

Experimental overview of CKM metrology from kaon physics



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Contents:

- Experimental status and Cabibbo Angle Anomaly
- Future experimental prospects

CKM matrix and 1st row unitarity

- SM coupling of quarks and leptons to W :

$$\frac{g}{\sqrt{2}} W_\alpha^+ (\mathbf{U}_L V_{CKM} \gamma^\alpha \mathbf{D}_L + \bar{e}_L \gamma^\alpha \nu_{eL} + \bar{\mu}_L \gamma^\alpha \nu_{\mu L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau L}) + \text{h.c.}$$

Single gauge coupling, g

Unitary CKM matrix

- Unitarity condition for 1st row:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$\therefore \Delta_{CKM}^u = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{td} \end{pmatrix}$$

$$|V_{CKM}^{ij}| = \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.042 \\ 0.009 & 0.041 & 0.9991 \end{pmatrix}$$

$$|V_{ud}|^2 \approx 0.949$$

$$|V_{us}|^2 \approx 0.05$$

$$|V_{ub}|^2 \approx 2 \times 10^{-5}$$

small

$$\therefore \Delta_{CKM}^u \approx |V_{ud}|^2 + |V_{us}|^2 - 1 = 0$$

Focus on measurements of V_{ud} and V_{us}

[PDG 2023]

CKM matrix and 1st row unitarity

- SM coupling of quarks and leptons to W :

$$\frac{g}{\sqrt{2}} W_{\alpha}^{+} (\mathbf{U}_L V_{CKM} \gamma^{\alpha} \mathbf{D}_L + \bar{e}_L \gamma^{\alpha} \nu_{eL} + \bar{\mu}_L \gamma^{\alpha} \nu_{\mu L} + \bar{\tau}_L \gamma^{\alpha} \nu_{\tau L}) + \text{h.c.}$$

Single gauge coupling, g

Unitary CKM matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{td} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Where $\lambda = V_{us} = \sin \theta_{12} = \sin \theta_{\text{Cabibbo}}$

CKM matrix and 1st row unitarity

- SM coupling of quarks and leptons to W :

$$\frac{g}{\sqrt{2}} W_\alpha^+ (\mathbf{U}_L V_{CKM} \gamma^\alpha \mathbf{D}_L + \bar{e}_L \gamma^\alpha \nu_{eL} + \bar{\mu}_L \gamma^\alpha \nu_{\mu L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau L}) + \text{h.c.}$$

Single gauge coupling, g (points to g)

Unitary CKM matrix (points to V_{CKM})

- Most precise test of CKM unitarity, unitarity of for 1st row:

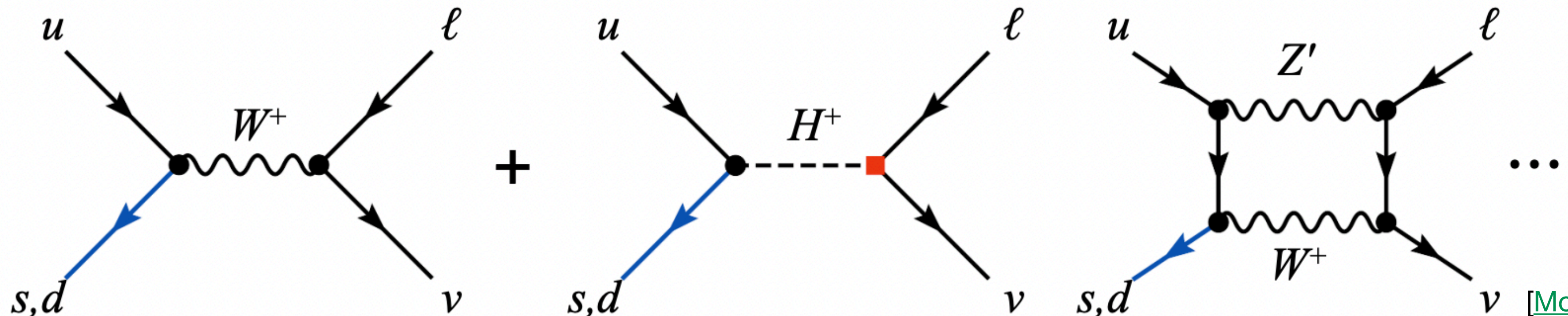
$$\therefore \Delta_{CKM}^u = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$$

$|V_{ub}|^2 \approx 2 \times 10^{-5}$

- Universality: is the coupling the same for quarks and leptons?

$$G_\ell^2 = \frac{g_e g_\mu}{m_W^4} \stackrel{?}{=} G_{CKM}^2 = \frac{(g_q g_\ell)^2 (|V_{ud}|^2 + |V_{us}|^2)}{m_W^4}$$

- BSM physics can break gauge universality:



CKM matrix and 1st row unitarity

- SM coupling of quarks and leptons to W :

$$\frac{g}{\sqrt{2}} W_\alpha^+ (\mathbf{U}_L V_{CKM} \gamma^\alpha \mathbf{D}_L + \bar{e}_L \gamma^\alpha \nu_{eL} + \bar{\mu}_L \gamma^\alpha \nu_{\mu L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau L}) + \text{h.c.}$$

Single gauge coupling, g Unitary CKM matrix

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- BSM physics can break gauge universality:

$$G_{CKM,ij} \sim G_\ell V_{ij} \sim \frac{g^2}{m_W^2} V_{ij} \quad \delta G_{CKM,ij} \sim \frac{1}{\Lambda^2} \quad \longrightarrow \quad \text{BSM effects scale as } \frac{m_W^2}{g^2} \frac{1}{\Lambda^2}$$

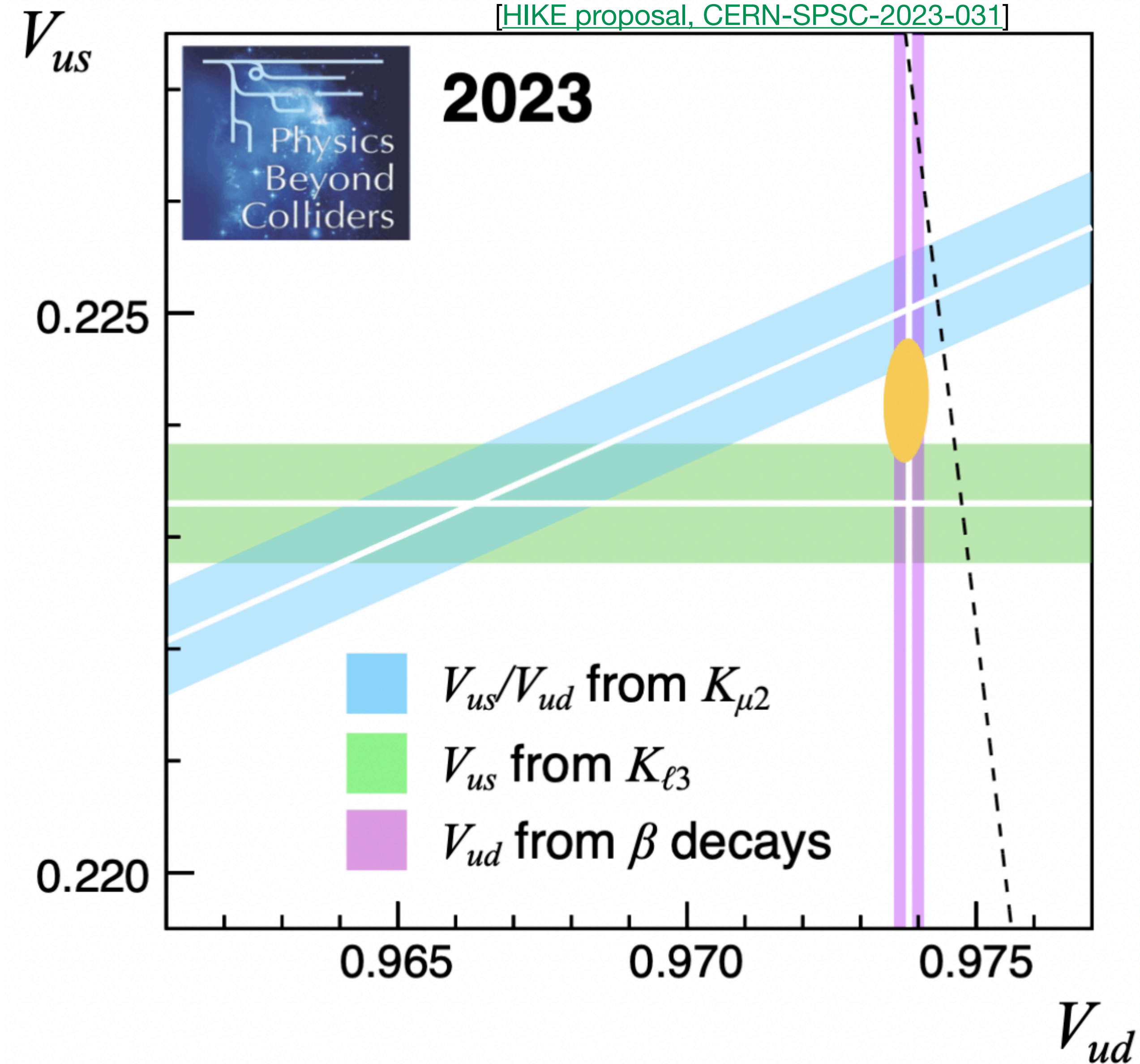
- Measurement of Δ_{CKM}^u with uncertainty σ probes scale $\Lambda \sim \frac{m_W}{g} \frac{1}{\sqrt{\sigma}}$

- e.g. for $\sigma \sim 10^{-4}$ probe scale $\Lambda \sim 20 \text{ TeV}$

Status of 1st row unitarity test

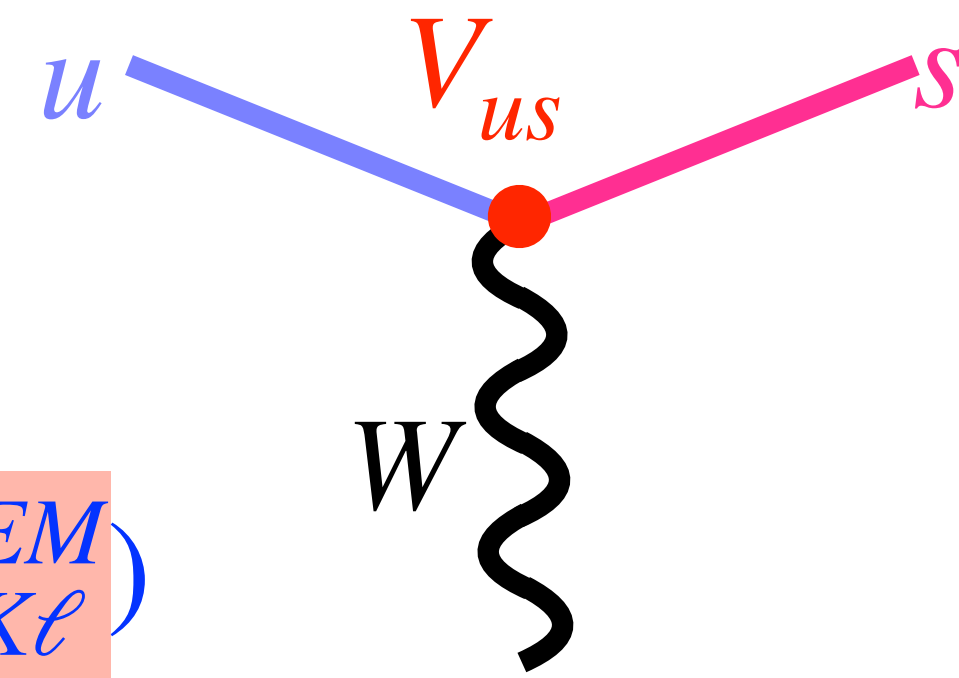
$$\Delta_{CKM}^u \approx |V_{ud}|^2 + |V_{us}|^2 - 1 = 0$$

- $\therefore |V_{ud}|^2 + |V_{us}|^2 \approx 1$ defines a circle [dotted line]
- Independent methods measuring $|V_{ud}|$ and $|V_{us}|$, and their ratio, should all meet at a point on this circle...
- Discrepancy between measurements constitutes a **3 σ deficit** in 1st row CKM unitarity : **Cabibbo angle anomaly**.
- A tension with similar significance exists between rates of $K \rightarrow \ell \nu$ and $K \rightarrow \pi \ell \nu$.



| V_{us} | Measurement

$|V_{us}|$ from $K_{\ell 3}$ decays



$$\Gamma(K \rightarrow \pi \ell \nu(\gamma)) = |V_{us}|^2 \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |f_+^{K^0\pi^-}(0)|^2 I_{K_L}(\lambda_{K\ell}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM})$$

- Where $K \in [K^+, K^0]$ and $\ell \in [e, \mu]$. $C_K^2 = \begin{cases} 0.5 & \text{for } K^+ \\ 1 & \text{for } K^0 \end{cases}$.
- $S_{EW} = 1.0232$: universal Short-Distance EW correction

Experimental inputs

- $\Gamma(K \rightarrow \pi \ell \nu(\gamma))$: decay rates with well-determined radiative decay treatment:
 - $K^\pm, K_{L,S}$ Branching ratios
 - Kaon lifetimes
- $I_{K_L}(\lambda_{K\ell})$: integral of form factor over phase-space.
 - λ terms parameterise t evolution.
 - K_{e3} : only λ_+ (or λ'_+, λ''_+), $K_{\mu 3}$: need λ_+ and λ_0

Theoretical inputs

- $f_+^{K^0\pi^-}(0)$: hadronic metric element (**form factor**) at 0 momentum transfer ($t=0$)
- $\Delta_K^{SU(2)}$: **Form-factor correction** for $SU(2)$ breaking
- $\Delta_{K\ell}^{EM}$: **Form-factor correction** for Long-Distance EM effects.

$|V_{us}|$: Kaon decay rates experimental inputs

$$\Gamma(K \rightarrow \pi \ell \nu(\gamma)) = |V_{us}|^2 \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |f_+^{K^0\pi^-}(0)|^2 I_{K_L}(\lambda_{K\ell}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM})$$

K_S decays:

- $\mathcal{B}(K_S \rightarrow \pi^0 \pi^0)$
- $\mathcal{B}(K_S \rightarrow \pi^+ \pi^-(\gamma))$
- $\mathcal{B}(K_S \rightarrow \pi \ell \nu), \ell = e, \mu$
- Lifetime τ_s

K_L decays:

- $\mathcal{B}(K_L \rightarrow \pi \ell \nu), \ell = e, \mu$
- $\mathcal{B}(K_L \rightarrow \pi^0 \pi^0 \pi^0)$
- $\mathcal{B}(K_L \rightarrow \pi^+ \pi^- \pi^0)$
- $\mathcal{B}(K_L \rightarrow \pi^+ \pi^- (\gamma_{(IB,DE)}))$
- $\mathcal{B}(K_L \rightarrow \pi^0 \pi^0)$
- $\mathcal{B}(K_L \rightarrow \gamma\gamma)$
- Lifetime τ_L

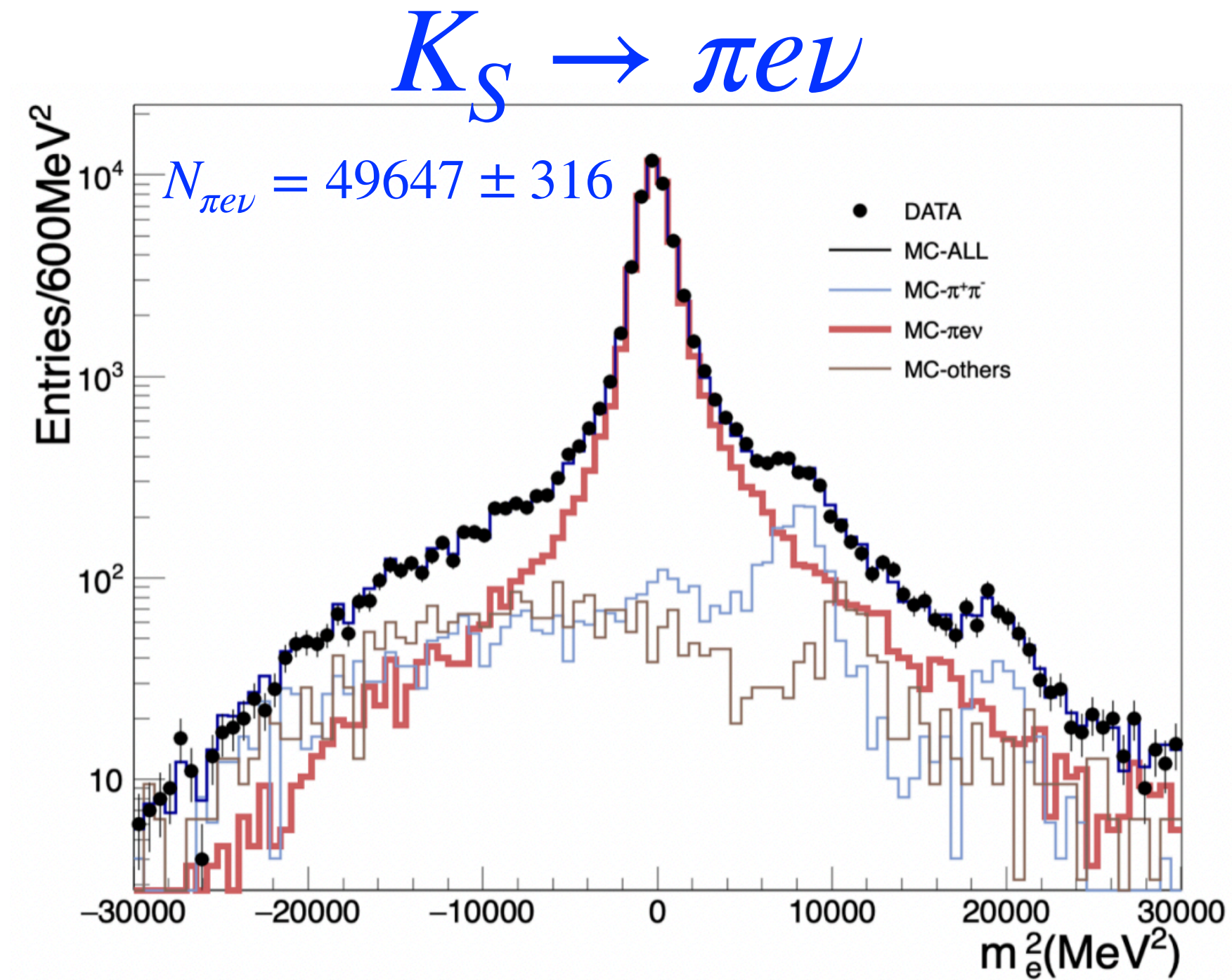
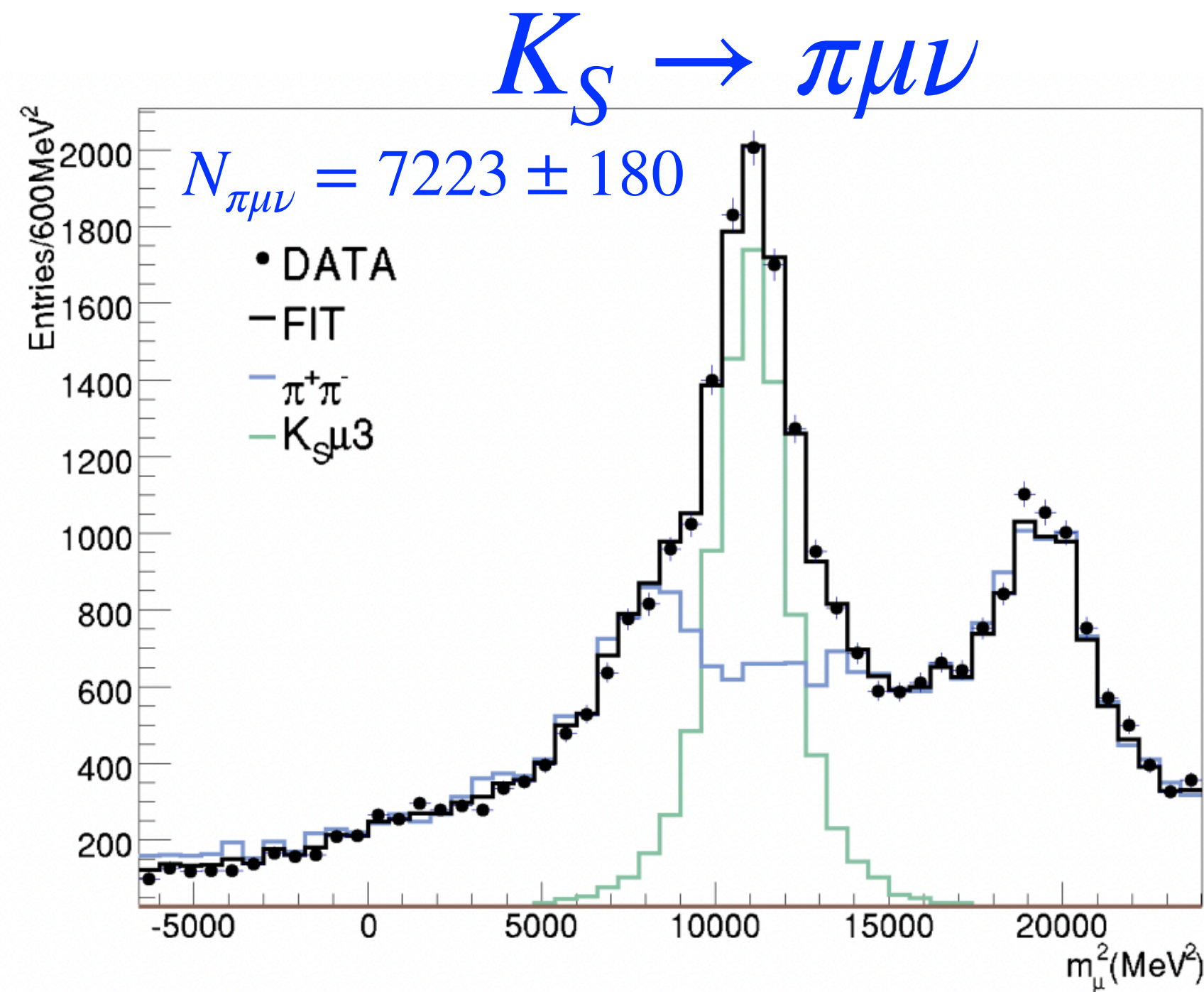
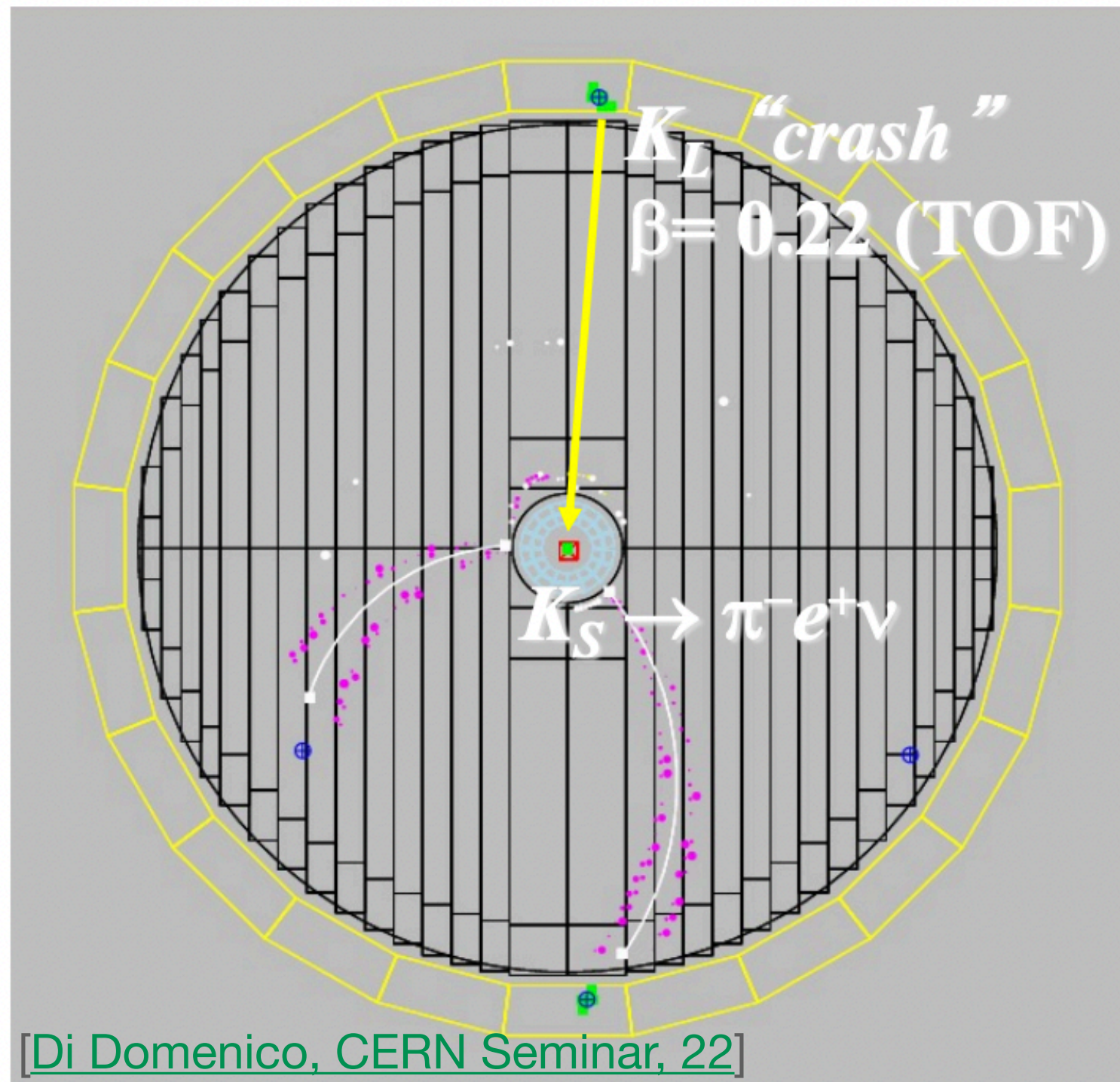
K^\pm decays:

- $\mathcal{B}(K^\pm \rightarrow \mu^\pm \nu)$
- $\mathcal{B}(K^\pm \rightarrow \pi^\pm \pi^0)$
- $\mathcal{B}(K^\pm \rightarrow \pi^\pm \pi^+ \pi^-)$
- $\mathcal{B}(K^\pm \rightarrow \pi^0 \ell^\pm \nu), \ell = e, \mu$
- $\mathcal{B}(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0)$
- Lifetime τ_\pm

- Use various experimental inputs to fit...

KLOE/KLOE-2 : $\mathcal{B}(K_S \rightarrow \pi \ell \nu)$

- DAΦNE ϕ -factory at LNF: $e^+e^- \rightarrow \phi \rightarrow K_S K_L$: tag K_L from “crash” interaction in calorimeter.
- Select signal with kinematic BDT & time-of-flight $\pi \ell$ assignment: fit to $m_\ell^2 = (E_{K_S} - E_\pi - p_{miss})^2 - p_\ell^2$.



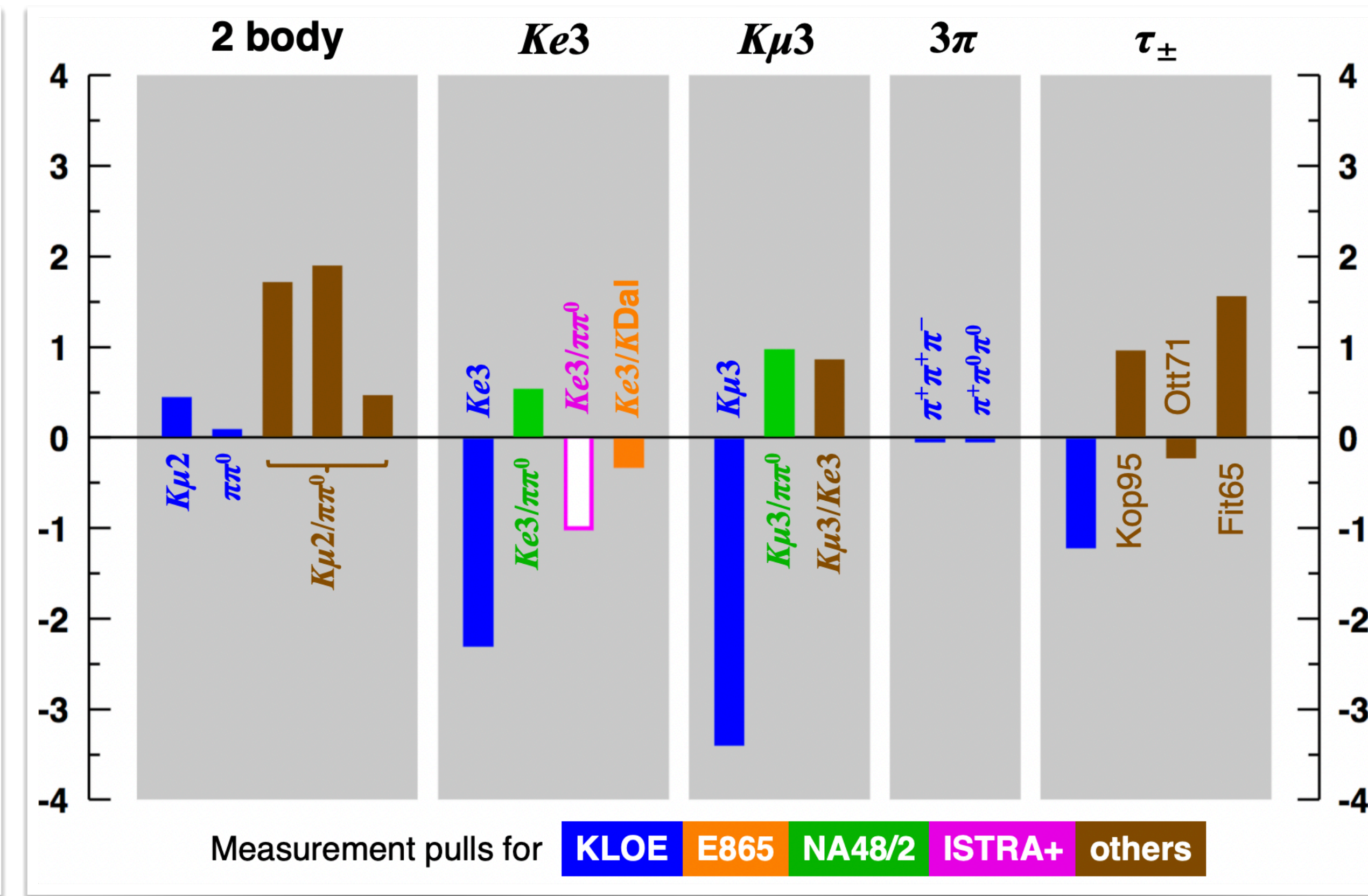
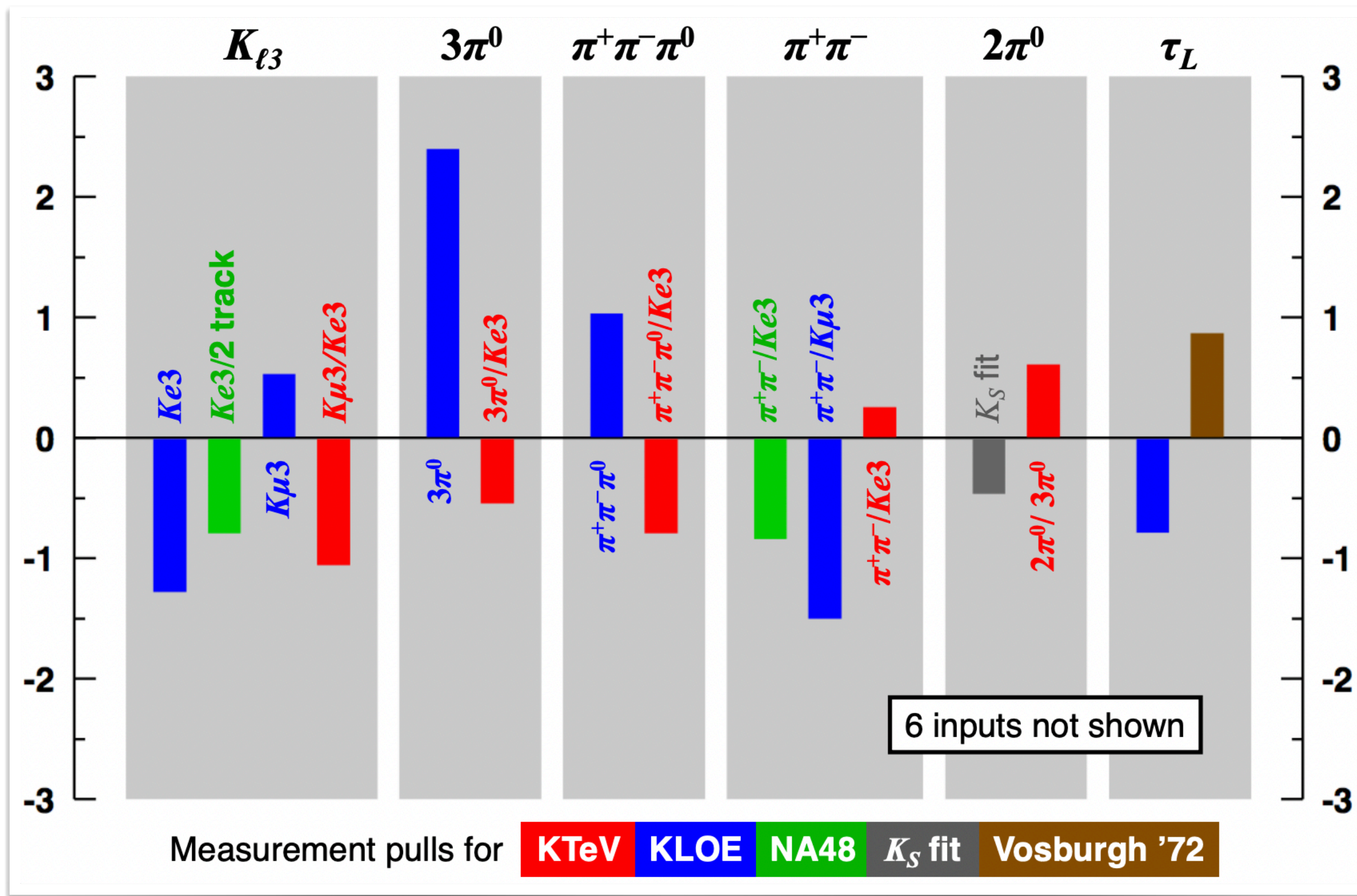
- [PLB 804 (2020) 135378] First measurement of BR: $\mathcal{B}(K_S \rightarrow \pi^0 \mu \nu) = (4.56 \pm 0.20) \times 10^{-4}$, based on 1.6 fb^{-1} .
- [JHEP 02 (2023) 098] **recent result**: $\mathcal{B}(K_S \rightarrow \pi^0 e \nu) = (7.153 \pm 0.037_{\text{stat}} \pm 0.044_{\text{syst}}) \times 10^{-4}$ [0.8% unc.], based on 1.6 fb^{-1} .
- Both normalised to $\mathcal{B}(K_S \rightarrow \pi^+ \pi^-)$ [also measured by KLOE]. Used to extract measurements of $|V_{us}| f_+(0)$ (see later).

Fits to K_L and K^\pm rate data : input data vs fit

[Moulson, CKM23]

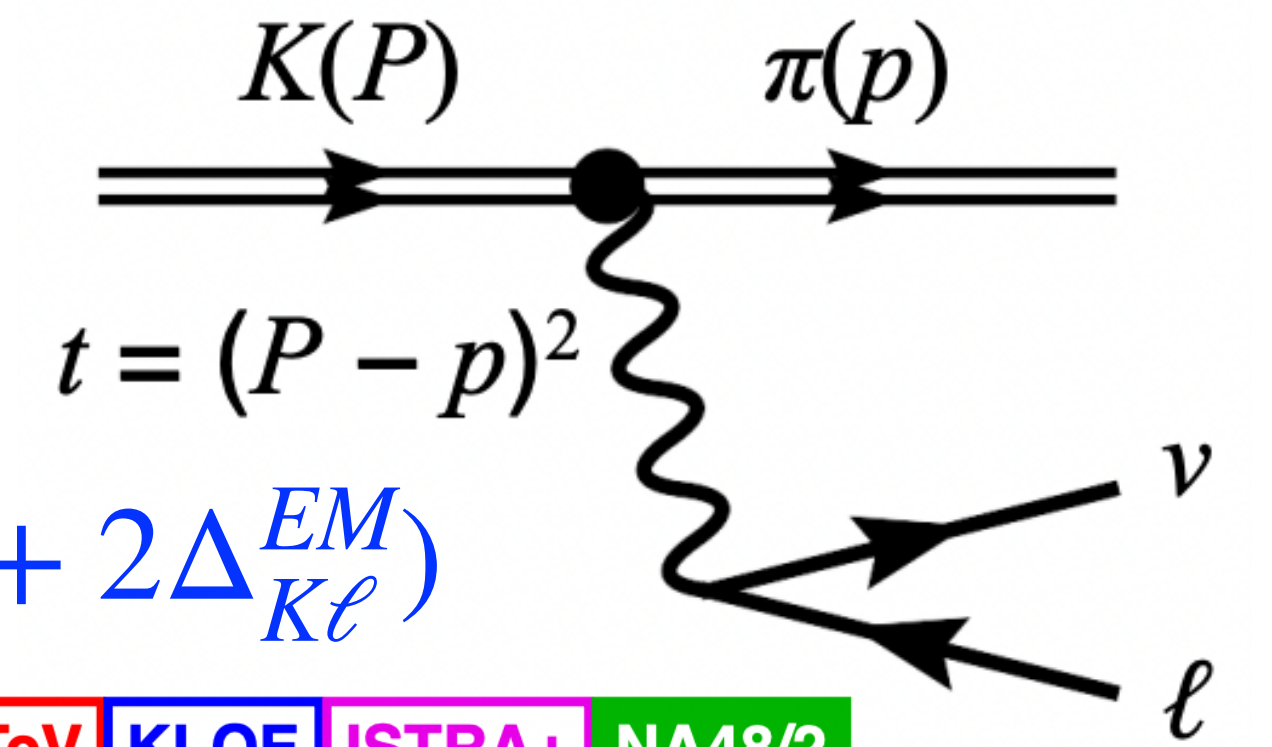
$$\Gamma(K \rightarrow \pi \ell \nu(\gamma)) = |V_{us}|^2 \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |f_+^{K^0\pi^-}(0)|^2 I_{K_L}(\lambda_{K\ell}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM})$$

K_L Fit K^\pm Fit



$K_{\ell 3}$ Form factors

[Moulson, CKM23]



$$\Gamma(K \rightarrow \pi \ell \nu(\gamma)) = |V_{us}|^2 \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |f_+^{K^0\pi^-}(0)|^2 I_{K_L}(\lambda_{K\ell}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM})$$

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**
 NA48 $K_{\ell 3}$ data included in fits but not shown

- Form factor parameterisations

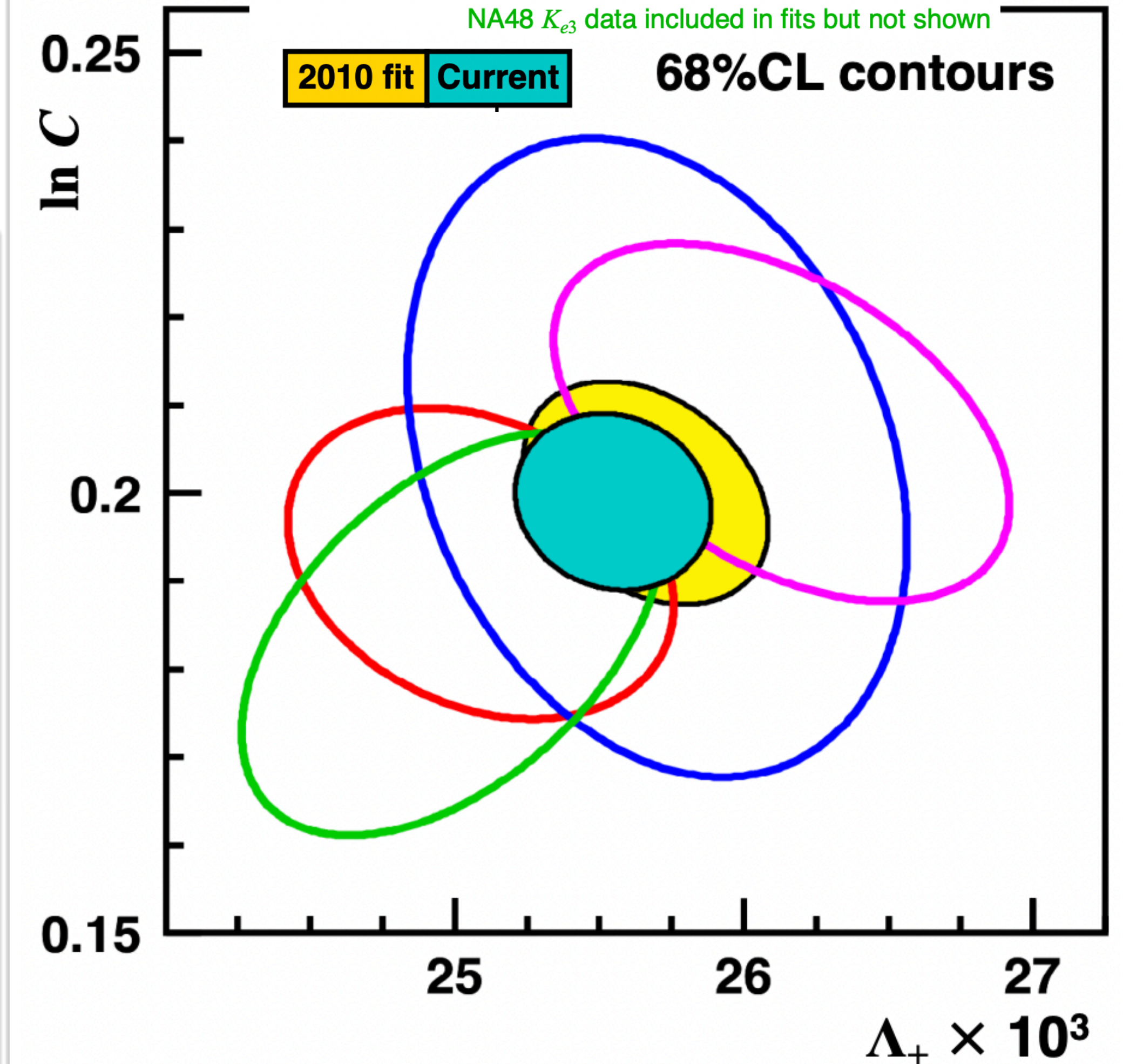
$$\tilde{f}_+(t) = \exp\left[\frac{t}{m_\pi^2(\Lambda_+ - H(t))}\right], \tilde{f}_0(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2(\ln C - G(t))}\right]$$

- Current fit:

$$\begin{aligned} \Lambda_+ \times 10^3 &= 25.55 \pm 0.38 \\ \ln C &= 0.1992(78) \\ \rho(\Lambda_+, \ln C) &= -0.110 \\ \chi^2/\text{ndf} &= 7.5/7 \text{ (38\%)} \end{aligned}$$

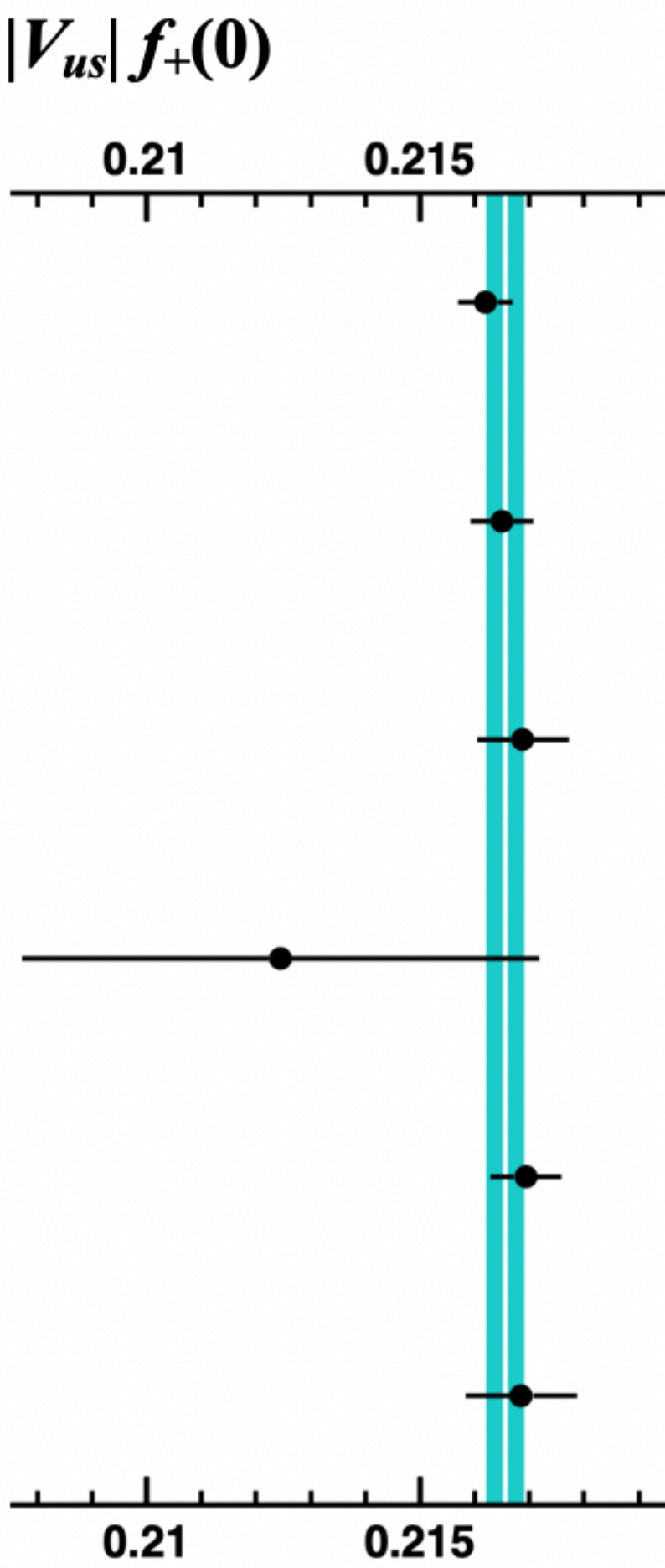
Integrals

Mode	Update	2010
K^0_{e3}	0.15470(15)	0.15476(18)
K^+_{e3}	0.15915(15)	0.15922(18)
$K^0_{\mu 3}$	0.10247(15)	0.10253(16)
$K^+_{\mu 3}$	0.10553(16)	0.10559(17)



$|V_{us}| f_+(0)$ and $f_+(0)$ from world data

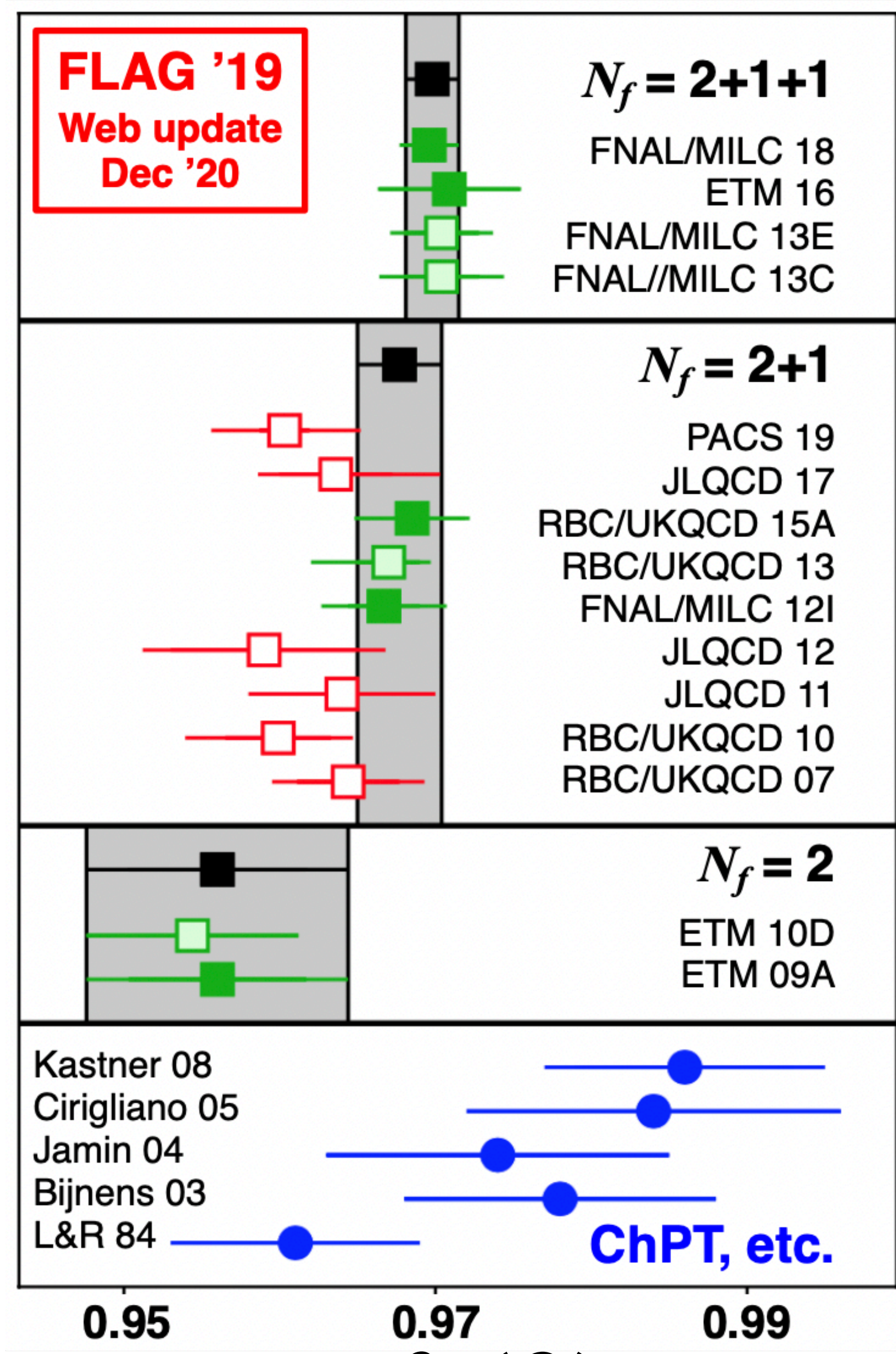
[Moulson, CKM23]



$K_{\ell 3}$ decays

		% err	Approx. contrib. to % err from:			
			BR	τ	Δ	Int
$K_L e 3$	0.2162(5)	0.23	0.09	0.20	0.02	0.05
$K_L \mu 3$	0.2165(6)	0.26	0.15	0.18	0.02	0.07
$K_S e 3$	0.2169(8)	0.39	0.38	0.02	0.02	0.05
$K_S \mu 3$	0.2125(47)	2.2	2.2	0.02	0.02	0.08
$K^\pm e 3$	0.2169(6)	0.30	0.27	0.06	0.11	0.05
$K^\pm \mu 3$	0.2168(10)	0.47	0.45	0.06	0.11	0.08

Average: $|V_{us}| f_+(0) = 0.21656(35)$ $\chi^2/\text{ndf} = 1.89/5$ (86%)



• See also: [FLAG '21].

$$\frac{|V_{us}|}{|V_{ud}|} \text{ Measurement}$$

$\frac{|V_{us}|}{|V_{ud}|}$ from $K^\pm \rightarrow \mu^\pm \nu$ and $\pi^\pm \rightarrow \mu^\pm \nu$ decays

$$\frac{|V_{us}|}{|V_{ud}|} \frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2}(\gamma)} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2}(\gamma)} m_{K^\pm}} \right)^{\frac{1}{2}} \begin{pmatrix} 1 - \frac{m_\mu^2}{m_{\pi^\pm}^2} \\ 1 - \frac{m_\mu^2}{m_{K^\pm}^2} \end{pmatrix} \left(1 - \frac{1}{2} \delta_{EM} - \frac{1}{2} \delta_{SU(2)} \right)$$

Experimental inputs

- $\Gamma(K_{\mu 2})$ and $\Gamma(\pi_{\mu 2})$: decay rates.
- Branching ratios $\mathcal{B}(K_{\mu 2})$ and $\mathcal{B}(\pi_{\mu 2})$ plus lifetimes τ_{K^\pm} and τ_{π^\pm} .
 - Use K^\pm info from fits.
 - Use π^\pm info from PDG.

Theoretical inputs

- f_K/f_π : ratio of decay constants, lattice-scale uncertainties cancel.
- δ_{EM} : Long-Distance EM corrections for $SU(2)$ breaking.
- $\delta_{SU(2)}$: Strong isospin breaking $f_K/f_\pi \rightarrow f_{K^\pm}/f_{\pi^\pm}$.

$$\frac{|V_{us}|}{|V_{ud}|} \frac{f_K}{f_\pi} = 0.27679(28)_{BR}(20)_{corr}$$

Summary : $|V_{us}|$ and $\frac{|V_{us}|}{|V_{ud}|}$ from Kaon decays

- From $K_{\ell 3}$: $|V_{us}| = 0.22330(35)_{\text{exp}}(39)_{\text{lat}}(8)_{\text{IB}}$ [total frac. unc. = 0.24%]
 - With $f_+(0) = 0.9698(17)$.

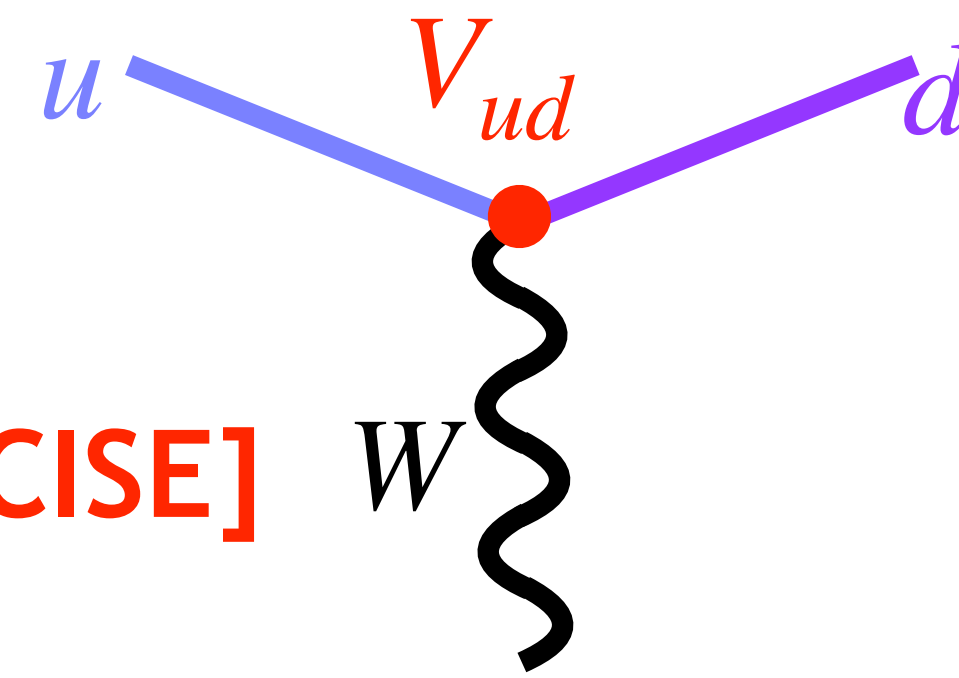
- From $K_{\mu 2}$: $\frac{|V_{us}|}{|V_{ud}|} = 0.23108(23)_{\text{exp}}(42)_{\text{lat}}(16)_{\text{IB}}$ [total frac. unc. = 0.22%]
 - With $f_K/f_\pi = 1.1978(22)$

- Are these results consistent with unitarity? : $\Delta_{CKM}^{(3)} = |V_{us}|_{(K_{\ell 3})}^2 \left(\frac{1}{|V_{us}/V_{ud}|_{(K_{\mu 2})}^2} + 1 \right) - 1$
 - Unitarity test: $\Delta_{CKM}^{(3)} = 0$?
 - Find: $\Delta_{CKM}^{(3)} = -0.0164(63)$ i.e. -2.6σ .

Hint of anomaly only from $K_{\mu 2}$ vs $K_{\mu 3}$

| V_{ud} | Measurement

Determinations of $|V_{ud}|$



- $|V_{ud}|$ from super-allowed $0^+ \rightarrow 0^+$ nuclear beta decays **[MOST PRECISE]**
 - e.g. $^{14}\text{O}(14,8) \rightarrow ^{14}\text{N}(14,7) + e^+ + \bar{\nu}_e$
 - Measure half-lives and Q-values, determine decay rate factors and then $|V_{ud}|$.
 - Using 15 most precisely measured decays [[Phys.Rev.C 102 \(2020\) 4, 045501](#)] and up-to-date set of radiative corrections [[PLB 838 \(2023\) 137748](#)]:

$$|V_{ud}|_{\beta} = 0.97367(11)_{\text{exp}(13)}\text{Rad.Cor.}(27)_{\text{Nucl.Struc.}}$$

Fractional uncertainty: $\sigma_{|V_{ud}|}/|V_{ud}| = 3.3 \times 10^{-4}$

- $|V_{ud}|$ from neutron lifetime τ_n and axial-vector/vector coupling $g_A = G_A/G_V$, (recent precision measurements avoiding inconsistencies [[PLB 838 \(2023\) 137748](#)]):

$$|V_{ud}|_n = 0.97413(13)_{\text{Rad.Cor.}(35)}g_A(20)\tau_n$$

Fractional uncertainty: $\sigma_{|V_{ud}|}/|V_{ud}| = 4.4 \times 10^{-4}$

- Good agreement, use average: $|V_{ud}| = 0.97384(26)$

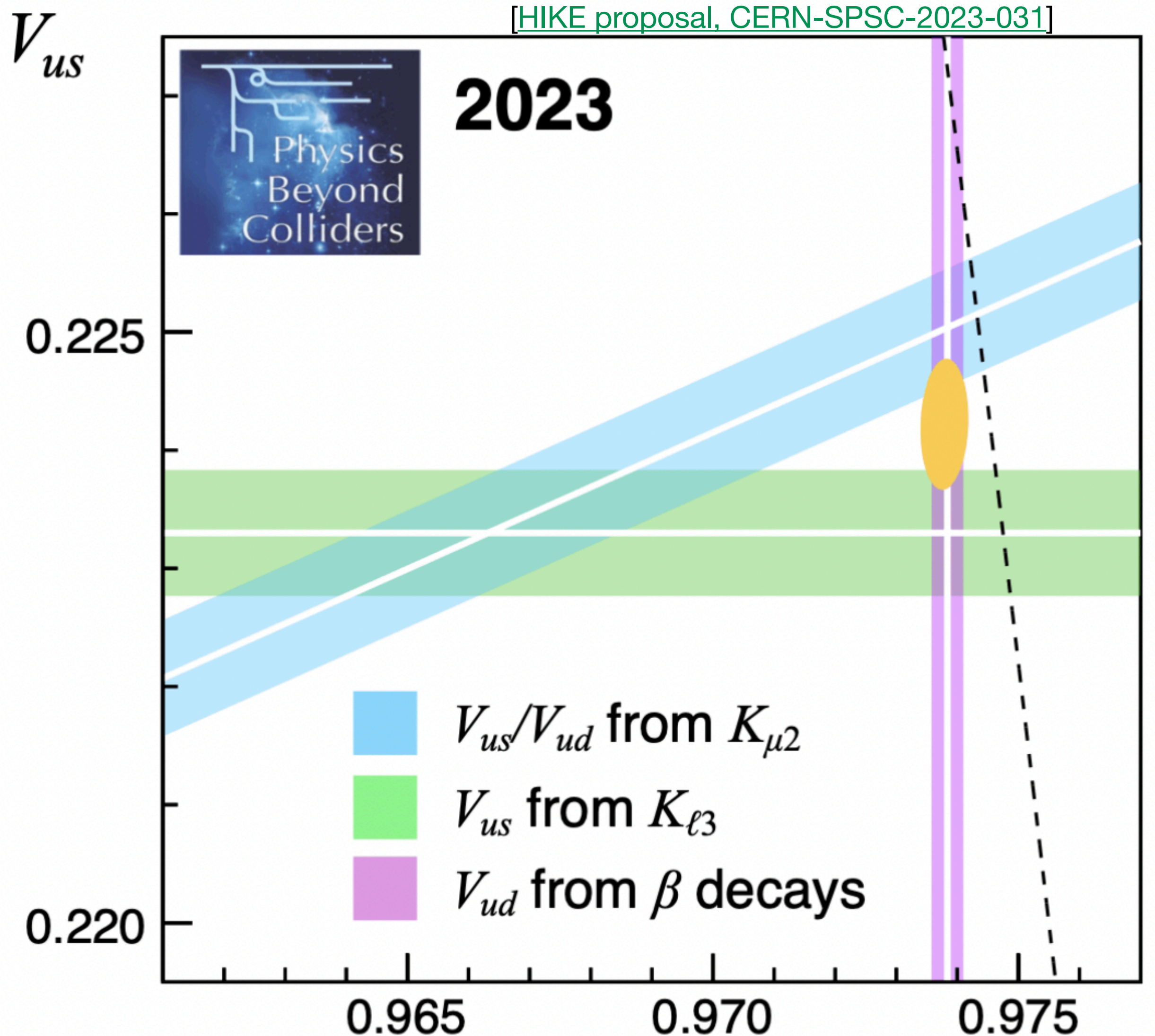
Fractional uncertainty: $\sigma_{|V_{ud}|}/|V_{ud}| = 2.7 \times 10^{-4}$

Global picture

Status of 1st row unitarity test

$$\Delta_{CKM}^u \approx |V_{ud}|^2 + |V_{us}|^2 - 1 = 0$$

- $\therefore |V_{ud}|^2 + |V_{us}|^2 \approx 1$ defines a circle [dotted line]
- Just from kaon decays: **-2.6 σ discrepancy**
- $$\Delta_{CKM}^{(3)} = |V_{us}|_{(K_{\ell 3})}^2 \left(\frac{1}{|V_{us}/V_{ud}|_{(K_{\mu 2})}^2} + 1 \right) - 1 = -0.0164(63)$$
- Include $|V_{ud}|$ from β decays: **-2.8 σ discrepancy**
 - From fit (yellow blob) without constraint: $[\chi^2/\text{ndf} = 6.4/2 \text{ (4.1\%)}]$
 - $\Delta_{CKM}^u = -0.0018(6)$
 - $V_{ud} = 0.97378(26)$, $V_{us} = 0.22422(36)$
 - With scale factor $S = 2.6$:
 - $V_{ud} = 0.9737(8)$, $V_{us} = 0.2242(10)$



- Discrepancy between measurements constitutes a **3 σ deficit** in 1st row CKM unitarity : **Cabibbo angle anomaly.**

1st row unitarity tests

$$\Delta_{CKM}^u = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$$

$|V_{ub}|^2 \approx 2 \times 10^{-5}$

3 observables: $|V_{us}|_{(K_{\ell 3})}^2$, $\left| \frac{V_{us}}{V_{ud}} \right|_{(K_{\mu 2})}^2$ and $|V_{ud}|_{(\beta)}$, and 2 unknowns V_{ud} and V_{us} .

Therefore construct 3 tests (each $\Delta_{CKM}^{(i)} = 0$ if unitarity holds...)

$$\Delta_{CKM}^{(1)} = |V_{ud}|_{(\beta)}^2 + |V_{us}|_{(K_{\ell 3})}^2 - 1 = -0.00176(56) \quad [-3.1\sigma]$$

$$\Delta_{CKM}^{(2)} = |V_{ud}|_{(\beta)}^2 \left[1 + |V_{us}/V_{ud}|_{K_{\mu 2}}^2 \right] - 1 = -0.00098(58) \quad [-1.7\sigma]$$

$$\Delta_{CKM}^{(3)} = |V_{us}|_{(K_{\ell 3})}^2 \left(|V_{us}/V_{ud}|_{(K_{\mu 2})}^{-2} + 1 \right) - 1 = -0.0164(63) \quad [-2.6\sigma]$$

Not using any inputs from β decays

1st row non-unitarity: BSM with RH currents?

- In SM W only couples to Left-Handed chiral fermion states.
- BSM physics with couplings to Right-Handed currents could explain deficits in 1st row unitarity and $K_{\ell 3} - K_{\mu 2}$ difference.

- Define:

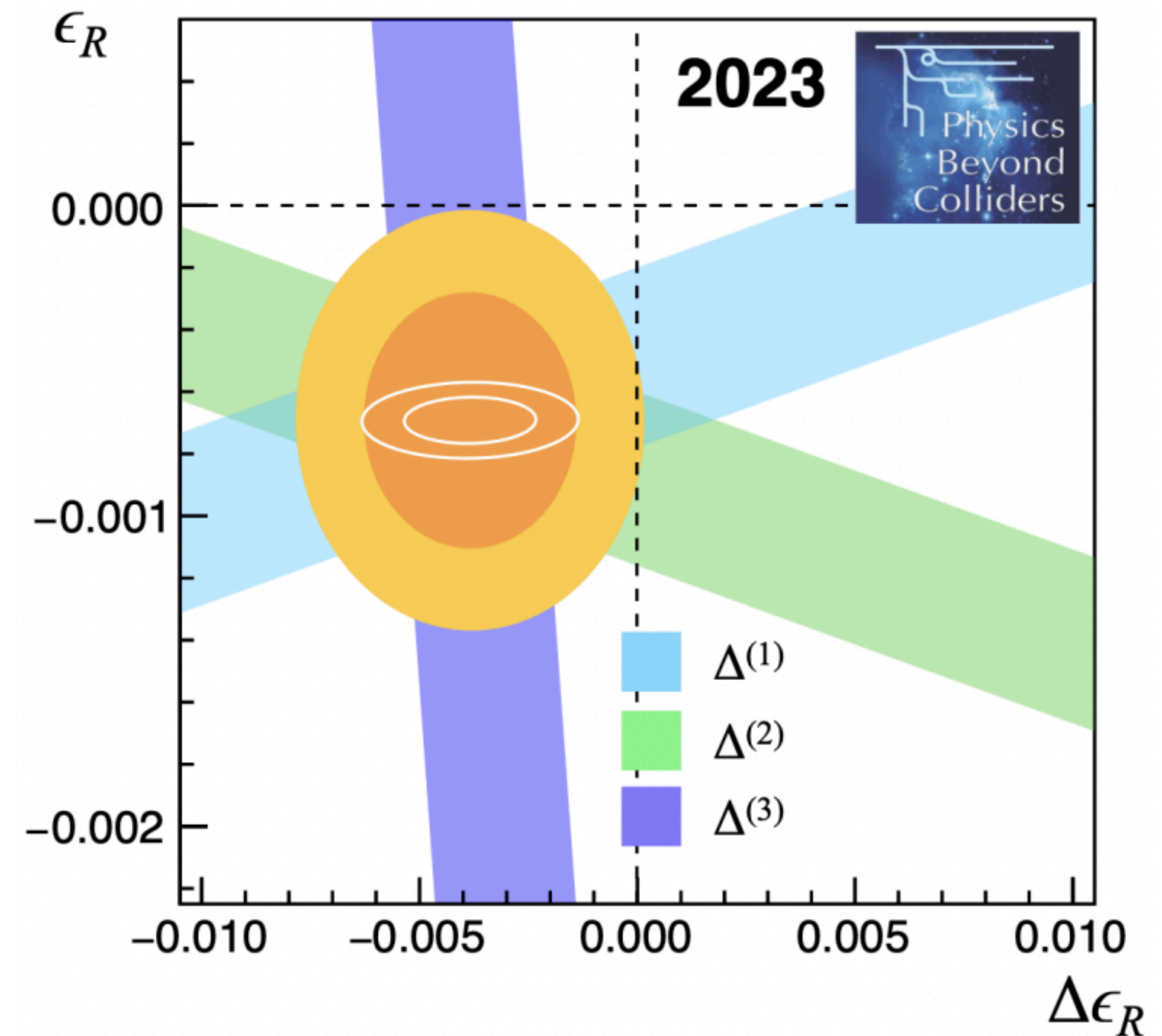
$$\Delta_{CKM}^{(1)} = 2\epsilon_R + 2\Delta\epsilon_R |V_{us}|^2$$

$$\Delta_{CKM}^{(2)} = 2\epsilon_R - 2\Delta\epsilon_R |V_{us}|^2$$

$$\Delta_{CKM}^{(3)} = 2\epsilon_R + 2\Delta\epsilon_R(2 - |V_{us}|^2)$$

- Where:
 - ϵ_R = RH currents in u,d sector
 - $\Delta\epsilon_R$ = RH currents in strange sector

- From current fit:
 - $\epsilon_R = -0.69(27) \times 10^{-3}$ (2.5σ)
 - $\Delta\epsilon_R = -3.9(1.6) \times 10^{-3}$ (2.4σ)
 - $\epsilon_R = \Delta\epsilon_R = 0$ excluded with 3.1σ significance.



[PLB 838 (2023) 137748]

Future: NA62/HIKE

NA62 and CKM 1st row unitarity : $R^{K_{\mu 3}/K_{\mu 2}}$

- NA62 is ideally placed to make a high precision measurement of

$$R^{K_{\mu 3}/K_{\mu 2}} = \frac{\mathcal{B}(K^+ \rightarrow \pi^0 \mu^+ \nu)}{\mathcal{B}(K^+ \rightarrow \mu^+ \nu)}, \text{ with many systematics cancelling.}$$

- This single measurement can have a significant impact: $(R^{K_{\mu 3}/K_{\mu 2}})^{-1/2} \propto r$ where:

$$r \equiv \left(\frac{1 + \Delta_{CKM}^2}{1 - \Delta_{CKM}^{(3)}} \right)^{-1/2} = \frac{\left| \frac{V_{us}}{V_{ud}} \right|_{(K\mu 2)}^2}{|V_{us}|_{(K\ell 3)}^2 / |V_{ud}|_{(\beta)}} = 1 - 2\Delta\epsilon_R$$

- Uses input from β decays, but provides a qualified statement about consistency of data set.
- Sensitive search for right-handed currents ($\Delta\epsilon_R \neq 0$).

NA62 hypothetical $K_{\mu 3}/K_{\mu 2}$ to 0.5%:

Result	$\Delta\epsilon_R$		Remarks
Same as fit	$-4.0(1.9) \times 10^{-3}$	2.1σ	Almost same precision as result from world average
$+ 1.5\sigma$	$-0.4(1.9) \times 10^{-3}$	0.2σ	$K_{\mu 2}, K_{\mu 3}, V_{ud}$ consistent: current tensions have experimental origin?
$- 1.5\sigma$	$-7.6(1.9) \times 10^{-3}$	4.0σ	Evidence for right-handed currents contributing to CKM non-unitarity

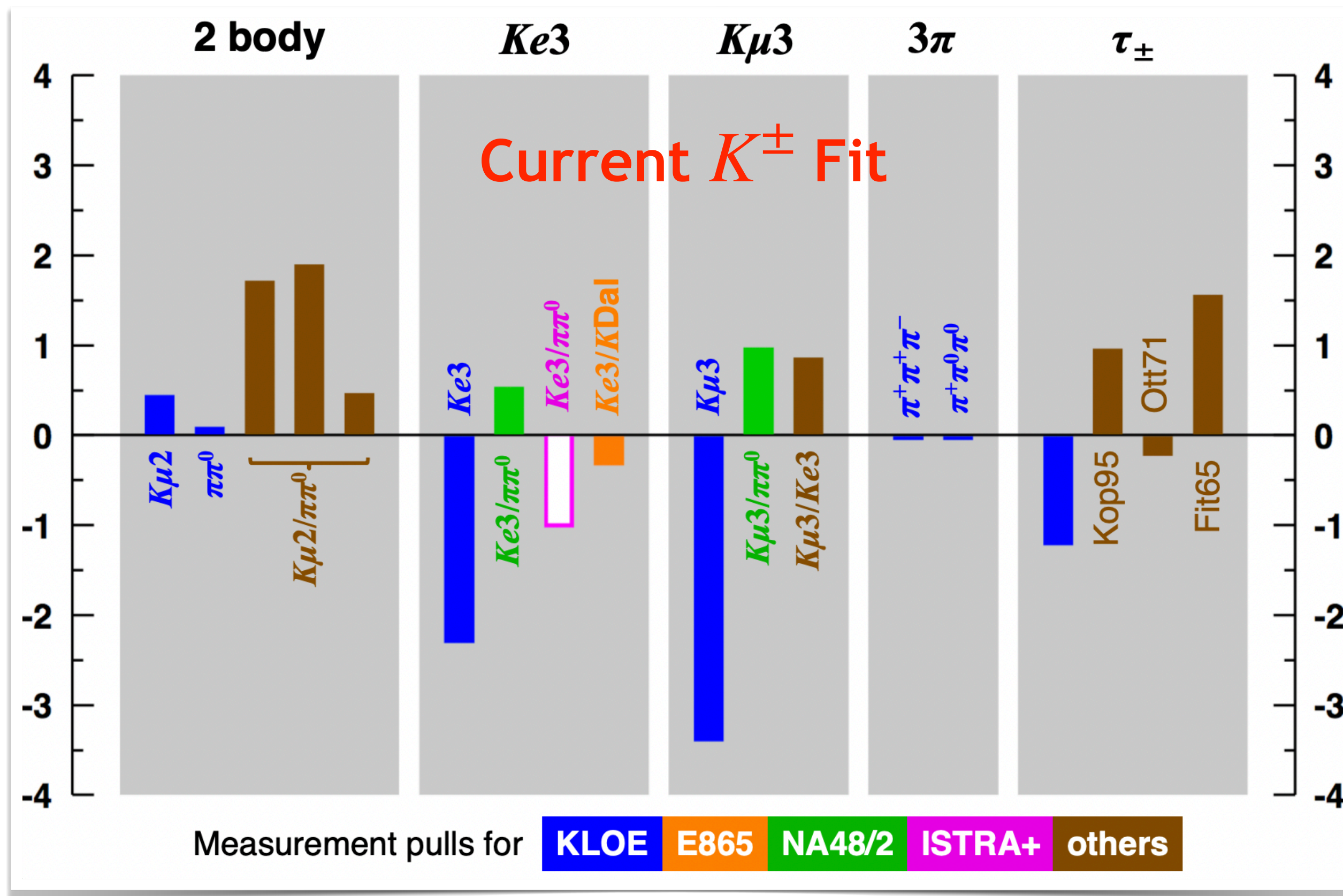
[[PLB 838 \(2023\) 137748](#)]

[[Moulson, CKM23](#)]

NA62 and CKM 1st row unitarity: Many opportunities



- NA62 can perform a suite of measurements of common kaon decays:
 - Add several new ratios to over-constrain fits with good control of systematics.
 - Use single analysis framework, data-set ... to maximise systematics cancellations.



- e.g. Target measurements:
 - $K_{\mu 2}/K_{\pi^+\pi^0}$: verify KLOE result with quoted 0.27% uncertainty.
 - $K_{e3}/K_{\pi^+\pi^0}$, $K_{\mu 3}/K_{\pi^+\pi^0}$: currently the $K_{\ell 3}$ have large scale factors in fit.
 - $K_{e3}/K_{\mu 3}$: test of lepton universality

- With a dedicated, stable, low-intensity data-taking with minimum-bias triggers statistical uncertainties <0.1% in 2 weeks.

NA62 and CKM 1st row unitarity: Many opportunities



- NA62 can perform a suite of measurements of common kaon decays:
 - Add several new ratios to over-constrain fits with good control of systematics.
 - Use single analysis framework, data-set ... to maximise systematics cancellations.

- Alternative strategy, mostly independent of other inputs:

- $R_A^{K_{\mu 2}} = \frac{K^+ \rightarrow \mu^+ \nu}{K^+ \rightarrow \pi^0 \pi^+ \rightarrow \pi^0 \mu^+ \nu}$

- $R_A^{K_{\mu 3}} = \frac{K^+ \rightarrow \pi^0 \mu^+ \nu}{K^+ \rightarrow \pi^0 \pi^+ \rightarrow \pi^0 \mu^+ \nu}$

- $R^{K_{\mu 3}/K_{\mu 2}} = \frac{K^+ \rightarrow \pi^0 \mu^+ \nu}{K^+ \rightarrow \mu^+ \nu}$ [see previous]

- Minimise systematics looking at modes with/without π^0 and perform simultaneous fit to m_{miss}^2 spectra.
- Systematics evaluation ongoing (2017-18 data): expected $<0.6\%$.

- e.g. Target measurements:

- $K_{\mu 2}/K_{\pi^+ \pi^0}$: verify KLOE result with quoted 0.27% uncertainty.

- $K_{e 3}/K_{\pi^+ \pi^0}$, $K_{\mu 3}/K_{\pi^+ \pi^0}$: currently the $K_{\ell 3}$ have large scale factors in fit.

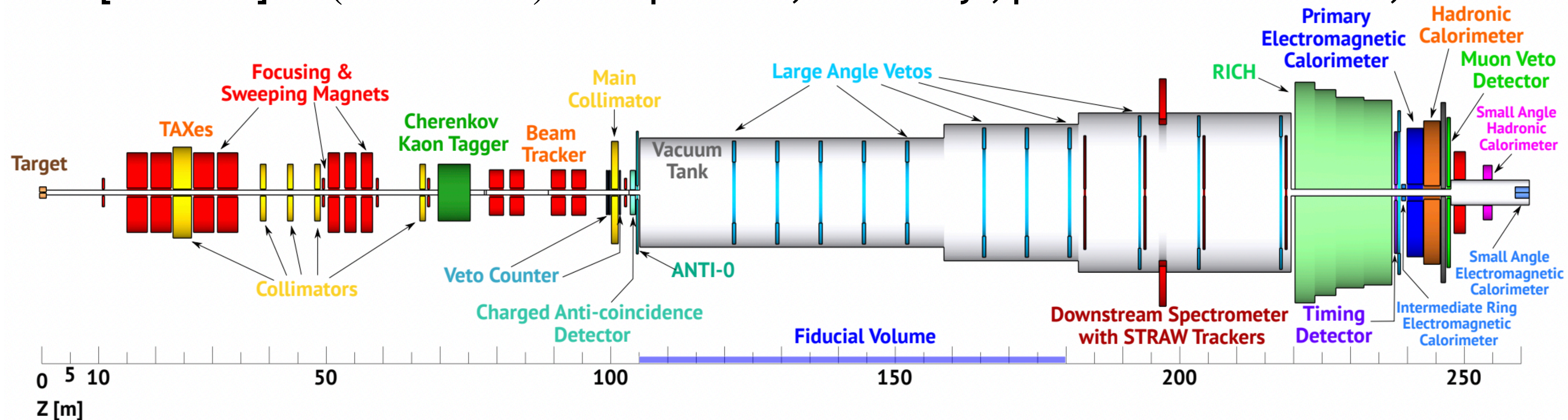
- $K_{e 3}/K_{\mu 3}$: test of lepton universality

- **With a dedicated, stable, low-intensity data-taking with minimum-bias triggers statistical uncertainties $<0.1\%$ in 2 weeks.**

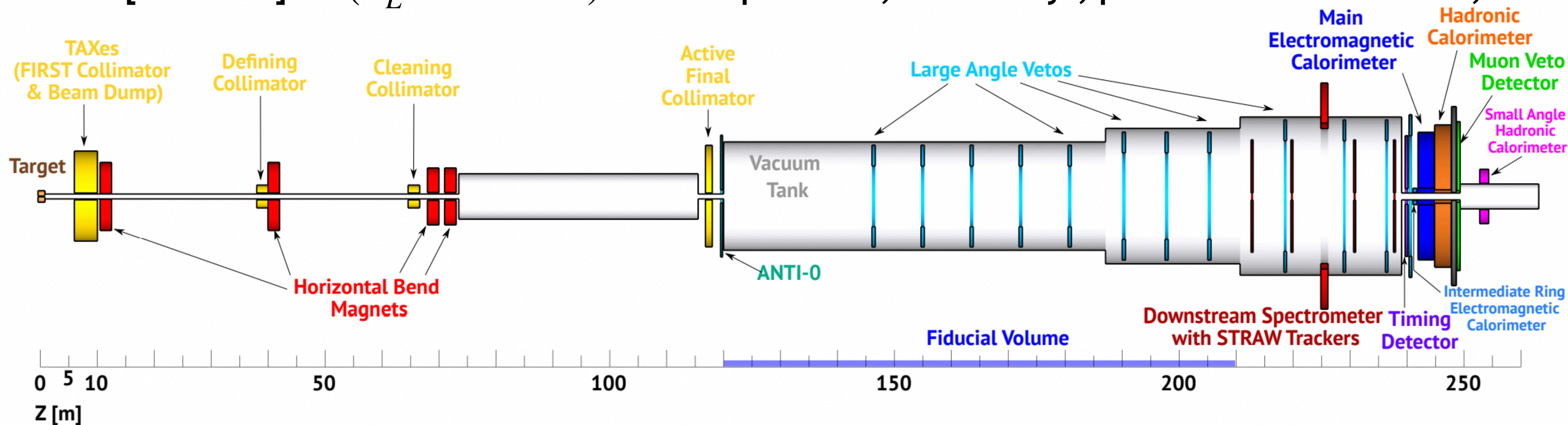
HIKE: long-term kaon programme at CERN



Phase1 [K^+ beam]: $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ to 5% precision, rare decays, precision measurements, LNV/LFV



Phase2 [K^0 beam]: $\mathcal{B}(K_L \rightarrow \pi^0 \ell^+ \ell^-)$ to ~15% precision, rare decays, precision measurements, LNV/LFV



- + FIPs searches with periodic beam dump running.

- + synergy with SHADOWS [[CERN-SPSC-2023-029](#)]

Hypothetical fits after HIKE Phase 1+2



K^+ fits after HIKE phase 1

11 input measurements:

3 old τ values in PDG

KLOE τ

KLOE BR $\pi\pi\pi, \pi\pi^0\pi^0$

HIKE $\pi\pi^0/\mu\nu$ to 0.4%

HIKE $K_{e3}/\pi\pi^0$ to 0.4%

HIKE $K_{\mu3}/\mu\nu$ to 0.2%

HIKE $K_{\mu3}/\pi\pi^0$ to 0.4%

HIKE $K_{\mu3}/K_{e3}$ to 0.2%

K_L fits after HIKE phase 2

24 input measurements:

21 inputs from current fit

Hypothetical HIKE

measurements chosen to agree with $V_{us} = 0.22417$:

HIKE $K_{\mu3}/K_{e3}$ to 0.3%

HIKE $\pi^+\pi^-/K_{e3}$ to 0.4%

HIKE $\pi^+\pi^-/\pi^+\pi^-\pi^0$ to 0.6%

- Can remove almost all old data.
- [Assume HIKE measurements consistent with $V_{us} = 0.22417$]
- Constraint $\sum \mathcal{B}(K^\pm) = 1$ significantly increases fit result for $\mathcal{B}(K_{\mu2})$.
- Results:

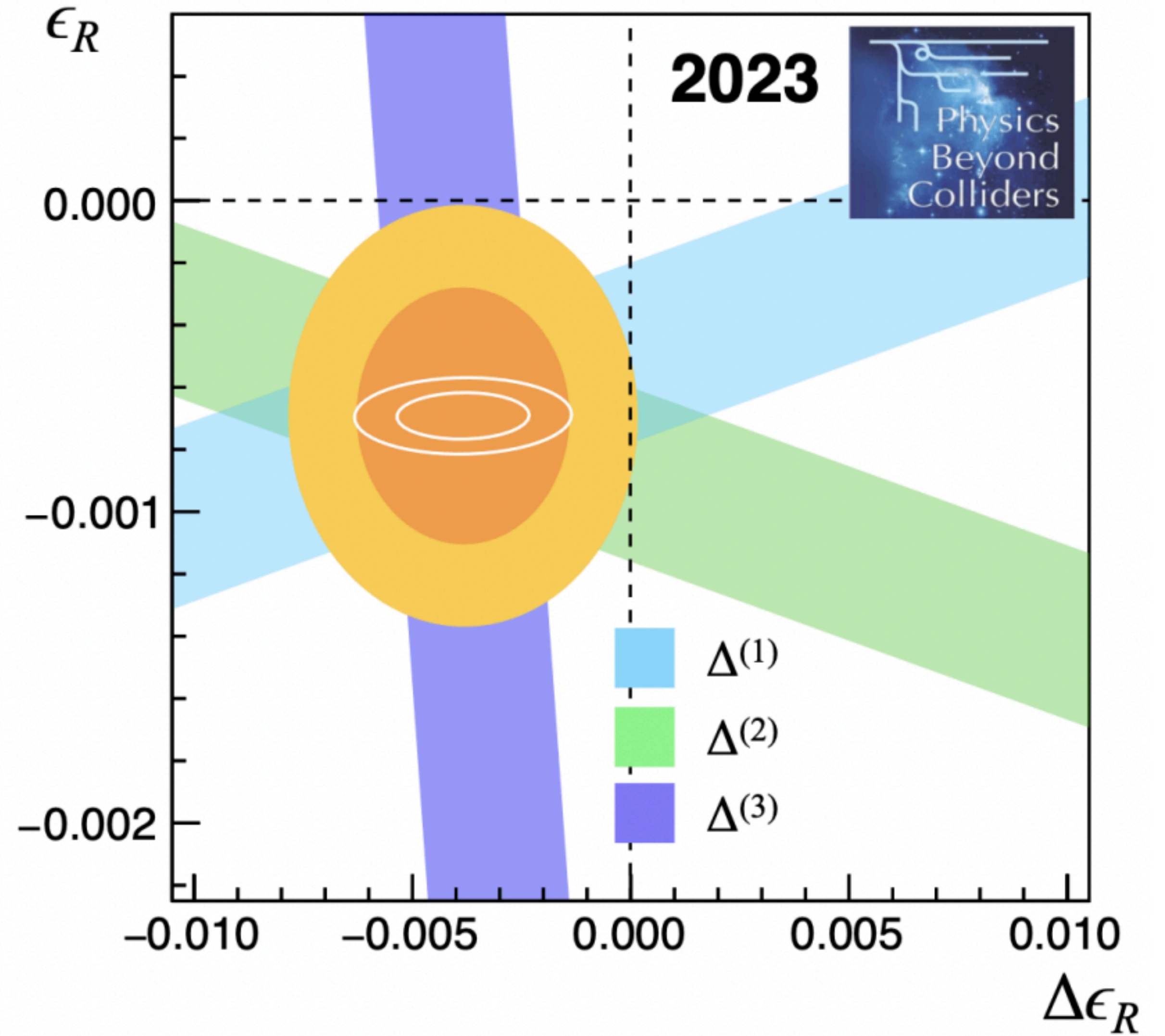
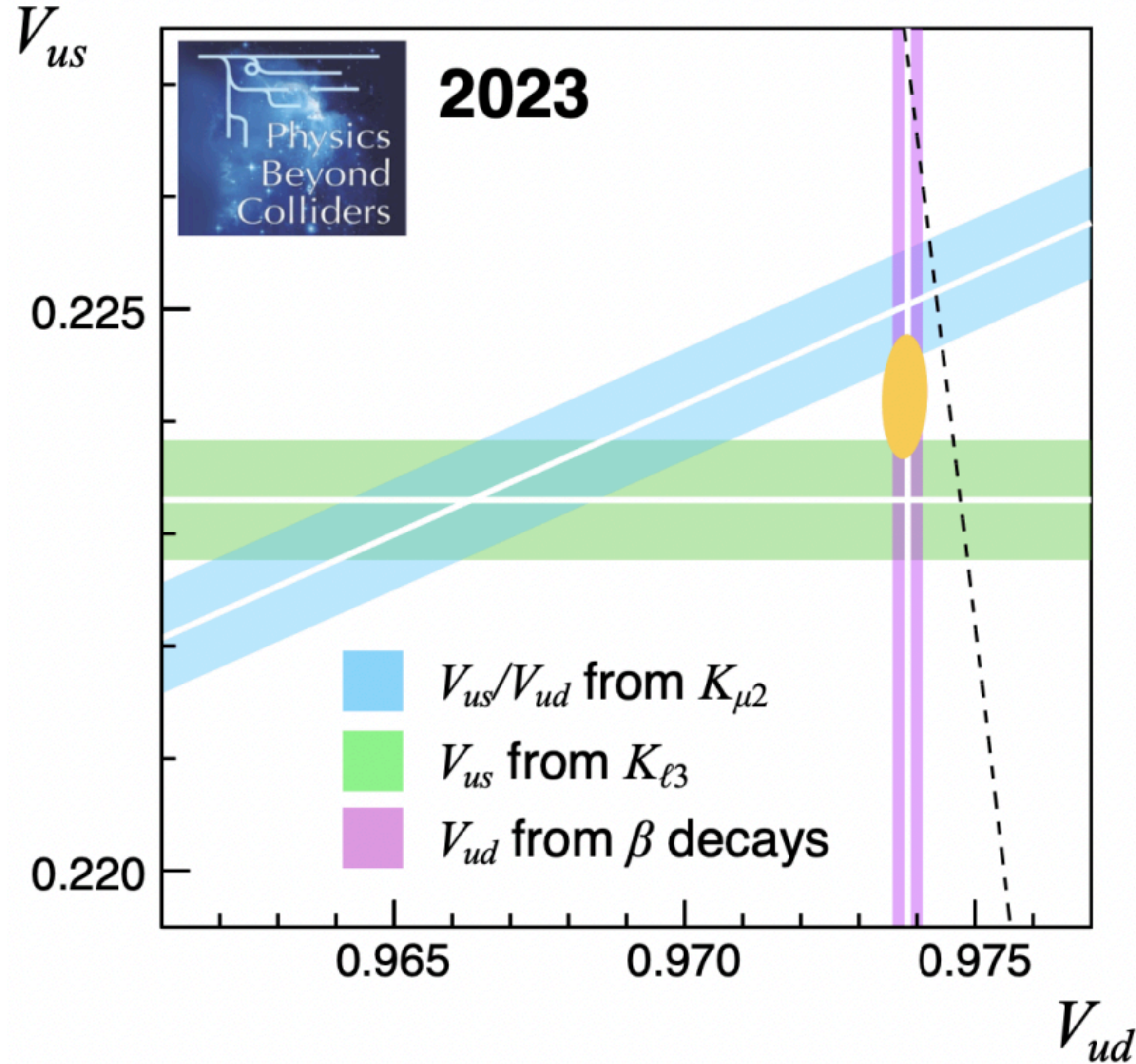
$\chi^2/\text{ndf} = 4.90/5$ (Prob = 42.8%)
compare current: 25.5/11 (0.78%)

- Harder to improve overall fit.
 - $K_{\ell3}$ modes dominate but $3\pi^0$ and $\pi^+\pi^-\pi^0$ critical for normalisation - with poor systematic cancellation.
- Results:

$\chi^2/\text{ndf} = 30.7/15$ (Prob = 0.94%)

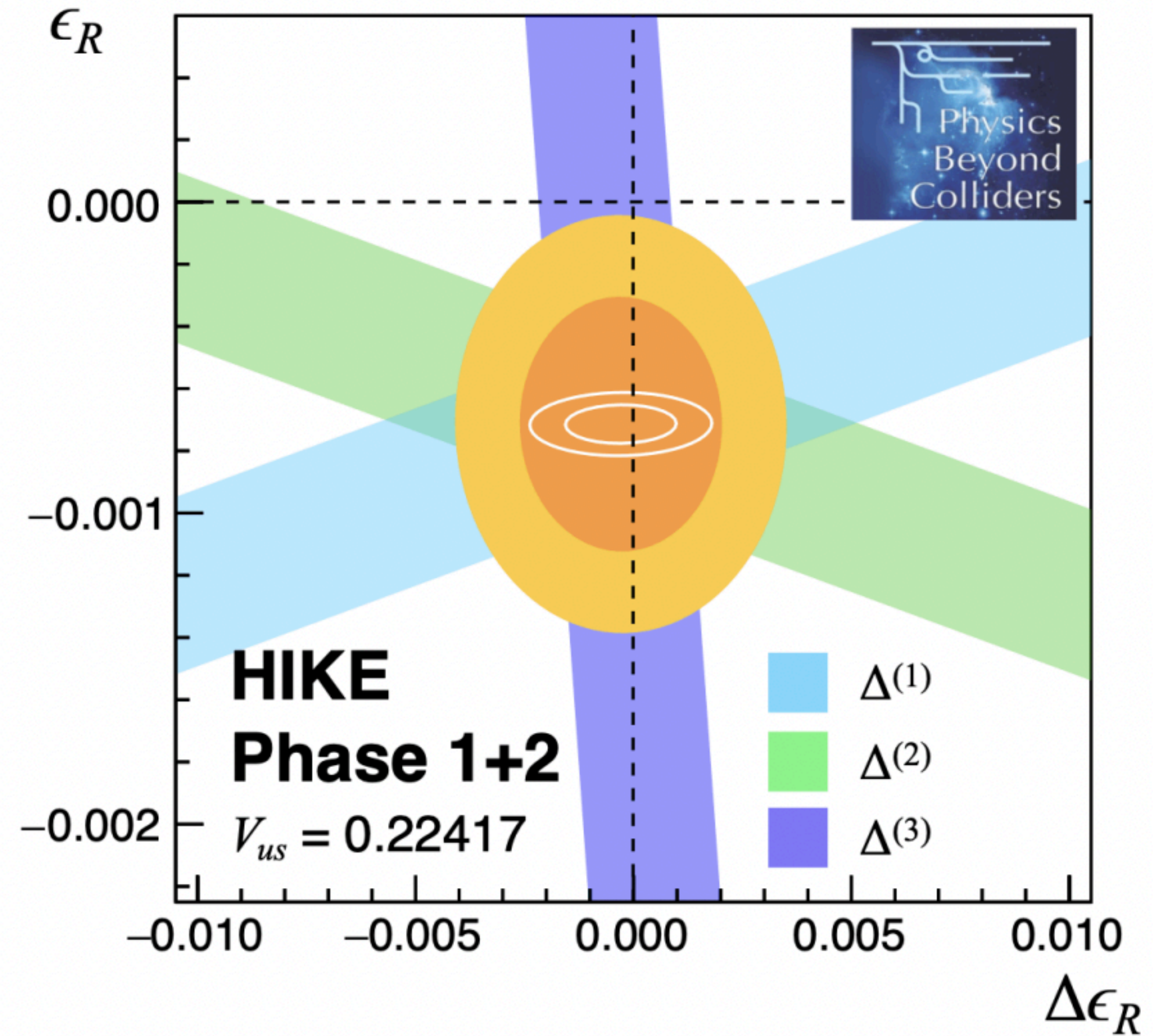
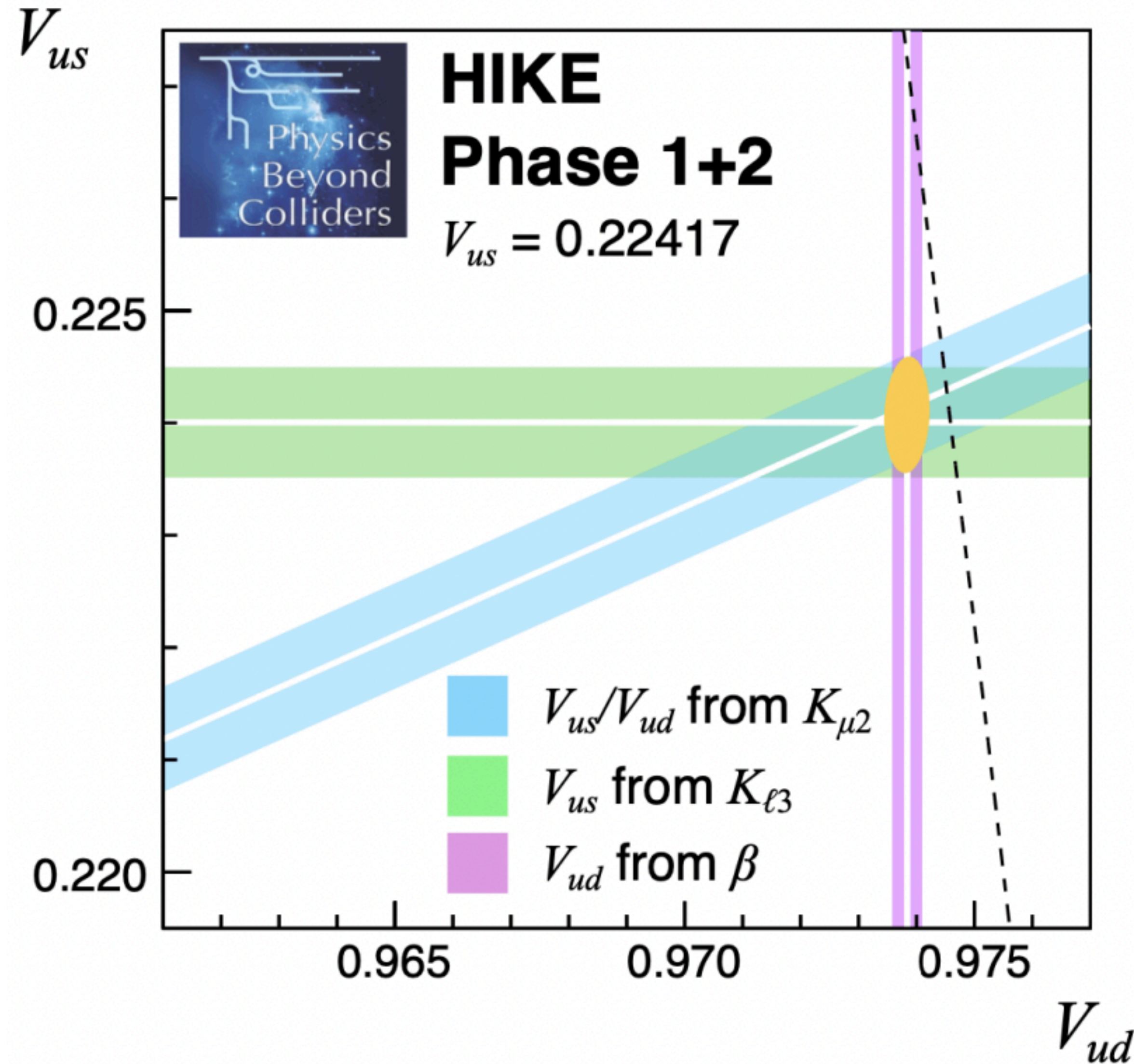
c.f. current 19.8/12 (Prob=7.0%)

Current status



- Poor agreement of $K_{\mu 3}$ vs $K_{\mu 2}$, unitarity deficit $\approx 2.8\sigma$
- $\epsilon_R = \Delta\epsilon_R = 0$ excluded with 3.1σ significance.

After HIKE Phase 1+2 [e.g. hypothetical scenario]



- Good agreement of $K_{\mu 3}$ vs $K_{\mu 2}$, unitarity deficit $\approx 2.7\sigma$ remains.
- $\epsilon_R = \Delta\epsilon_R = 0$ excluded with 2.2σ significance.

Future: Kaon/Pion Experiments

CKM constraints from $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ (“pion beta decay”)

- PIBETA experiment at PSI: $\mathcal{B} = (1.036 \pm 0.004_{\text{stat}} \pm 0.005_{\text{syst}}) \times 10^{-8}$ (0.6% precision) [[PRL 93 \(2004\) 181803](#)].

- Gives:

$$|V_{ud}| = 0.9739(27) \left[\frac{BR(\pi^+ \rightarrow e^+ \nu_e(\gamma))}{1.2325 \times 10^{-4}} \right]^{\frac{1}{2}}$$

Normalising to measurement:

$$BR(\pi^+ \rightarrow e^+ \nu_e(\gamma)) = 1.2325(23) \times 10^{-4}$$

Or theory:

$$1.2350(2) \times 10^{-4}$$

- > Fractional unc. of 0.3% on $|V_{ud}|$ [[PDG](#)], but theory uncertainties are small...
- PIONEER experiment at PSI [[Snowmass21](#)][[proposal, 2203.01981](#)] Approved - starting data-taking in ~5 years...

CKM constraints from $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ (“pion beta decay”)

- PIONEER experiment at PSI [[Snowmass21](#)] [[proposal, 2203.01981](#)] Approved - starting data-taking in ~5 years. Focusing on $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ in phases 2 and 3.
 - Phase1 : $R_{e\mu} = \frac{\Gamma(\pi^+ \rightarrow e^+ \nu(\gamma))}{\Gamma(\pi^+ \rightarrow \mu^+ \nu(\gamma))}$, stopped pions, look for positrons from $\pi^+ \rightarrow e^+ \nu$ or $\pi^+ \rightarrow \mu^+ \nu, \mu^+ \rightarrow e^+ \nu \bar{\nu}$.
 - Phase2 : measure $\mathcal{B}(\pi^+ \rightarrow \pi^0 e^+ \nu_e)$ with 0.2% precision, looking for back-to-back $\gamma\gamma$ from stopped pions, normalising to $\pi^+ \rightarrow e^+ \nu$ [same concept as PIBETA].
 - Thus improve $|V_{ud}|$ and $|V_{ud}/V_{us}|$ precision (using ratio $R_V = \frac{\Gamma(K \rightarrow \pi l \nu(\gamma))}{\Gamma(\pi^+ \rightarrow \pi^0 e^+ \nu(\gamma))}$) by factor ~3.

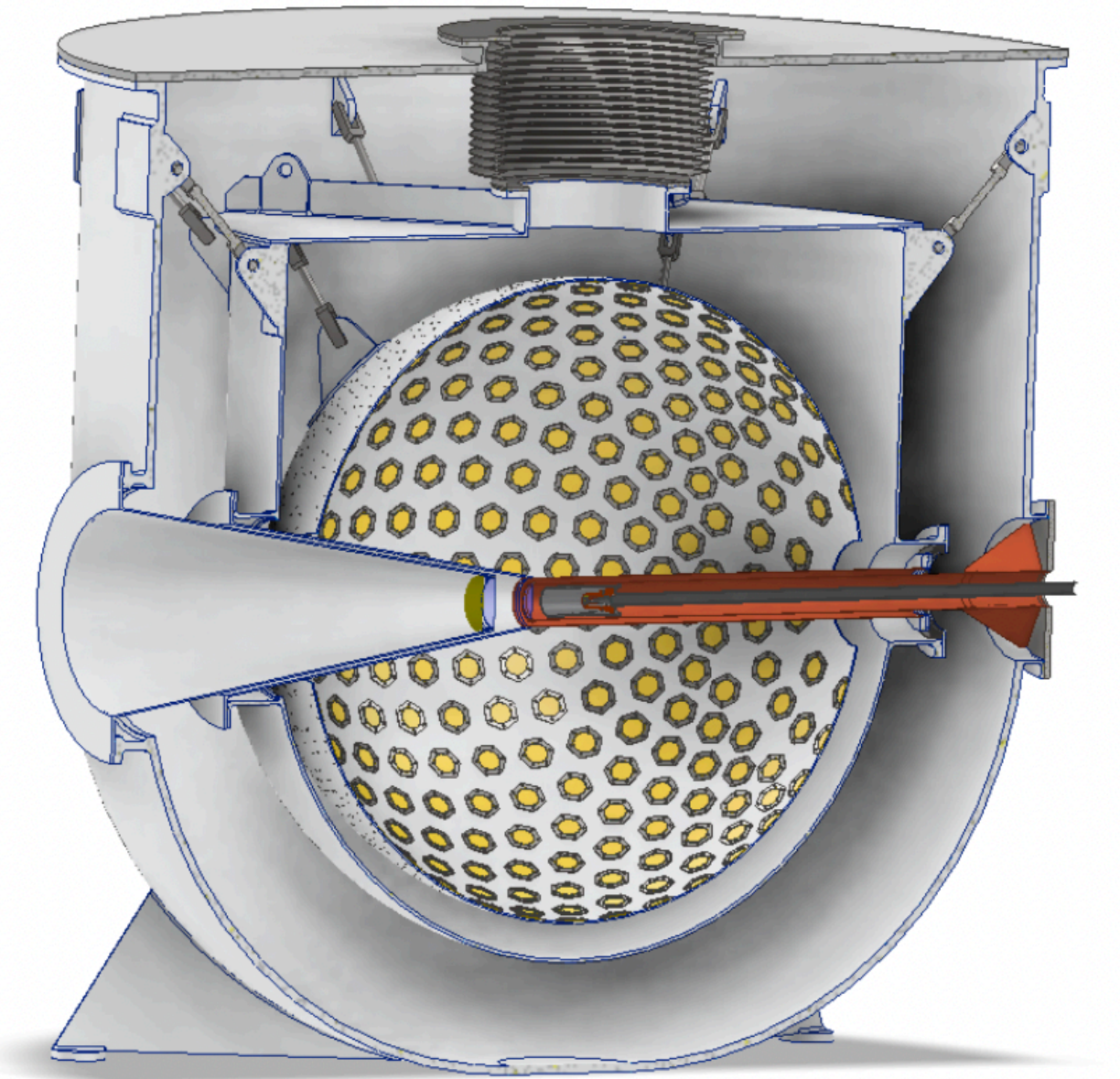


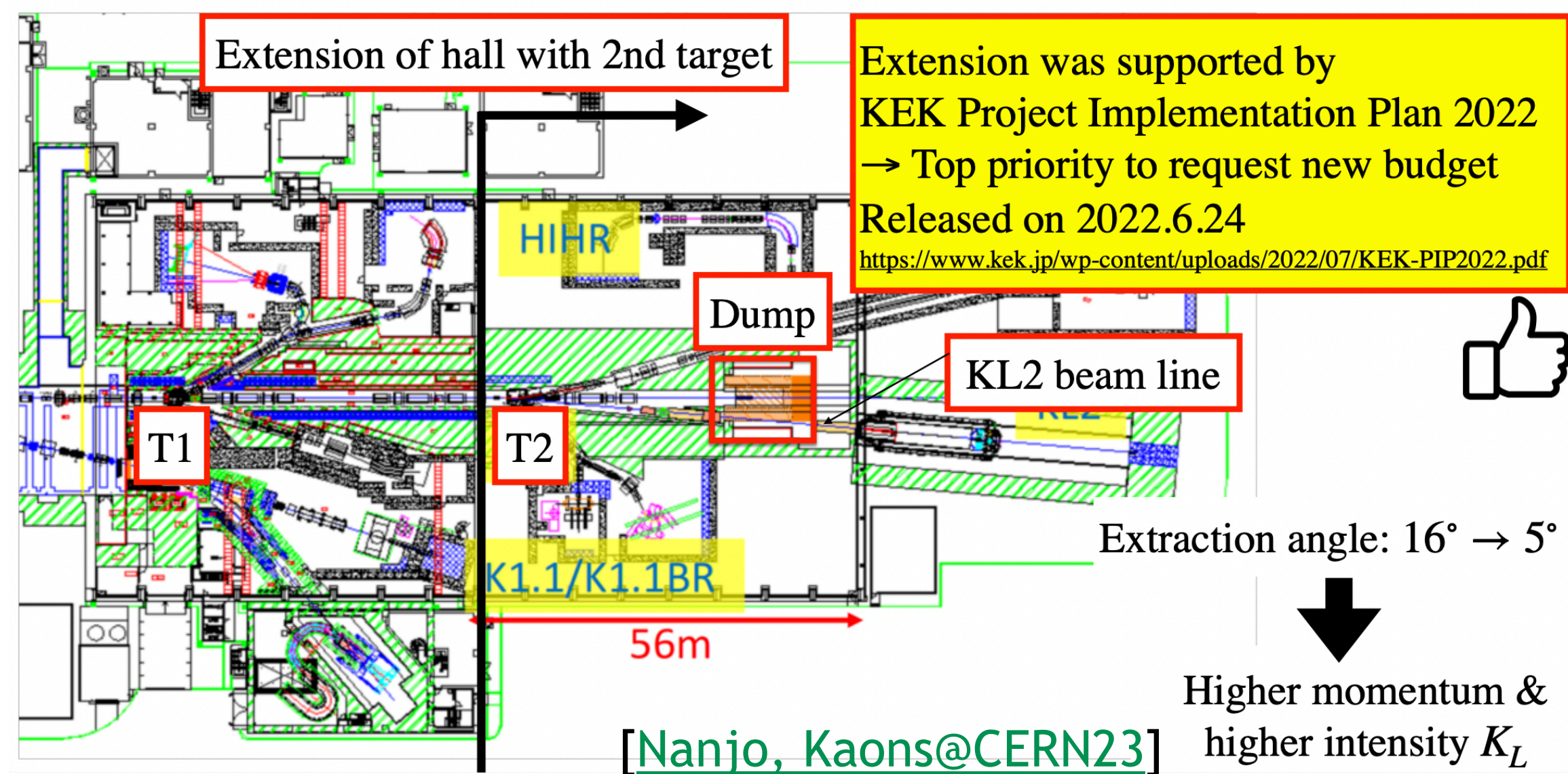
FIG. 4 – PIONEER conceptual design. The sphere is a LXe calorimeter, with the beam entering from the left, and the ATAR at the center. For scale, the lid is 3.05 m diameter. The yellow circles are merely representative of the LXe photosensors; they are not placed accurately.

Challenging long-term goal: reach precision of 0.02% on $|V_{ud}|$ (competitive with super-allowed β decays) by end of phase 3.

Future for neutral kaons?

- KOTO currently running at J-PARC, will be followed by KOTO-2.
 - Bespoke, highly specialised, design to study $K_L \rightarrow \pi^0 \nu \bar{\nu}$.
 - Limited precision for other K_L decays, unlikely to significantly impact fits.

KOTO II with extension of hadron experimental facility



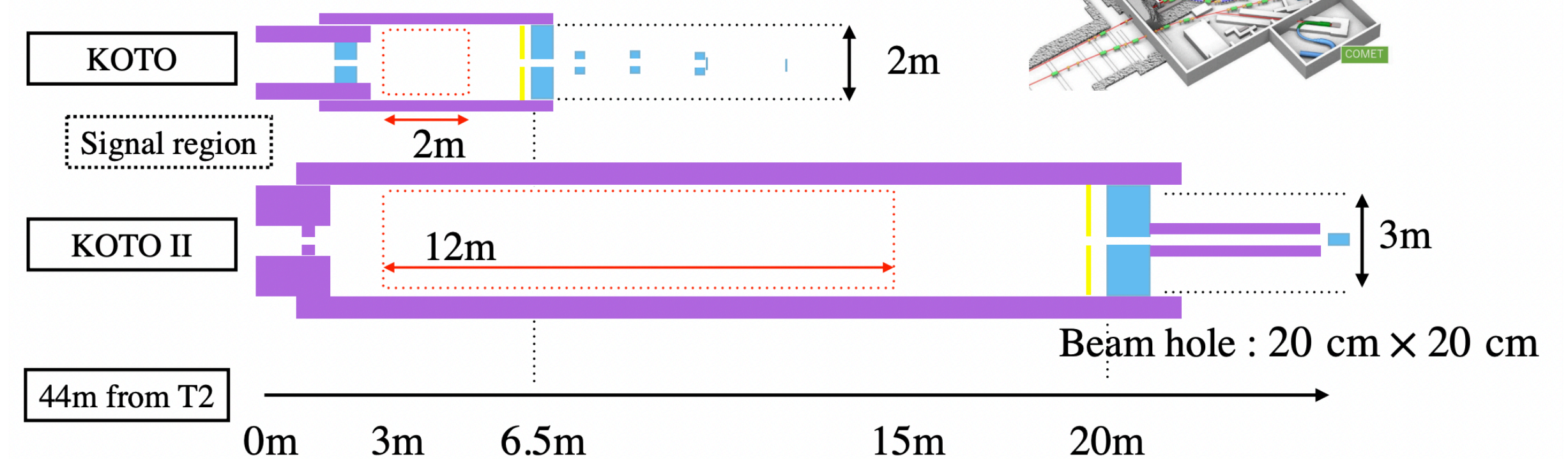
KOTO-II detector

[Nanjo, Kaons@CERN23]

Peak K_L momentum : 1.4 GeV/c (step-1) → 3 GeV/c (step-2)

Possible to use longer decay volume (2 m → 12 m)

Larger diameter calorimeter (2 m → 3 m)



- HIKE phase2: Primary objective: $K_L \rightarrow \pi^0 \ell^+ \ell^-$, but notable impacts expected.

$$(K_L \rightarrow \pi^0 \ell^+ \nu, K_L \rightarrow \pi^0 \pi^+ \pi^-, K_L \rightarrow \pi^+ \pi^-)$$

Summary

Summary

- Cabibbo angle anomaly: tension with 1st row unitarity [currently at $\sim 3\sigma$].
 - + tension in $K_{\mu 3}$ vs $K_{\mu 2}$
- Most precise tests by studying $|V_{us}|^2_{(K_{\ell 3})}$, $|V_{us}/V_{ud}|^2_{(K_{\mu 2})}$ and $|V_{ud}|_{(\beta)}$.
 - Key results from combined fits of K_S, K_L, K^\pm data.
- Is current tension **experimental** in nature?
 - NA62/HIKE can provide new, high-precision measurements of $K_{\mu 3}/K_{\mu 2}$: **directly tests the tensions** and can have **significant impact on fits**.
 - + Future pion experiments (PIONEER) may challenge $|V_{ud}|$ precision.
- Is tension due to **BSM physics**?
 - Test for right-handed currents established, currently weakly favoured (exclude 0 RH currents at $\sim 3\sigma$)

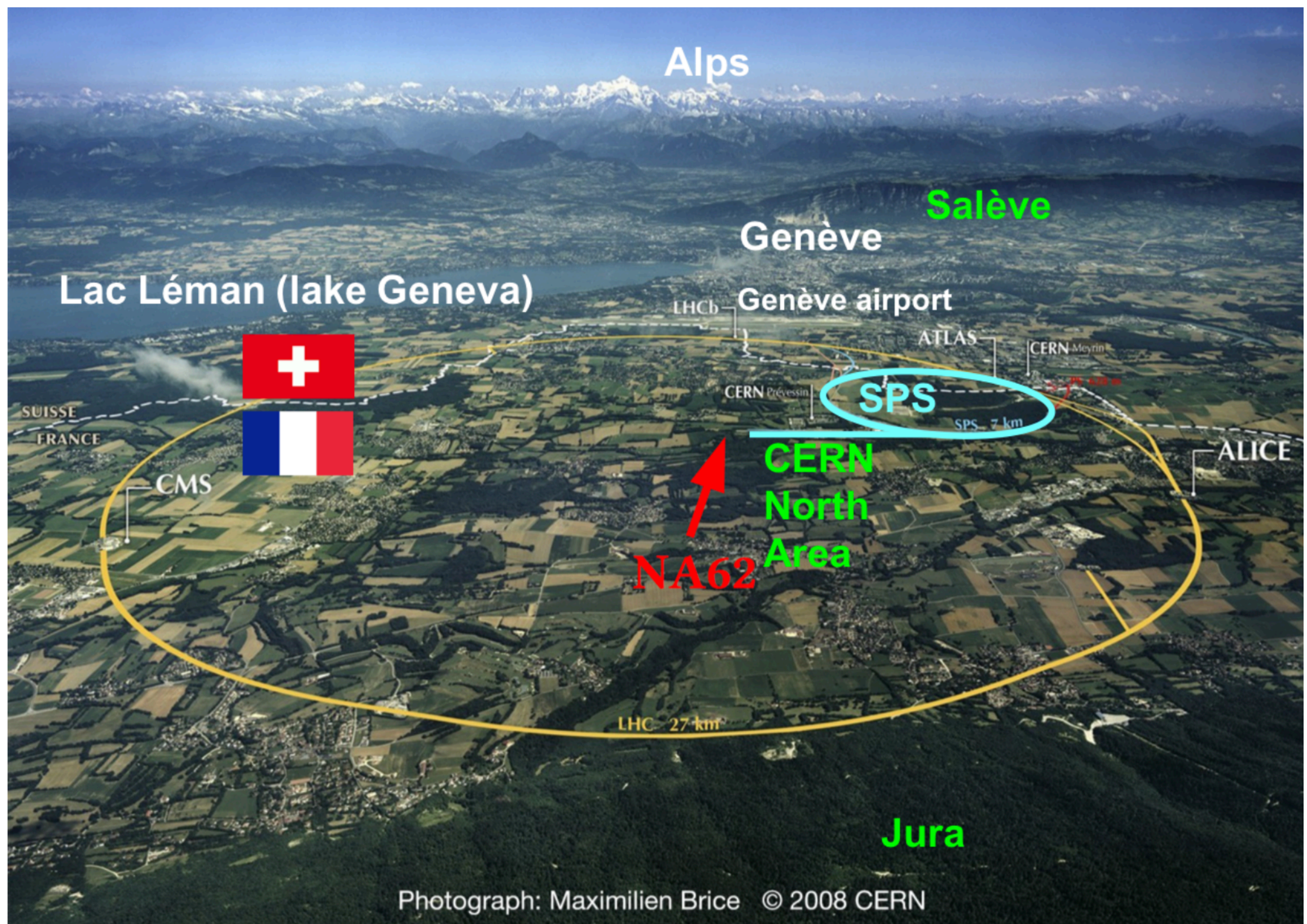
Supplemental



The NA62 Experiment at CERN

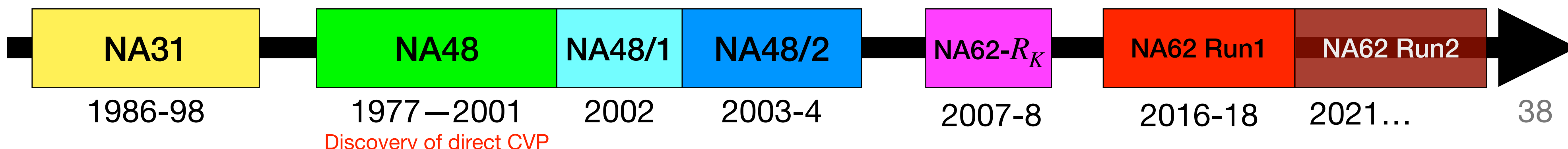


~300 collaborators from ~30 institutions.



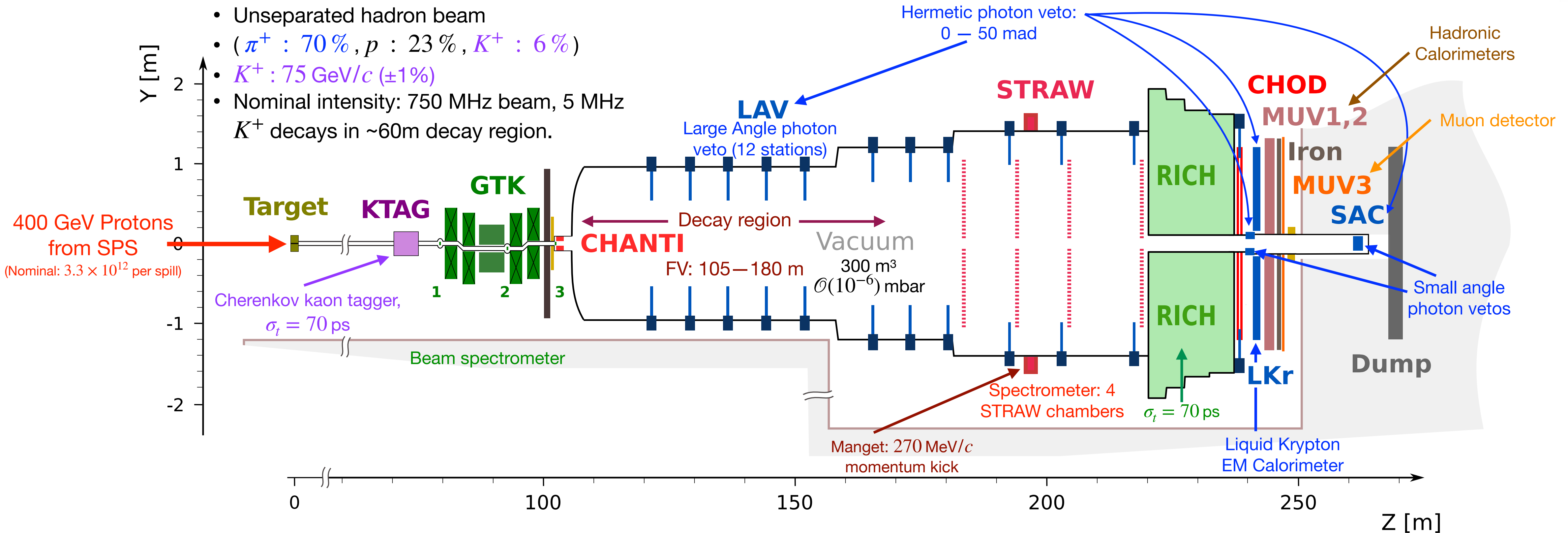
- Primary goal: measurement of $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$
- New Technique: K^+ decay-in-flight
- Results: [[PLB 791 \(2019\) 156](#)] [[JHEP 11 \(2020\) 042](#)] [[JHEP 06 \(2021\) 093](#)]
- Broader physics programme:
 - Rare K^+ decays (e.g. $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ [[JHEP 11 \(2022\) 011](#)])
 - LNV/(c)LFV decays
 - Exotics (e.g. Dark photon [[arXiv:2303.08666](#)])
- Data taking
 - 2016 Commissioning + Physics run (45 days).
 - 2017 Physics run (160 days).
 - 2018 Physics run (217 days).
 - 2021 Physics run (85 days [10 beam dump]).
 - 2022 Physics run (215 days).
 - 2023 Physics run (just finished!).

Continues long history of Kaon physics at CERN :



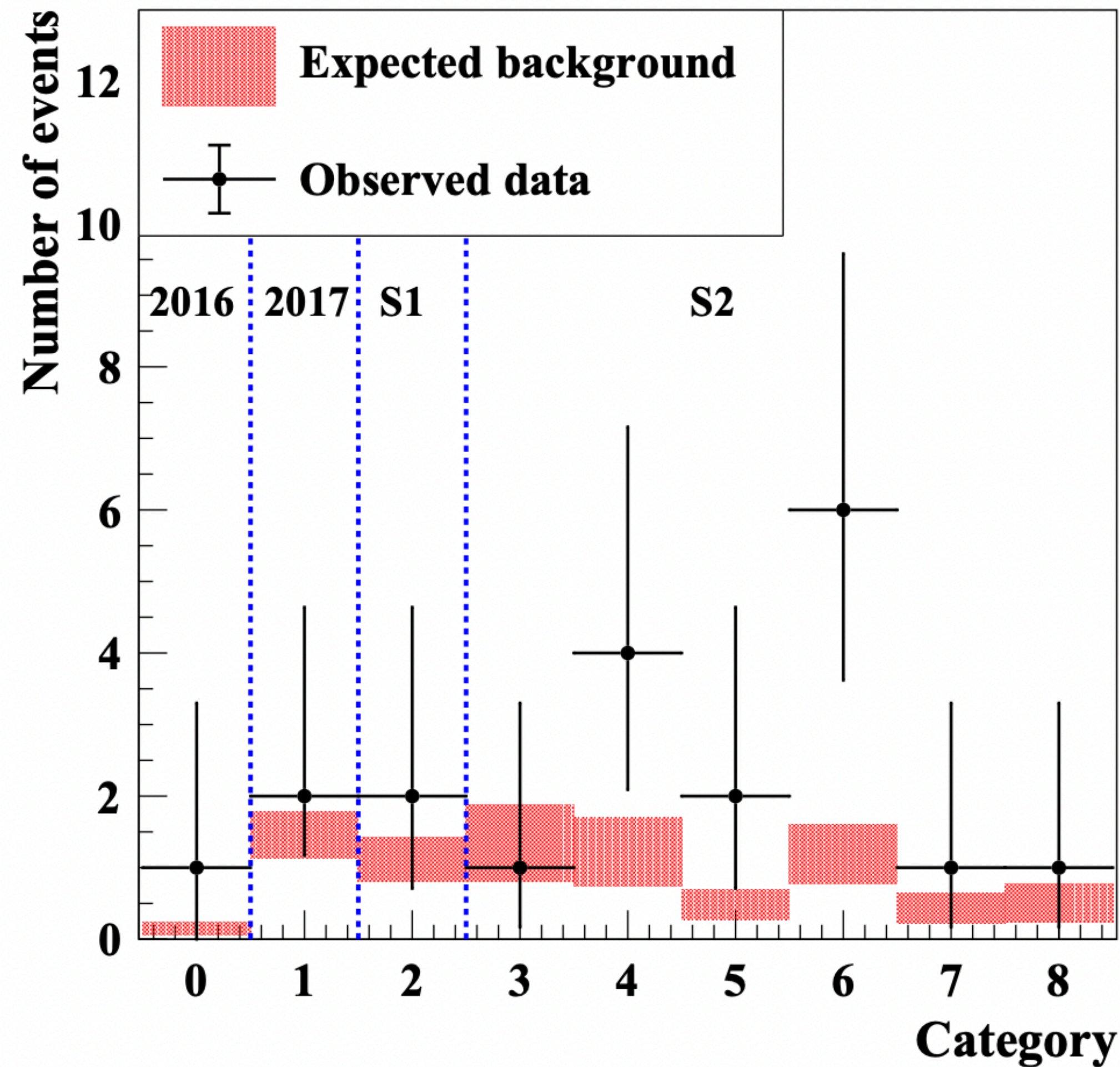
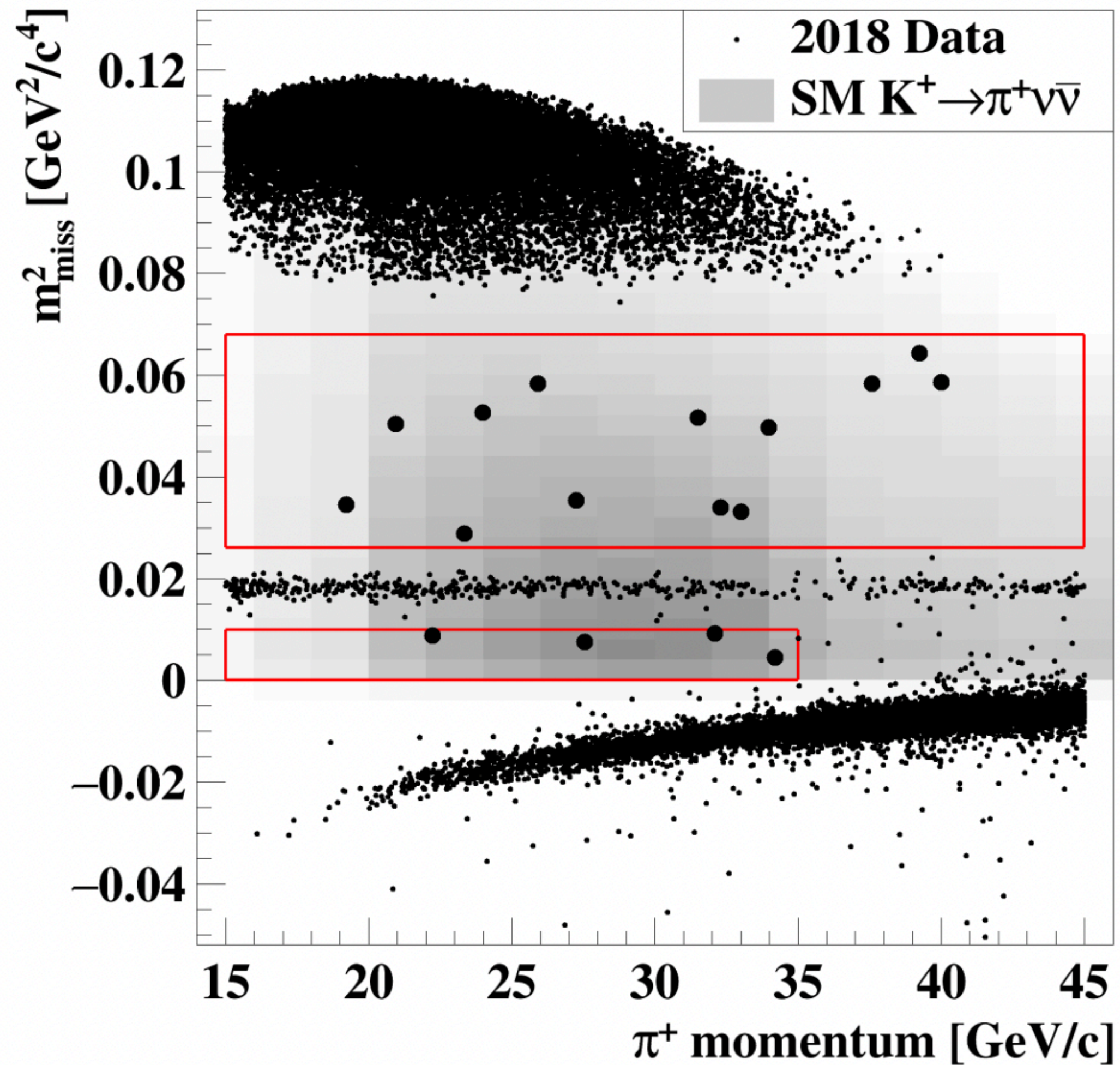
NA62 Beamline & Detector

[JINST 12 (2017) 05, P05025]



- Designed & optimised for study of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$:
 - Particle tracking: beam particle (GTK) & downstream tracks (STRAW)
 - PID: K^+ - KTAG, π^+ - RICH, Calorimeters (LKr, MUV1,2), MUV3 (μ detector)
 - Comprehensive veto systems: CHANTI (beam interactions), LAV, IRC, SAC (γ)

Run 1 (2016–18) $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ Results:

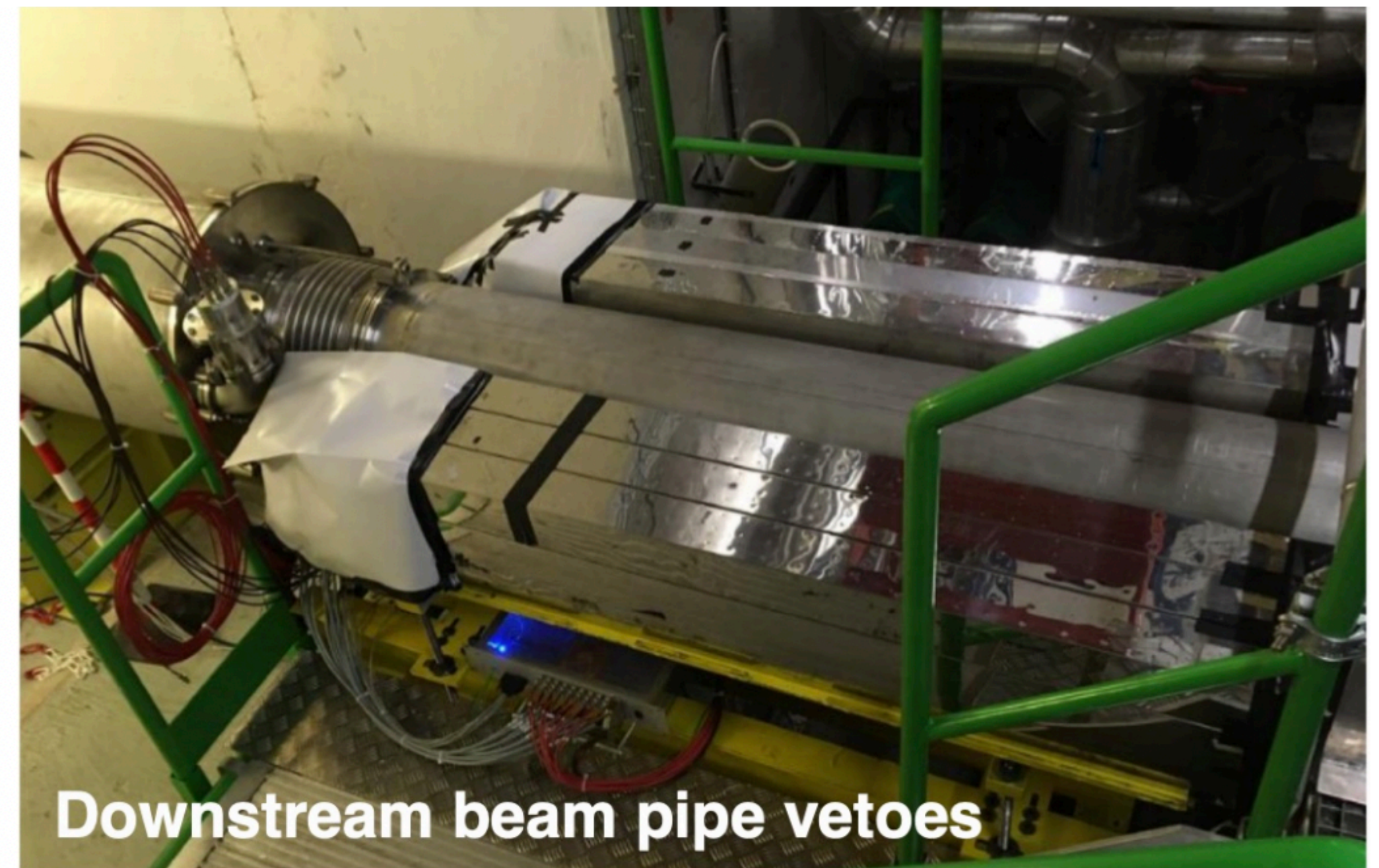
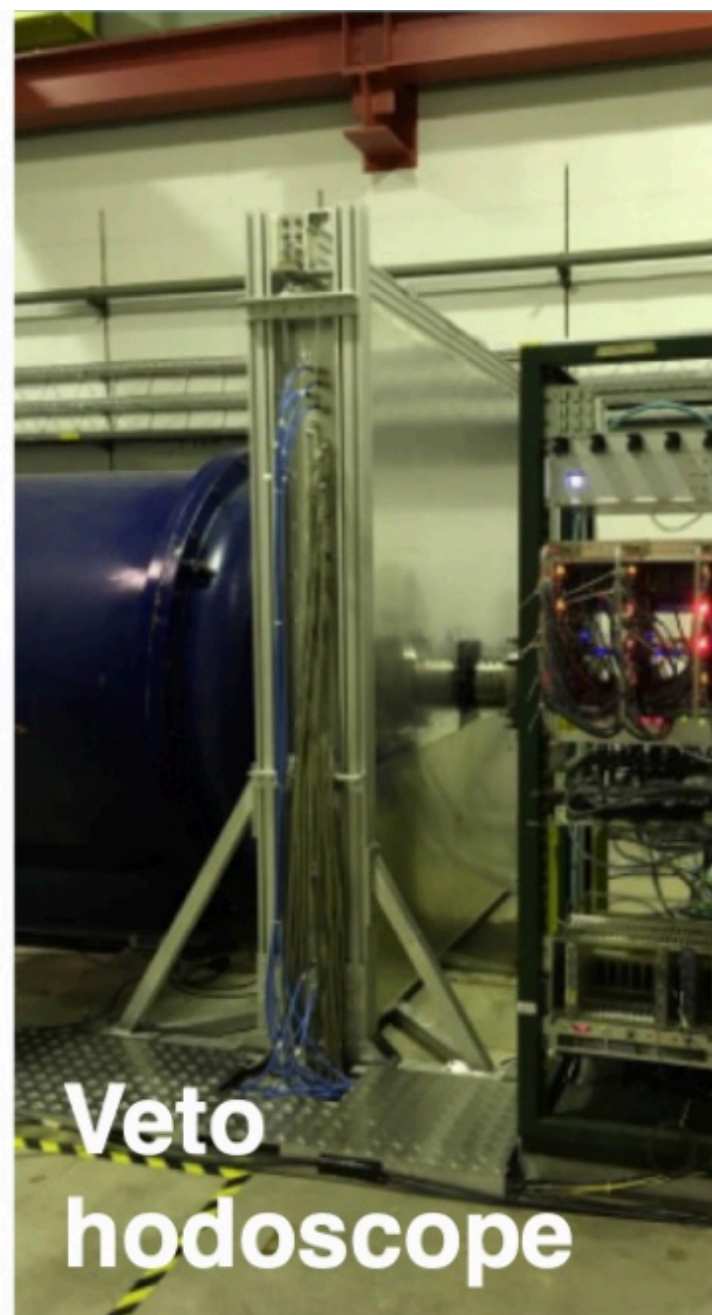
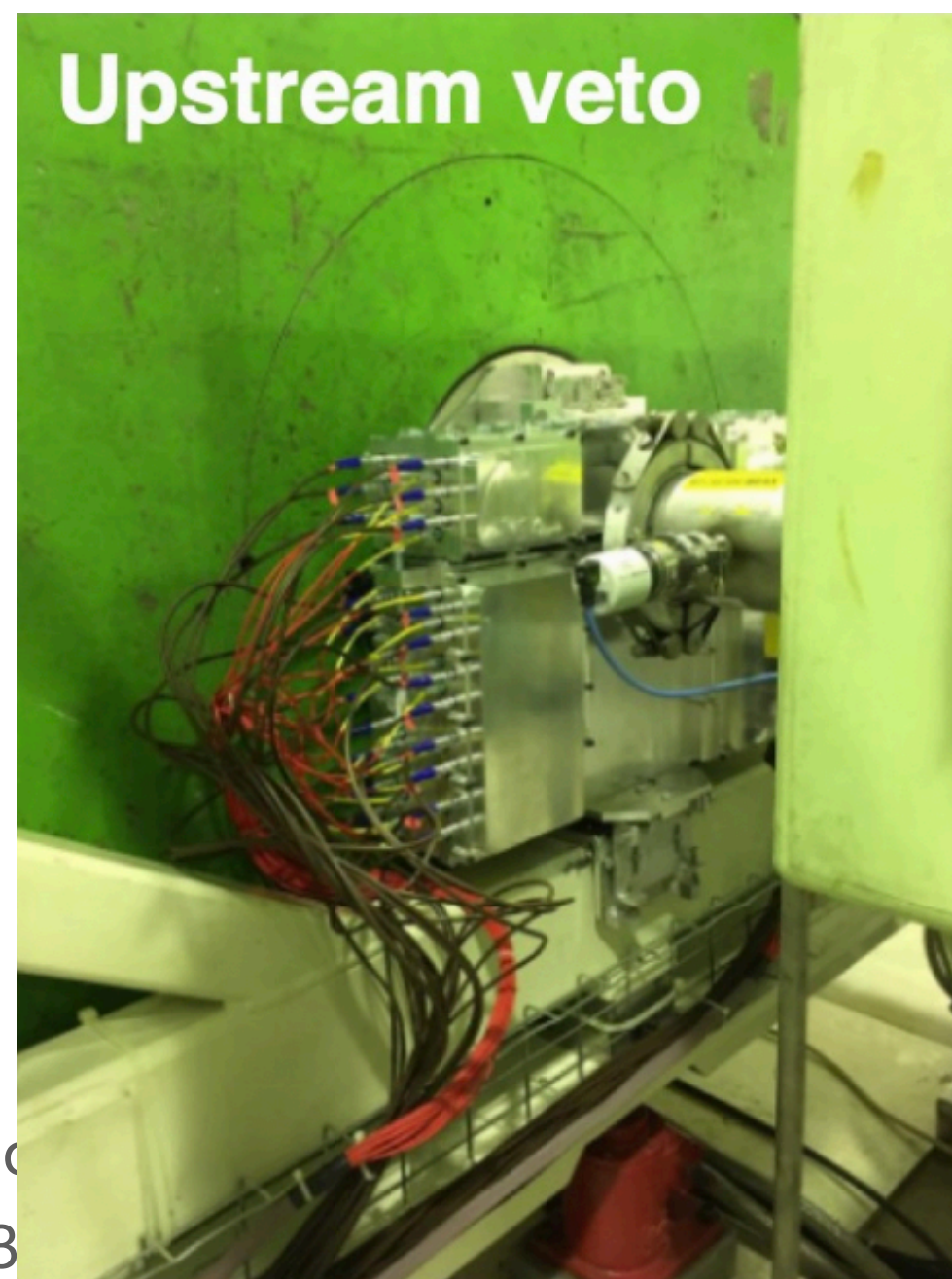


- $N_{\pi\nu\bar{\nu}}^{exp} = 10.01 \pm 0.42_{syst} \pm 1.19_{ext}$, $N_{bkg.}^{exp} = 7.03^{+1.05}_{-0.82}$: $n_{obs} = 20$
- In background-only hypothesis: $p = 3.4 \times 10^{-4} \Rightarrow$ signal significance: 3.4σ .

- $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (10.6^{+4.0}_{-3.4}|_{stat} \pm 0.9_{syst}) \times 10^{-11} @ 68 \% CL$

NA62 Run2 & The Future

- NA62 technique is firmly established.
- Run2 - target $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ measurement: $\mathcal{O}(10)\%$ precision.
 - 4th GTK (Kaon beam tracker) & rearrange beam line elements around GTK achromat.
 - New upstream veto & veto hodoscope upstream of decay volume.
 - Additional veto detector at end of beam-line.
 - Intensity increased by $\sim 30\%$ with respect Run1. Matched by trigger updates.
- Improvements to the trigger have led to smaller trigger downscaling factors for multi-track triggers: more data available for CLFV/LNV searches.
- Beyond LS3: High Intensity Kaon Experiment (HIKE) program under development at CERN SPS. [\[arXiv:2204.13394\]](https://arxiv.org/abs/2204.13394)



New detectors
installed in 2021
for NA62 Run2:

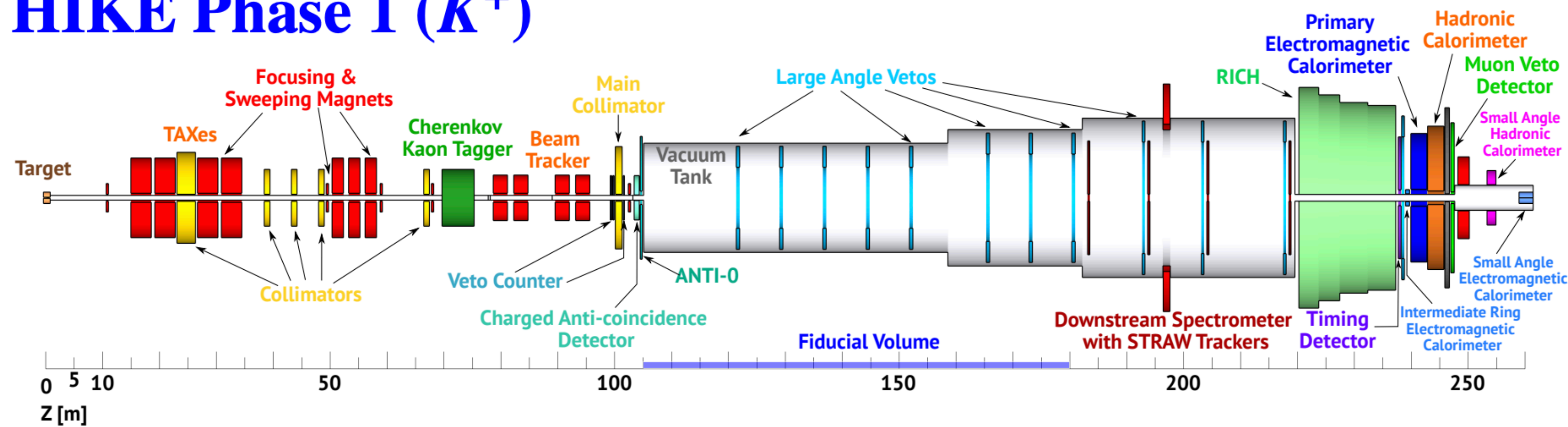
HIKE: High Intensity Kaon Experiments



[HIKE LOI: [arXiv:2211.16586](https://arxiv.org/abs/2211.16586), proposal to SPSC under preparation]

- Long-term programme of kaon physics at the CERN SPS beyond LS3.
- Beam intensity 4-6 times higher than NA62, detectors with $\mathcal{O}(20)$ ps time resolution.
- Multi-phase physics program (K^+ and K_L beams foreseen in different phases):
 - Ultimate precision for $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ measurement, target: $\mathcal{O}(5\%)$ i.e. approx. SM theory precision.
 - $K_L \rightarrow \pi^0 \ell^+ \ell^-$ observation and BR measurement.
 - LFUV tests with sub-percent precision.
 - LFV/CLFV searches with $\mathcal{O}(10^{-12})$ sensitivity
 - Measurements of V_{us} and main kaon decay modes.
 - Beam dump physics in synergy with SHADOWS experiment.

HIKE Phase 1 (K^+)



CKM

Fit to K_S rate data

Fit inputs & constraints

7 input measurements:

KLOE '06 BR $\pi^0\pi^0/\pi^+\pi^-$

NA48 $\Gamma(K_S \rightarrow \pi e \nu)/\Gamma(K_L \rightarrow \pi e \nu)$, τ_S

KLOE '11 τ_S

KTeV '11 τ_S

KLOE-2 '22 BR $\pi e \nu/\pi^+\pi^-$ **New!**

KLOE-2 '20 BR $\pi\mu\nu/\pi^+\pi^-$

Fit results

Parameter	Value
BR($\pi^+\pi^-(\gamma)$)	69.20(5)%
BR($\pi^0\pi^0$)	30.69(5)%
BR(K_{e3})	$7.15(6) \times 10^{-4}$
BR($K_{\mu3}$)	$4.56(20) \times 10^{-4}$
τ_S	89.58(4) ns

$\chi^2/\text{ndf} = 0.36/3$ (Prob = 95%)

- Constraint: $\sum \mathcal{B}(K_S) = 1$

Fit to K_L rate data

Fit inputs & constraints

21 input measurements:

5 KTeV ratios

NA48 BR($K_{e3}/2$ track)

4 KLOE BRs

with dependence on τ_L

KLOE, NA48 BR($\pi^+\pi^-/K_{\ell 3}$)

KLOE, NA48 BR($\gamma\gamma/3\pi^0$)

BR($2\pi^0/\pi^+\pi^-$) from K_S fit, Re ε'/ε

KLOE τ_L from $3\pi^0$

Vosburgh '72 τ_L

KTeV BR($\pi^+\pi^-\gamma/\pi^+\pi^-(\gamma)$)

E731, 2 KTeV BR($\pi^+\pi^-\gamma_{DE}/\pi^+\pi^-\gamma$)



Fit results

Parameter	Value	S
BR(K_{e3})	0.4056(9)	1.3
BR($K_{\mu 3}$)	0.2704(10)	1.5
BR($3\pi^0$)	0.1952(9)	1.2
BR($\pi^+\pi^-\pi^0$)	0.1254(6)	1.3
BR($\pi^+\pi^-(\gamma_{IB})$)	$1.967(7) \times 10^{-3}$	1.1
BR($\pi^+\pi^-\gamma$)	$4.15(9) \times 10^{-5}$	1.6
BR($\pi^+\pi^-\gamma_{DE}$)	$2.84(8) \times 10^{-5}$	1.3
BR($2\pi^0$)	$8.65(4) \times 10^{-4}$	1.4
BR($\gamma\gamma$)	$5.47(4) \times 10^{-4}$	1.1
τ_L	51.16(21) ns	1.1

$\chi^2/\text{ndf} = 19.8/12$ (Prob = 7.0%)

- c.f. current PDG result (since 09):
 $\chi^2/\text{ndf} = 37.4/17$ (Prob = 0.30%)

- Constraint: $\sum \mathcal{B}(K_L) = 1$

Fit to K^\pm rate data

[Moulson, CKM23]

Fit inputs & constraints

17 input measurements:

3 old τ values in PDG

KLOE τ

KLOE BR $\mu\nu, \pi\pi^0$

KLOE BR $K_{e3}, K_{\mu3}$

with dependence on τ

NA48/2 BR $K_{e3}/\pi\pi^0, K_{\mu3}/\pi\pi^0$

E865 BR $K_{e3}/KDal$

3 old BR $\pi\pi^0/\mu\nu$

KEK-246 $K_{\mu3}/K_{e3}$

KLOE BR $\pi\pi\pi, \pi\pi^0\pi^0$

(Bisi '65 BR $\pi\pi^0\pi^0/\pi\pi\pi$ removed)

• Constraint: $\sum \mathcal{B}(K^\pm) = 1$

• Note: restricted set of inputs wrt PDG fit (35 inputs, 8 parameters).

Fit results

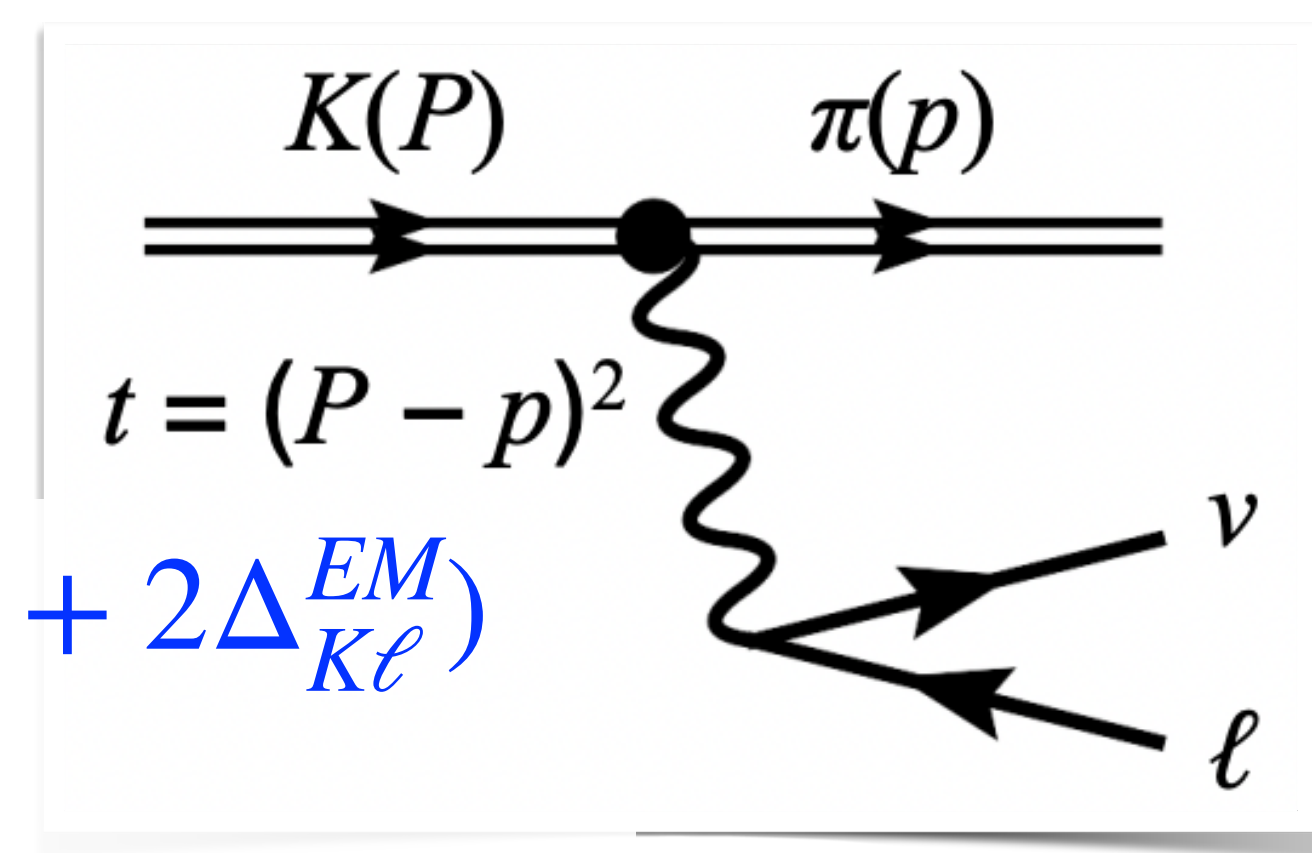
Parameter	Value	S
BR($\mu\nu$)	63.58(11)%	1.1
BR($\pi\pi^0$)	20.64(7)%	1.1
BR($\pi\pi\pi$)	5.56(4)%	1.0
BR(K_{e3})	5.088(27)%	1.2
BR($K_{\mu3}$)	3.366(30)%	1.9
BR($\pi\pi^0\pi^0$)	1.764(25)%	1.0
τ_\pm	12.384(15) ns	1.2

$\chi^2/\text{ndf} = 25.5/11$ (Prob = 0.78%)

- c.f. current PDG result (2016):
 $\chi^2/\text{ndf} = 53/28$ (Prob = 0.26%)
- Note: negligible change when including **ISTRA+** measurements.

$K_{\ell 3}$ Form factors

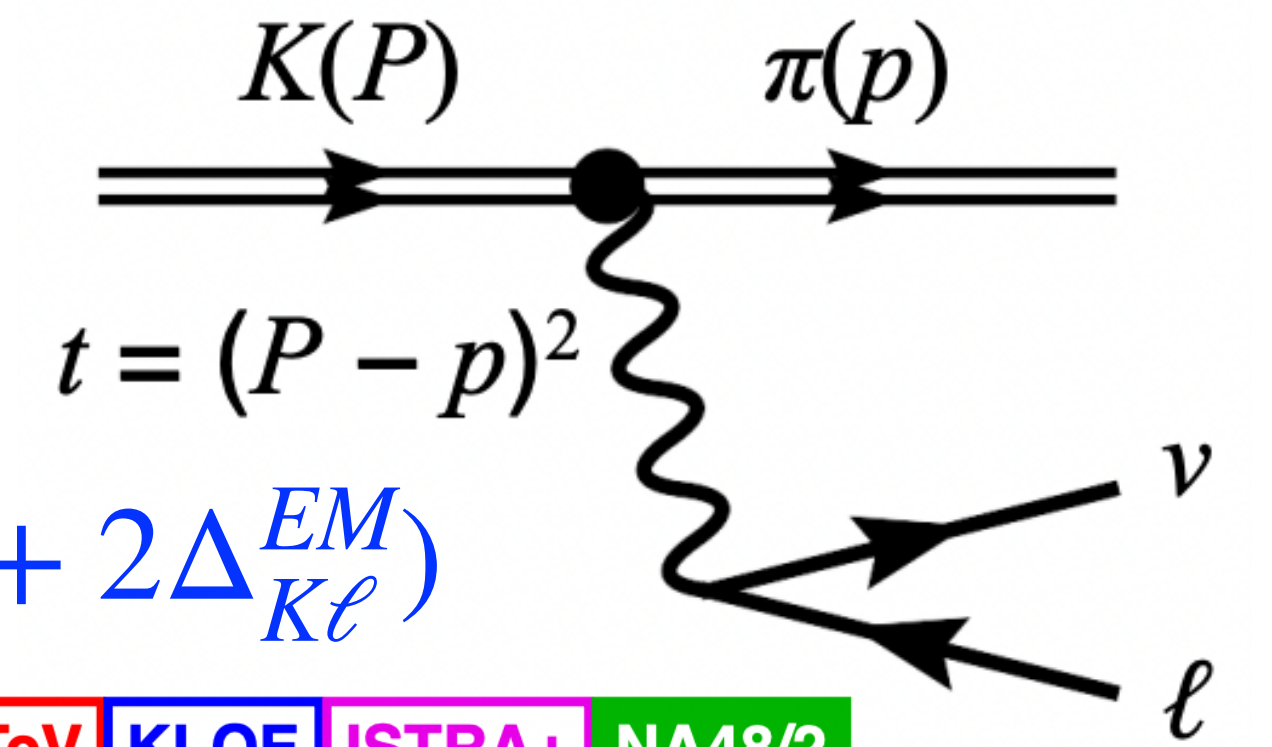
$$\Gamma(K \rightarrow \pi \ell \nu(\gamma)) = |V_{us}|^2 \frac{C_K^2 G_F^2 m_K^5}{192 \pi^3} S_{EW} |f_+^{K^0 \pi^-}(0)|^2 I_{K_L}(\lambda_{K\ell}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM})$$



- Hadronic matrix element: $\langle \pi | J_\alpha | K \rangle = f(0) [\tilde{f}_+(t)(P + p)_\alpha + \tilde{f}_-(t)(P - p)_\alpha]$
 - K_{e3} decays: only vector form factor $\tilde{f}_+(t)$
 - $K_{\mu 3}$ decays: also need scalar form factor $\tilde{f}_0(t) = \tilde{f}_+ + \tilde{f}_- \frac{t}{m_K^2 - m_\pi^2}$
- Parameterise using dispersion relations $\tilde{f}_+(t) = \exp\left[\frac{t}{m_\pi^2(\Lambda_+ - H(t))}\right]$, $\tilde{f}_0(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2(\ln C - G(t))}\right]$.
 - Where $H(t)$ and $G(t)$ are polynomials from $K\pi$ scattering data, contributing to (but not much) uncertainty in fits.
- Evaluate Integral of form factors over phase-space: $I_{K_L}(\lambda_{K\ell})$ [where λ parameterises t evolution]

$K_{\ell 3}$ Form factors

[Moulson, CKM23]



$$\Gamma(K \rightarrow \pi \ell \nu(\gamma)) = |V_{us}|^2 \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |f_+^{K^0 \pi^-}(0)|^2 I_{K_L}(\lambda_{K\ell}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM})$$

- Form factor parameterisations

$$\tilde{f}_+(t) = \exp\left[\frac{t}{m_\pi^2(\Lambda_+ - H(t))}\right], \tilde{f}_0(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2(\ln C - G(t))}\right]$$

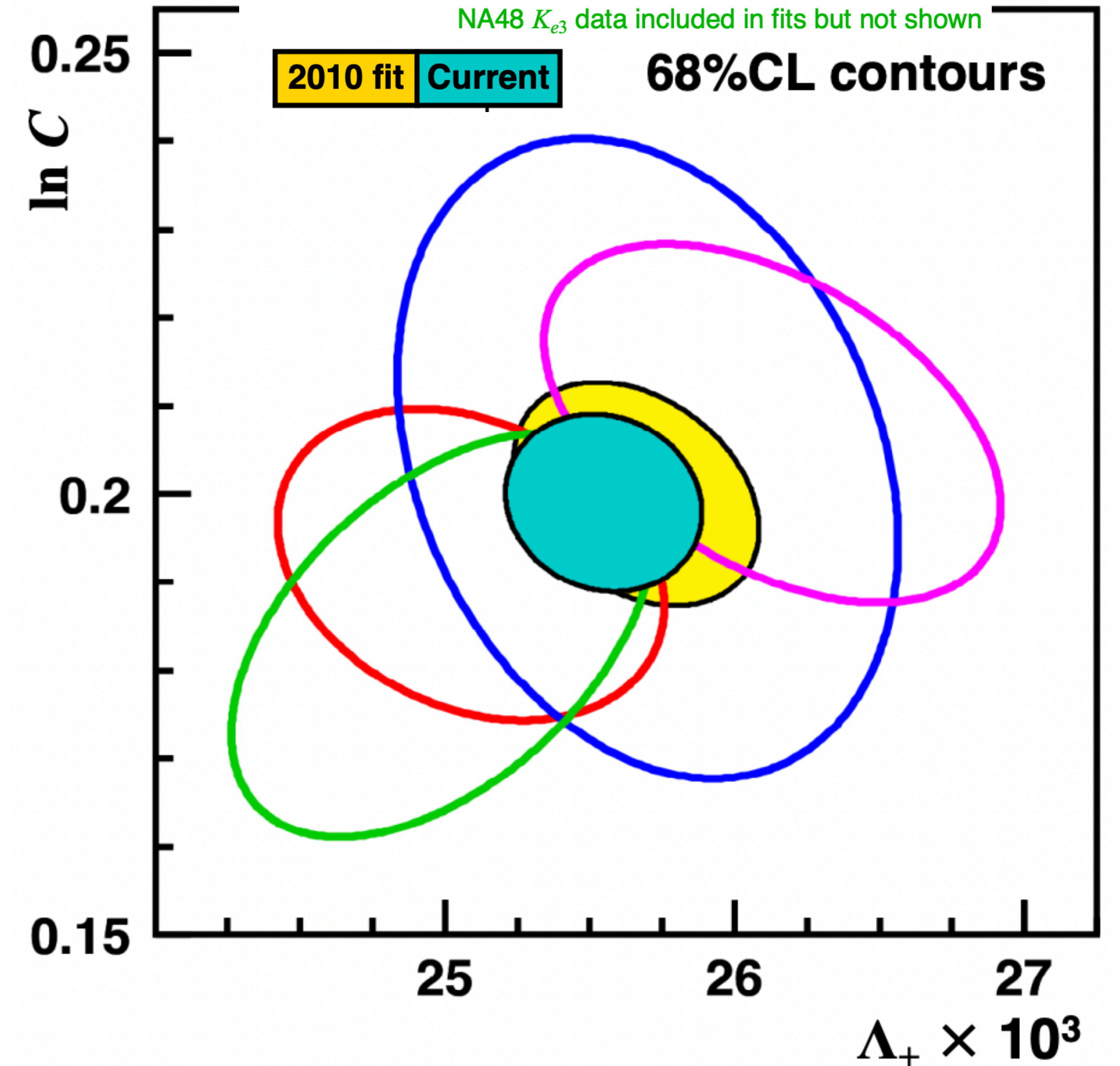
- Current fit:

$$\begin{aligned} \Lambda_+ \times 10^3 &= 25.55 \pm 0.38 \\ \ln C &= 0.1992(78) \\ \rho(\Lambda_+, \ln C) &= -0.110 \\ \chi^2/\text{ndf} &= 7.5/7 \text{ (38\%)} \end{aligned}$$

Integrals

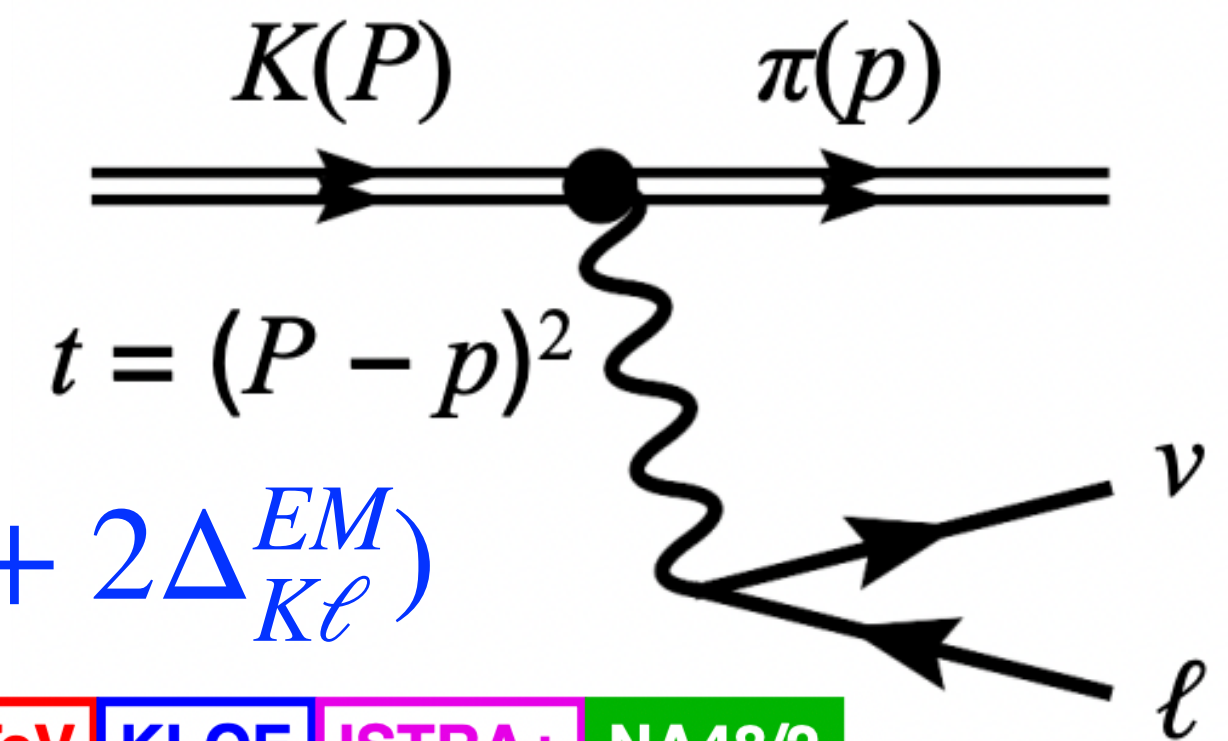
Mode	Update	2010
K^0_{e3}	0.15470(15)	0.15476(18)
K^+_{e3}	0.15915(15)	0.15922(18)
$K^0_{\mu 3}$	0.10247(15)	0.10253(16)
$K^+_{\mu 3}$	0.10553(16)	0.10559(17)

K_{e3} avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**
 NA48 K_{e3} data included in fits but not shown



$K_{\ell 3}$ Form factors

[Moulson, CKM23]



$$\Gamma(K \rightarrow \pi \ell \nu(\gamma)) = |V_{us}|^2 \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |f_+^{K^0 \pi^-}(0)|^2 I_{K_L}(\lambda_{K\ell}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM})$$

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**
 NA48 $K_{\ell 3}$ data included in fits but not shown

- Form factor parameterisations

$$\tilde{f}_+(t) = \exp\left[\frac{t}{m_\pi^2(\Lambda_+ - H(t))}\right], \tilde{f}_0(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2(\ln C - G(t))}\right]$$

- Current fit:

$\Lambda_+ \times 10^3 = 25.55 \pm 0.38$
 $\ln C = 0.1992(78)$
 $\rho(\Lambda_+, \ln C) = -0.110$
 $\chi^2/\text{ndf} = 7.5/7$ (38%)

Integrals

Mode	Update	2010
K^0_{e3}	0.15470(15)	0.15476(18)
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$K^0_{\mu 3}$	0.10247(15)	0.10253(16)
$K^+_{\mu 3}$	0.10553(16)	0.10559(17)

