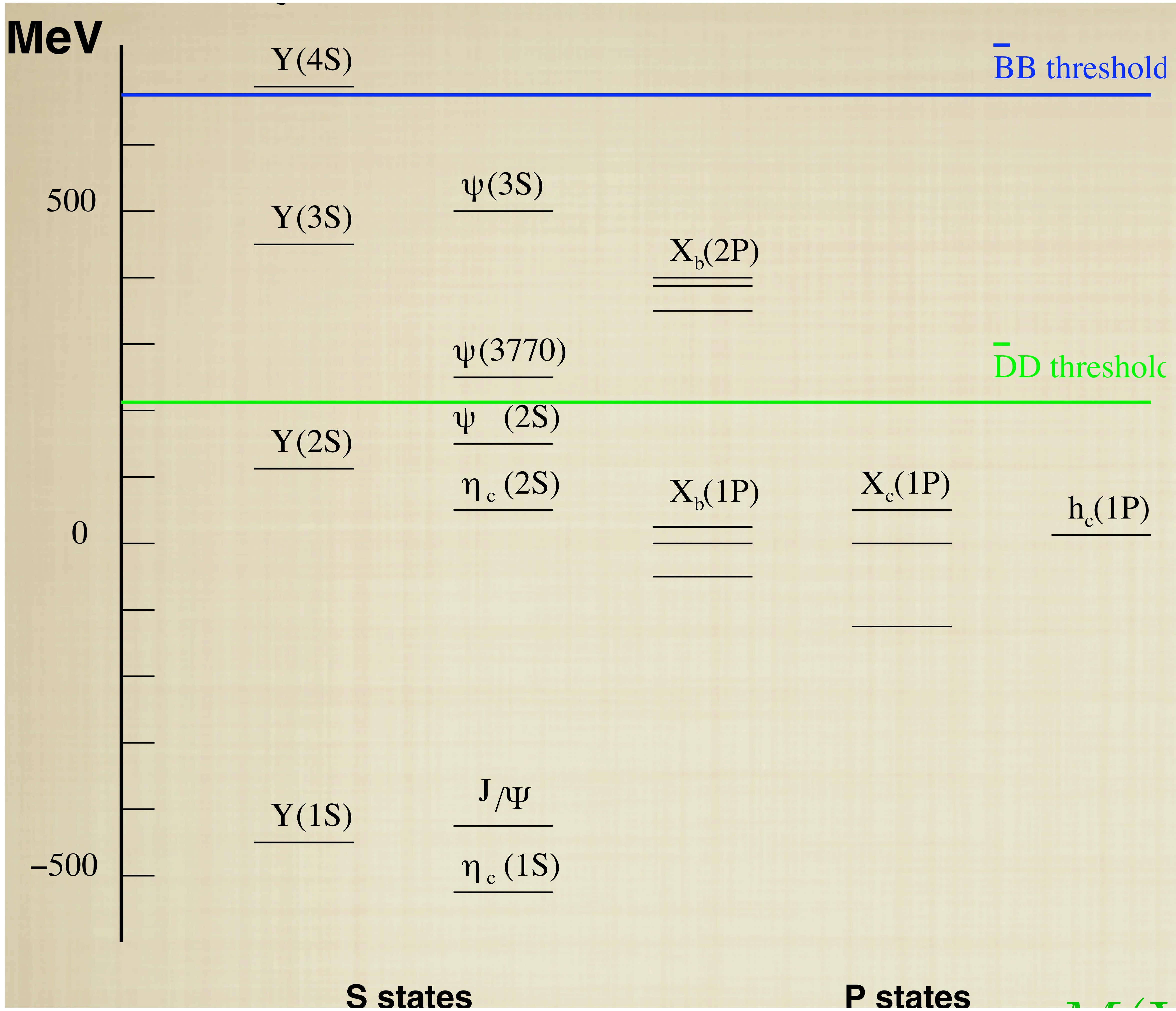

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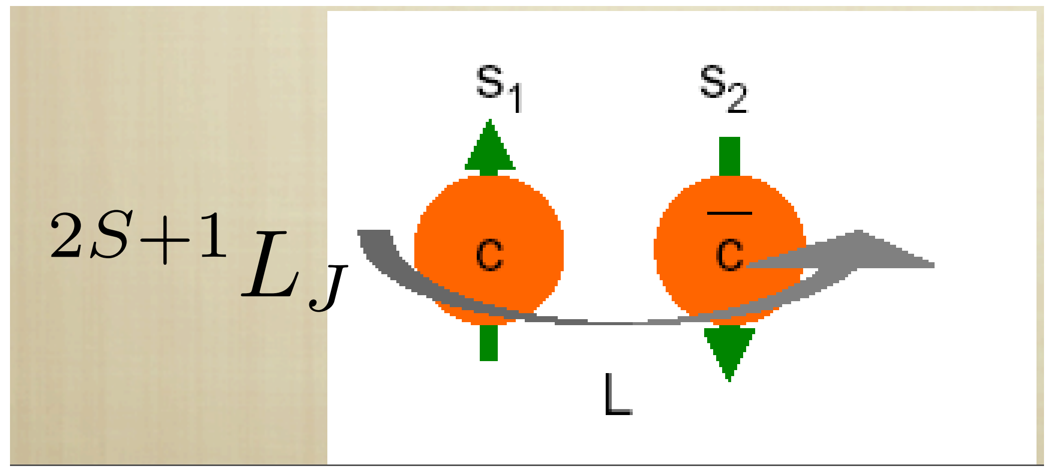

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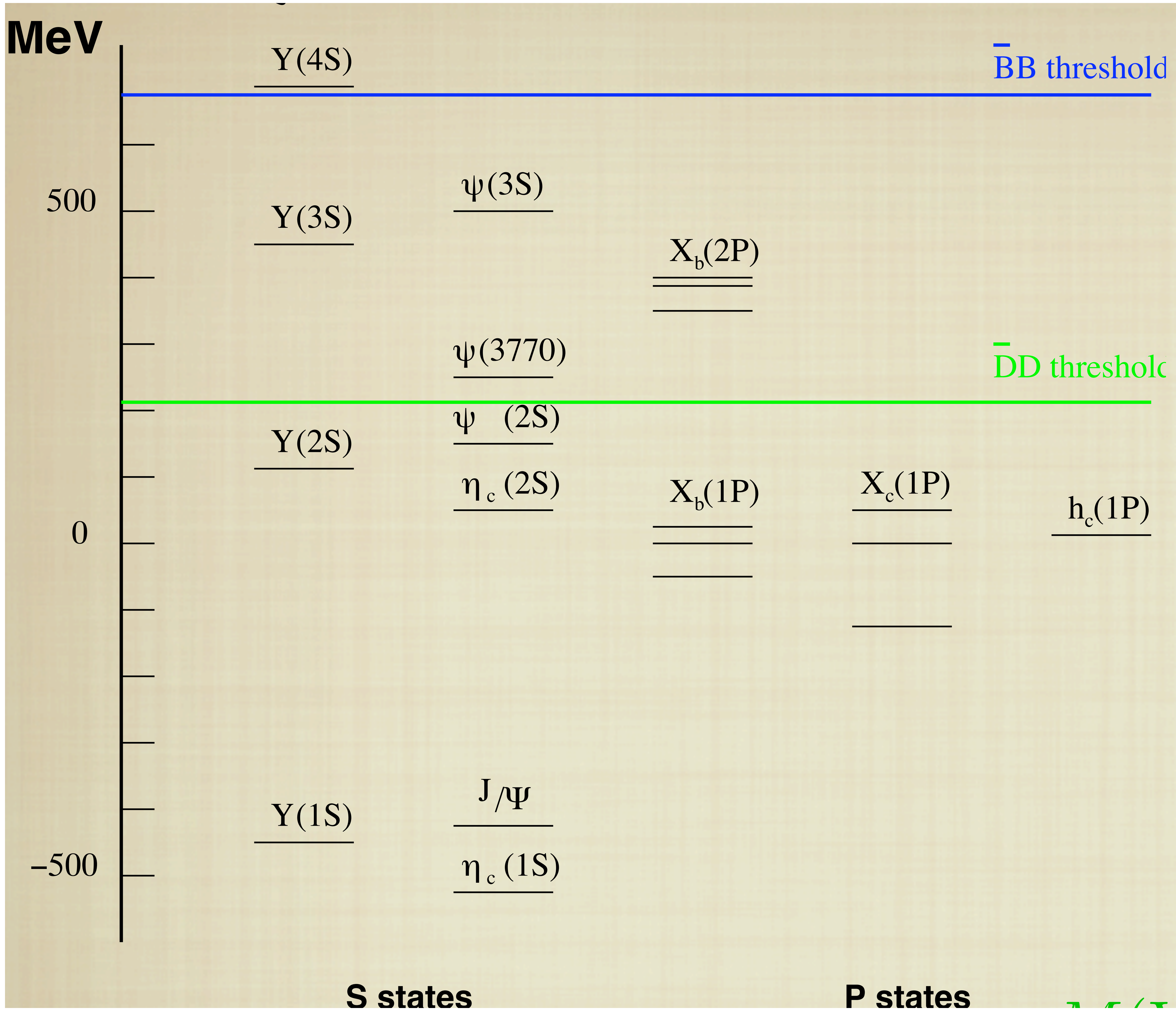

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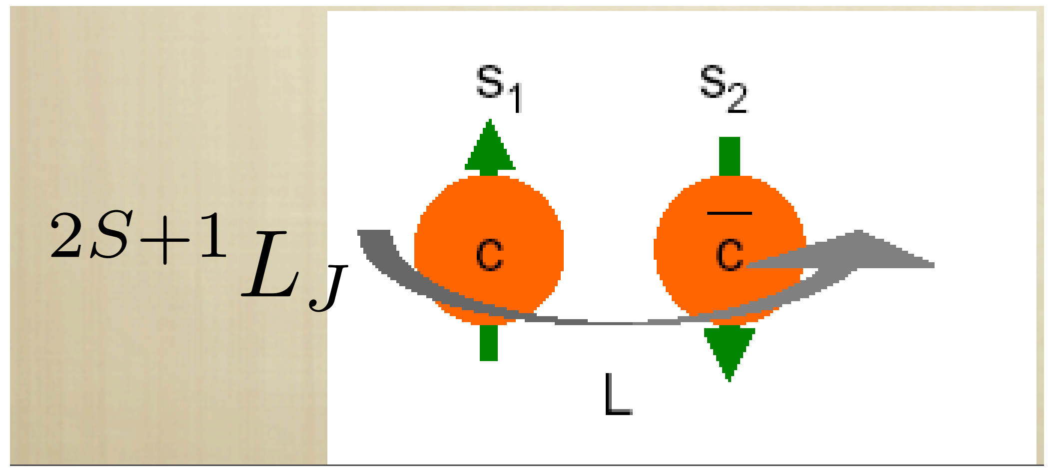

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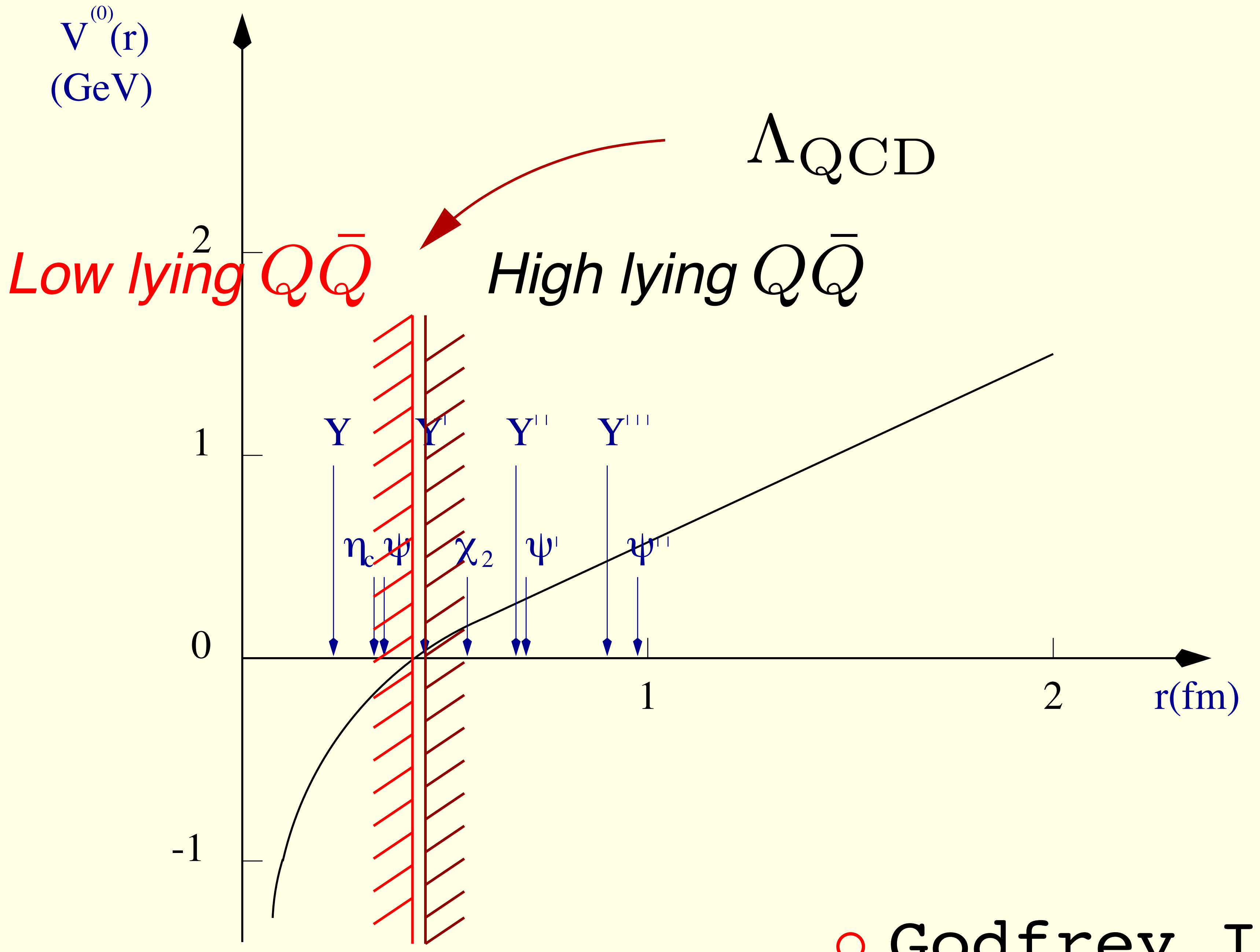
The rich structure of separated energy scales makes QQbar systems an ideal probe of strong interactions

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.



○ Godfrey Isgur PRD 32(85)189

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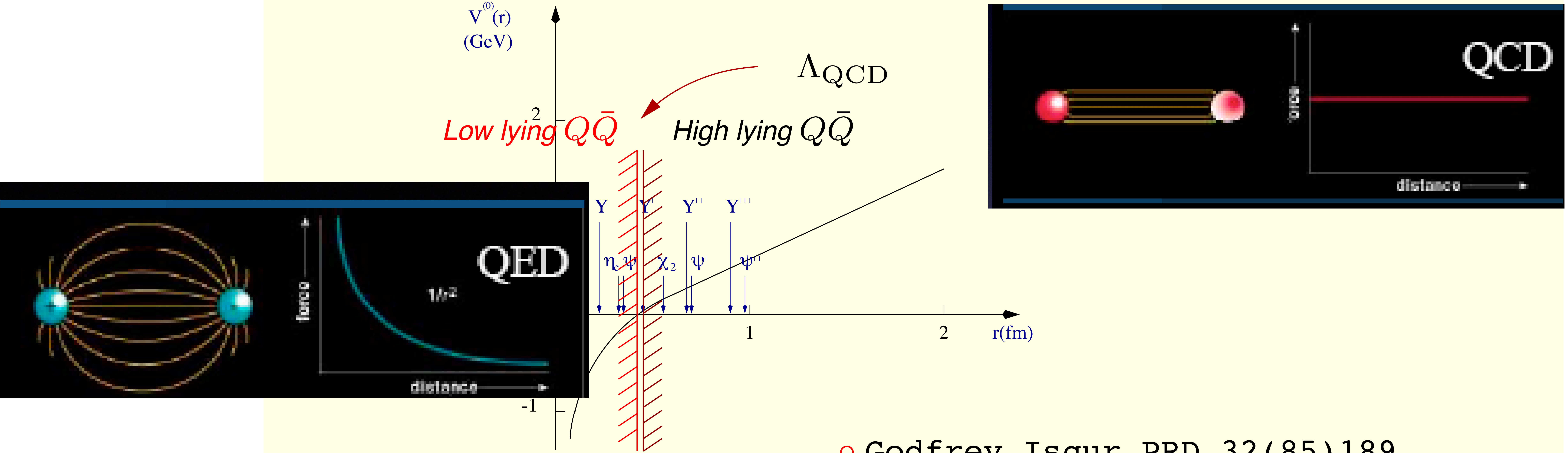

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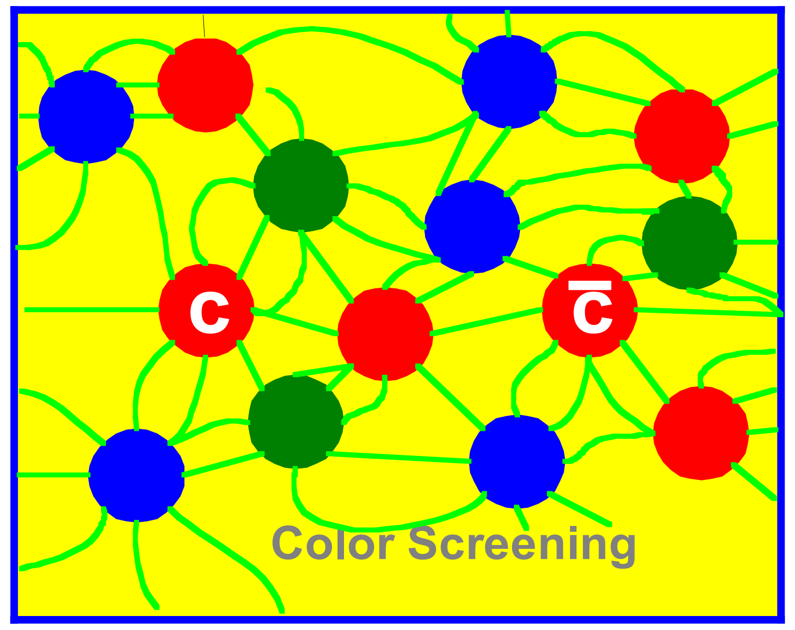
The present revolutions: nuclear matter phase diagram

Quarkonia are probe of QGP formation

Matsui Satz 1986 idea of **color screening in medium**

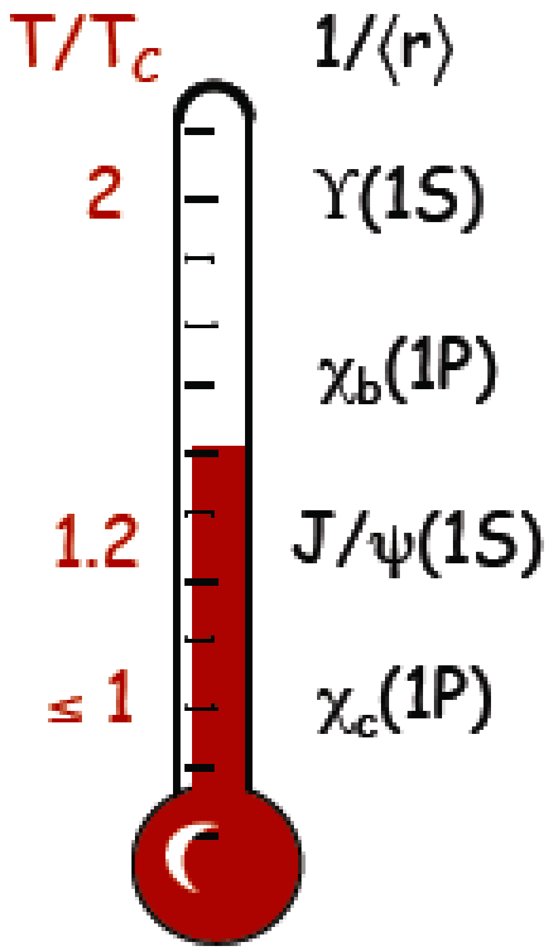


Debye charge screening

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$m_D \sim gT$$

$$r \sim \frac{1}{m_D}$$ → Bound state dissolve



Sequential Melting at different Temperature

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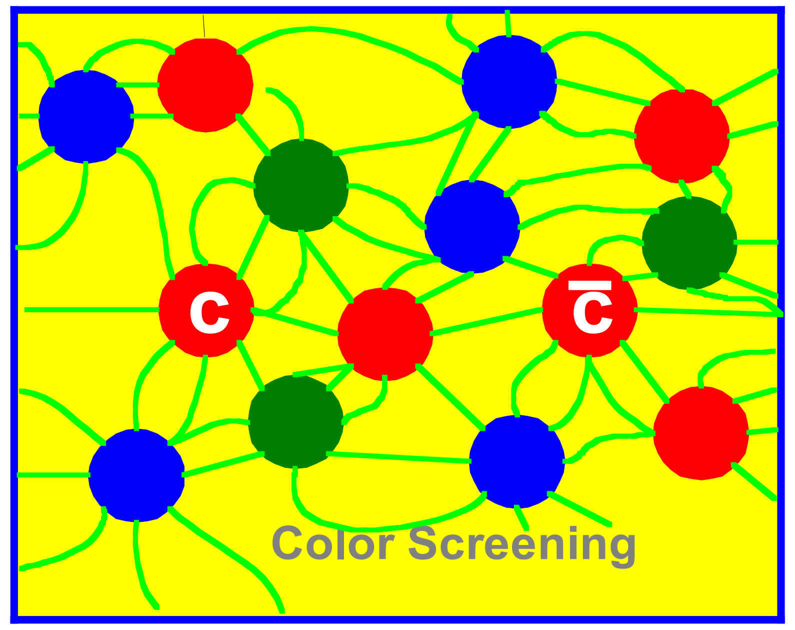

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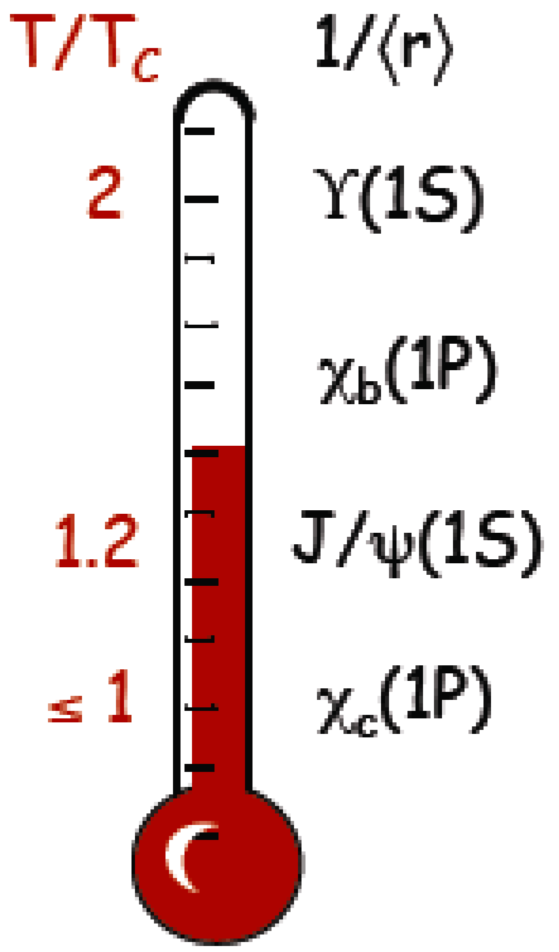

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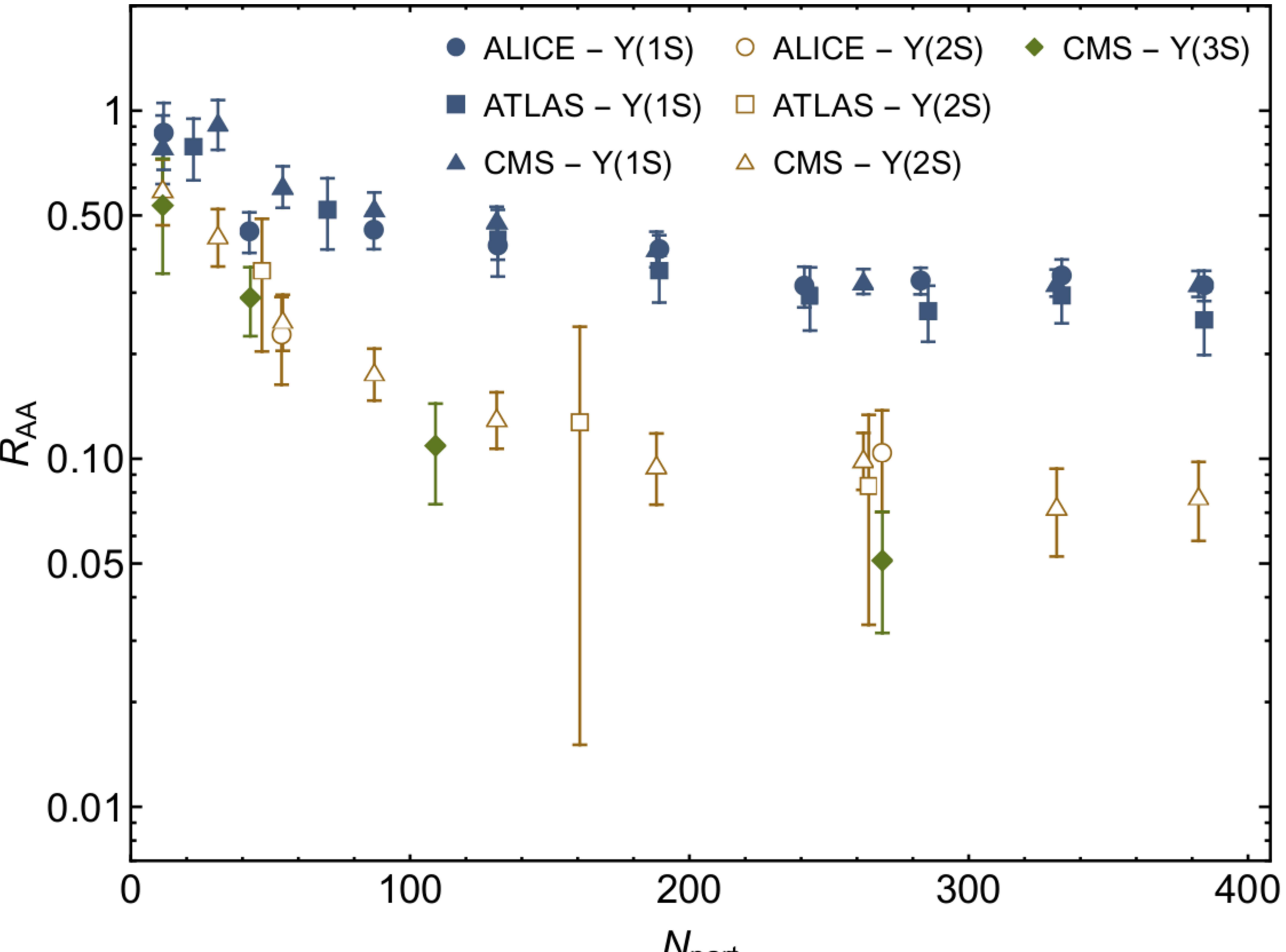
Experimental measurements:

R_{AA} is the nuclear modification factor = yield of quarkonium in PbPb / yield in pp.



CMS PLB 790 (2019) 270
ALICE PLB 822 (2021) 136579
ATLAS PRC 107 (2023) 054912

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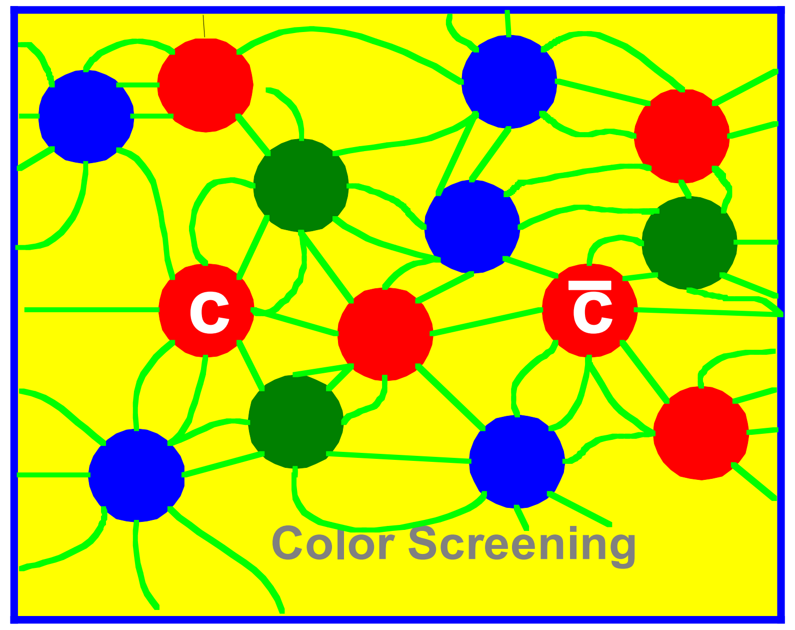

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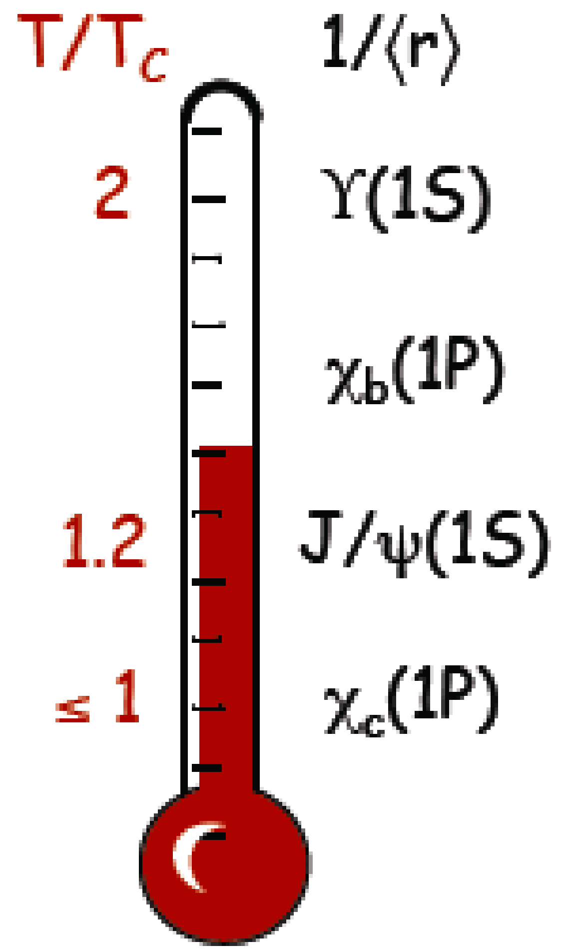

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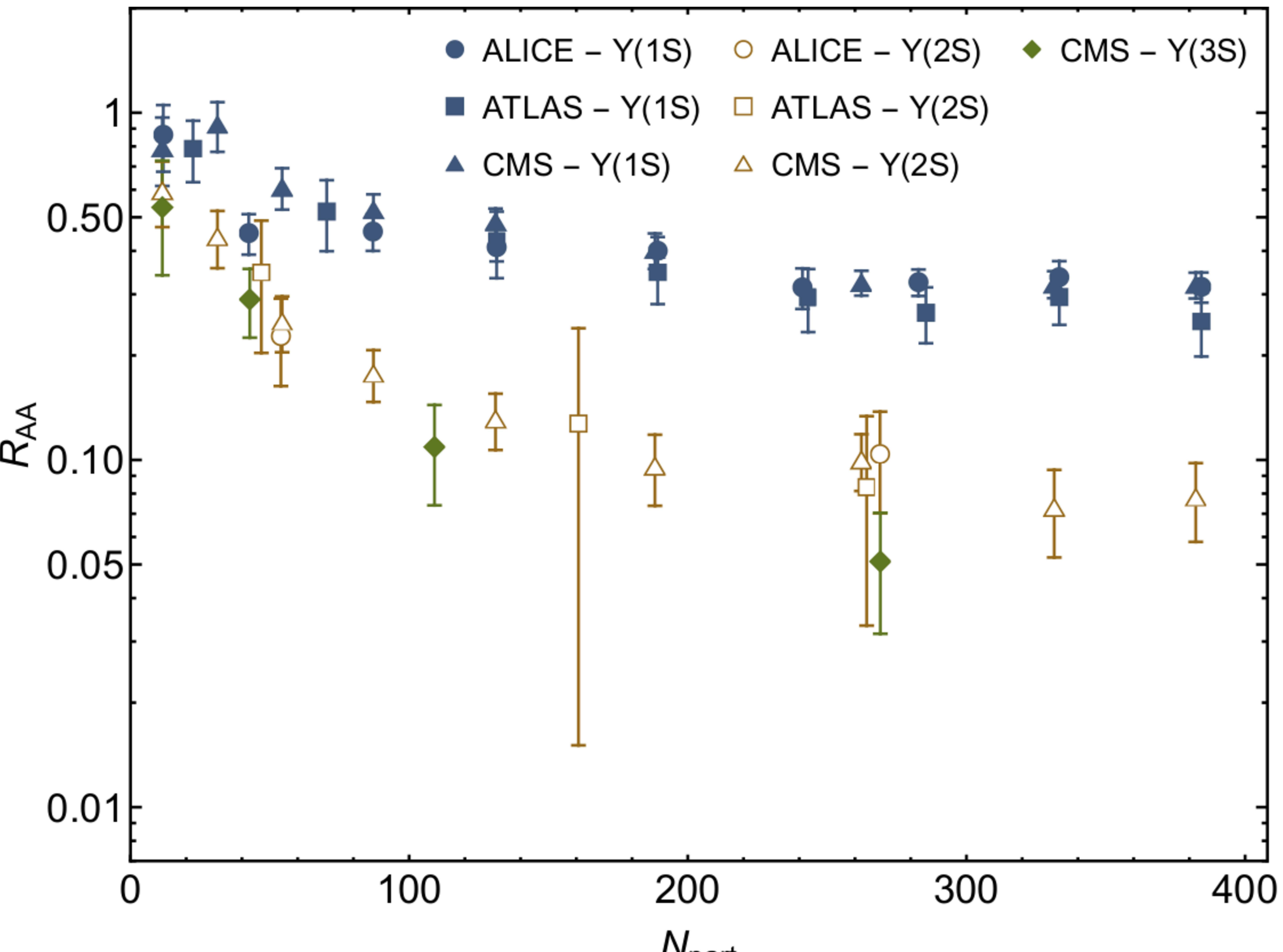

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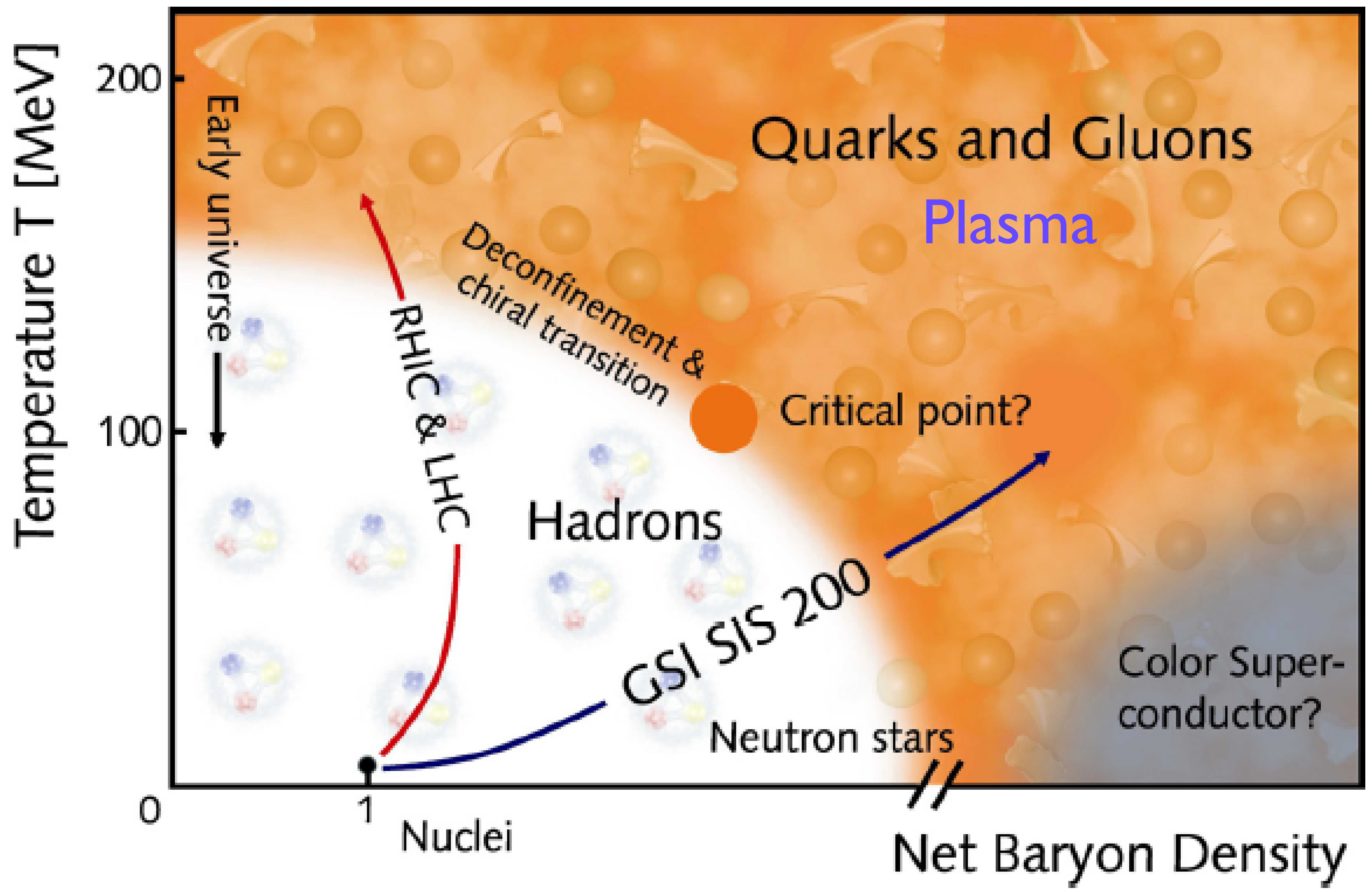
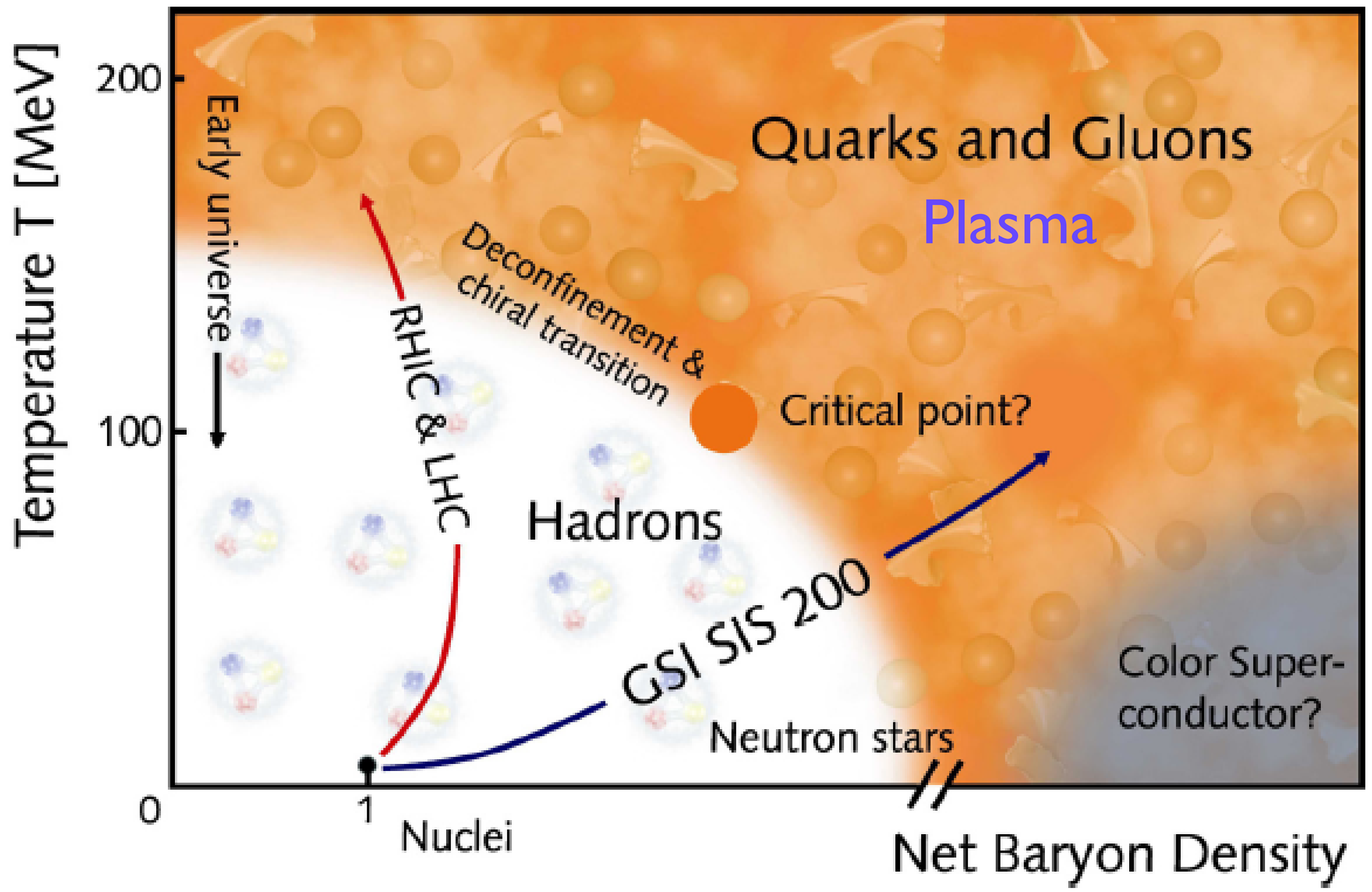
Today a new paradigm emerged **beyond screening** relating the R_AA to the **nonequilibrium evolution of the heavy pair in medium:** medium induced dissociation and color singlet/octet recombination. **Quantum phenomenon to be addresses with quantum master equations**

The present revolutions: nuclear matter phase diagram



Nuclear Matter

The present revolutions: nuclear matter phase diagram



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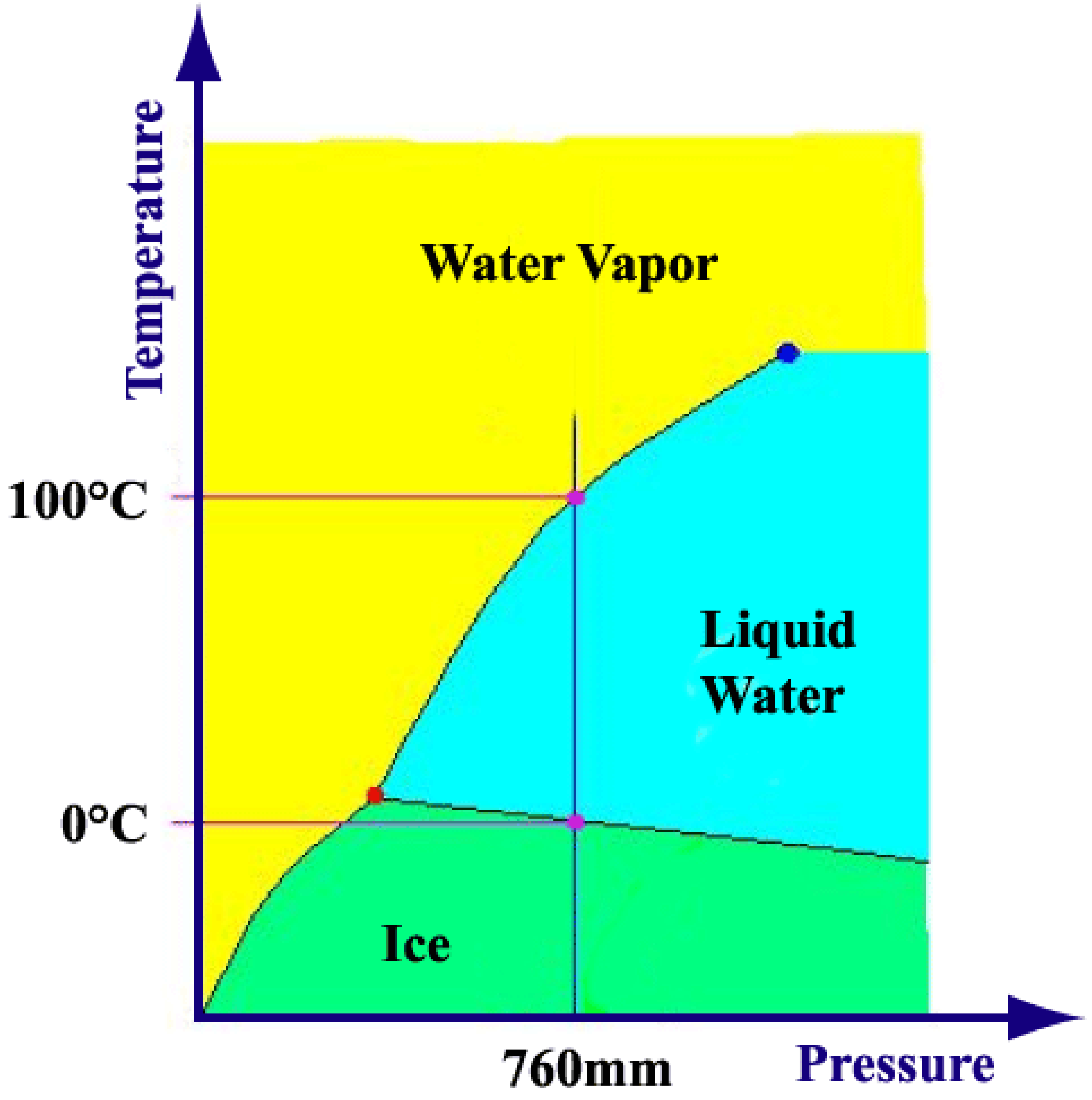

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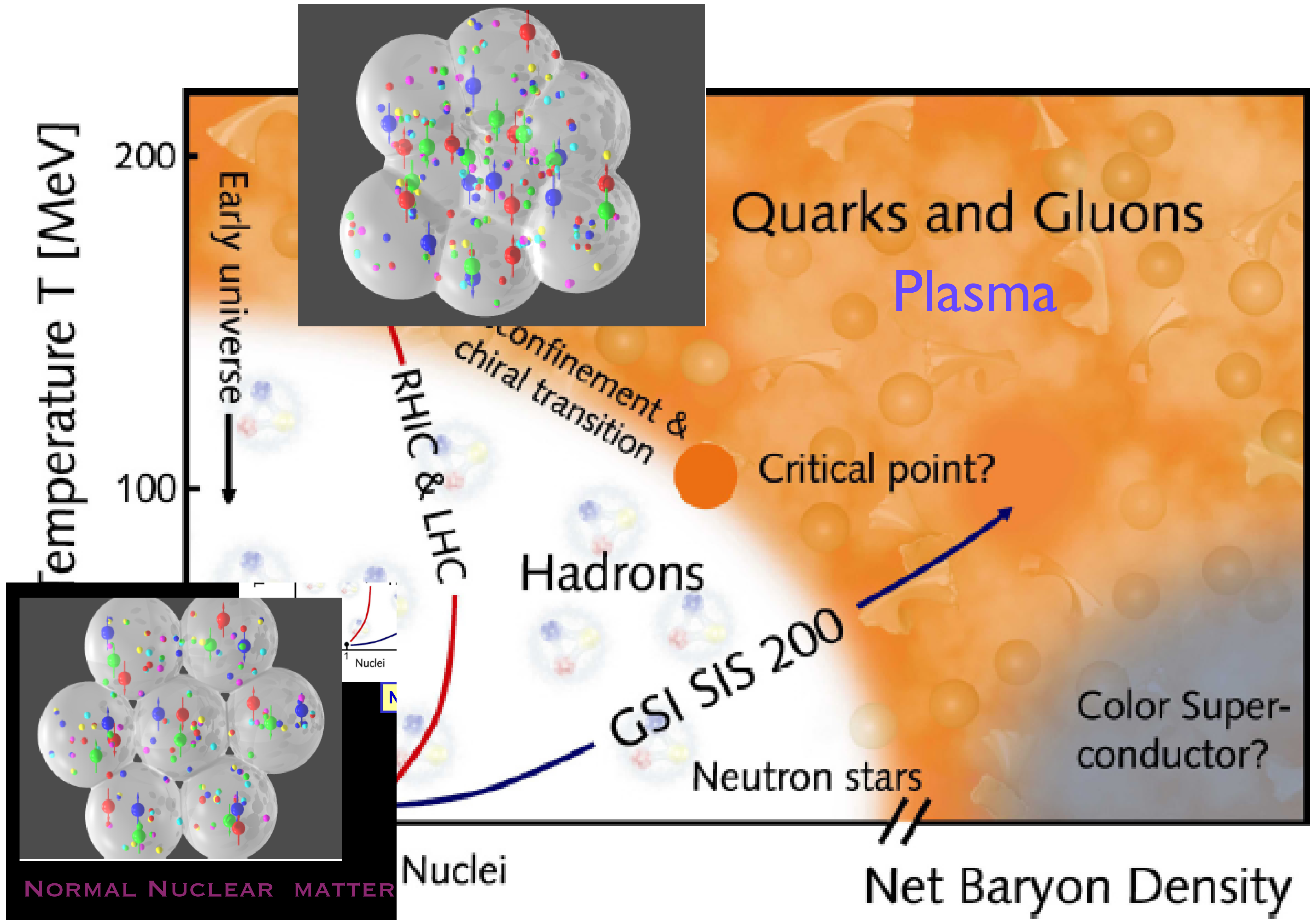

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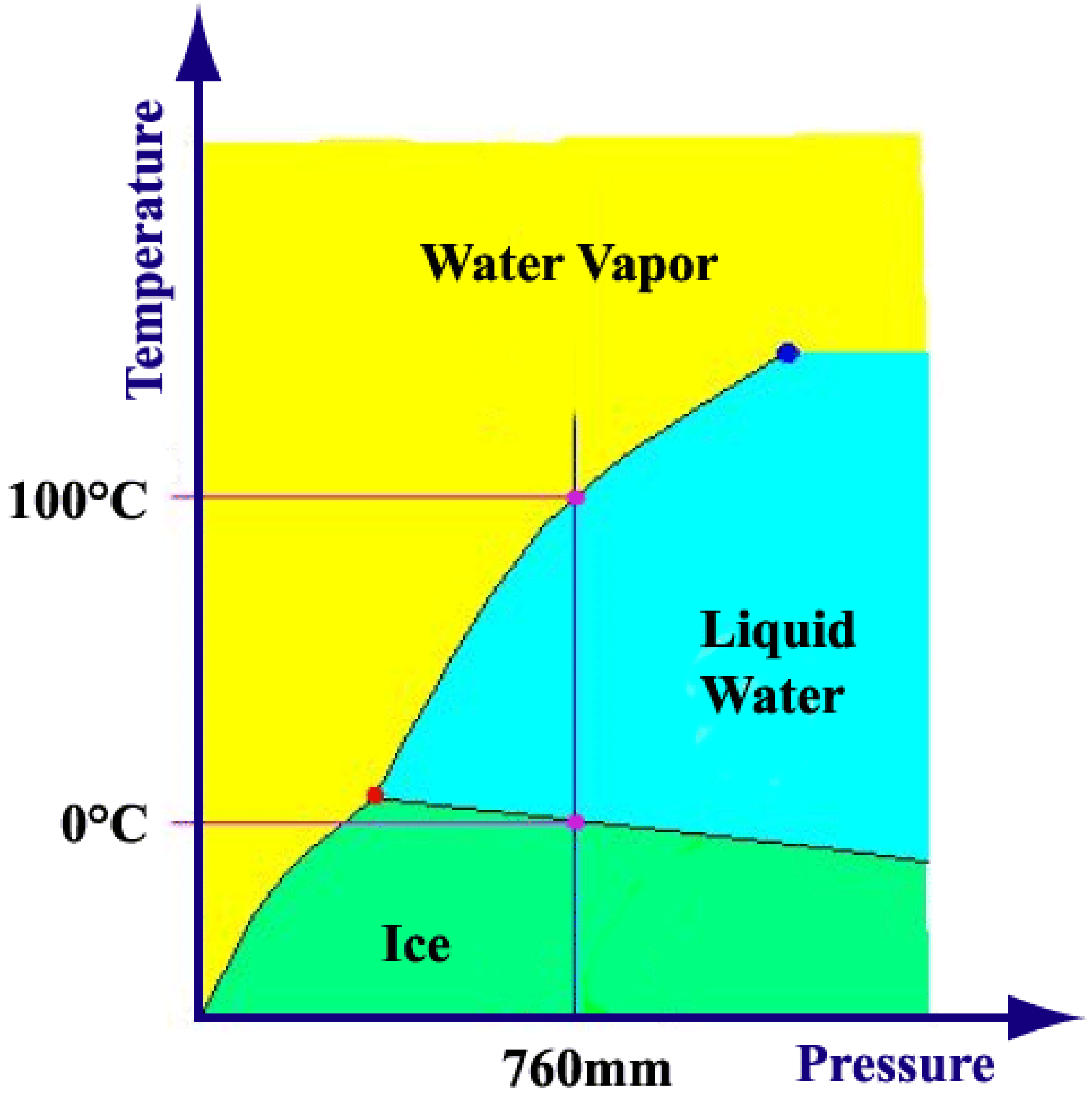

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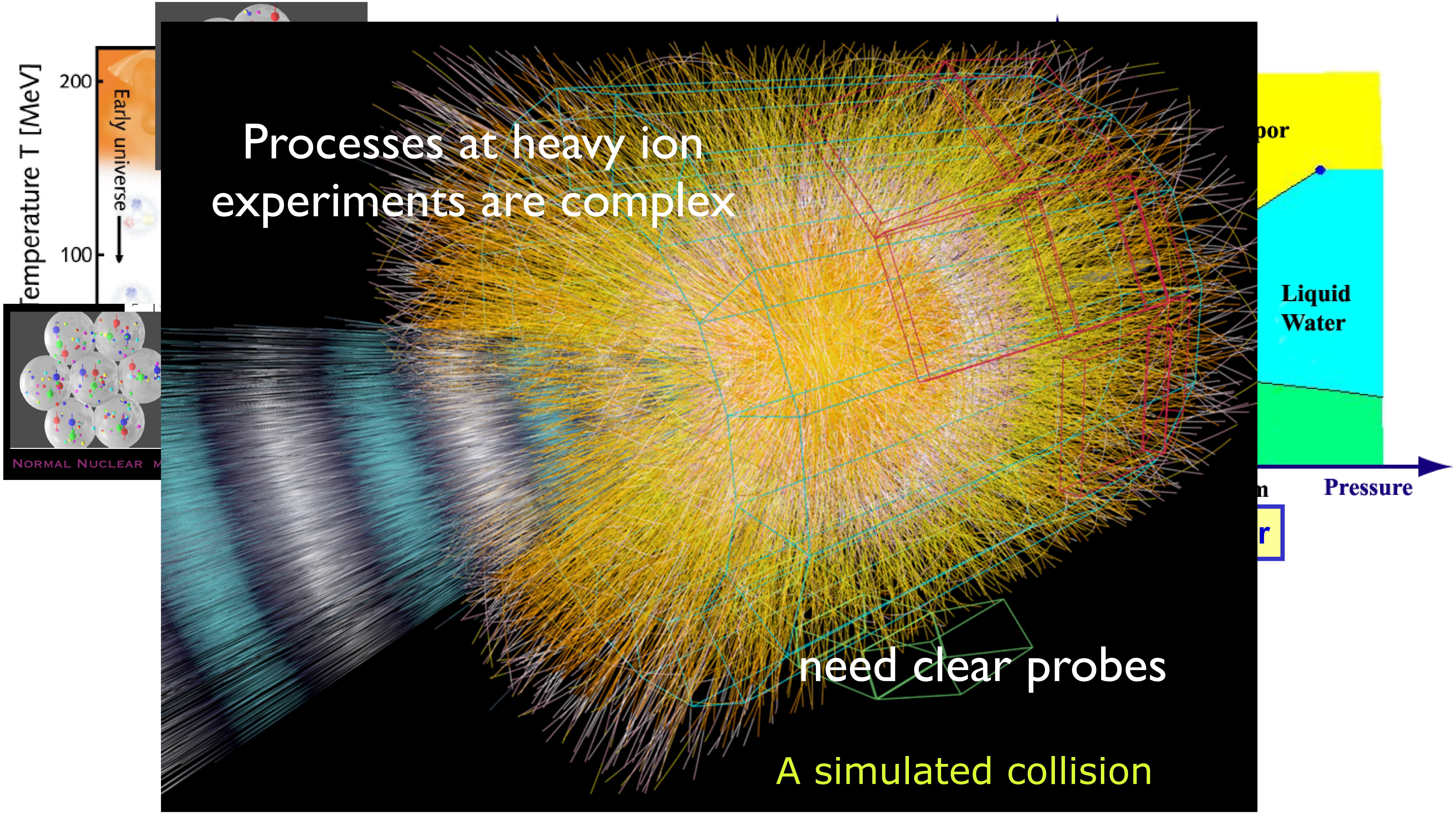
investigated in heavy ions collision at the LHC at CERN and RHIC USA



The present revolution: new particles discoveries beyond the Quark Model

XYZs share similar characteristics to quarkonium and being strongly correlated exotics systems are golden and new probe of QCD

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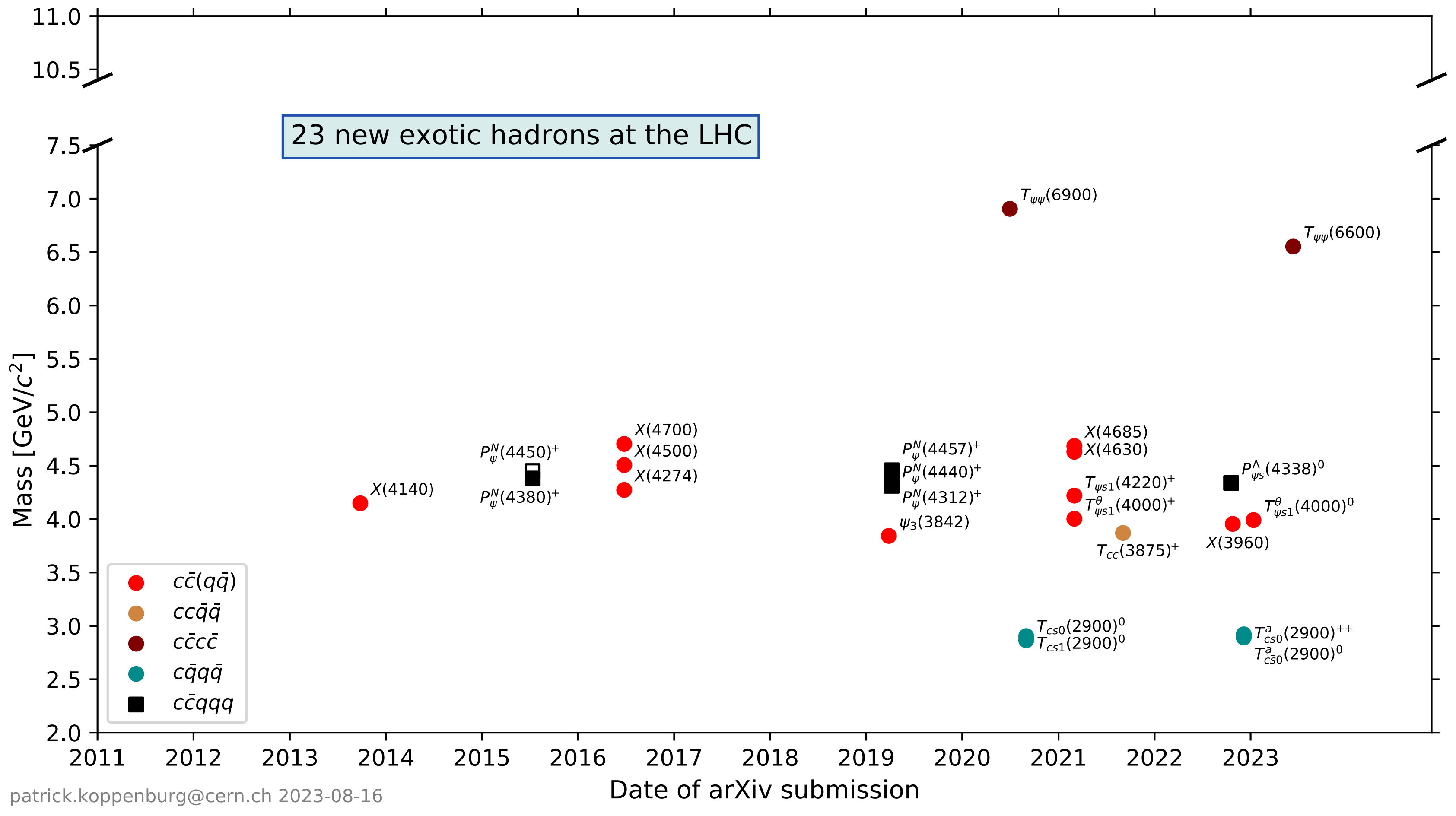
Not only the spectroscopy of XYZs



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but also the XYZ production in medium

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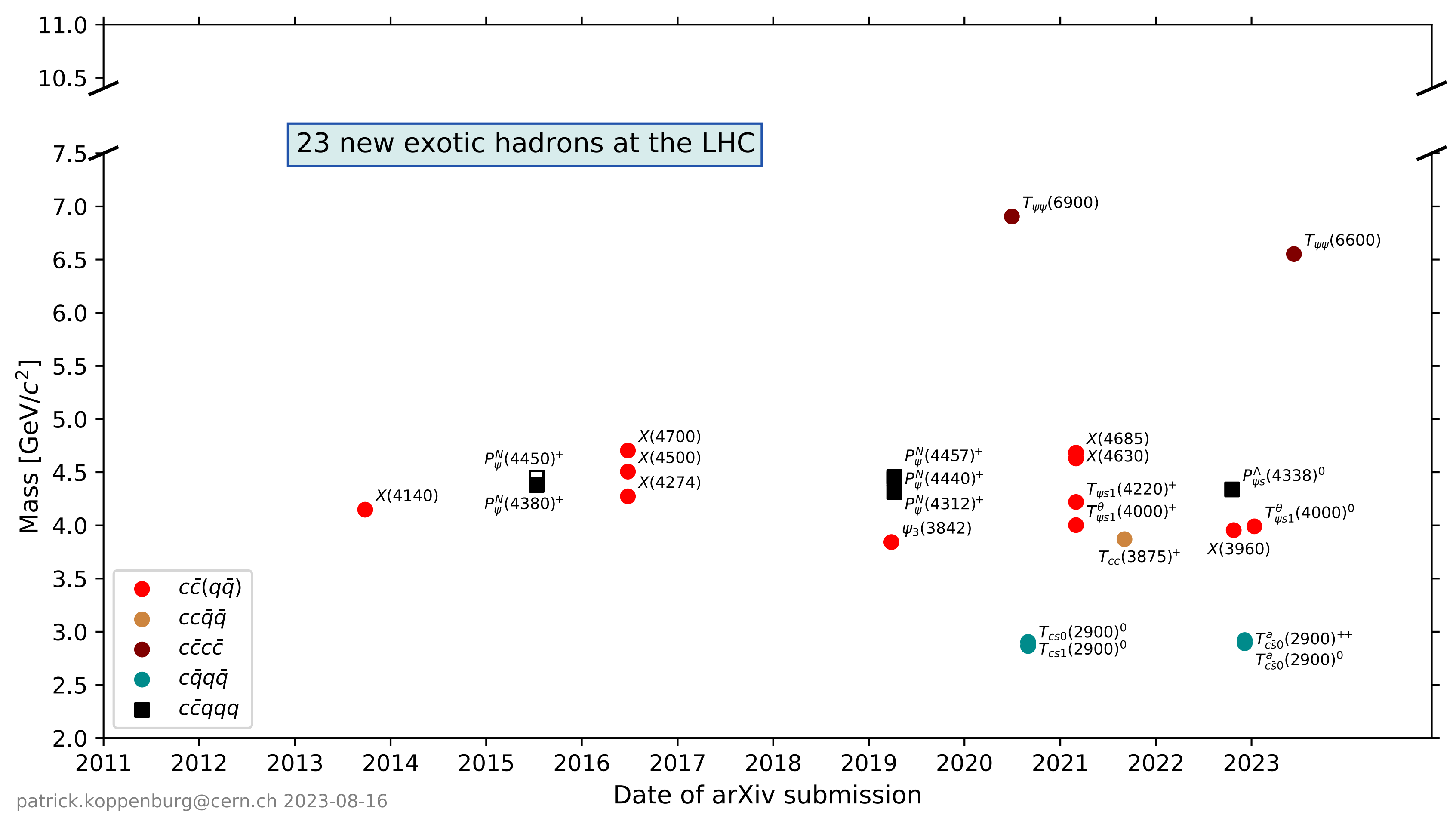
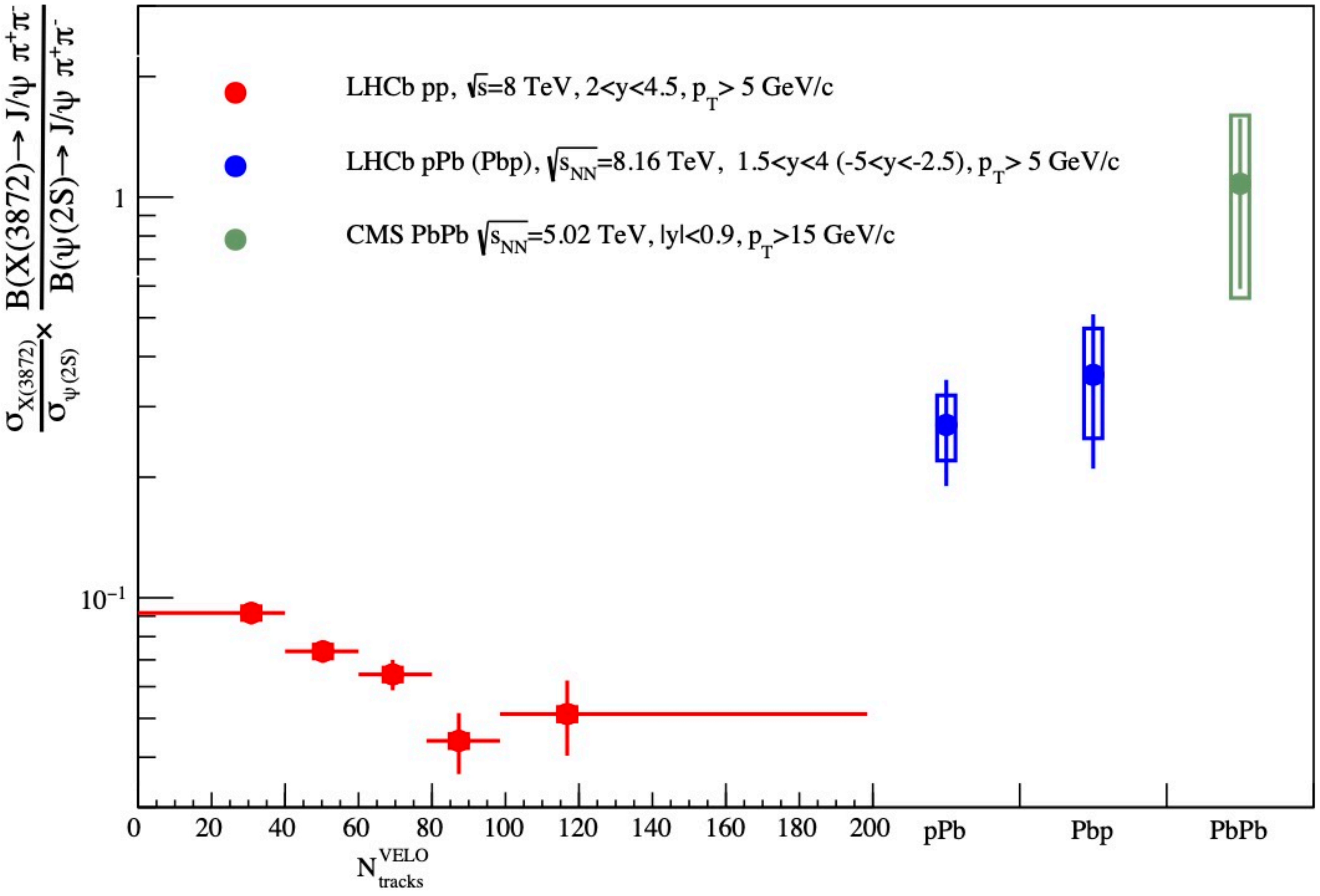
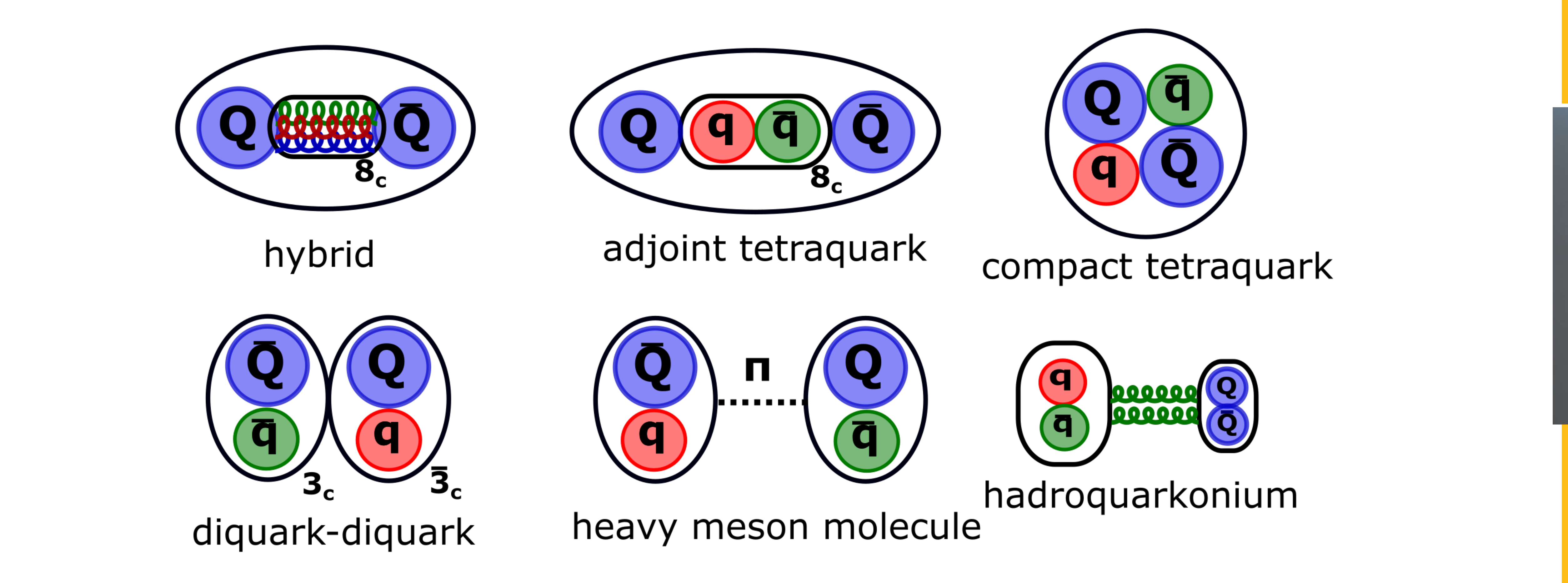


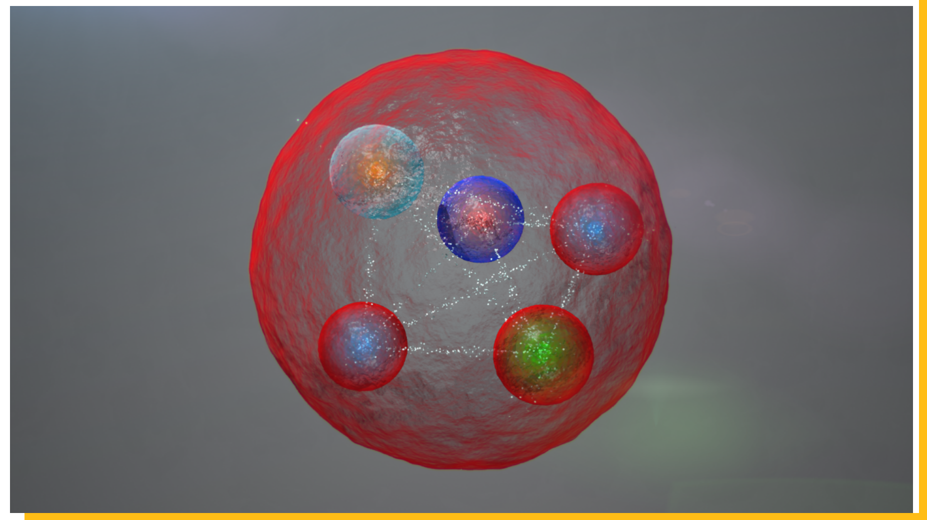


opens completely new perspectives!

The present revolution: new particles discoveries beyond the Quark Model

- This is ‘new exotics physics’ predicted by QCD but never observed before—> emerged only in the heavy sector
- XYZs appear at or above the strong decay threshold where many light degrees of freedom (light quarks, glue) become active
- Exotics strongly interacting states of a new nature ->allow to explore the nature of the strong force
- Tens of models in the literature! based on selecting some configurations/degrees of freedom and attaching to them a model hamiltonian





Pentaquarks

Pioneering ab initio lattice calculations-> challenging, close to threshold, scattering effects, many channels

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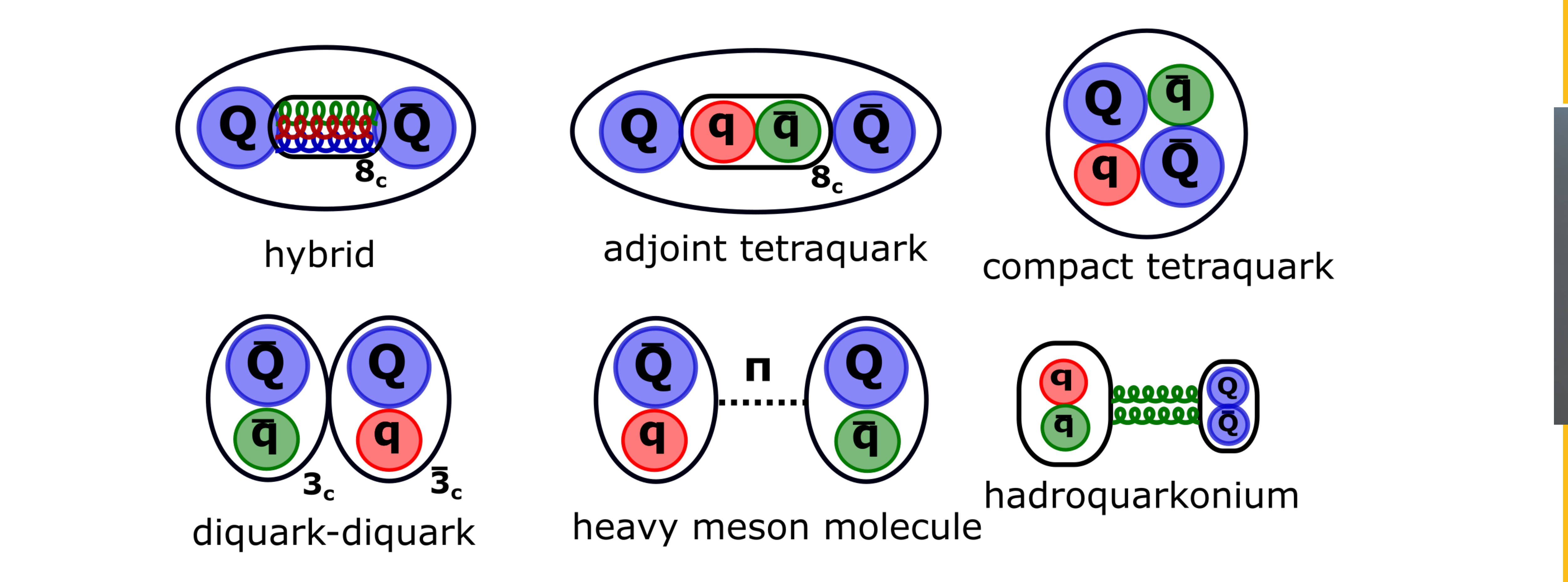

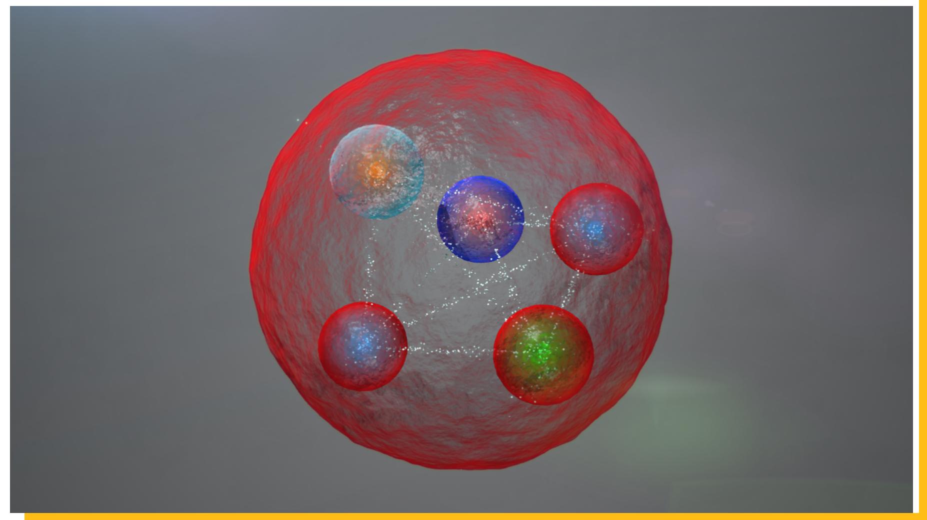

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I will present here the BOEFT (Born Oppenheimer effective field theory) for XYZ, obtained from QCD , encompassing models, with a direct contact to phenomenology

For quarkonium to become a probe of strong interactions, it should be treated in QCD :a very hard problem

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Close to the bound state $\alpha_s \sim v$

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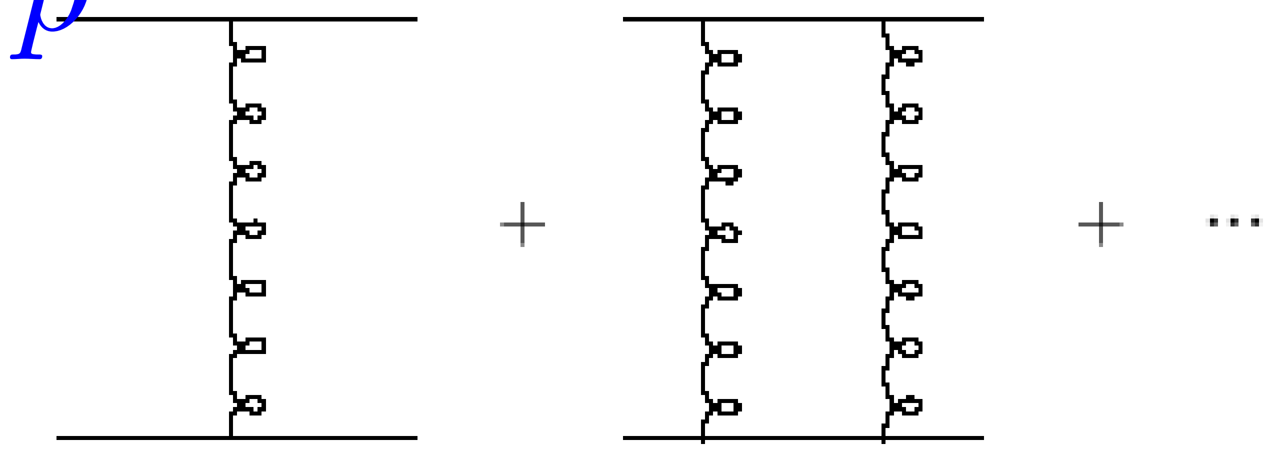
$p \sim m\alpha_s$

$\bar{Q}$



$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

- From $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

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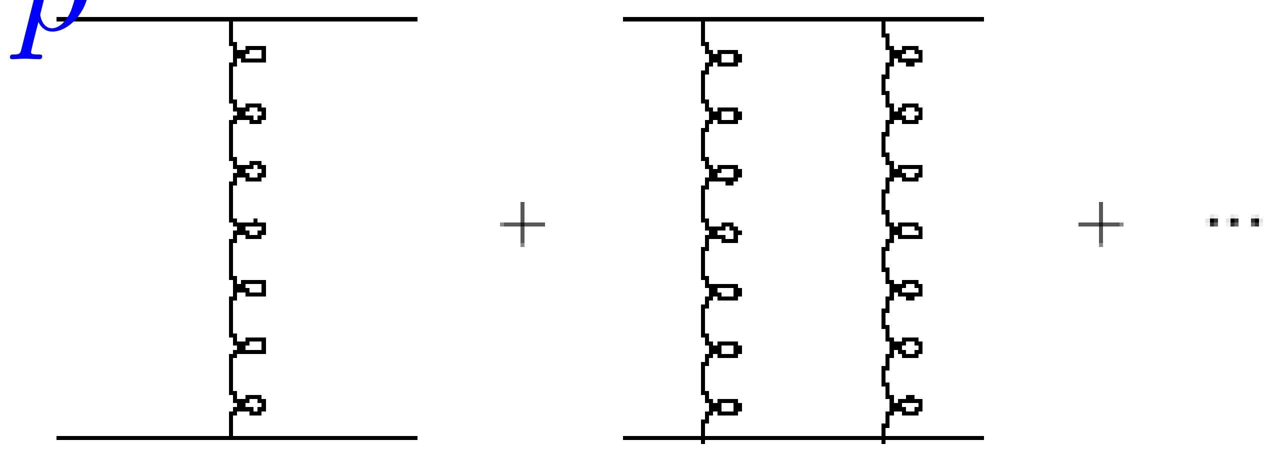

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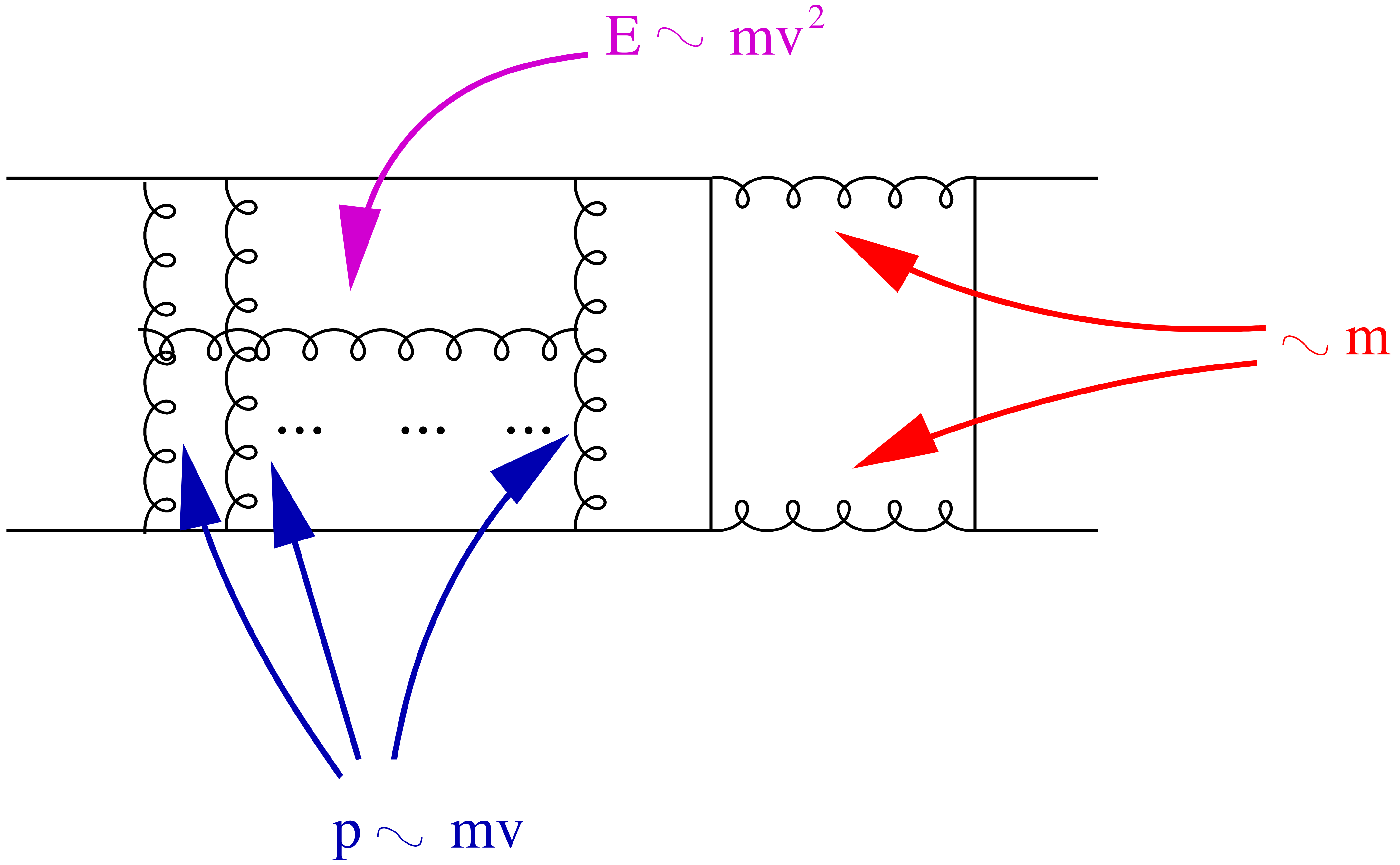

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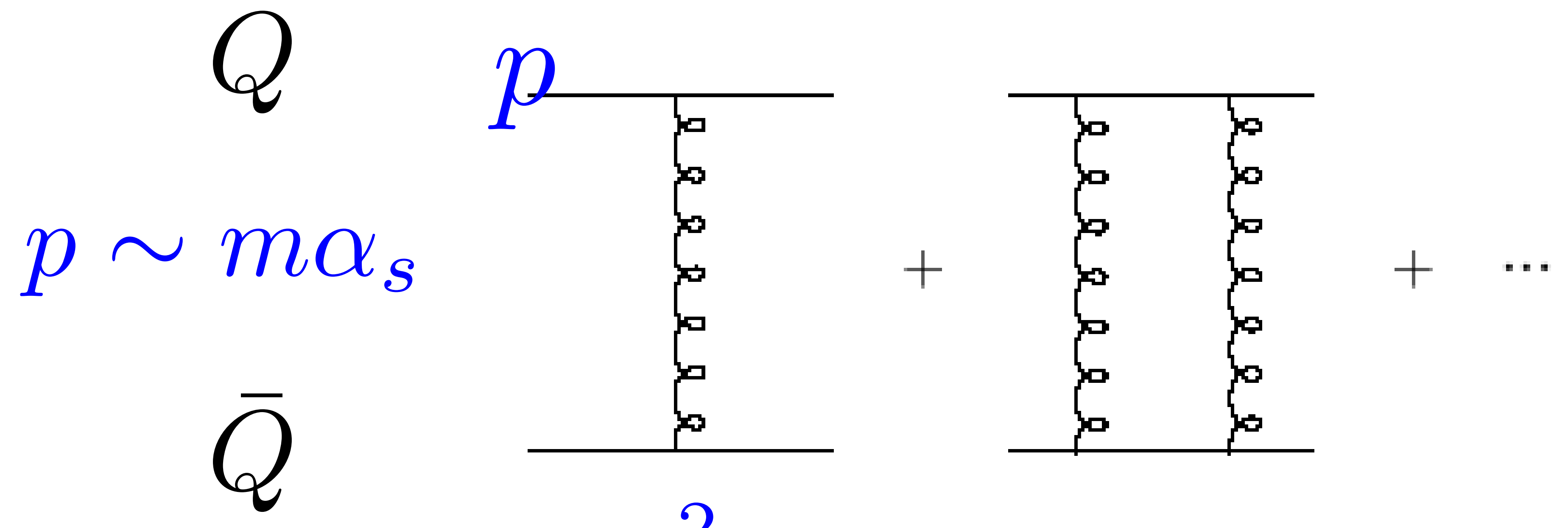

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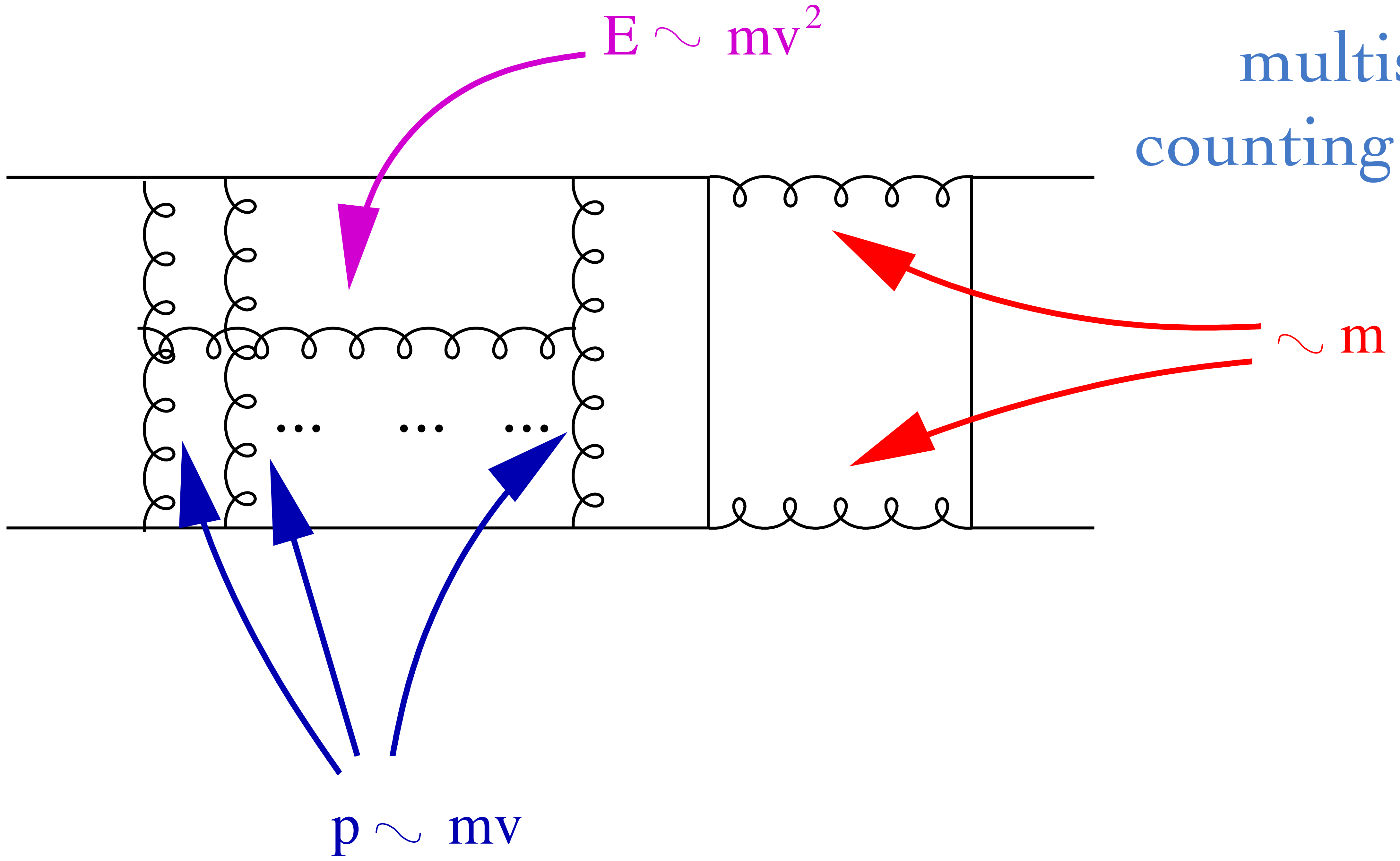


multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

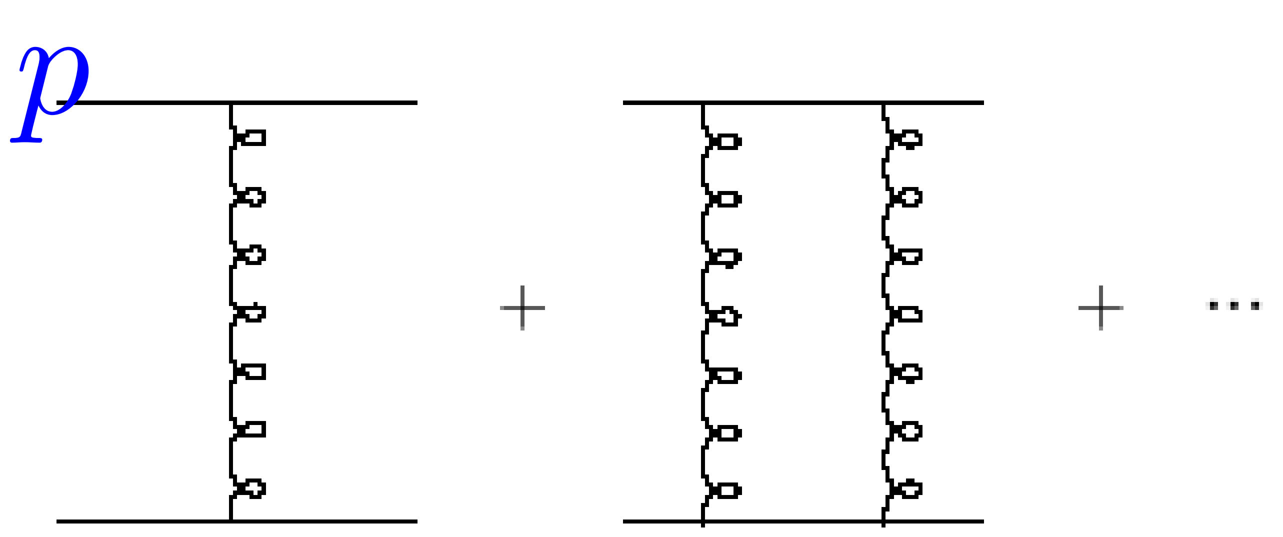
For quarkonium to become a probe of strong interactions, it should be treated in QCD :a very hard problem

Close to the bound state $\alpha_s \sim v$

Q

$p \sim m\alpha_s$

$\bar{Q}$



$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

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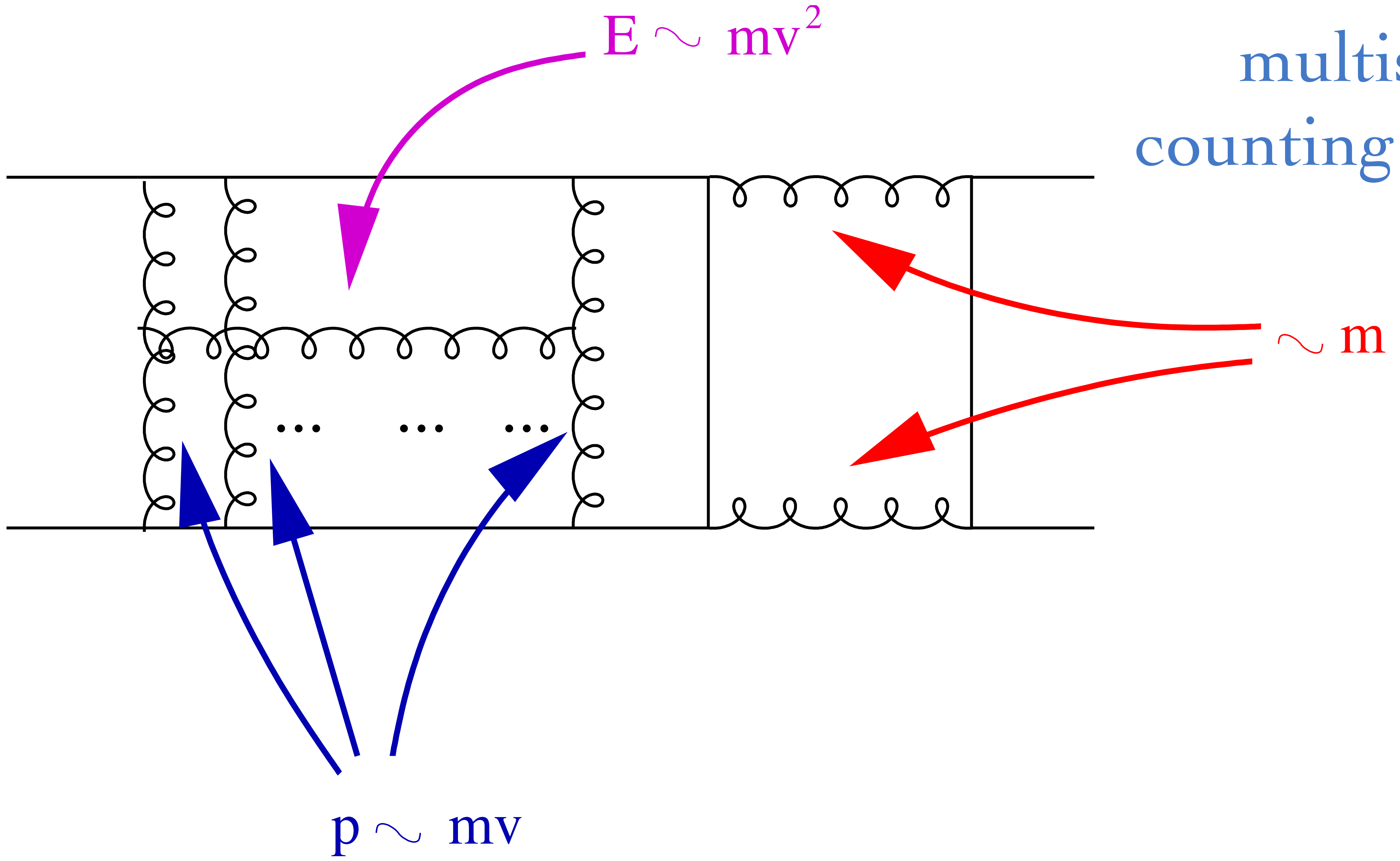

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Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

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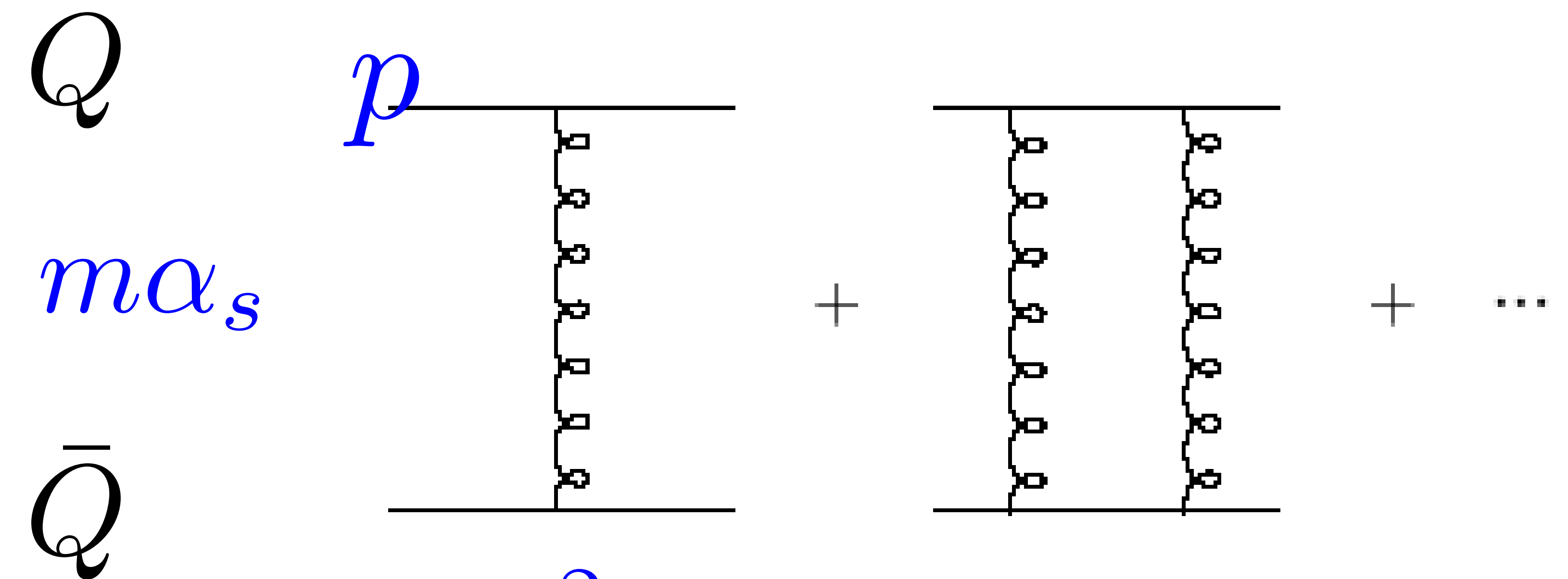

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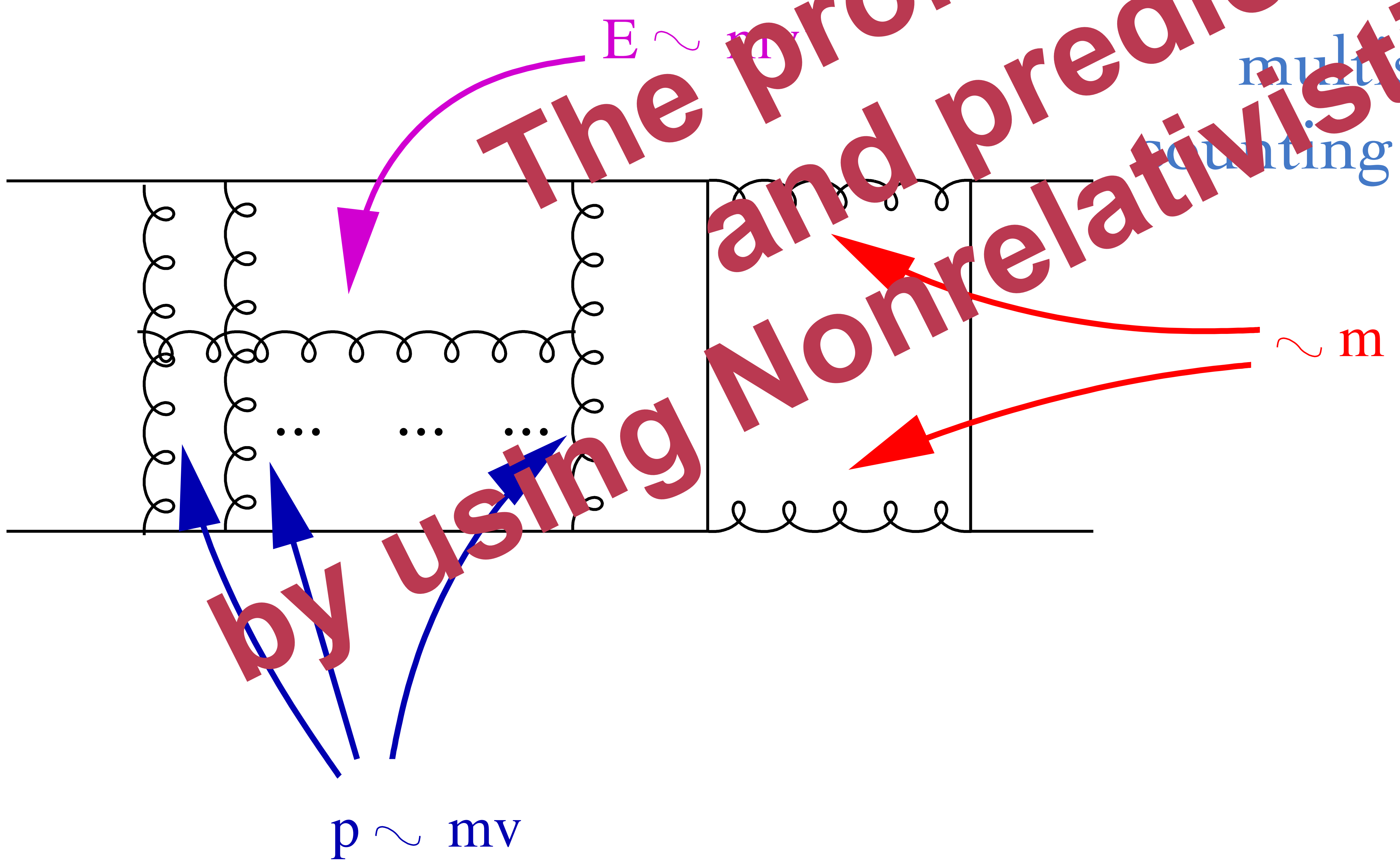
The problem is greatly simplified and predictivity is achieved by using Nonrelativistic Effective Field Theories



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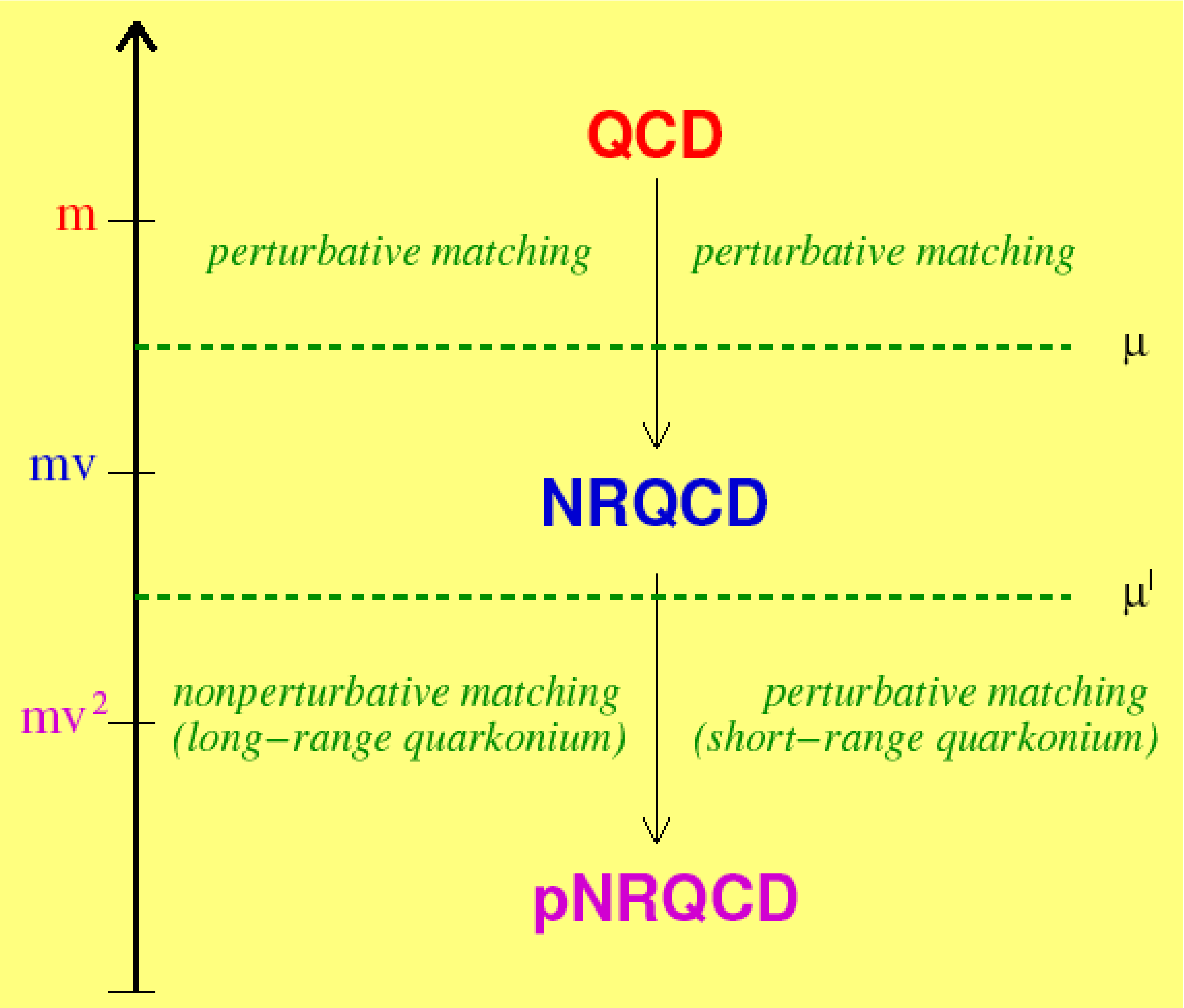
QQbar systems with NR EFT



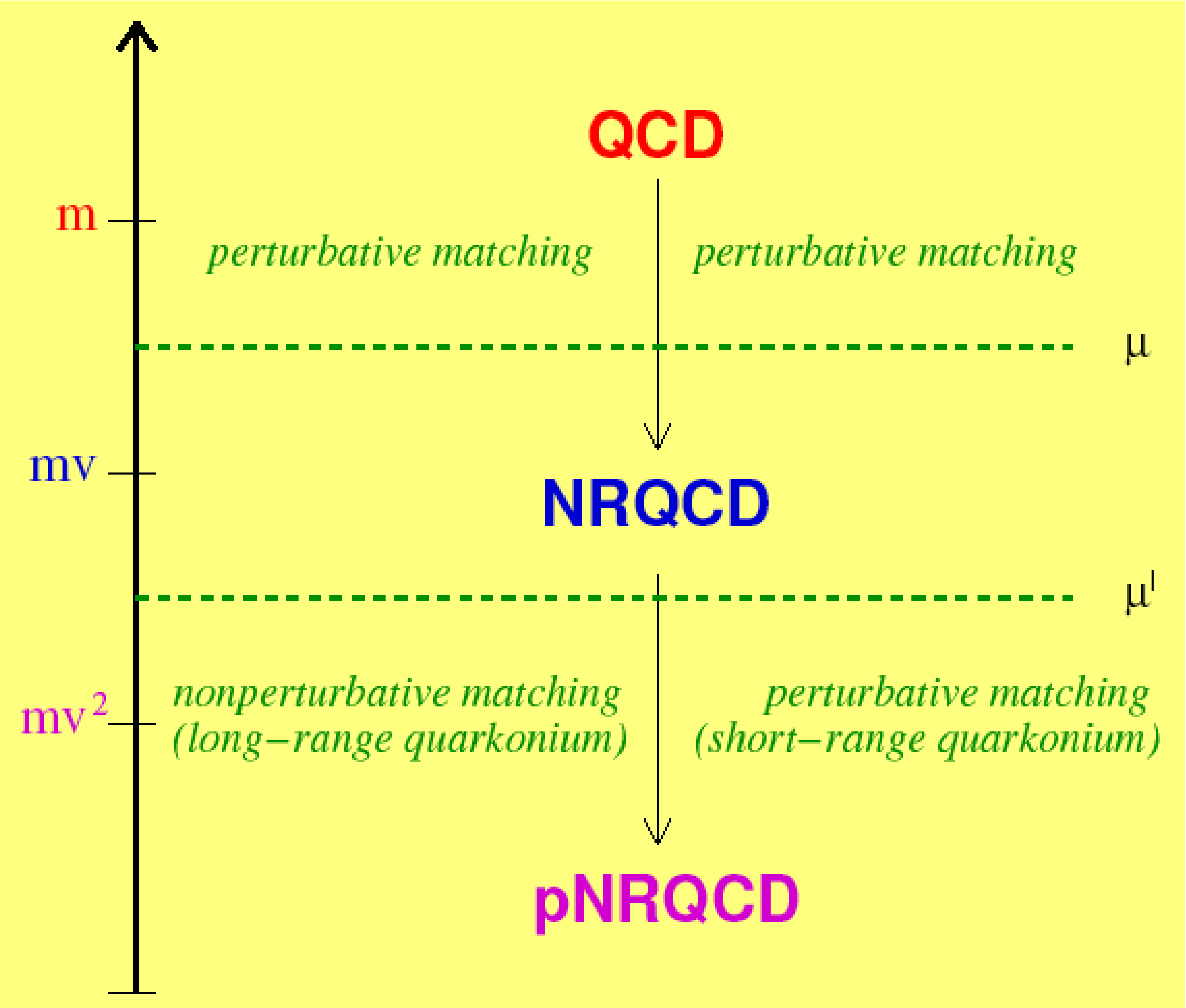
Color degrees of freedom
3X3=1+8
singlet and octet QQbar

Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

QQbar systems with NR EFT



$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

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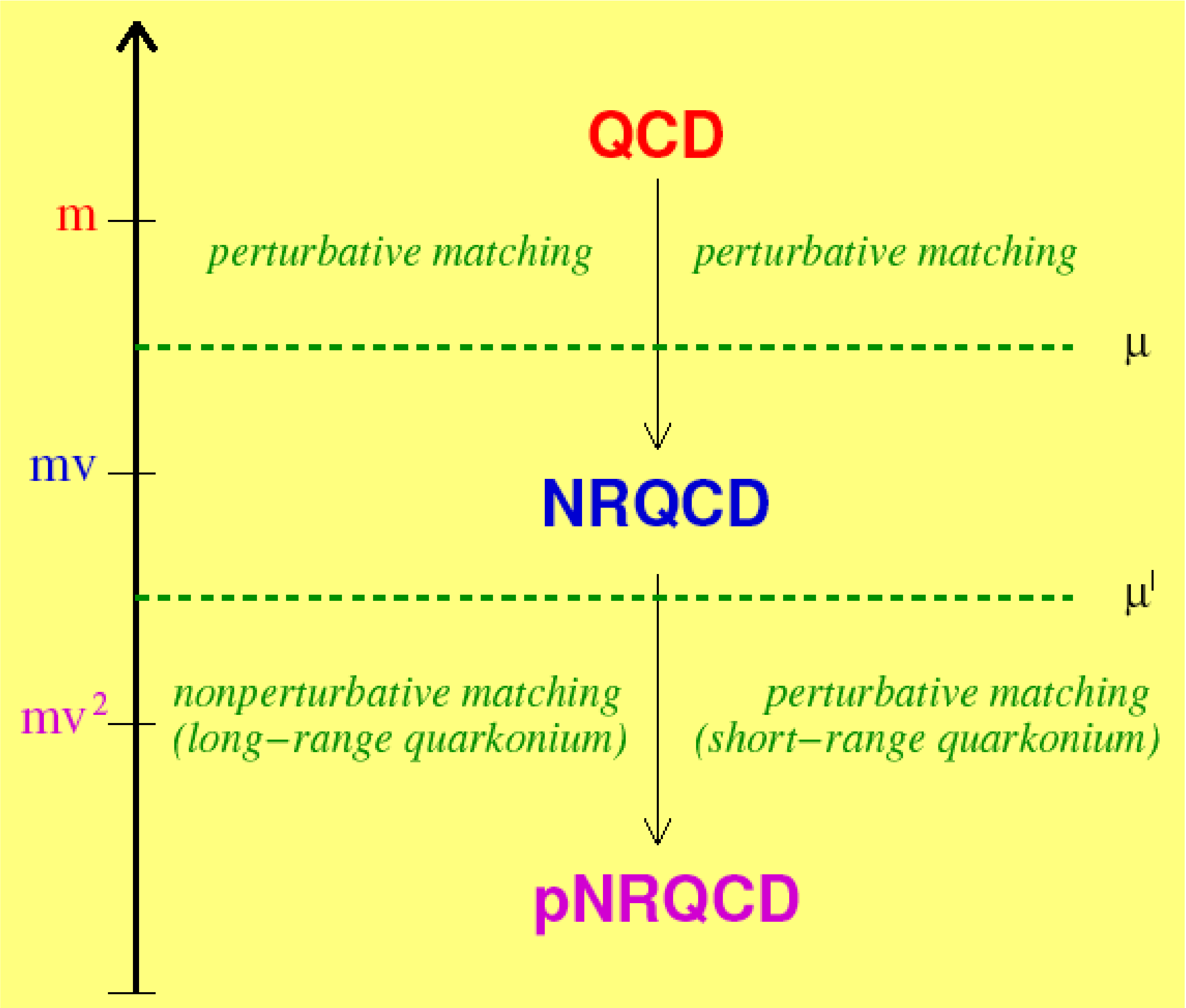

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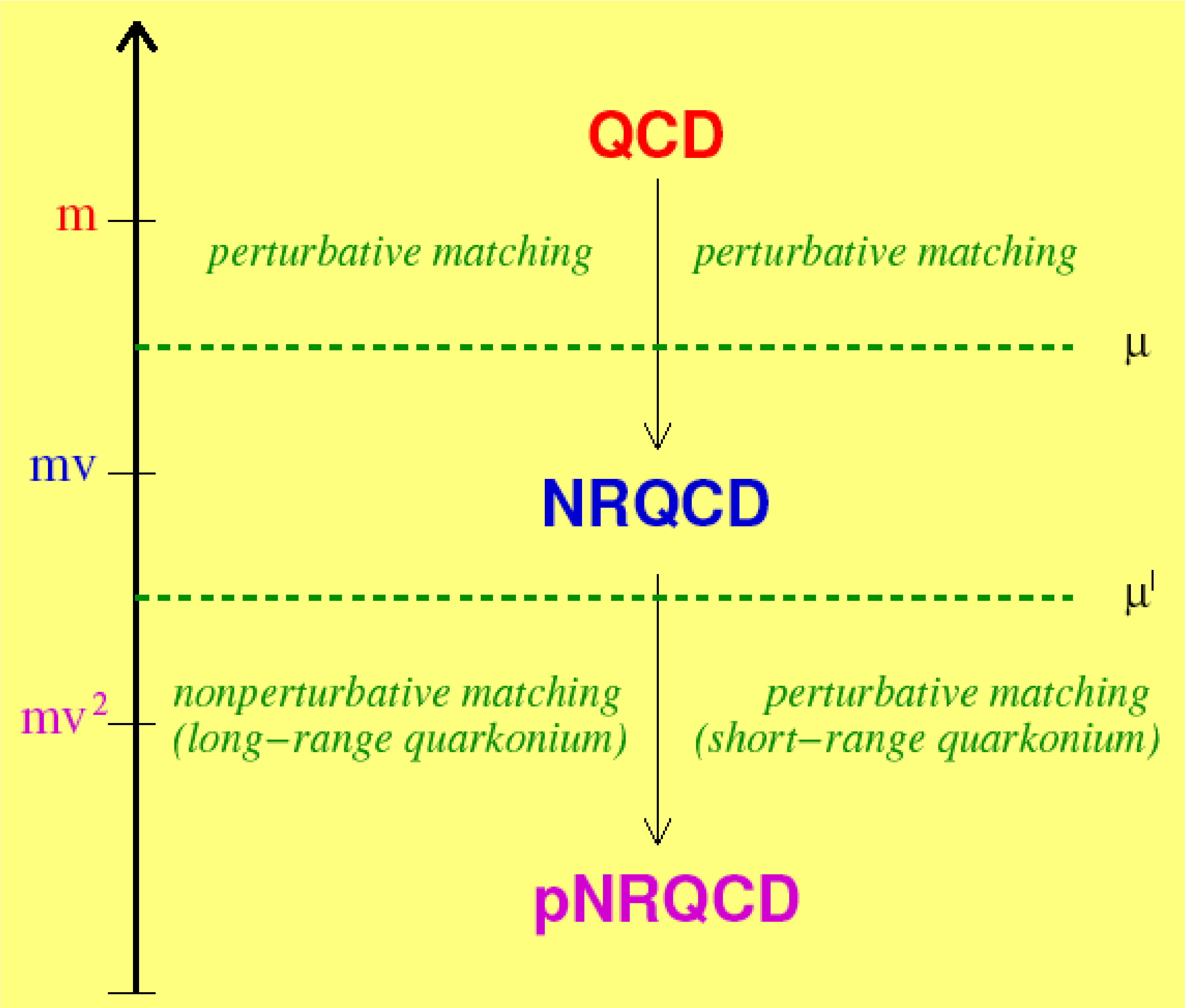

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$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

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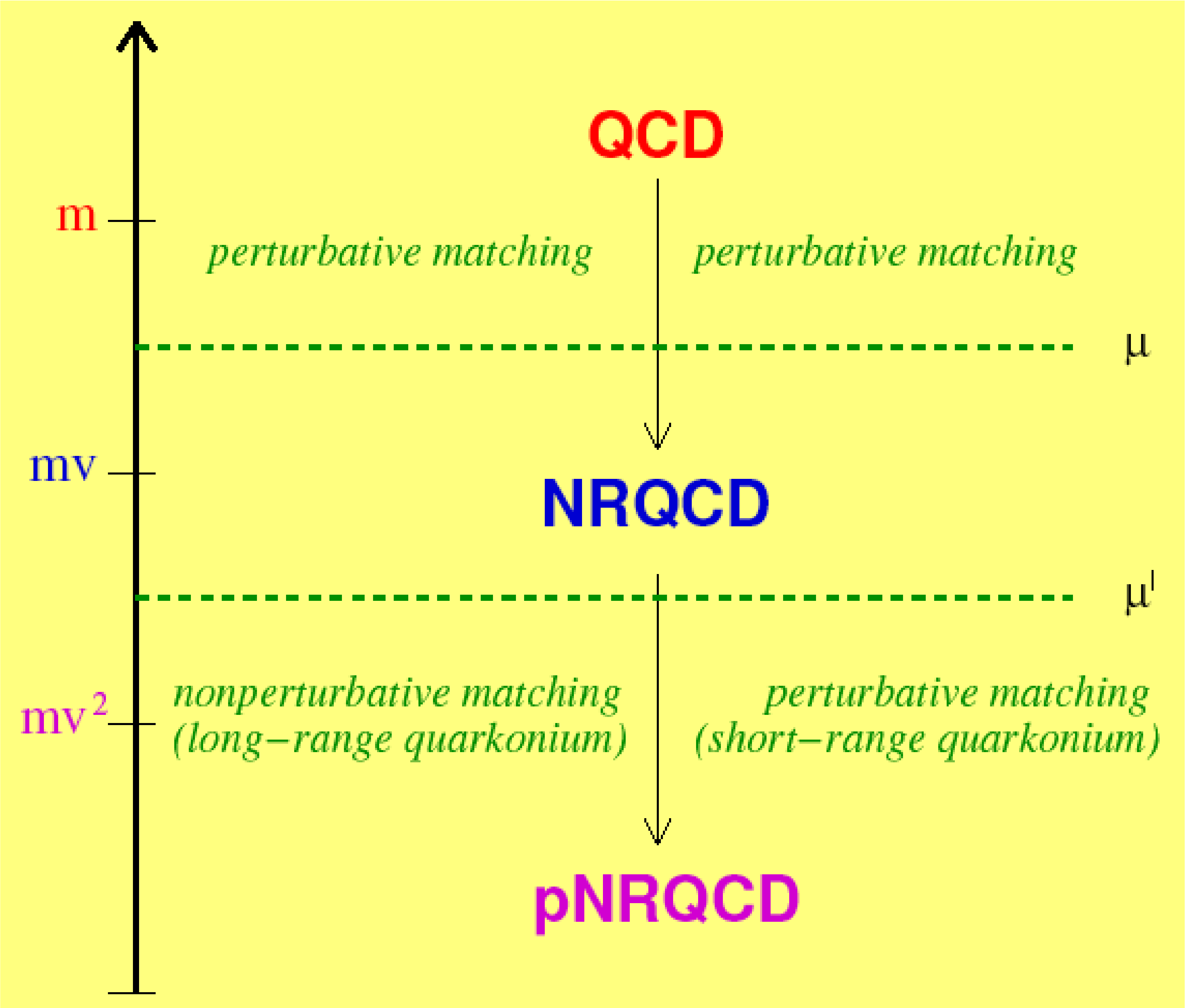

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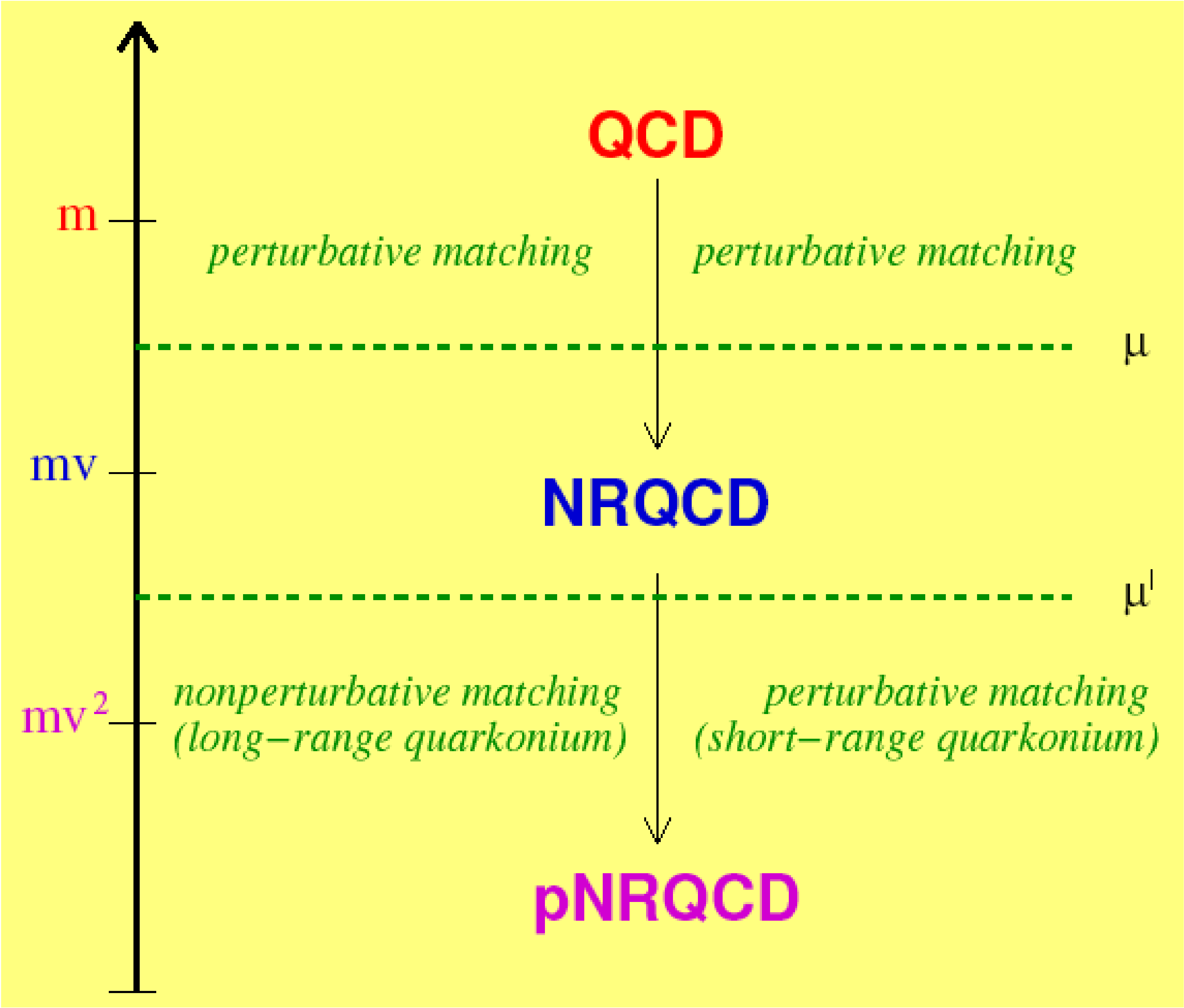
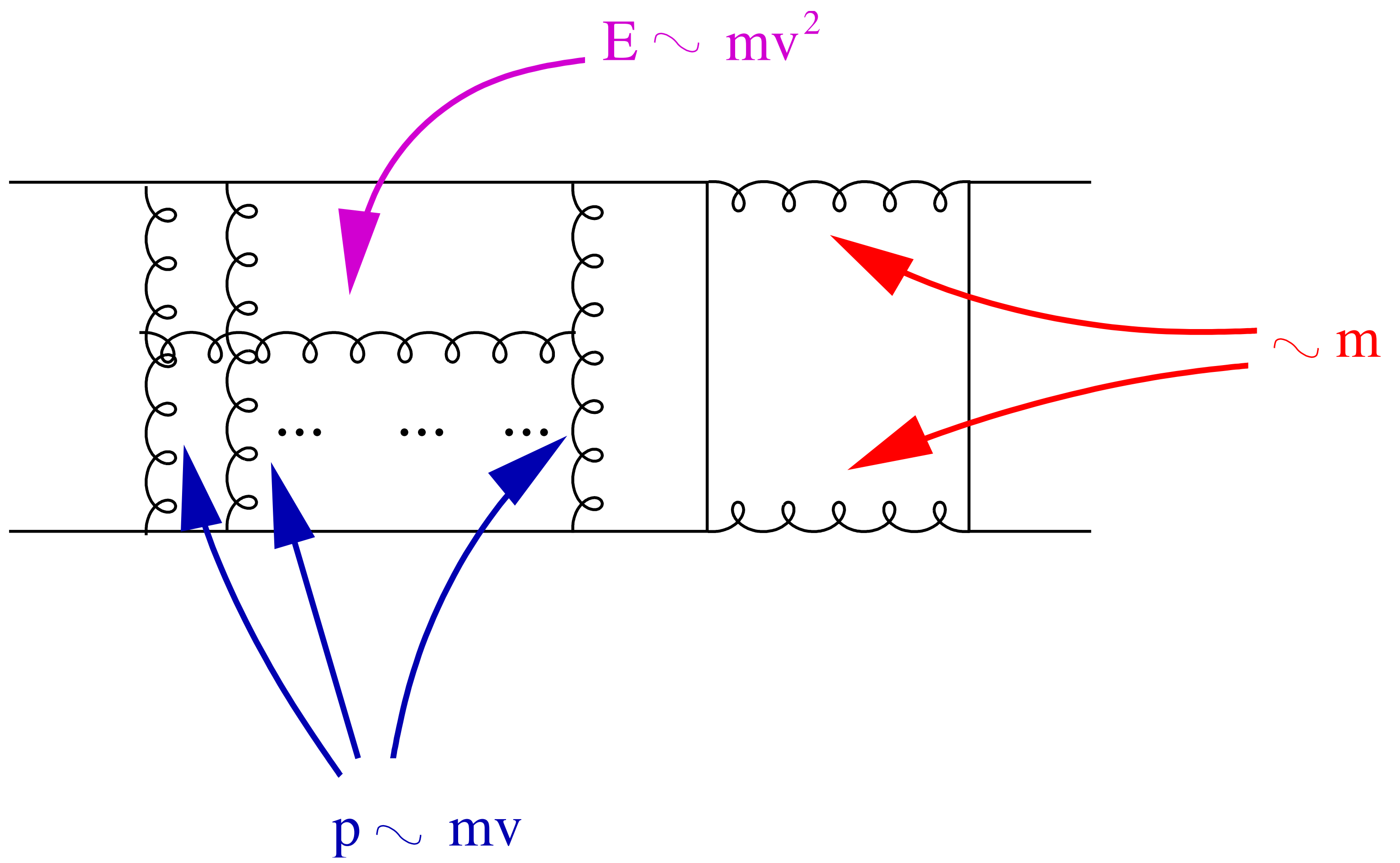
QQbar systems with NR EFT: Non Relativistic QCD (NRQCD)

Caswell, Lepage 86, Lepage Thacker 88, Bodwin, Braaten, Lepage 95





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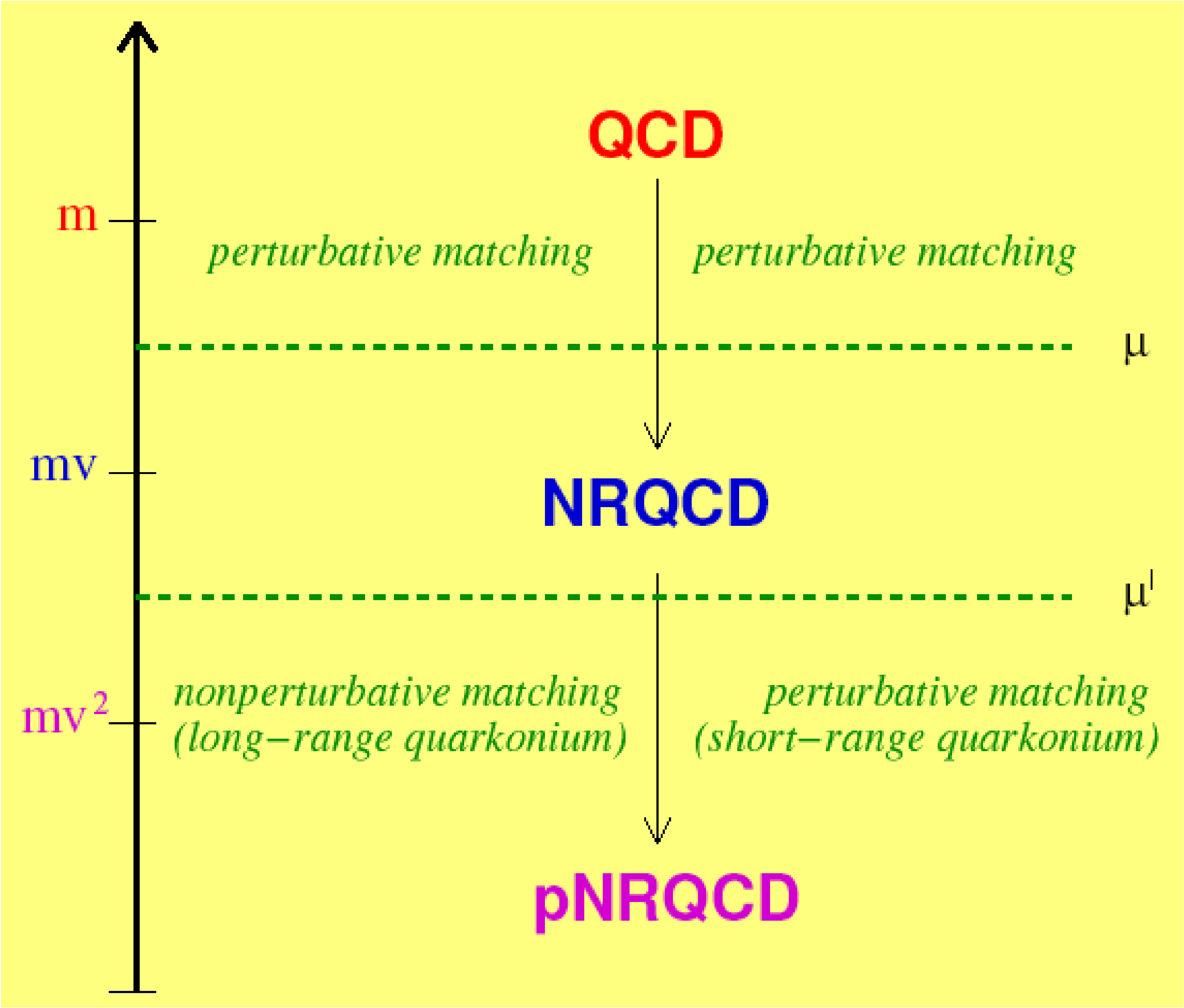

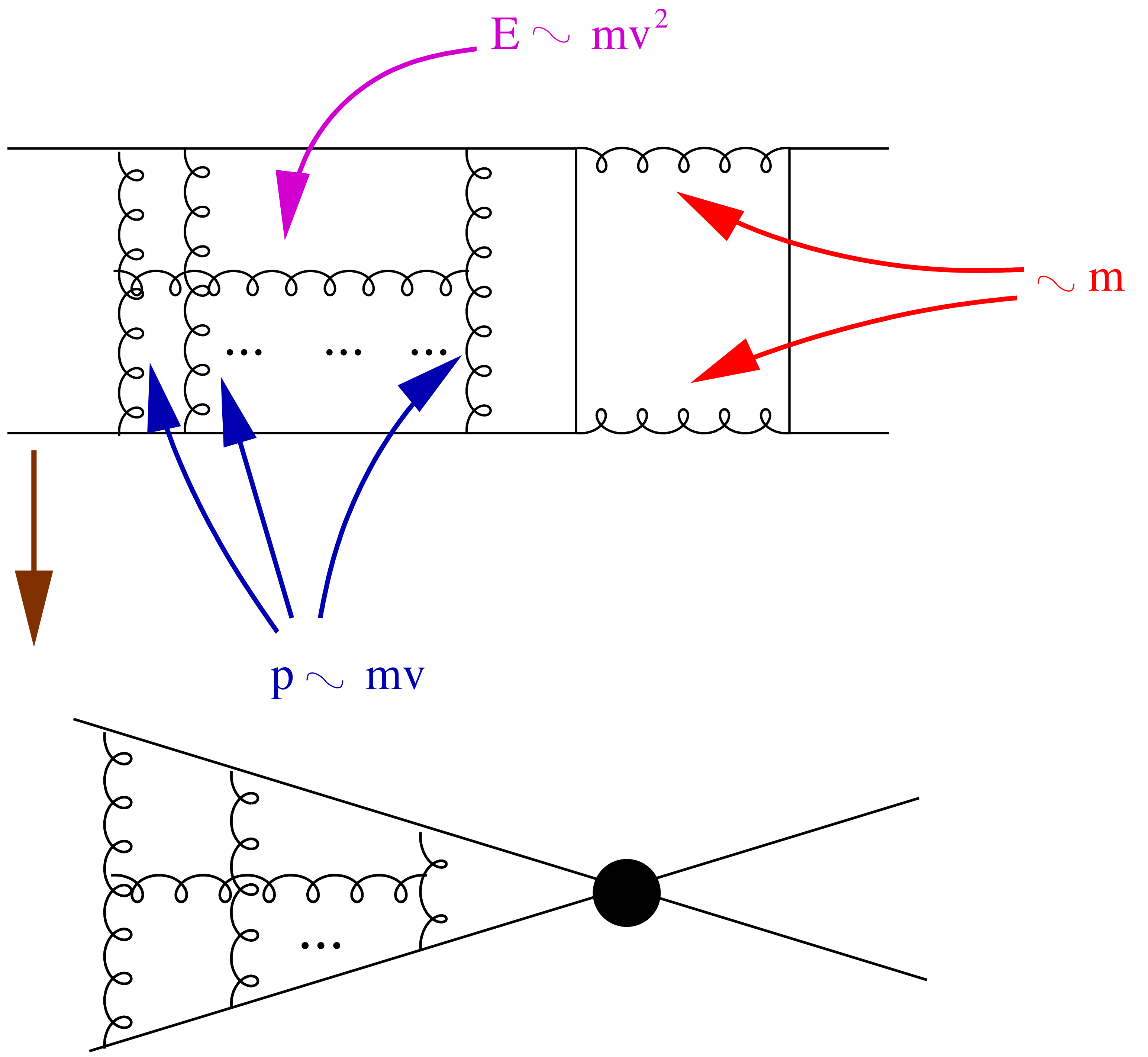

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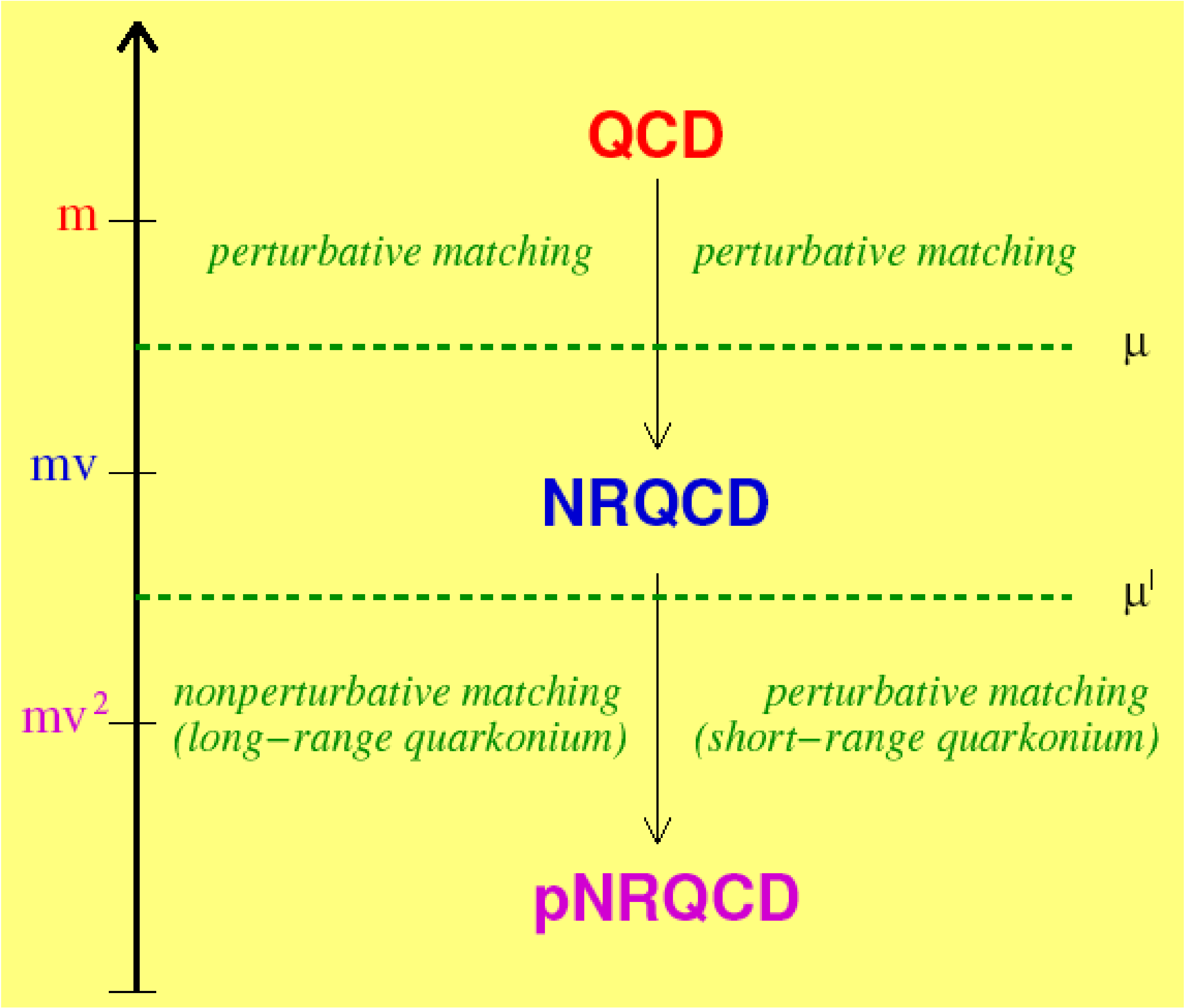

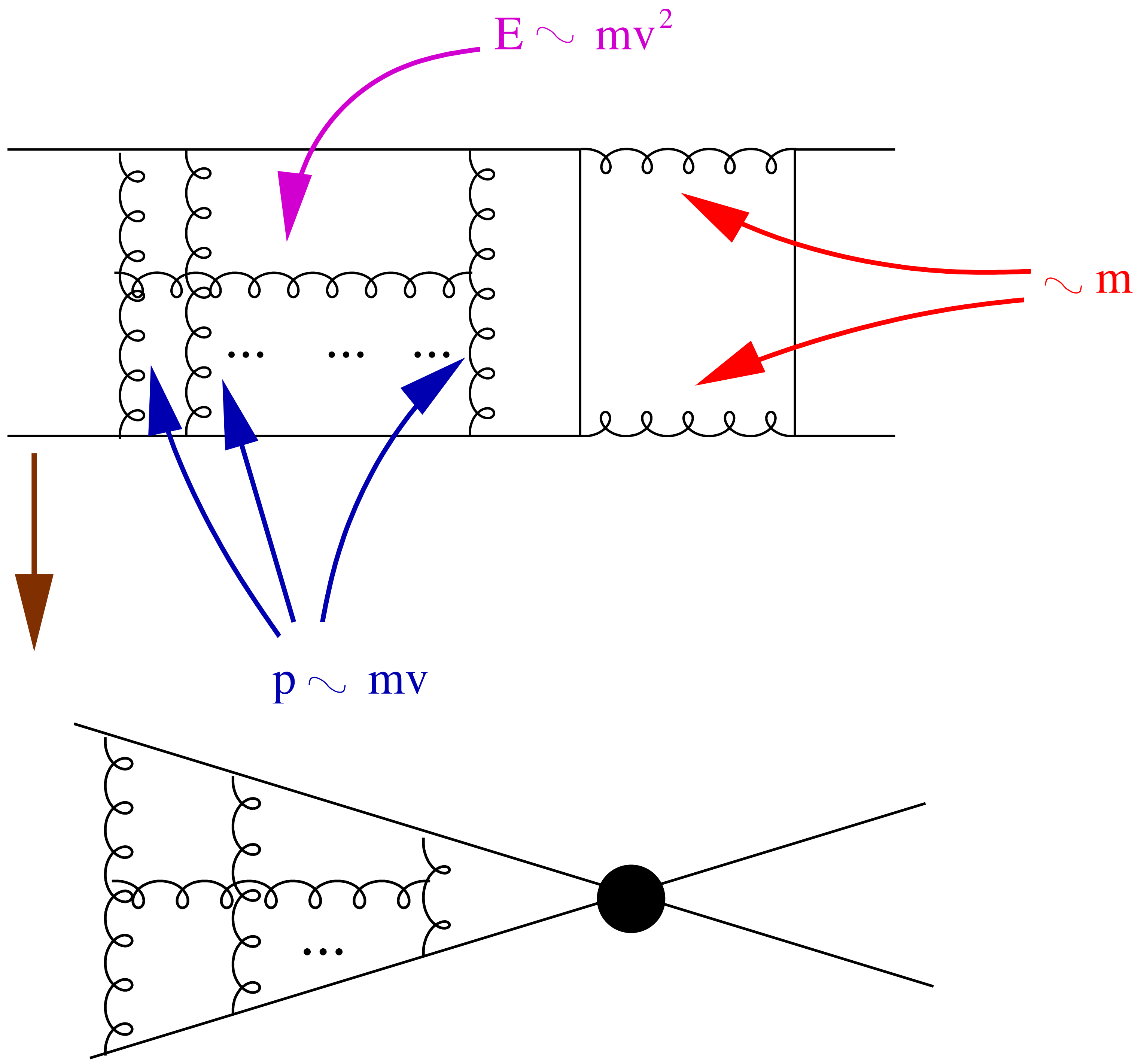

$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_{\rm s}(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

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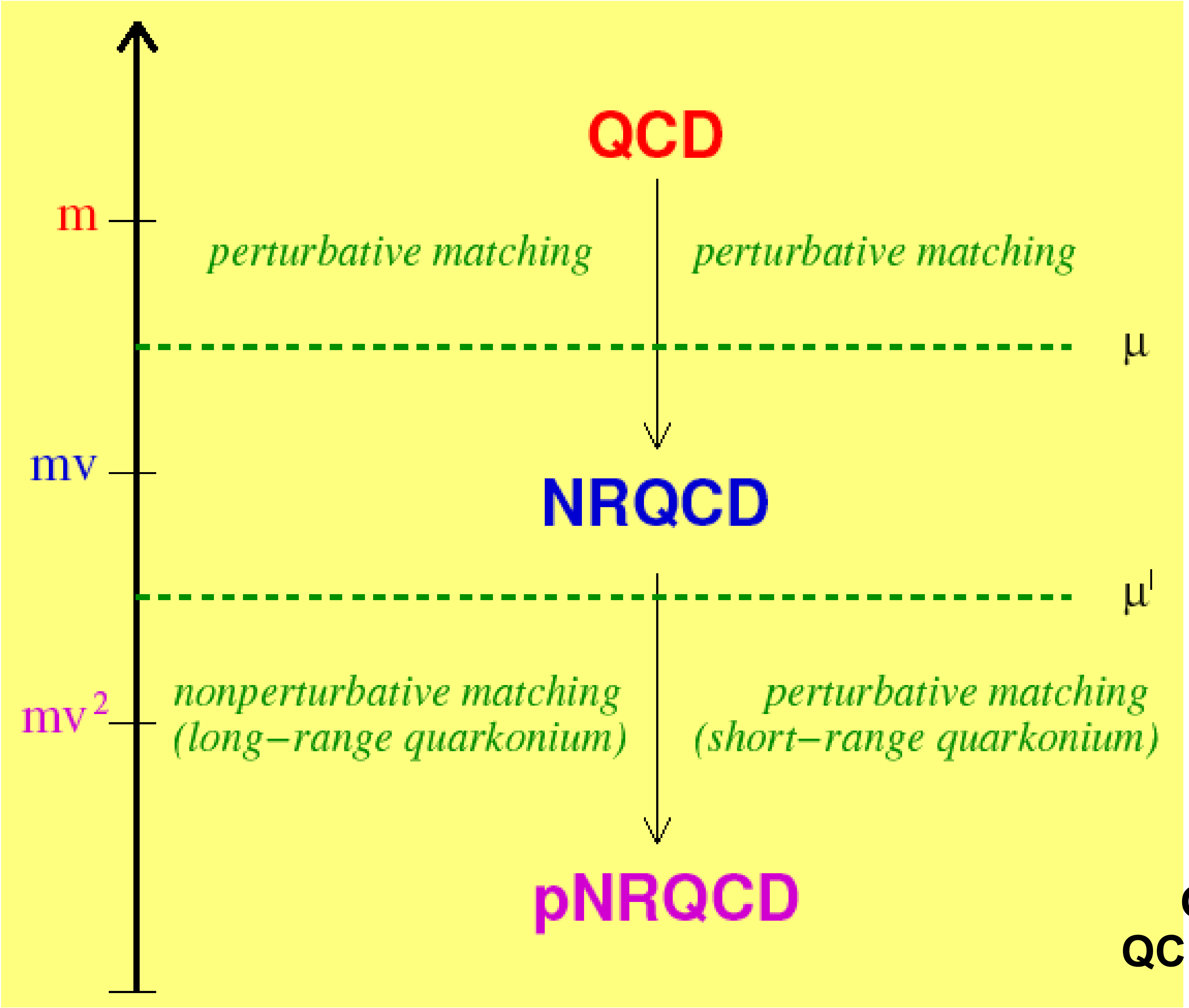

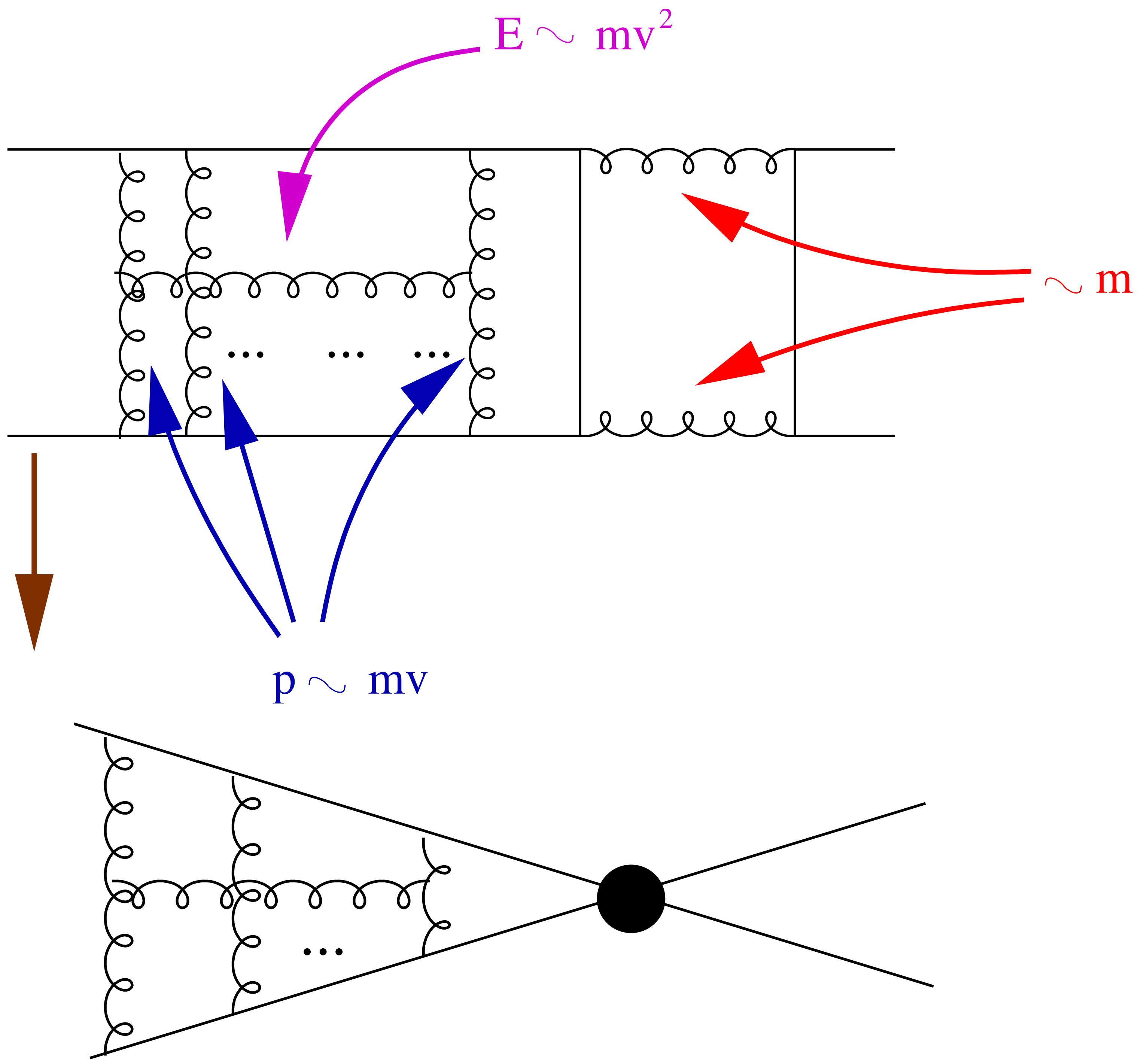

Only at the level of pNRQCD we obtain the potentials from QCD and the zero order problem is the Schroedinger equations

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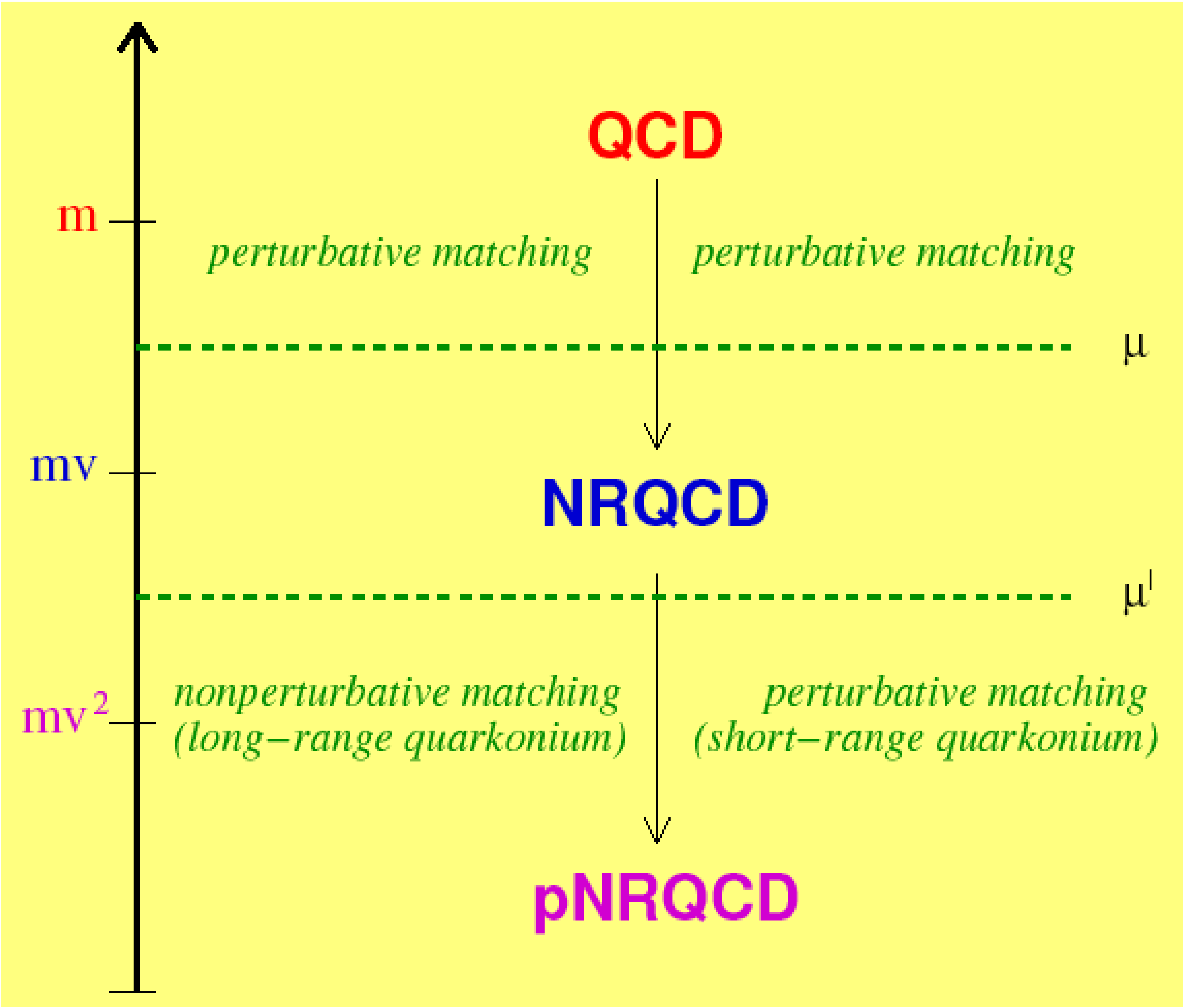
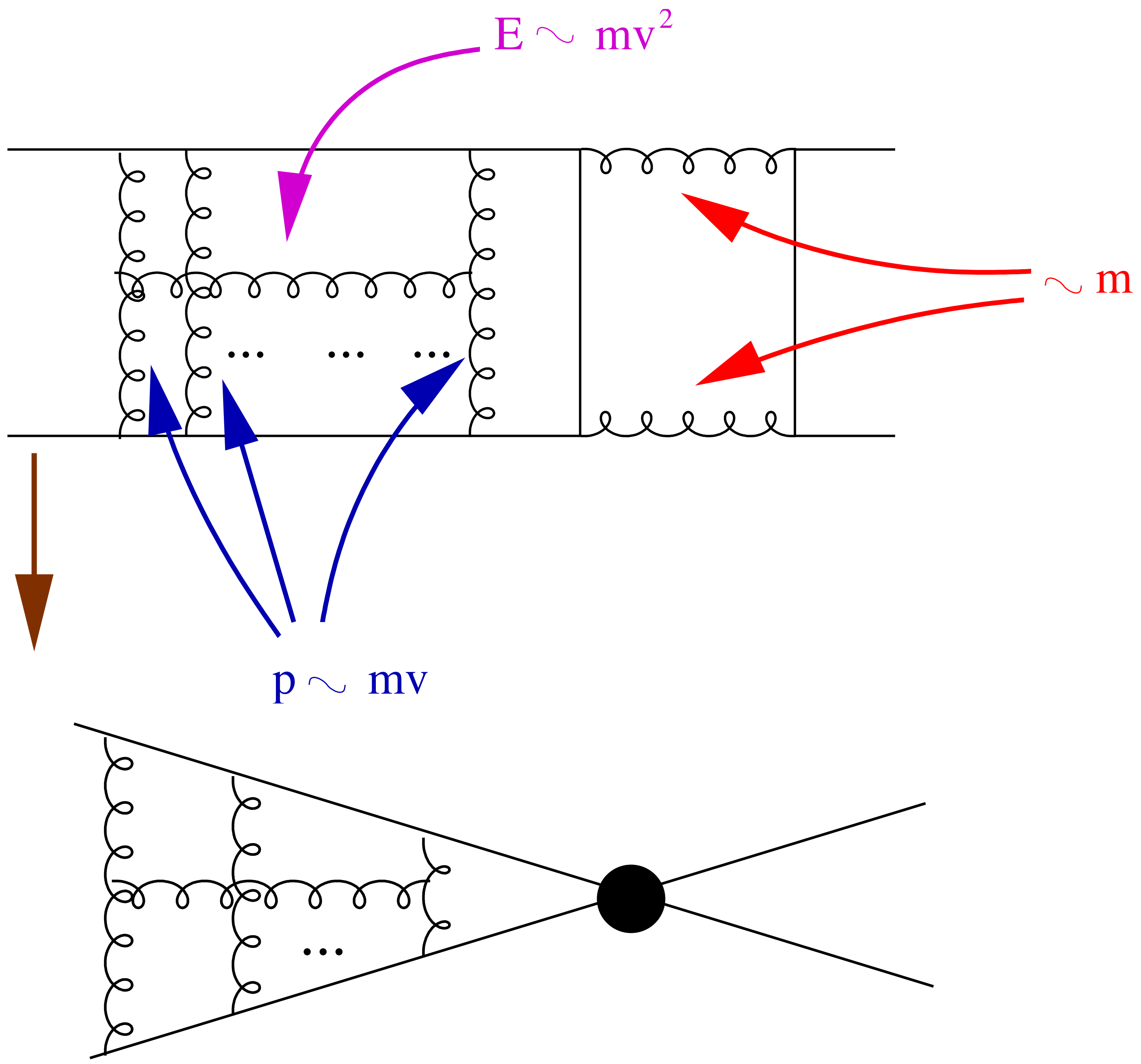
Quarkonium with NR EFT: potential Non Relativistic QCD (pNRQCD)

Pineda Soto 97, N. B., Pineda, Soto, Vairo 98





For mv >>Lambda_QCD

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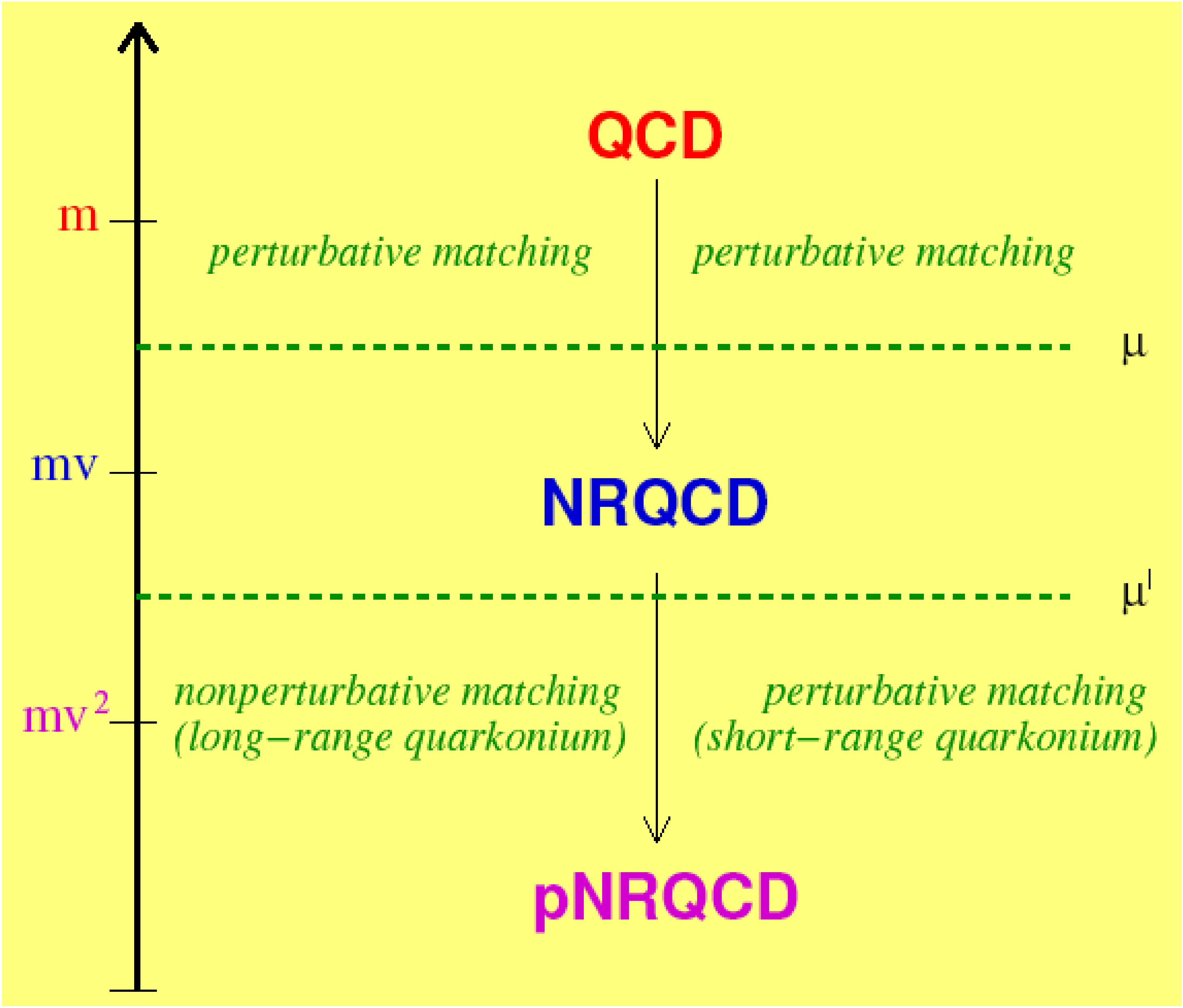

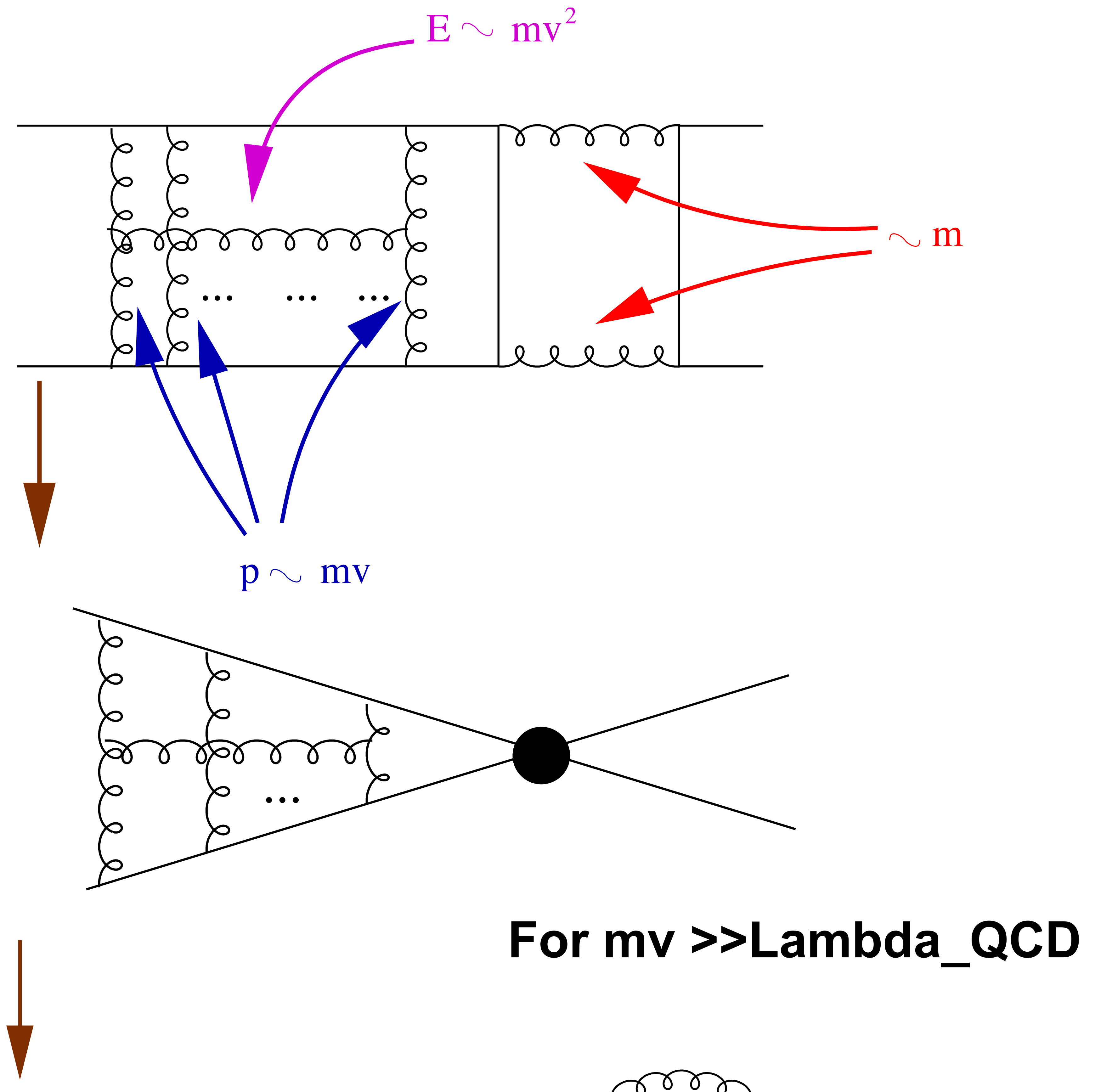

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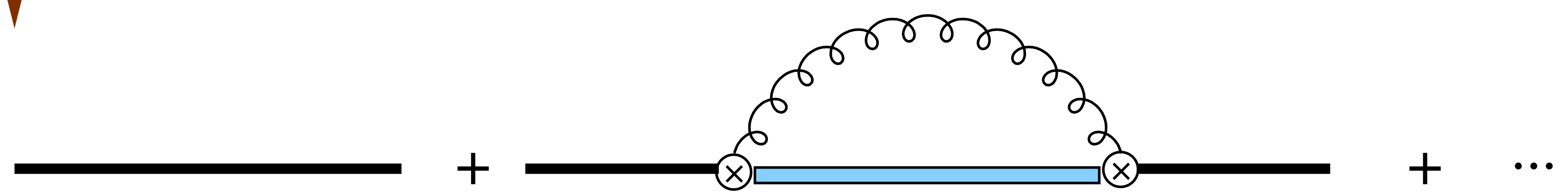

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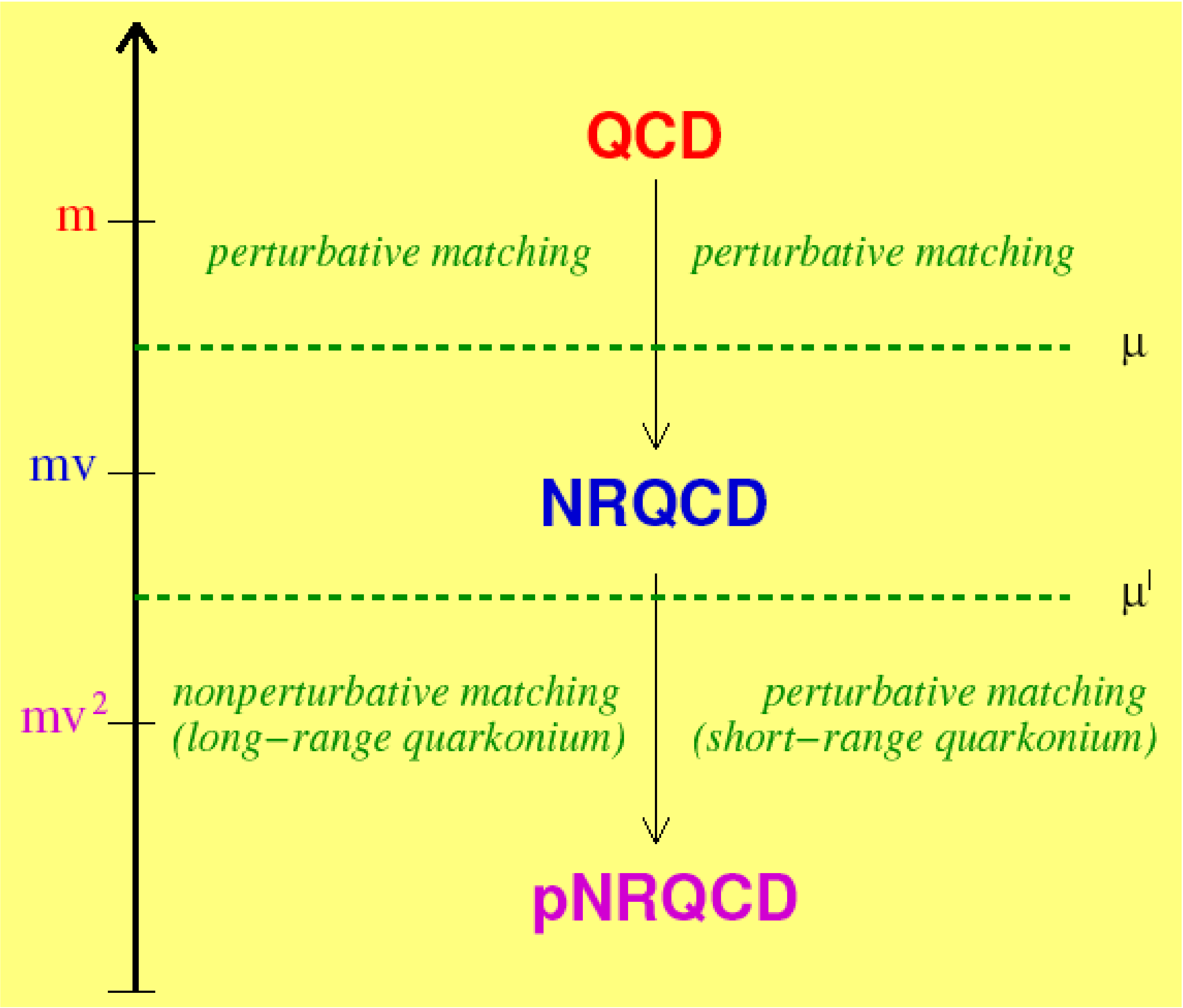

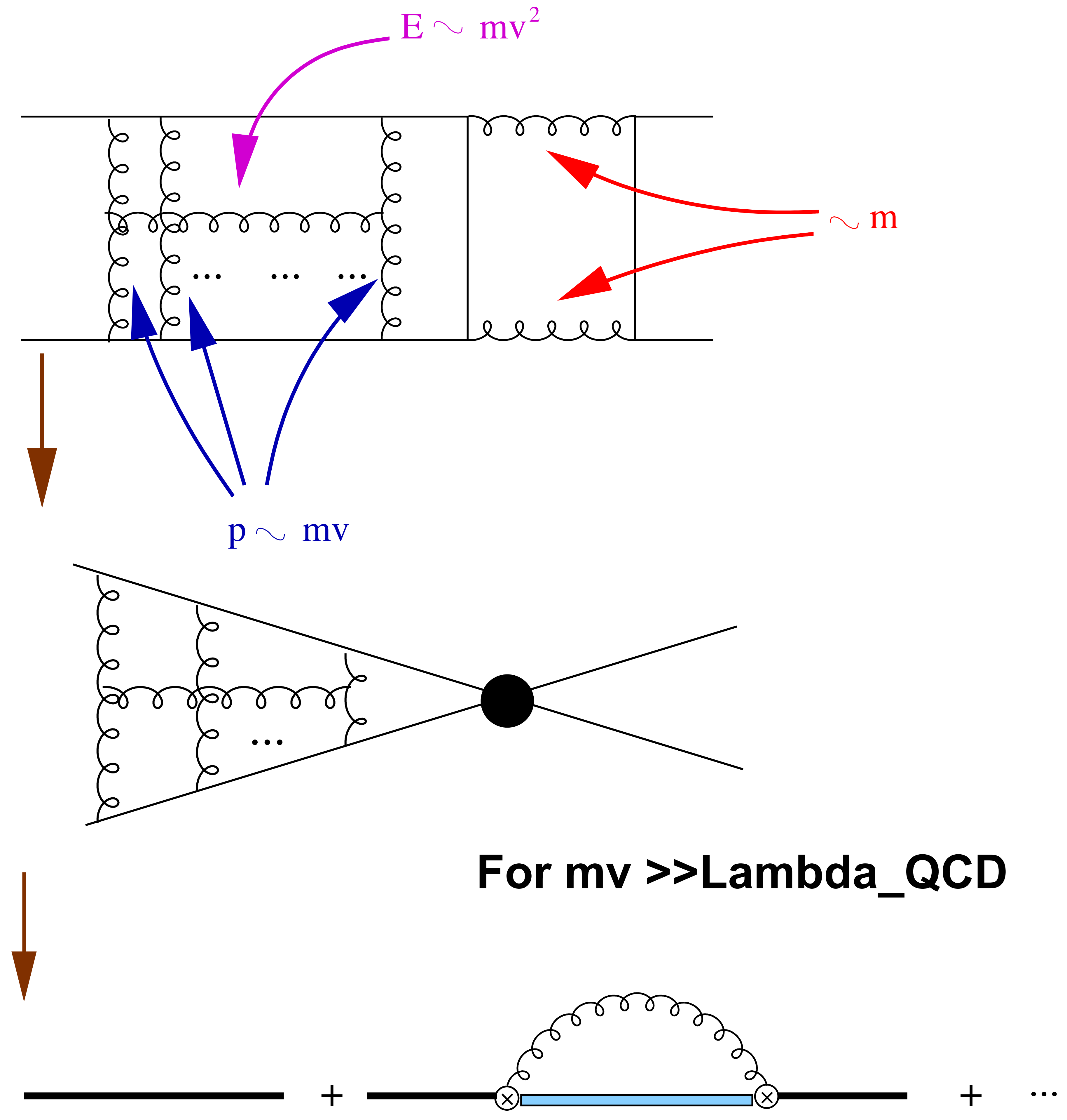

$$\mathcal{L}_{\mathrm{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

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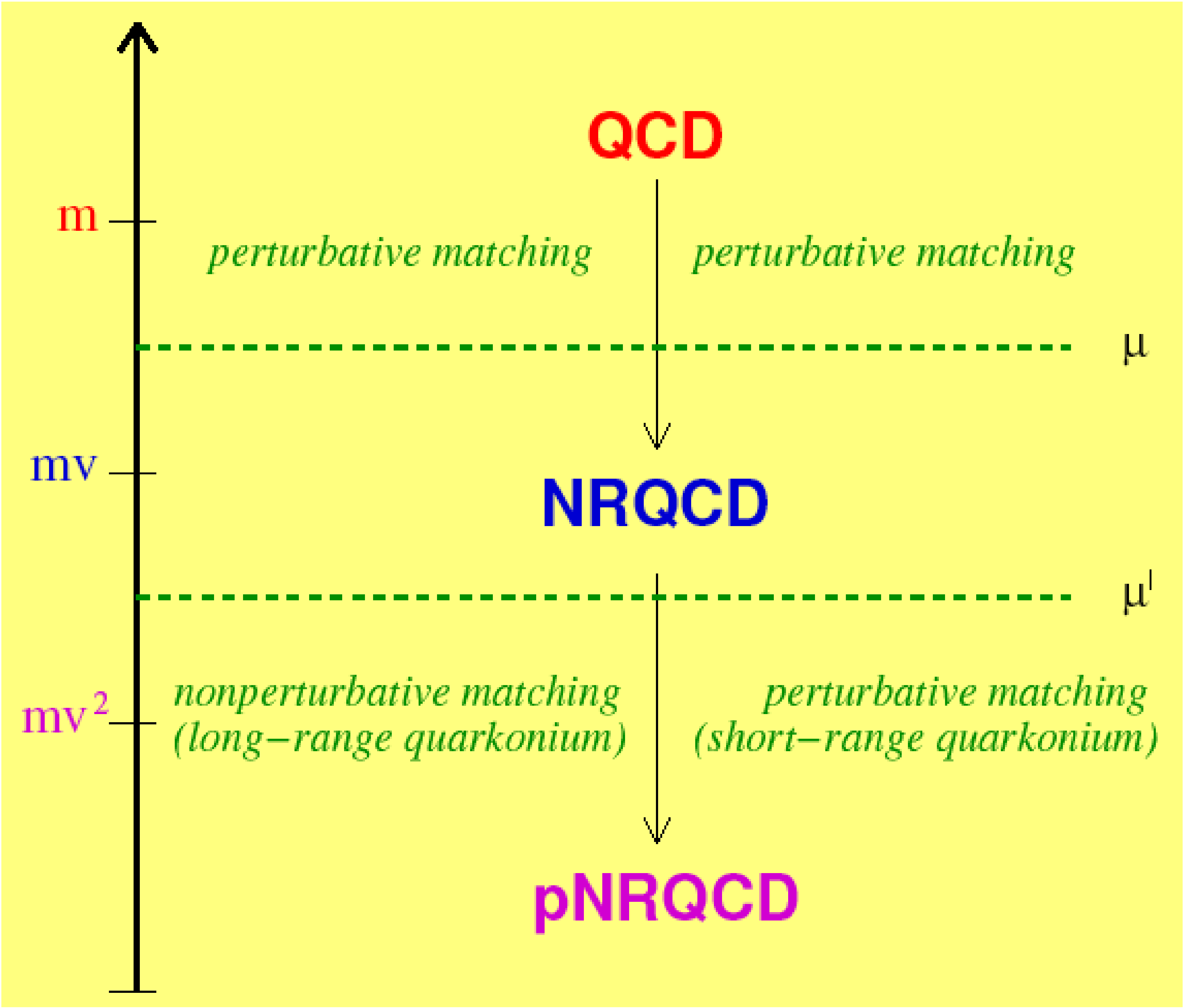

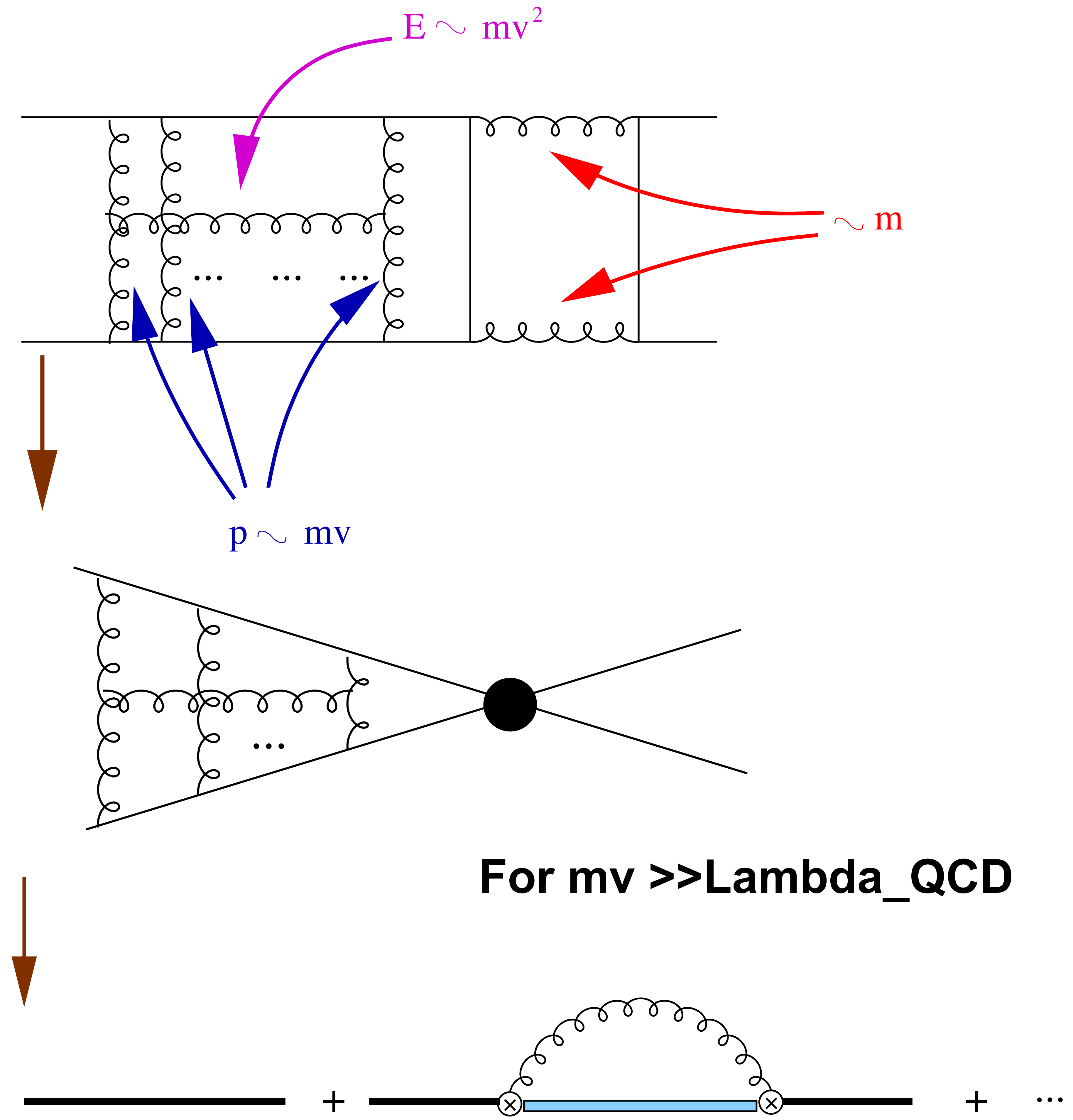

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Weakly coupled pNRQCD

○ Pineda Soto NP PS 64 (1998) 428
Brambilla Pineda Soto Vairo NPB 566 (2000) 275

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

  Non-analytic behaviour in $r \to$ matching coefficients V

The gauge fields are multipole expanded:
$A(R,r,t) = A(R,t) + \mathbf{r}\cdot\boldsymbol{\nabla}A(R,t) + \dots$

$\mathbf{R}$ = center of mass
$\mathbf{r}$ = $Q\bar{Q}$ distance

$$\mathcal{L}^{\rm pNRQCD} = \int d^3r \, {\rm Tr}\,\{S^\dagger(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \cdots)S + O^\dagger(iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \cdots)O +$$

LO in r

$$+V_A(S^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger\mathbf{r}\cdot g\mathbf{E}S) + \frac{V_B}{2}(O^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E})\} + \dots$$

NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

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The matching coefficients are the Coulomb potential

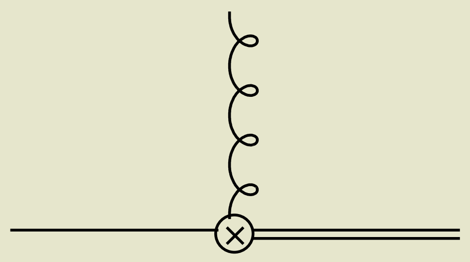
$$V_S(r) = -C_F\frac{\alpha_s}{r} + \dots, \quad V_o(r) = \frac{1}{2N}\frac{\alpha_s}{r} + \dots,$$

$$V_A = 1 + \mathcal{O}(\alpha_s^2),\ V_B = 1 + \mathcal{O}(\alpha_s^2).$$

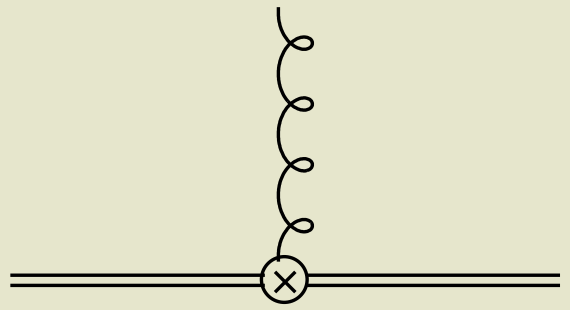
Feynman rules

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m+V)}$$

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m+V_o)}\left(e^{-i\int dt\, A^{\rm adj}}\right)$$



$= {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm S}$



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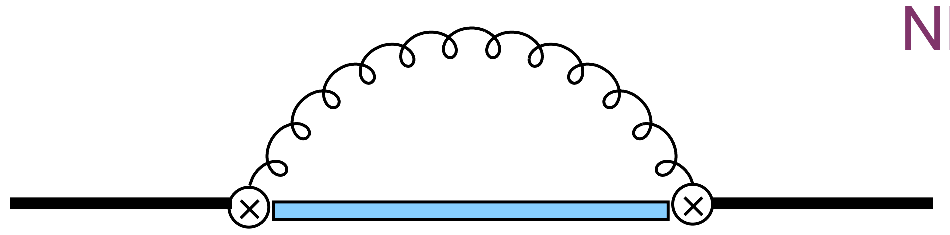
Using weakly coupled pNRQCD quarkonium becomes a tool for precision physics in QCD

$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02
NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

Energies at order m alpha^5 (NNNLO)

$$E_n = 2m + \langle n | \frac{p^2}{m} + V_s | n \rangle + \langle n |$$  $$| n \rangle$$

$$\sim e^{i\Lambda_{\rm QCD} t}$$

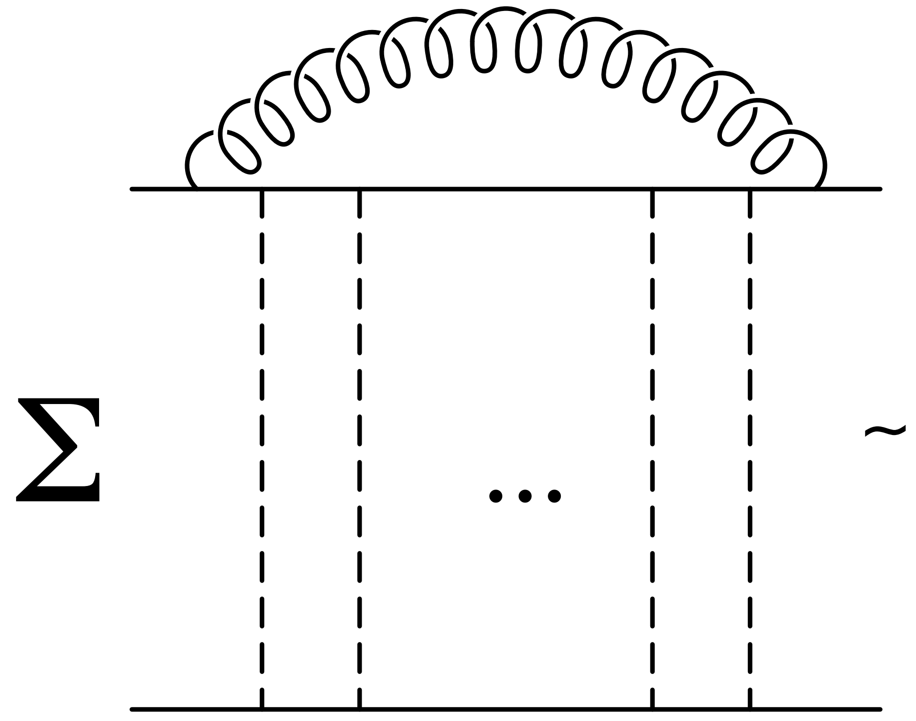
$$E_n = \langle n | H_s(\mu) | n \rangle - i \frac{g^2}{3N_c} \int_0^\infty dt \, \langle n | \mathbf{r} e^{it(E_n^{(0)} - H_o)} \mathbf{r} | n \rangle \, \langle \mathbf{E}(t)\, \mathbf{E}(0) \rangle(\mu)$$

$E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0) \rangle$

local condensates as predicted in a paper by Misha Voloshin in 1979

—>used to extract precise (NNNLO) determination of m_c and m_b

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED



$\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

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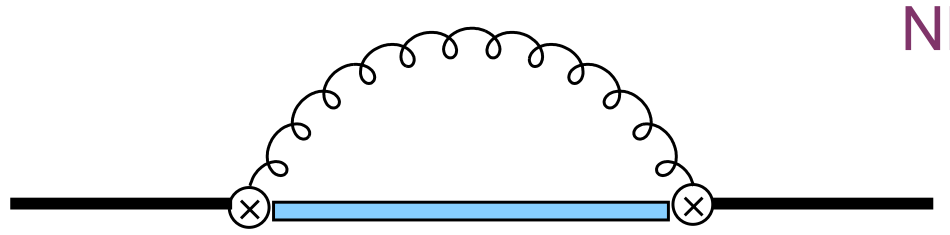
$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

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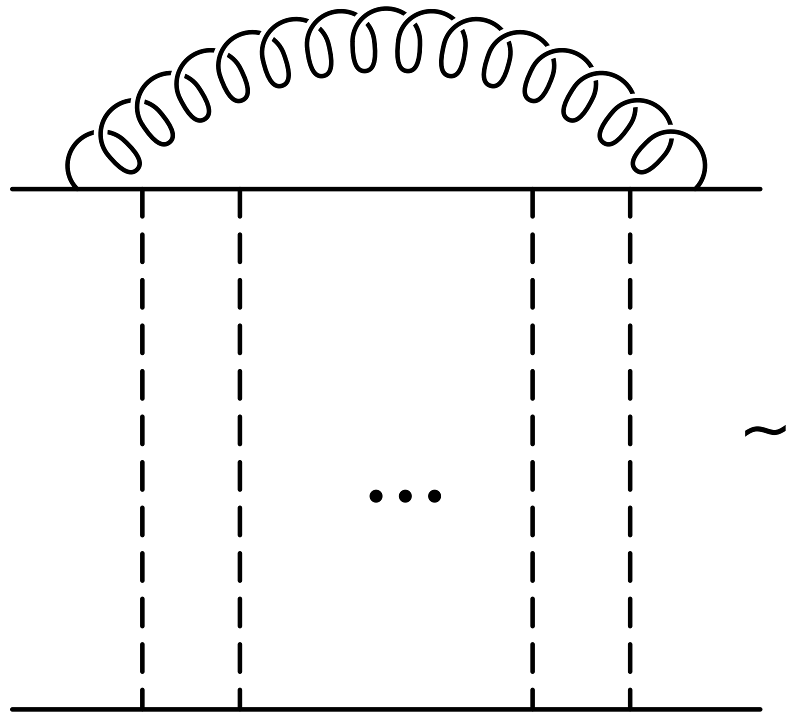
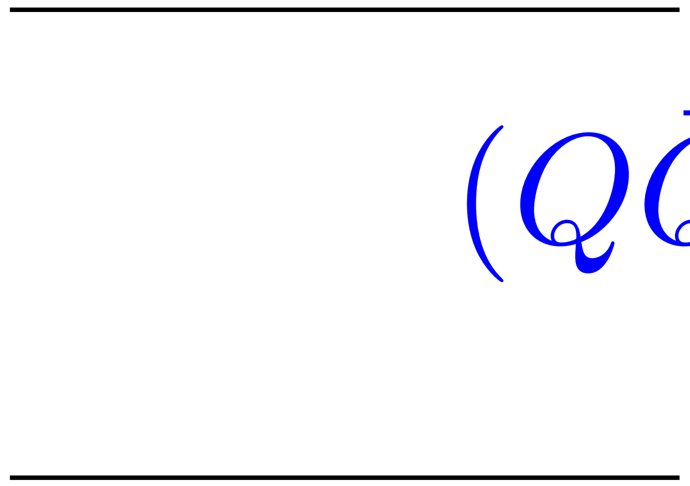
Σ  $\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

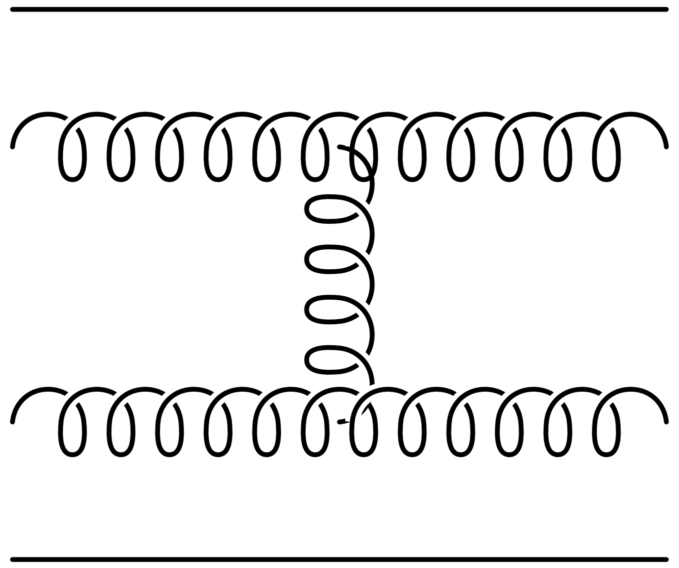
Applications of weakly coupled pNRQCD include: precise alphas extraction from the static energy,ttbar production, quarkonia spectra, decays, E1 and M1 transitions, QQq and QQQ energies, thermal masses and potentials

Strongly coupled pNRQCD — Hitting the scale $\Lambda_{\rm QCD}$ $\quad r \sim \Lambda_{QCD}^{-1}$

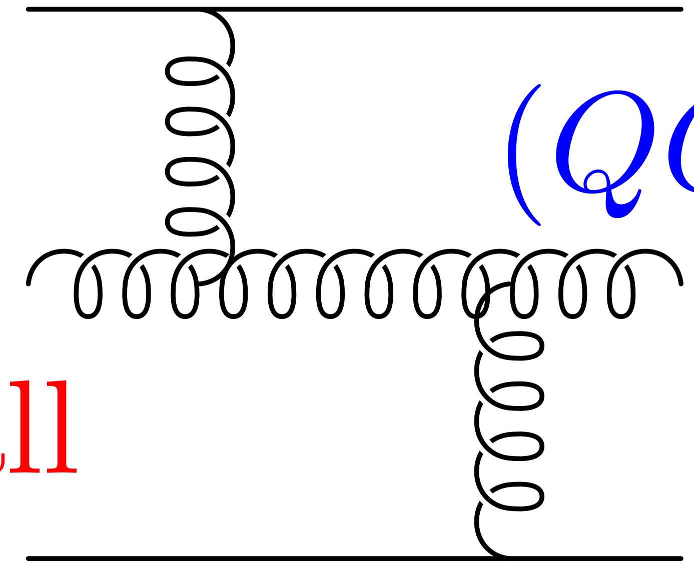
The degrees of freedoms now are



$(Q\bar{Q})_1$



$(Q\bar{Q})_1 + \mathrm{Glueball}$



$(Q\bar{Q})_8 G$

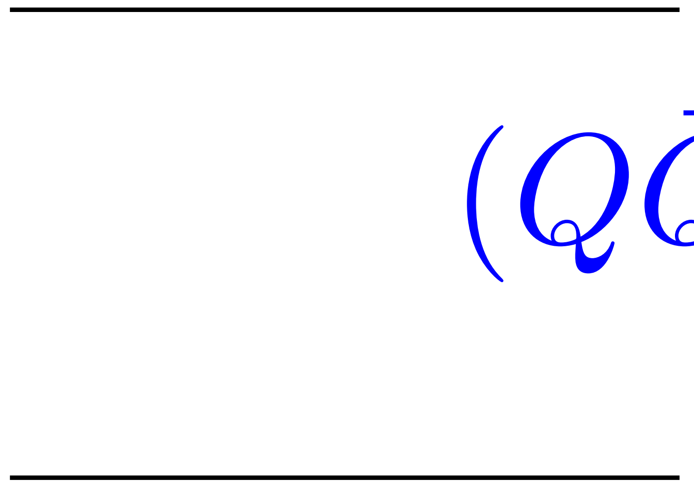
hybrid /tetraquarks

with gluons/light quarks at the scale $\Lambda_{\rm QCD}$ —>nonperturbative problem, use lattice

Strongly coupled pNRQCD

Hitting the scale Λ_{QCD} $r \sim \Lambda_{QCD}^{-1}$

The degrees of freedoms now are



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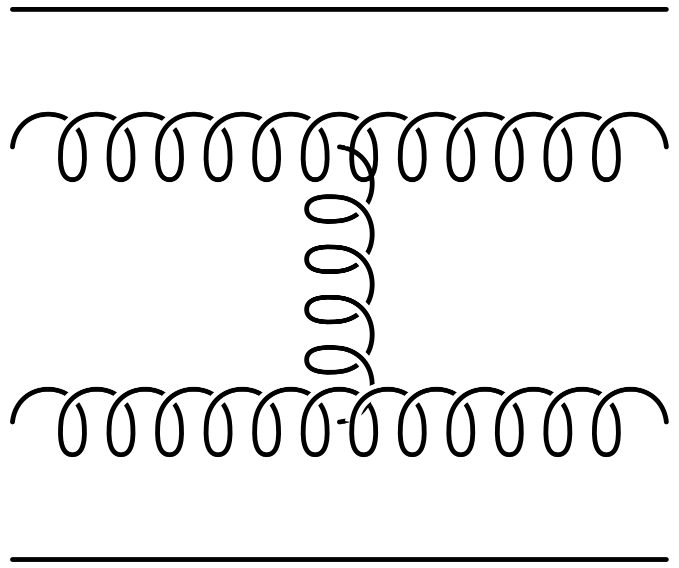

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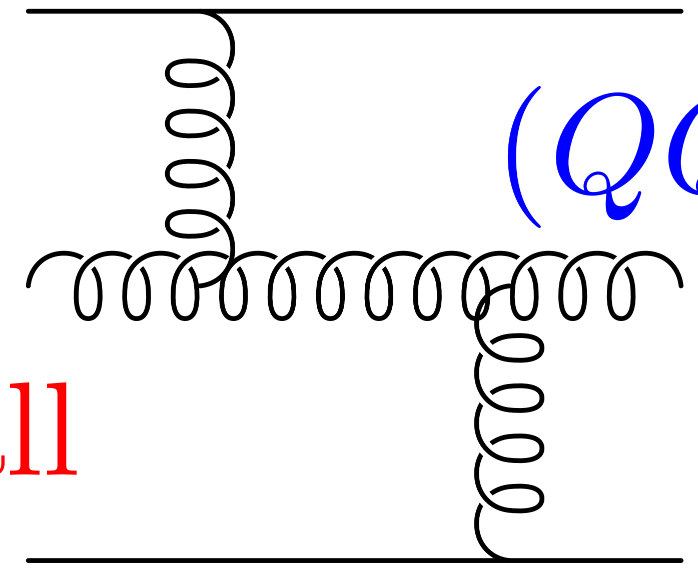


$(Q\bar{Q})_8 G$

hybrid /tetraquarks

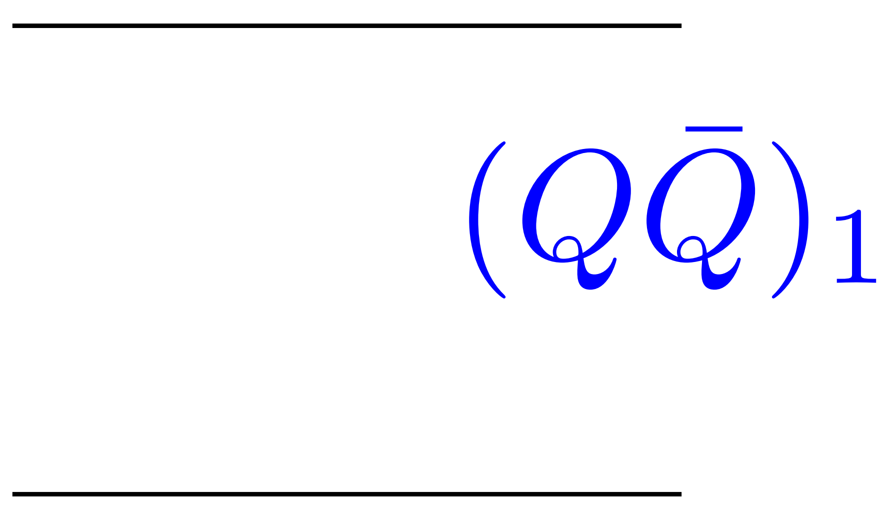
with gluons/light quarks at the scale Λ_{QCD} —>nonperturbative problem, use lattice

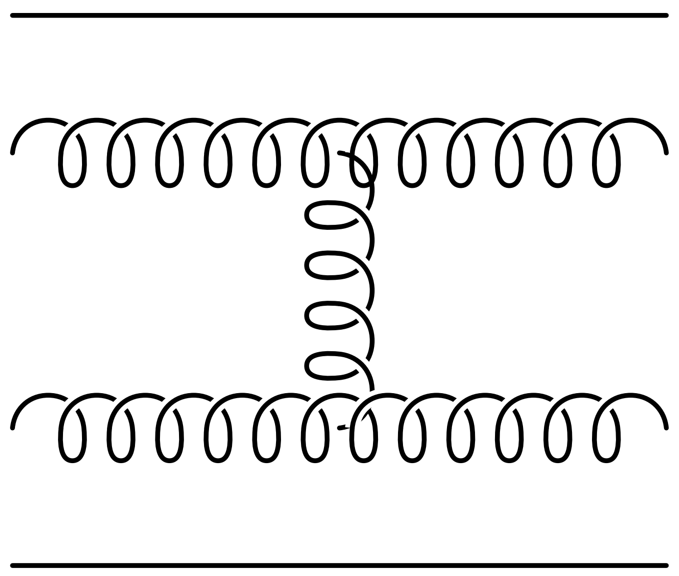
At or close to the the strong decay threshold XYZ EXOTICS appear: light quark and glue degrees of freedom become dynamical at this nonperturbative scale and can participate in the binding

Strongly coupled pNRQCD

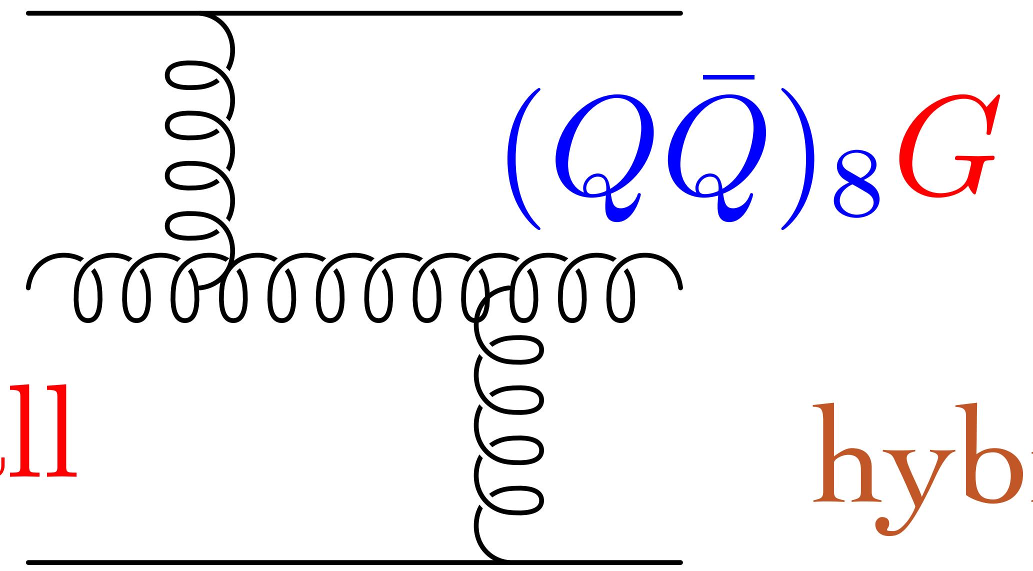
Hitting the scale $\Lambda_{\rm QCD}$ $r \sim \Lambda_{QCD}^{-1}$

The degrees of freedoms now are





$(Q\bar{Q})_1$ + Glueball



hybrid /tetraquarks

with gluons/light quarks at the scale $\Lambda_{\rm QCD}$ —>nonperturbative problem, use lattice

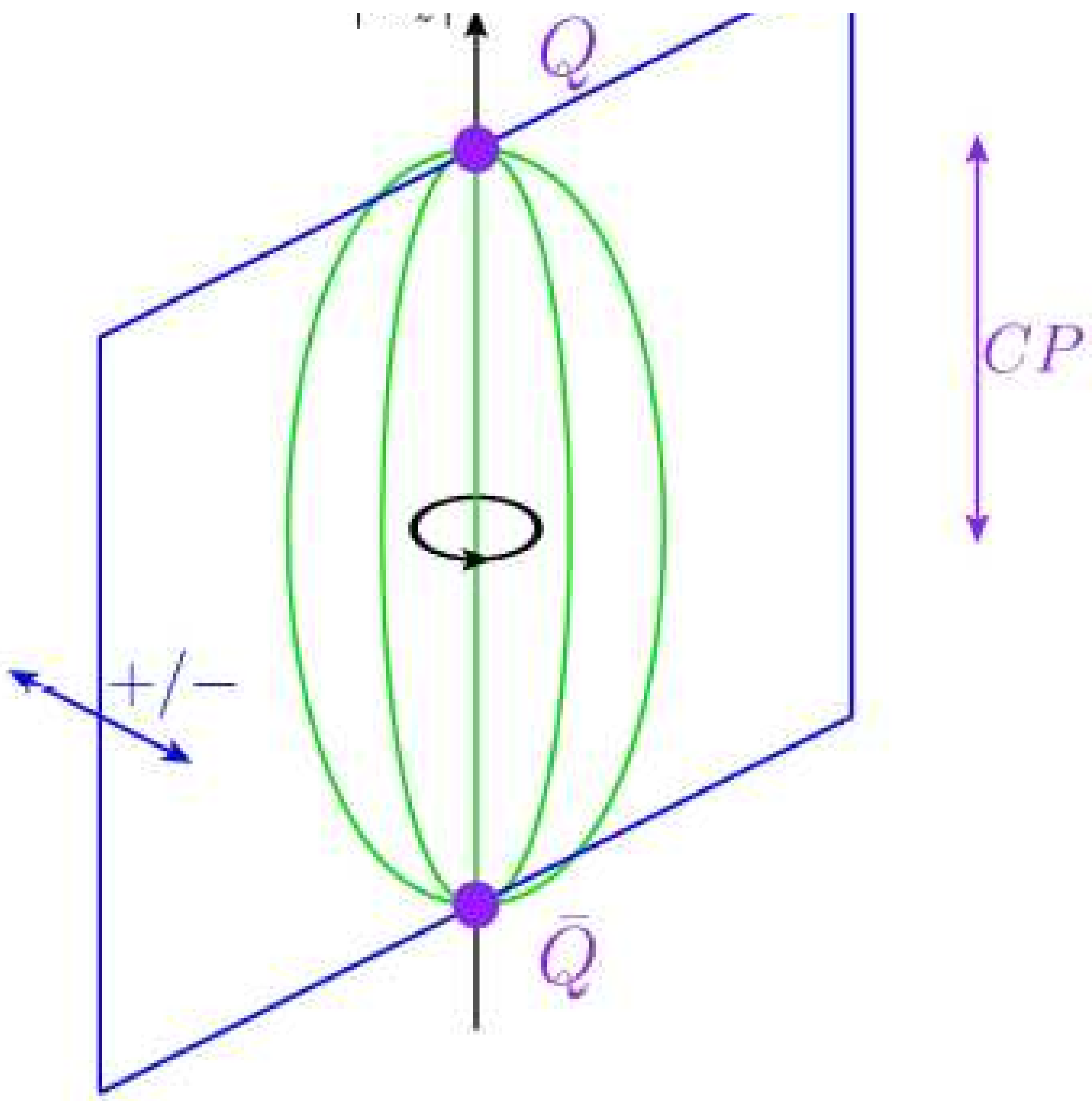
At or close to the the strong decay threshold XYZ EXOTICS appear: light quark and glue degrees of freedom become dynamical at this nonperturbative scale and can participate in the binding

Use symmetry and scale separation: $m > \Lambda_{QCD}$ NRQCD holds

$\Lambda_{QCD} > mv^2$ fast (gluons, light quarks) and slow (heavy quarks)

like in molecular physics (fast-electrons, slow nuclei)

The spectrum of static energies can be calculated in NRQCD



Symmetry of a system with a static Q in x_1 and a Qbar in x_2

Irreducible representations of $D_{\infty h}$

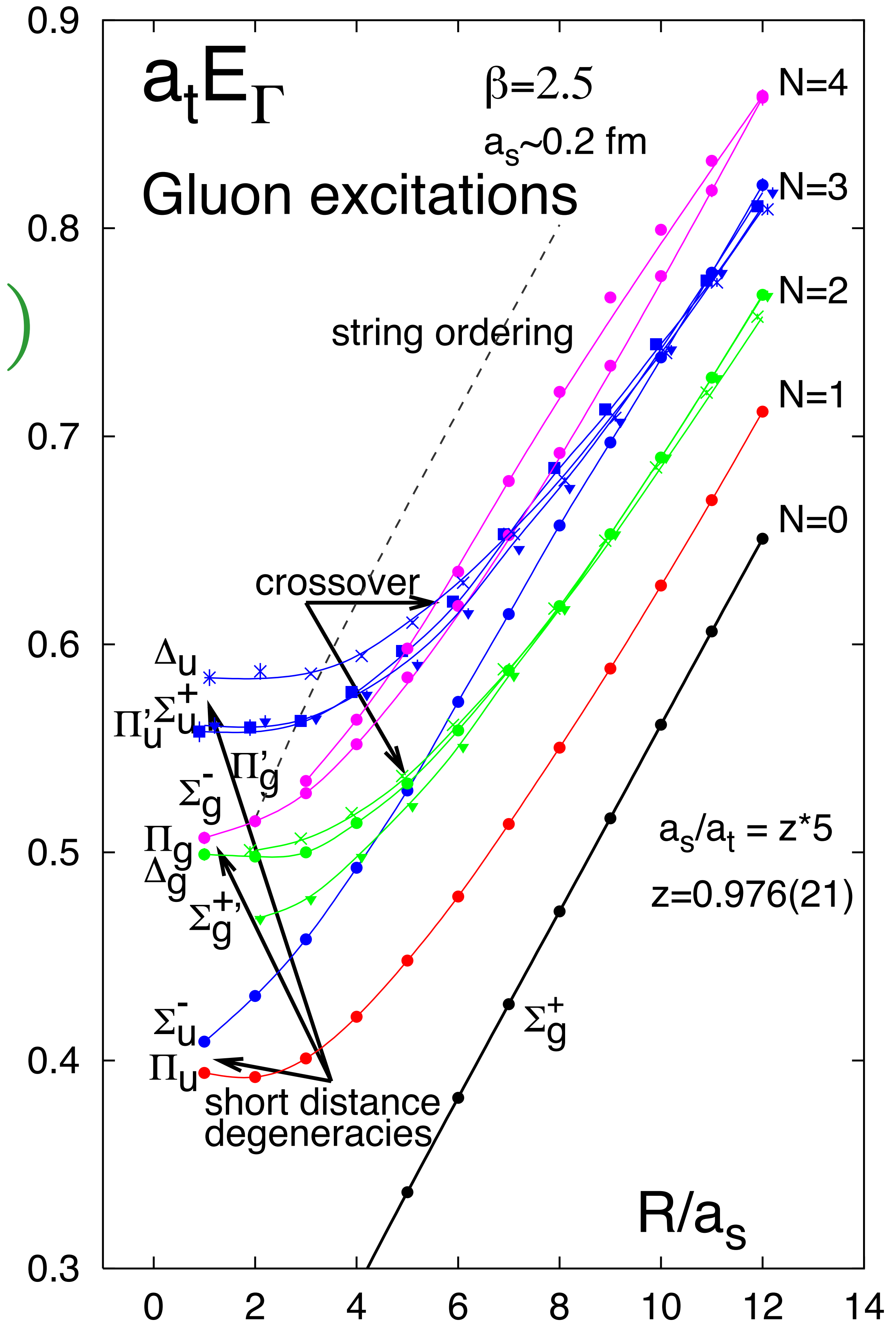
- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \; (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g), \; -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

$$\Lambda^{\sigma}_{\eta}$$

The spectrum of static energies can be calculated in NRQCD

Lattice Spectrum of NRQCD hybrid static energies E^0_n

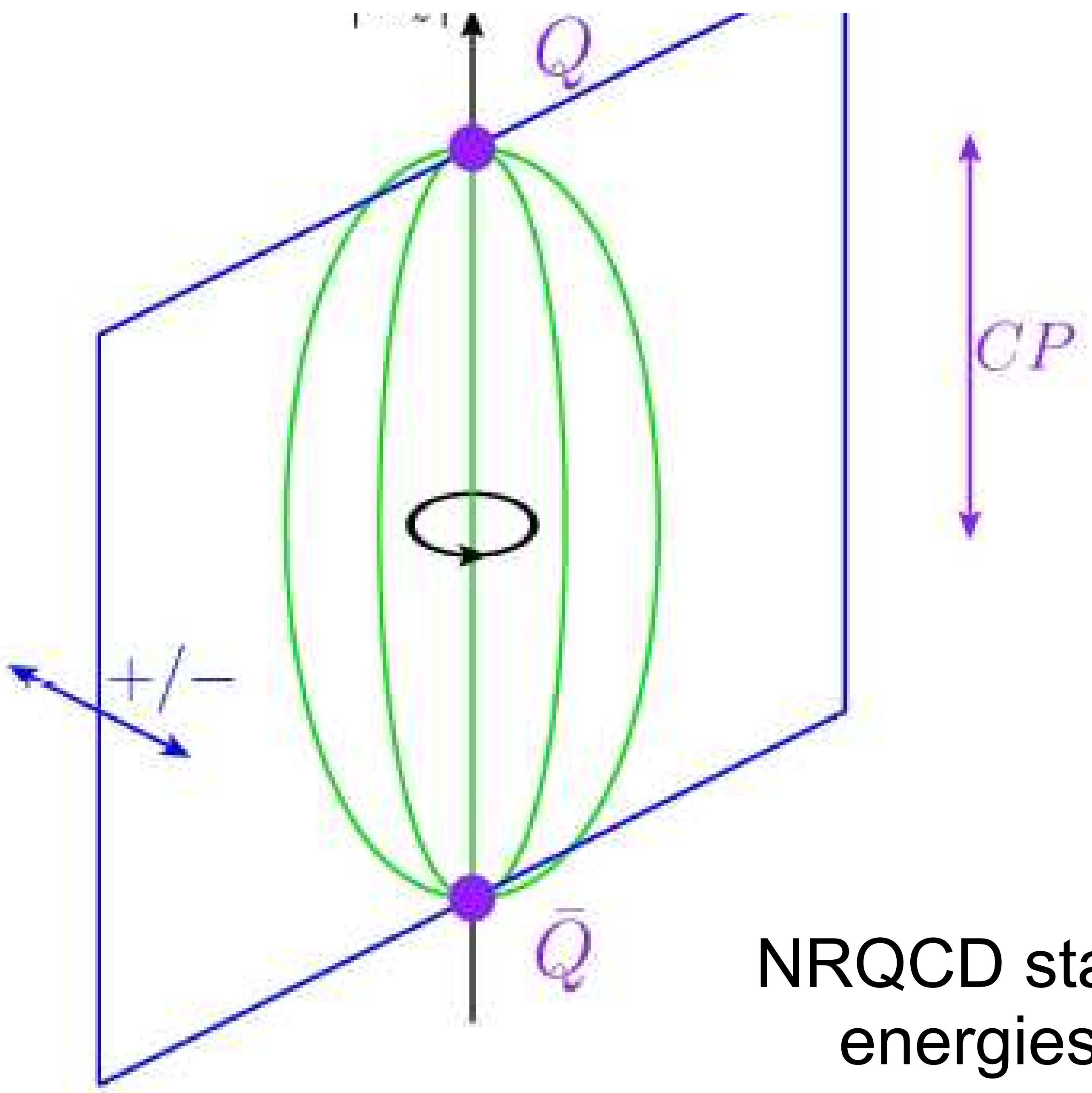
$E_n^{(0)}$



$r = |\mathbf{x}_1 - \mathbf{x}_2|$

Juge Kuti Mornigstar 98-06

Schlosser, Wagner 2111.00741, Bali Pineda 2004



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$$\Lambda^\sigma_\eta$$

$$\mathcal{H}^{(0)} = \int d^3\mathbf{x}\, \frac{1}{2}\left(\mathbf{\Pi}^a\mathbf{\Pi}^a + \mathbf{B}^a\mathbf{B}^a\right) - \sum^{n_f} \bar{q}\, i\mathbf{D}\cdot\boldsymbol{\gamma}\, q$$

NRQCD static energies

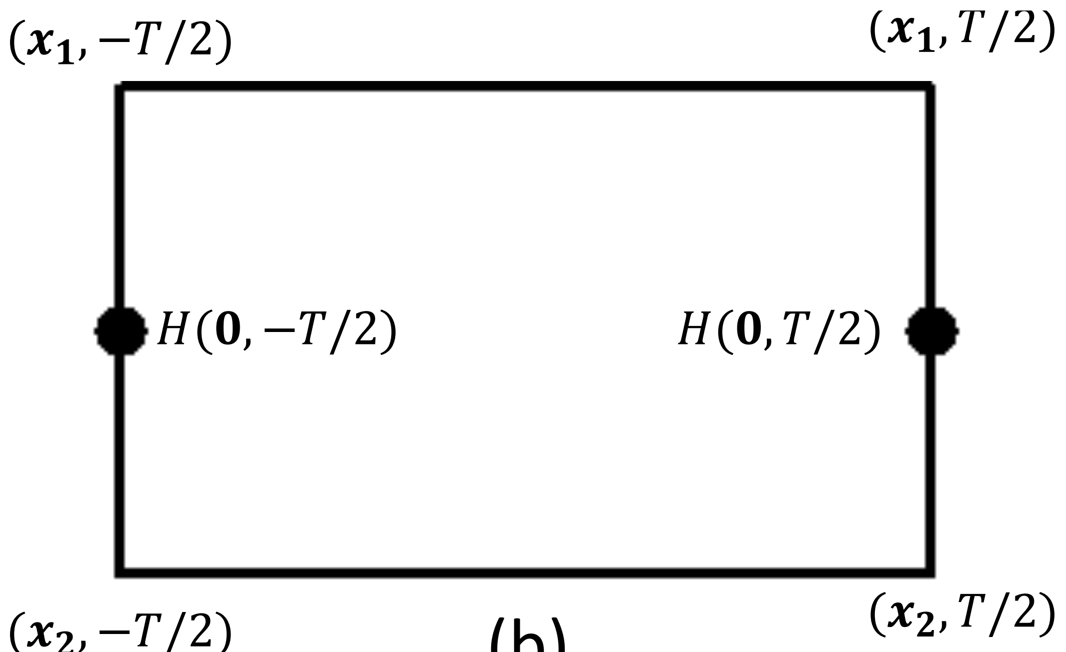
$$\mathcal{H}^{(0)}|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2)|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$E_n^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_n, T/2 | X_n, -T/2\rangle$$

$$|X_n\rangle = \chi(\mathbf{x_2})\phi(\mathbf{x_2}, \mathbf{R})T^a H^a(\mathbf{R})\phi(\mathbf{R}, \mathbf{x}_1)\psi^\dagger(\mathbf{x_1})|\mathrm{vac}\rangle$$

Phi =Wilson lines and H= gluonic and light quarks



(b)

o Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 034502

The spectrum of static energies can be calculated in NRQCD

Symmetry of a system with a static Q in x_1 and a Qbar in x_2

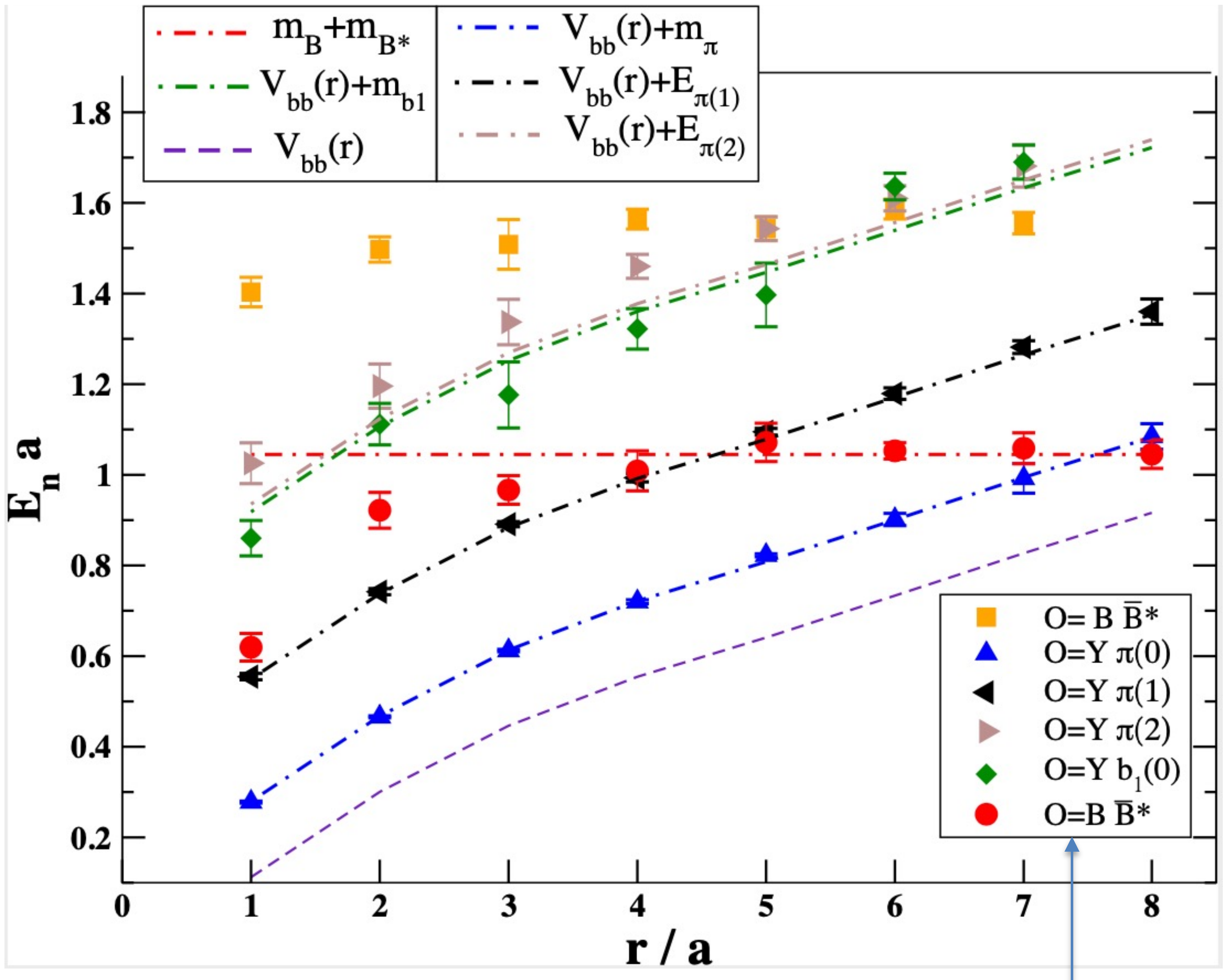
Tetraquark static energies

$\bar{b}\,b\,\bar{d}\,u$

Z_b channe $\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$

Sadl, Prevlosek,

Eigen-energies $E_n(r)$: channel S_h=1, CP=-1, ε=-1



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$$\Lambda_\eta^\sigma$$

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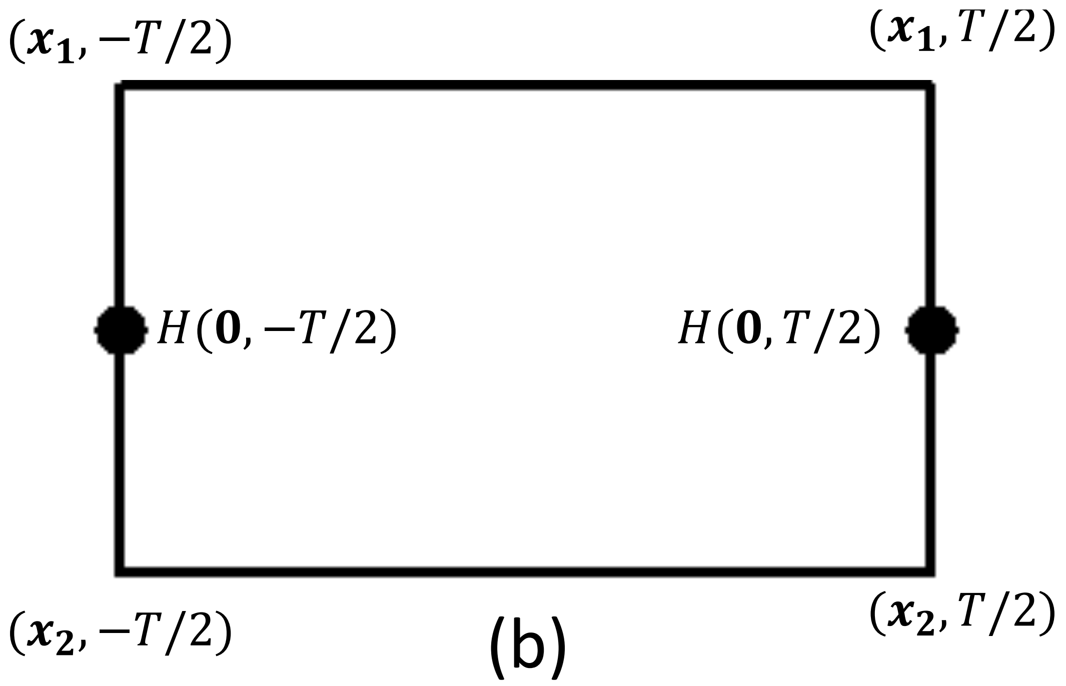
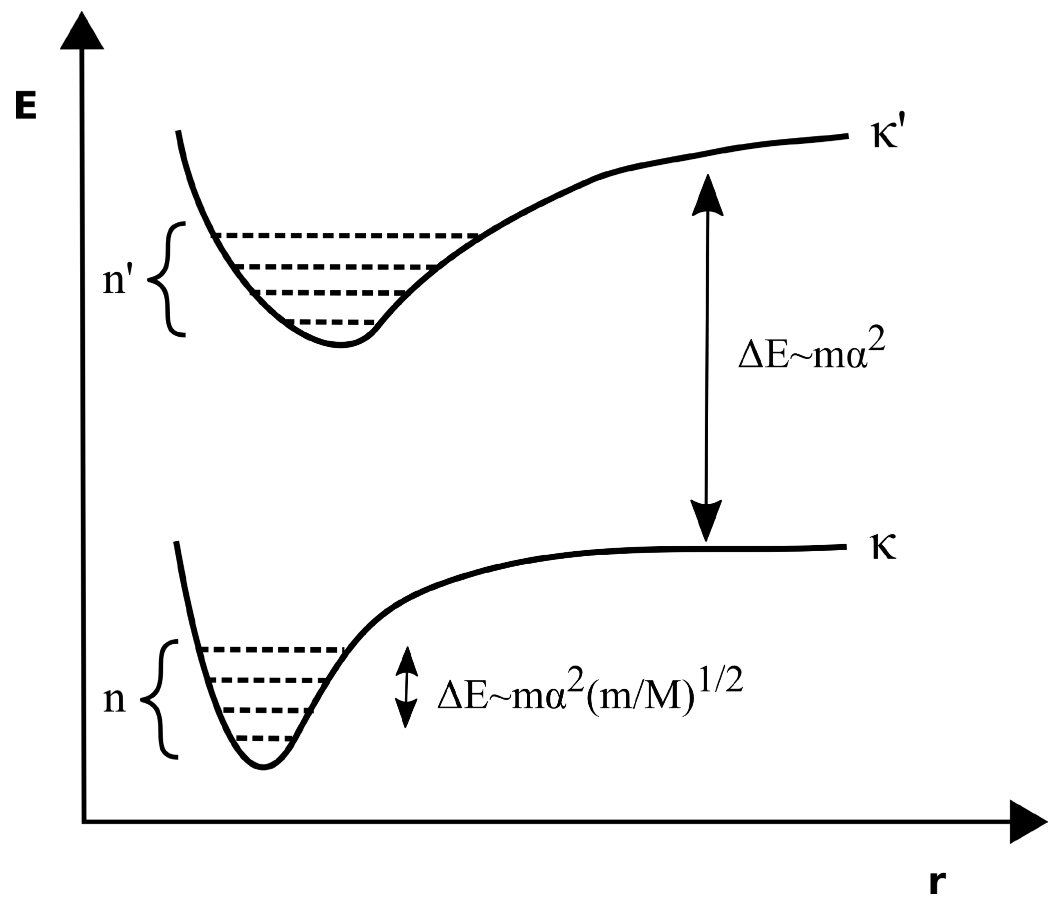


(b)

Notice: in presence of light quark in the binding one adds ispospin quantum numbers and measure tetraquark static energies



QED

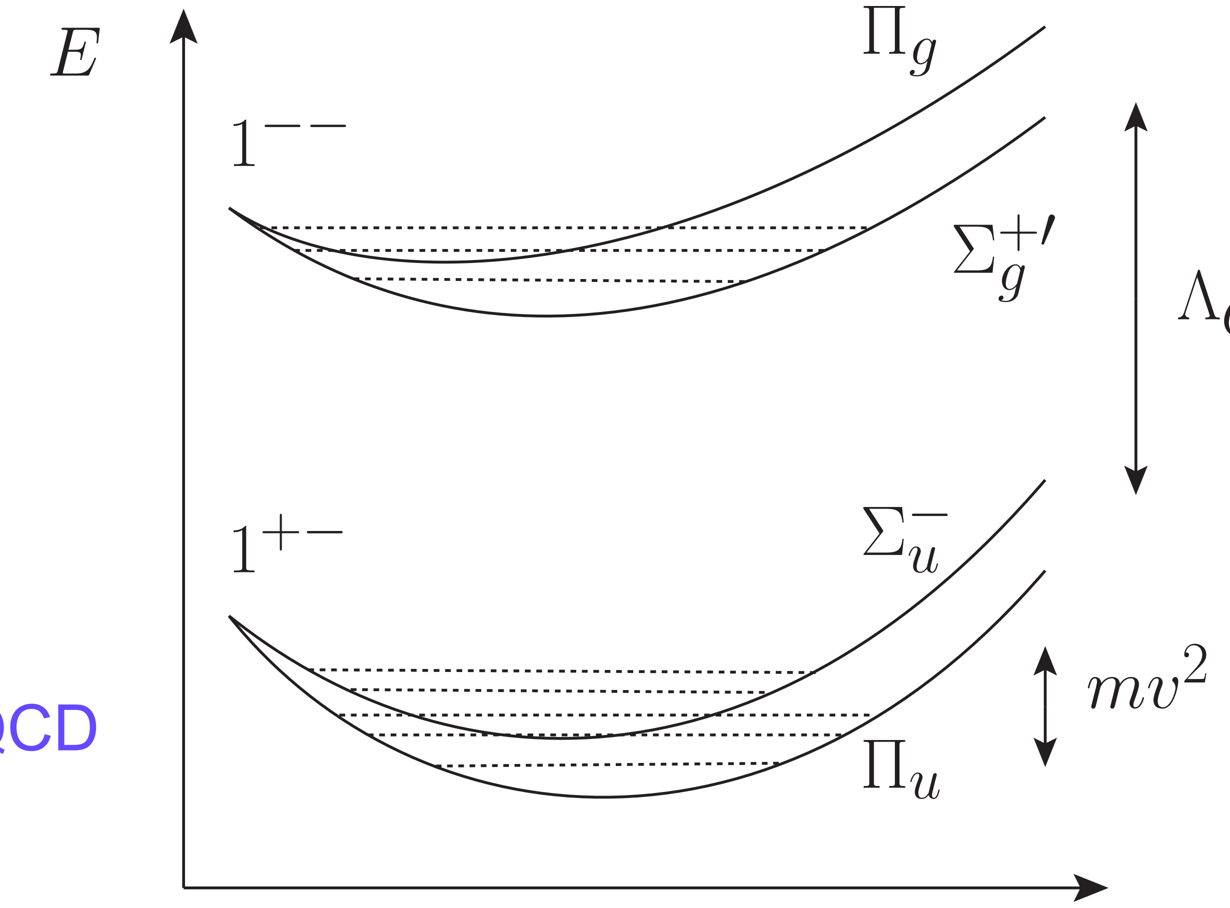
$$\Lambda_{\rm QCD} \gg mv^2$$

Braaten PRL 111 (2013) 162003

Braaten Langmack Smith PRD 90 (2014) 014044

Born Oppenheimer Description

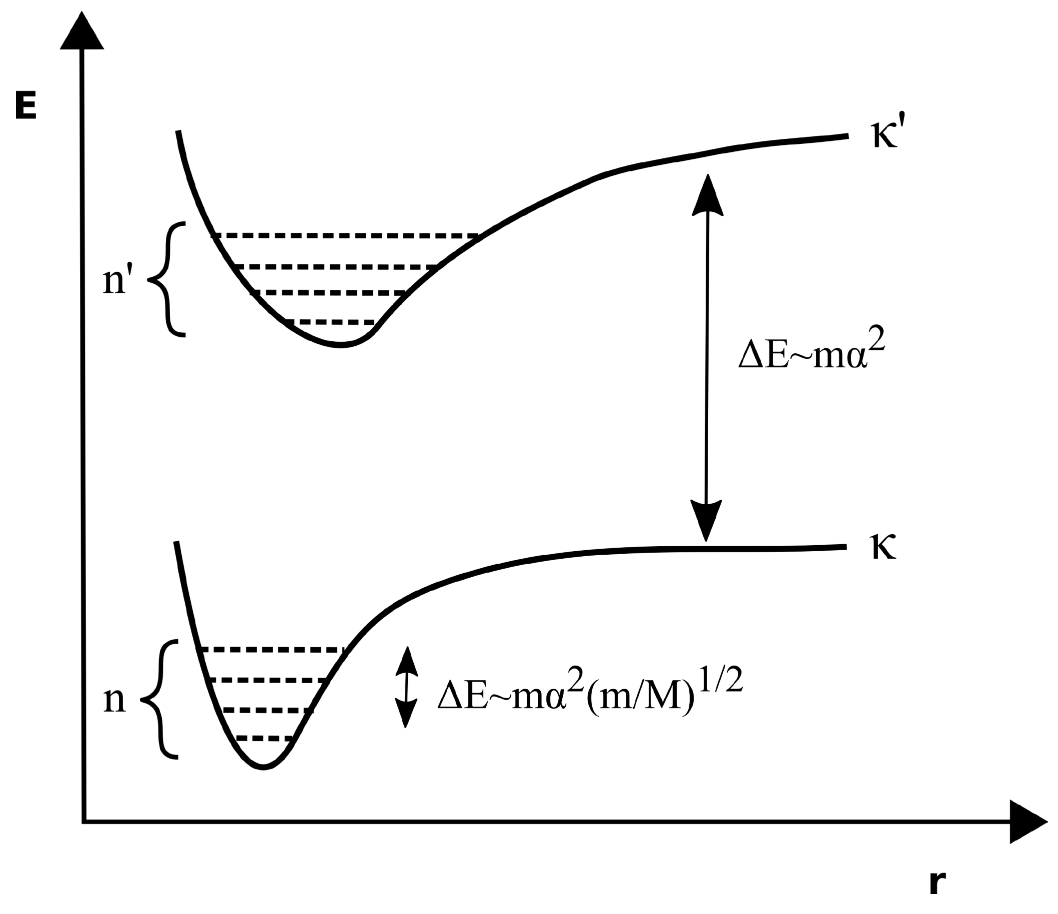
Higher excitations develop a gap of order Lambda_QCD



QCD

Introducing a finite mass m:

- The spectrum of the mv^2 fluctuations around the lowest static energy is the quarkonium spectrum
- The spectrum of the mv^2 fluctuations around the higher excitations is the exotic spectrum (hybrids and tetraquarks)



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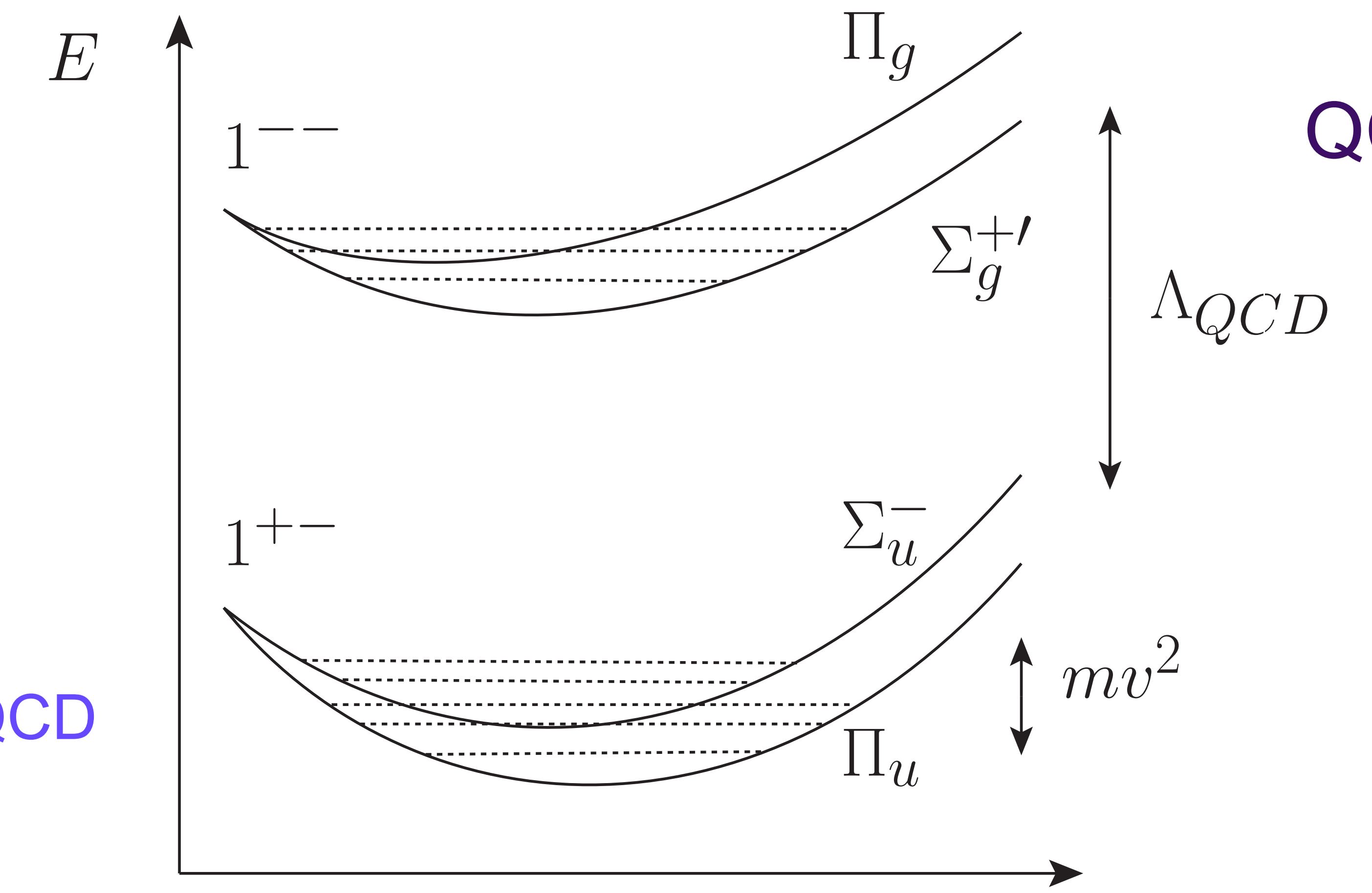

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Nonperturbative matching to the pNREFT

$|\underline{0}; \mathbf{x}_1\mathbf{x}_2\rangle -> \ |(Q\bar{Q})_1\rangle$ → Quarkonium Singlet

$E_0(r) -> V_0(r)$ pNRQCD

$|\underline{n} > 0; \mathbf{x}_1\mathbf{x}_2\rangle -> \ |(Q\bar{Q})g^{(n)}\rangle$ → Higher Gluonic Excitations

$|Q\bar{Q}q\bar{q}\rangle$ Tetraquarks

$E_n^{(0)}(r) -> V_n^{(0)}(r)$ BOEFT

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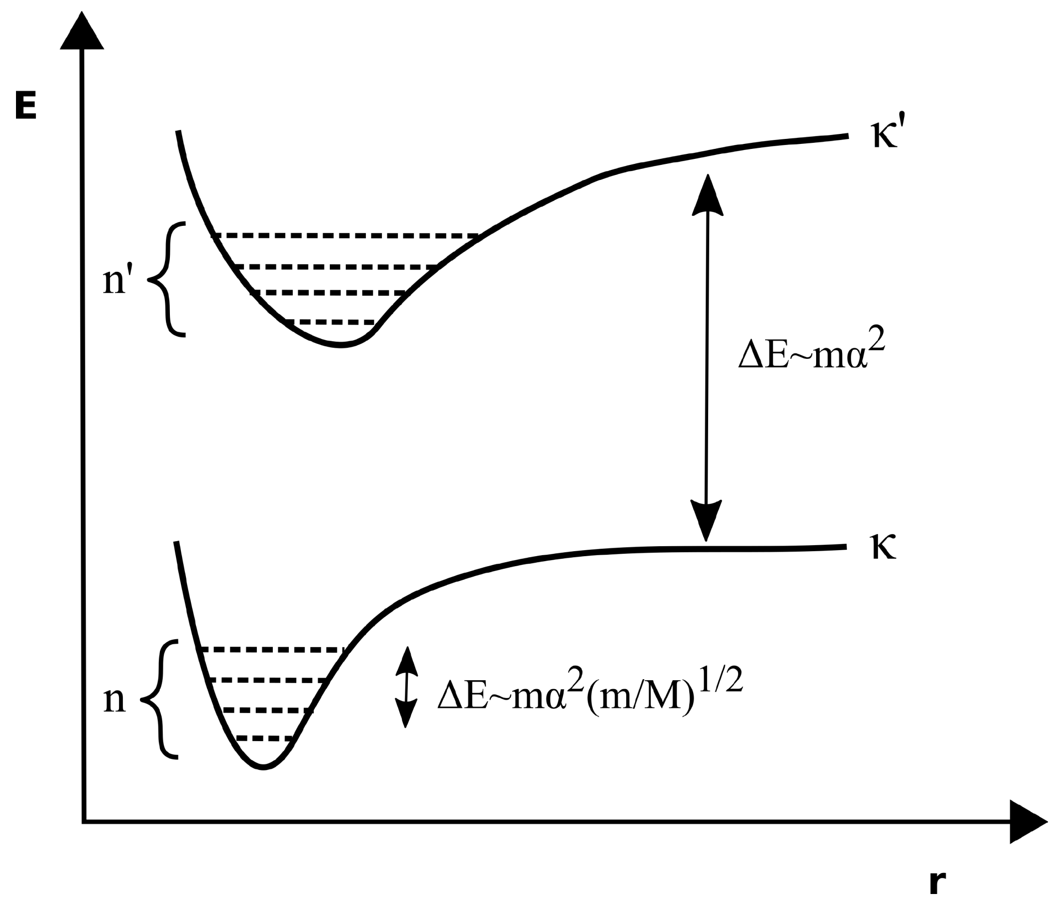

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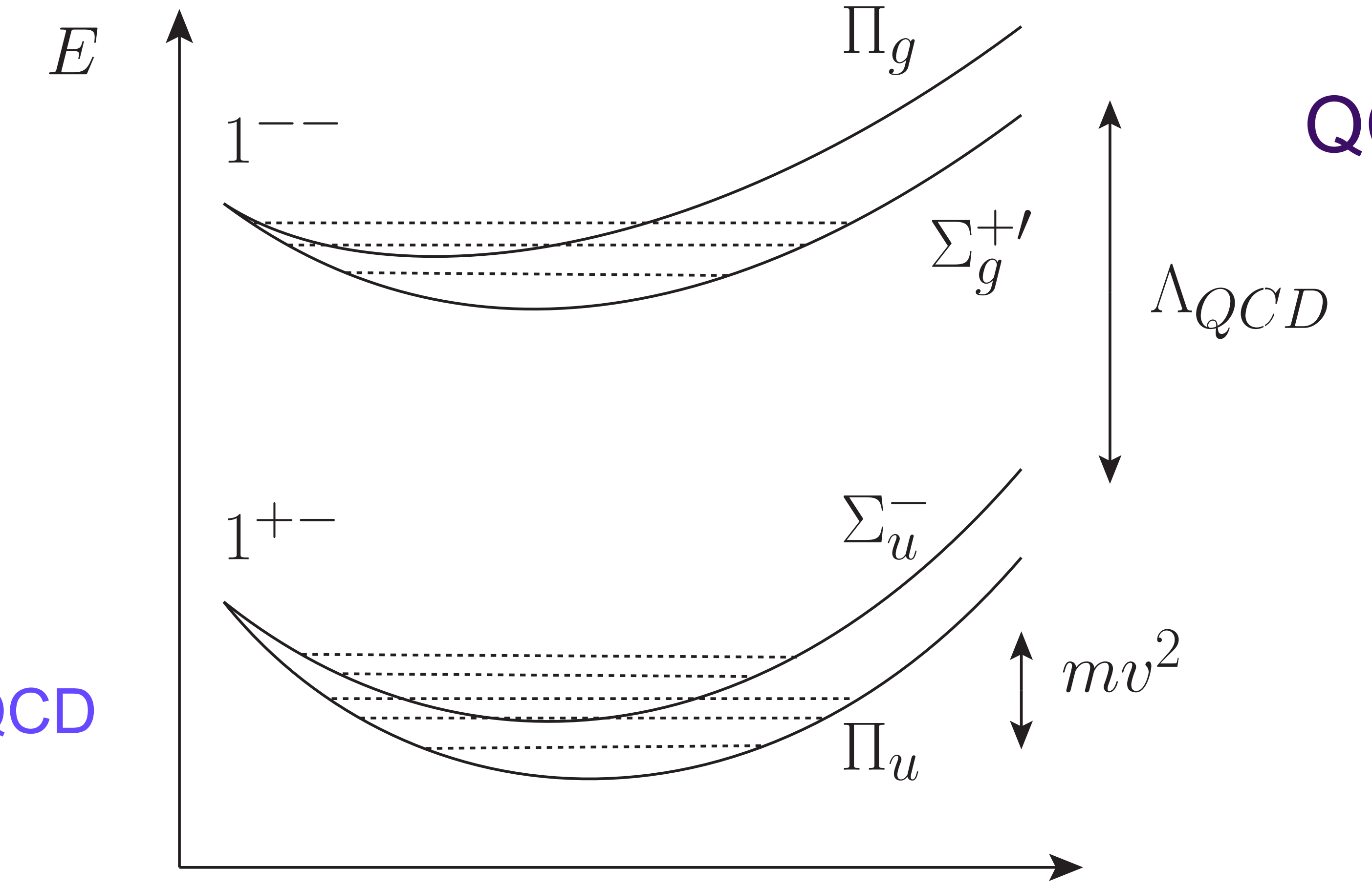

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Nonperturbative matching to the pNREFT

systematically

$$\langle H|\mathcal{H}|H\rangle = \langle nljs|\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}|nljs\rangle$$

expand quantomechanically NRQCD states and energies in 1/m around the zero order and identify the QCD potentials

$|\underline{0};\mathbf{x}_1\mathbf{x}_2\rangle -> \quad |(Q\bar{Q})_1\rangle \rightarrow$ Quarkonium Singlet

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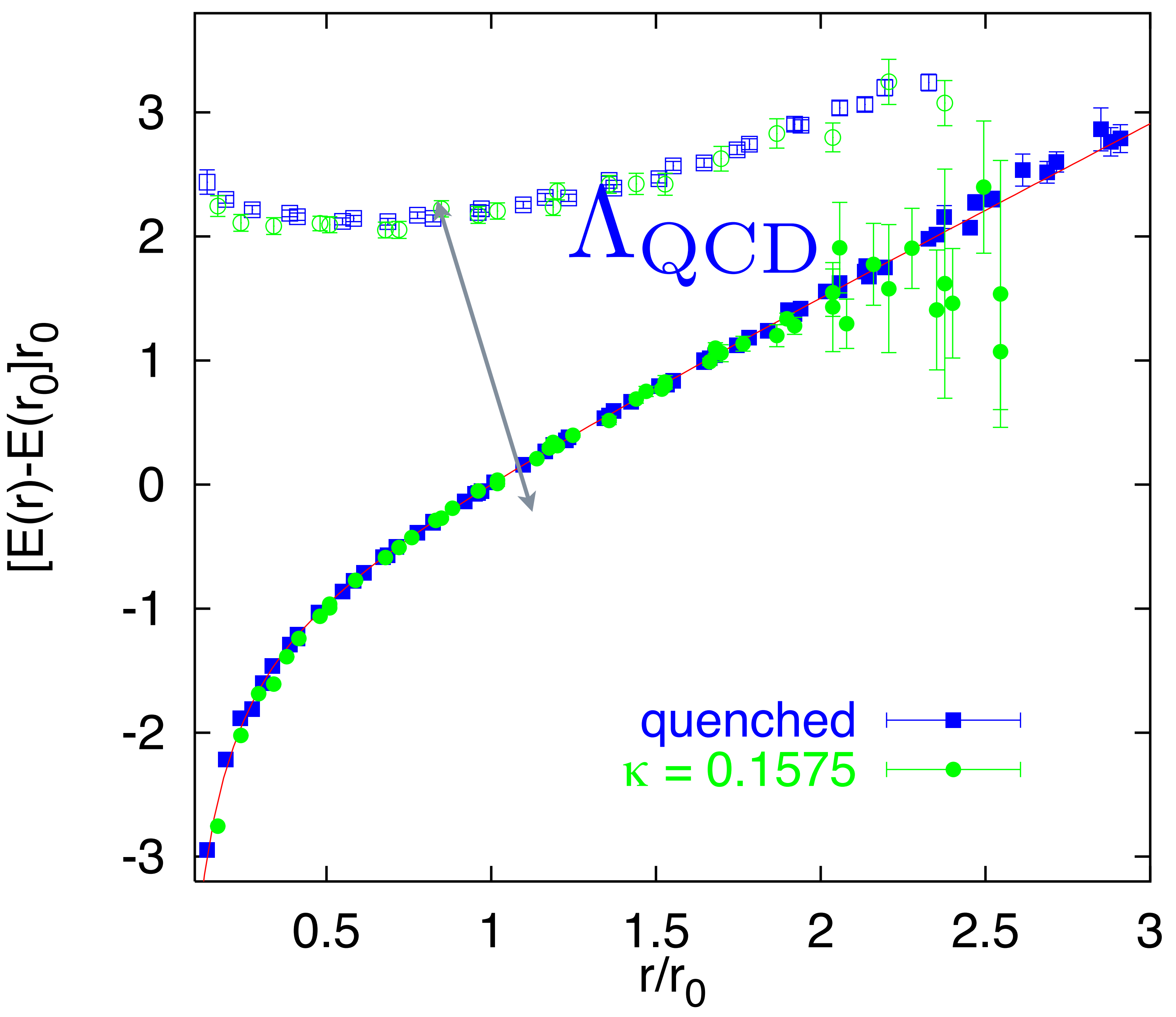
Strongly coupled pNRQCD for quarkonium

$mv \sim \Lambda_{QCD}$



Bali et al. 98

pNRQCD and the potentials come from integrating out all scales up to mv^2

- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

Brambilla Pineda Soto Vairo 00

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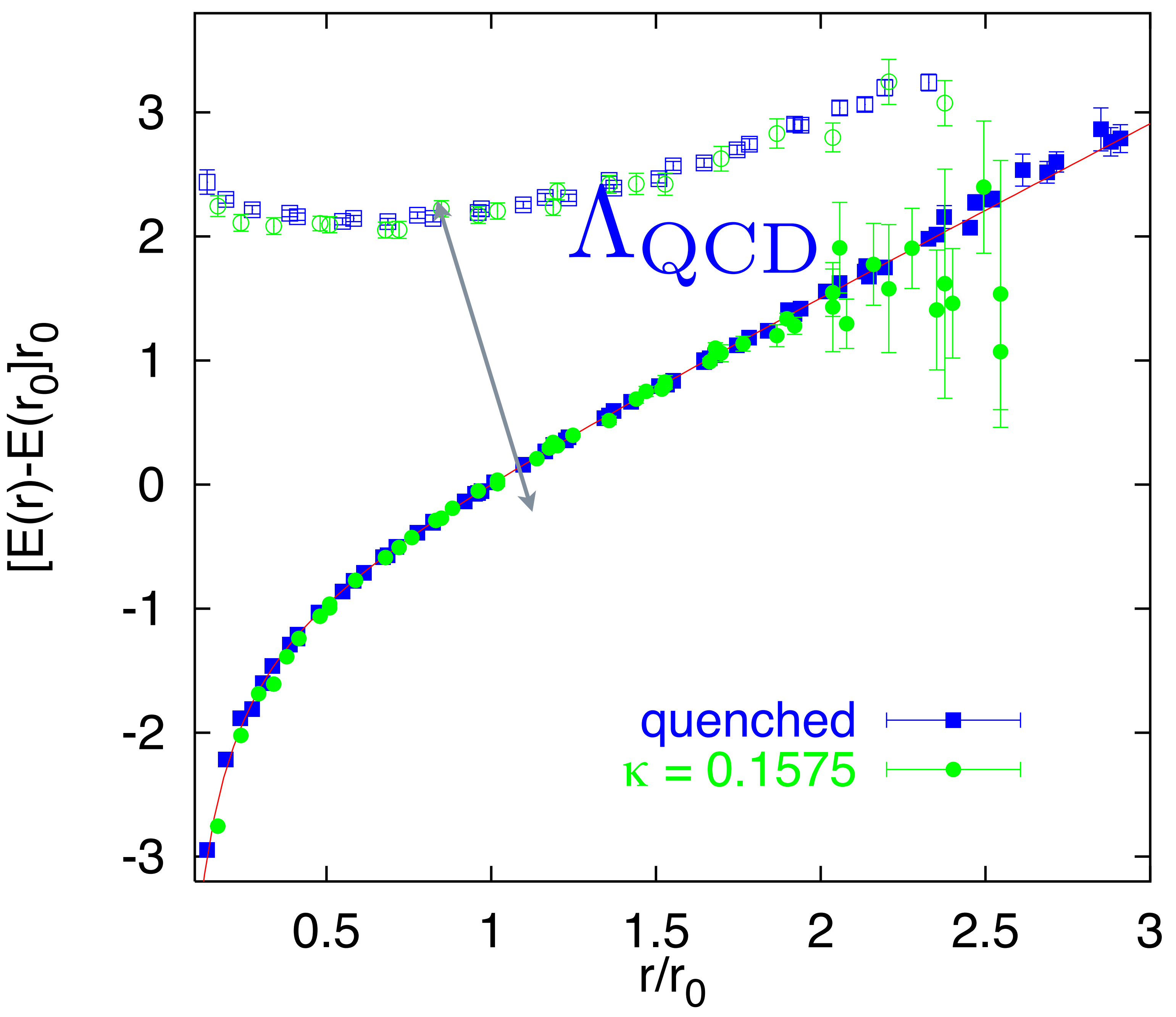

Bali et al. 98

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$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

Brambilla Pineda Soto Vairo 00

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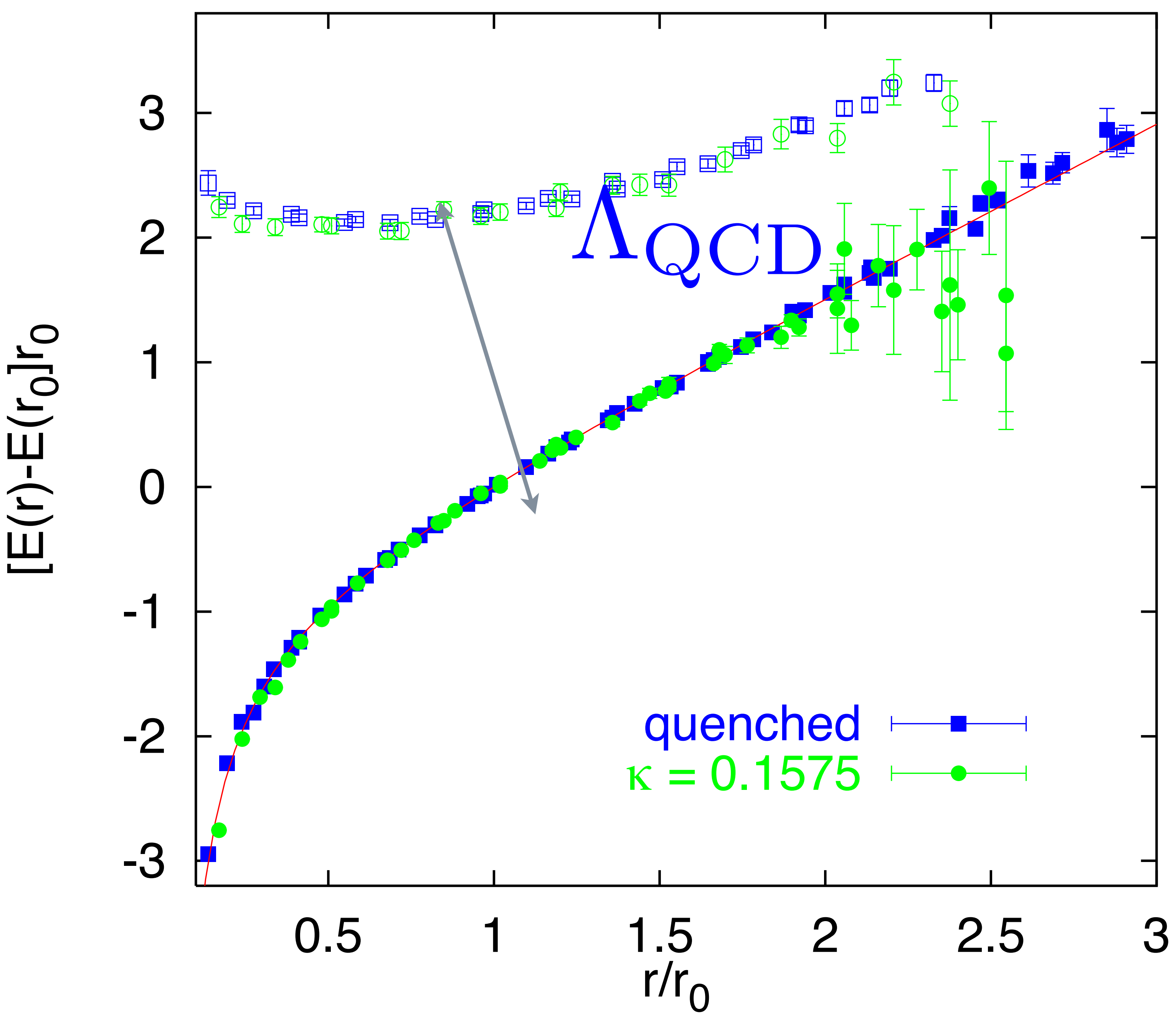

Bali et al. 98

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Brambilla Pineda Soto Vairo 00

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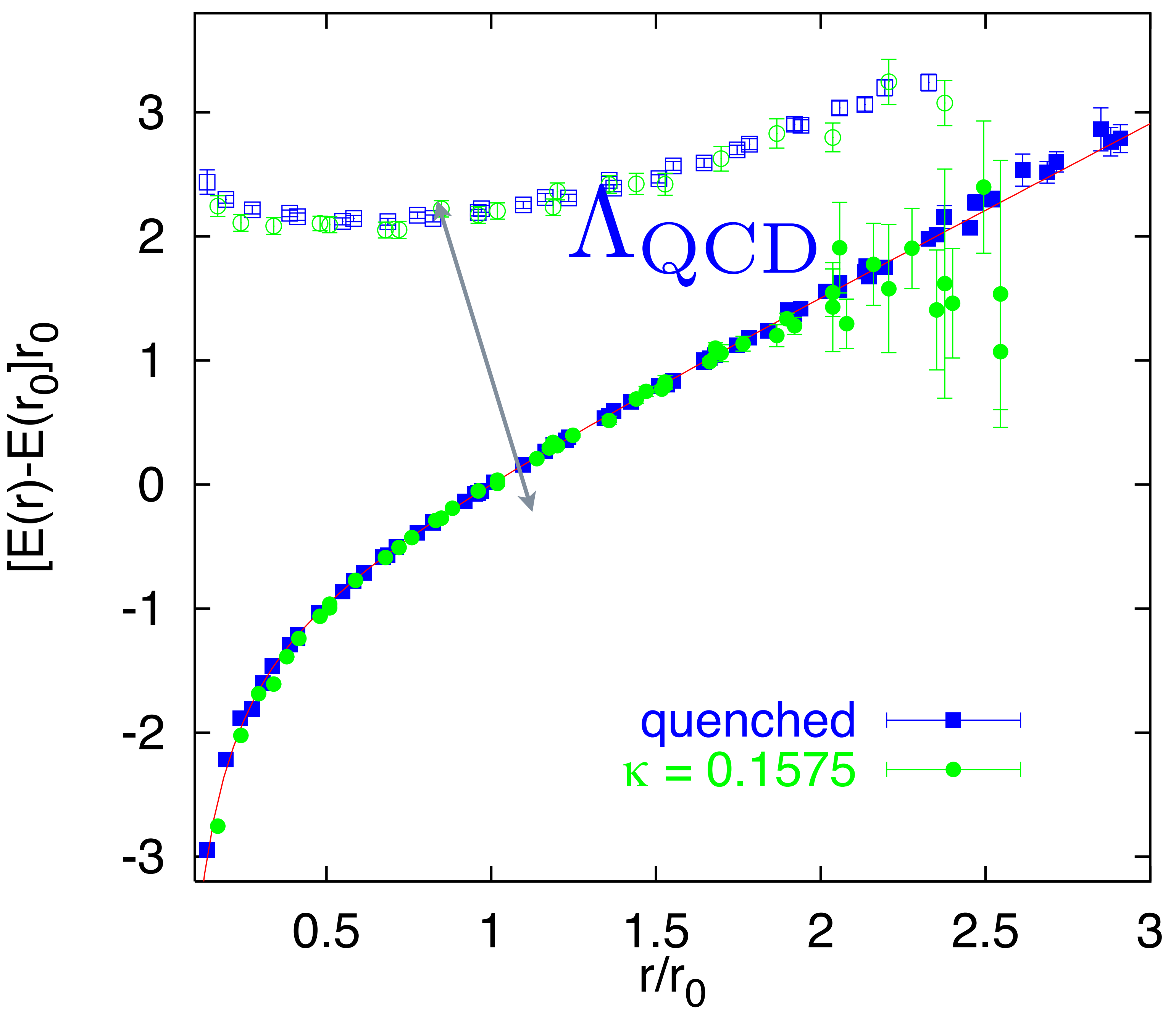

Bali et al. 98

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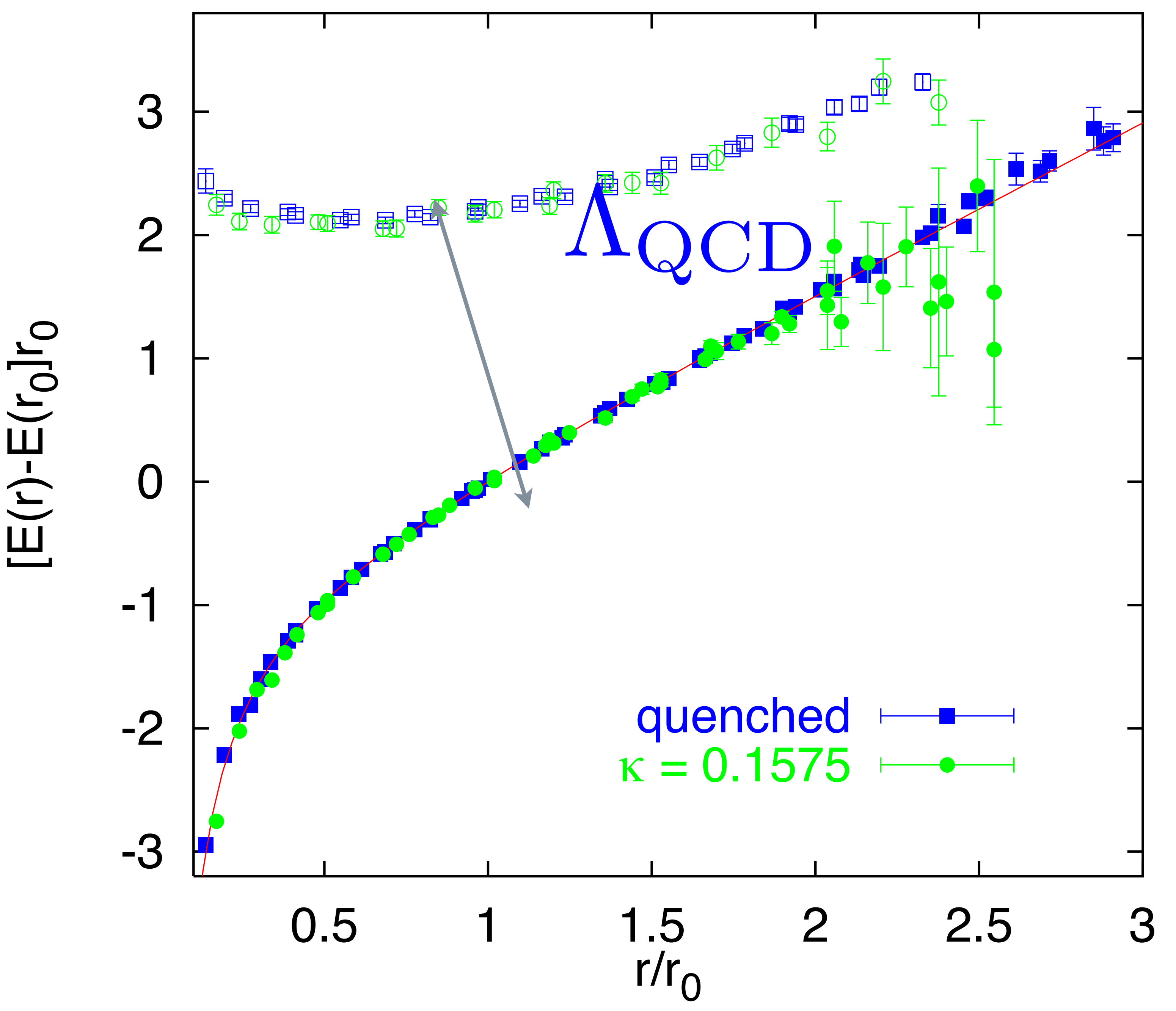
- A pure potential description emerges from the EFT **however this is not the constituent quark model, alphas and masses are the QCD fundamental parameters**
- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays
- We obtain the form of the nonperturbative potentials V in terms of generalized Wilson loops (stat that are low energy pure gluonic correlators: all the flavour dependence is pulled out

Strongly coupled pNRQCD for quarkonium

$mv \sim \Lambda_{QCD}$



Bali et al. 98

pNRQCD and the potentials come from integrating out all scales up to mv^2

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Brambilla Pineda Soto Vairo 00

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Applications regard: Spectrum, decays, production at LHC, studies of confinement

The singlet potential has the general structure

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static — spin dependent — velocity dependent

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

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$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t\, \langle \square \rangle$$

gauge invariant wilson loops can be calculated also in QCD vacuum model and large N

Pineda Vairo PRD 63 (2001) 054007

Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

$$V^{(2)}_{\rm SD} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \langle \square \rangle\, \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2) \; |V_{LS}^{(2)}$$

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t\, \langle \square \rangle - \frac{2c_F - 1}{2}\nabla^k V^{(0)}\right)\mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2) \; |V_{LS}^{(1)}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \square \rangle - \frac{\delta_{ij}}{3} \langle \square \rangle \right)\left(\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}})\right) \; |V_T$$

$$+\left(\frac{2}{3}c_F^2 i \int_0^\infty dt \langle \square \rangle - 4\left(d_{sv} + \frac{4}{3}d_{vv}\right)\delta^{(3)}(\mathbf{r})\right)\mathbf{S}_1 \cdot \mathbf{S}_2 \; |V_S$$

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Pineda Vairo PRD 63 (2001) 054007

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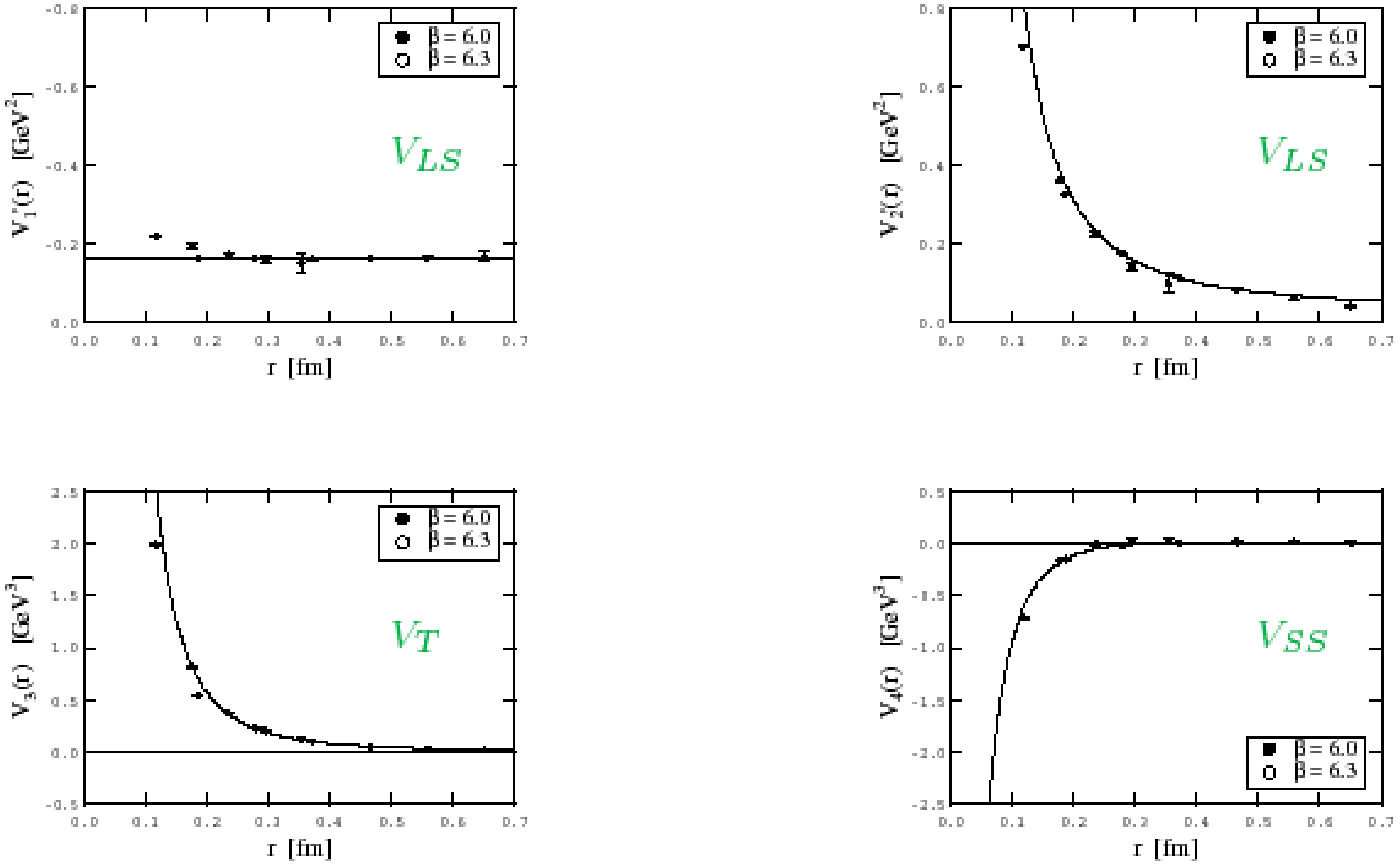
- the potentials contain the contribution of the scale m inherited from NRQCD matching coefficients—> they cancel any QM divergences, good UV behaviour
- the flavour dependent part is extracted in the NRQCD matching coefficients
- the nonperturbative part is factorized and depends only on the glue —> only one lattice calculation to get the dynamics and the observables instead of an ab initio calculation of multiple Green functions

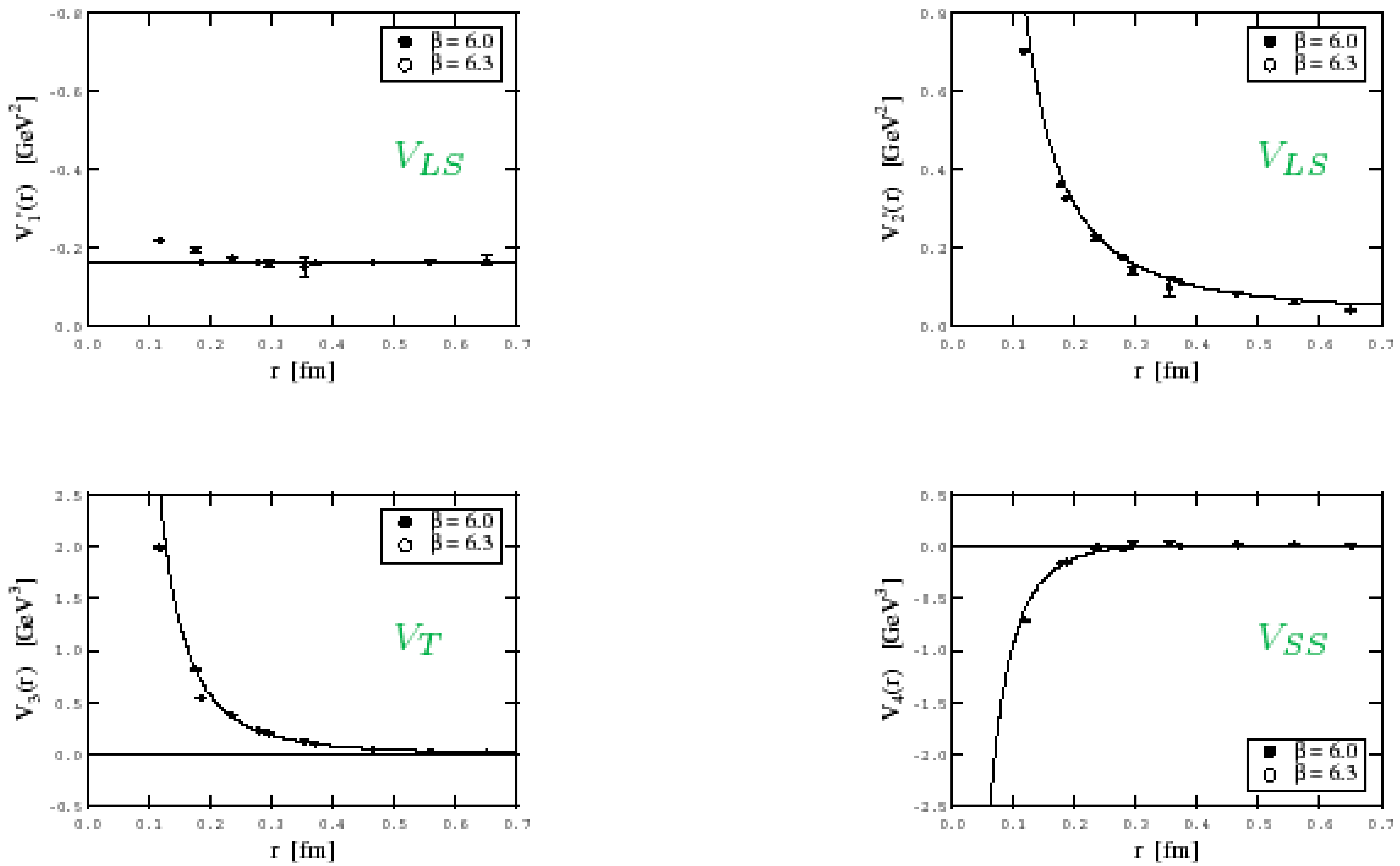
Lattice evaluation of the spin dependent potentials



Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

Lattice evaluation of the spin dependent potentials



Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

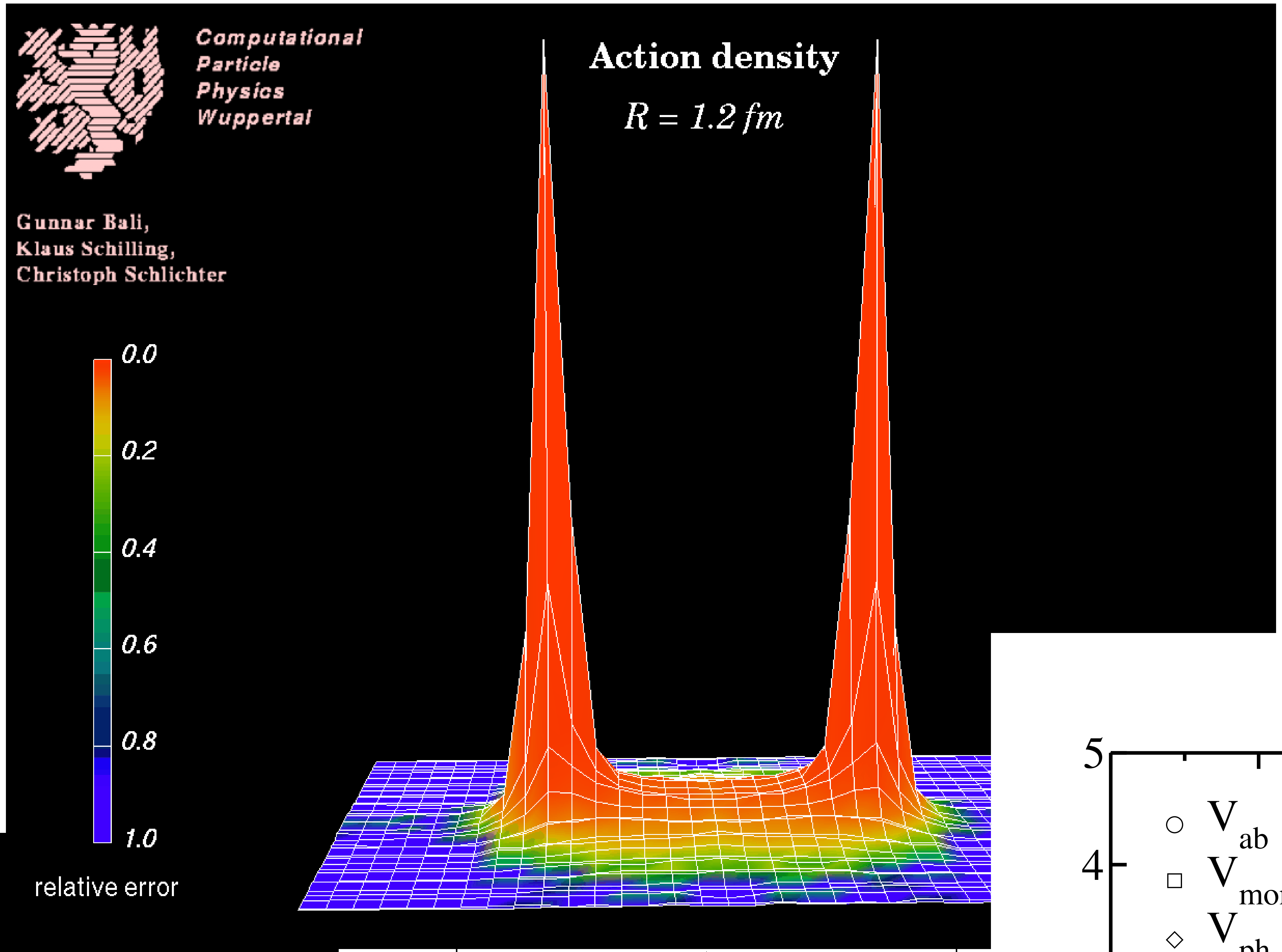
Such data can distinguish different models for the dynamics of low energy QCD e.g. effective string model

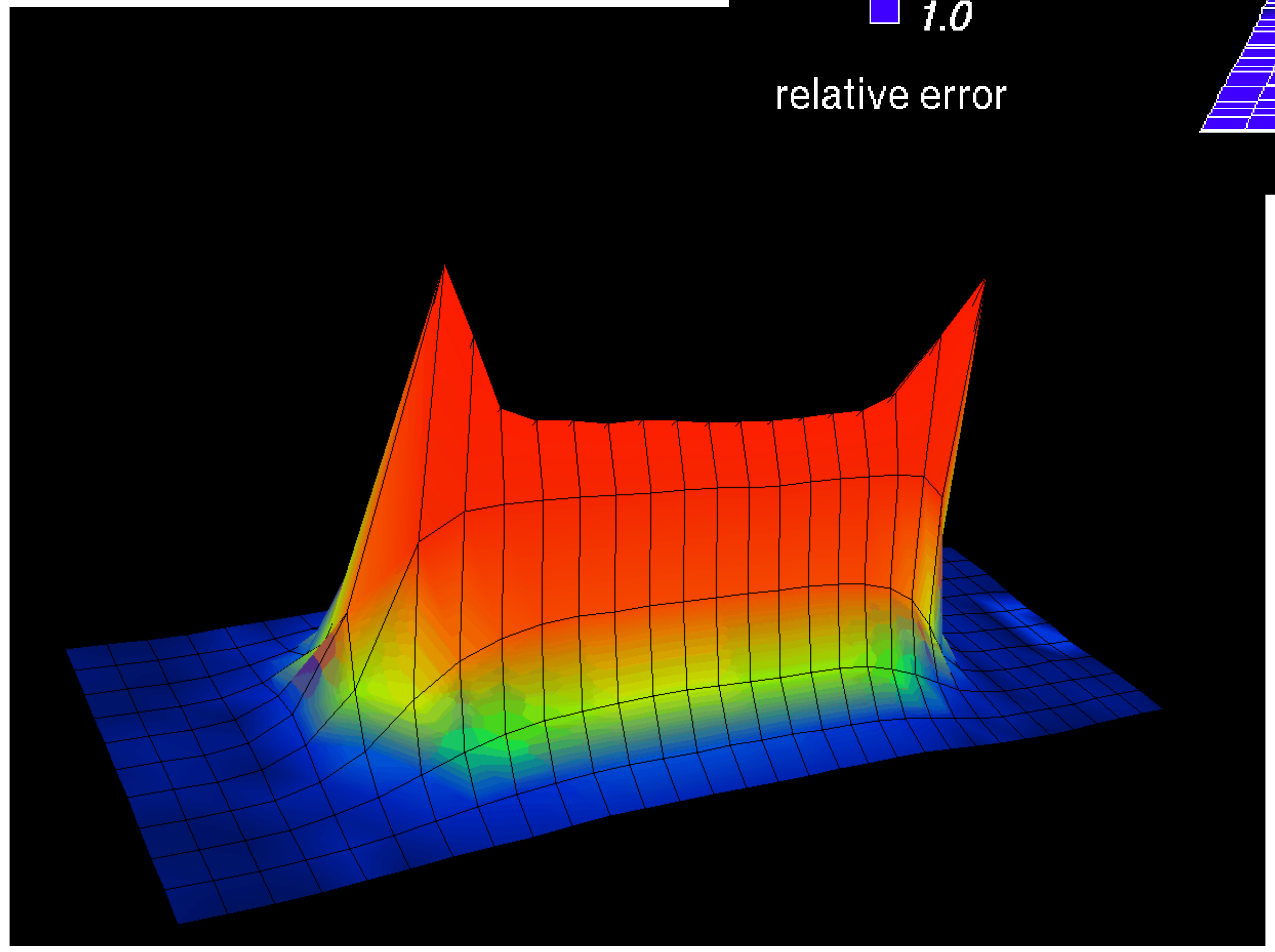
N. B., Martinez, Vairo 2014

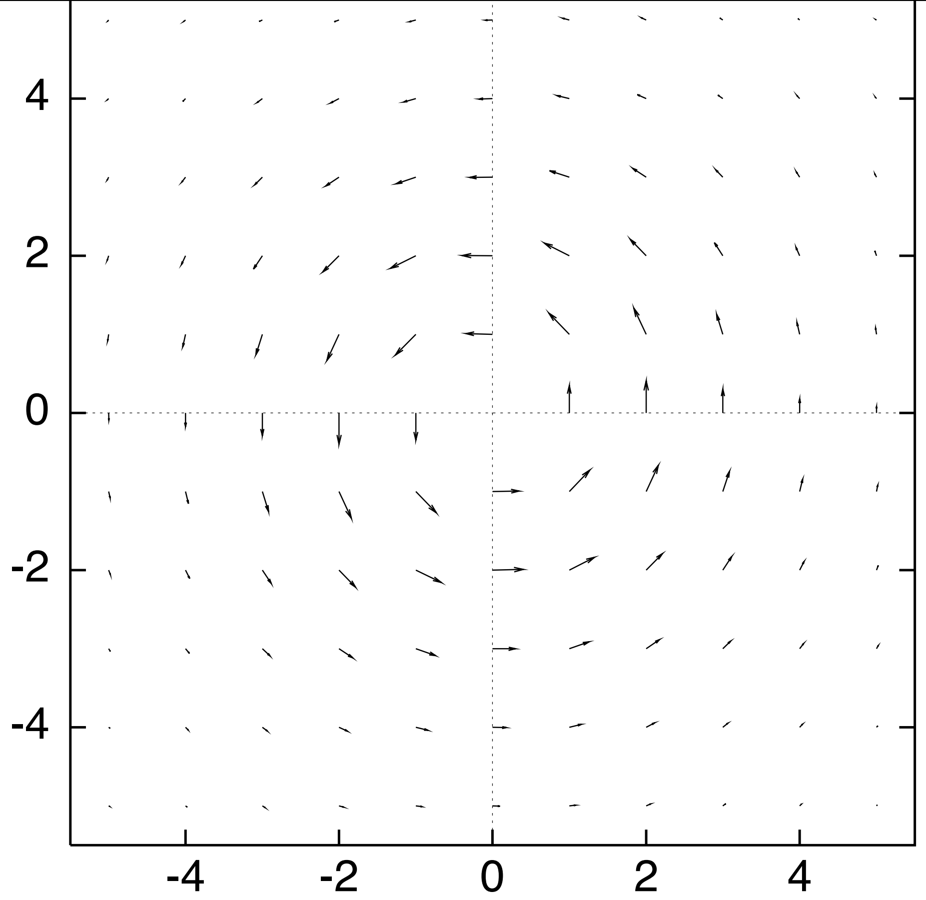
Low energy physics factorized in Wilson loops: can be used to probe the confinement mechanism

any QCD vacuum model is an assumption on the behaviour of the Wilson loop

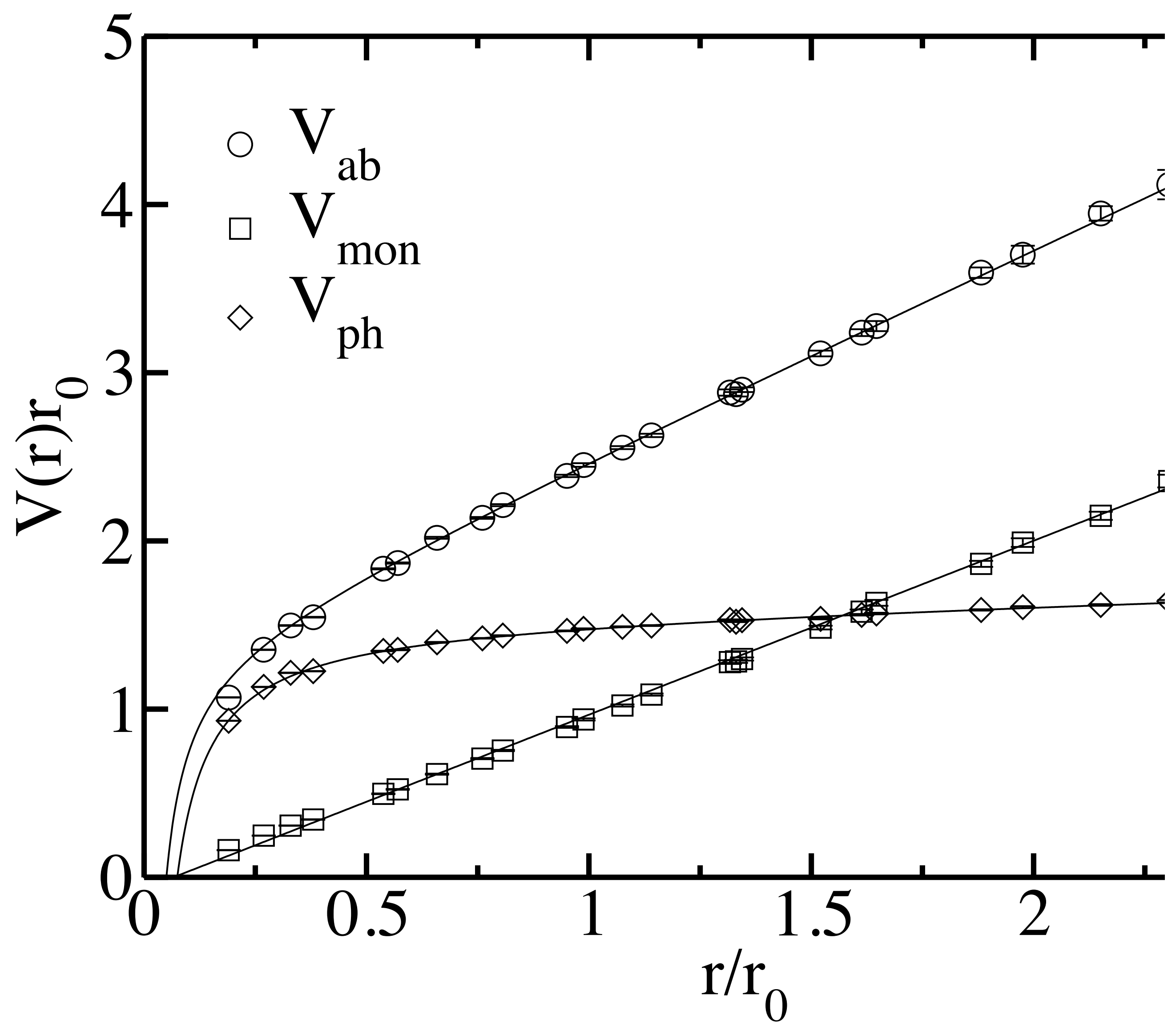
Bali et al





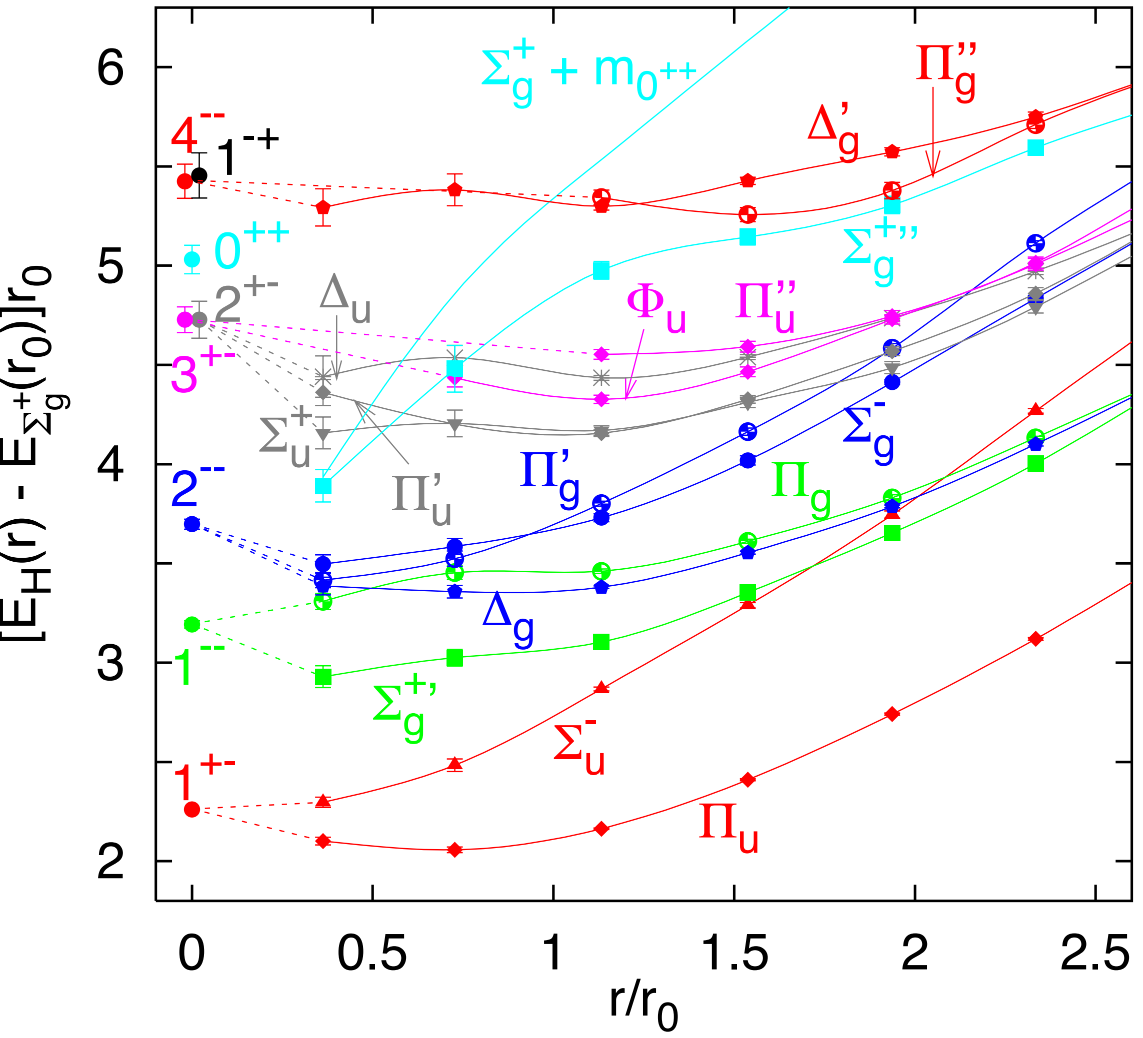


Boryakov et al. 04



Let us focus on hybrids

Hybrids static energies at short distances



The BOEFT characterises the hybrids static energy for short distance

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.

the hybrid static energy can be written as a (multipole) expansion in r:

octet potential → non perturbative coefficient

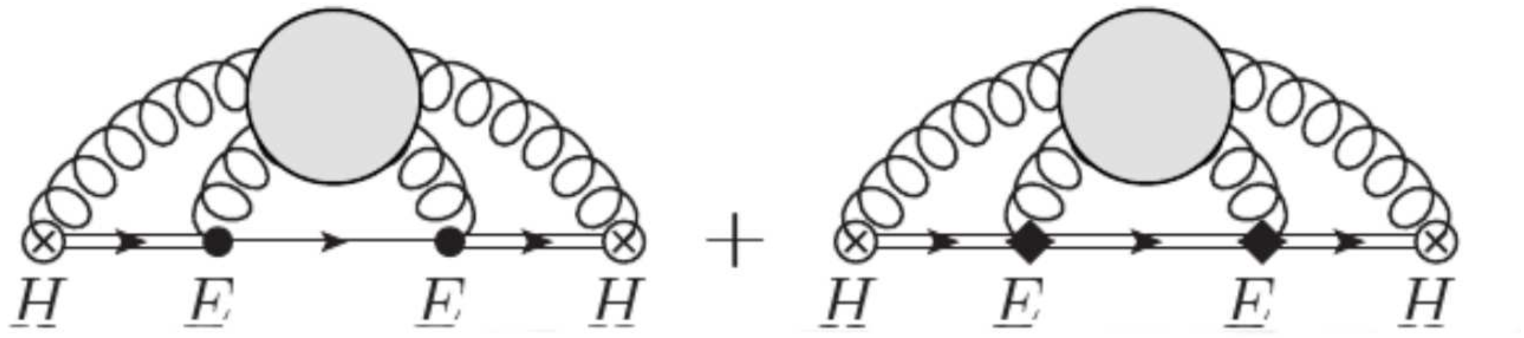
$$E_g = \frac{\alpha_s}{6r} + \Lambda_g + a_g r^2 + \ldots$$

Λ_g is the gluelump mass: $\Lambda_g = \lim_{T\to\infty} \frac{i}{T} \ln\langle H^a(T/2)\phi^{\text{adj}}_{ab}(T/2,-T/2)H^b(-T/2)\rangle$

calculated on the lattice

Foster Michael PRD 59 (1999) 094509
Bali Pineda PRD 69 (2004) 094001
Lewis Marsh PRD 89 (2014) 014502

a_g can be expressed as field correlators (single line = singlet, double line = octet), e.g.,



In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ^σ_η representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.

The gluelump multiplets Σ_u^-, Π_u; $\Sigma_g^{+\prime}$, Π_g; Σ_g^-, Π_g', Δ_g; Σ_u^+, Π_u', Δ_u are degenerate.

Gluonic excitation operators up to dim 3

Λ^σ_η	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g'	2^{--}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Π_u'	2^{+-}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u	2^{+-}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  Oncala Soto PRD 96 (2017) 014004
  Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$$P^{i\dagger}_{\kappa\lambda} O^a(\mathbf{r},\mathbf{R},t) H^{ia}_\kappa(\mathbf{R},t) = Z_\kappa \Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

$$\mathcal{L}_{\text{BOEFT for } 1+-} = \int d^3r \sum_{\lambda\lambda'} \text{Tr}\left\{\Psi^\dagger_{1+-\lambda}\left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m}\hat{r}^i_{\lambda'}\right)\Psi_{1+-\lambda'}\right\}$$

- $\lambda = \pm 1, 0; \quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \dfrac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \dfrac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$.

fitted from the lattice hybrids static energies

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

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$$\mathcal{L}_{\text{BOEFT for } 1+-} = \int d^3r \sum_{\lambda\lambda'} \text{Tr}\left\{ \Psi^\dagger_{1+-\lambda} \left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m} \hat{r}^i_{\lambda'} \right) \Psi_{1+-\lambda'} \right\}$$

- $\lambda = \pm 1, 0$; $\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp \left(\hat{\theta}^i \pm i\hat{\phi}^i \right)/\sqrt{2}$.
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fitted from the lattice hybrids static energies

The LO e.o.m. for the fields $\Psi^\dagger_{1+-\lambda}$ are a set of coupled Schrödinger equations:

$$i\partial_0 \Psi_{1+-\lambda} = \left[\left(-\frac{\boldsymbol{\nabla}^2_r}{m} + V^{(0)}_{1+-\lambda} \right) \delta_{\lambda\lambda'} - \sum_{\lambda'} C^{\text{nad}}_{1+-\lambda\lambda'} \right] \Psi_{\kappa\lambda'}$$

The eigenvalues $\mathcal{E}_N$ give the masses M_N of the states as $M_N = 2m + \mathcal{E}_N$.

$$\hat{r}^{i\dagger}_\lambda \left(\frac{\boldsymbol{\nabla}^2_r}{m} \right) \hat{r}^i_{\lambda'} = \delta_{\lambda\lambda'} \frac{\boldsymbol{\nabla}^2_r}{m} + C^{\text{nad}}_{1+-\lambda\lambda'}$$

with $C^{\text{nad}}_{1+-\lambda\lambda'} = \hat{r}^{i\dagger}_\lambda \left[\frac{\boldsymbol{\nabla}^2_r}{m}, \hat{r}^i_{\lambda'} \right]$ called the nonadiabatic coupling.

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
- Oncala Soto PRD 96 (2017) 014004
- Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$$\mathcal{L}_{\text{BOEFT for } 1^{+-}} = \int d^3r \sum_{\lambda\lambda'} \text{Tr}\left\{\Psi^\dagger_{1^{+-}\lambda}\left(i\partial_0 - V_{1^{+-}\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m}\hat{r}^i_{\lambda'}\right)\Psi_{1^{+-}\lambda'}\right\}$$

- $\lambda = \pm 1, 0$; $\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1^{+-}\lambda\lambda'} = V^{(0)}_{1^{+-}\lambda\lambda'} + \frac{V^{(1)}_{1^{+-}\lambda\lambda'}}{m} + \frac{V^{(2)}_{1^{+-}\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1^{+-}\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1^{+-}\lambda}$, with $V^{(0)}_{1^{+-}0} = E_{\Sigma_u^-}$, $V^{(0)}_{1^{+-}\pm 1} = E_{\Pi_u}$.

fitted from the lattice hybrids static energies

$$\left[-\frac{1}{mr^2}\,\partial_r r^2 \partial_r + \frac{1}{mr^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E^{(0)}_\Sigma & 0 \\ 0 & E^{(0)}_\Pi \end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_\Sigma \\ \psi^{(N)}_{-\Pi}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi^{(N)}_\Sigma \\ \psi^{(N)}_{-\Pi}\end{pmatrix}$$

$$\left[-\frac{1}{mr^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{mr^2} + E^{(0)}_\Pi\right]\psi^{(N)}_{+\Pi} = \mathcal{E}_N\, \psi^{(N)}_{+\Pi}$$

Mixing remove the degeneration among opposite parity states: ->Lambda doubling existing also in molecular physics

- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = \left(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g\right)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$

Spectrum: general consideration

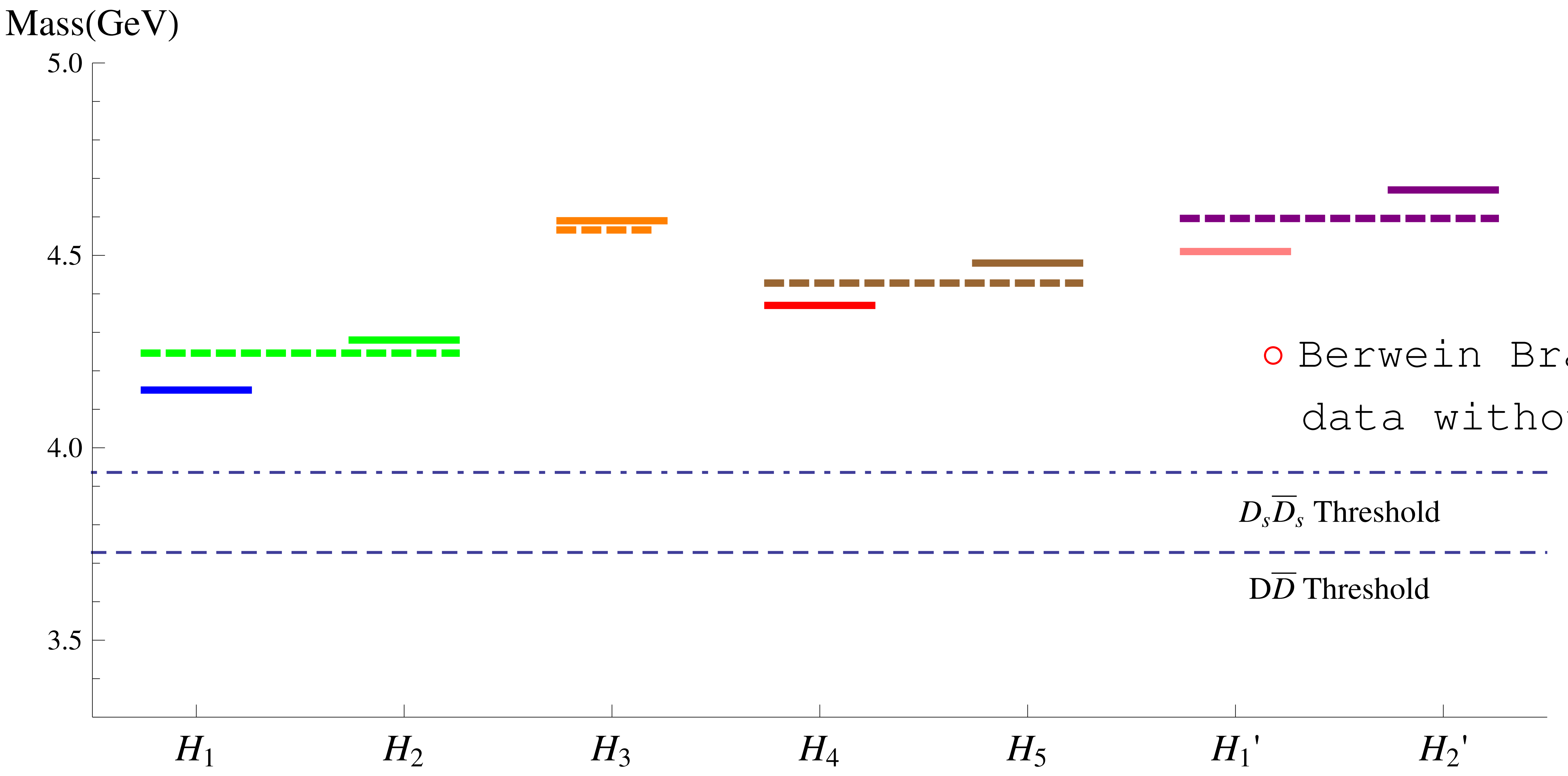
Multiplet	T	$J^{PC}(S=0)$	$J^{PC}(S=1)$	E_Γ
H_1	1	1^{--}	$(0,1,2)^{-+}$	$E_{\Sigma_u^-}, E_{\Pi_u}$
H_2	1	1^{++}	$(0,1,2)^{+-}$	E_{Π_u}
H_3	0	0^{++}	1^{+-}	$E_{\Sigma_u^-}$
H_4	2	2^{++}	$(1,2,3)^{+-}$	$E_{\Sigma_u^-}, E_{\Pi_u}$

Spin degenerated

Spectrum: with mixing and Λ-doubling

- The Schrödinger equation mixes states with the same parity.
  A consequence is Λ-doubling, i.e., the lifting of degeneracy between states with opposite parity. This happens also in molecular physics, however, there Λ-doubling is a subleading effect, while it is a LO effect in the quarkonium hybrid spectrum.
- The eigenstates are organized in the multiplets H_1, H_2, ... Neglecting off-diagonal terms, the multiplets H_1 and H_2 would be degenerate.
- We compute the spectrum using quark masses in the renormalon subtraction (RS) scheme: $m_{c\,\mathrm{RS}} = 1.477(40)$ GeV and $m_{b\,\mathrm{RS}} = 4.863(55)$ GeV.

  The gluelump masses, which enter in the normalization of the hybrid potentials, have been computed in the same scheme and assigned an uncertainty of ± 0.15 GeV, which is the largest source of uncertainty in the hybrid masses.

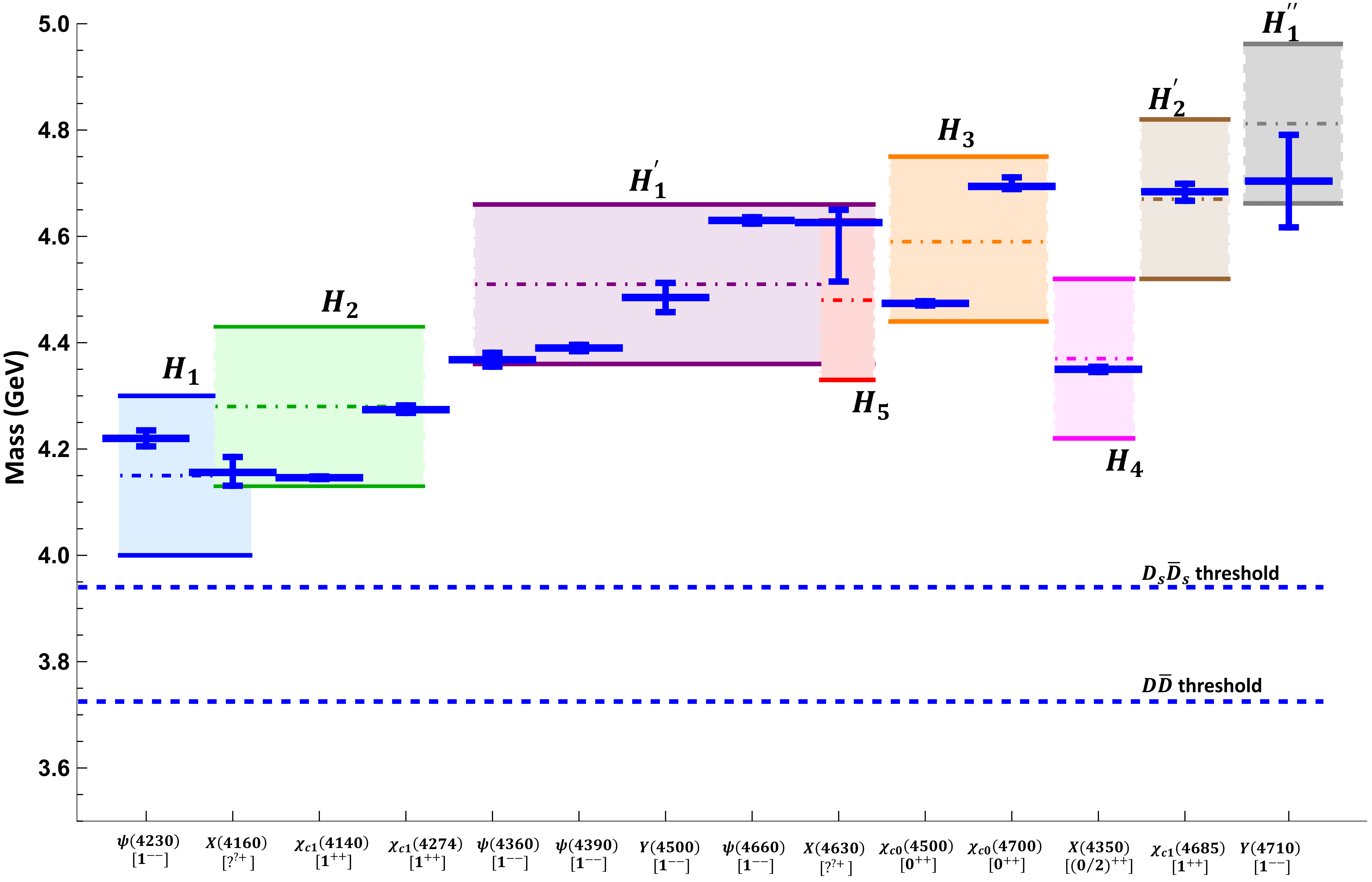


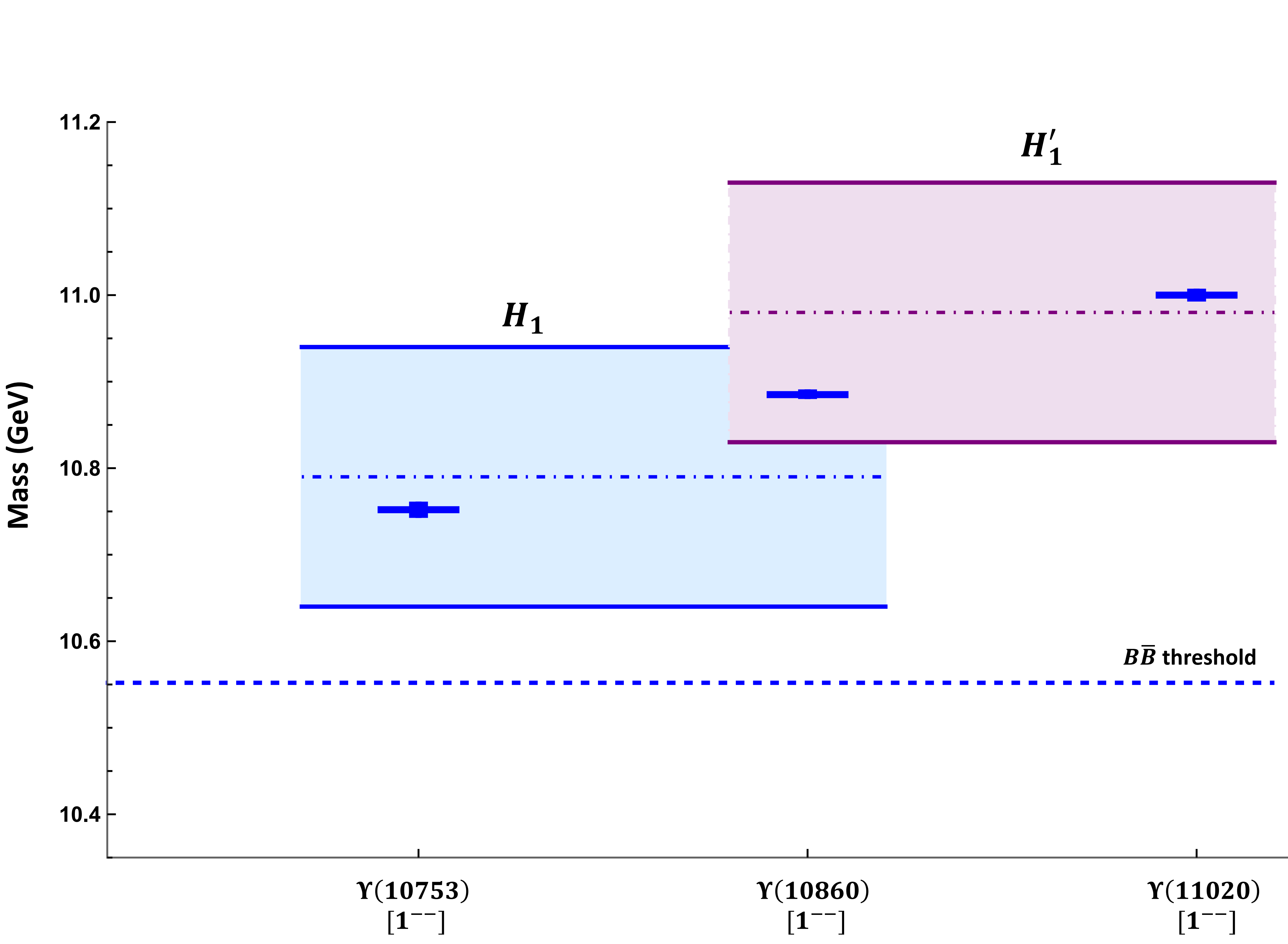
charmonium hybrids

○ Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
data without mixing (dashed) from Braaten et al PRD 90 (2014)

Without Lambda-doubling masses of opposite parity states are degenerate

Hybrid multiplets as predicted by BOEFT (coloured rectangles) compared to the neutral isoscalar states observed in charmonium/bottomonium sector (crosses)





Note: Band in the mass value for each multiplet is due to the error (150 Mev) on the gluelump mass measured on the lattice

Multiplet	T	$J^{PC}(S=0)$	$J^{PC}(S=1)$	E_Γ
H_1	1	1^{--}	$(0,1,2)^{-+}$	$E_{\Sigma_u^-}, E_{\Pi_u}$
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H_4	2	2^{++}	$(1,2,3)^{+-}$	$E_{\Sigma_u^-}, E_{\Pi_u}$

N. B. W. K. Lai, A- Mohapatra, A. Vairo 2212.09187

The **BOEFT** gives a prescription to calculate the **hybrids spin dependent potentials at order 1/m and 1/m^2**

1/m

$$V^{(1)}_{1^{+-}\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$S_{12} = 12(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) - 4(\mathbf{S}_1\cdot\mathbf{S}_2)$$

1/m^2

$$V^{(2)}_{1^{+-}\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S} + V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j} + S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'} + V^{(2)}_{S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2} + S^{i}_{2}S^{j}_{1}\right)$$

$\left(K^{ij}\right)^{k} = i\epsilon^{ikj}$ is the angular momentum of the spin one gluons

$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

The **BOEFT** gives a prescription to calculate the **hybrids spin dependent potentials at order 1/m and 1/m^2**

1/m

$$V^{(1)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

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$\left(K^{ij}\right)^{k} = i\epsilon^{ikj}$ is the angular momentum of the spin one gluons

$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

Features:

- New spin structures with respect to the quarkonium case: all terms at order 1/m and two terms at order 1/m^2

Differently from the quarkonium case, the hybrid potential gets a first contribution already at order $\Lambda^2_{\mathrm{QCD}}/m_h$. The corresponding operator does not contribute at LO to matrix elements of quarkonium states as its projection on quark-antiquark color singlet states vanishes. Hence, spin splittings are remarkably less suppressed in heavy quarkonium hybrids than in heavy quarkonia.

Hybrid spin dependent potentials at order 1/m and 1/m^2

1/m

$$V^{(1)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

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1/m^2

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$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

Features:

- The nonperturbative part in V_i (r) depend on nonperturbative gluonic correlators non local in time not yet calculated on the lattice: six unknowns, the octet perturbative part can be calculated in perturbation theory
- The only flavor dependence is carried by the perturbative NRQCD matching coefficients

Hybrid spin dependent potentials at order 1/m and 1/m^2

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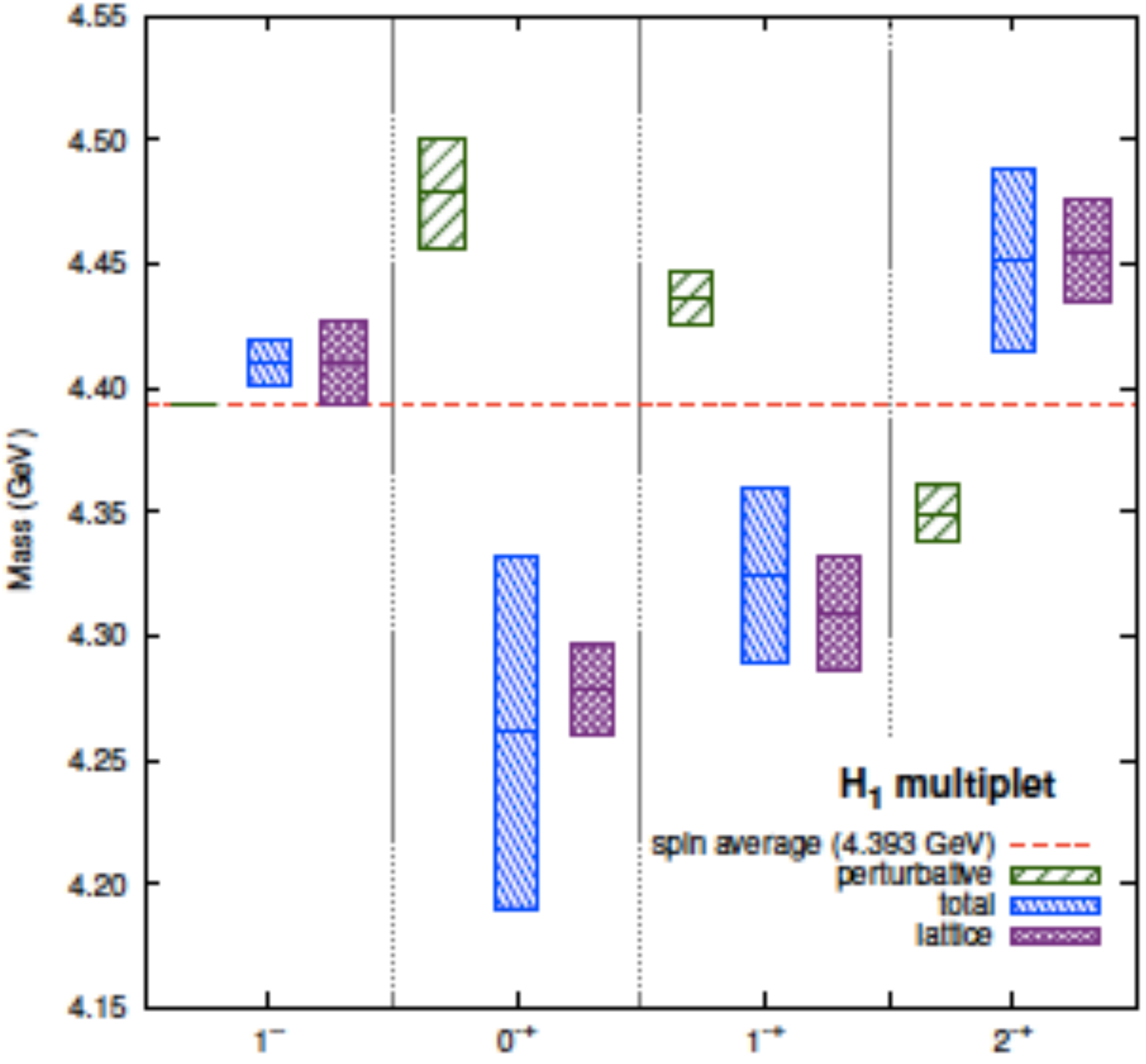
USE LATTICE CALCULATION OF THE CHARMONIUM SPIN MULTIPLETS TO EXTRACT the 6 UNKNOWNs and PREDICT THE BOTTOMONIUM SPIN MULTIPLETS, learn also about the DYNAMICS

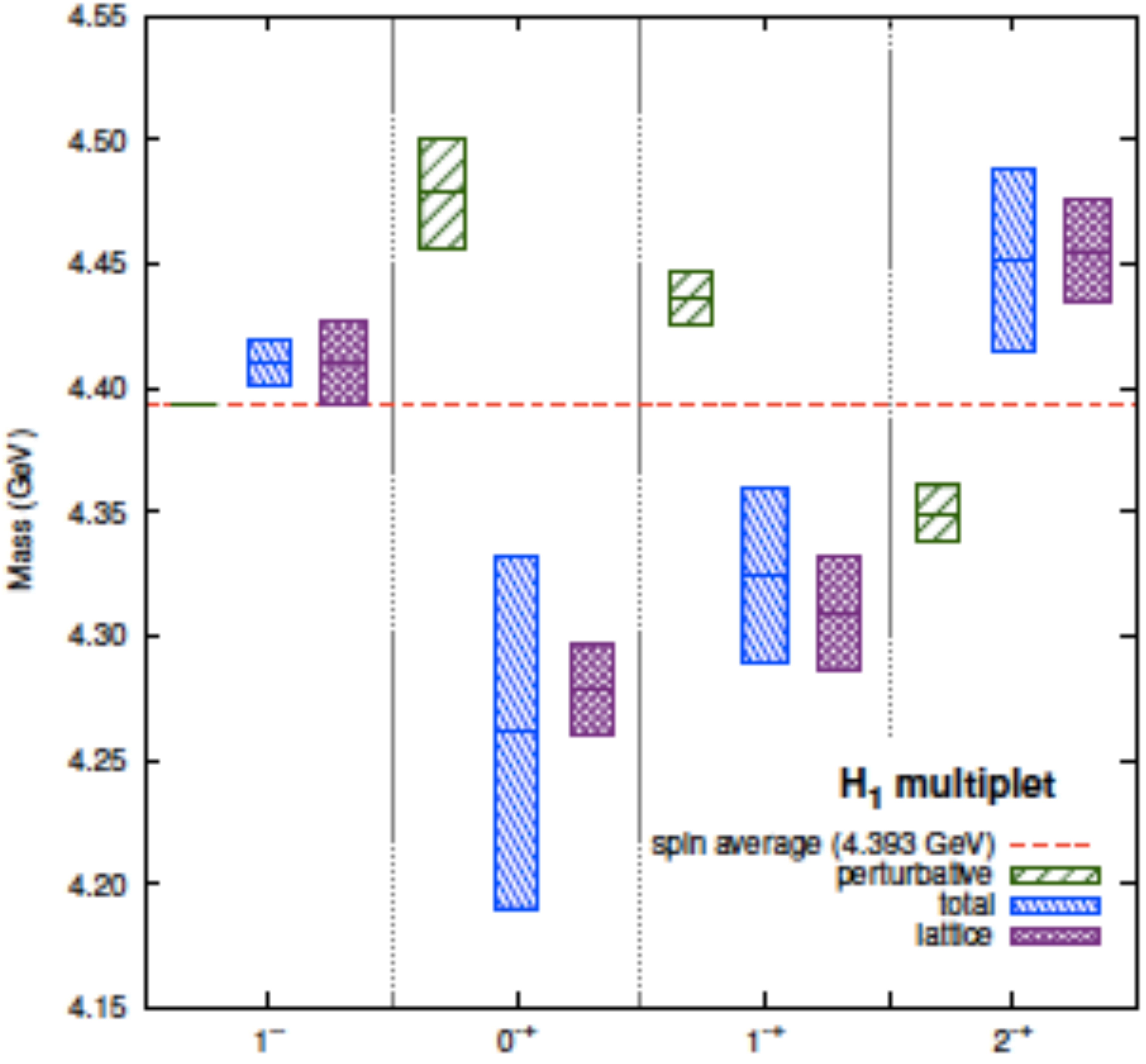
Charmonium Hybrids Multiplets H_1

lattice data from (violet) from
G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP **12**, 089 (2016), arXiv:1610.01073 [hep-lat].
with a pion of about 240 MeV



- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

Charmonium Hybrids Multiplets H_1



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with a pion of about 240 MeV

height of the boxes is an estimate of the uncertainty:
estimated by the parametric size of higher order corrections, m alpha_s^5 for the perturbative part, powers of Lambda_qcd/m for the nonperturbative part, plus the statistical error on the fit

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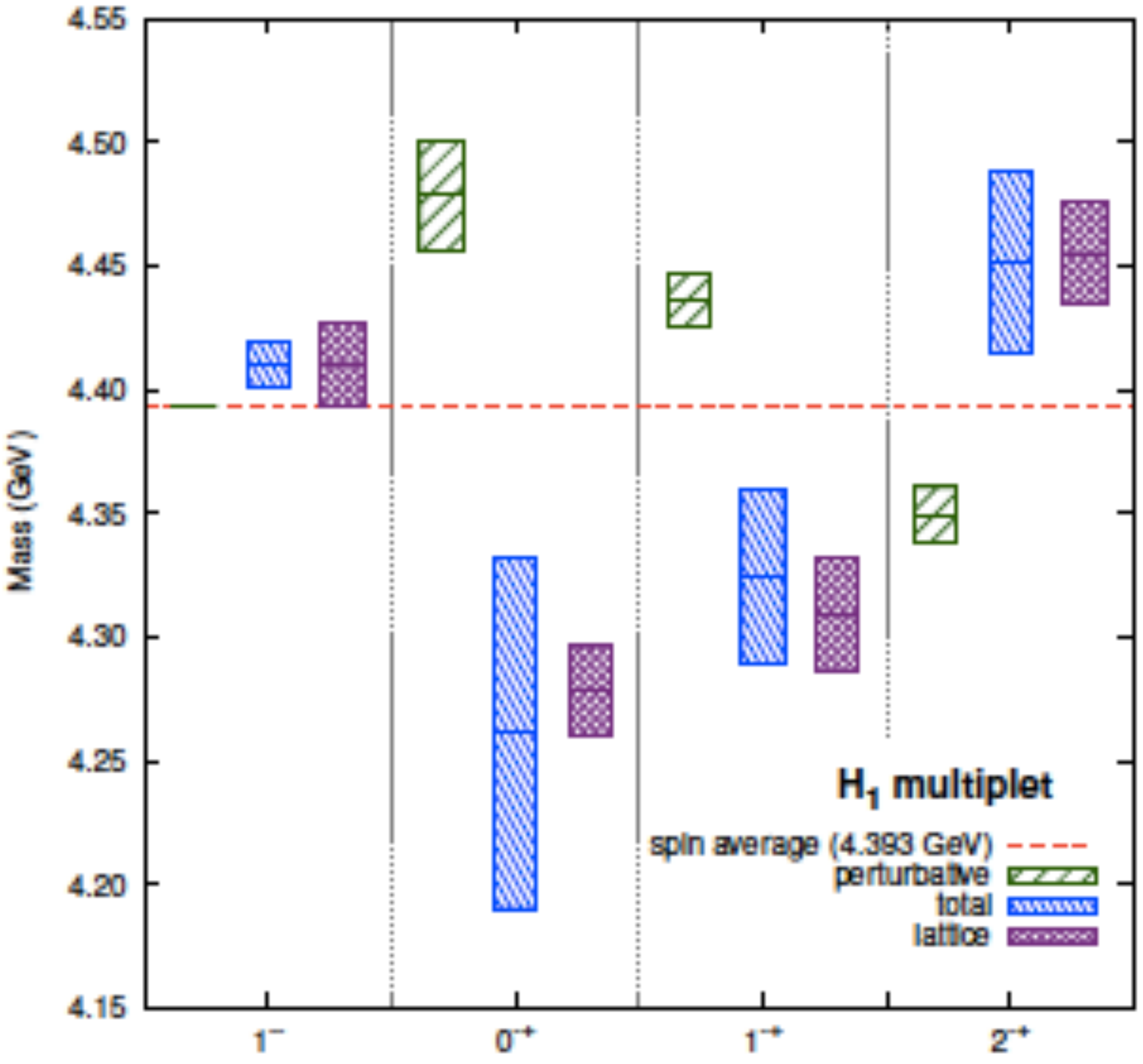
Charmonium Hybrids Multiplets H_1



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G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP **12**, 089 (2016), arXiv:1610.01073 [hep-lat].
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height of the boxes is an estimate of the uncertainty:
estimated by the parametric size of higher order corrections, m alpha_s^5 for the perturbative part, powers of Lambda_qcd/m for the nonperturbative part, plus the statistical error on the fit

the perturbative part produces a pattern opposite to the lattice and to ordinary quarkonia —> discrepancy can be reconciled thanks to the nonperturbative parts, especially the one at order 1/m which goes like Lambda^2/m and is parametrically larger than the perturbative contribution at order m v^4

which is interesting as some models are taking the spin interaction from perturbation theory with a constituent gluon

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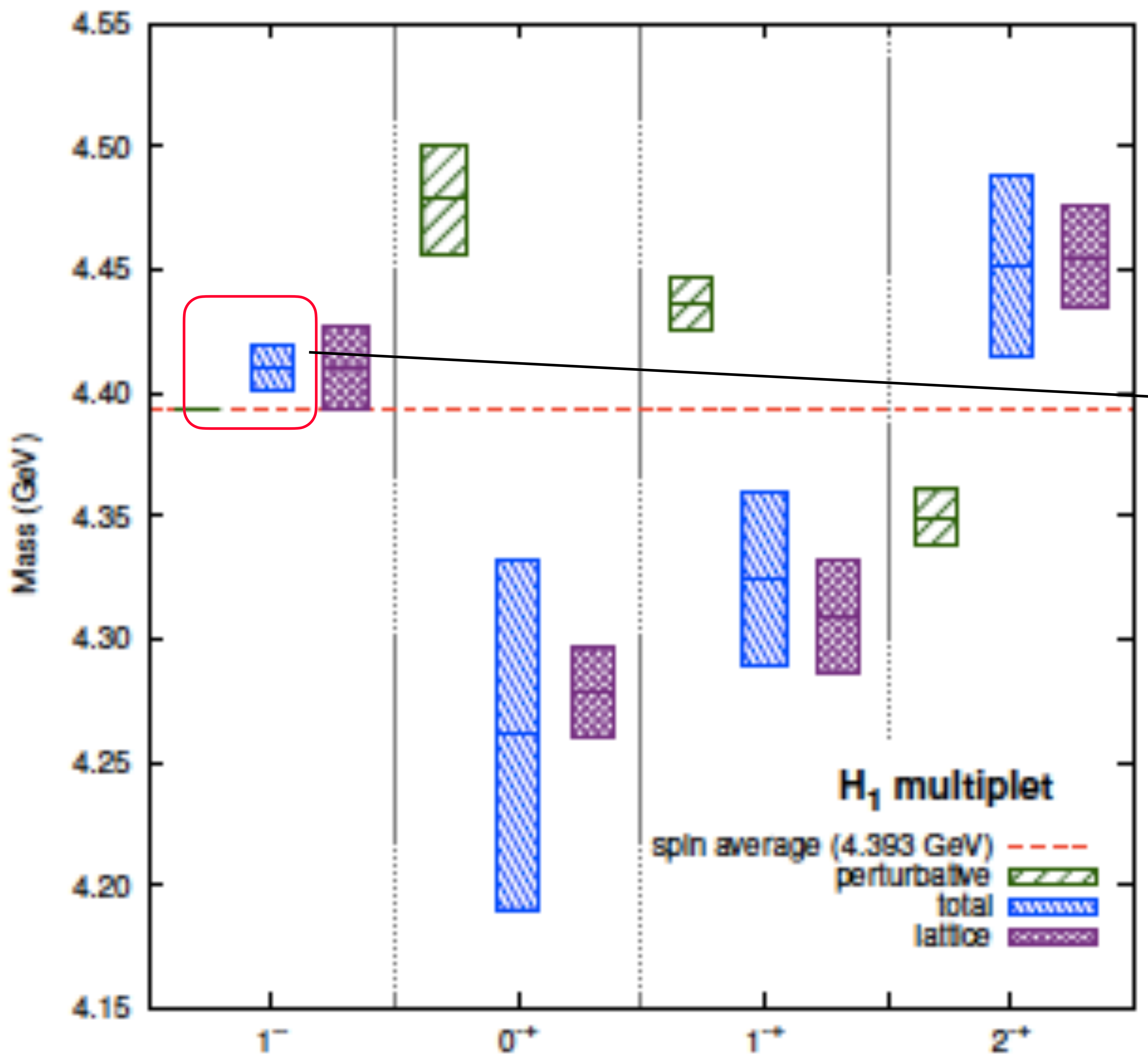
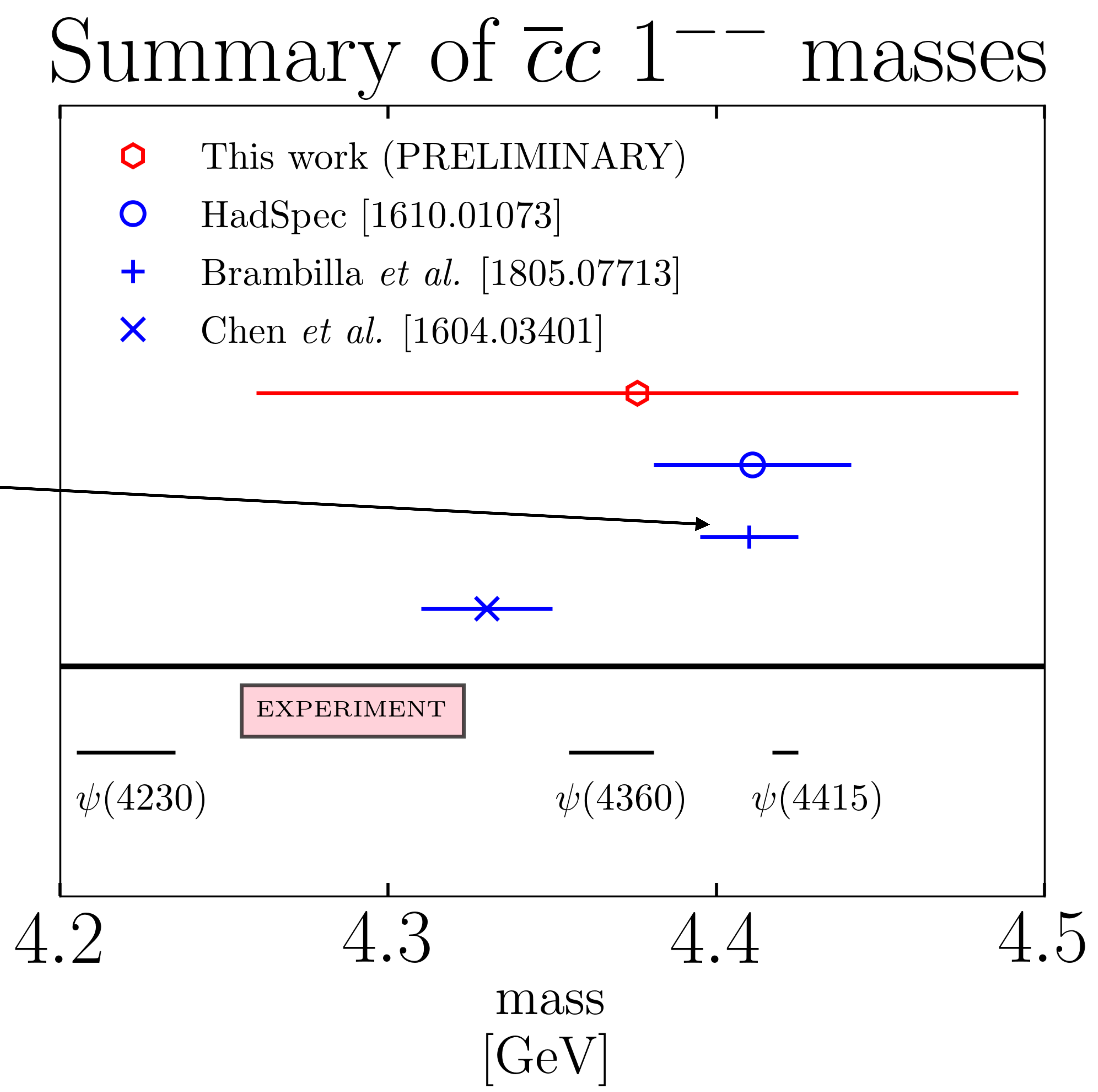
Charmonium Hybrids Multiplets H_1

HISQ lattice action with 2+1+1 sea quarks

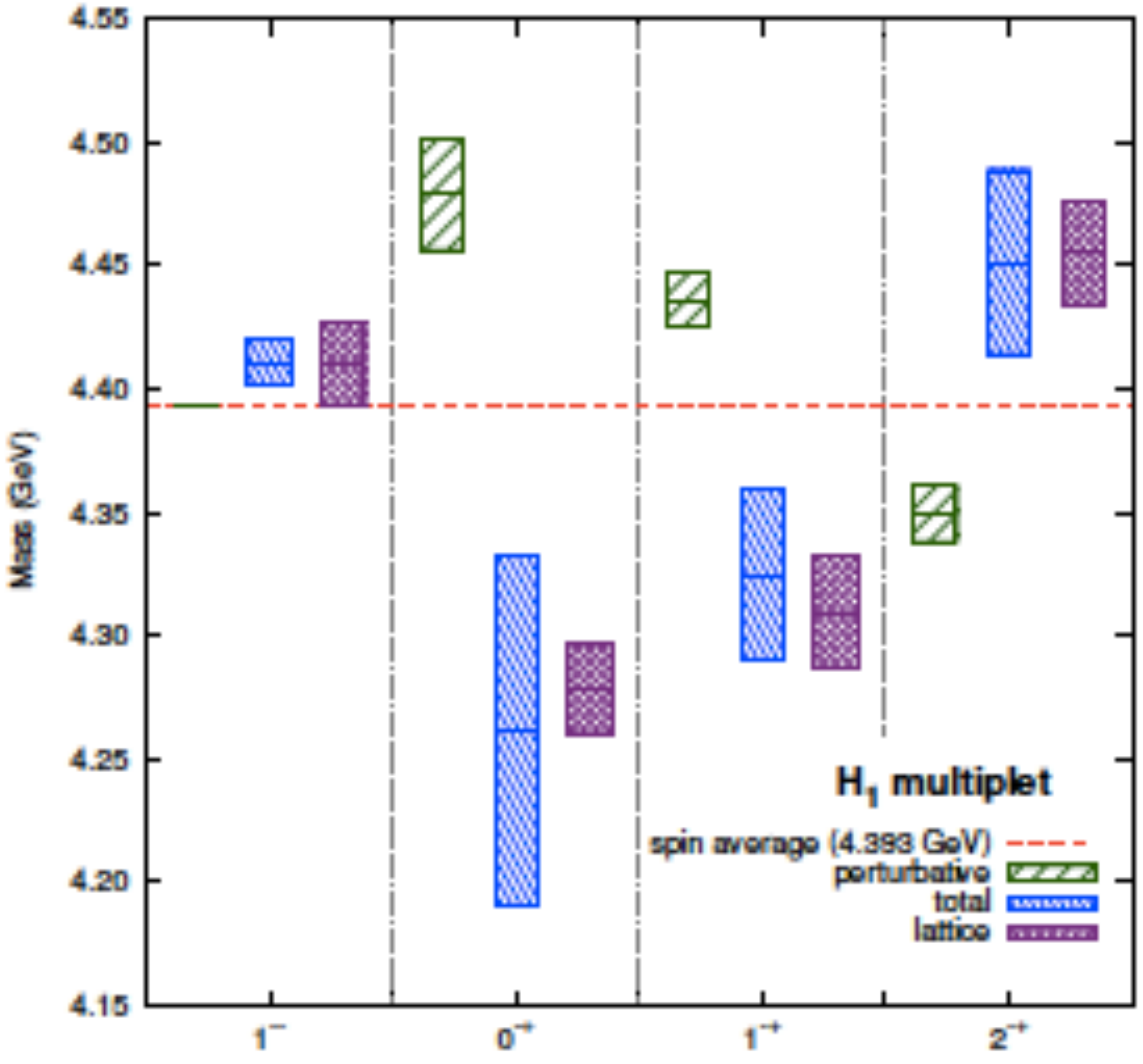


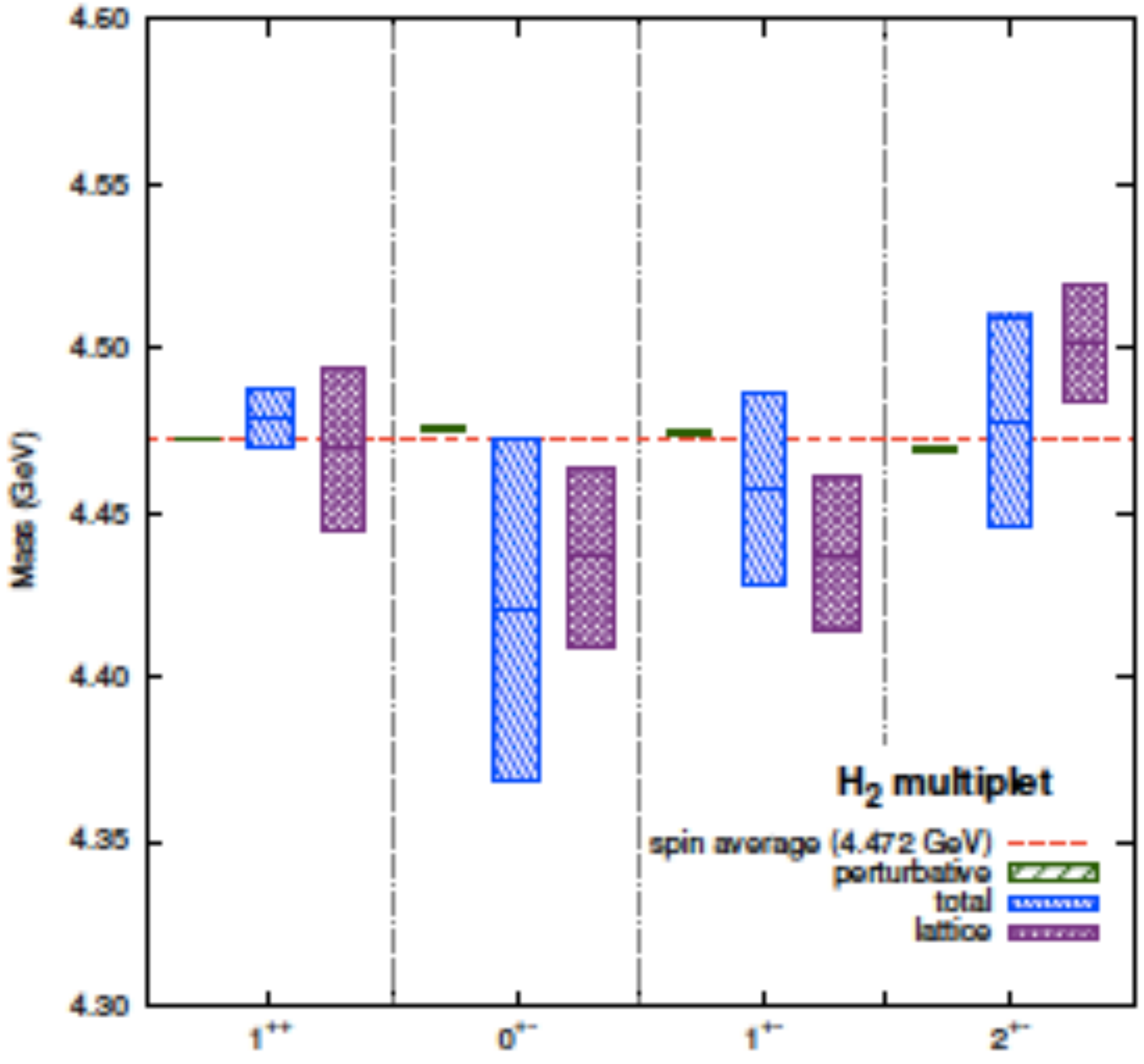


G. Ray, C. McNeile, 2110.14101

○ Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

Charmonium Hybrids Multiplets H_1 and H_2





H_1 and H_2 corresponds to l=1 and are negative and positive parity resp. The mass splitting between H_1 and H_2 is a result of lambda-doubling

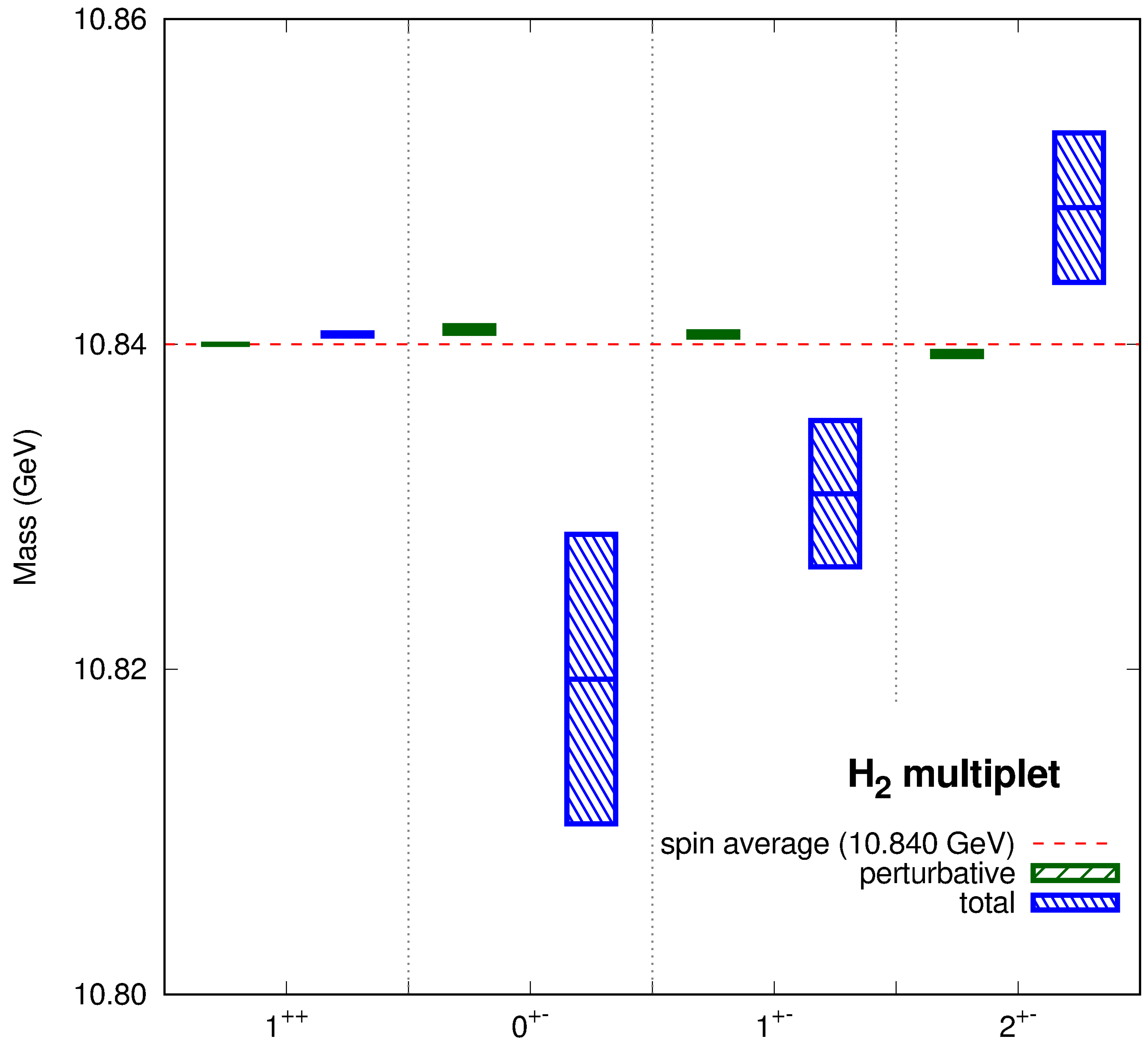
H_3 and H_4 are also calculated

- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

here you find predictions for all H multiplets

Bottomonium hybrid spin splittings

thanks to the BOEFT factorizatio we can fix the nonperturbative unknowns from a charmonium hybrid calculationthe nonperturbative low energy unknownsdo not depend on the flavor: we can predict the bottomonium hybrids splin splittings



and also the other H multiplets

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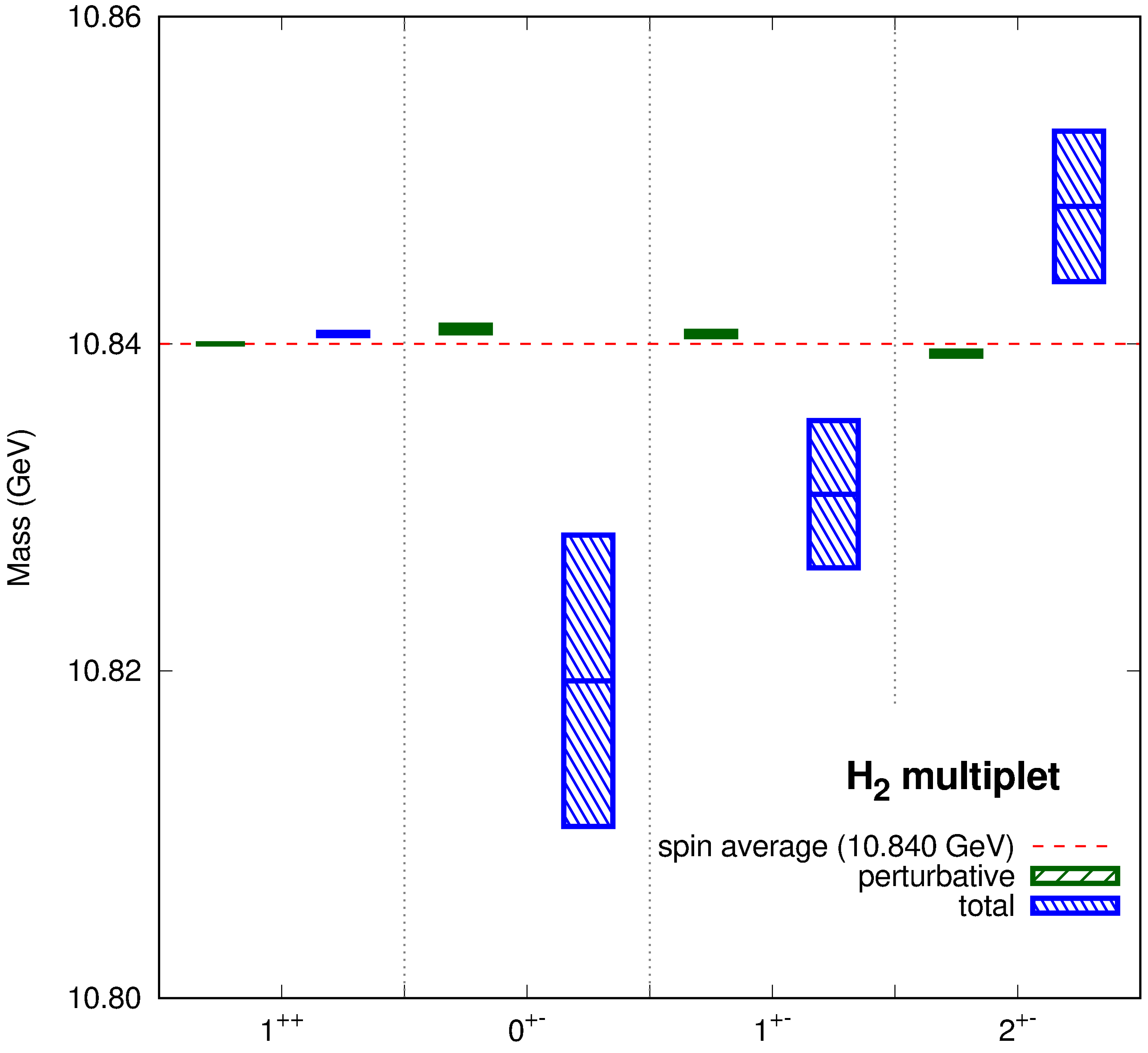
Bottomonium hybrid spin splittings

thanks to the BOEFT factorizatio we can fix the nonperturbative unknowns from a charmonium hybrid calculationthe nonperturbative low energy unknownsdo not depend on the flavor: we can predict the bottomonium hybrids splin splittings

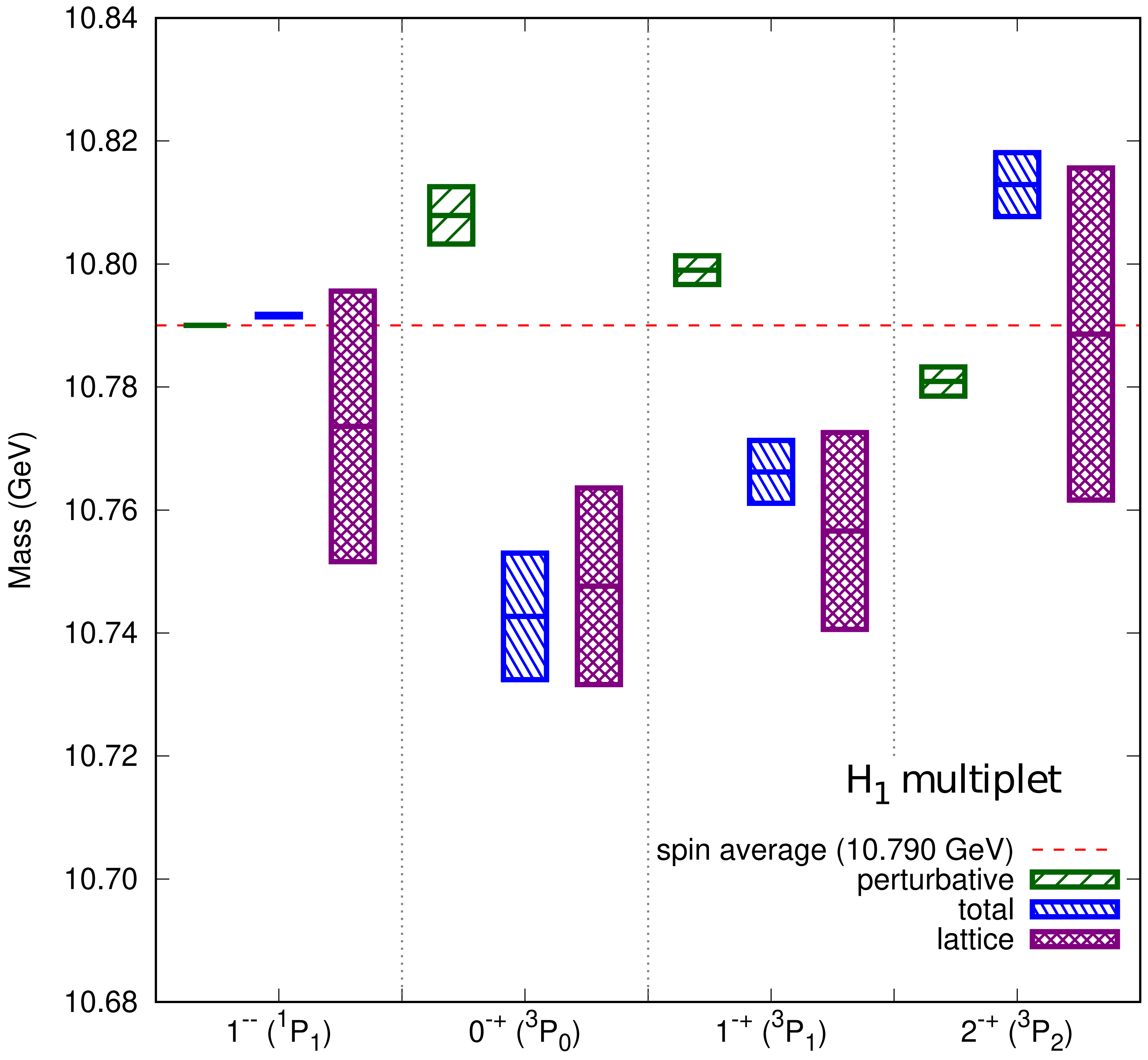


and also the other H multiplets

○ Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

Comparison of our prediction to the existing lattice data on H1

Bottomonium H_1 hybrid spin splittings



blue BOEFT predictions (more precise), violet actual lattice calculation

○ Ryan et al arXiv:2008.02656 [2+1 flavors, $m_\pi = 400$ MeV] unpublished plot by J. Segovia and J. Tarrus

Spin effects

->difficult to insert in models

->this spin structure has huge impact in phenomenology : larger spin multiplets separation than in quarkonium

->less spin symmetry in decays due to quarkonium-hybrids mixing via a spin operator at 1/m

Oncala & Soto, Phys. Rev. D. 96, (2017)

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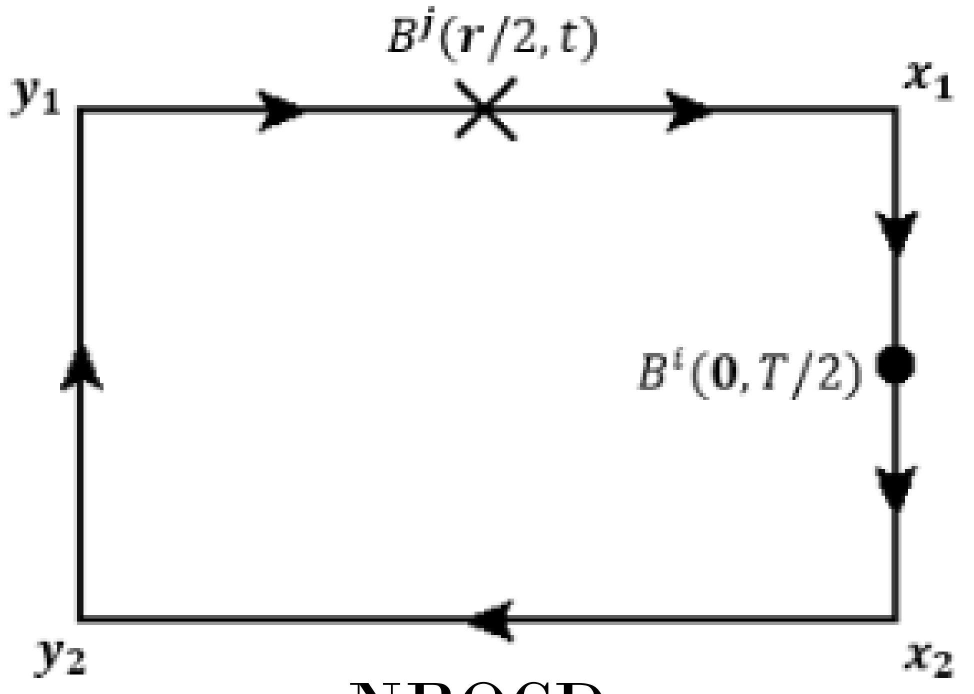
Oncala & Soto, Phys. Rev. D. 96, (2017)

- Hybrid states in the same energy range and same quantum #'s as quarkonium can mix.

Ex. $H_1\left[1^{--}\right](4155) \leftrightarrow c\bar{c}\left[1^{--}\right](3S)$

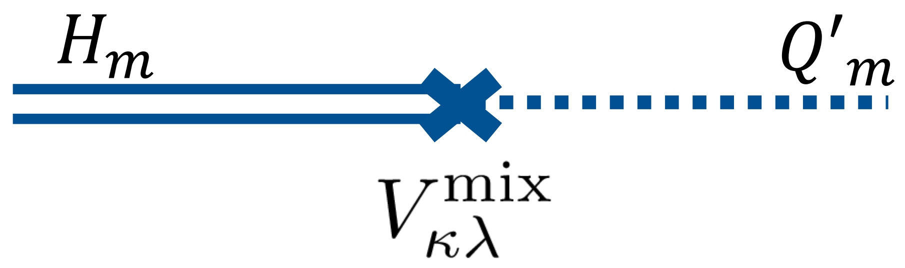
Effect on decay: $H_m \leftrightarrow Q'_m \rightarrow (\eta_c, J/\psi, \cdots) + (\gamma, \cdots)$

- Mixing potential $V^{\mathrm{mix}}_{\kappa\lambda}$: determined from matching NRQCD and BOEFT at $O(1/m)$



NRQCD

=



BOEFT

Expression after matching:

$$V^{\mathrm{mix}}_{|\lambda|} = -\frac{g c_F}{2m_Q}\,{}^{(0)}_{\lambda}\langle 1|B^j\left(\mathbf{r}/2,0\right)|0\rangle^{(0)} P^j_{\lambda},$$

Above expression can be computed on lattice if we identify:

$$|0\rangle^{(0)} = |\Sigma_g^+\rangle$$

$$|1\rangle^{(0)}_{\lambda=0} = |\Sigma_u^-\rangle, |1\rangle^{(0)}_{|\lambda|=1} = |\Pi_u\rangle$$

BOEFT calculation of semi inclusive hybrids decays to quarkonium

$$\Gamma_{H\to S} = -2\,\langle H|\mathrm{Im}\Delta V|H\rangle.$$

we calculated all spin conserving and spin flipping decays

Decay to open threshold states not accounted

BOEFT calculation of semi inclusive hybrids decays to quarkonium

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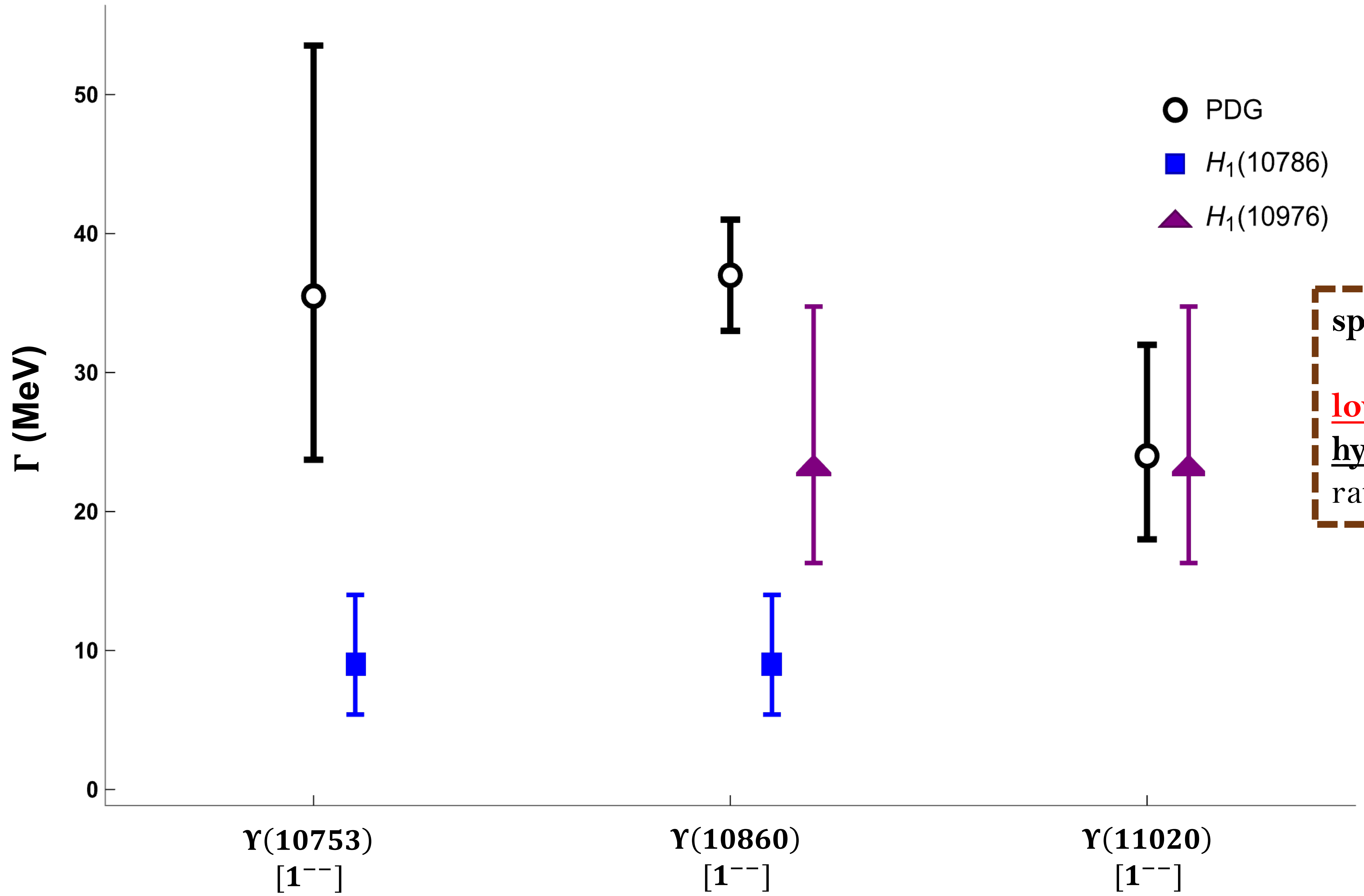
Decay to open threshold states not accounted

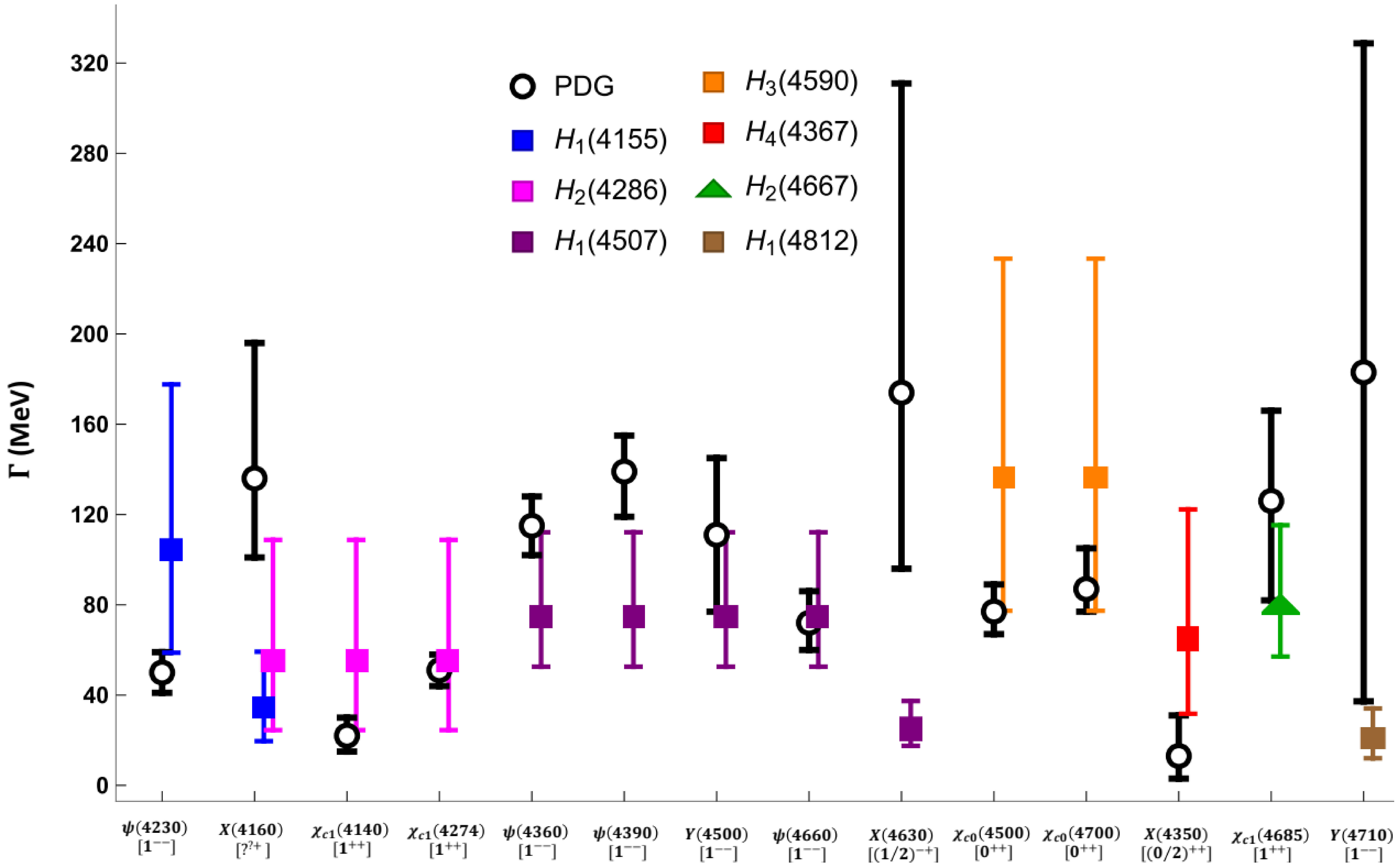
- Comparison: bottom exotic states with corresponding bottomonium hybrid state:



spin-conserving + spin-flipping decays
=
lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

BOEFT calculation of semi inclusive hybrids decays to quarkonium

Comparison: charm exotic states with corresponding charmonium hybrid state:



$$H_m \to Q_n + X$$

spin-conserving + spin-flipping decays

=

lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Hybrid: Summary

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Hybrids ($Q\bar{Q}g$): Color singlet state of color octet $Q\bar{Q}$ + gluon. (Q = c, b)

✓ **Isoscalar neutral mesons (Isospin=0)**

✓ Candidates for hybrids based on **mass, quantum numbers,** and **decays** to **quarkonium**:

Charm sector:

- $X(4160)$: could be **charm hybrid** $H_1[2^{-+}](4155)$.
- $X(4630)$: could be **charm hybrid** $H_1[(1/2^{-+})](4507)$.
- $\psi(4390)$: could be **charm hybrid** $H_1[1^{--}](4507)$.
- $\psi(4710)$: could be **charm hybrid** $H_1[(1^{--})](4812)$.
- $X(4630)$: could be **charm hybrid** $H_1[(1/2^{-+})](4507)$.
- $\chi_{c1}(4685)$: could be **charm hybrid** $H_2[(1^{++})](4667)$.

Bottom sector:

- $\Upsilon(10753)$: could be **bottom hybrid** $H_1[(1^{--})](10786)$.

DISCLAIMER!!!
All the above interpretation can differ accounting for decays to meson-pair threshold states and hybrid-quarkonium mixing.

Hybrid Decays

Hybrid decays to meson-pair threshold states:

Conventional Wisdom: Hybrid decays to two S-wave mesons forbidden! $H_m \nrightarrow D^{(*)}\bar{D}^{(*)}$

Kou & Pene, Phys Lett B 631 (2005) Page, Phys Lett B 407 (1997) Farina, Tecocoatzi, Giachino, Santopinto & Swanson, Phys Rev D 102 (2020)

Born Oppenheimer quantum numbers for hybrids and ground state meson pair **does allow for decay to two s-wave mesons**. Bruschini 2306.17120

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Most quarkonium hybrids can decay into pair of s-wave mesons !

forbidden for decay into pair of s-wave mesons

N.B. G. Krein, J. Tarrus, A. Vairo 1707.09647.
J. Tarrus 1901. 09761, J. Soto, J. Tarrus 2005.00552 ,

BOEFT may be used to describe any system made by two heavy quarks bound adiabatically with some light quarks degrees of freedom (tetraquarks QQlight quarks, QQbar light quarks, pentaquarks)

In case of light quarks isospin quantum numbers should be added

Tetraquarks and Pentaquarks

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Steps go as before:

—identify the symmetries, identify the interpolating operators O_n

$$\mathcal{O}_n(t,\,\boldsymbol{r},\,\boldsymbol{R}) = \chi(t,\,\boldsymbol{R}-\boldsymbol{r}/2)\phi(t,\,\boldsymbol{R}-\boldsymbol{r}/2,\boldsymbol{R})H_n(t,\,\boldsymbol{R})\phi(t,\,\boldsymbol{R},\boldsymbol{R}+\boldsymbol{r}/2)\psi^\dagger(t,\,\boldsymbol{R}+\boldsymbol{r}/2)$$

.. Examples of gluonic operators and light-quark operators for quarkonium hy and tetraquarks respectively, $\boldsymbol{q}=(u,\,d)$ and τ^a are isospin Pauli matrices.

Λ_η^σ	κ	H	$H = H^a T^a (I=0,\, I=1)$
Σ_g^+	0^{++}	$\mathbb{1}$	$\bar{\boldsymbol{q}}T^a(\mathbb{1},\,\boldsymbol{\tau})\boldsymbol{q}$
Σ_u^-	1^{+-}	$\hat{\mathbf{r}}\cdot\mathbf{B}$	$\bar{\boldsymbol{q}}\,[(\hat{\boldsymbol{r}}\times\boldsymbol{\gamma})\cdot,\,\gamma]\,T^a(\mathbb{1},\,\boldsymbol{\tau})\boldsymbol{q}$
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Π_g	1^{--}	$\hat{\mathbf{r}}\times\mathbf{E}$	$\bar{\boldsymbol{q}}\,(\hat{\boldsymbol{r}}\times\boldsymbol{\gamma})T^a(\mathbb{1},\,\boldsymbol{\tau})\boldsymbol{q}$

—define the static energies

$$E_n^{(0)}(r) = \lim_{T\to\infty}\frac{i}{T}\log\langle\mathcal{O}_n(T,\,\boldsymbol{r},\,\boldsymbol{R})|\mathcal{O}_n(0,\,\boldsymbol{r},\,\boldsymbol{R})\rangle$$

-obtain the coupled Schroedinger equations in BOEFT

N. B. Mohapatra Vairo in preparation

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-obtain the coupled Schroedinger equations in BOEFT

N. B. Mohapatra Vairo in preparation

-the structure of the spin corrections will be similar to the hybrids case (with a 1/m spin correction)
calculation of decays will usethe same technology

Tetraquarks and Pentaquarks: lattice input needed

Tetraquark static energies

I=1 S. Prevlosek, H. Bahtiyar, J. Petrovich eprint: 1912.02656

I=0 Bicudo Cichy Peters Wagner PRD 93 (2016) 034501

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Mixing between quarkonium, hybrids; hybrids, tetraquarks

Preliminary studies N. B. , Schlosser, Wagner, Vairo

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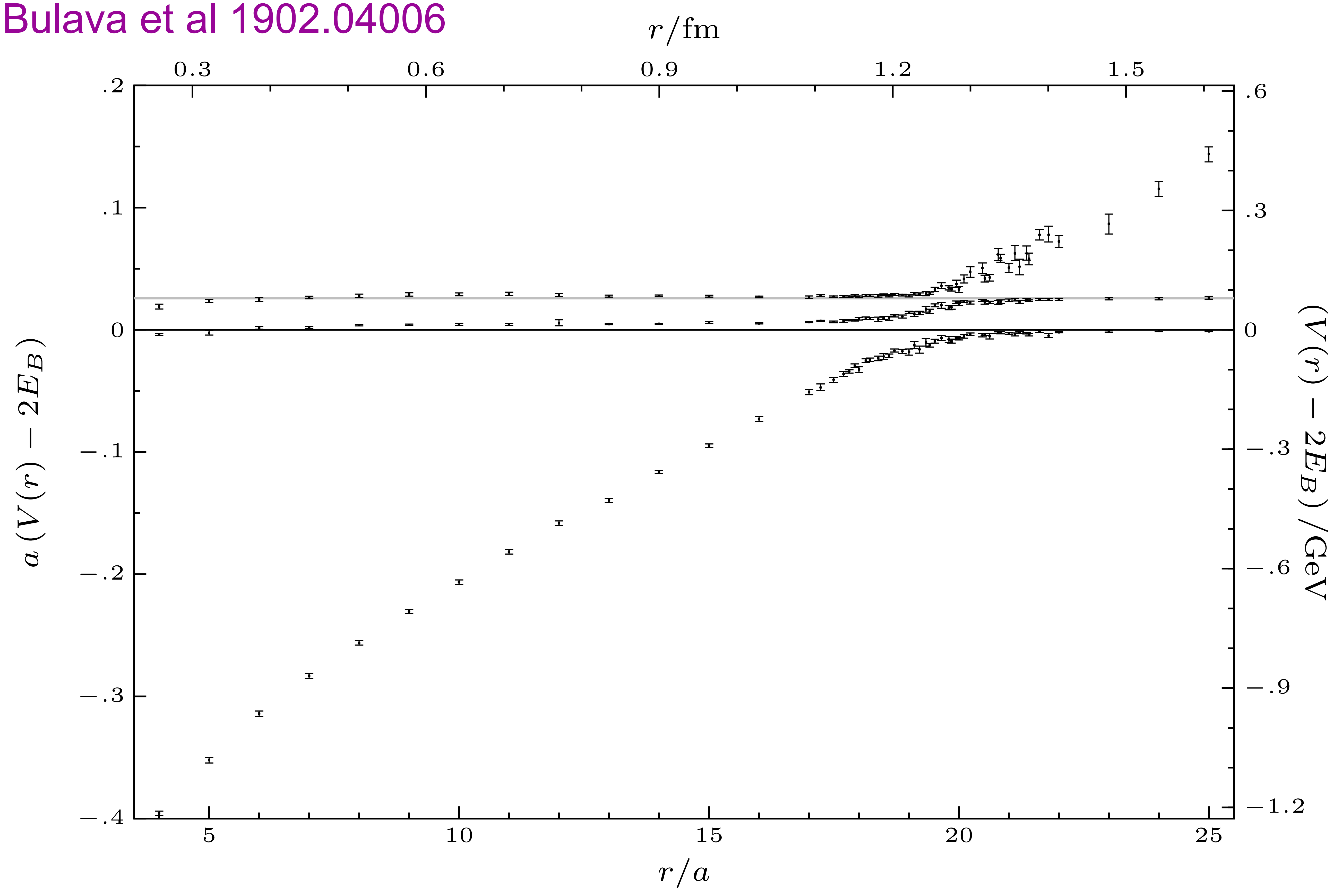
Cross talk with the heavy light static energies

Cross talk with the heavy light static energies

Adiabatic energy levels of the static energy of quarkonium and heavy-light, heavylight strange-> avoided level crossing

Bulava et al 1902.04006



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In the diabatic picture gives the coupling between quarkonium and heavy light allowing to calculate the decay to heavy light states

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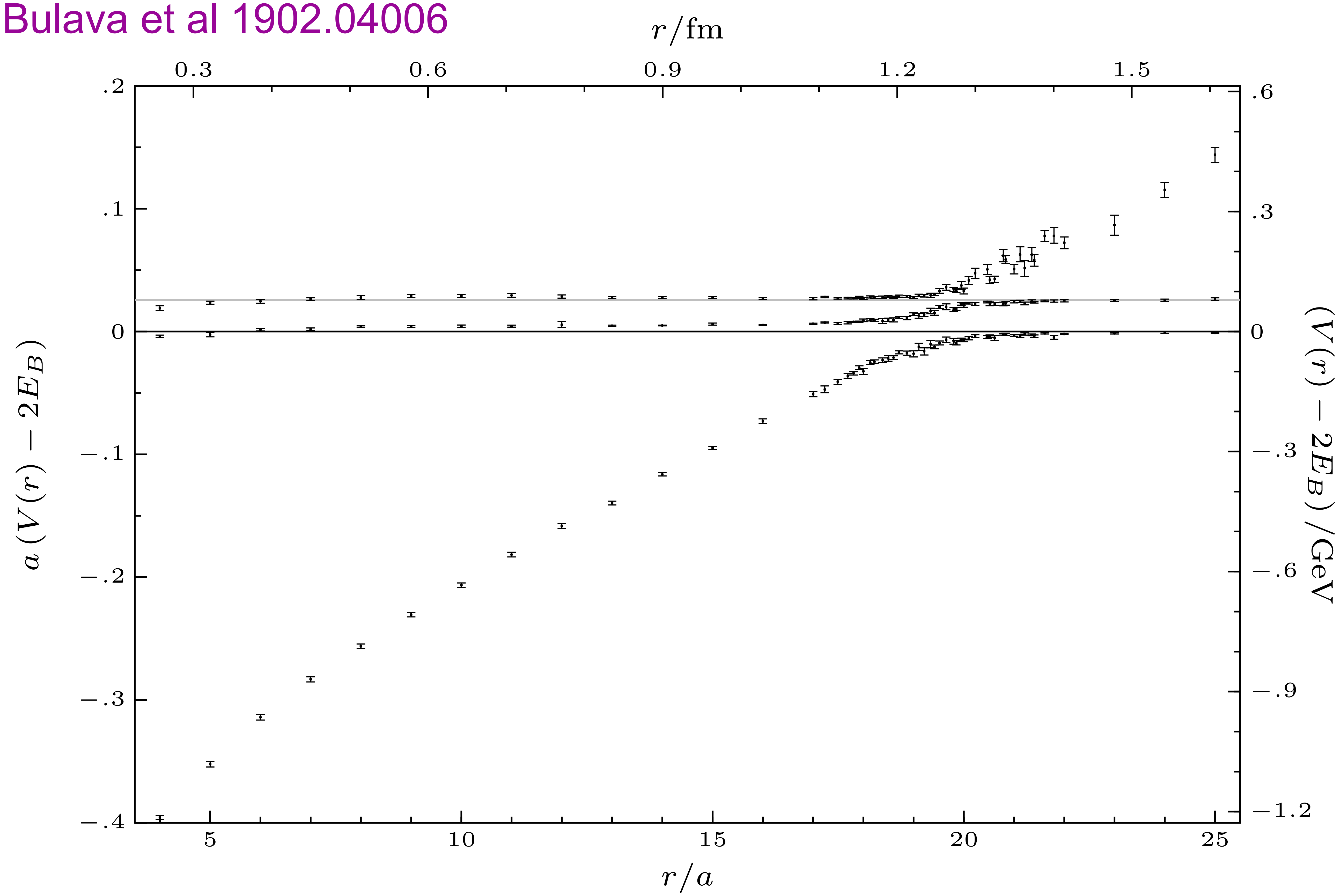

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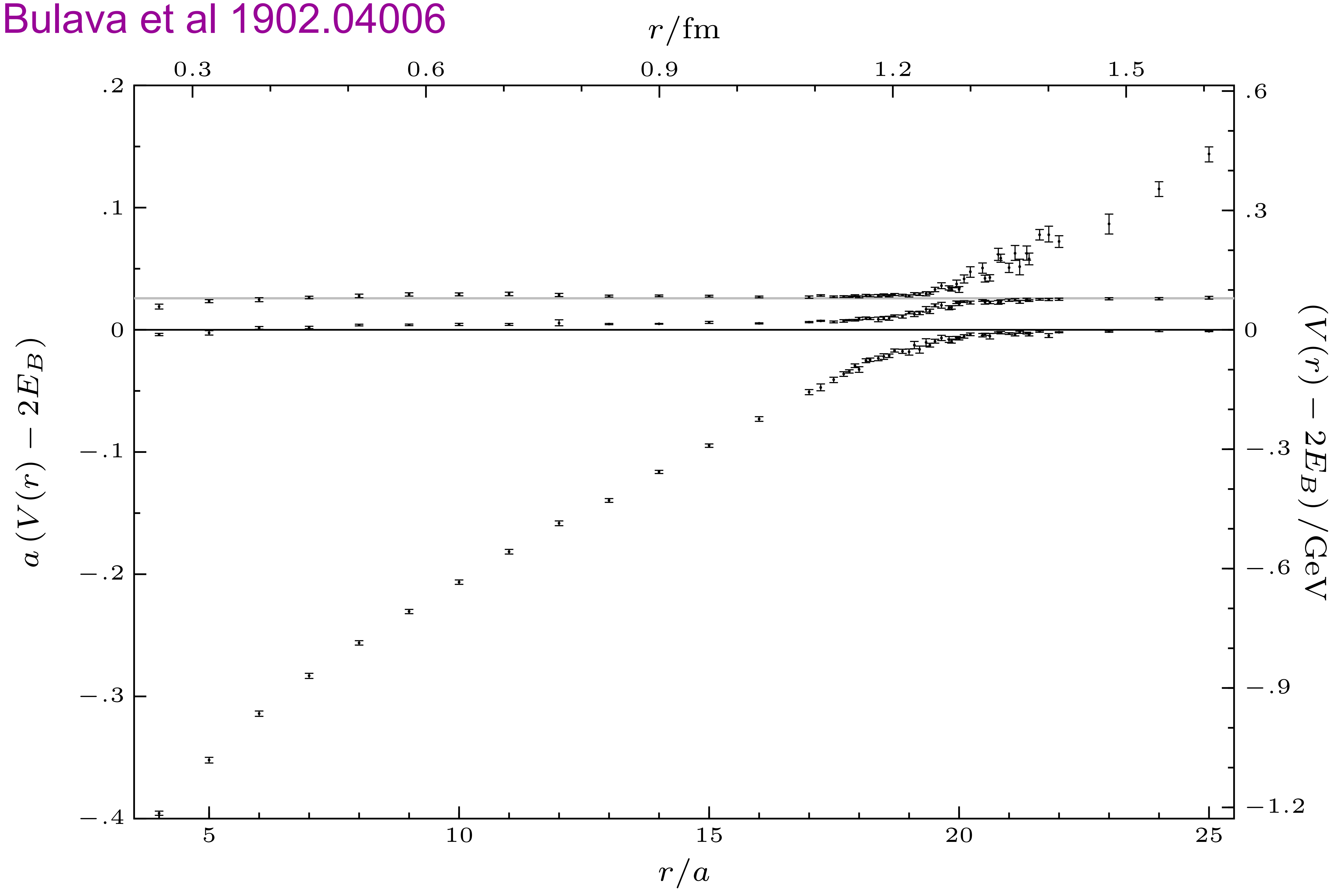

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The same cross talk with the heavy light static energies can be studied for hybrids and tetraquarks

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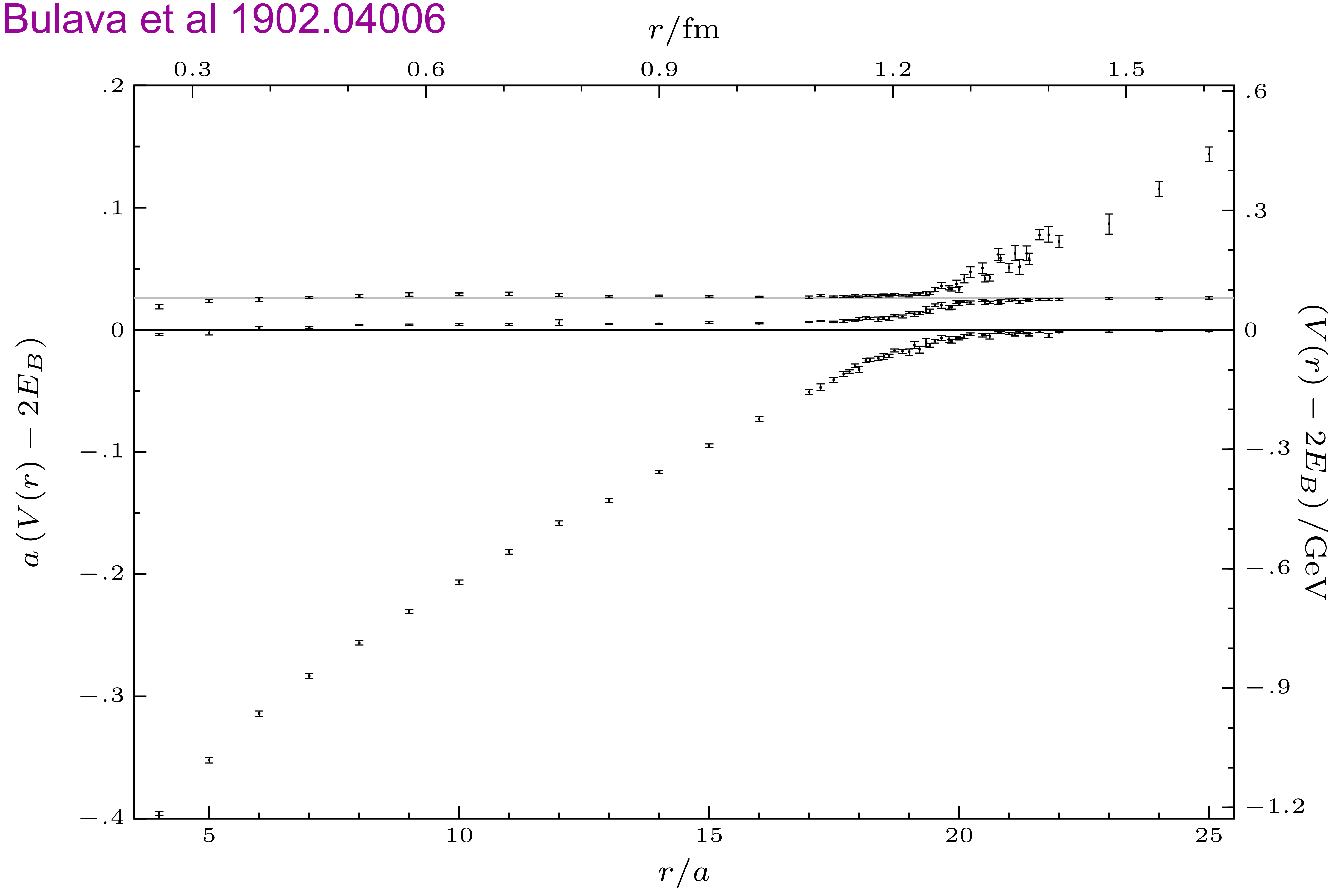

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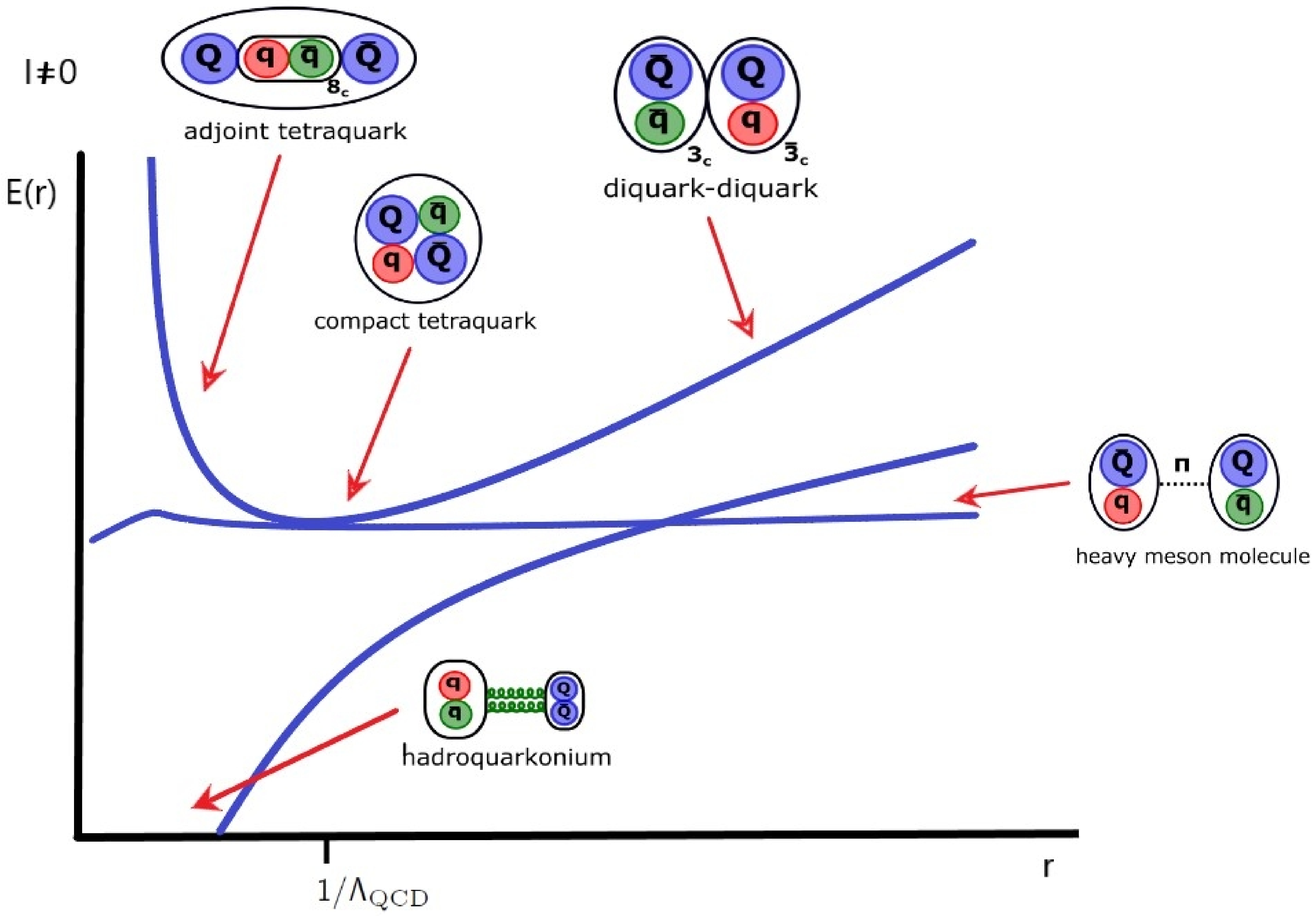
The same cross talk with the heavy light static energies can be studied for hybrids and tetraquarks

In this way special states with strong molecular characteristics like the X(3872) can be originated

Bruschini, Gonzalez 2111.07653

The BOEFT contains all models: what dominates and where depends on the QCD dynamics

Static energies for $I \neq 0$ (schematic):



The static energies are defined in BOEFT that gives the appropriate set of operators to be used and could describe the short distance limit.
Being nonperturbative objects E(r) should be calculated on the lattice (or in QCD vacuum models)

Figure from J. Tarrus

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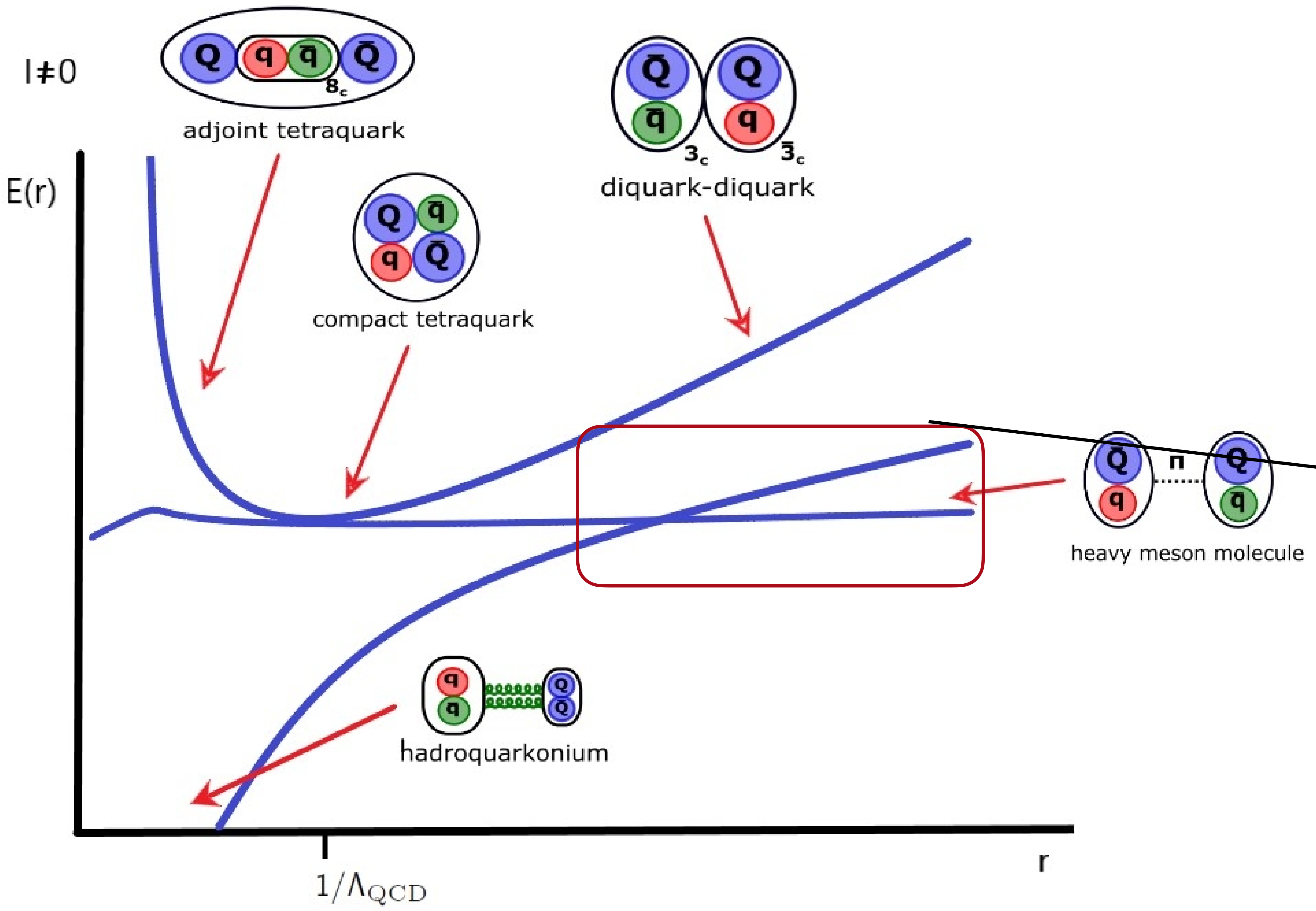


avoided crossing of the energy levels, mixing with open flavour meson-meson configurations

Bruschini, Gonzalez 2021

The static energies are defined in BOEFT that gives the appropriate set of operators to be used and could describe the short distance limit.
Being nonperturbative objects E(r) should be calculated on the lattice (or in QCD vacuum models)

Figure from J. Tarrus

Fully heavy Tetraquarks

Assi and Wagman 2311.01498

Tetraquarks	Order	$M_{Q\bar{Q}Q\bar{Q}}$ [GeV]	$B_{Q\bar{Q}Q\bar{Q}}$ [MeV]
$T_{cc\bar{c}\bar{c}}$	LO	6.1276(3)	16.6(4)
	NLO	6.078(2)	67.9(1)
	NNLO′	6.018(3)	144(2)
$T_{cc\bar{c}\bar{b}}/T_{bc\bar{c}\bar{c}}$	LO	9.294(3)	23.0(4)
	NLO	9.312(4)	72(2)
	NNLO′	9.259(5)	139(2)
$T_{bb\bar{c}\bar{c}}/T_{cc\bar{b}\bar{b}}$	LO	12.503(1)	23.7(4)
	NLO	12.457(4)	79(2)
	NNLO′	12.386(3)	157(3)
$T_{bc\bar{b}\bar{c}}$	LO	12.471(5)	19.5(8)
	NLO	12.417(5)	69(2)
	NNLO′	12.354(6)	139(2)
$T_{bb\bar{b}\bar{c}}/T_{bc\bar{b}\bar{b}}$	LO	15.652(6)	27.9(7)
	NLO	15.50(2)	87(2)
	NNLO′	15.37(7)	169(4)
$T_{bb\bar{b}\bar{b}}$	LO	18.8693(5)	31.2(6)
	NLO	18.8207(6)	83.6(1)
	NNLO′	18.7598(6)	151(1)

TABLE II. Predictions for tetraquark masses and binding energies for all combinations of tetraquarks involving only b and c quarks at each order of pNRQCD indicated. Pairs of tetraquarks in the same row have identical binding energies in our calculations due to charge conjugation.

Variational and Green function Monte Carlo method based on Weakly coupled pNRQCD potential calculated at LO NLO and NNLO’ (prime means only two body forces are considered)

Decays may be calculated in the same framework

XYZ production and evolution in medium can be studied with the tools developed for quarkonium

Bottomonium Nuclear Modification factor

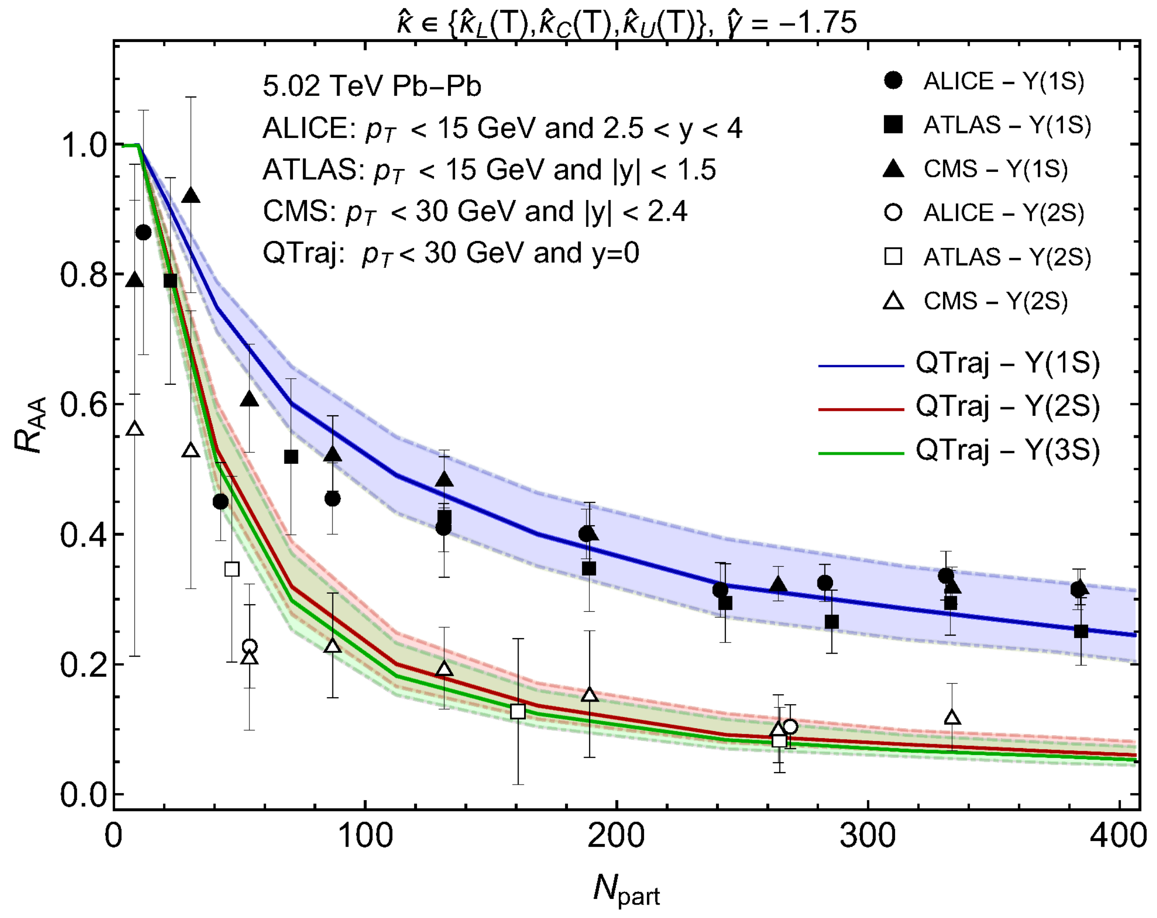
can be obtained using pNRQCD at finite temperature, density matrix, and open quantum systems

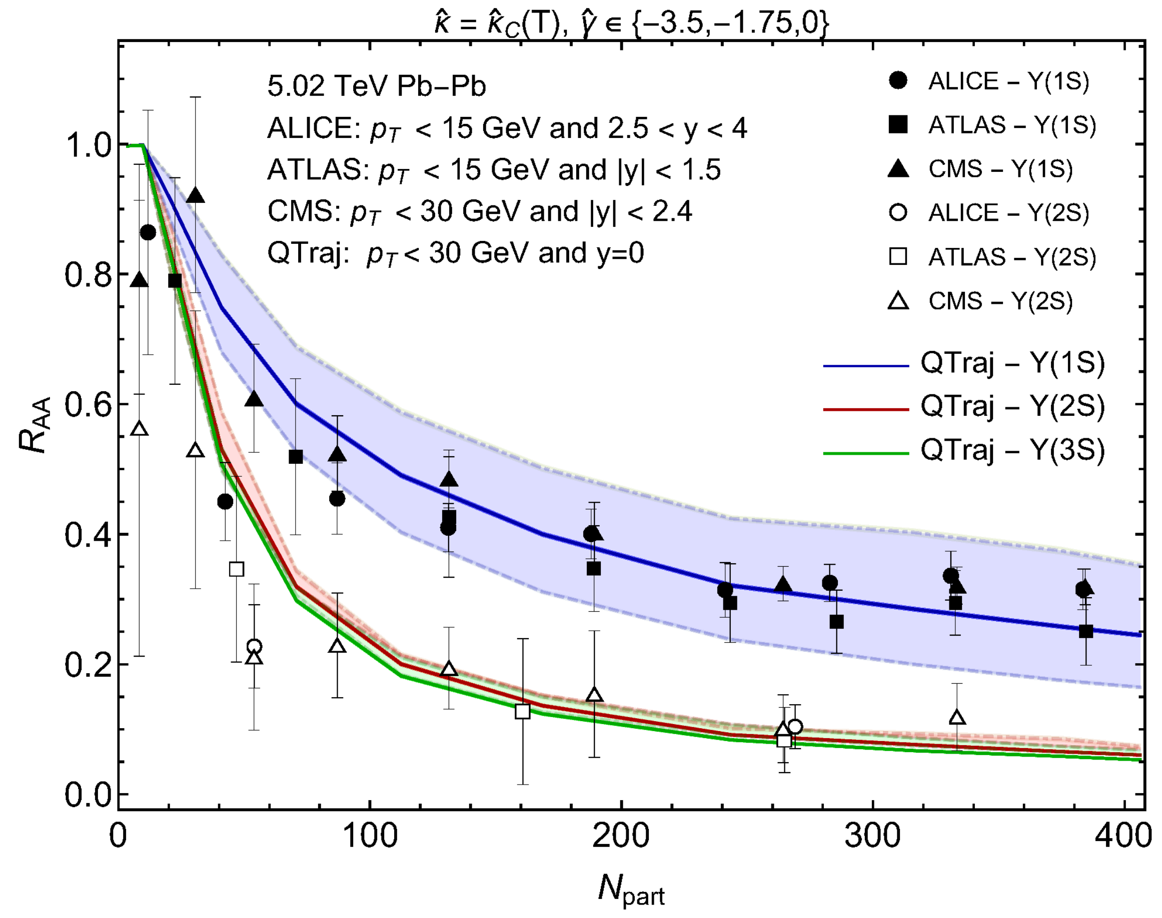
$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$

Bands are from the dependence in kappa and gamma parameters

N.B., M. Escobedo, M. Strickland, A. Vairo P. VanderGriend et al

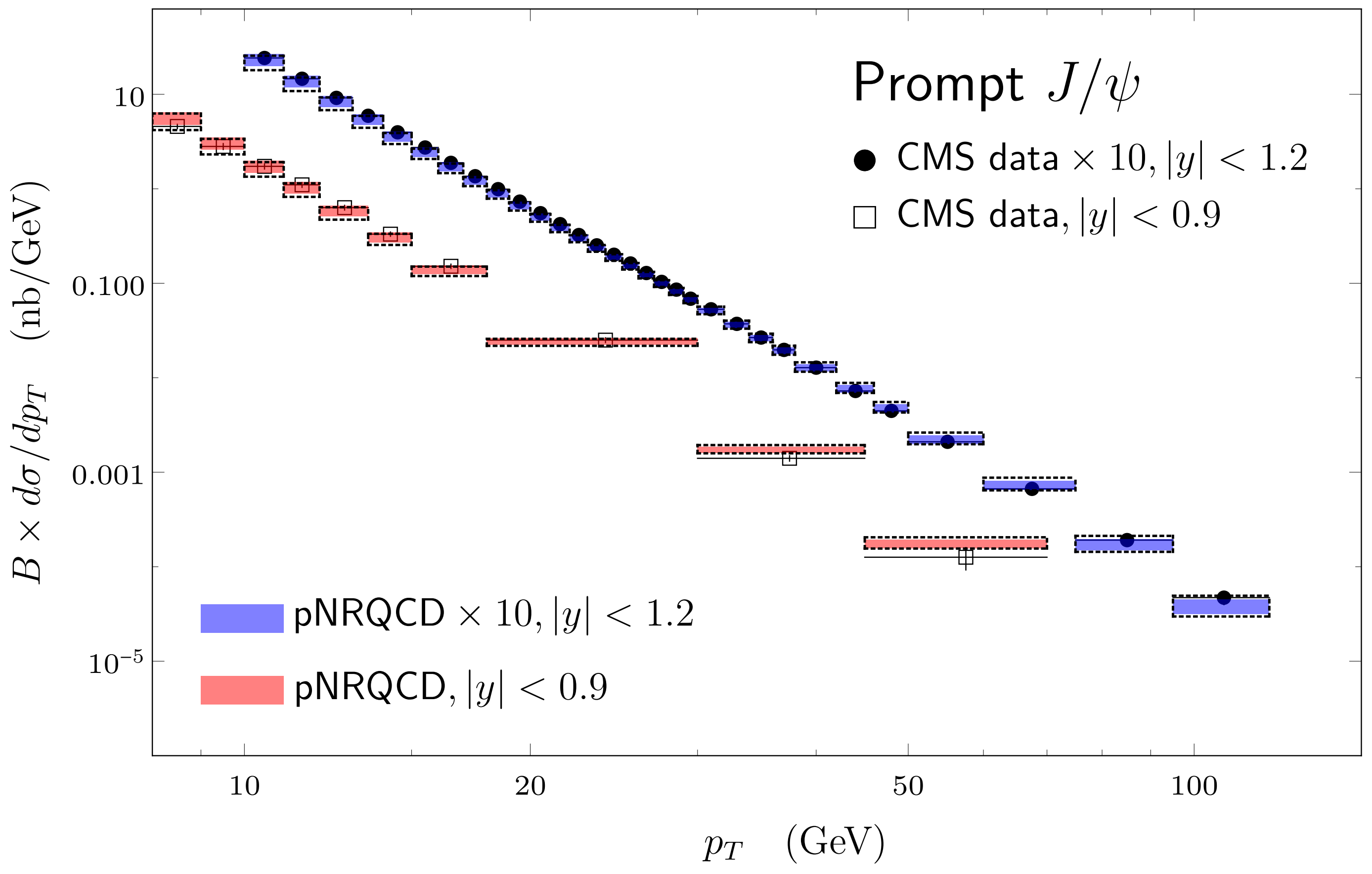
arXiv:2107.06222 arXiv:1711.04515 arXiv:2302.11826 arXiv:2205.10289

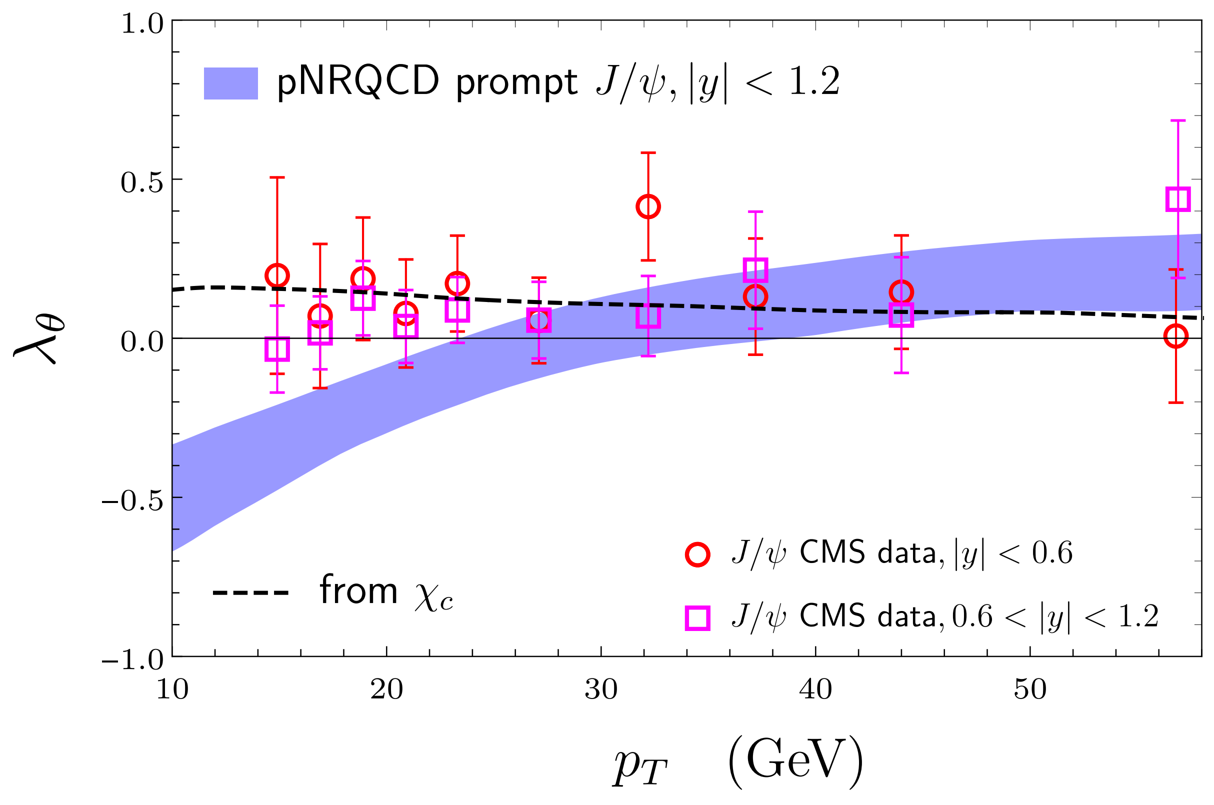


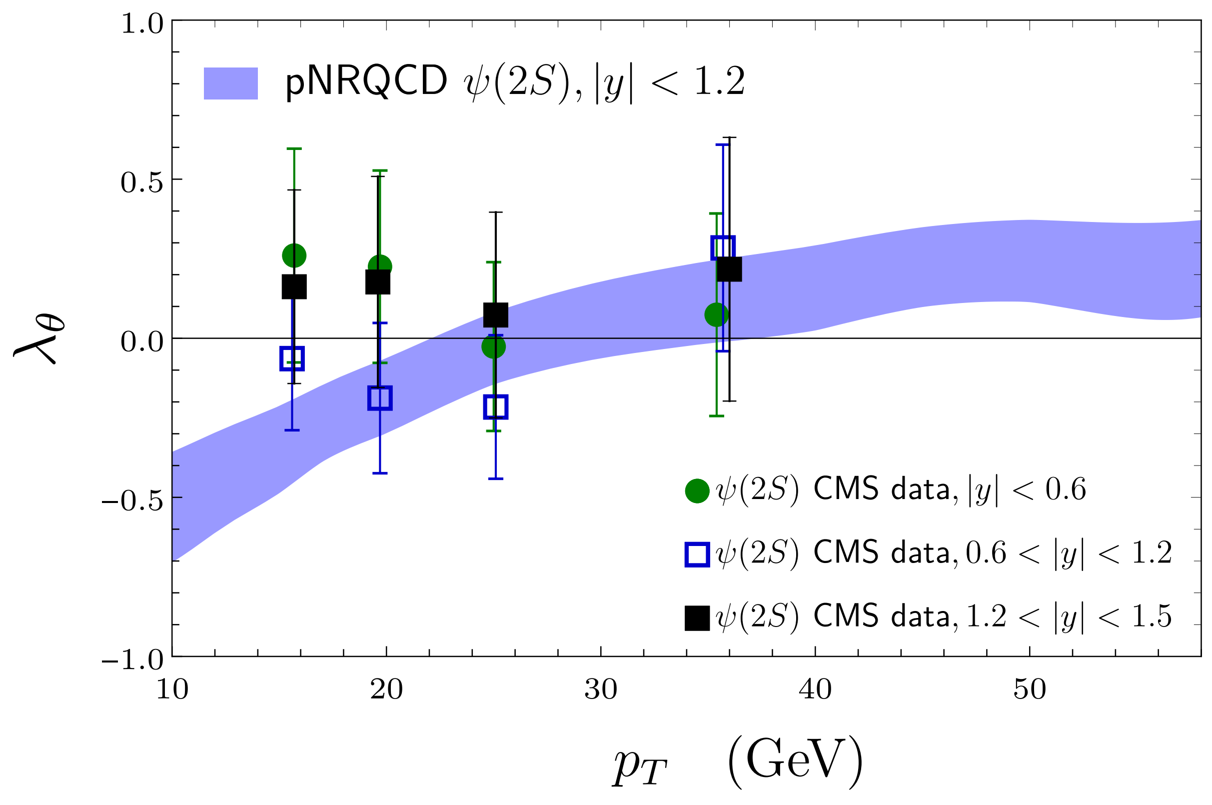


Quarkonium production can factorized and calculated in pNRQCD

N.B., Chung, Vairo, Wang 2210.17345







Outlook

NREFTs and lattice allows us to describe the physics of quarkonium away from the strong decay threshold in quantum field theory: higher order perturbative calculation can be performed and quarkonium can be used for precision physics/ factorisation allows to systematically studyconfinement

BOEFT allows to describe hybrids: new unexpected features are found (Lambda doubling, Spin structure, decays, mixing) that have important impact on the phenomenology

BOEFT allows to describe hybrids and calculate multiplets, mixing and decays: on going work

The same picture can be extended to tetraquarks and pentaquarks, once some lattice input on relevant correlators will be available.

NOTICE that the needed lattice calculations are simpler than the direct calculations of the X Y Z properties on the lattice, the knowledge of few correlators together with the BOEFTwill allow to obtain many phenomenological information

NREFTs and lattice and open quantum system allows us to describe the nonequilibrium evolution quarkonium in the quark gluon plasma and production processes: same theory could be then used for XYZ production and evolution in medium in heavy ion collisions

This picture has the possibility to give a unified description to exotics and to leave the dynamics decide which configuration will dominate in a given range

Material for discussion/references

pNRQCD

N. Brambilla, A. Pineda, J. Soto, A. Vairo
Rev. Mod . Phys 77 (2005) 1423 • hep-ph/0410047

N. Brambilla, A. Pineda J. Soto, A. Vairo
Nucl. Phys. B566 (2000) 275• hep-ph/9907240

A. Pineda J. Soto, hep-ph/9707481

N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan
The XYZ states: experimental and theoretical status and perspectives
Phys.Rept. 873 (2020) 1-154 • e-Print: 1907.07583 [hep-ex]

BOEFT

QCD spin effects in the heavy hybrid potentials and spectra Nora Brambilla, Wai Kin Lai, J. Segovia, J. Tarrus *Phys.Rev.D* 101 (2020) 5, 054040 • e-Print: 1908.11699

Quarkonium Hybrids with Nonrelativistic Effective Field Theories
Matthias Berwein , Nora Brambilla, Jaume Tarrús Castellà, Antonio Vairo
Phys.Rev. D92 (2015) no.11, 114019
e-Print: arXiv:1510.04299

Born-Oppenheimer approximation in an effective field theory language
Nora Brambilla , Gastão Krein, Jaume Tarrús Castellà, Antonio Vairo
Phys.Rev. D97 (2018) no.1, 016016
e-Print: arXiv:1707.09647

Spin structure of heavy-quark hybrids
N. Brambilla, Wai KIn Lai, J. Segovia, J. Tarrus A. Vairo
Phys.Rev.D 99 (2019) 1, 014017, e-Print: 1805.07713 [hep-ph]

Long range properties of 1S bottomonium states
N. Brambilla, G. Krein, J.. Tarrus, A. Vairo
Phys.Rev.D 93 (2016) 5, 054002 • e-Print: 1510.05895

BOEFT

Heavy hybrids decays to quarkonia
N. Brambilla, A. Mohapatra A. Vairo
Phys.Rev.D 107 (2023) 5, 054034, e-Print: 2212.09187[hep-ph]

Oncala and Soto
Phys.Rev.D 96 (2017) 1, 014004 •
Heavy hybrids: spectrum, decay and mixing

Nonrelativistic effective field theory for heavy exotic hadrons
J. Soto, J. Tarrus Published in: *Phys.Rev.D* 102 (2020) 1, 014012 •

Material for discussion/references

Inclusive production of J/psi psi(2s) Y states in pNRQCD
N. Brambilla, H. S. Chung, A. Vairo, X.P. Wang
JHEP 03 (2023) 242 2210.17345

Quarkonium Production

Gradient Flow Perturbative calculations

QCD static force in gradient flow
N. Brambilla et al
JHEP 01 (2022) 184
e-Print: 2111.07811[hep-ph]

alpha_s Extraction

Determination of the QCD coupling from the static energy and the free energy
A. Bazavov N. Brambilla, et al TUMQCD
Phys.Rev.D 100 (2019) 11, 114511 e-Print: 1907.11747[hep-ph]

Gradient Flow Lattice calculations

QCD Force
J. Mayer-Steudte N. Brambilla et al TUMQCD
e-Print: 2212.12400[hep-ph]

LATTICE

Static energy in (2+1+1) flavor lattice QCD
N. Brambilla, et al TUMQCD
Phys.Rev.D 107 (2023) 7, 074503, e-Print: 2206.03156[hep-ph]

Lattice computation of the static force
N. Brambilla, V. Leino et al TUMQCD
Phys.Rev.D 105 (2022) 5, 054514 e-Print: 2106.01794 hep-ph]

EFTs and Lattice for XYZ
N. Brambilla
e-Print: 2111.10788[hep-ph]

Material for discussion/references

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **96** (2017) no.3, 034021 [arXiv:1612.07248 [hep-ph]].

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **97** (2018) no.7, 074009 [arXiv:1711.04515 [hep-ph]].

N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, Phys. Rev. D **100** (2019) no.5, 054025 [arXiv:1903.08063 [hep-ph]].

N. Brambilla, M. A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. H. Weber, [arXiv:2012.01240 [hep-ph]]. and arXiv: 2107 .06222

M. Escobedo *Phys.Rev.D* 103 (2021) 3, 034010 • e-Print: 2010.10424 [
N. Brambilla, M. Escobedo, A. Islam, M. Strickland . A. Tiwari, A. Vairo arXiv 2205.10289

Regeneration of bottomonia in an open quantum systems approach
N. Brambilla, M. Escobedo, M Strickland et al **2302.11826**

Non-equilibrium evolution in QGP

N. Brambilla, J. Ghiglieri, A. Vairo, Peter Petreczky
Phys.Rev.D 78 (2008) 014017 • e-Print: 0804.0993 [hep-ph]

N. Brambilla, M. Escobedo, J. Ghiglieri, A. Vairo
JHEP 05 (2013) 130 • e-Print: 1303.6097 [hep-ph]

N. Brambilla, M. Escobedo, J. Ghiglieri, A. Vairo
JHEP 12 (2011) 116 • e-Print: 1109.5826 [hep-ph]

N. Brambilla, M. Escobedo, J. Ghiglieri, J. Soto, A. Vairo
JHEP 09 (2010) 038 • e-Print: 1007.4156 [hep-ph]

Potential and energies in medium

N. Brambilla, V. Leino, Peter Petreczky, A. Vairo e-Print 2007.10078 [hep-ph],
N. Brambilla, V. Leino, J. Meyer Steudte,P. Petreczky 2206.02861

Lattice calculation of the heavy quark transport coefficient