



Kaon rare decays: theory overview

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Based on 2206.14748 and 2311.04878 in collaboration with G. D'Ambrosio, A. Iyer, F. Mahmoudi

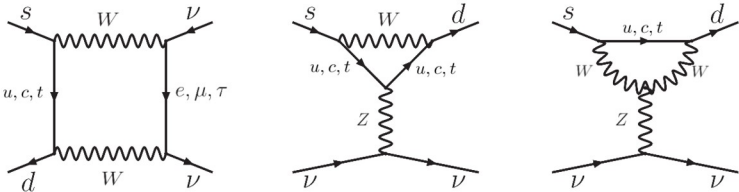
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8-10 November 2023

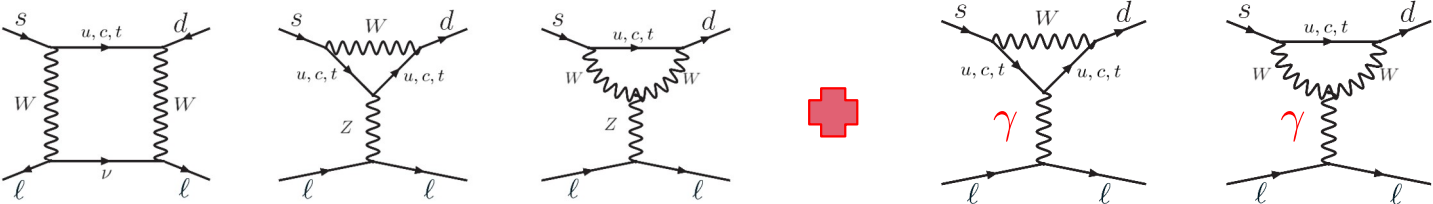
Rare kaon decays

- Take place via FCNC
- Suppressed in the Standard Model
- Interesting probe of New Physics
 - ↳ Requires reliable prediction in the SM

- SD dominated
 - $K^+ \rightarrow \pi^+ \nu \nu$ and $K_L \rightarrow \pi^0 \nu \nu$ (golden channels)



- LD dominated
 - $K_L \rightarrow \mu \mu$, $K_S \rightarrow \mu \mu$, $K^+ \rightarrow \pi^+ \ell \ell$ and $K_L \rightarrow \pi^0 \ell \ell$, ...



Rare kaon decays

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Weak effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{td} \frac{\alpha_e}{4\pi} \sum_k C_k^\ell O_k^\ell$$

$$O_L^\ell = (\bar{s}\gamma_\mu P_L d)(\bar{\nu}_\ell \gamma^\mu (1 - \gamma_5) \nu_\ell), \quad O_9^\ell = (\bar{s}\gamma_\mu P_L d)(\bar{\ell} \gamma^\mu \ell), \quad O_{10}^\ell = (\bar{s}\gamma_\mu P_L d)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

NP contributions: $C_k \rightarrow C_k^{\text{SM}} + \delta C_k$

$K^+ \rightarrow \pi^+ \nu \nu$

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \frac{\kappa_+(1 + \Delta_{\text{EM}})}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_{\ell} \left[\text{Im}^2 (\lambda_t C_L^\ell) + \text{Re}^2 \left(-\frac{\lambda_c X_c}{s_w^2} + \lambda_t C_L^\ell \right) \right] \quad (\lambda_i = V_{is}^* V_{id})$$

- top loop: $C_{L,\text{SM}}^\ell = -X_{\text{SM}}(x_t)/s_W^2$ NNLO QCD and 2-loop EW [Buchalla, Buras, '99; Misiak, Urban '99, Brod et al. '10]
- charm contribution: $X_c = \lambda^4 [P_c^{\text{SD}} + \delta P_{c,u}^{\text{LD}}]$ SD: NNLO QCD and NLO EW; LD: ChPT SD:[Buras et al. '05; Brod et al. '08]
LD:[Isidori et al.'05]
- O_L matrix elements known from $K_{3\ell}$ branching ratios \rightarrow included in κ_+ [Mescia, Smith '17]
- $\Gamma_{\text{SD}}/\Gamma > 90\%$

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$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (7.86 \pm 0.61) \times 10^{-11} \quad \text{[D'Ambrosio, Iyer, Mahmoudi, SN '22]}$$

Sources of uncertainty:

SD $\sim 2\%$, LD $\sim 3\%$, Parametric $\sim 7\%$

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$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (7.73 \pm 0.61) \times 10^{-11} \quad \text{[Brod, Gorbahn, Stamou '21]}$$

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (8.60 \pm 0.42) \times 10^{-11} \quad \text{[Buras, Venturini '22]}$$

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$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{NA62}} = (10.6_{-3.5}^{+4.0} \pm 0.9) \times 10^{-11} \quad [\text{D'Ambrosio, Iyer, Mahmoudi, SN '22}]$$

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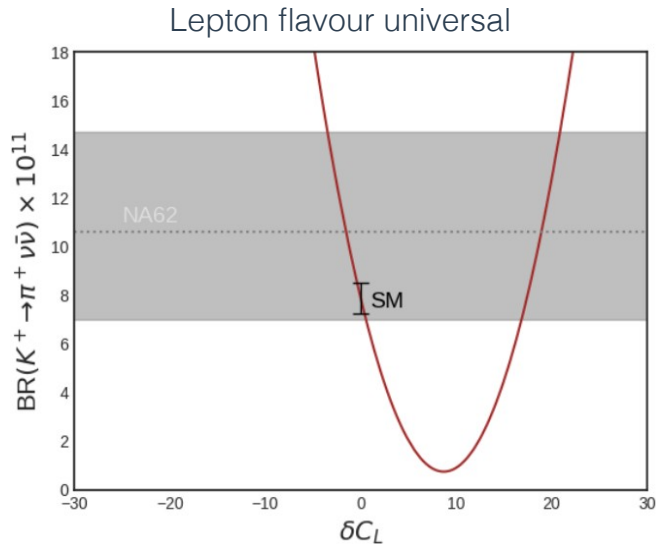
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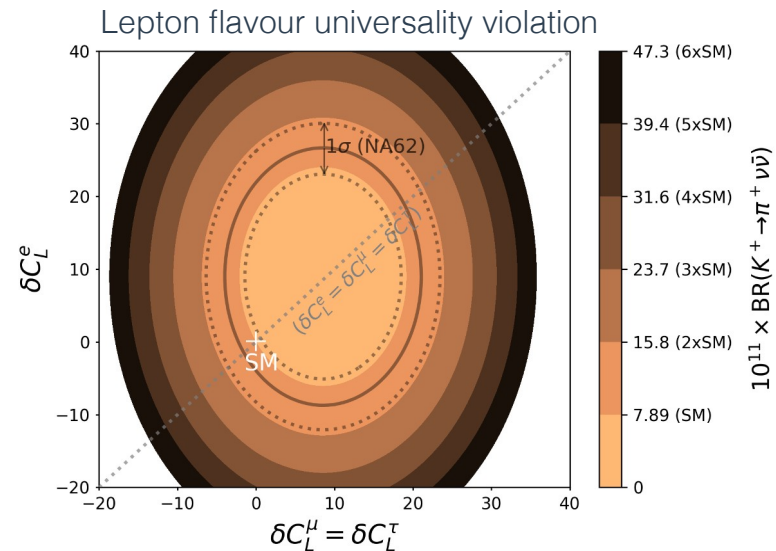
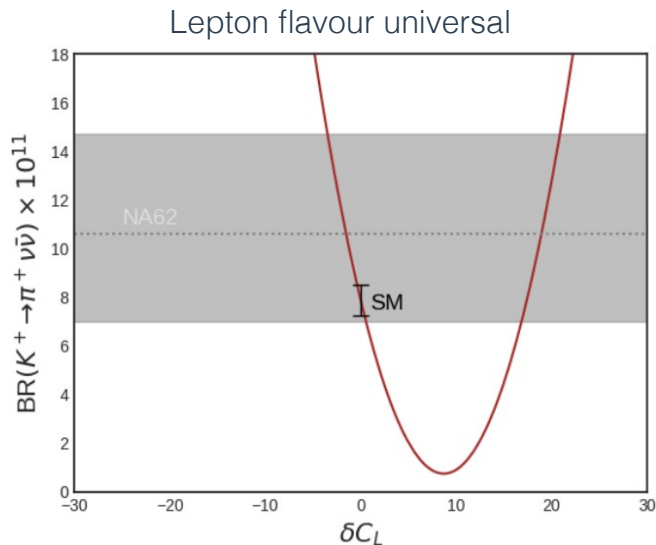
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- $C_{L,SM}$ same as for $K^+ \rightarrow \pi^+ \nu \nu$
- charm contributions below 1%
- 99% SD distance

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{SM} = (2.68 \pm 0.30) \times 10^{-11} \quad [\text{D'Ambrosio, Iyer, Mahmoudi, SN '22}]$$

Sources of uncertainty:

SD $\sim 2\%$, LD $\sim 1\%$, Parametric $\sim 11\%$

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{SM} = (2.59 \pm 0.29) \times 10^{-11} \quad [\text{Brod, Gorbahn, Stamou '21}]$$

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{SM} = (2.94 \pm 0.15) \times 10^{-11} \quad [\text{Buras, Venturini '22}]$$

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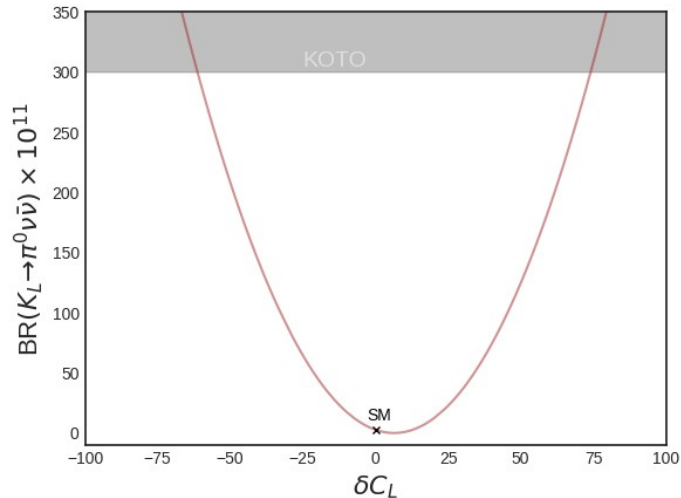
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$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{KOTO}} < 3.0 \times 10^{-11} \text{ at 90\% CL [KOTO Coll., Ahn et al. '18]}$$

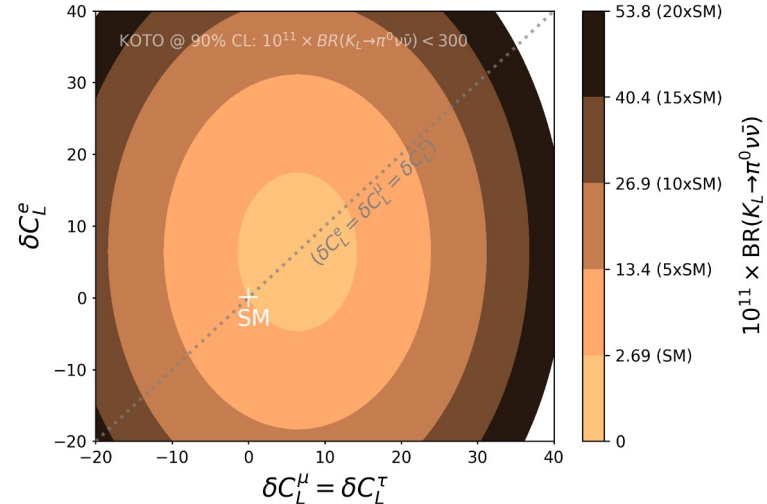
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New Physics effects:

Lepton flavour universal

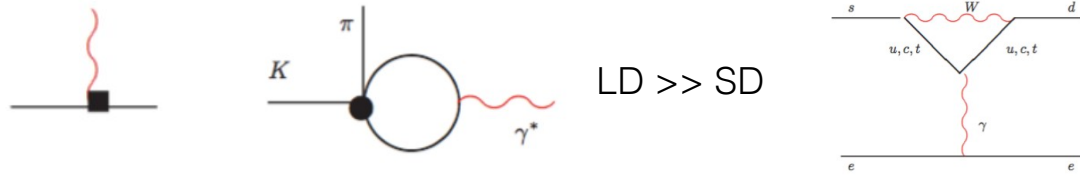


Lepton flavour universality violation



LFUV in $K^+ \rightarrow \pi^+ \ell \ell$

$K^+ \rightarrow \pi^+ \ell \ell$ is long distance dominated, mediated by single photon exchange $K^+ \rightarrow \pi^+ \gamma^*$



LD \gg SD

\Rightarrow precise SM prediction not yet possible

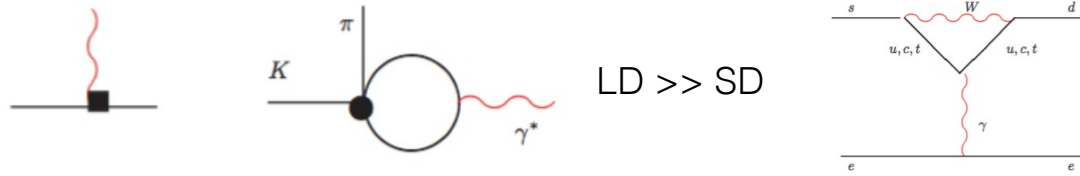
$$\mathcal{A}(z) \propto G_F M_K^2 (a + bz) + W^{\pi\pi}(z) \quad z = m^2(\ell\ell) / M_K^2$$

form factors

loop term

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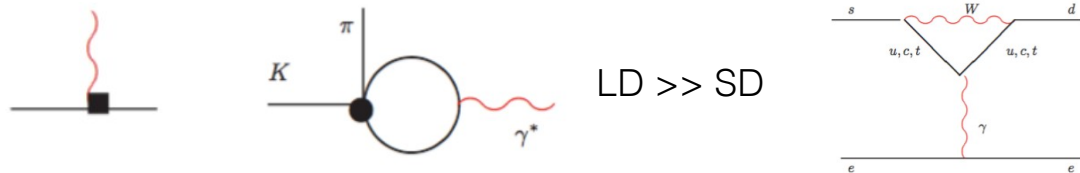
$\downarrow \quad \downarrow$
 form factors loop term

LFU predicts the same form factors a and b , for $\ell = e, \mu$

$a^{ee} \neq a^{\mu\mu}$ indicates LFUV NP: $a_+^{\mu\mu} - a_+^{ee} = -\sqrt{2} \operatorname{Re} [V_{td} V_{ts}^* (C_9^\mu - C_9^e)]$
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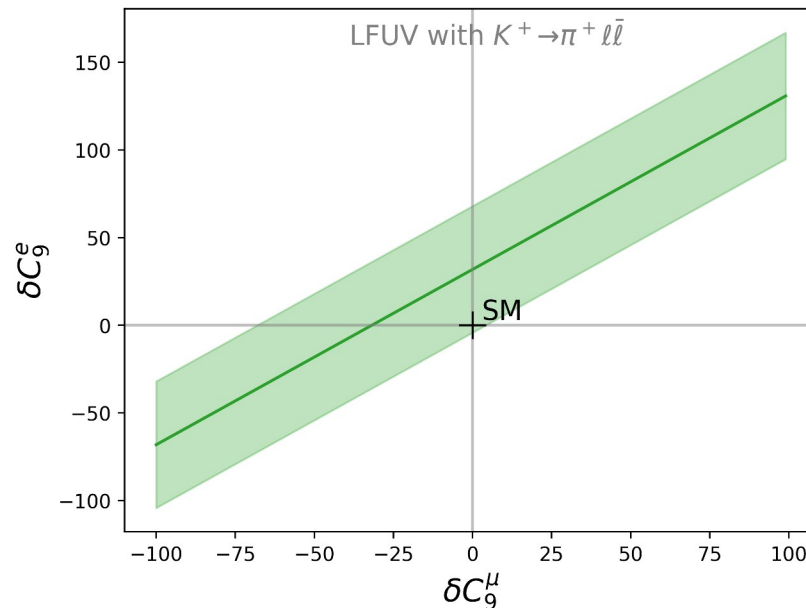
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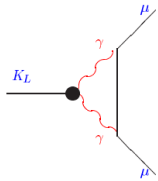
Channel	a_+	b_+	Reference
ee	-0.561 ± 0.009	-0.694 ± 0.040	E865 '99 and NA48/2 '09 comb. [D'Ambrosio, Greynat, Knecht '18]
$\mu\mu$	-0.575 ± 0.013	-0.722 ± 0.043	NA62 Coll. '22



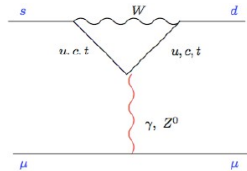
$K_L \rightarrow \mu \mu$

$K_L \rightarrow \mu \mu$ is long distance dominated, mediated by two photons via $K_L \rightarrow \gamma^* \gamma^*$

$$\text{BR}(K_L \rightarrow \mu \bar{\mu}) = \tau_L \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left| N_L^{\text{LD}} - \left(\frac{2m_\mu G_F \alpha_e}{m_K \sqrt{2}\pi} \right) \text{Re} \left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell \right] \right|^2$$



LD \gg SD



$$N_L^{\text{LD}} \propto (\chi_{\text{disp}} + i\chi_{\text{abs}}) \longrightarrow N_L^{\text{LD}} = \pm [0.54(77) - 3.95i] \times 10^{-11} (\text{GeV})^{-2}$$

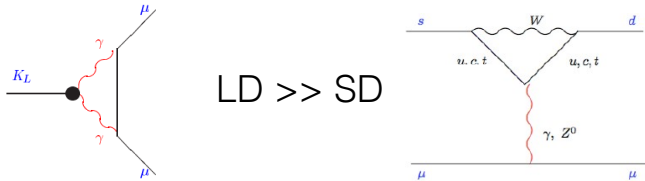
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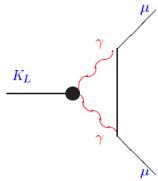
[D'Ambrosio et al. '17]

Prediction depends on the sign of $A(K_L \rightarrow \gamma\gamma)$ contribution
determining the effect of the SD-LD interference

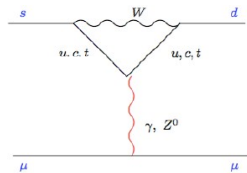
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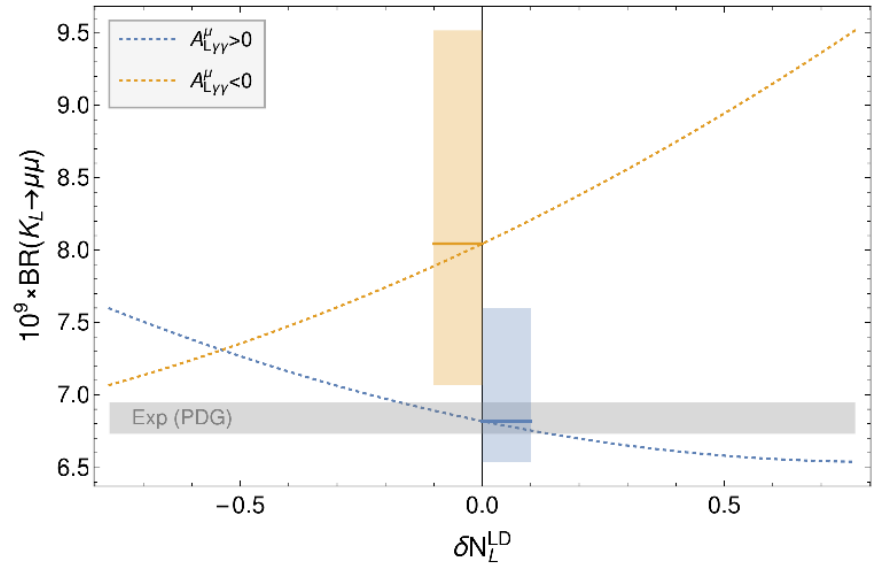
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Prediction depends on the sign of $A(K_L \rightarrow \gamma\gamma)$ contribution determining the effect of the SD-LD interference

$$\text{BR}(K_L \rightarrow \mu \bar{\mu})_{\text{SM}} = \begin{cases} \text{LD}(+): (6.82_{-0.24}^{+0.77} \pm 0.04) \times 10^{-9} \\ \text{LD}(-): (8.04_{-0.97}^{+1.46} \pm 0.09) \times 10^{-9} \end{cases}$$

[D'Ambrosio, Iyer, Mahmoudi, SN '22]



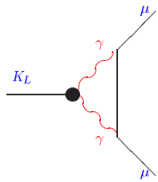
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[PDG]

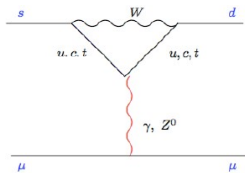
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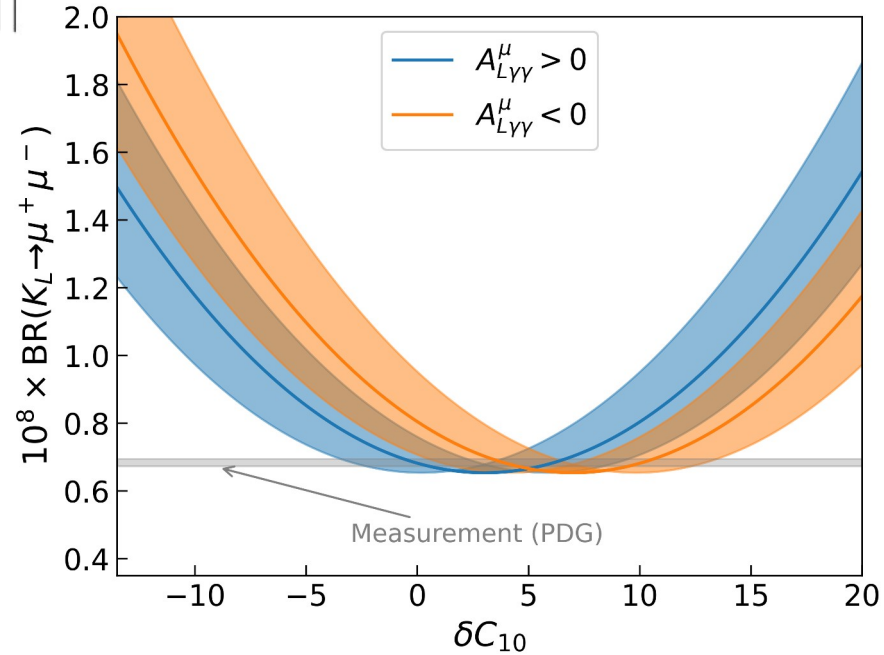
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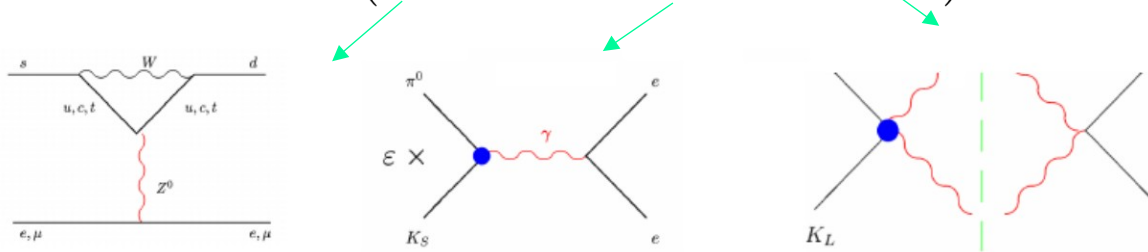


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[PDG]

$K_L \rightarrow \pi^0 \ell \bar{\ell}$

$$\text{BR}(K_L \rightarrow \pi^0 \ell \bar{\ell}) = \left(C_{\text{dir}}^\ell \pm C_{\text{int}}^\ell |a_S| + C_{\text{mix}}^\ell |a_S|^2 + C_{\gamma\gamma}^\ell \right)$$



[Dambrosio et al. '98; Isidor et al. '04; Mescia, Smith, Trine '06]

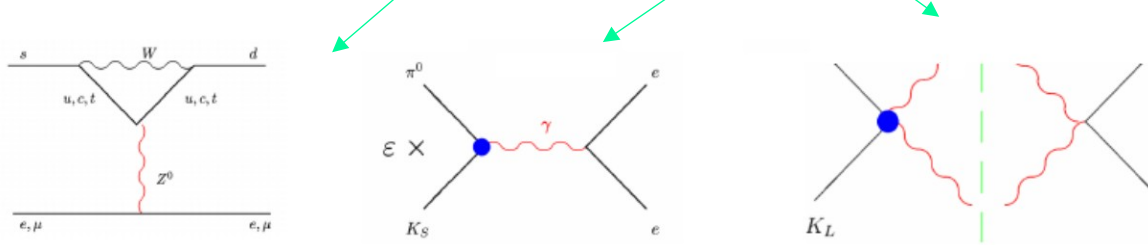
	C_{dir}^ℓ	C_{int}^ℓ	C_{mix}^ℓ	$C_{\gamma\gamma}^\ell$
$\ell = e$	$(4.62 \pm 0.24) (w_{7V}^2 + w_{7A}^2)$	$(11.3 \pm 0.3) w_{7V}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05) (w_{7V}^2 + 2.32 w_{7A}^2)$	$(2.63 \pm 0.06) w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

[Mescia, Smith, Trine '06]

$$w_{7V} = \frac{1}{2\pi} \text{Im} \left[\frac{\lambda_t^{sd}}{1.407 \times 10^{-4}} C_9 \right], \quad w_{7A} = \frac{1}{2\pi} \text{Im} \left[\frac{\lambda_t^{sd}}{1.407 \times 10^{-4}} C_{10} \right]$$

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	C_{dir}^ℓ	C_{int}^ℓ	C_{mix}^ℓ	$C_{\gamma\gamma}^\ell$
$\ell = e$	$(4.62 \pm 0.24) (w_{7V}^2 + w_{7A}^2)$	$(11.3 \pm 0.3) w_{7V}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05) (w_{7V}^2 + 2.32 w_{7A}^2)$	$(2.63 \pm 0.06) w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

[Mescia, Smith, Trine '06]

$$w_{7V} = \frac{1}{2\pi} \text{Im} \left[\frac{\lambda_t^{sd}}{1.407 \times 10^{-4}} C_9 \right], \quad w_{7A} = \frac{1}{2\pi} \text{Im} \left[\frac{\lambda_t^{sd}}{1.407 \times 10^{-4}} C_{10} \right]$$

$$\text{BR}^{\text{SM}}(K_L \rightarrow \pi^0 e \bar{e}) = 3.46_{-0.80}^{+0.92} (1.55_{-0.48}^{+0.60}) \times 10^{-11}$$

$$\text{BR}^{\text{SM}}(K_L \rightarrow \pi^0 \mu \bar{\mu}) = 1.38_{-0.25}^{+0.27} (0.94_{-0.20}^{+0.21}) \times 10^{-11}$$

[D'Ambrosio, Iyer, Mahmoudi, SN '22]

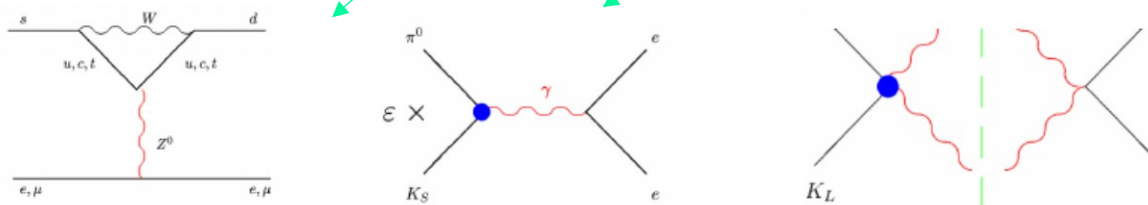
$$\text{BR}^{\text{exp}}(K_L \rightarrow \pi^0 e \bar{e}) < 28 \times 10^{-11} \quad \text{at 90\% CL}$$

$$\text{BR}^{\text{exp}}(K_L \rightarrow \pi^0 \mu \bar{\mu}) < 38 \times 10^{-11} \quad \text{at 90\% CL}$$

[KTeV '00 and '03]

$K_L \rightarrow \pi^0 \ell \bar{\ell}$

$$\text{BR}(K_L \rightarrow \pi^0 \ell \bar{\ell}) = \left(C_{\text{dir}}^\ell \pm C_{\text{int}}^\ell |a_S| + C_{\text{mix}}^\ell |a_S|^2 + C_{\gamma\gamma}^\ell \right)$$



[Dambrosio et al. '98; Isidor et al. '04; Mescia, Smith, Trine '06]

	C_{dir}^ℓ	C_{int}^ℓ	C_{mix}^ℓ	$C_{\gamma\gamma}^\ell$
$\ell = e$	$(4.62 \pm 0.24) (w_{7V}^2 + w_{7A}^2)$	$(11.3 \pm 0.3) w_{7V}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05) (w_{7V}^2 + 2.32 w_{7A}^2)$	$(2.63 \pm 0.06) w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

[Mescia, Smith, Trine '06]

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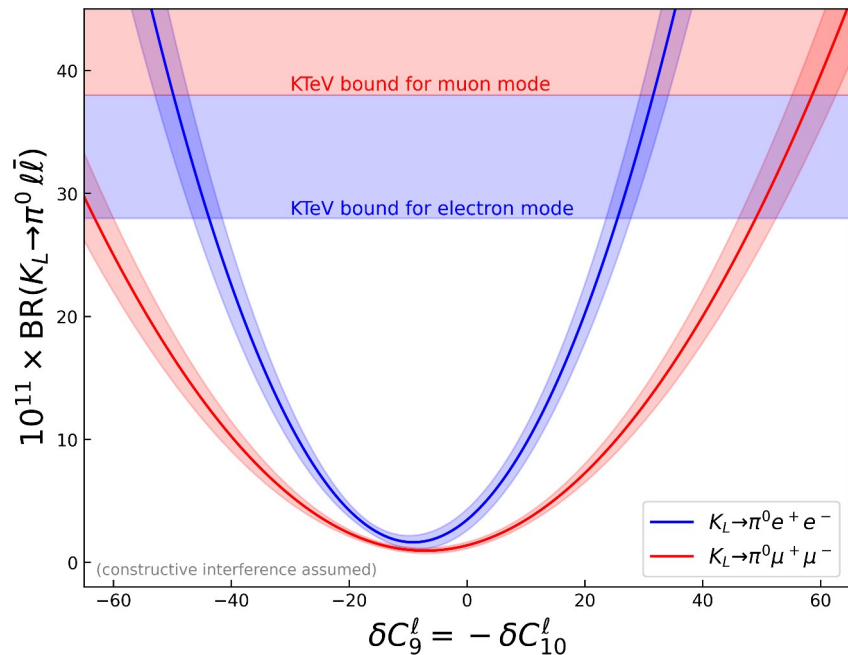
$$\text{BR}^{\text{SM}}(K_L \rightarrow \pi^0 \mu \bar{\mu}) = 1.38_{-0.25}^{+0.27} (0.94_{-0.20}^{+0.21}) \times 10^{-11}$$

[D'Ambrosio, Iyer, Mahmoudi, SN '22]

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$$\text{BR}^{\text{exp}}(K_L \rightarrow \pi^0 \mu \bar{\mu}) < 38 \times 10^{-11} \quad \text{at 90\% CL}$$

[KTeV '00 and '03]



Global analysis

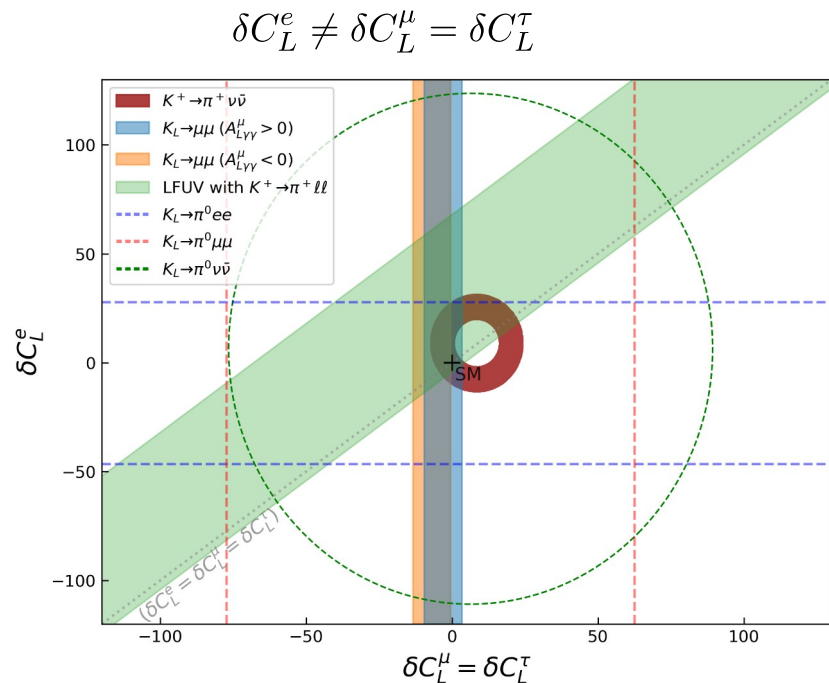
All observables

Rare kaon observables

Observable	SM prediction	Experimental results
$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6_{-3.5}^{+4.0} \pm 0.9) \times 10^{-11}$
$\text{BR}(K_L^0 \rightarrow \pi^0 \nu \bar{\nu})$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 3.0 \times 10^{-9} \text{ @90\% CL}$
$\text{LFUV}(a_+^{\mu\mu} - a_+^{ee})$	0	-0.014 ± 0.016
$\text{BR}(K_L \rightarrow \mu\mu) (+)$	$(6.82_{-0.29}^{+0.77}) \times 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$
$\text{BR}(K_L \rightarrow \mu\mu) (-)$	$(8.04_{-0.98}^{+1.47}) \times 10^{-9}$	
$\text{BR}(K_S \rightarrow \mu\mu)$	$(5.15 \pm 1.50) \times 10^{-12}$	$< 2.1(2.4) \times 10^{-10} \text{ @90(95)\%}$
$\text{BR}(K_L \rightarrow \pi^0 ee)(+)$	$(3.46_{-0.80}^{+0.92}) \times 10^{-11}$	$< 28 \times 10^{-11} \text{ @90\% CL}$
$\text{BR}(K_L \rightarrow \pi^0 ee)(-)$	$(1.55_{-0.48}^{+0.60}) \times 10^{-11}$	
$\text{BR}(K_L \rightarrow \pi^0 \mu\mu)(+)$	$(1.38_{-0.25}^{+0.27}) \times 10^{-11}$	$< 38 \times 10^{-11} \text{ @90\% CL}$
$\text{BR}(K_L \rightarrow \pi^0 \mu\mu)(-)$	$(0.94_{-0.20}^{+0.21}) \times 10^{-11}$	

We assume NP contributions of the charged and neutral leptons related to each other by the $\text{SU}(2)_L$ gauge symmetry and we work in the chiral basis

$$\delta C_L^\ell \equiv \delta C_9^\ell = -\delta C_{10}^\ell$$



Bounds from individual observables:

Coloured regions: 68% CL measurements

Dashed lines: 90% upper limits

All observables

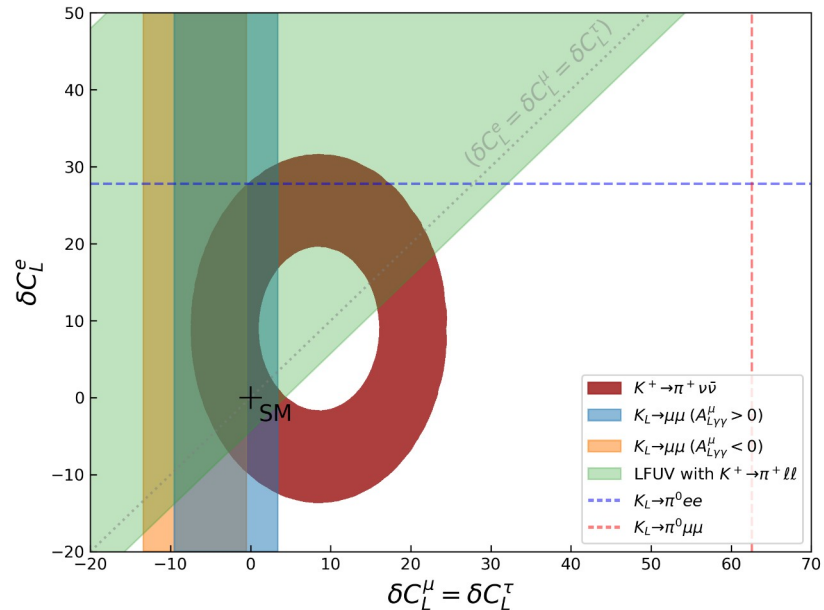
Rare kaon observables

Observable	SM prediction	Experimental results
$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6_{-3.5}^{+4.0} \pm 0.9) \times 10^{-11}$
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$\text{BR}(K_L \rightarrow \pi^0 \mu\mu)(+)$	$(1.38_{-0.25}^{+0.27}) \times 10^{-11}$	$< 38 \times 10^{-11} \text{ @90\% CL}$
$\text{BR}(K_L \rightarrow \pi^0 \mu\mu)(-)$	$(0.94_{-0.20}^{+0.21}) \times 10^{-11}$	

We assume NP contributions of the charged and neutral leptons related to each other by the $\text{SU}(2)_L$ gauge symmetry and we work in the chiral basis

$$\delta C_L^\ell \equiv \delta C_9^\ell = -\delta C_{10}^\ell$$

$$\delta C_L^e \neq \delta C_L^\mu = \delta C_L^\tau$$



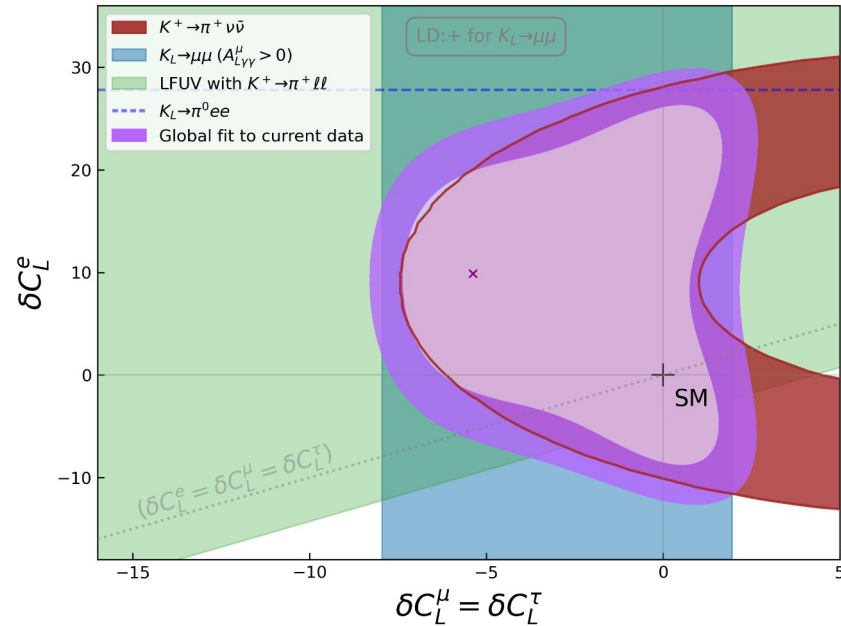
Bounds from individual observables:

Coloured regions: 68% CL measurements

Dashed lines: 90% upper limits

All observables / Global fit

Fit (with SuperIso public program) for positive LD contributions to $K_L \rightarrow \mu\mu$



Lighter / darker purple region: **68% / 95%** CL of global fit

Main constraining observables $\text{BR}(K^+ \rightarrow \pi^+ \nu\nu)$ followed by $\text{BR}(K_L \rightarrow \mu\mu)$

Prospects for future measurements

Numerical values used in the projections

Observable	SM prediction	Experimental results	NA62 final
$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6_{-3.5}^{+4.0} \pm 0.9) \times 10^{-11}$	20%
$\text{BR}(K_L^0 \rightarrow \pi^0 \nu \nu)$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 3.0 \times 10^{-9}$ @90% CL	current
$\text{LFUV}(a_+^{\mu\mu} - a_+^{ee})$	0	-0.014 ± 0.016	current
$\text{BR}(K_L \rightarrow \mu\mu) (+)$	$(6.82_{-0.29}^{+0.77}) \times 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$	current
$\text{BR}(K_L \rightarrow \mu\mu) (-)$	$(8.04_{-0.98}^{+1.47}) \times 10^{-9}$		
$\text{BR}(K_S \rightarrow \mu\mu)$	$(5.15 \pm 1.50) \times 10^{-12}$	$< 2.1(2.4) \times 10^{-10}$ @90(95)% CL	current
$\text{BR}(K_L \rightarrow \pi^0 ee)(+)$	$(3.46_{-0.80}^{+0.92}) \times 10^{-11}$	$< 28 \times 10^{-11}$ @90% CL	current
$\text{BR}(K_L \rightarrow \pi^0 ee)(-)$	$(1.55_{-0.48}^{+0.60}) \times 10^{-11}$		
$\text{BR}(K_L \rightarrow \pi^0 \mu\mu)(+)$	$(1.38_{-0.25}^{+0.27}) \times 10^{-11}$	$< 38 \times 10^{-11}$ @90% CL	current
$\text{BR}(K_L \rightarrow \pi^0 \mu\mu)(-)$	$(0.94_{-0.20}^{+0.21}) \times 10^{-11}$		

Numerical values used in the projections

Observable	SM prediction	Experimental results	NA62 final	HIKE Phase 2	HIKE P2 + KOTO-II
$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6_{-3.5}^{+4.0} \pm 0.9) \times 10^{-11}$	20%	5%	5%
$\text{BR}(K_L^0 \rightarrow \pi^0 \nu \nu)$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 3.0 \times 10^{-9}$ @90% CL	current	current	20%
$\text{LFUV}(a_+^{\mu\mu} - a_+^{ee})$	0	-0.014 ± 0.016	current	± 0.007	± 0.007
$\text{BR}(K_L \rightarrow \mu\mu) (+)$	$(6.82_{-0.29}^{+0.77}) \times 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$	current	1%	1%
$\text{BR}(K_L \rightarrow \mu\mu) (-)$	$(8.04_{-0.98}^{+1.47}) \times 10^{-9}$				
$\text{BR}(K_S \rightarrow \mu\mu)$	$(5.15 \pm 1.50) \times 10^{-12}$	$< 2.1(2.4) \times 10^{-10}$ @90(95)% CL	current	current	current
$\text{BR}(K_L \rightarrow \pi^0 ee)(+)$	$(3.46_{-0.80}^{+0.92}) \times 10^{-11}$	$< 28 \times 10^{-11}$ @90% CL	current	20%	20%
$\text{BR}(K_L \rightarrow \pi^0 ee)(-)$	$(1.55_{-0.48}^{+0.60}) \times 10^{-11}$				
$\text{BR}(K_L \rightarrow \pi^0 \mu\mu)(+)$	$(1.38_{-0.25}^{+0.27}) \times 10^{-11}$	$< 38 \times 10^{-11}$ @90% CL	current	20%	20%
$\text{BR}(K_L \rightarrow \pi^0 \mu\mu)(-)$	$(0.94_{-0.20}^{+0.21}) \times 10^{-11}$				

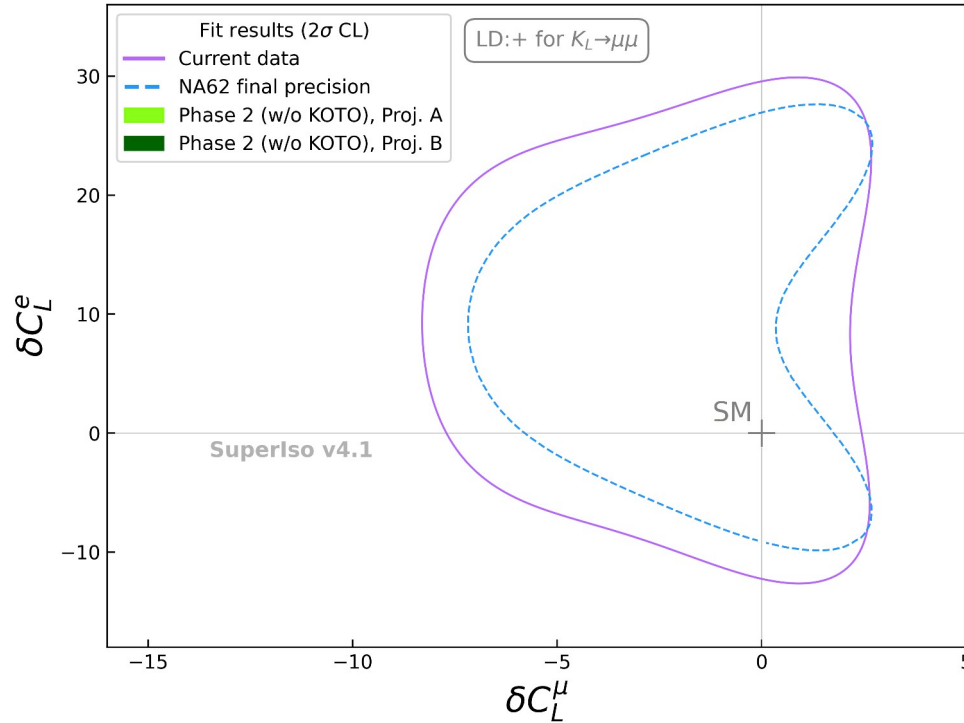
Projection A

Observables already measured are kept, others assumed to be match SM, all with target precision of HIKE (+ KOTO-II)

Projection B

All measurements give current best-fit point with target precision of HIKE (+ KOTO-II)

NA62 final precision



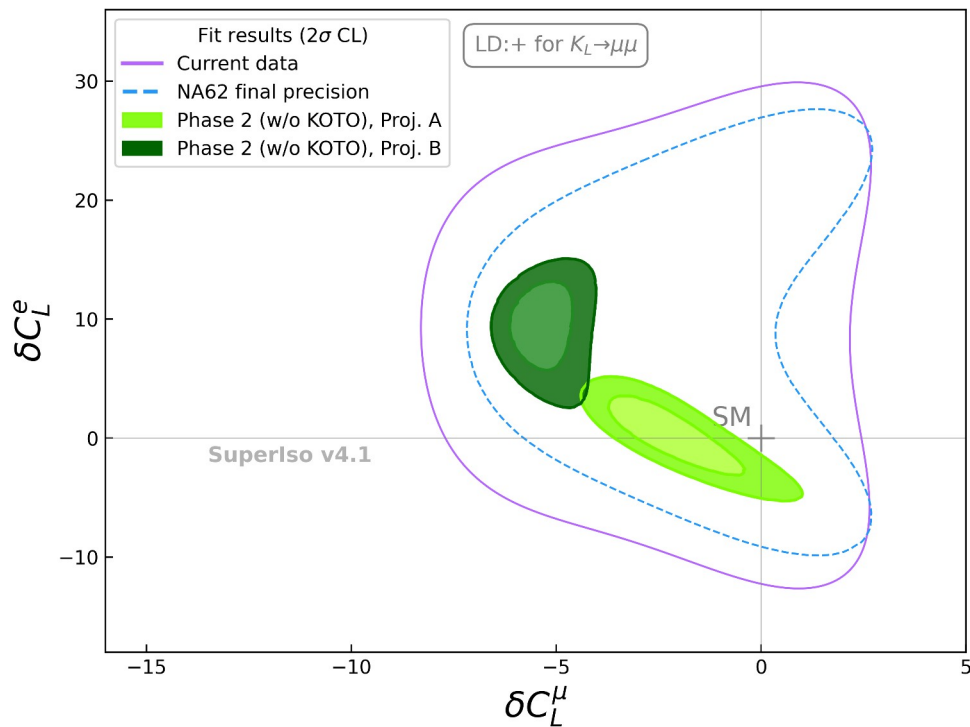
Projection A

Observables already measured are kept, others assumed to be match SM, all with target precision of HIKE (+ KOTO-II)

Projection B

All measurements give current best-fit point with target precision of HIKE (+ KOTO-II)

HIKE without KOTO



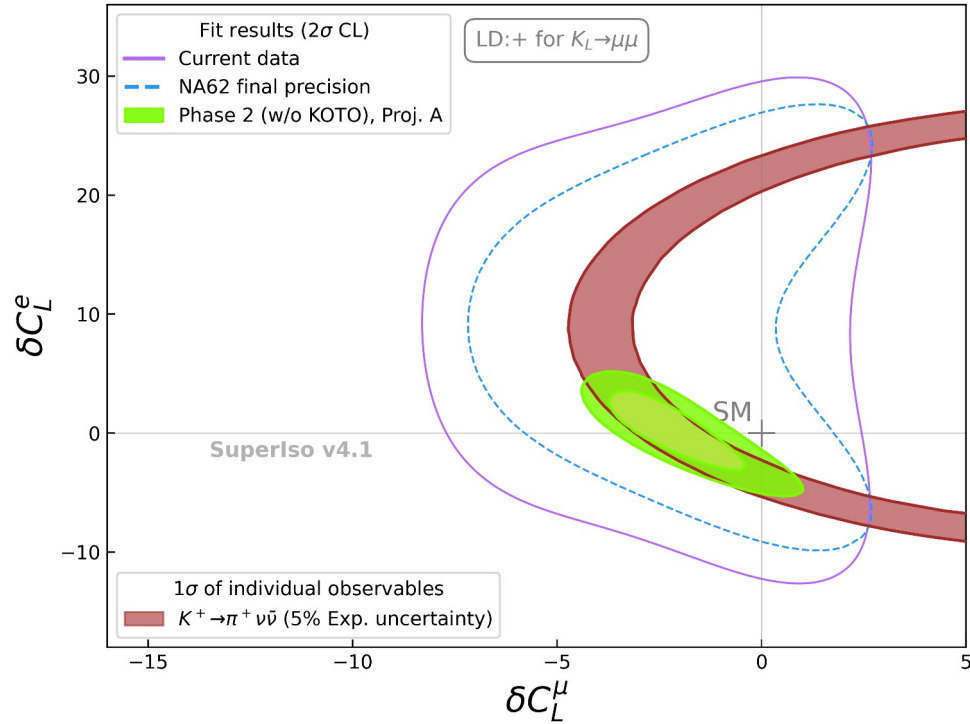
Projection A

Observables already measured are kept, others assumed to be match SM, all with target precision of HIKE (+ KOTO-II)

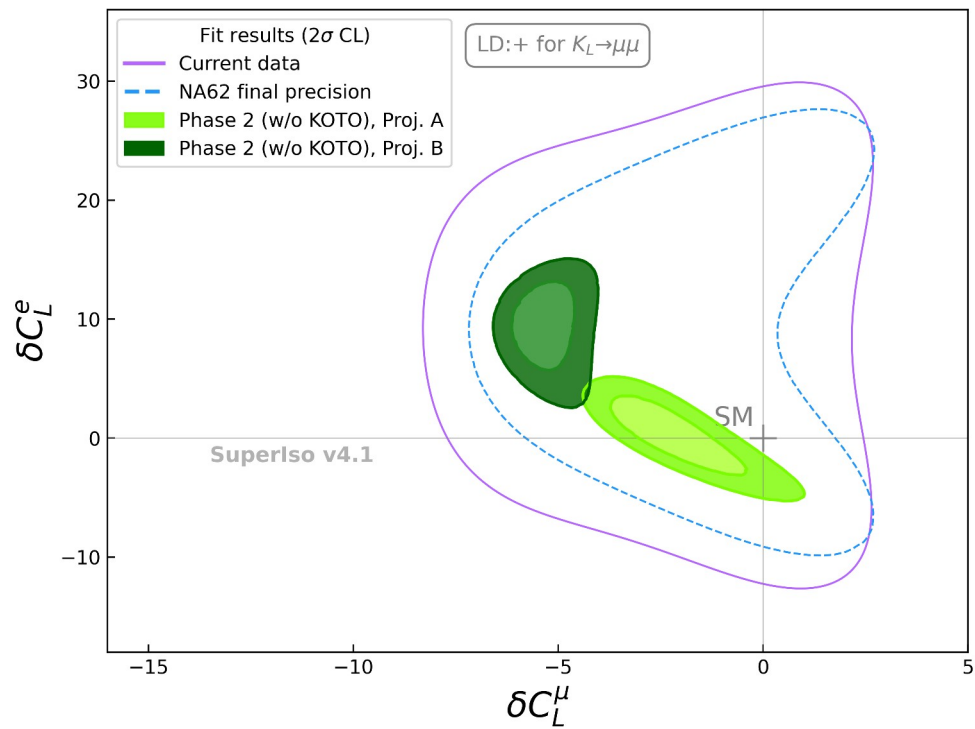
Projection B

All measurements give current best-fit point with target precision of HIKE (+ KOTO-II)

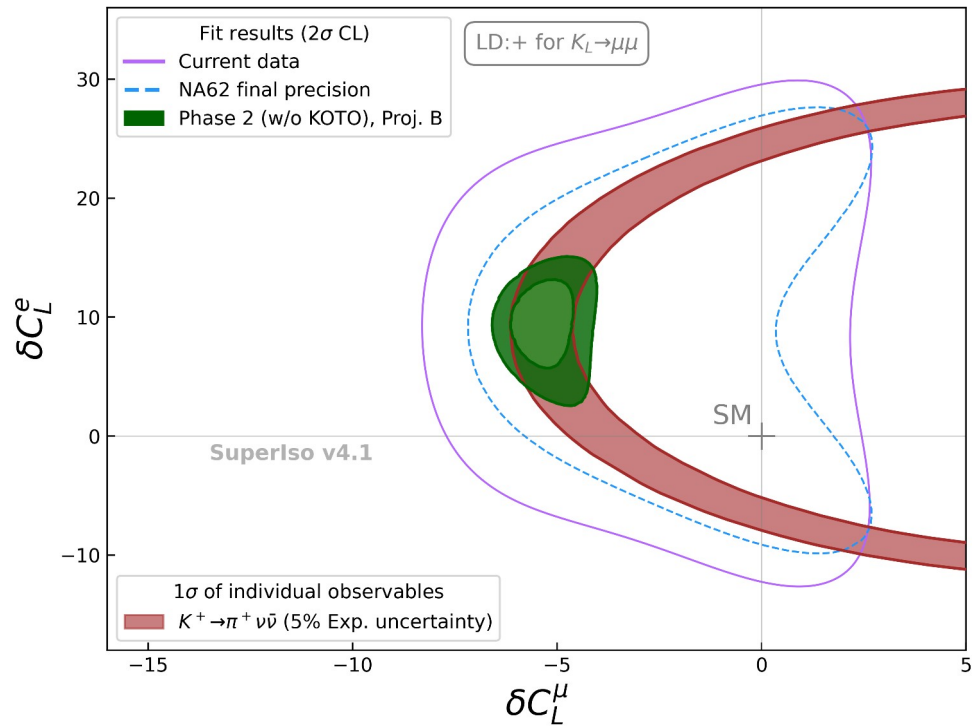
HIKE without KOTO – impact of $K^+ \rightarrow \pi^+ \nu \nu$



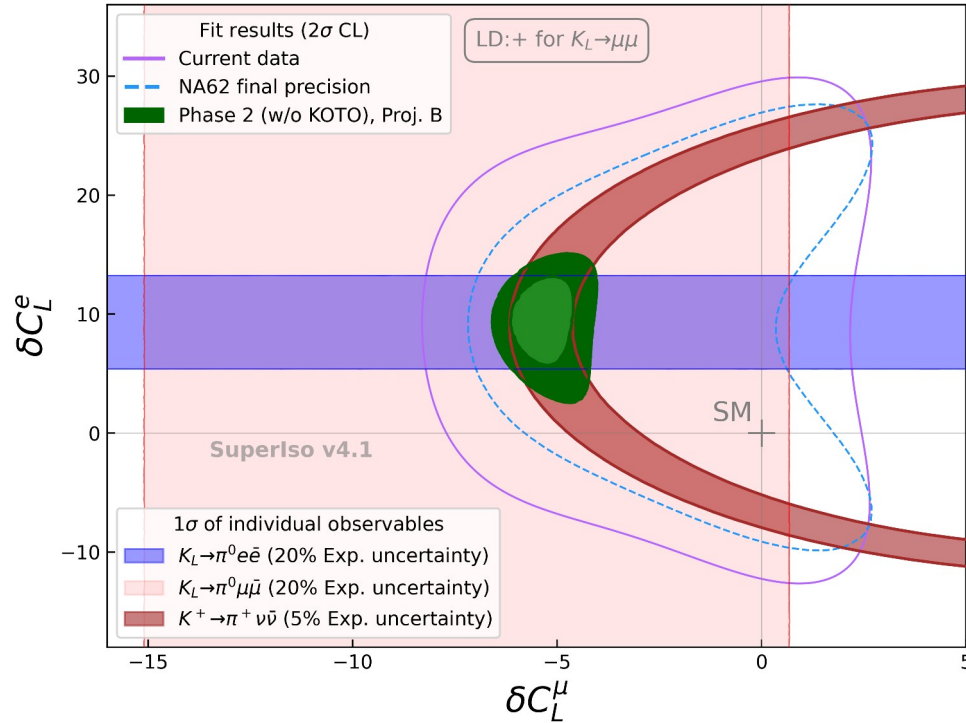
HIKE without KOTO



HIKE without KOTO – impact of $K^+ \rightarrow \pi^+ \nu \nu$

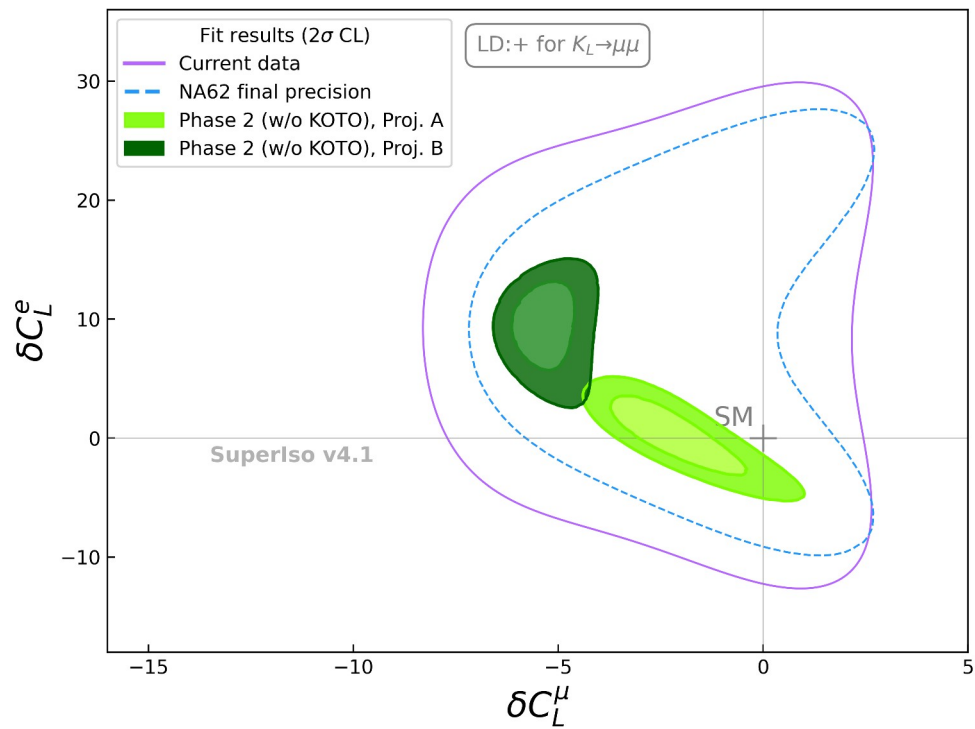


HIKE without KOTO – impact of $K^+ \rightarrow \pi^+ \nu \nu$ and $K_L \rightarrow \pi^0 \ell \ell$

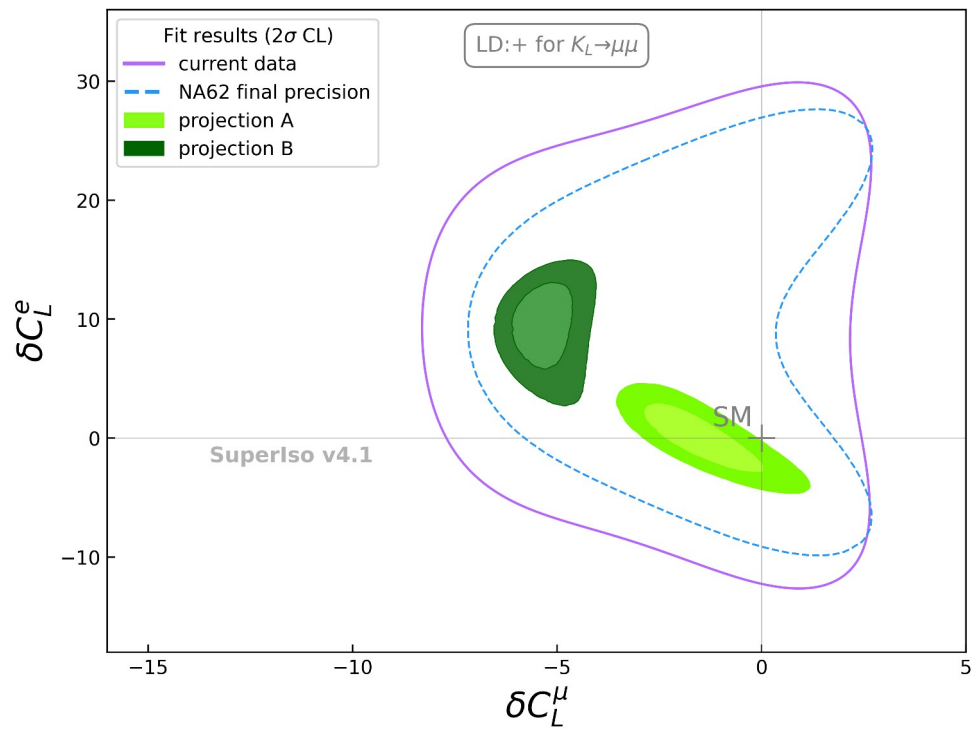


Main constraining observables $\text{BR}(K^+ \rightarrow \pi^+ \nu \nu)$ followed by $\text{BR}(K_L \rightarrow \pi^0 e^+ e^-)$

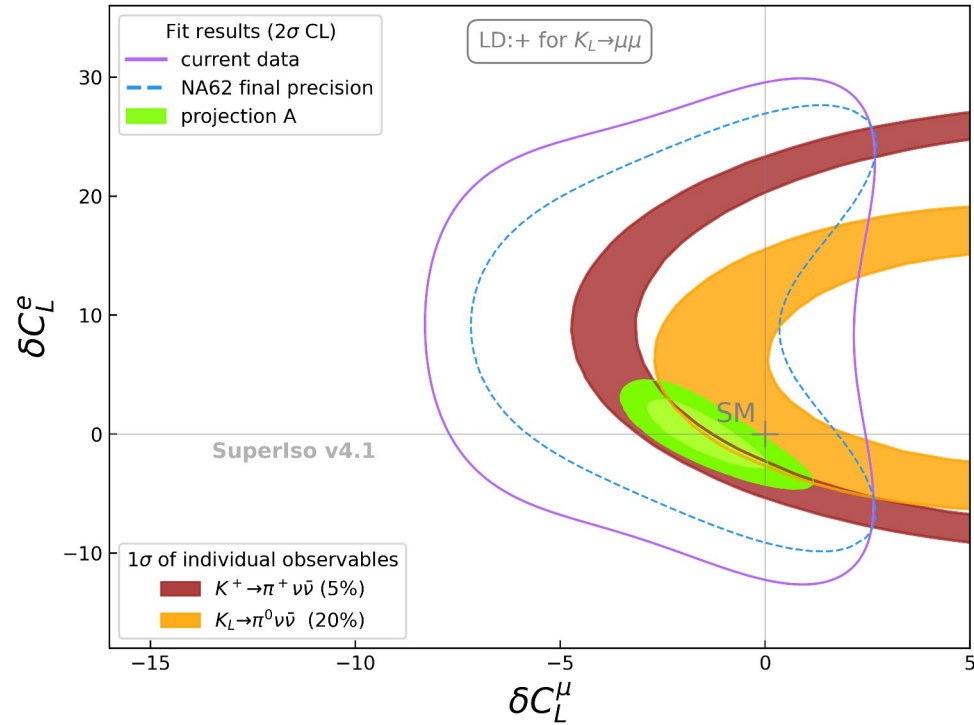
HIKE without KOTO



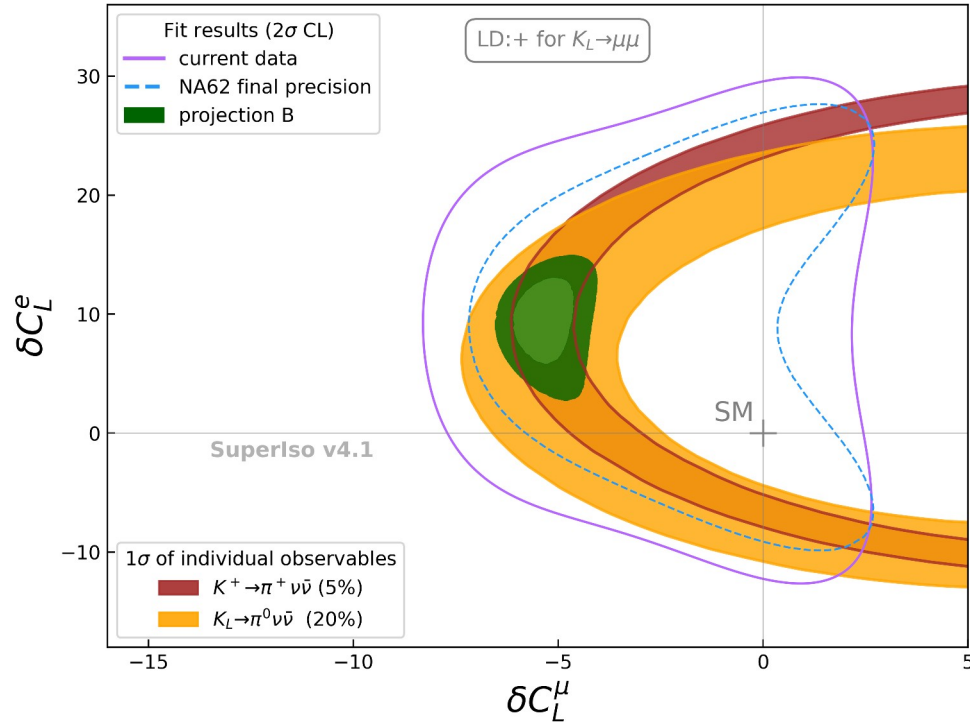
HIKE with KOTO



HIKE with KOTO – impact of $K^+ \rightarrow \pi^+ \nu \nu$ and $K_L \rightarrow \pi^0 \nu \nu$



HIKE with KOTO – impact of $K^+ \rightarrow \pi^+ \nu \nu$ and $K_L \rightarrow \pi^0 \nu \nu$



Summary

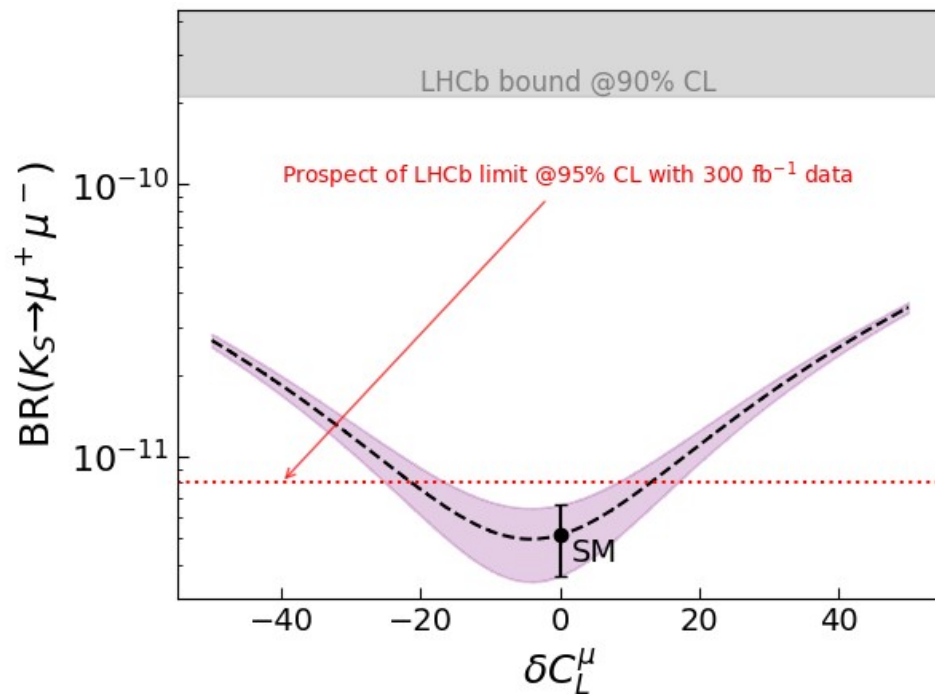
- Rare kaon decays offer interesting information on short distance physics, even those which are long-distance dominated
- Hike (Phase 1 and 2) offers excellent sensitivity to new physics
- KOTO-II measurement of $K_L \rightarrow \pi^0 \nu\nu$ will improve this sensitivity
- Measurement of $K_L \rightarrow \pi^0 \ell\ell$, especially in the electron sector gives a very effective probe of new physics

Backup

$$\text{BR}(K_S \rightarrow \mu\bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 |N_S^{\text{LD}}|^2 + \left(\frac{2m_\mu G_F \alpha_e}{m_K \sqrt{2}\pi} \right)^2 \text{Im}^2 \left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell \right] \right\}$$

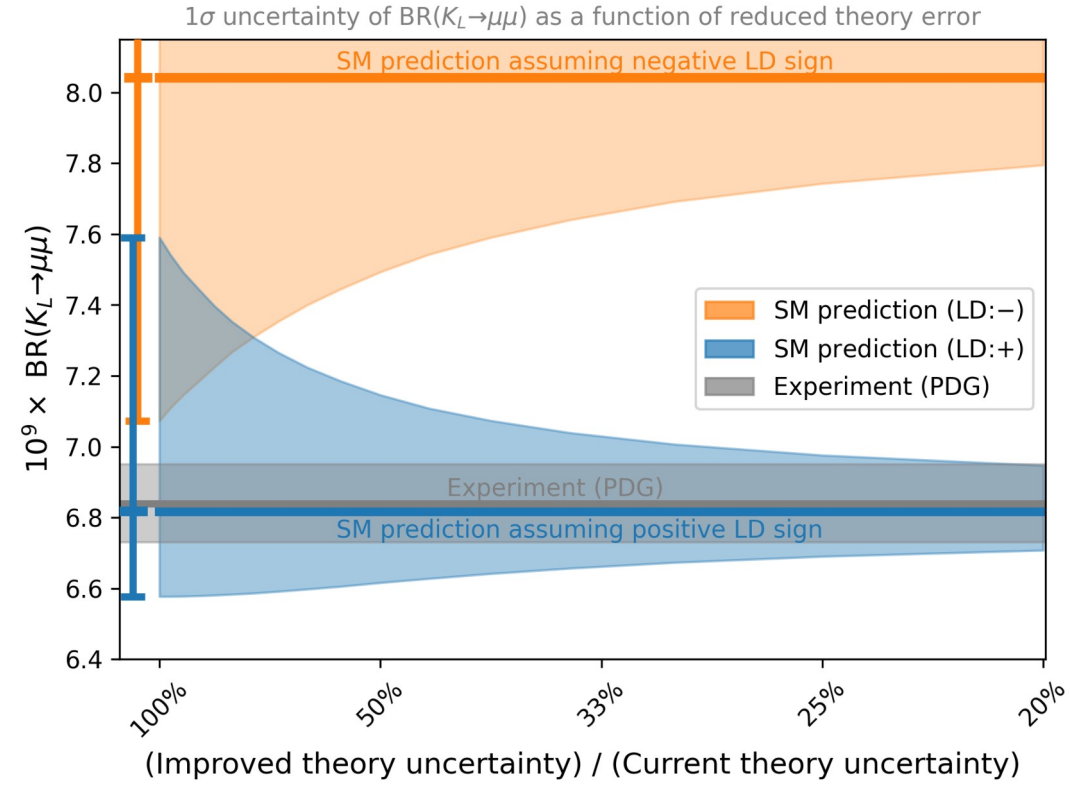
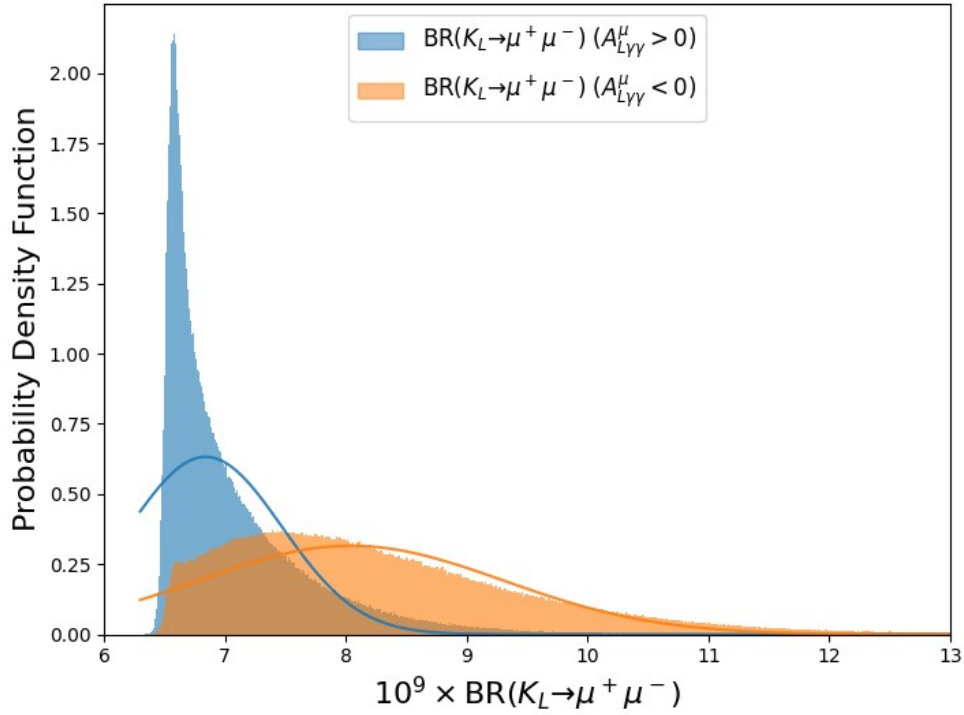
$$N_S^{\text{LD}} = (-2.65 + 1.14i) \times 10^{-11} (\text{GeV})^{-2}$$

$$\text{BR}(K_S \rightarrow \mu\bar{\mu})^{\text{SM}} = (5.15 \pm 1.50) \times 10^{-12}$$



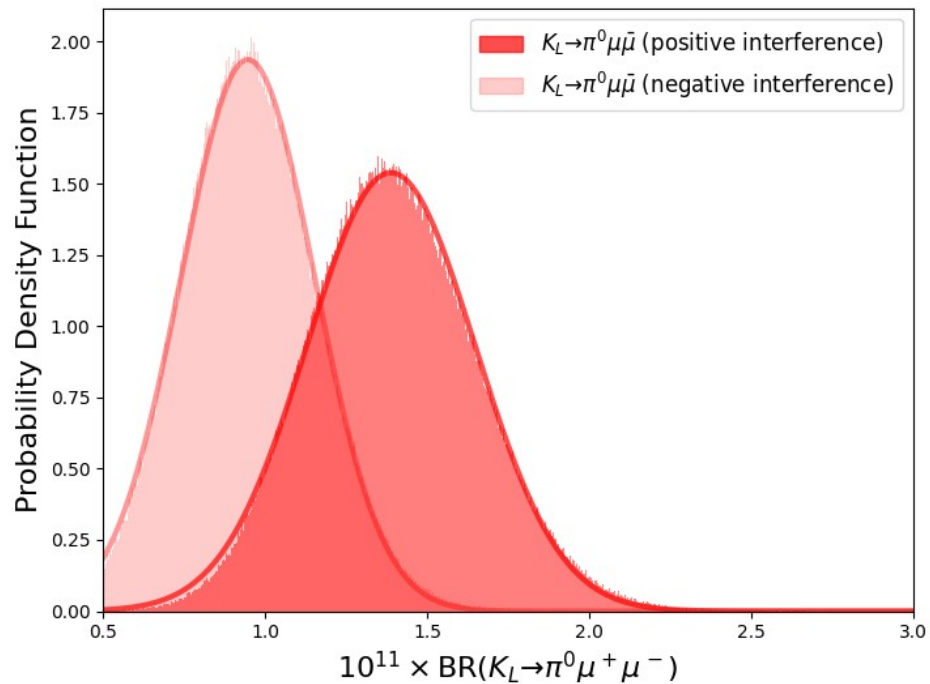
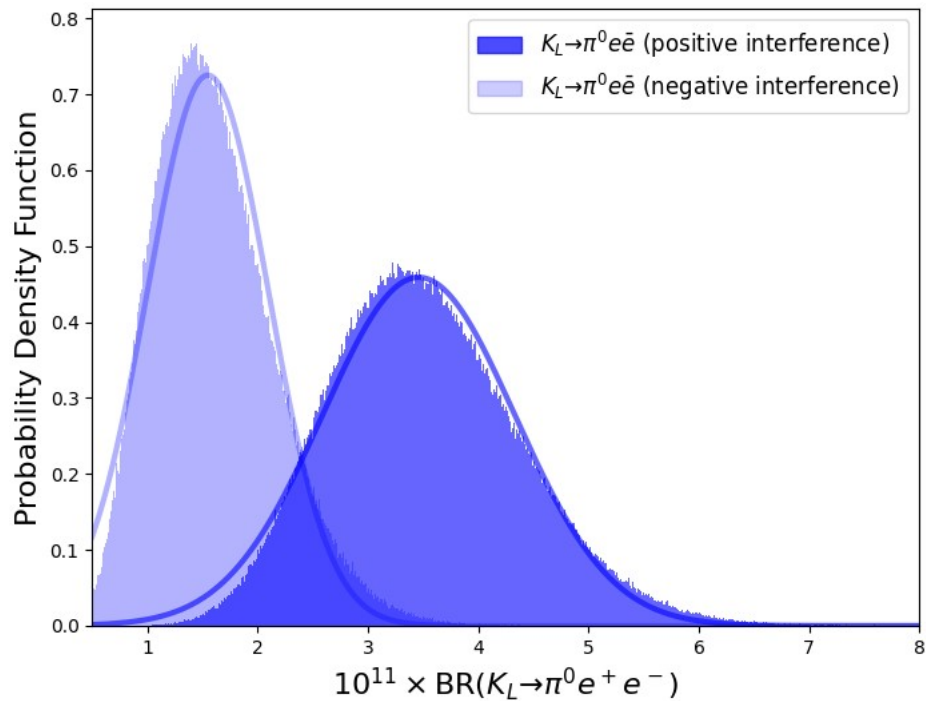
Backup

Asymmetric theoretical uncertainty of $K_L \rightarrow \mu\mu$



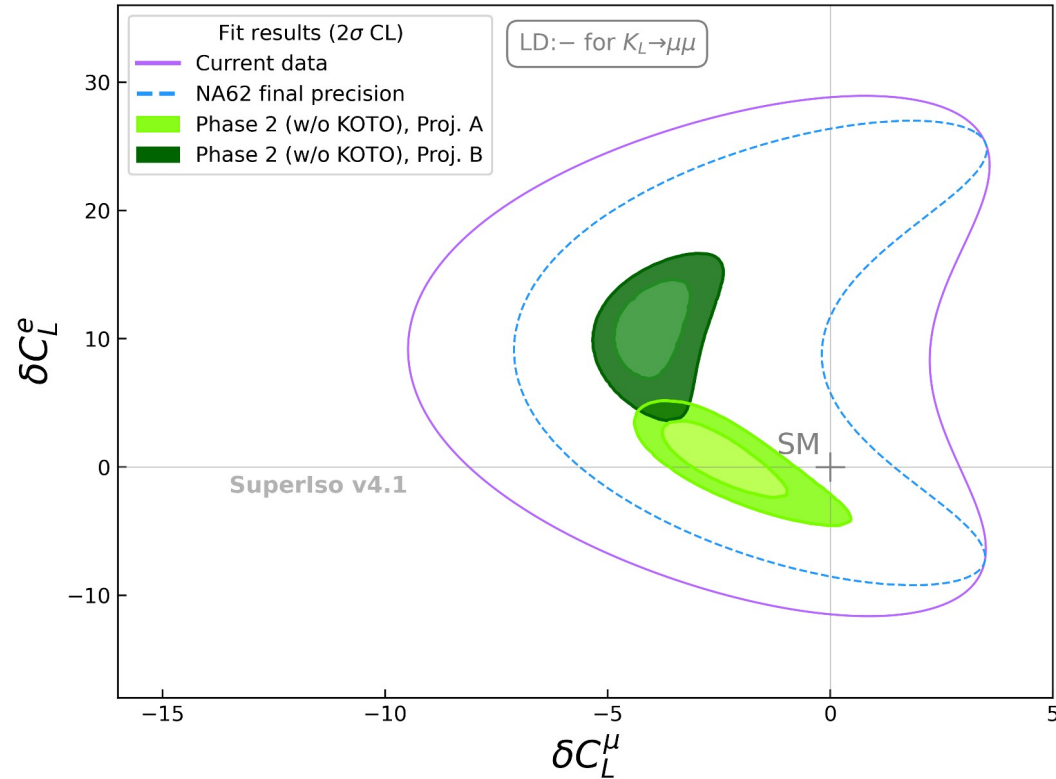
Backup

Asymmetric theoretical uncertainty of $K_L \rightarrow \pi^0 \ell \ell$



Backup

Fit (with SuperIso public program) for negative LD contributions to $K_L \rightarrow \mu\mu$



Backup

Fit (with SuperIso public program) for negative LD contributions to $K_L \rightarrow \mu\mu$

