

# Three body systems in Strong Gravity Regime

Consiglio della sezione INFN di Perugia



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● *Tidal deformations of a binary system induced by an external Kerr black hole*

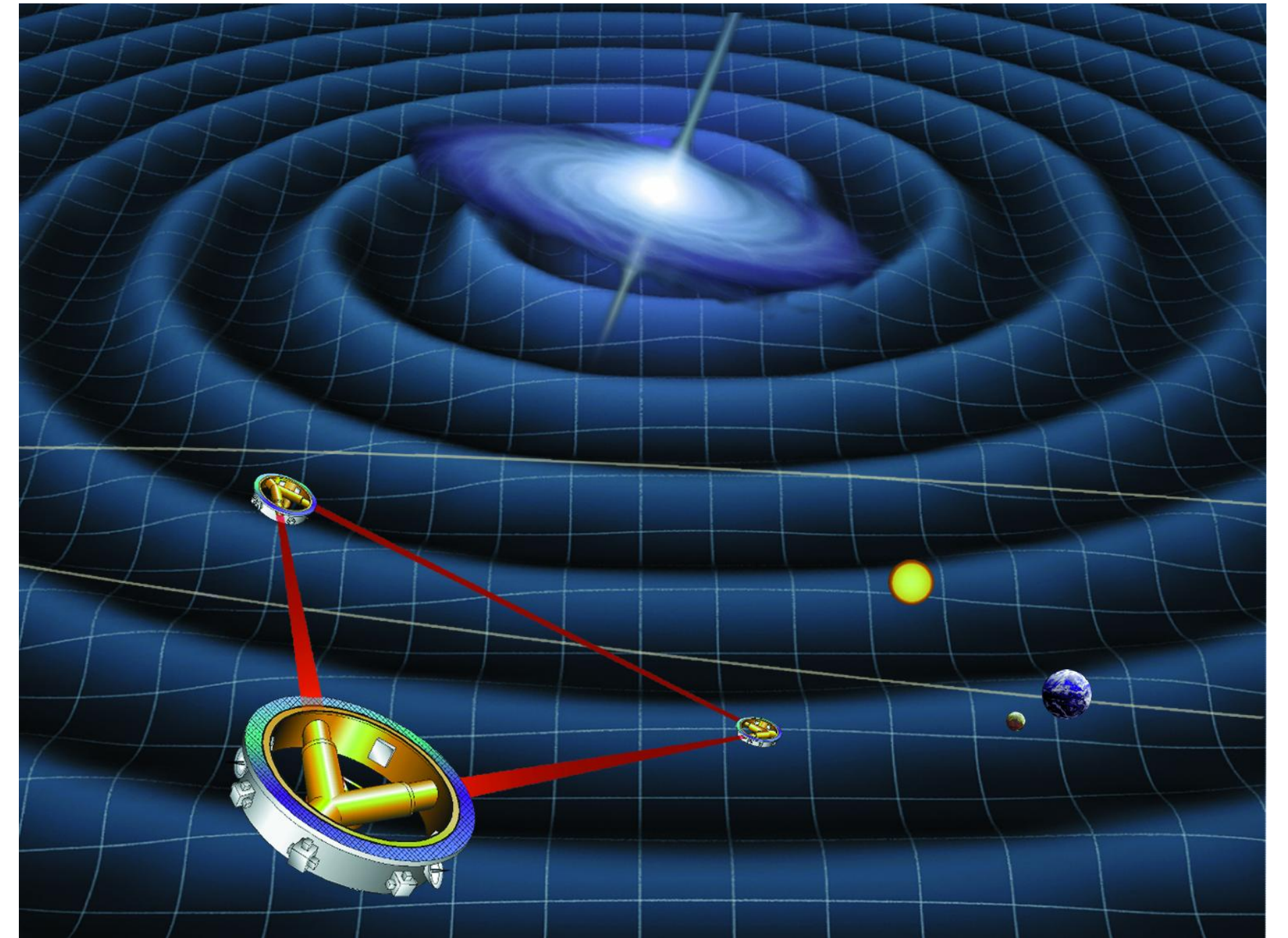
<https://doi.org/10.1103/PhysRevD.107.084011> - [F. Camilloni, G. Grignani, T. Harmark, R. Oliveri, M. Orselli, D. Pica]

● *Binary mergers in strong gravity background of Kerr black hole*

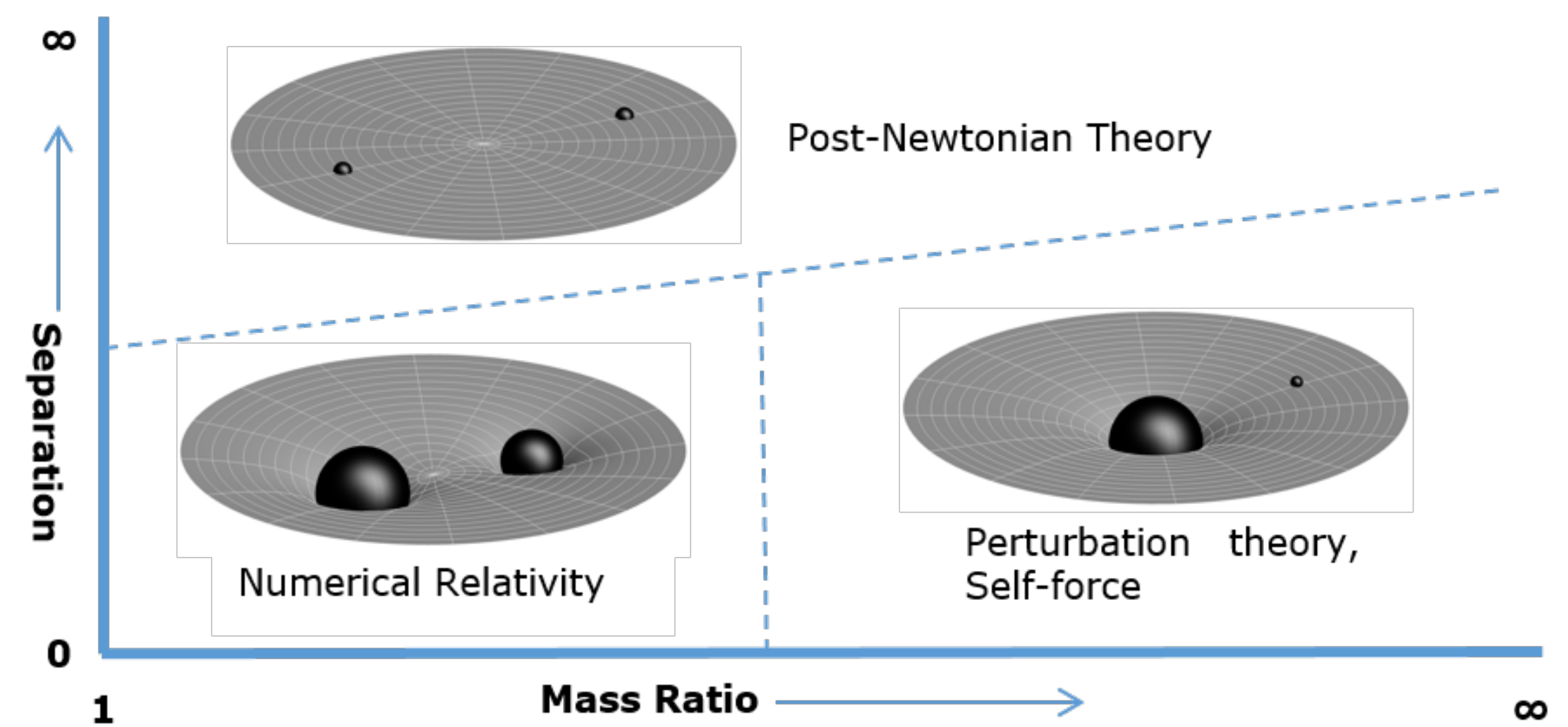
In preparation - [F. Camilloni, G. Grignani, T. Harmark, M. Orselli, D. Pica]

# Why Three Body Systems?

- ▶ **Abundance of sources:** the density of stars in galactic nuclei can be  $10^6$  times the one in our solar system neighborhood ( $0.1 \text{ stars/pc}^3$ )
- ▶ Primary targets for **future GWs observations**
  - ▶  $\omega \sim 10^{-2} - 10 \text{ kHz} \rightarrow \text{LIGO/VIRGO}$
  - ▶  $\omega \sim 0.1 - 100 \text{ mHz} \rightarrow \text{LISA}$
- ▶ **Kozai-Lidov Mechanism**  $\rightarrow$  very short merger timescale!
  - ▶ A circular binary of two  $10M_{\odot}$  black holes orbiting a SMBH, with a separation of 1AU it would take  $10^{14}$  years to merge.
  - ▶ A binary of two  $10M_{\odot}$  black holes with eccentricity  $e = 0.9995$  orbiting a SMBH, with a separation of 1AU it would take  $4 \times 10^3$  years to merge.



# Strong Gravity Regime

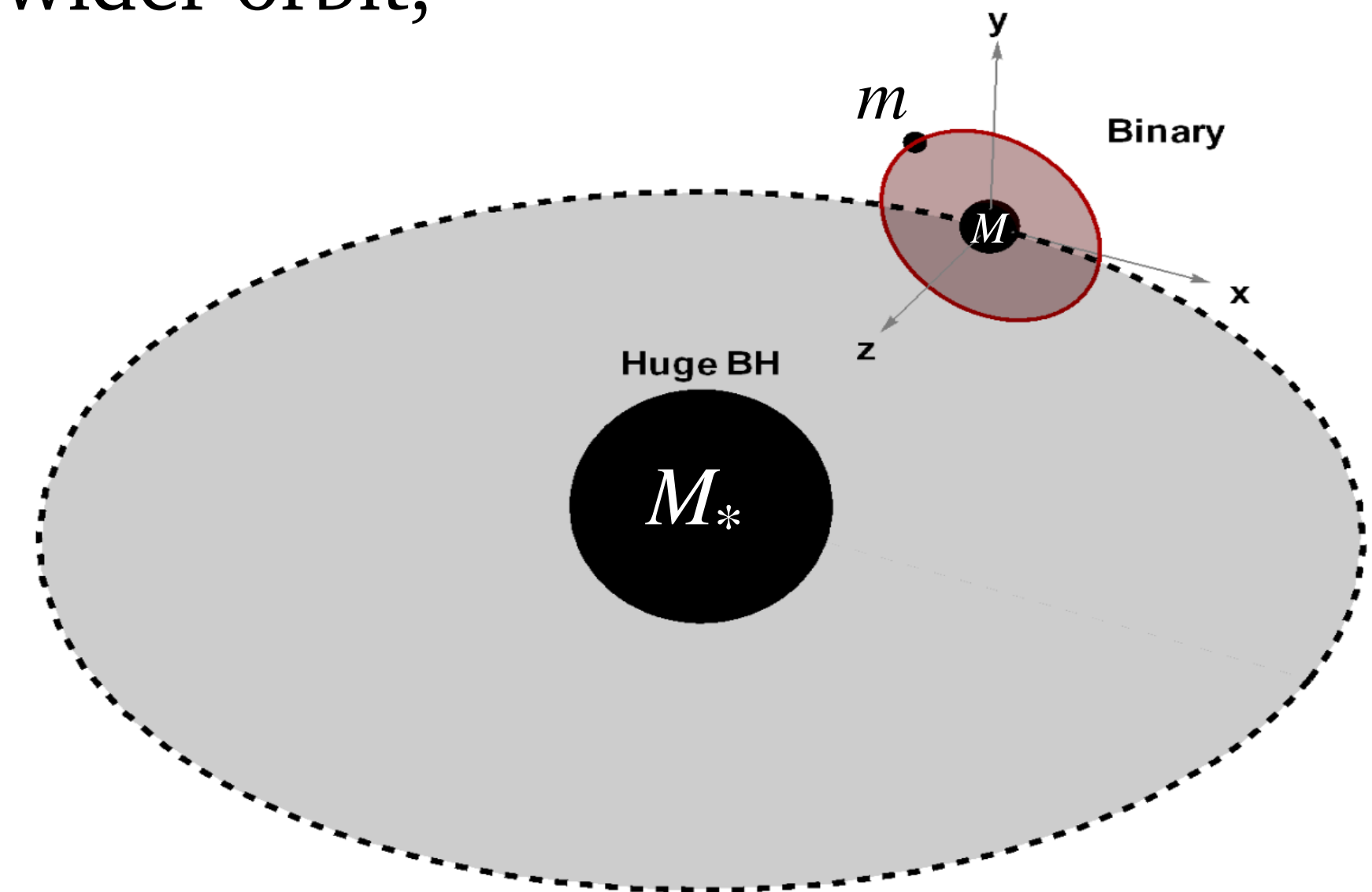


- ▶ **Strong Gravity:** when GR effects become relevant
- ▶ **Small-tide approximation:** when the characteristic scale of the binary is much smaller than the radius of the curvature generated by the third body evaluated in the position of the binary system
- ▶ *Weak-field approximation* → the orbital velocity of the binary system must be small → binary system far away from the perturber.
- ▶ *Small-hole approximation* → dimension of the binary system is assumed to be much smaller than the external mass → the binary system can be close to the source of the tidal fields.
- ▶ Three body systems are usually studied as point particles in the weak-field approximation

# Tidal deformation of a binary system induced by a Kerr black hole

[F. Camilloni, G. Grignani, T. Harmark, R. Oliveri, M. Orselli, D. Pica]

- ▶ **Hierarchical three body system:** a binary system under the gravitational interaction of another massive astrophysical object.
- ▶ Hierarchical in scale, a tight inner binary orbited by a tertiary on a wider orbit, forming the outer binary:  $m \ll M \ll M_*$
- ▶  $m$  is a test particle,  $M$  is a Schwarzschild black hole,  $M_*$  is a Kerr black hole
- ▶ In the limit  $m \rightarrow 0$ , extreme mass ratio (EMR)
- ▶  $\mu = \frac{m}{M} \ll 1 \Rightarrow$  Natural perturbative approach to include all possible GR effects!



# Tidal deformation of a binary system induced by a Kerr black hole

- ▶ **Two different length scales:** the mass of the intermediate black hole  $M$  and the radius of the curvature generated by the third body evaluated in the position of the binary system  $\mathcal{R}$
- ▶ *Small-hole approximation* → black hole mass is assumed to be much smaller than the external mass,  $M \ll M_*$  → the binary system can be close to the source of the tidal fields.
- ▶ *Small-hole approximation* → EMR could be on the Innermost Stable Circular Orbit (**ISCO**) of the Kerr black hole → “migration trap”!
- ▶ **Tidally deformed metric:**  $ds^2 \simeq - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + h_{\mu\nu} dx^\mu dx^\nu + \mathcal{O}(r^3/\mathcal{R}^3)$
- ▶ At leading order the Tidal Environment is specified by **Quadrupole Tidal Moments**
- ▶ We neglect the motion of the binary system around the SMBH

# Tidal deformation of a binary system induced by a Kerr black hole

▶ Tidal fields generated by the outer body deform the orbits of the unperturbed Schwarzschild metric

▶ **Secular dynamics** → integrate over one orbit of the test particle

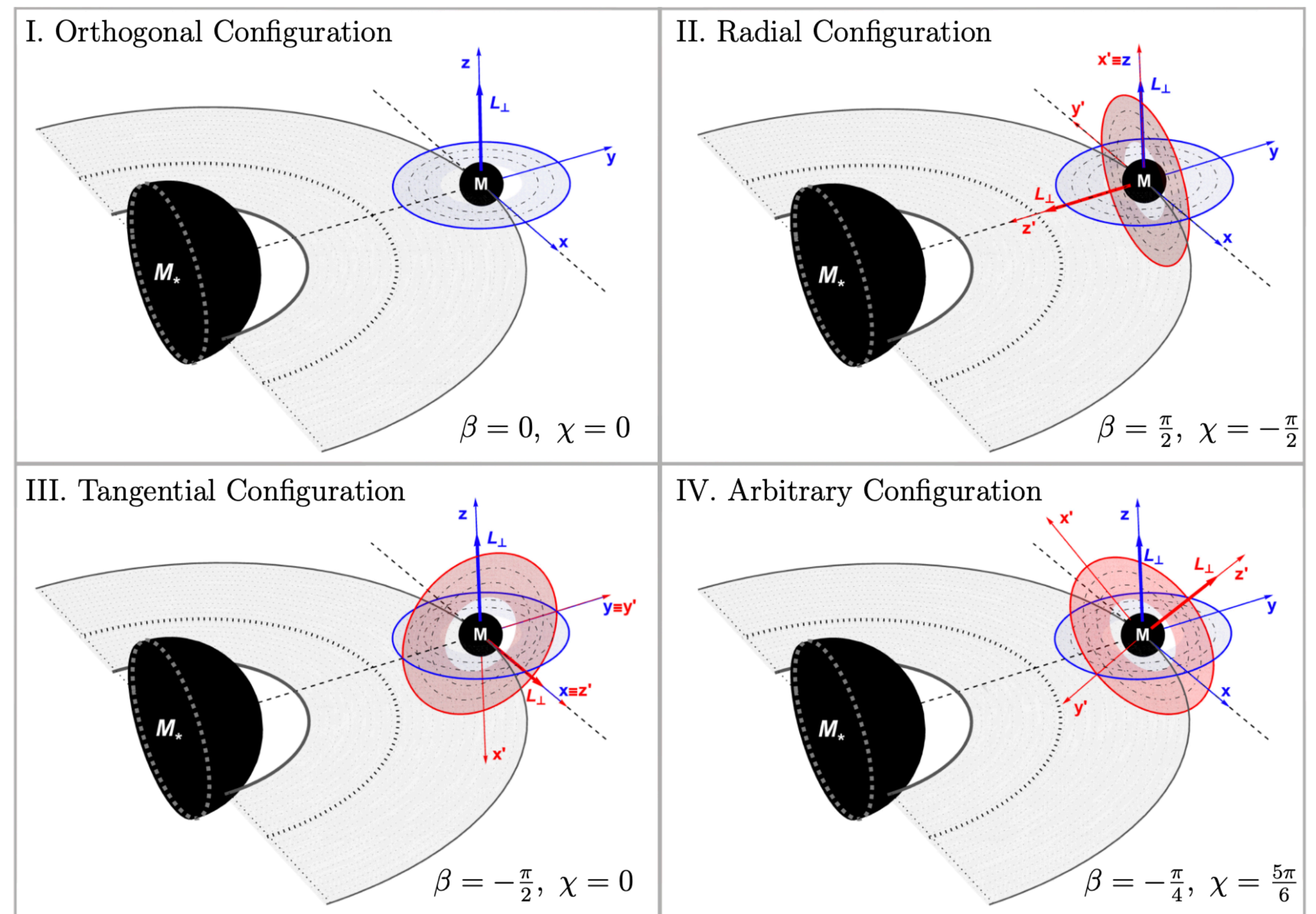
▶ **3 Euler angles** → All possible orientation for the binary system

▶ **ISCO shifts!**

$$\begin{aligned} r &= r_{ISCO} + \eta r_1, \\ E &= E_{ISCO} + \eta E_1, \\ L &= L_{ISCO} + \eta L_1, \\ \Omega &= \Omega_{ISCO} + \eta \Omega_1 \end{aligned}$$

$$\eta = -\frac{M^2}{2} \langle \mathcal{E}^q \rangle$$

▶ **Effective perturbative parameter  $\eta$**



# Binary mergers in strong gravity background of Kerr black hole

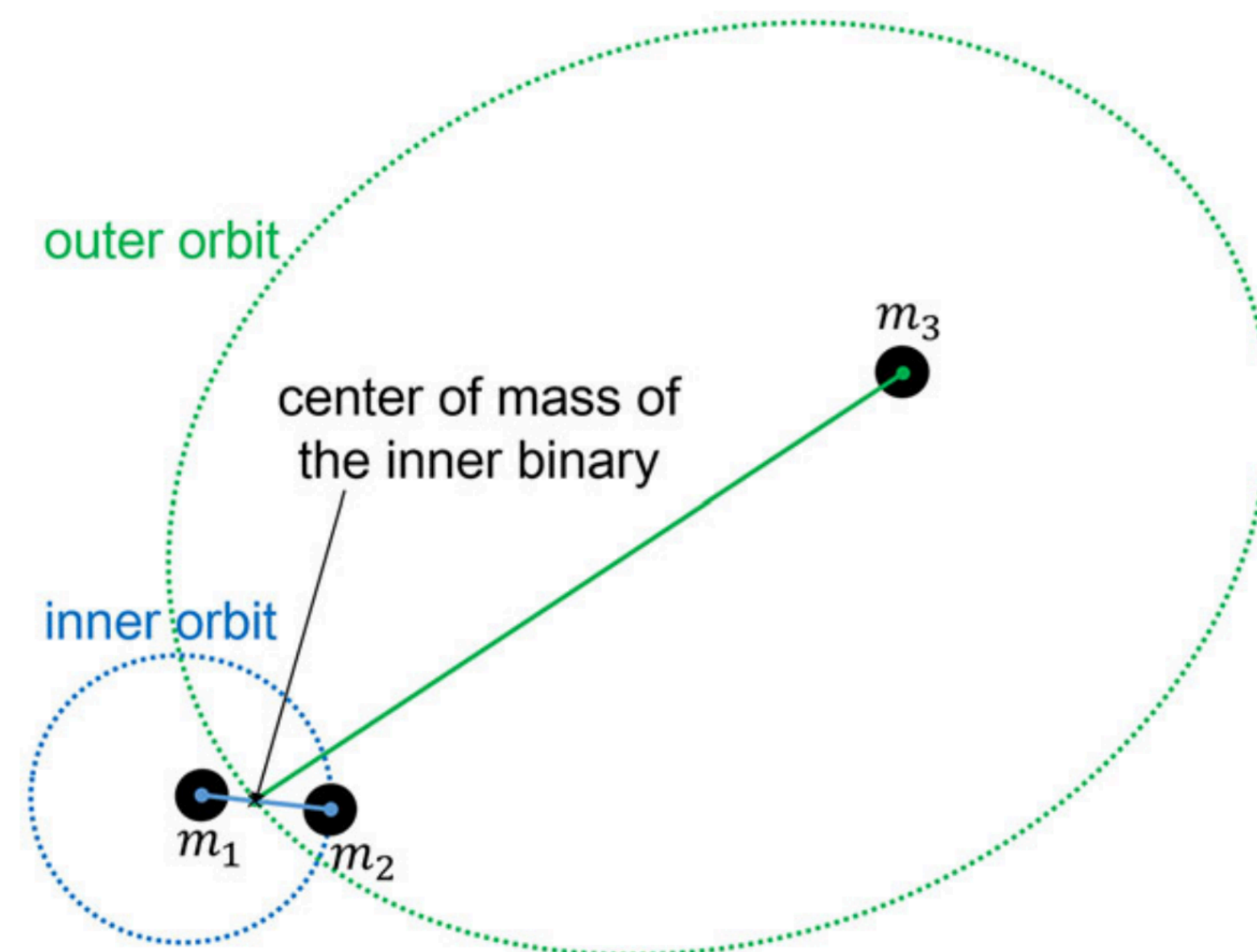
[F. Camilloni, G. Grignani, T. Harmark, M. Orselli, D. Pica]

- ▶ Binary system of two black holes  $m_1$  and  $m_2$  moving along a geodesic around a SM Kerr BH  $m_3$ 
  - ▶ *Inner orbit*
  - ▶ *Outer orbit*
- ▶  $a$  semi-major axis of the binary,  $\hat{r}$  distance between the binary and the SMBH

## ▶ Two approximations:

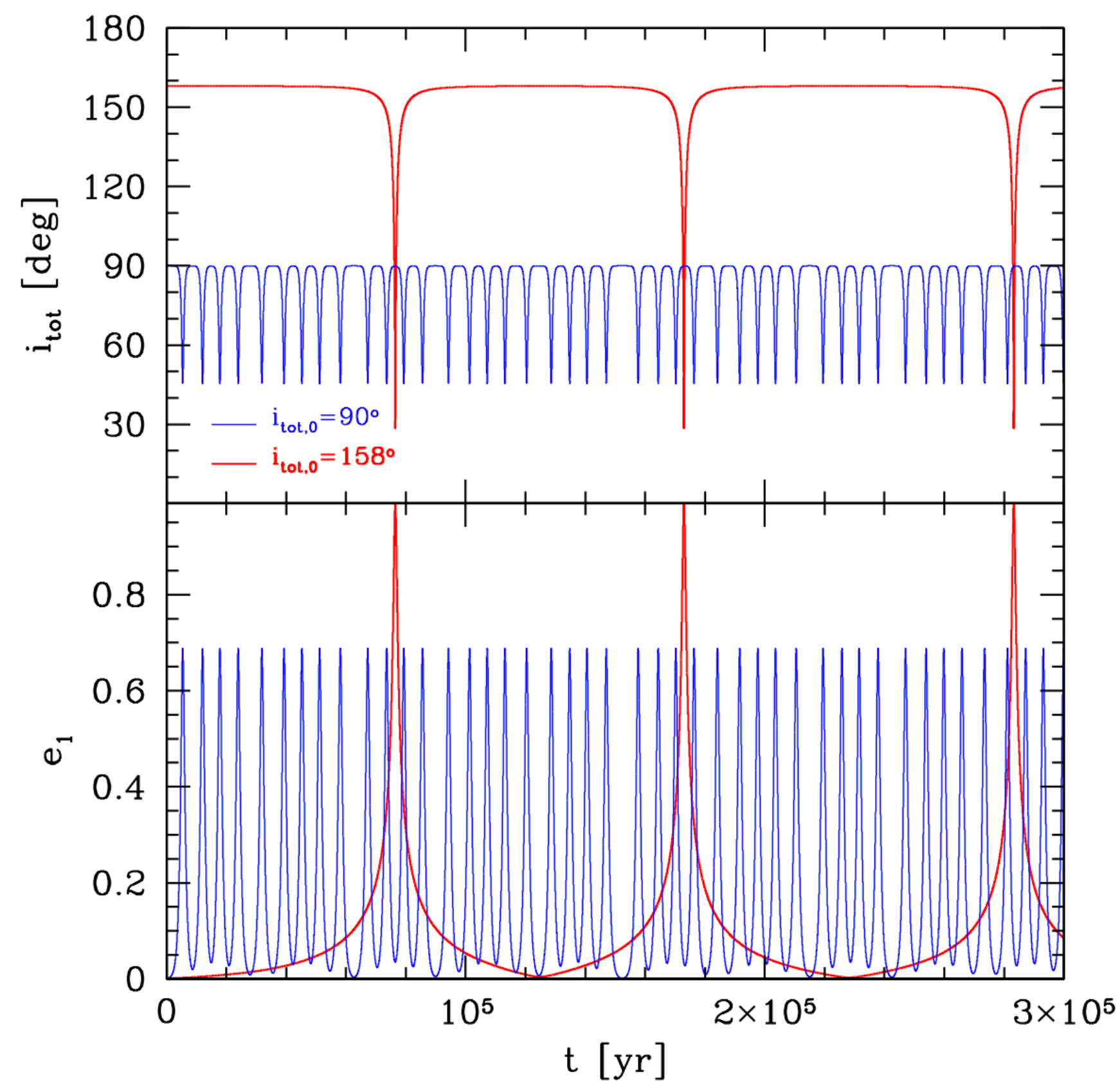
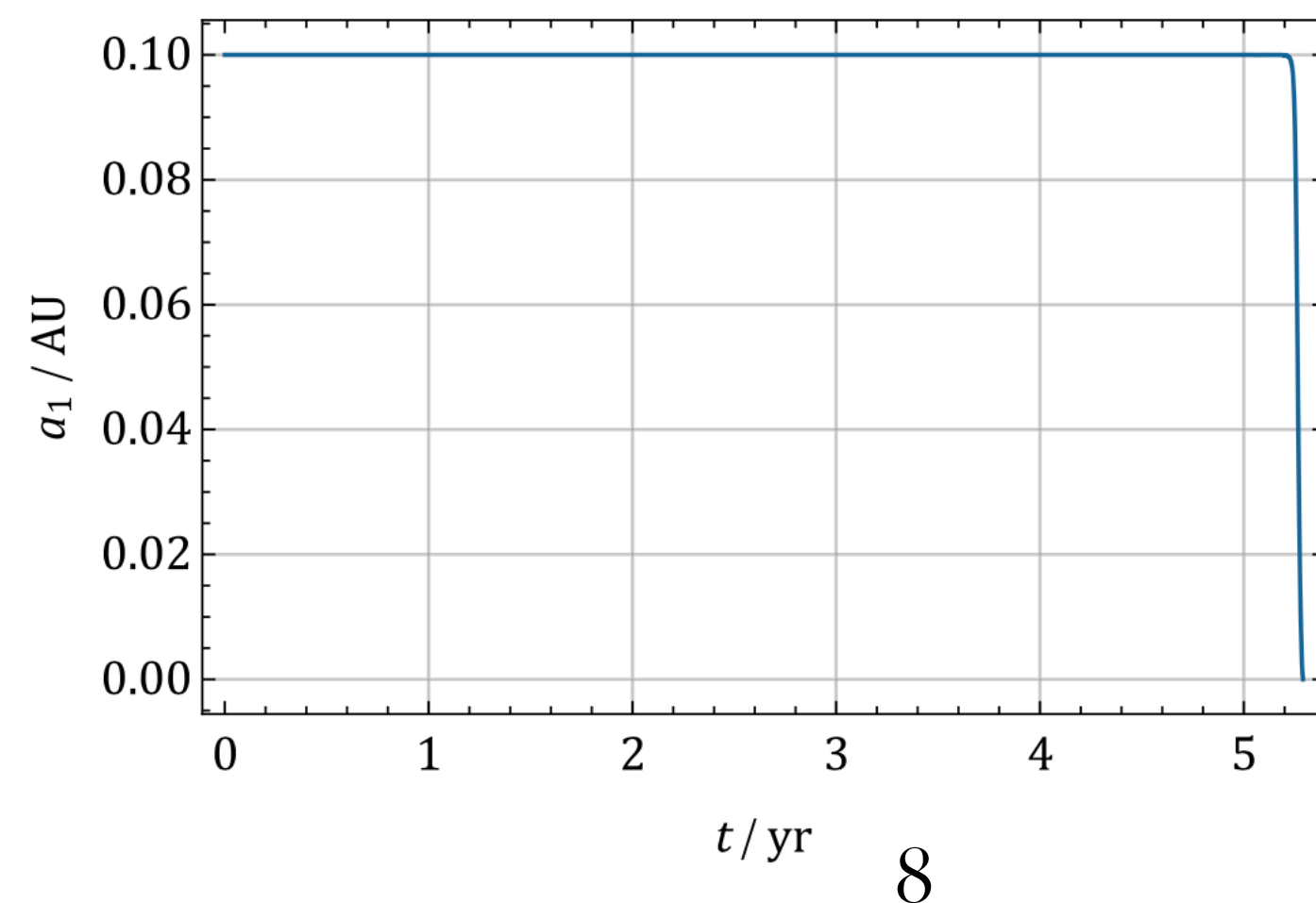
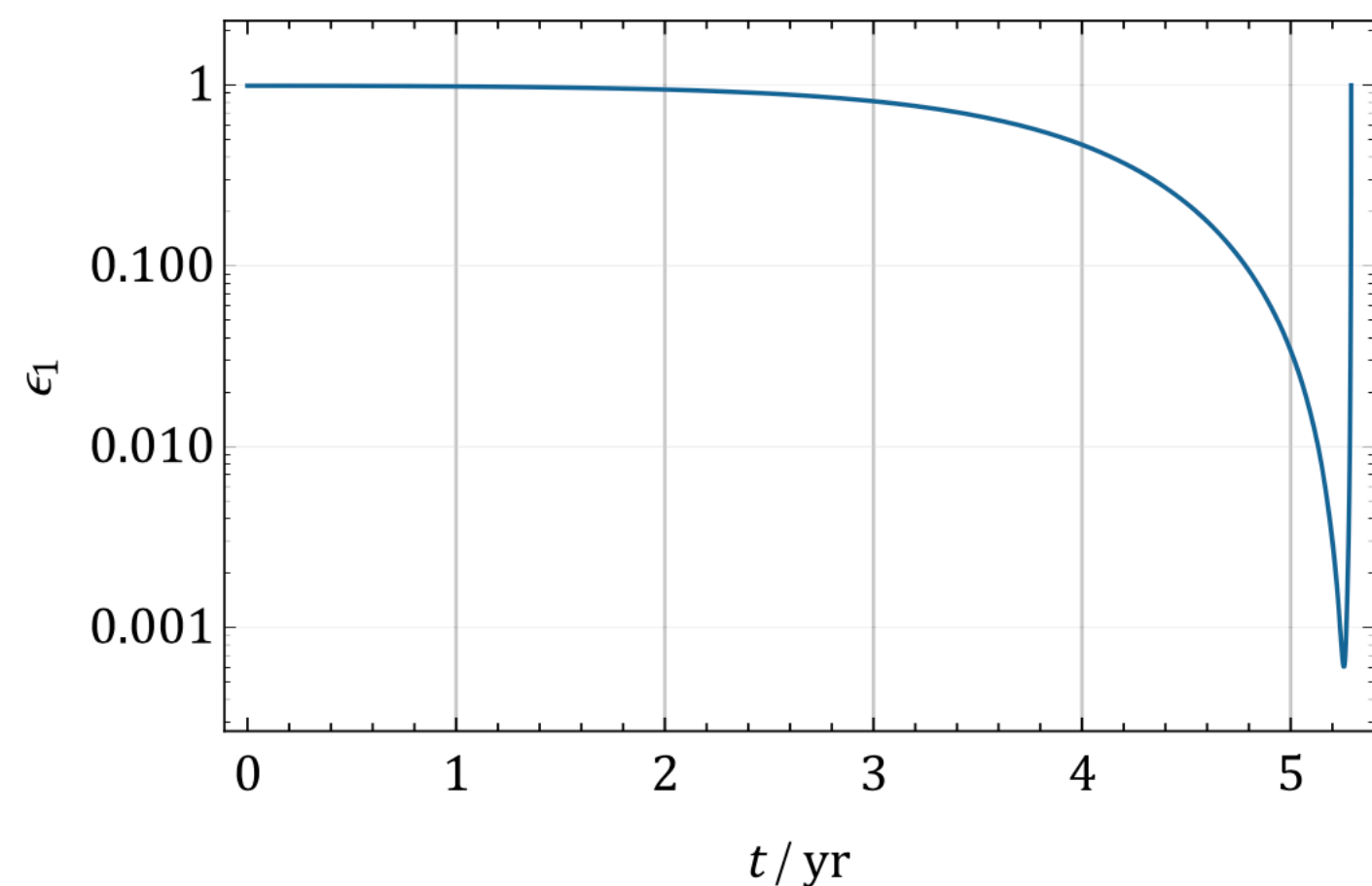
▶ *Tidal Limit*  $\rightarrow a \ll \frac{Gm_3}{c^2}$

▶ *Point Particle Limit*  $\rightarrow \frac{Gm_{1,2}}{c^2} \ll a$



# Kozai-Lidov Mechanism - Point Particle Approximation

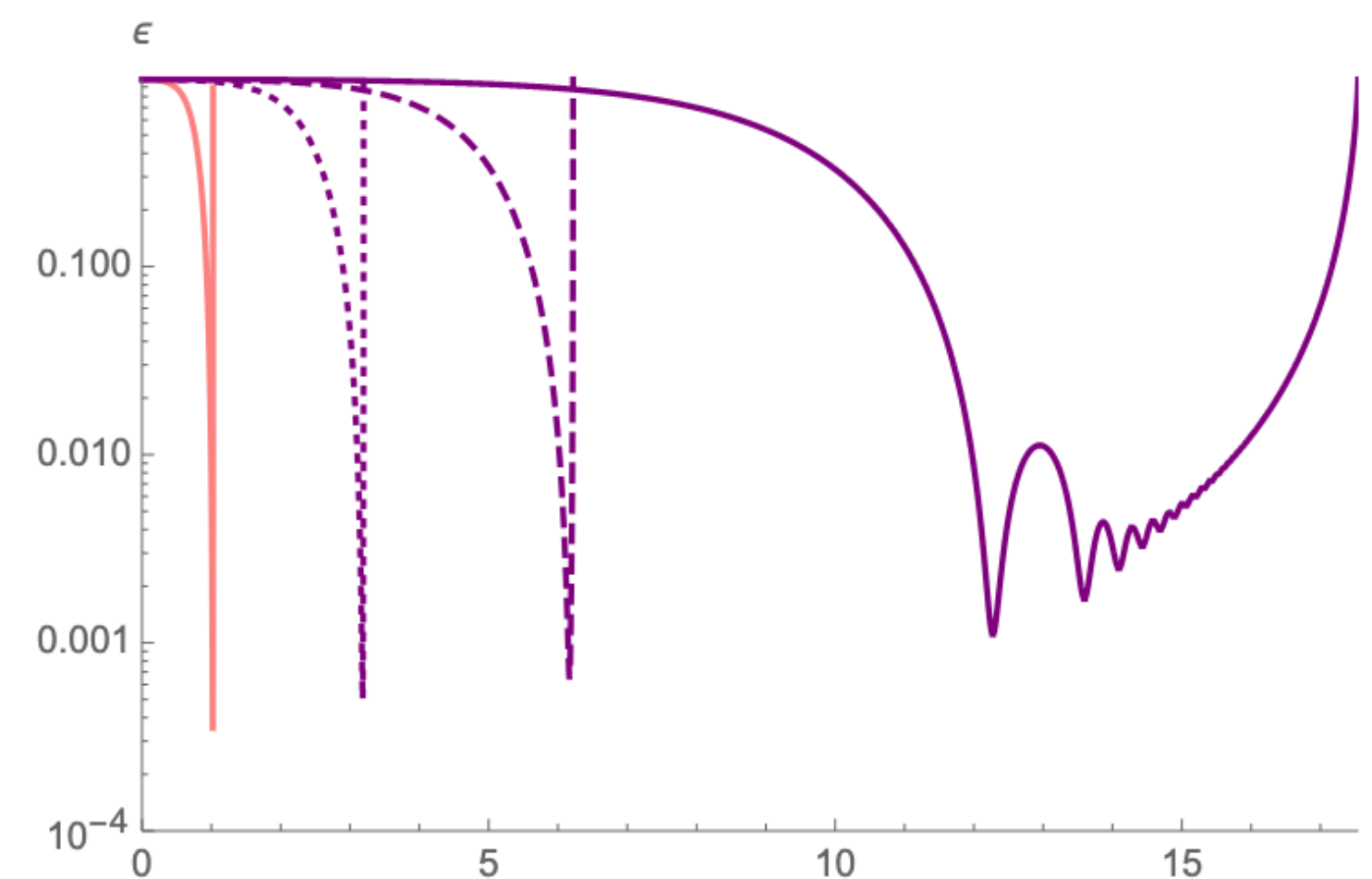
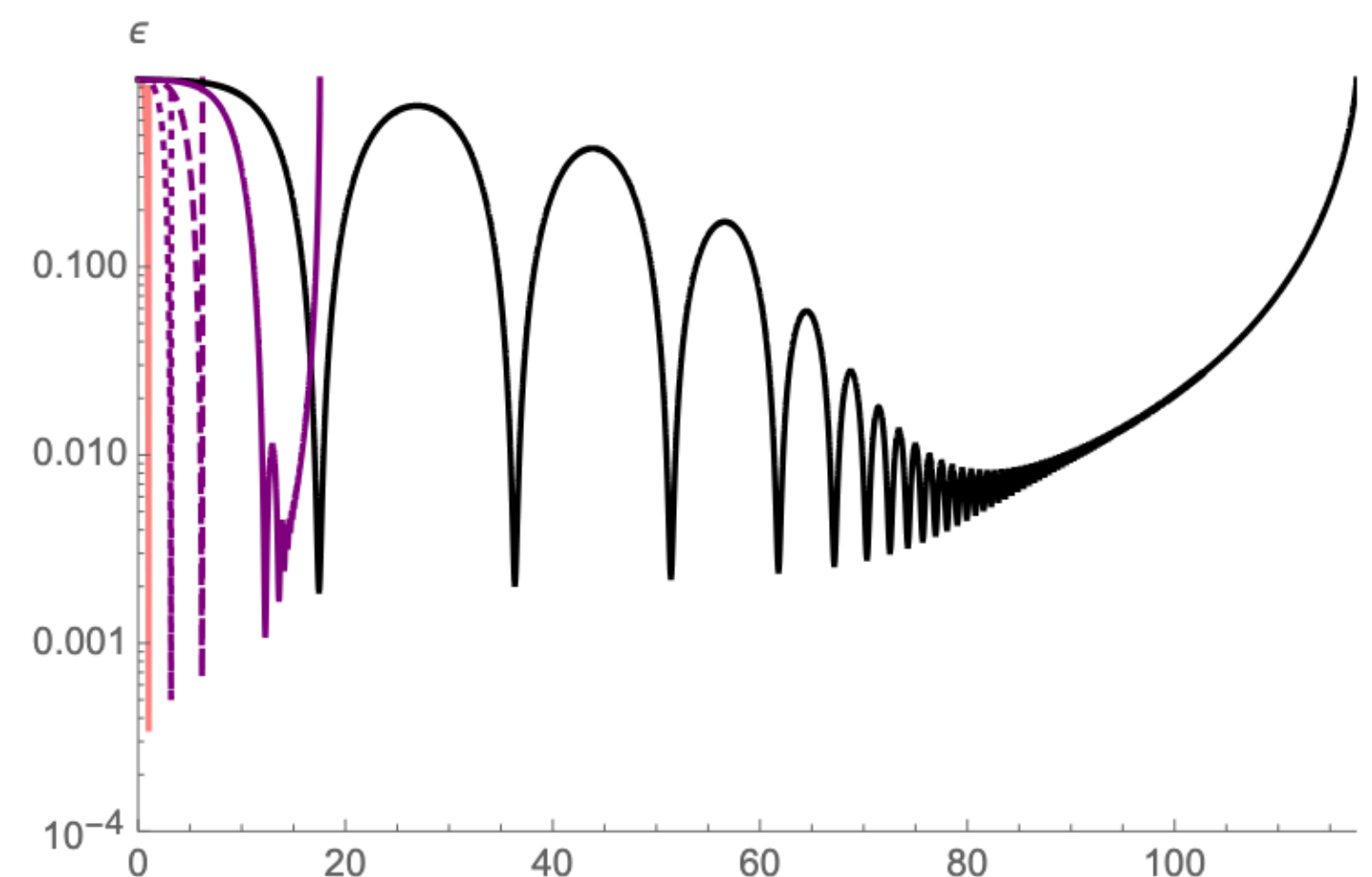
- ▶ When the binary system has a high relative inclination to its orbit around the SMBH it will evolve over many orbits trading eccentricity for inclination in a periodic fashion
- ▶ Secular average over both the inner and outer orbit  $\rightarrow$  Long timescale
- ▶ The BHB can reach eccentricity close to 1  $\rightarrow e \sim 1$
- ▶ When including the emission of GW from the BHB the KL mechanism can speed up the merger!





# Kozai-Lidov Mechanism - Strong Gravity Regime

- ▶ The black holes in the binary are treated as point particles, the perturber as a rotating black hole
- ▶ **How the spin affects the merger**
- ▶  $m_3 = 4 \times 10^6 M_\odot, m_1 = m_2 = 10 M_\odot, \hat{r} = 500 r_{ISCO}, a = 0.1 \text{ AU}$
- ▶ As the spin of the SMBH increases
  - ▶ Merger time decreases
  - ▶ Fewer KL cycles
  - ▶ Higher eccentricity
  - ▶ **Faster merger!**



# Kozai-Lidov Mechanism - Strong Gravity Regime

- ▶ The black holes in the binary are treated as point particles, the perturber as a rotating black hole
- ▶ Binary close to the supermassive (ISCO - Innermost Stable Circular Orbit)

▶  $m_3 = 4 \times 10^8 M_\odot, m_1 = m_2 = 10 M_\odot, \hat{r} = r_{ISCO}, a = 0.1 \text{ AU}$

- ▶ Point Particle result  $\rightarrow$  blue line
- ▶ Full GR result  $\rightarrow$  black line

- ▶ Point Particle approximation breaks down on the ISCO!

- ▶ The spin speed up the merger as expected even on the ISCO

