

# Testing leptogenesis with *CMB, Gravitational Waves & Non-gaussianity*

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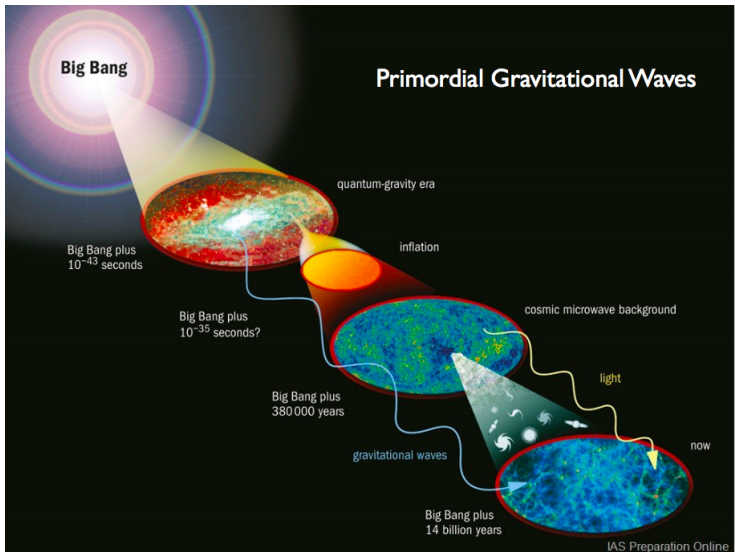
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June 2023  
Florence, Italy

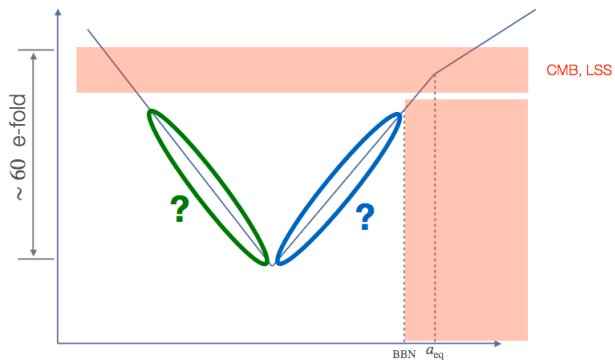
Collaboration: Hitoshi Murayama, Anupam Mazaumdar, Graham White, Marek Lewicki, and others..

2301.05672, 2210.14176, 2206.07032, 2301.05672, 2205.06260, 2208.01670, 2210.14176

- 1 Cosmic Observables: CMB spectral indices, inflationary GW & Non-Gaussianity
- 2 Novel probes of BSM physics, particularly for weakly coupled & high-scale BSM.
- 3 Baryogenesis via Leptogenesis: need for high scale testability
- 4 Presence of RH neutrino on Inflationary tensor perturbations
  - Propagation of Inflationary GW
  - Matter domination via RH neutrino
  - Testing seesaw and leptogenesis via GW detectors
- 5 Leptogenesis during inflationary re-heating
  - Non-thermal Leptogenesis via Inflaton Decay
  - CMB spectral index imprints on Inflaton couplings
  - Impact of leptogenesis on CMB predictions
- 6 Non-gaussianity as novel probe of baryogenesis.
- 7 Conclusions
- 8 Can NanoGrav Signal for Stochastic GW background say something about baryogenesis ?

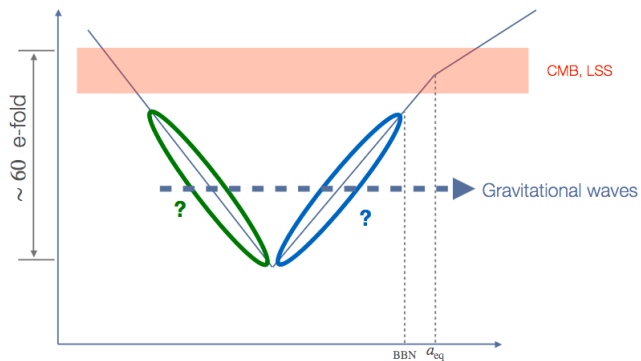


## Early universe

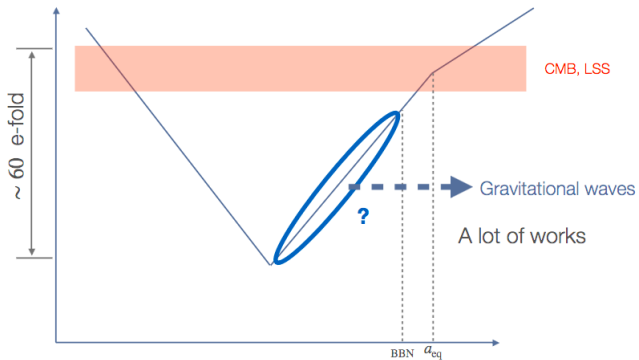


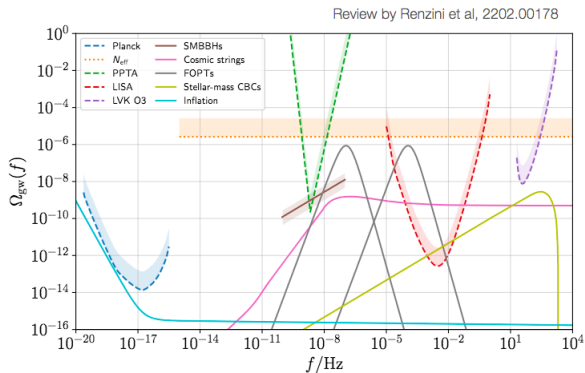


## Early universe



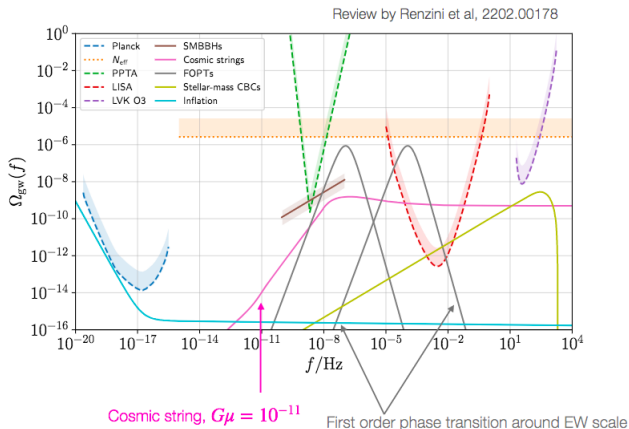
## Early universe



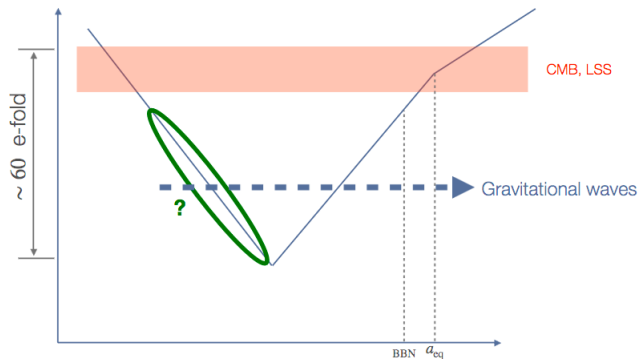


Typically, need something quite dramatic.

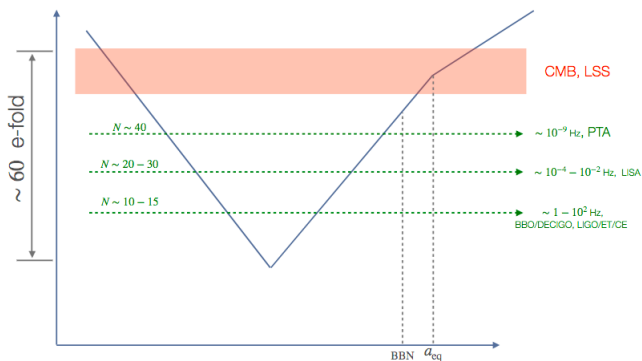
# History of the Universe



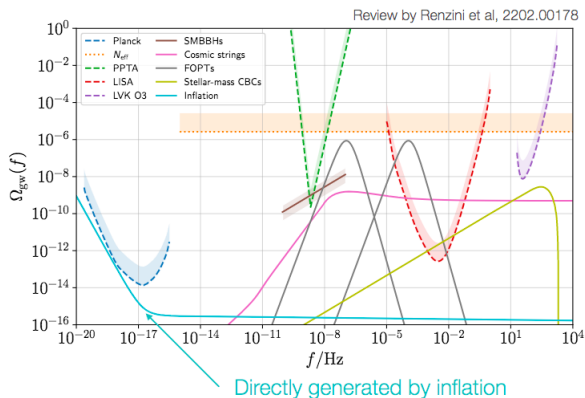
## Early universe



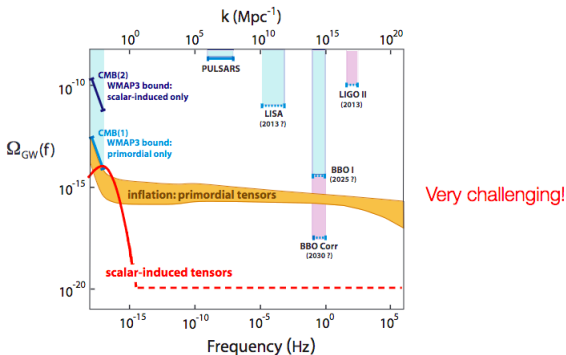
## Early universe



## Example: primordial tensor



## Secondary GW from primordial scalar perturbation



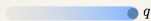


- Gravitational Waves (GW) first detected in 2016.
- New Window into the Early Universe.
- New Probes of Particle Phenomenology beyond TeV (LHC scale).
- Robust predictions of GW signatures from UV-completion conditions.
- Sources of GW of cosmological origin & corresponding GW spectrum:
  - Inflation: Primordial GW.
  - Inflation: Secondary GW.
  - Strong First-order Phase Transition.
  - (P)reheating.
  - Re-heating.
  - Graviton bremsstrahlung.
  - Topological Defects like cosmic strings, domain walls, etc.
  - Oscillon.
  - Q-balls.
  - Primordial BH-induced GW.

# Gravitational Wave Basics: Emission

## Electromagnetism

- Accelerated electric charges → Electromagnetic radiation



- If  $q$  &  $\gamma$  increase → Emitted power increases

- Electromagnetic waves oscillate charges up and down



## Gravity

- Accelerated masses → Gravitational radiation



- If  $M$  &  $\gamma$  increase → Emitted power increases

- Gravitational waves stretches and squeezes space



perturbations of the background metric:  $ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu}(\mathbf{x}, \tau))dx^\mu dx^\nu$

↑
↑
↑  
 scale factor: cosmological expansion    background metric    GW

governed by linearized Einstein equation ( $\tilde{h}_{ij} = ah_{ij}$ , TT - gauge)

$$\tilde{h}_{ij}''(\mathbf{k}, \tau) + \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\sim a^2 H^2} \tilde{h}_{ij}(\mathbf{k}, \tau) = \underbrace{16\pi G a \Pi_{ij}(\mathbf{k}, \tau)}_{\text{source term from } \delta T_{\mu\nu}}$$

source: anisotropic stress-energy tensor

$$k \gg aH : h_{ij} \sim \cos(\omega\tau)/a, \quad k \ll aH : h_{ij} \sim \text{const.}$$

a useful plane wave expansion:  $h_{ij}(\mathbf{x}, \tau) = \sum_{P=+, \times} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int d^2\hat{\mathbf{k}} h_P(\mathbf{k}) \underbrace{T_k(\tau)}_{\sim a(\tau_i)/a(\tau)} e_{ij}^P(\hat{\mathbf{k}}) e^{-ik(\tau - \hat{\mathbf{k}}\mathbf{x})}$

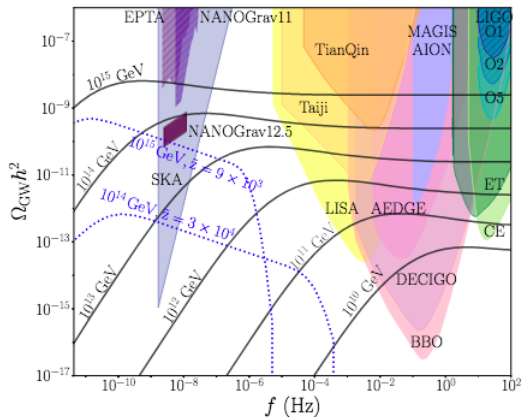
transfer function , expansion coefficients , polarization tensor  $P = +, \times$

Spectrum of GW background,

$$\Omega_{GW}(k, \tau) = \frac{1}{\rho_c} \frac{\partial \rho_{GW}(k, \tau)}{\partial \ln k},$$

$$\int_{-\infty}^{\infty} d \ln k \frac{\partial \rho_{GW}(k, \tau)}{\partial \ln k} = \frac{1}{32\pi G} \langle \dot{h}_{ij}(\mathbf{x}, \tau) \dot{h}^{ij}(\mathbf{x}, \tau) \rangle$$

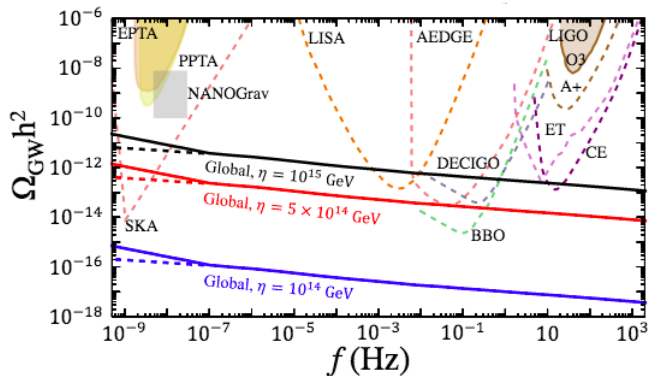
Topological defects like cosmic strings can be formed in early universe when some gauge  $U(1)_X$  symmetry is broken in early universe. It give rise to scale invariant GW spectrum. Detection prospects lies on the symmetry breaking scale  $\text{vev}$ .



King (2020)

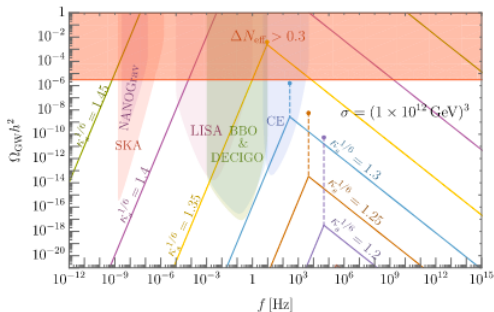
# GW - - Global Cosmic String

Topological defects like cosmic strings can be formed in early universe when some global  $U(1)_X$  symmetry is broken in early universe. Detection prospects lies on the symmetry breaking scale  $v_{ev}$  which needs to be very high.



Cui (2021)

Topological defects like Domain Walls are formed when a discrete symmetry is broken and give rise to GW spectrum may look something like this (still under active research topic). Detection prospects lies of symmetry breaking scale as well as the asymmetry term in the potential, like cubic term.



Dunsky et al. (2021)

Or life in early universe maybe lead to more rich and complex structures: hybrid defects various  $SO(10)$  breakin chains may give unique GW signals:

## Gravitational Wave Gastronomy

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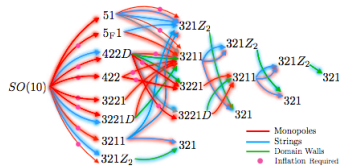
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(Dated: November 18, 2021)*



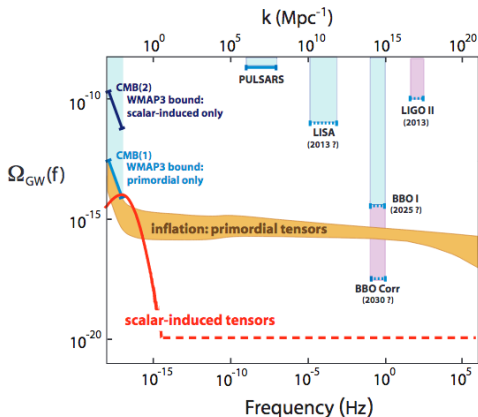
Ghoshal (2021)



# GW - - Primordial and Scalar Induced Secondary GW

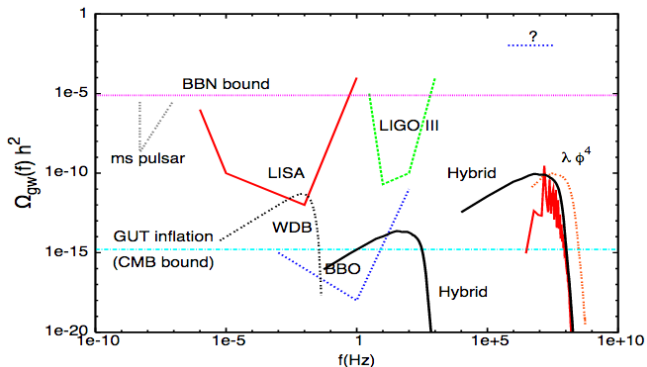
Primary Tensor Perturbations and Secondary Tensor Spectrum induced by first-order scalar perturbation via mixing. Can be tuned to generate high amplitude in high frequency regions.

Acts as natural probes of particle models like Higgs inflation, axion inflation, MSSM inflation, etc.



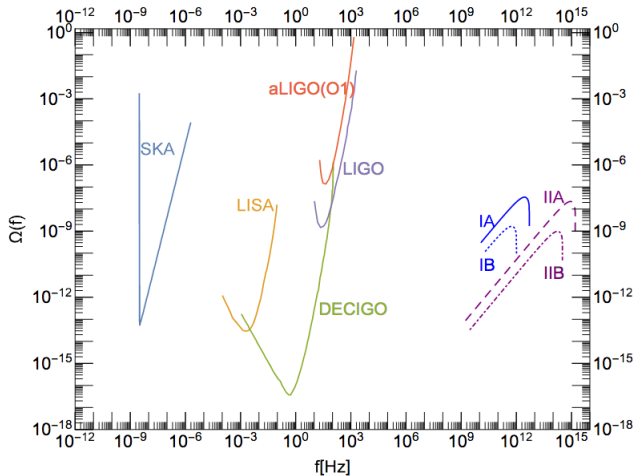
# GW - - (P)-reheating

Excitation of tensor perturbations during inflaton oscillating in FRW background. Backreaction and effects of metric fluctuations. Enhancement mechanism: Bose-resonance, tachyonic growth, parametric resonance.



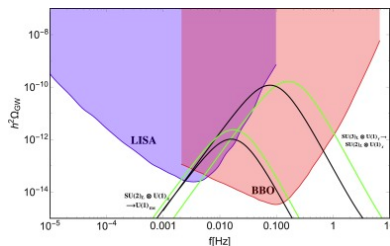
Figuera (2007)

Inflaton radiating away gravitons forming Stochastic GW background.



Nakayama (2018)

Typical GW spectrum from thermal first-order phase transition:



Huang (2018)

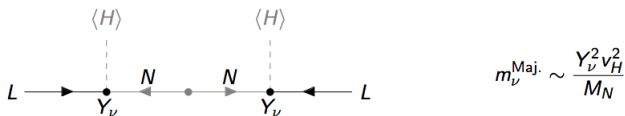
Let us recap leptogenesis.

Introductory baryogenesis via Leptogenesis:

## Type I Seesaw

[Minkowski 77; Gell-Mann, Ramond, Slanski 79; Yanagida 80]

$$\text{SM} + 3 \text{ copies of } N \sim (1, 1, 0) : \quad \mathcal{L} \supset Y_\nu L \epsilon H N + \frac{1}{2} M_N N N, \quad \epsilon \equiv i\sigma_2$$



$M_N \gg Y_\nu v_H$  small neutrino mass and mixing with RHN

$$m_\nu \simeq 0.05 \text{ eV} \cdot Y_\nu^2 \cdot \frac{10^{15} \text{ GeV}}{M_N}, \quad |V_{\nu N}|^2 = \frac{Y_\nu^2 v_H^2}{M_N^2} = \frac{m_\nu}{M_N}$$

## Baryon asymmetry and baryon-to-photon-ratio

Observation typically phrased in terms of

$$\eta \equiv \frac{n_B}{n_\gamma} \Big|_{\text{today}} = \begin{cases} 5.93 \times 10^{-10} & \text{BBN} \\ 6.12 \times 10^{-10} & \text{CMB} \end{cases}$$

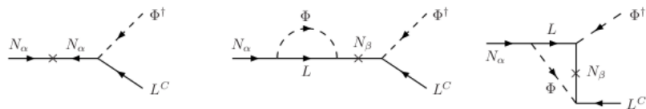
or alternatively  $n_B - n_{\bar{B}}$ :

$$\Delta_B \equiv \frac{n_B - n_{\bar{B}}}{s} = \begin{cases} 8.47 \times 10^{-11} & \text{BBN} \\ 8.74 \times 10^{-11} & \text{CMB} \end{cases}$$

- $T < 100 \text{ GeV}$ :  $n_B - n_{\bar{B}} = \text{const.}$  (up to redshift)
- $n_B|_{\text{today}} \gg n_{\bar{B}}|_{\text{today}}$  and  $n_\gamma|_{\text{today}} = 413 \text{ cm}^{-3} \simeq s/7|_{\text{today}}$
- Why is  $\Delta_B \neq 0$ ?

## Vanilla Leptogenesis

[Fukugita, Yanagida 1986] Decays of **lightest** RHN  $N \rightarrow LH$ ,  $N \rightarrow \bar{L} H^*$



source: 0904.1182

$$\frac{n_B - n_{\bar{B}}}{s} = \underbrace{C_{\text{sph.}}}_{\dot{B}} \cdot \underbrace{\epsilon_L}_{CP} \cdot \underbrace{\kappa}_{\text{out of. equil.}} \cdot \frac{n_N^{\text{rel}}}{s} \stackrel{!}{=} 8 \times 10^{-11}$$

3 (small) numbers from **Sakharov conditions**



## Davidson Ibarra bound

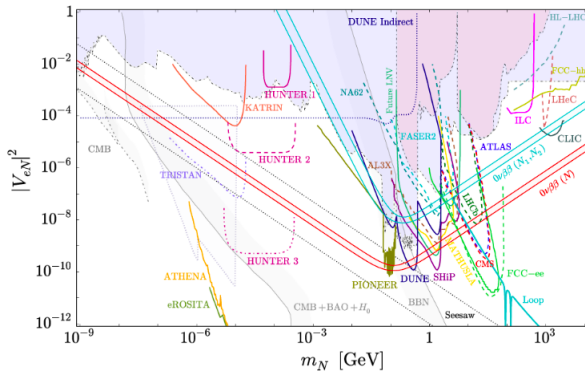
①  $\beta$ :  $c_{\text{sph.}} \simeq 0.3$

②  $CP$ :  $\epsilon_L \leq \frac{3}{8\pi} \frac{M_1(m_3 - m_1)}{v_H^2}$

③ out of eq.:  $\kappa \simeq \text{Min}[1, \underbrace{K}_{n_N^i=0}, \underbrace{g_{\text{SM}}}_{\text{dom.}}, 1/K]$

[Davidson, Ibarra 2002]  $M_1 \gtrsim \left( \frac{0.05 \text{ eV}}{m_3} \right) \cdot \begin{cases} 5 \times 10^8 \text{ GeV} & n_N^i = n_N^{\text{eq.}} \\ 2.4 \times 10^9 \text{ GeV} & n_N^i = 0 \\ 1.74 \times 10^7 \text{ GeV} & n_N^i = \frac{\rho_{\text{Rad}}}{M_N} \end{cases}$

## Experimental probes



source: Bolton et al. 2019, 1912.03058, [www.sterile-neutrino.org](http://www.sterile-neutrino.org)

Lots of literature now in the past 4 years on GW tests for leptogenesis (I only concentrate on inflationary GW):

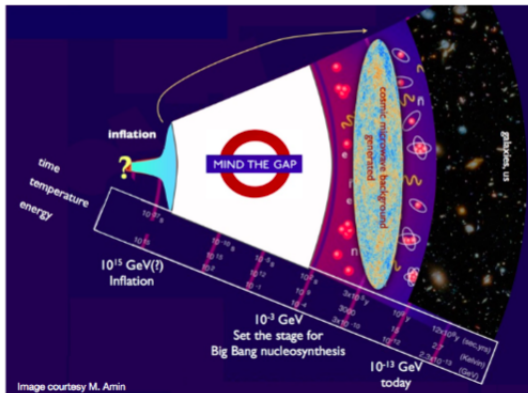
## RHN + GWs

- lots of literature on GWs and neutrino masses
- realistic models  $Y_{N\sigma}NN$  with  $M_N = Y_{N\sigma}v_\sigma$
- almost all of them: **phase transition** from  $\sigma$
- GWs from 1st order, 2nd order PT, cosmic strings
- **probes the underlying theory (e.g. SO(10))**  
but not dynamics of RHN
- absent for  $v_\sigma > \text{Max} \left[ \frac{H_I}{2\pi}, T_{\text{max.}} \right]$

Here we will be the most minimal: SM + 3 RHNs.

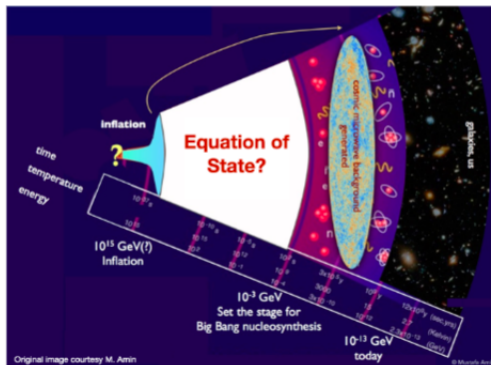
# History of the Universe

However huge gap in our knowledge in between the inflation ending & the beginning of radiation-dominance era:



# History of the Universe

What dominated this era ? New gravity ? New Matter ? Predictions from UV-completions.



Propagation of Primordial GW generated during Inflation:

$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + \frac{k^2}{a^2} h_{ij} = 16\pi G \Pi_{ij}^{TT},$$

Solution:

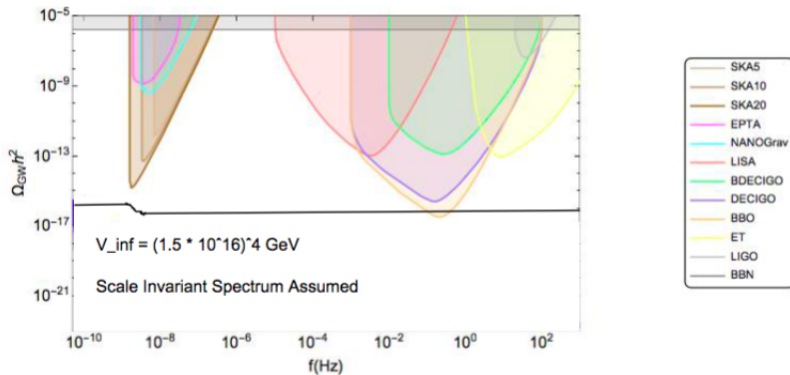
$$h_{ij}(t, \vec{x}) = \sum_P \int \frac{d^3k}{(2\pi)^3} h^P(t, \vec{k}) \epsilon_{ij}^P(\vec{k}) e^{i\vec{k}\cdot\vec{x}},$$

$$h_{\vec{k}}^P = h_{\vec{k},0}^P U(t, k),$$

$$\Pi_{ij} = \frac{T_{ij} - p g_{ij}}{a^2}$$

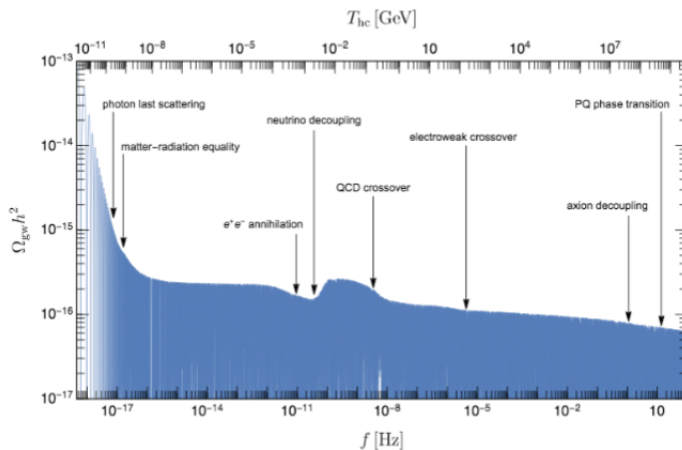
$$\Omega_{GW}(\eta, k) = \frac{1}{12 a^2(\eta) H^2(\eta)} \mathcal{P}_T(k) [U'(\eta, k)]^2$$

# History of the Universe



$$P_T = \frac{2}{3\pi^2} \frac{V_{\text{inf}}}{M_{\text{pl}}^4}. \text{ UV-completion: Trans-Planckian Censorship may constrain } V_{\text{inf}}.$$

# History of the Universe



Impact on PGW spectrum from thermal history of the Universe.

Ringwald (2020)



## Tensor modes from inflation

- quantum fluctuations of inflaton:  
scalar & **tensor** ( $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ ) perturb.

- power spectra:  $k_* = 0.05$  /Mpc

$$P_S = A_S \left( \frac{k}{k_*} \right)^{n_S - 1}, \quad P_T = A_T \left( \frac{k}{k_*} \right)^{n_T}$$

- well measured [Planck 2018; BICEP2-KECK 2021]

$$A_S = 2.0989 \times 10^{-9}, \quad n_S = 0.9649 \pm 0.0042, \quad r = A_T/A_S < 0.035$$

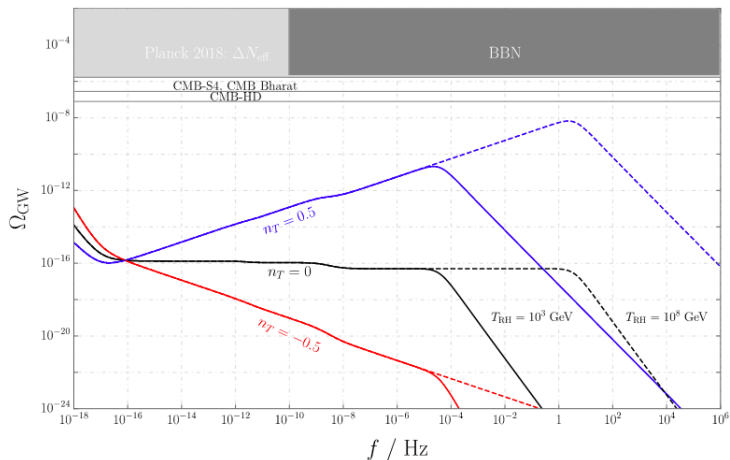
- **single inflaton field:**  $n_T = -r/8 = -2\epsilon \sim (V'_\phi/V_\phi)^2 \lesssim -4 \times 10^{-3}$

- 1 **Blue-Tilt:**  $n_T > 0$  (model/scenario dependent)

- 2 **Red-Tilt:**  $n_T < 0$

## Blue- vs Red-Tilt

$$f \simeq 10^{-10} \text{ Hz} \cdot (T/10 \text{ MeV})$$



## Intermediate RHN domination

- use **RHN to generate MD** to distort GWs
- inflation  $\rightarrow$  reheating (MD)  $\rightarrow$  early RD  $\rightarrow$  **RHN MD**  $\rightarrow$  2nd RD
- $\rho_N \sim M_1 T^3$  larger than  $\rho_{\text{rad}} \sim T^4 \Rightarrow$  **RHN dominates** energy dens.

$$T_{\text{dom.}} \simeq \frac{7}{4} \frac{M_1}{g_*(T_{\text{dom.}})} \simeq 2\% M_1$$

- decay when  $\Gamma \sim M_1^2 \tilde{m}_1 / v_H^2 = H(T_{\text{dec.}})$

$$T_{\text{dec.}} = 3 \times 10^8 \text{ GeV} \cdot \sqrt{\frac{\tilde{m}_1}{10^{-6} \text{ eV}}} \cdot \left( \frac{M_1}{10^{10} \text{ GeV}} \right)$$

- $T_{\text{dec.}} < T_{\text{dom.}}$  for  $\tilde{m}_1 < 2 \times 10^{-7} \text{ eV}$  implies  $\text{Min}[m_\nu] \simeq 0$

## Modified DI bound from RHN reheating

- out-of-equ. decay **releases entropy**

$$\Delta \equiv \frac{s_{\text{after}}}{s_{\text{before}}} \simeq \frac{\rho_N / T_{\text{dec.}}}{s_{\text{before}}} \sim \frac{M_1}{T_{\text{dec.}}} \gg 1$$

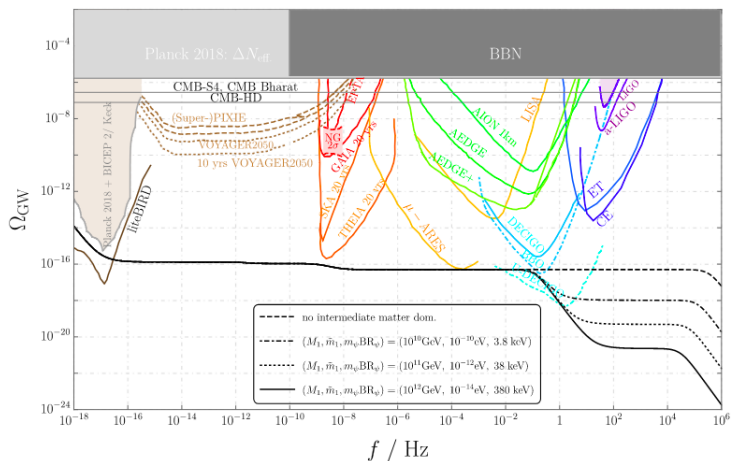
- during domination:  $n_N \sim T^4 / M_N$

$$\frac{n_B - n_{\bar{B}}}{s} \sim \frac{n_N}{n_N^{\text{rel.}}} \sim \frac{T_{\text{dec.}}}{M_1} \sim \frac{1}{\Delta}$$

- DI bound  $M_1 \gtrsim 2 \times 10^8 \text{ GeV} \cdot \sqrt{\frac{2 \times 10^{-7} \text{ eV}}{\tilde{m}_1}}$  fixes  $T_{\text{dec}} \sim M_1 \sqrt{\tilde{m}_1}$
- frequency of GW  $f \sim T_x$ ,  $T_x$ : horizon crossing

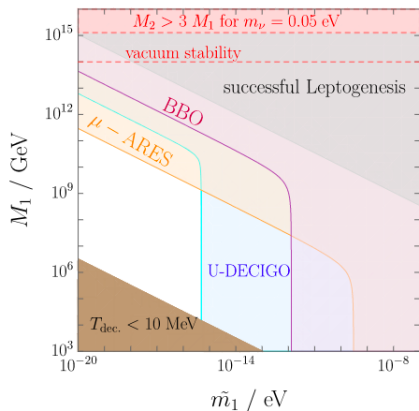
$$f_{\text{sup.}} \gtrsim 9 \times 10^{-2} \text{ Hz} \cdot \left( \frac{n_B/s}{8 \times 10^{-11}} \right)$$

## Spectrum (1)



$$n_T = 0, T_{\text{RH}} = 10^{13} \text{ GeV}, r = 0.035$$

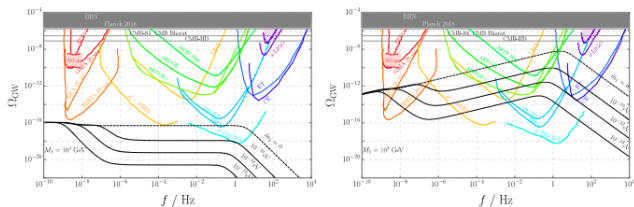
## Signal to Noise Ratio (1)



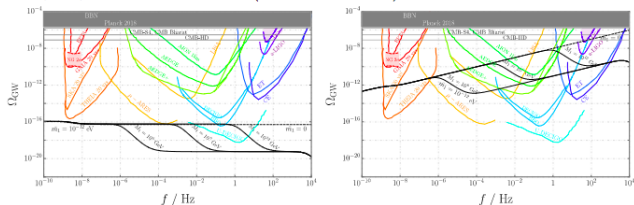
$$\text{SNR} \equiv \sqrt{\tau \int_{f_{\min}}^{f_{\max}} df \left( \frac{\Omega_{\text{GW}}(f) h^2}{\Omega_{\text{exp}}(f) h^2} \right)^2}$$

$$\geq 10$$

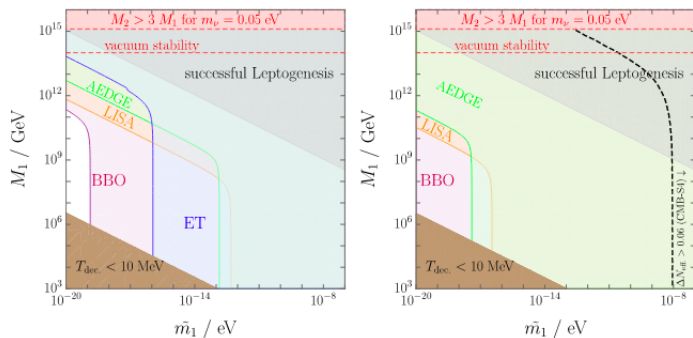
$$n_T = 0, T_{\text{RH}} = 10^{16} \text{ GeV}, r = 0.035$$



**Figure 2.** Example GW spectra for  $T_{RH} = 10^8$  GeV,  $M_1 = 10^4$  GeV and  $n_T = 0$  (left) as well as  $n_T = 0.5$  (right). Here we varied  $\tilde{m}_1 = (10^{-10}, 10^{-12}, 10^{-14})$  eV.



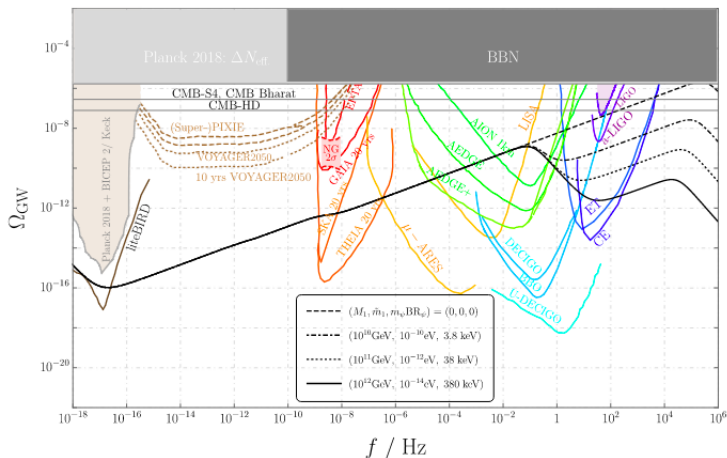
**Figure 3.** Example spectra for  $T_{RH} = 10^{12}$  GeV,  $\tilde{m}_1 = 10^{-12}$  eV and  $n_T = 0$  (left) as well as  $n_T = 0.5$  (right). Here we varied  $M_1 = (10^6, 10^9, 10^{12})$  GeV.



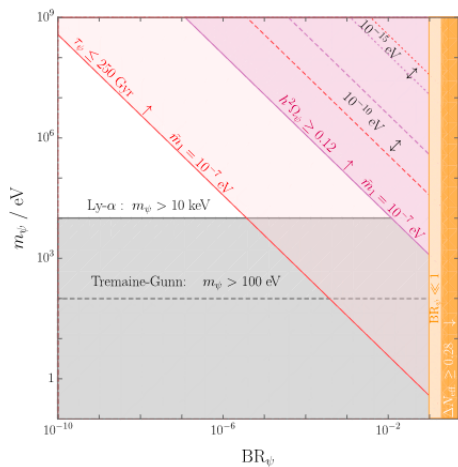
**Figure 9.** Parameter space in the  $M_1$  versus  $\tilde{m}_1$  plane with contours for SNR = 10 for  $n_T = 0.3$  (left) and  $n_T = 0.5$  (right). In both plots we fixed  $T_{\text{RH}} = 10^{16}$  GeV. See the main text for details on the constraints. The SNR is larger than 10 in the colored regions. Note that the colored lines from the experiments do not correspond to constraints, but to projections of future sensitivities.



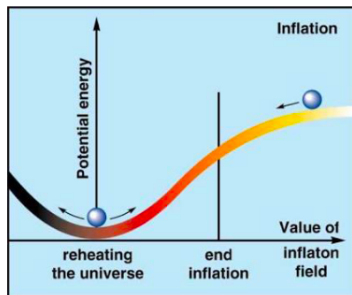
## Dark and Dark Radiation from RHN decay:



**Figure 5.** We fix  $M_1$  as a function of  $\bar{m}_1 = (10^{-10}, 10^{-12}, 10^{-14})$  eV for successful leptogenesis and set  $T_{\text{RH}} = 10^{13}$  GeV,  $n_T = 0.5$ . Furthermore we show which value of  $m_\psi \text{BR}_\psi$  would be required for a given  $\bar{m}_1$  to generate the observed dark matter relic abundance.

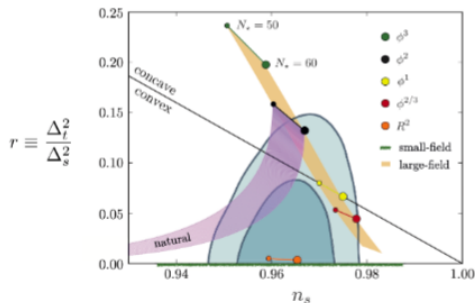


CMB test for leptogenesis



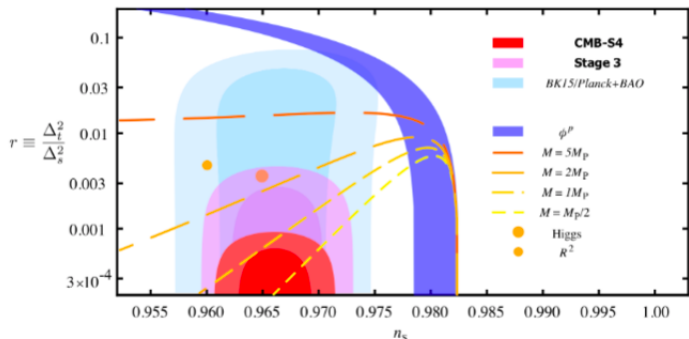
- Alleviates flatness and horizon problems etc.
- Quantum fluctuations present in the early universe are amplified spatially. The fluctuations then act as seeds for cosmic structure formation

Well known story about the inflation scalar field inflaton rolling down a potential with little or no interactions:



Little or no interactions

Going to be more well-known in very near future:



Non-Gaussianity parameters:

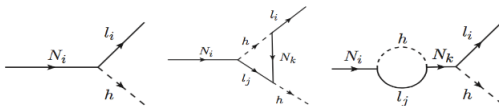
$$f_{NL}^{\text{local}} = -0.9 \pm 5.1, f_{NL}^{\text{equilibrium}} = -26 \pm 47, f_{NL}^{\text{orthogonal}} = -38 \pm 24.$$

Detection of CMB BB-modes will tell us the scale of inflation.

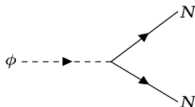
Leptogenesis is possible in a simple type-I seesaw framework.

$$-\mathcal{L} = y_\nu \bar{L} \tilde{H} N + M \bar{N}^c N. \quad (1)$$

Annals Phys. 315 (2005) 305-351



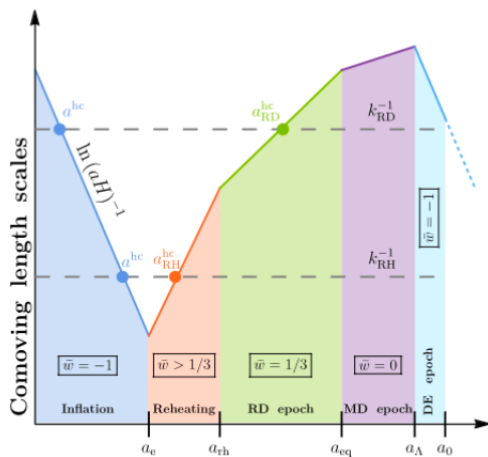
Leptogenesis could be thermal ( $T_R > M_N$ ) or non-thermal ( $T_R < M_N$ ).



In thermal leptogenesis, wash out effects are important.



Leptogenesis during the re-heating era:





For representation purpose, we chose to work with Starobinsky-like inflation potential

$$V(\phi) = \Lambda^4 \left( 1 - e^{-\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_P}} \right)^{2n}. \quad (2)$$

During inflation, potential is slow rolling hence  $H = \frac{da/dt}{a} \sim \text{const.}$  and hence  $a(t) \sim \exp(Ht)$ . Slow roll conditions are ensured by  $\epsilon, \eta \ll 1$ .

$$\epsilon = \frac{1}{2} \times \left( \frac{\partial V(\phi)/\partial \phi}{V(\phi)} \right)^2, \quad \eta = \left( \frac{\partial^2 V(\phi)}{\partial \phi^2} \right)$$

Important parameters that Planck measures are, [Planck collb 1807.06209](#)

$$n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon, \quad A_s = \frac{V_{\text{inf}}}{24\pi^2 \epsilon}. \quad (3)$$

Inflation ends when  $\max[\epsilon, \eta] = 1$ . We find

$$V_{\text{end}} = \Lambda^4 \left( \frac{2n}{2n + \sqrt{3\alpha}} \right)^{2n} \quad (4)$$

$$r = \frac{192\alpha n^2 (1 - n_s)^2}{\left[ 4n + \sqrt{16n^2 + 24\alpha n(1 - n_s)(1 + n)} \right]^2}. \quad (5)$$

The observed value of  $A_s^{\text{obs}} = 2.2 \times 10^{-9}$  precisely fixes one of the model parameters  $\Lambda$  as,

$$\Lambda = M_P \left( \frac{3\pi^2 r A_s^{\text{obs}}}{2} \right)^{1/4} \times \left[ \frac{2n(1 + 2n) + \sqrt{4n^2 + 6\alpha(1 + n)(1 - n_s)}}{4n(1 + n)} \right]^{n/2}. \quad (6)$$

The number of e-fold during inflation is

$$N_k = \frac{3\alpha}{4n} \left[ e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_P}} - e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi_{\text{end}}}{M_P}} - \sqrt{\frac{2}{3\alpha}} \frac{(\phi_k - \phi_{\text{end}})}{M_P} \right]$$

where  $\phi_k$  is the inflaton field value at horizon exit. From the expression of  $n_s$ , one can write  $\phi_k$  as function of  $n_s$ .

$$\phi_k = \sqrt{\frac{3\alpha}{2}} M_P \ln(1 + \Delta(n_s)), \quad (7)$$

with  $\Delta(n_s) = \frac{4n + \sqrt{16n^2 + 24\alpha n(1-n_s)(1+n)}}{3\alpha(1-n_s)}$ .

Note: Parameters  $\{N_k, r, V_{\text{end}}\}$  are important which are all function of  $n_s$ . We have considered  $n = 1$  in our analysis.

We also propose the following Lagrangian in a model independent manner:

$$-\mathcal{L} \supset y_N \phi \overline{N^c} N + y_R \phi \overline{X} X + y_\nu \overline{l}_L \tilde{H} N + M_N \overline{N^c} N + h.c.. \quad (8)$$

The set of Boltzmann equations that govern the evolution of energy densities of various species, number densities for N and the yield of lepton asymmetry is given by

$$\frac{d\rho_\phi}{dt} + 3H(\rho_\phi + p_\phi) = -\Gamma_\phi^N \rho_\phi - \Gamma_\phi^R \rho_\phi, \quad (9)$$

$$\frac{d\rho_R}{dt} + 3H(\rho_R + p_R) = \Gamma_\phi^R \rho_\phi + \Gamma_N \rho_N, \quad (10)$$

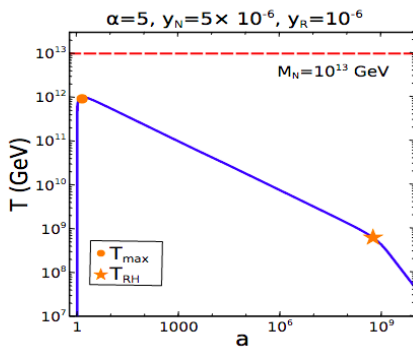
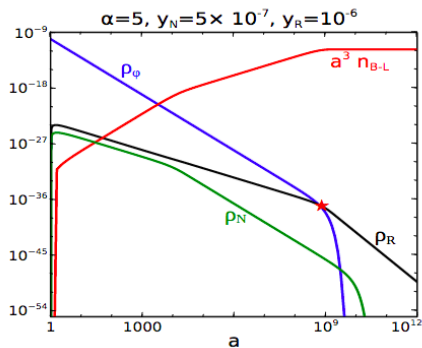
$$\frac{d\rho_N}{dt} + 3H(\rho_N + p_N) = \Gamma_\phi^N \rho_\phi - \Gamma_N \rho_N, \quad (11)$$

$$\frac{dn_{B-L}}{dt} + 3Hn_{B-L} = -\frac{\epsilon \rho_N \Gamma_N}{M_N}. \quad (12)$$

$$\epsilon = \frac{1}{8\pi} \sum_{j \neq 1} \frac{\text{Im} \left[ (y_\nu^\dagger y_\nu)_{1j}^2 \right]}{(y_\nu^\dagger y_\nu)_{11}} \mathcal{F} \left( \frac{M_j^2}{M_1^2} \right),$$

$$y_\nu = \frac{\sqrt{2}}{v} U_{\text{PMNS}} \sqrt{m_\nu^d} \mathcal{R}^T \sqrt{M_N}, \quad \mathcal{R} = \begin{pmatrix} 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \\ 1 & 0 & 0 \end{pmatrix}$$

# History of the Universe



We considered  $M_N > T_{\max}$ .

At horizon exit  $k = a_k H_k$ , and one can write,

$$\ln \left( \frac{k}{a_k H_k} \right) = \ln \left( \frac{a_{\text{end}}}{a_k} \frac{a_{\text{re}}}{a_{\text{end}}} \frac{a_0}{a_{\text{re}}} \frac{k}{a_0 H_k} \right) = 0, \quad (13)$$

Considering FRW ansatz, the e-folding number from the end of inflation to the end of reheating epoch is written as

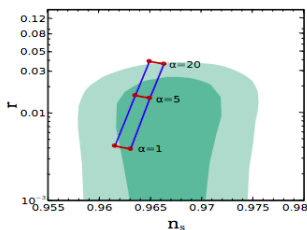
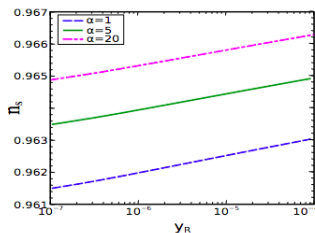
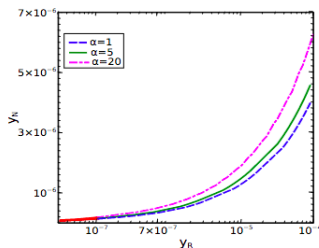
$$N_{\text{re}} = \ln \left( \frac{a_{\text{re}}}{a_{\text{end}}} \right) = -\frac{1}{3(1 + \bar{\omega}_{\text{re}})} \ln \left( \frac{\rho_{\text{re}}}{\rho_{\text{end}}} \right), \quad (14)$$

We obtain, obtain,

$$N_{\text{re}} = \frac{4}{3\omega_{\text{re}} - 1} \left[ N_k + \ln \left( \frac{k}{a_0 T_0} \right) + \frac{1}{4} \ln \left( \frac{40}{\pi^2 g_*} \right) + \frac{1}{3} \ln \left( \frac{11g_{s*}}{43} \right) - \frac{1}{2} \ln \left( \frac{\pi^2 M_P^2 r A_s}{2V_{\text{end}}^{1/2}} \right) \right]. \quad (15)$$

Case I:  $y_N \lesssim y_R$ , free parameters:  $\{y_N, y_R, \theta, M_N\}$ 

$$\theta = 0.1 + 0.1i, M_1 = 10^{13} \text{ GeV}$$

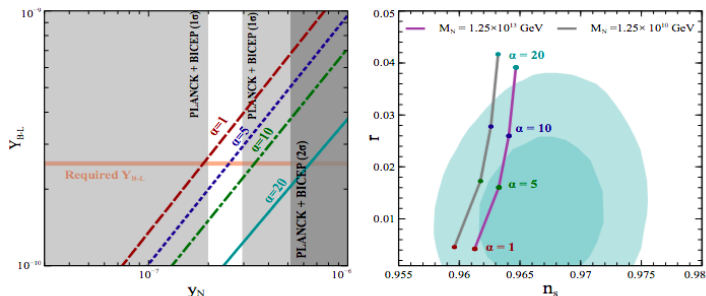


$$\begin{aligned} \frac{n_{B-L}}{s} &\sim \frac{3}{2} \times \text{Br}_{\phi \rightarrow NN} \frac{\epsilon T_R}{m_\phi} \\ &\sim \sqrt{M_{PE}} \frac{y_N^2}{y_R \sqrt{m_\phi}} \end{aligned}$$

$$\text{where } m_\phi = \frac{2\Lambda}{3\sqrt{\alpha} M_P}$$

Case II:  $y_R \ll y_N$  free parameters:  $\{y_N, \theta, M_N\}$ 

$$\theta = 0.1 + 0.1i$$



$$\frac{n_{B-L}}{s} \sim \sqrt{M_{Pl} \epsilon} \frac{y_N}{\sqrt{m_\phi}}. \quad (16)$$

# Non-Gaussianity as probe of leptogenesis:

Non-Gaussianity  $f_{\text{NL}}$  as novel probe of leptogenesis: via curvaton-induced neutrino mass !



## What is a curvaton model?

### Single field inflaton:

A **single field**  $\phi$  both drives inflation and is the source of the perturbations.

### Curvaton paradigm:

One field  $\phi$  drives inflation but has negligible perturbations; a **second field**  $\sigma$  is the source of perturbations but is negligible during inflation.

## Why study curvaton models?

- ▶ because they exist!
- ▶ because they can give measurable non-Gaussianity and isocurvature
- ▶ because light scalar fields ( $m < H$ ) might exist and it is important to calculate their consequences
- ▶ because the curvaton mechanism gives more freedom for the inflation model
- ▶ because they have interesting, constrainable dynamics after inflation

**Curvaton:** a light scalar field which is partially or totally responsible for the density fluctuation.

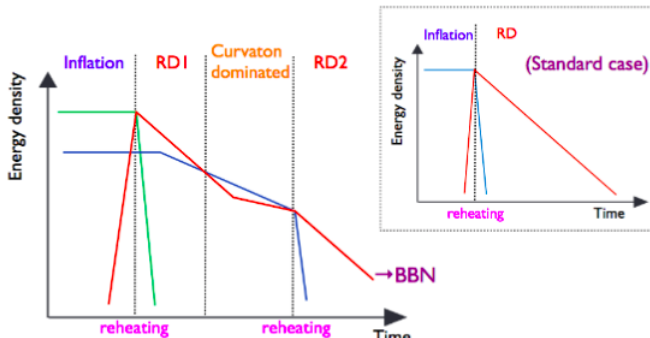
(Usually, quantum fluctuation of the inflaton is assumed to be responsible for that.)

## Curvaton scenario

Inflaton → causes the inflation,  
not fully responsible for the cosmic fluctuation

Curvaton → is not responsible for the inflation,  
is fully or partially responsible for the cosmic fluctuation

## Thermal history of the universe with the curvaton

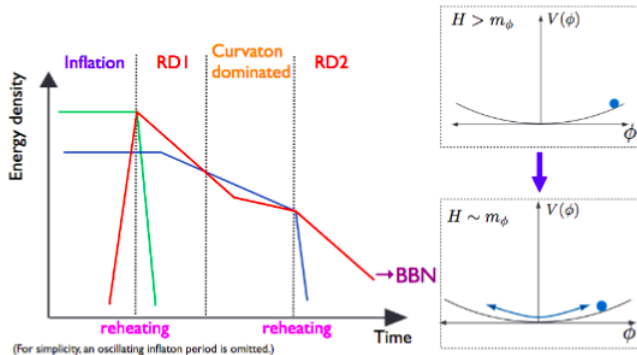


(For simplicity, an oscillating inflaton period is omitted.)

The potential of the curvaton is assumed as  $V_{\text{curvaton}} = \frac{1}{2} m_\phi^2 \phi^2$

→ The oscillating field behaves as matter.

## Thermal history of the universe with the curvaton



The potential of the curvaton is assumed as  $V_{\text{curvaton}} = \frac{1}{2} m_\phi^2 \phi^2$

→ The oscillating field behaves as matter.

## Density Perturbation

- Fluctuation of the inflaton  $\rightarrow$  Curvature perturbation

$$\left(\delta\chi \sim \frac{H}{2\pi}\right) \quad \mathcal{R} \sim -\frac{H}{\dot{\chi}}\delta\chi$$

- Fluctuation of the curvaton  $\rightarrow$  No curvature perturbation  
(The curvaton is subdominant component during inflation.)

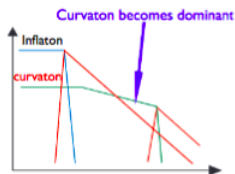
$\rightarrow$  However, isocurvature fluctuation can be generated.

$$S_{\phi\chi} \sim \delta\chi - \delta\phi = \frac{2\delta\phi_{\text{init}}}{\phi_{\text{init}}}$$

where  $\delta_i \equiv \frac{\delta\rho_i}{\rho_i}$

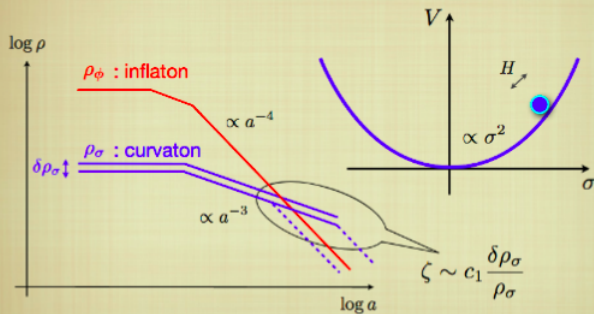
At some point, the curvaton dominates the universe.

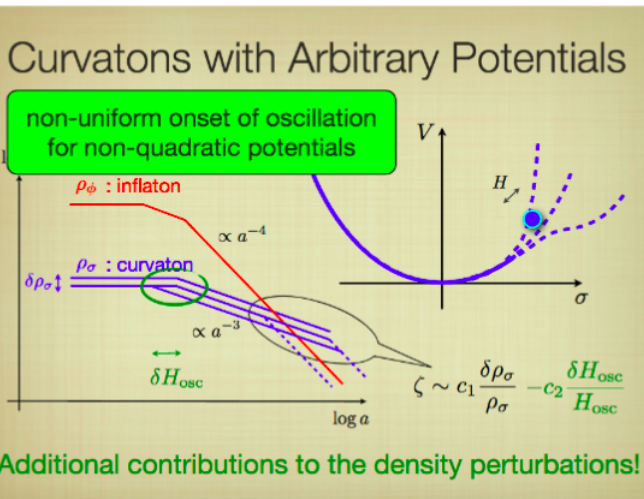
$\rightarrow$  the isocurvature fluc.becomes adiabatic (curvature) fluc.



## The Quadratic Curvaton Scenario

: a light field with negligible energy density during inflation







## Density Perturbations

$$\mathcal{P}_\zeta = \left( \frac{\partial \mathcal{N}}{\partial \sigma_*} \frac{H_*}{2\pi} \right)^2$$

$$\frac{\partial \mathcal{N}}{\partial \sigma_*} = \frac{r}{4 + 3r} (1 - X(\sigma_{\text{osc}}))^{-1} \left\{ \frac{V'(\sigma_{\text{osc}})}{V(\sigma_{\text{osc}})} - \frac{3X(\sigma_{\text{osc}})}{\sigma_{\text{osc}}} \right\} \frac{V'(\sigma_{\text{osc}})}{V'(\sigma_*)}$$

$$r \equiv \frac{\rho_\sigma}{\rho_r} \quad \text{@ curvaton decay}$$

$$* : \text{@ horizon exit}$$

$$\text{osc} : \text{@ onset of curvaton oscillation}$$

$$X(\sigma_{\text{osc}}) \equiv \frac{1}{2(c-3)} \left( \frac{\sigma_{\text{osc}} V''(\sigma_{\text{osc}})}{V'(\sigma_{\text{osc}})} - 1 \right)$$

: effects due to non-uniform onset of oscillation

$$n_s - 1 \simeq \frac{2 m_0^2}{3 H_k^2} \cos \theta_k + 2 \frac{\dot{H}_k}{H_k^2}$$

## Density Perturbations

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$$* : @ \text{ horizon exit}$$

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: effects due to non-uniform onset of oscillation

$$n_s - 1 \simeq \frac{2 m_0^2}{3 H_k^2} \cos \theta_k + 2 \frac{\dot{H}_k}{H_k^2}$$

## Non-Gaussianity

$$f_{\text{NL}} = \frac{40(1+r)}{3r(4+3r)} + \frac{5(4+3r)}{6r} \left\{ \frac{V'(\sigma_{\text{osc}})}{V(\sigma_{\text{osc}})} - \frac{3X(\sigma_{\text{osc}})}{\sigma_{\text{osc}}} \right\}^{-1} \left[ (1-X(\sigma_{\text{osc}}))^{-1} X'(\sigma_{\text{osc}}) \right. \\ \left. + \left\{ \frac{V'(\sigma_{\text{osc}})}{V(\sigma_{\text{osc}})} - \frac{3X(\sigma_{\text{osc}})}{\sigma_{\text{osc}}} \right\}^{-1} \left\{ \frac{V''(\sigma_{\text{osc}})}{V(\sigma_{\text{osc}})} - \frac{V'(\sigma_{\text{osc}})^2}{V(\sigma_{\text{osc}})^2} - \frac{3X'(\sigma_{\text{osc}})}{\sigma_{\text{osc}}} + \frac{3X(\sigma_{\text{osc}})}{\sigma_{\text{osc}}^2} \right\} \right. \\ \left. \left. + \frac{V''(\sigma_{\text{osc}})}{V'(\sigma_{\text{osc}})} - (1-X(\sigma_{\text{osc}})) \frac{V''(\sigma_*)}{V'(\sigma_{\text{osc}})} \right] \right]$$

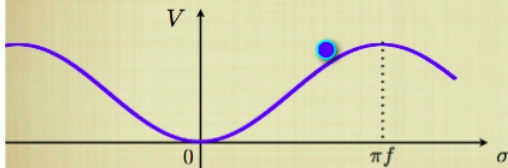
$$r \equiv \frac{\rho_\sigma}{\rho_r} \text{ @ curvaton decay}$$

\* : @ horizon exit  
osc : @ onset of curvaton oscillation

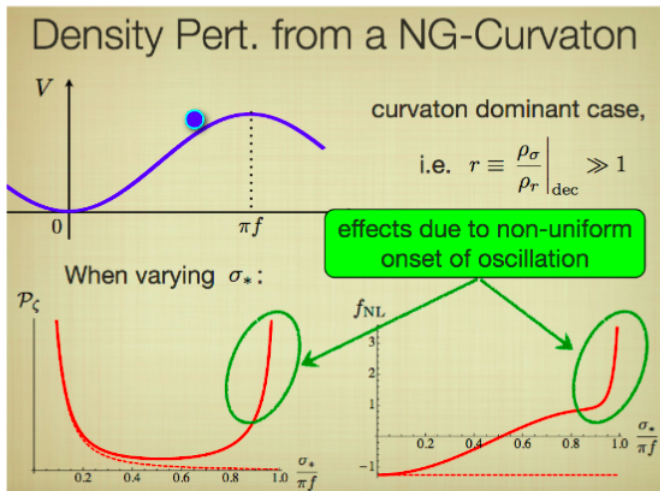
$$\left( \begin{array}{l} \text{cf. quadratic curvatons : } f_{\text{NL}} \sim \frac{1}{r} \\ f_{\text{NL}} \gg 1 \text{ only for curvatons decaying when subdominant ( } r \ll 1 \text{ )} \end{array} \right)$$

pseudo-NG boson of a broken U(1)  
symmetry

$$V(\sigma) = \Lambda^4 \left[ 1 - \cos\left(\frac{\sigma}{f}\right) \right]$$



$$\text{curvaton decay rate : } \Gamma_\sigma \sim \frac{1}{16\pi} \frac{m^3}{f^2} = \frac{1}{16\pi} \frac{\Lambda^6}{f^5}$$



Equations of motion to solve numerically

$$H_{inf}^2 = \frac{1}{3M_{\text{Pl}}^2} (\rho_r + \rho_a)$$

$$\dot{\rho}_r + 4H\rho_r = 0$$

$$\rho_a = \frac{1}{2}\dot{a}^2 + V(a)$$

$$\ddot{a} + 3H\dot{a} + \frac{dV(a)}{da} = 0$$

Convert these into the variable Nefolds. We then integrate from the time ALP starts to oscillate ( $m_a = H$ ) till the time when the ALP decays ( $\Gamma_a = H$ ) and then plug it into the definition of  $P_\zeta$ ,  $f_{NL}$  and  $g_{NL}$ .

Majoron neutrino mass model from strongly-coupled sector:

We will consider the simplest Majoron model [47, 48] with a global baryon minus lepton number  $U(1)_{B-L}$  symmetry under which three SM singlet fermions  $N_i$  carry charge  $-1$  and a complex scalar field  $\sigma$  carries charge  $2$ . The relevant new interactions for us is

$$-\mathcal{L} \supset \frac{1}{2}\xi_i\sigma\overline{N}_i^c N_i + \lambda_{\alpha i}\overline{\ell}_{\alpha i}\epsilon H^\dagger N_i + y_\alpha\overline{\ell}_\alpha H e_\alpha + \text{H.c.}, \quad (1)$$

where  $\ell_\alpha$  and  $H$  are respectively the SM lepton and Higgs doublet ( $\epsilon$  is the  $SU(2)$  anti-symmetry tensor) and without loss of generality, we work in the basis where dimensionless couplings  $\xi_i$  and  $y_\alpha$  are real and diagonal while  $\lambda$  remains a complex  $3\times 3$  matrix. After  $\sigma$  acquires a vacuum expectation value  $v_{B-L}$  [MHR: along the radial direction],

$$\sigma = (v_{B-L} + \rho) e^{i\chi/v_{B-L}}, \quad (2)$$

we have

$$-\mathcal{L} \supset \frac{1}{2}M_i\overline{N}_i^c N_i e^{i\chi/v_{B-L}} + \frac{1}{2}\xi_i\rho e^{i\chi/v_{B-L}}\overline{N}_i^c N_i + \lambda_{\alpha i}\overline{\ell}_{\alpha i}\epsilon H^\dagger N_i + y_\alpha\overline{\ell}_\alpha H e_\alpha + \text{H.c.} \quad (3)$$

Leptonic current:

$$\mathcal{L} \supset -\frac{\partial_\mu \chi}{2v_{B-L}} J_{B-L}^\mu, \quad (4)$$

where  $J_{B-L}^\mu = \sum_f q_f^{B-L} \bar{f} \gamma^\mu f$  is the  $B-L$  current. Carrying out integration by parts in the action and discarding the surface term, we have

$$\mathcal{L} \supset \frac{\chi}{2v_{B-L}} \partial_\mu J_{B-L}^\mu = \frac{i\chi}{v_{B-L}} M_i \bar{N}_i^c N_i, \quad (5)$$



Decay of the Majoranon as the curvaton:

mass  $m_\chi$  to  $\chi$  by assuming that  $U(1)_{B-L}$  is anomalous under a new gauge interaction which becomes strong at  $\Lambda < v_{B-L}$ . If  $m_\chi > 2M_i$  for some  $i$ , the decay width of  $\chi \rightarrow N_i N_i$  is given by<sup>3</sup>

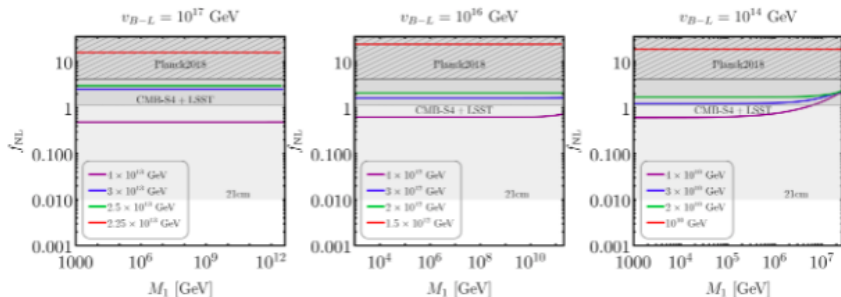
$$\Gamma_\chi = \frac{m_\chi M_i^2}{16\pi v_{B-L}^2} \sqrt{1 - \frac{4M_i^2}{m_\chi^2}}. \quad (6)$$

For definiteness, we will assume that  $m_\chi > 2M_1$  while  $m_\chi \ll M_2, M_3$  such that the dominant channel is  $\chi \rightarrow N_1 N_1$ . As we will discuss in the next section, the decay to the new sector through anomaly is either loop-suppressed or forbidden in the absence of light states.

Mass of the right-handed neutrino  $M_i = v_{B-L} \times \xi_i$

# f<sub>NL</sub> as probe of seesaw and leptogenesis

Probing seesaw breaking scale via Non-Gaussianity:



Large  $f_{NL}$  can be reached, and verified in next generation CMB, 21-cm and LSS experiments. Scale on Inflation given in the inset.

Testing leptogenesis via Non-Gaussianity:

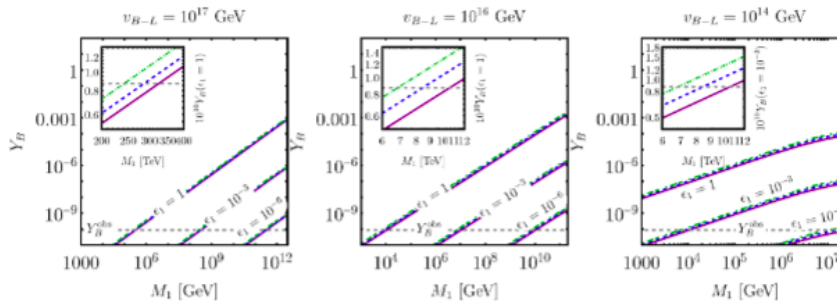





FIG. 4: *Left:*  $Y_B/d$  as a function of the right-handed neutrino mass  $M_1$  for  $v_{B-L} =$  and different  $\epsilon_1$  for different  $H_{inf}$ . Different colors and line styles correspond to different in Fig. 3. Dashed horizontal line corresponds to  $Y_B^{obs}$ . *Right:* zoomed near the region produced baryon asymmetry accounting for dilution matches the observed asymmetry, for

If you have interactions present for inflaton, you will have Non-gaussianity.  
Non-Gaussianity  $f_{\text{NL}}$  as novel probe of baryogenesis: ongoing work, stay tuned !

## Cosmological probes of Grand Unification:

*Primordial Blackholes & scalar-induced Gravitational Waves*

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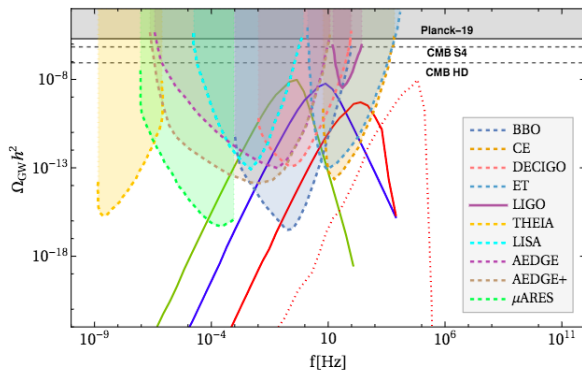


FIG. 5: The spectrum of scalar induced gravitational waves (solid red, blue and green curves for  $v = 0.1, 0.8, 10 M_{\text{Pl}}$ ) versus current and future GW detectors constraints. We have used the parameter values in Table I. The red dotted curve corresponds to  $v = 10 M_{\text{Pl}}$ , but with a modified value of the tensor to scalar ratio,  $r \sim 0.064$ .

## Gravitational Wave Pathway to Testable Leptogenesis

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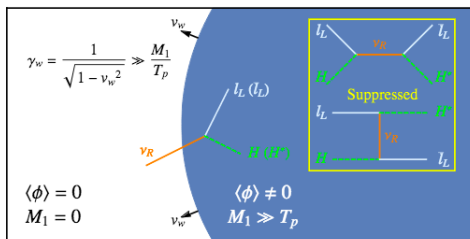
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We analyze the classically scale-invariant  $B - L$  model in the context of resonant leptogenesis with the recently proposed mass-gain mechanism. The  $B - L$  symmetry breaking in this scenario is associated with a strong first order phase transition that gives rise to detectable gravitational waves (GWs) via bubble collisions. The same  $B - L$  symmetry breaking also gives Majorana mass to right-handed neutrinos inside the bubbles, and their out of equilibrium decays can produce the observed baryon asymmetry of the Universe via leptogenesis. We show that the current LIGO-VIRGO limit on stochastic GW background already excludes part of the  $B - L$  parameter space, complementary to the collider searches for heavy  $Z'$  resonances. Moreover, future GW experiments like Einstein Telescope and Cosmic Explorer can effectively probe the parameter space of leptogenesis over a wide range of the  $B - L$  symmetry-breaking scales and gauge coupling values.



**Figure 1.** The sketch of leptogenesis triggered by a FOPT. The blue and white regions represent the new vacuum bubble (in which  $\langle \phi \rangle \neq 0$ ) and the old vacuum background (in which  $\langle \phi \rangle = 0$ ), respectively. The FOPT occurs at temperature  $T_p$ , and the bubble expands at a wall velocity  $v_w$ . Inside a bubble,  $\nu_R$  gains a huge mass  $M_1 \gg T_p$ , such that the  $\nu_R$ 's that have penetrated the bubble decay quickly, generating the BAU. The possible washout effects (some of which are illustrated inside the yellow rectangle) are suppressed since  $M_1/T_p \gg 1$ .



## Bremsstrahlung High-frequency Gravitational Wave Signatures of High-scale Non-thermal Leptogenesis

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Inflaton seeds non-thermal leptogenesis by pair producing right-handed neutrinos in the seesaw model. We show that the inevitable graviton bremsstrahlung associated with inflaton decay can be a unique probe of non-thermal leptogenesis. The emitted gravitons contribute to a high-frequency stochastic gravitational waves background with a characteristic fall-off below the peak frequency. Besides leading to a lower bound on the frequency ( $f \gtrsim 10^{11}$  Hz), the seesaw-perturbativity condition makes the mechanism sensitive to the lightest neutrino mass. For an inflaton mass close to the Planck scale, the gravitational waves contribute to sizeable dark radiation, which is within the projected sensitivity limits of future experiments such as CMB-S4 and CMB-HD.

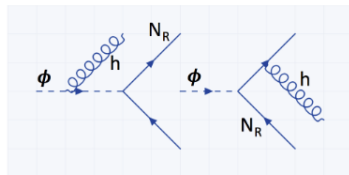


FIG. 1. *Diagrams representing the three-body decay of inflaton ( $\phi$ ) to right-handed neutrinos ( $N_R$ ) and graviton ( $h$ ) bremsstrahlung. A similar diagram with a graviton attached to the incoming fermion line also contributes to the total decay-width. However, the four-point interaction vanishes [44].*

- High-scale baryogenesis (via leptogenesis) testable via **novel Primordial GW spectral shape** analysis.
- LISA and ET GW detectors are going to cover the parameter space where it can test minimal type-I seesaw and successful leptogenesis.
- **Freeze-in Dark Matter** from RHN decay testable in similar fashion.
- Era to think about low scale as well as high scale baryogenesis and leptogenesis models with **complementary GW versus laboratory tests in future.**

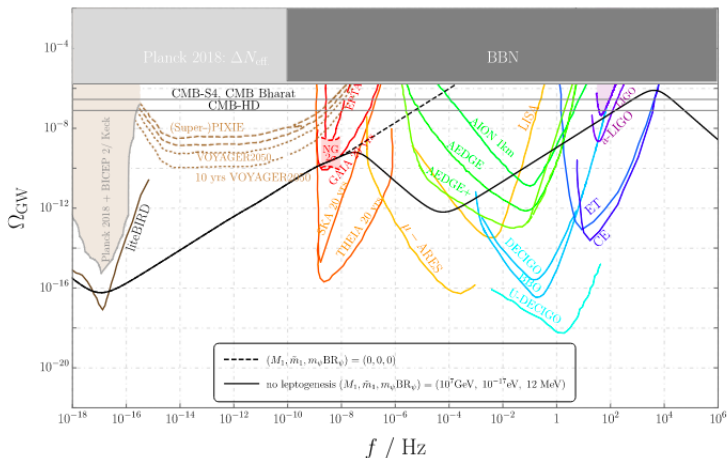
1. In this work, we reinforce the fact that the final amount of lepton asymmetry yield crucially depends on the reheating dynamics of the Universe.
2. We find that such correlations results into very predictive inflationary observable values  $(n_s, r)$ .
3. In the first case, we find that the corresponding bound as obtained, appears stronger than the recent Planck-BICEP data. For example, our analysis reveals that for  $\alpha = 5$ , successful baryogenesis via leptogenesis predicts  $0.9616 \lesssim n_s \lesssim 0.9630$  with  $0.0038 \lesssim r \lesssim 0.00437$ .
4. In the second case, we have a single independent parameter (involving inflaton) which is inflaton to RHN coupling coefficient. We obtain unique correlations between  $y_N$  and  $(n_s - r)$  values for a constant  $\alpha$  that leads to successful baryogenesis in the early Universe. For example,  $\alpha = 5$  requires  $y_N = 2.3 \times 10^{-7}$  to yield correct order of baryon asymmetry which implies  $n_s = 0.9632$  and  $r = 0.015$ .

- High-scale baryogenesis (via leptogenesis) testable via **novel Primordial Non-Gaussianity** signatures.
- Mojaronn which gives neutrino mass can act as the curvaton and generate the entire CMB.
- Next generation experiments measuring fNL will probe seesaw scales and leptogenesis.

THANK YOU

THANK YOU.

# What NanoGrav result may tell you about (testable) baryogenesis:



**Figure 6.** We fix  $M_1 = 10^7 \text{ GeV}$ ,  $\tilde{m}_1 = 10^{-17} \text{ eV}$ ,  $T_{\text{RH}} = 5 \times 10^{12} \text{ GeV}$  and  $n_T = 0.85$  to fit the NANOGRAV anomaly [205]. Furthermore we show the value of  $m_\psi \text{BR}_\psi = 12 \text{ MeV}$  required for the given  $\tilde{m}_1$  to generate the observed dark matter relic abundance.

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Thank You