10th Conference on PET, SPECT, and MR Multimodal Technologies, Total Body and Fast Timing in Medical Imaging

Comparison of Synergistic and Single Modality Anatomically-Informed Structural Priors for Yttrium-90 PET/CT and SPECT/CT Reconstruction

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Complexity

Naive

Solution only informed by proximity to data

- MLEM
- OSEM

Improved Image Quality?

Complexity

Naive

Solution only informed by proximity to data

- MLEM
- OSEM

A-priori

Solution informed by vague idea of how we would expect it to look

- Total Variation Prior
- Quadratic Prior

Improved Image Quality?

Complexity →

Naive	A-priori	Guided
Solution only informed by proximity to data	Solution informed by vague idea of how we would expect it to look	Solution informed by additional evidence
• MLEM • OSEM	• Total Variation Prior • Quadratic Prior	• Directional Total Variation Prior • (Hybrid) Kernelised Expectation Maximisation

Improved Image Quality? →

Complexity →

Naive	A-priori	Guided	Synergistic
Solution only informed by proximity to data	**Solution informed by vague idea of how we would expect it to look**	**Solution informed by additional evidence**	**Information shared across multiple modalities throughout reconstructioin**
• MLEM • OSEM	• Total Variation Prior • Quadratic Prior	• Directional Total Variation Prior • (Hybrid) Kernelised Expectation Maximisation	• Total Nuclear Variation • Mutually Weighted Quadratic Priors

Improved Image Quality? →

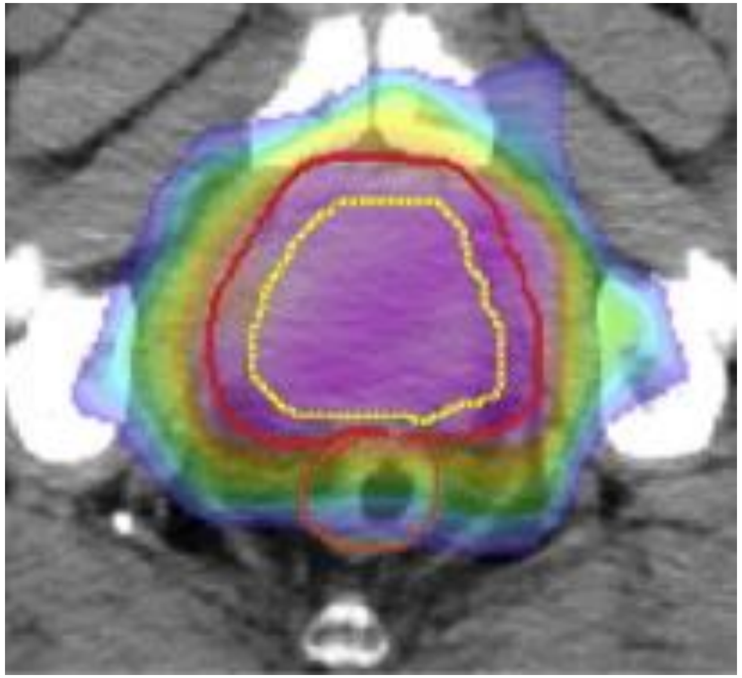
Potential for synergistic reconstruction in MRT

The importance of dosimetry:

Molecular radiotherapy: targeted cancer treatment where radioactive substances are used to destroy or damage cancer cells by binding to specific molecules within the body.

We need to plan & measure dose delivered:

- Patient-tailored treatment
- Reduce chance of side-effects
- Increase accuracy of prognosis



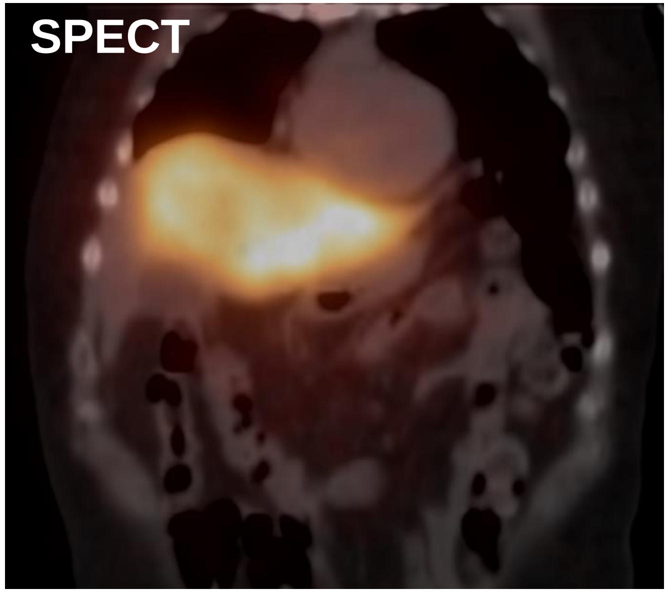
Radiotherapy prostate treatment plan: Vernaste et al (2019)

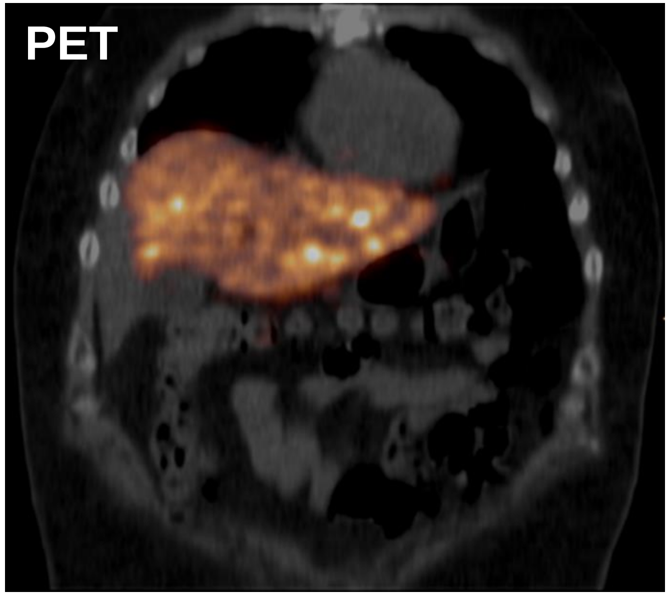
How can we improve quantification (bias and uncertainty) whilst maintaining image quality at fixed cost?

Yttrium-90 SIRT

- **SPECT – low resolution**
  - Spatial resolution ~10 mm
  - Wide spectrum of bremsstrahlung radiation
  - Compton down-scatter
  - ***Lots of photons***

- **PET – very low count**
  - Lots of noise
  - Difficult to distinguish features
  - ***Spatial Resolution ~ 4mm***





OSEM reconstruction of Yttrium 90microsphere treatment Wright, C.L. *et al.* (2015)

The inverse problem

$$\Phi(\mathbf{u})$$

Objective function

The inverse problem

$$\Phi(\mathbf{u}) = \sum_{m=1}^{M} \Psi_m(A_m(u_m), f_m)$$

Data fit (how closely does our estimate fit the measurements?)

The inverse problem

$$\Phi(\mathbf{u}) = \sum_{m=1}^{M} \Psi_m(A_m(u_m), f_m) + \Gamma(\mathbf{u})$$

Prior (how closely does our estimate fit our prior knowledge of the solution?)

The inverse problem

$$\Phi(\mathbf{u}) = \sum_{m=1}^{M} \Psi_m(A_m(u_m), f_m) + \Gamma(\mathbf{u})$$

$$\mathbf{u}^* = \arg\min_{\mathbf{u}} \{\Phi(\mathbf{u})\}$$

Minimise!

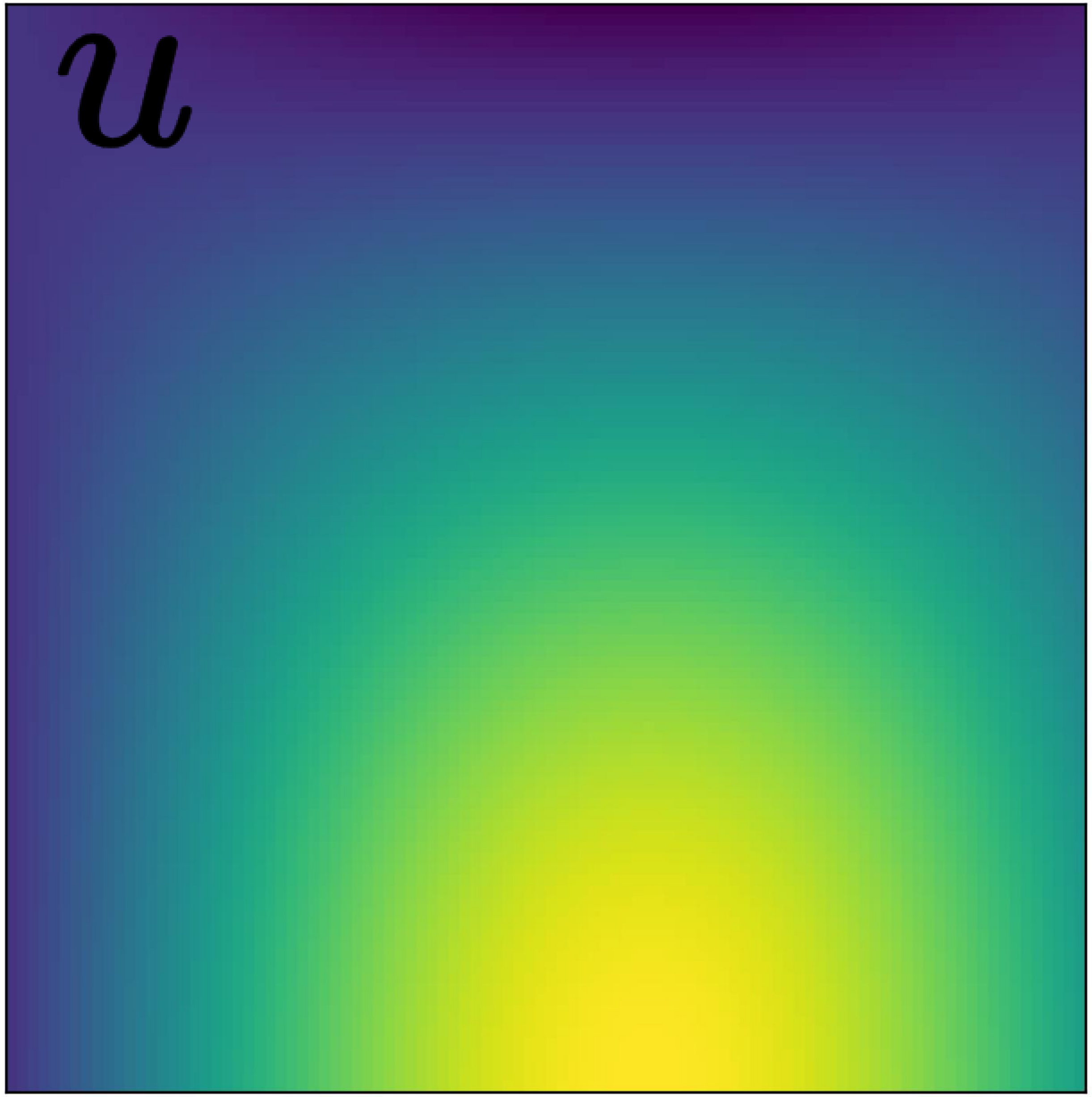
The Priors

Smoothed directional Total Variation (single modality)

$$\Gamma_{m,dTV} = \gamma_m \sum_j \Lambda(\|D_{v,j} \nabla u_m\|_2, \delta)$$

Smoothed directional Total Nuclear Variation (synergistic)

$$\Gamma_{dTNV} = \sum_j \sum_i \Lambda(s(\mathbf{J}_j)_i, \delta)$$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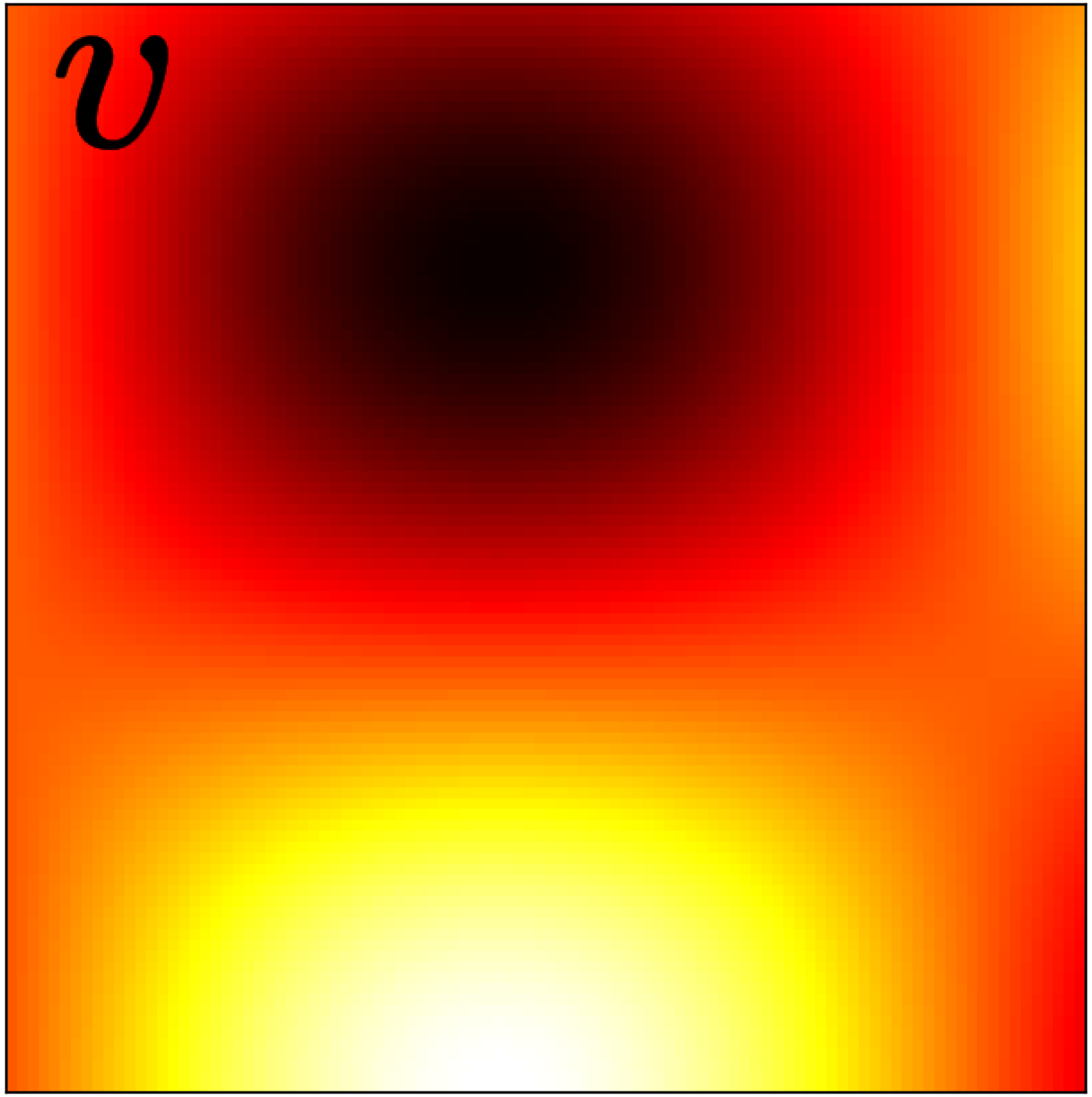
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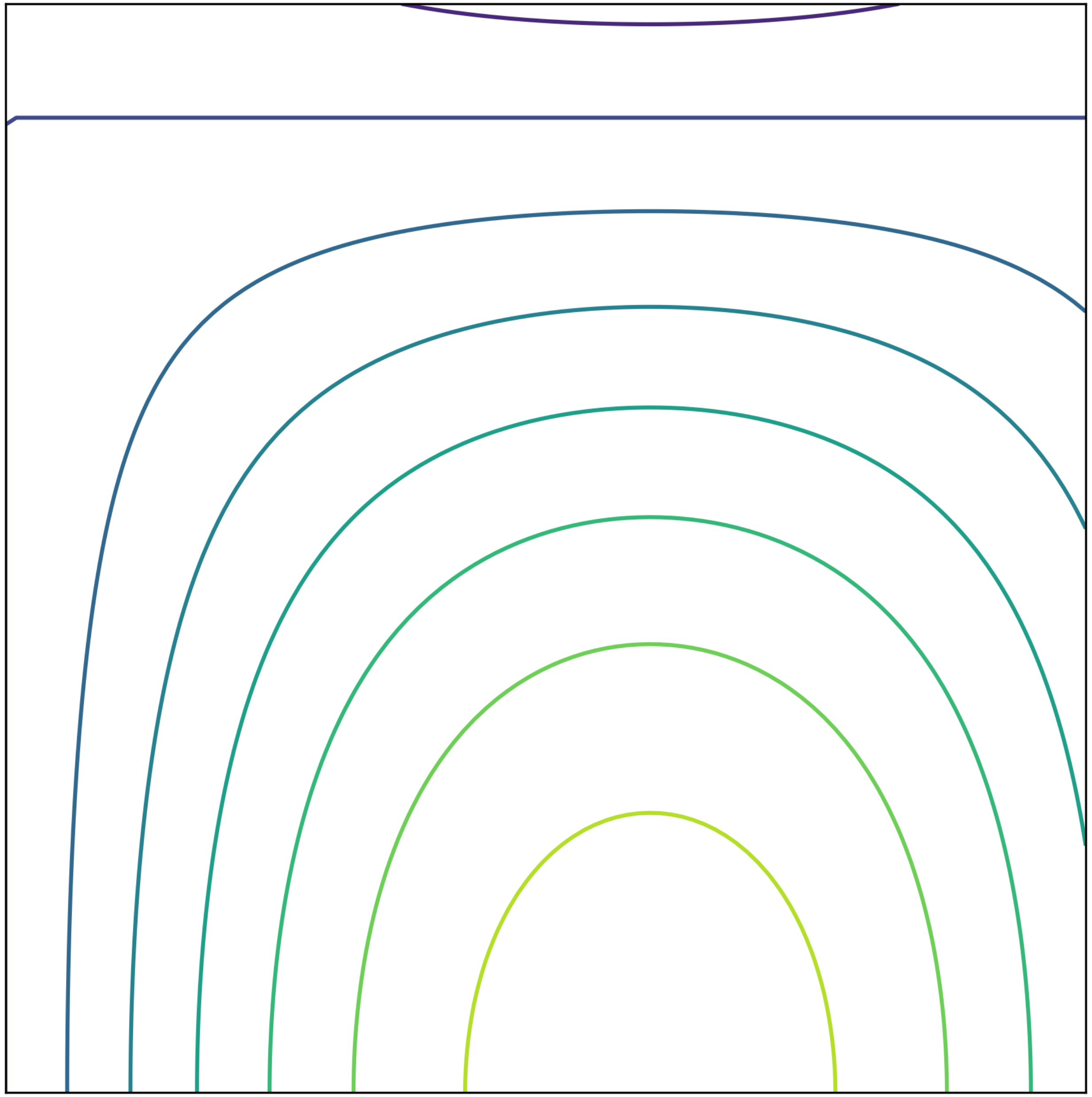
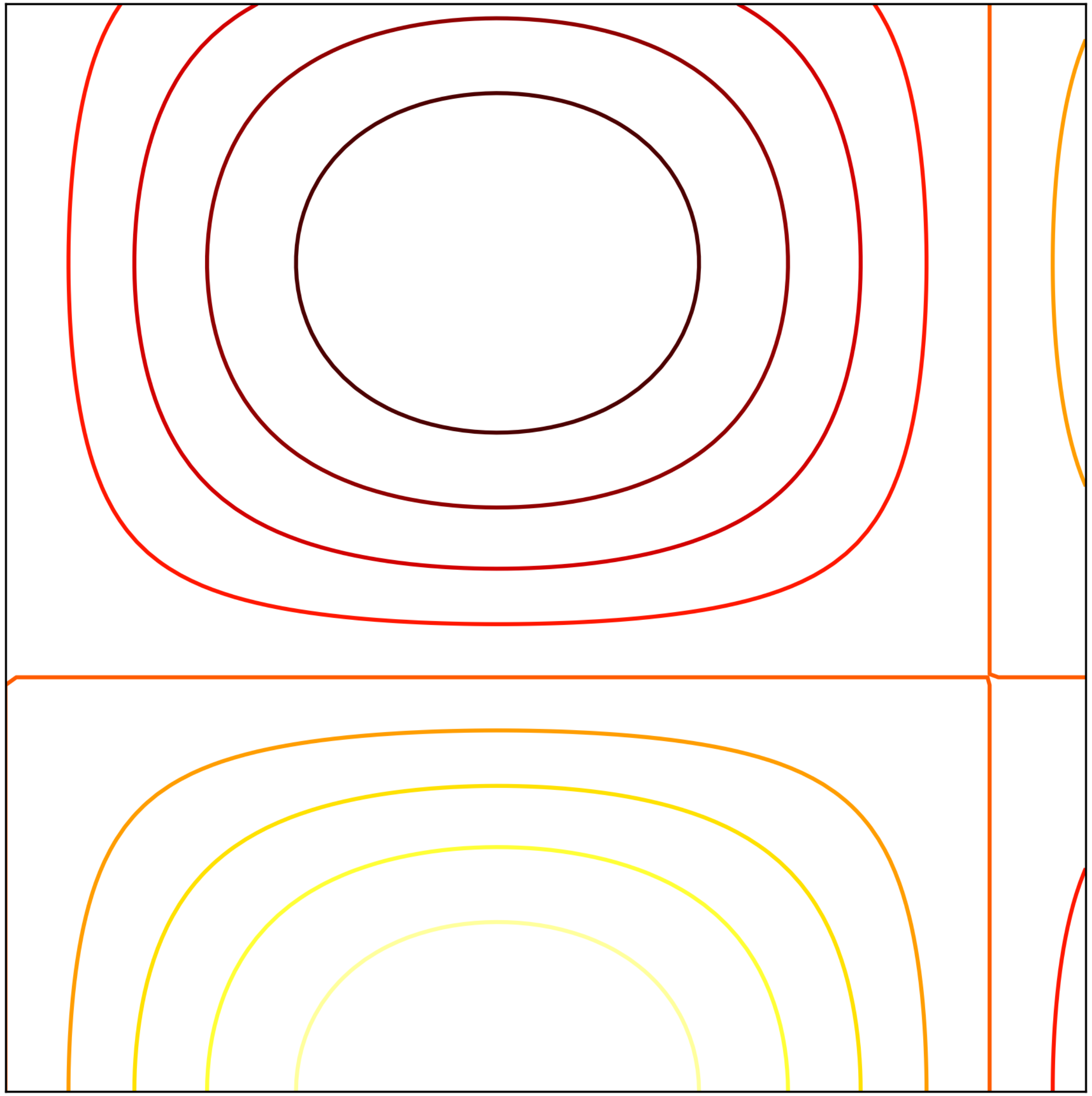
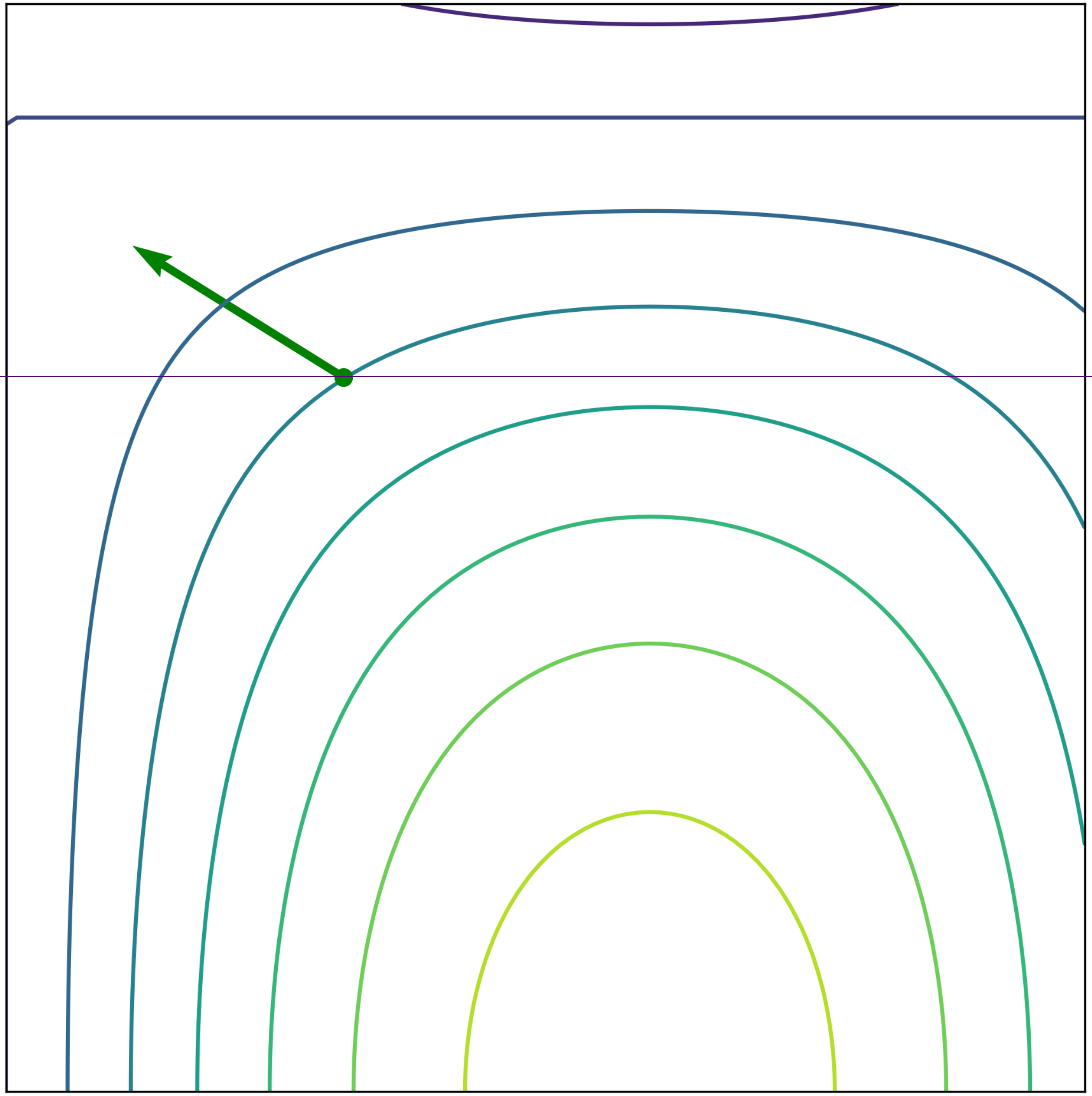
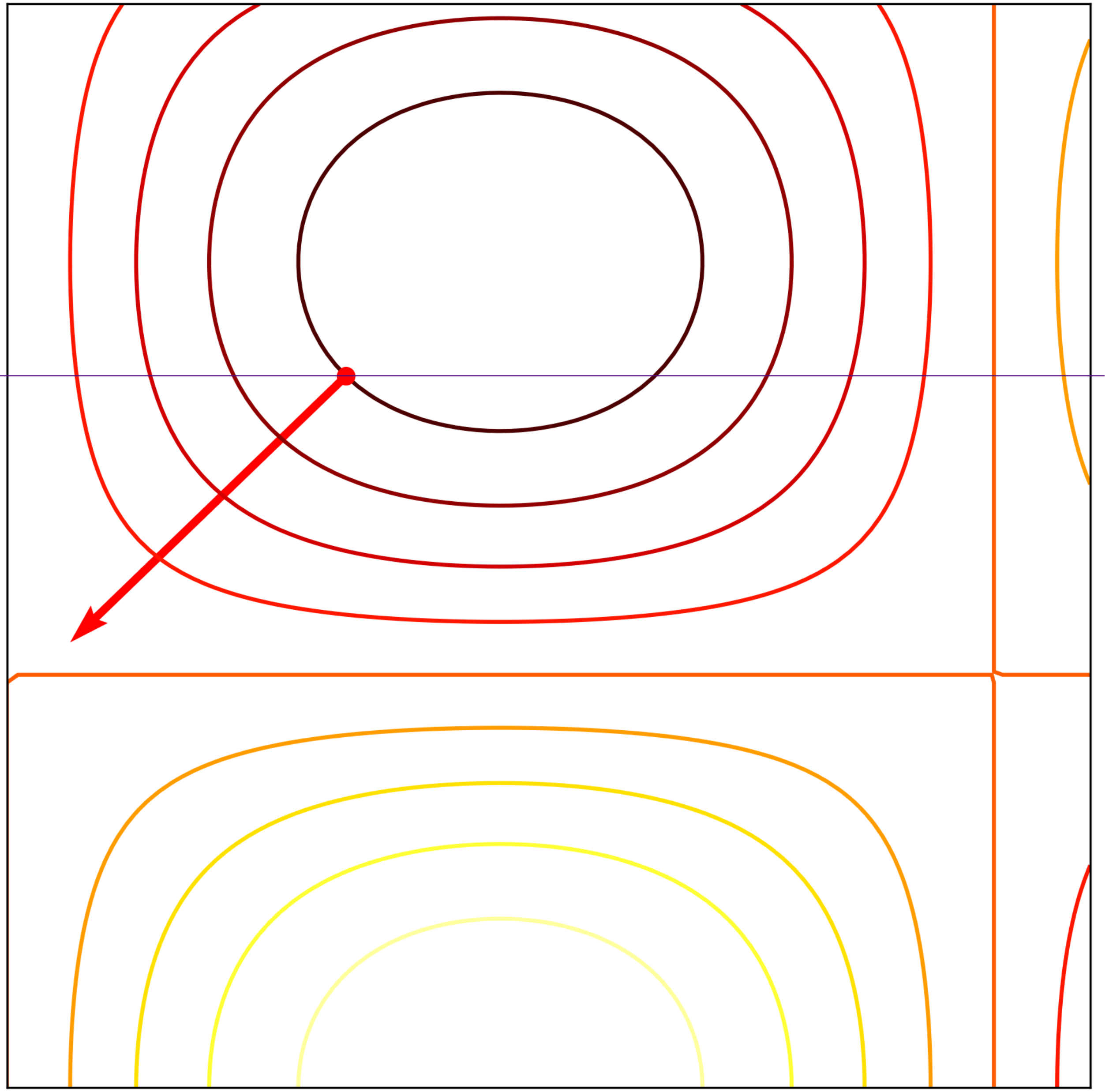
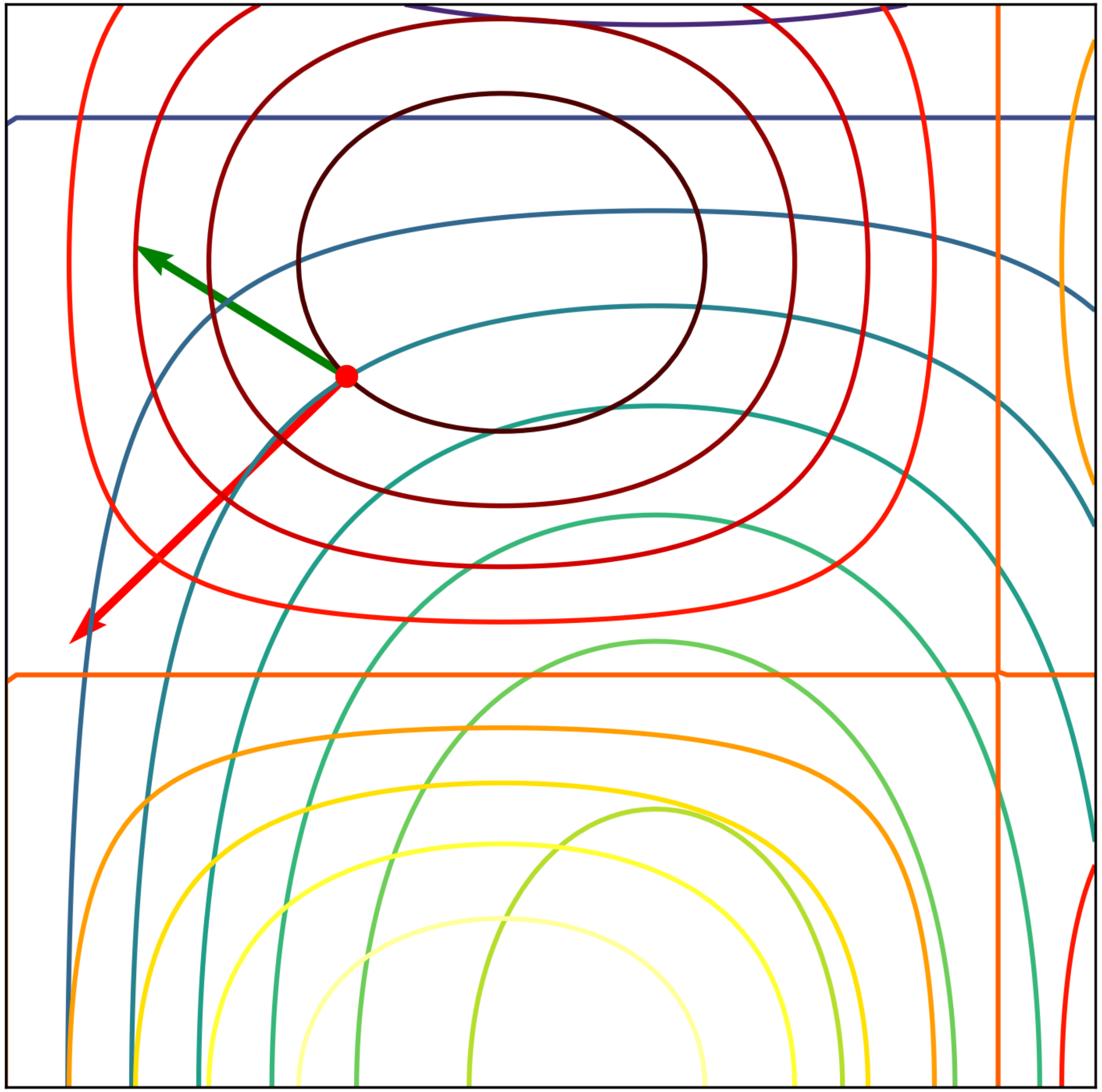
Methods – Parallel Level Sets

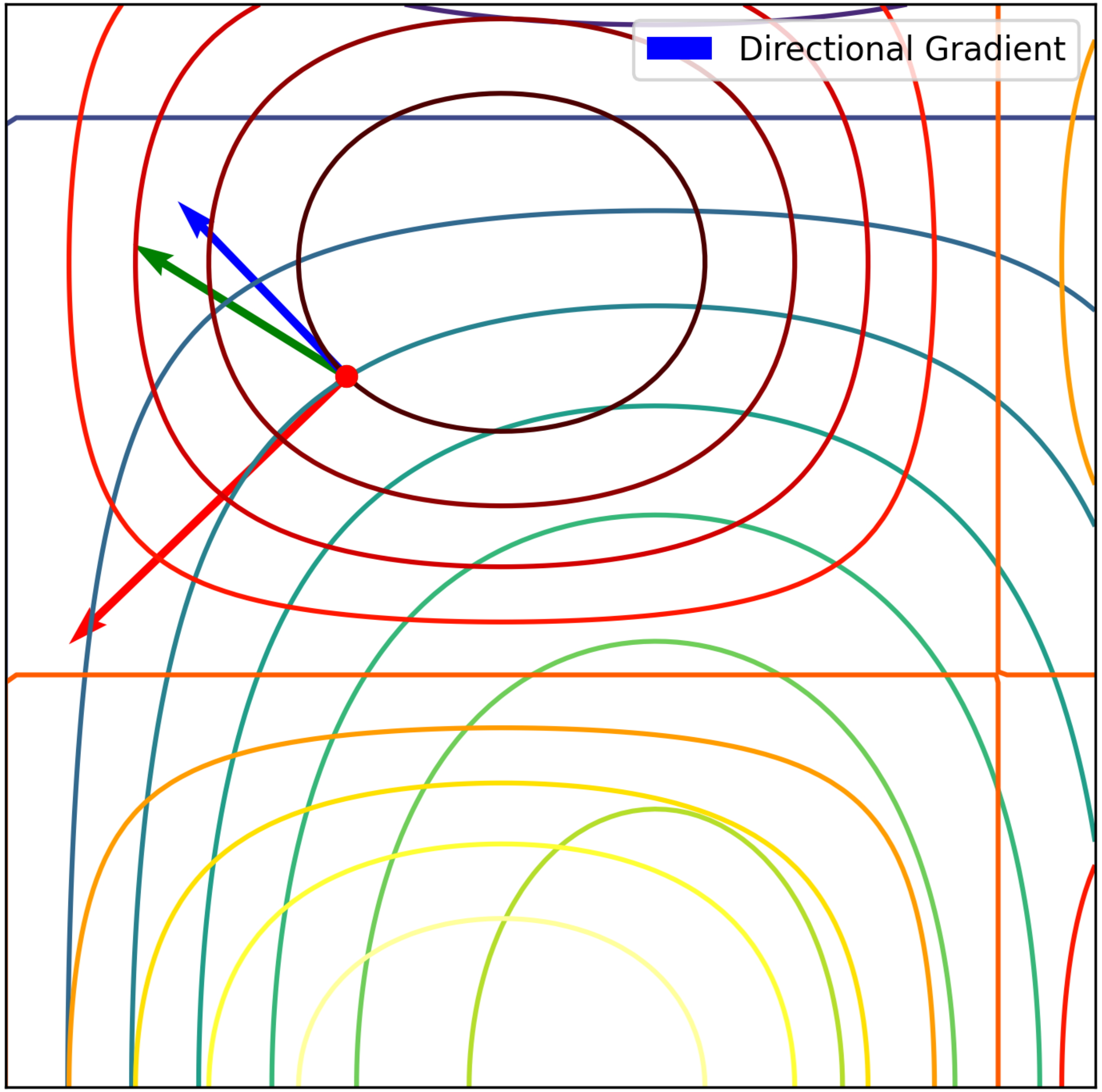






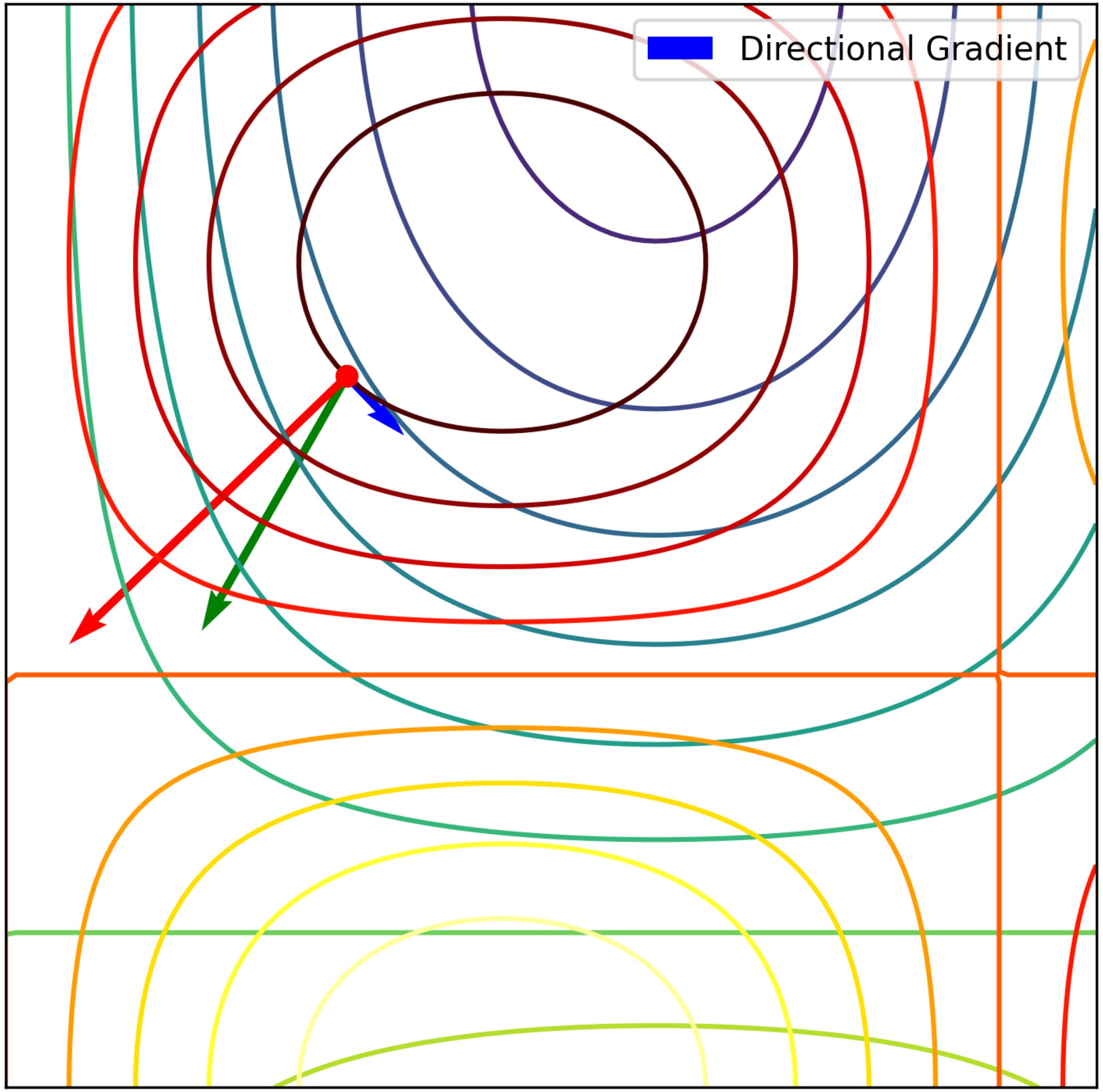






$$\left(1 - \frac{\nabla v}{\|\nabla v\|^2} \nabla v^\top\right) \nabla u$$

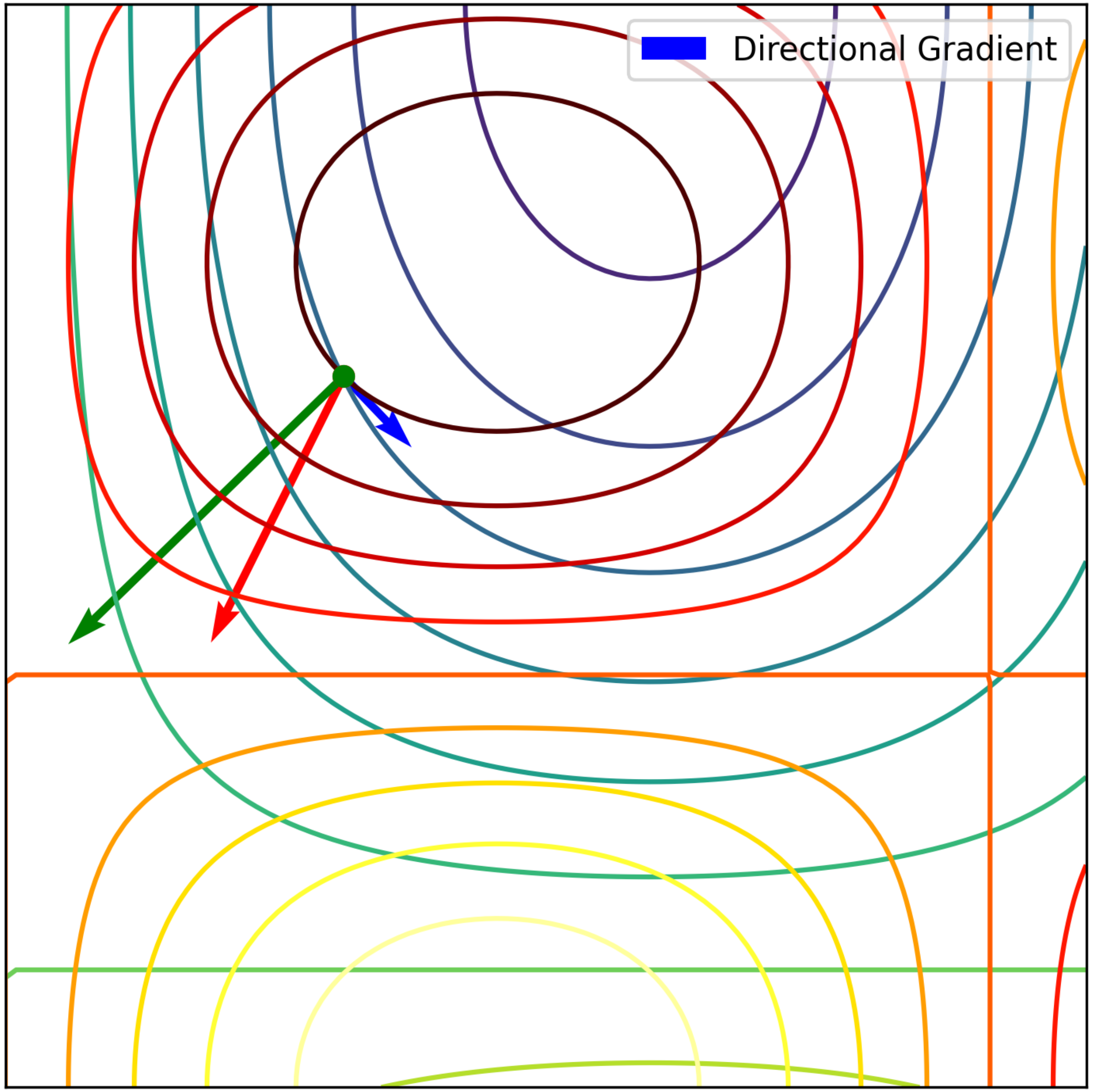
$$D_v \nabla u$$



$$\left(1 - \frac{\nabla v}{\|\nabla v\|^2} \nabla v^\top\right) \nabla u$$

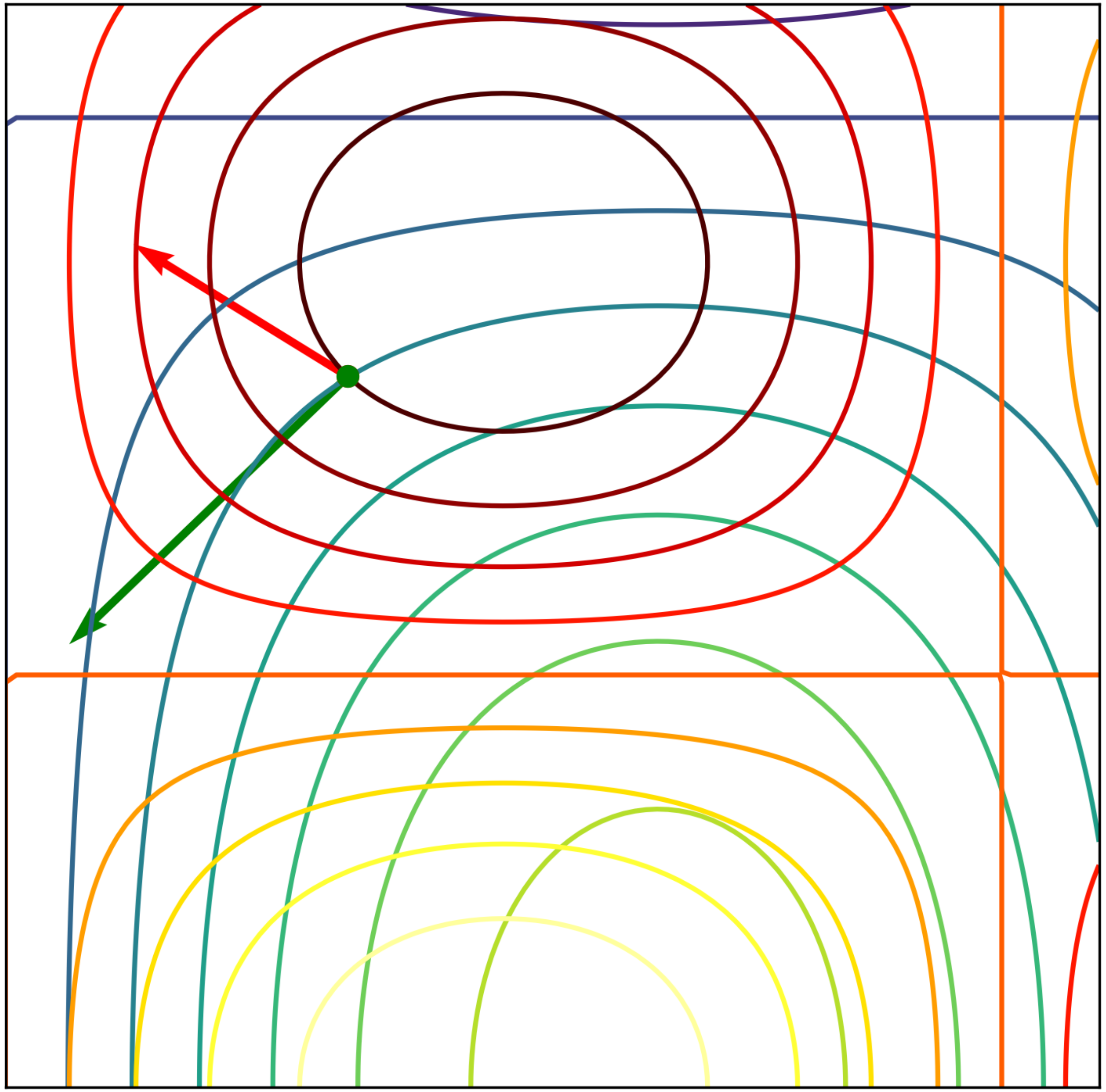
$$D_v \nabla u$$

Methods – directional Total Variation



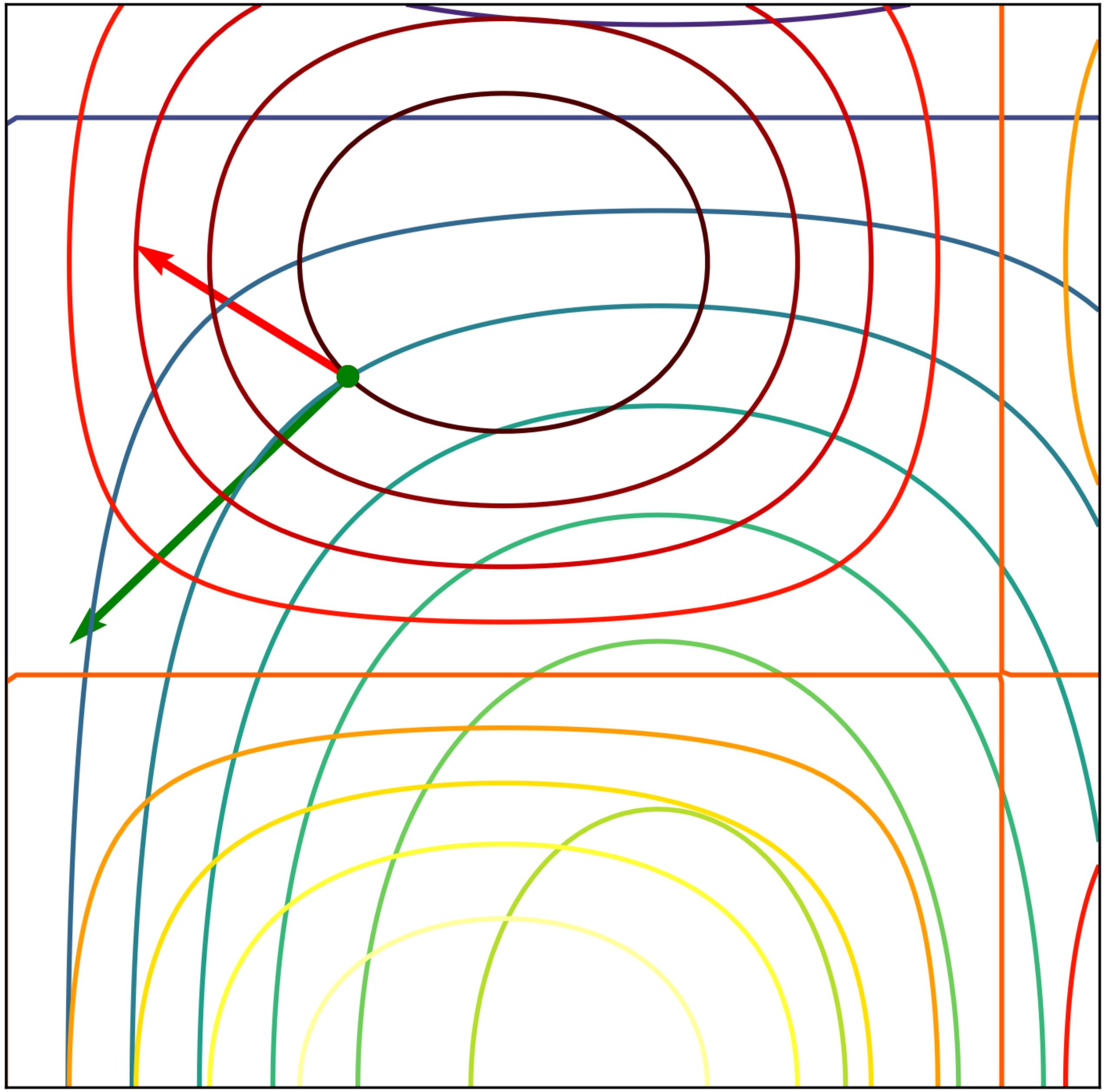
$$\left\| \left(1 - \frac{\nabla v}{\|\nabla v\|^2} \nabla v^\top \right) \nabla u \right\|_2$$

$$\| D_v \nabla u \|_2$$



$$\left\| \begin{matrix} \nabla u^\top \\ \nabla v^\top \end{matrix} \right\|_*$$

- Convex envelope of rank function
- Trying to make the rows/columns the same (or zero)



$$\sum_i s_i\left(\begin{bmatrix}\nabla u^\top \\ \nabla v^\top\end{bmatrix}\right)$$

- Convex envelope of rank function
- Trying to make the rows/columns the same (or zero)

Original

$$dTV(u) = \|D_v \nabla u\|_2$$

$$TNV(u) = \sum_i s_i(J(\nabla u_1, \nabla u_2))$$

* s(J) = singular value of matrix J

Original

$$dTV(u) = \|D_v \nabla u\|_2$$

$$TNV(u) = \sum_i s_i(J(\nabla u_1, \nabla u_2))$$

Combine

$$dTNV(u) = \sum_i s_i(J(D_v \nabla u_1, D_v \nabla u_2))$$

* s(J) = singular value of matrix J

Methods – The Priors

Original

$$dTV(u) = \|D_v \nabla u\|_2$$

$$TNV(u) = \sum_i s_i(J(\nabla u_1, \nabla u_2))$$

Combine

$$dTNV(u) = \sum_i s_i(J(D_v \nabla u_1, D_v \nabla u_2))$$

Smooth

$$dTV(u) = \Lambda(\|D_v \nabla u\|_2)$$

$$dTNV(u) = \sum_i \Lambda(s_i(J(D_v \nabla u_1, D_v \nabla u_2)))$$

* s(J) = singular value of matrix J

Data

- **NEMA phantom**
  - 6 hot spheres (11-37)mm diameter
  - Total Activity for all spheres 187±4MBq
  - Cold water background
- **Mediso Trio Anyscan triple-modality scanner**
  - **SPECT**:
    - 120 60s projections
    - Energy window (75-225)keV
    - MLEGP collimator
    - 54.5cm axial FoV
  - **PET**:
    - 60 minute scan
    - 38 ring, 15cm axial FoV

Reconstruction setup

$$\Gamma_{dTNV} = \sum_j \sum_i \Lambda \left(s \left(\begin{bmatrix} \alpha (D_{v,j} \nabla u_{1,j})^\top \\ \beta (D_{v,j} \nabla u_{2,j})^\top \end{bmatrix} \right) \right)$$

- Scatter:
  - PET: Mediso vendor scatter
  - SPECT SIMIND simulation of smoothed 10 epoch 12 subset OSEM recon.

- Forward Model:
  - STIR/ParallelProj PET projector
  - STIR SPECTUB
  - + Resolution modelling

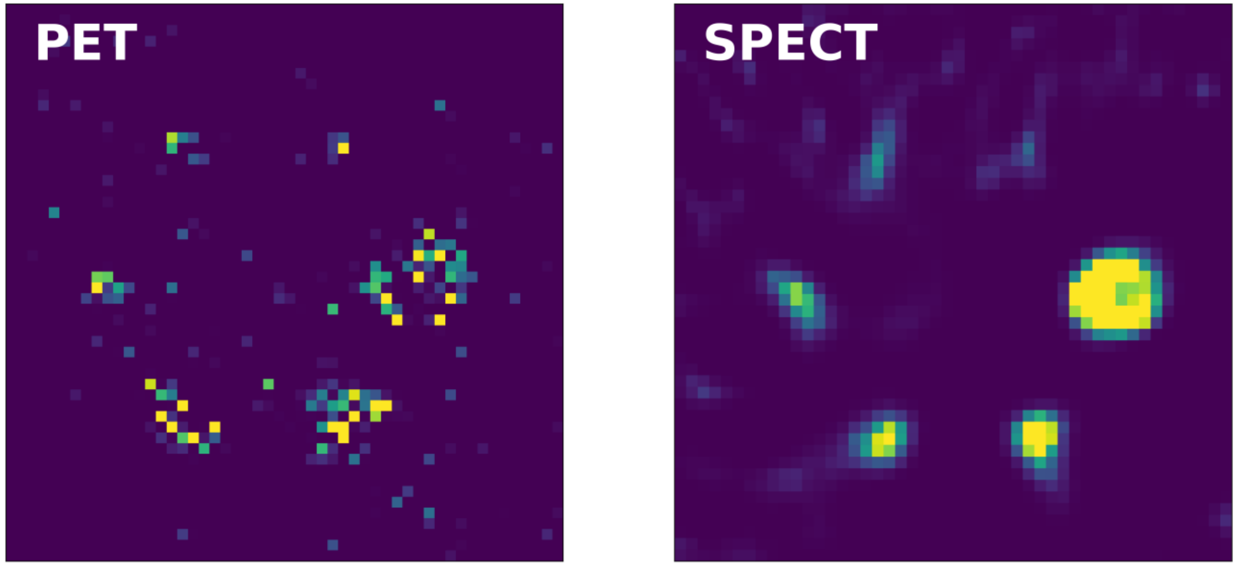
- Reconstructed with modified BSREM
  - Initial stepsize = 2
  - Relaxation parameter = 0.05
  - 12 subsets of PET/SPECT
  - 50 epochs reconstruction
  - α β shown in table

α	β
0.0	0.0
32.0	0.0625
64.0	0.125
96.0	0.25
128.0	0.5
192.0	0.75
256.0	1.0
512.0	2.0

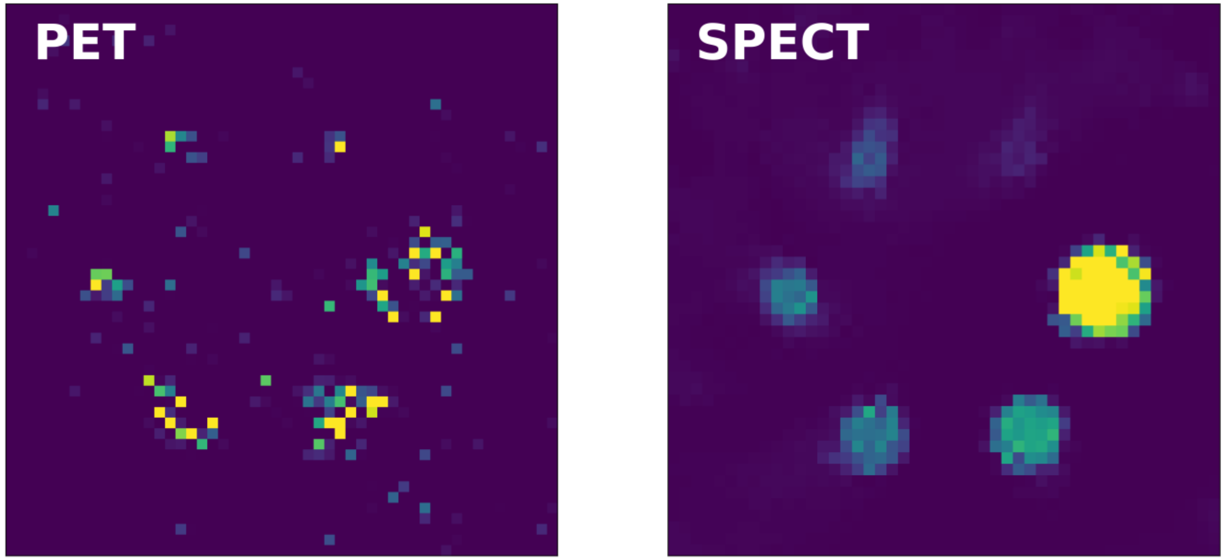
- Compared to dTNV reconstructions to dTV reconstructions with same α β values

Results - Visual

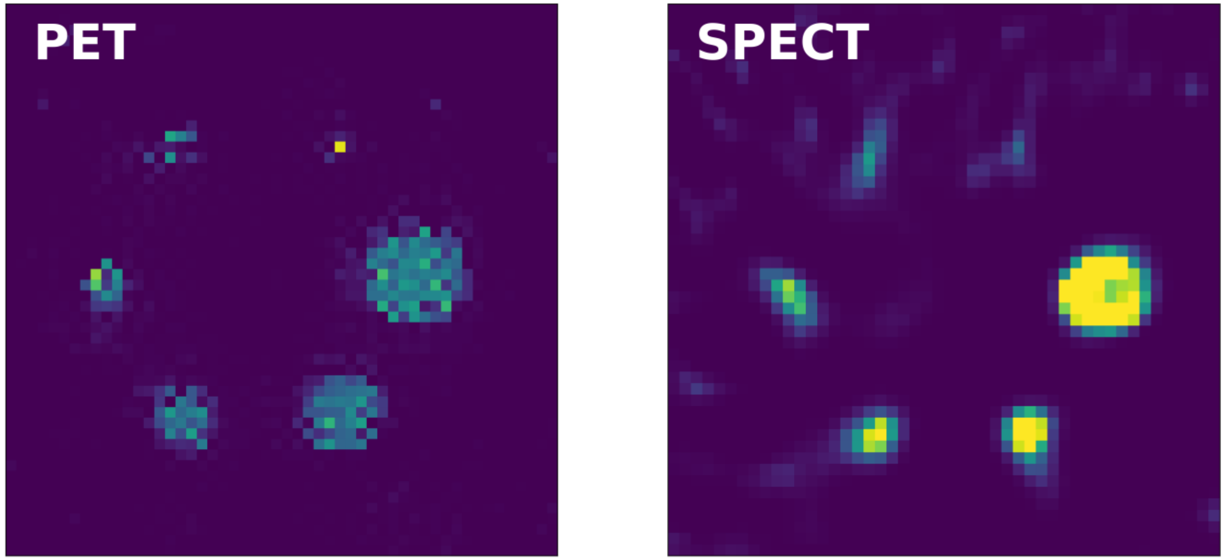
$\alpha = 32, \beta = 0.0625$



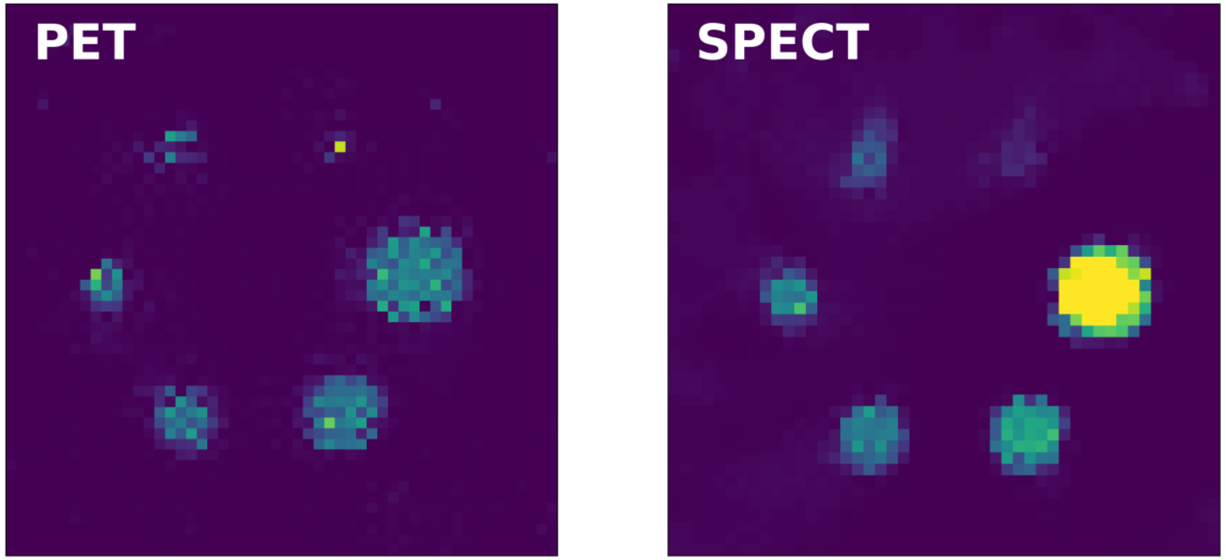
$\alpha = 32, \beta = 2$



$\alpha = 512, \beta = 0.0625$

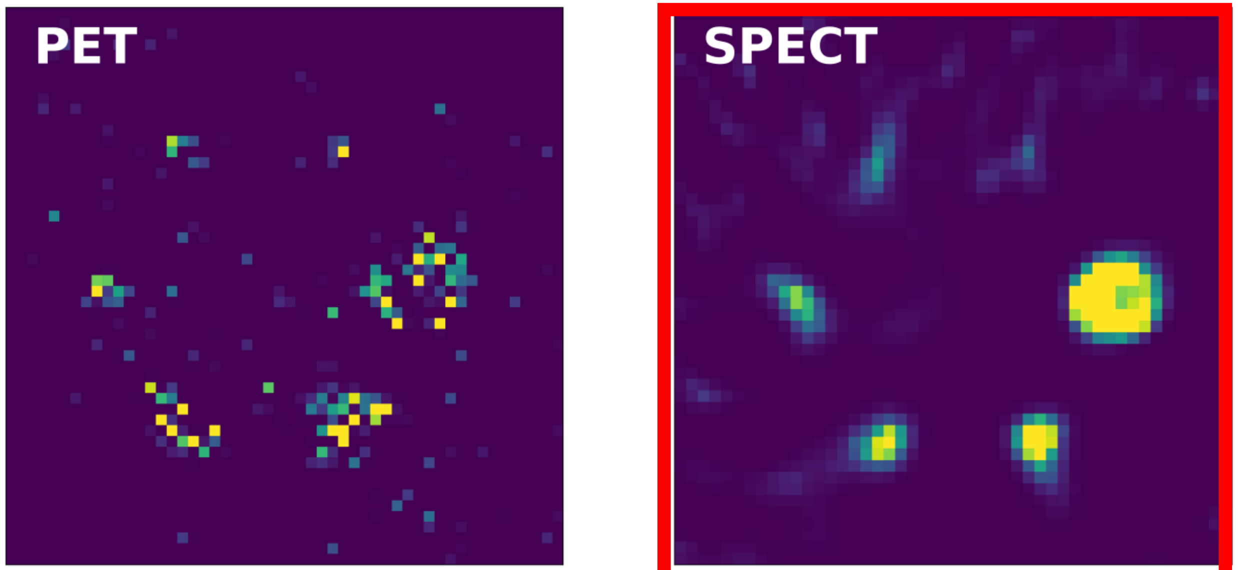


$\alpha = 512, \beta = 2$

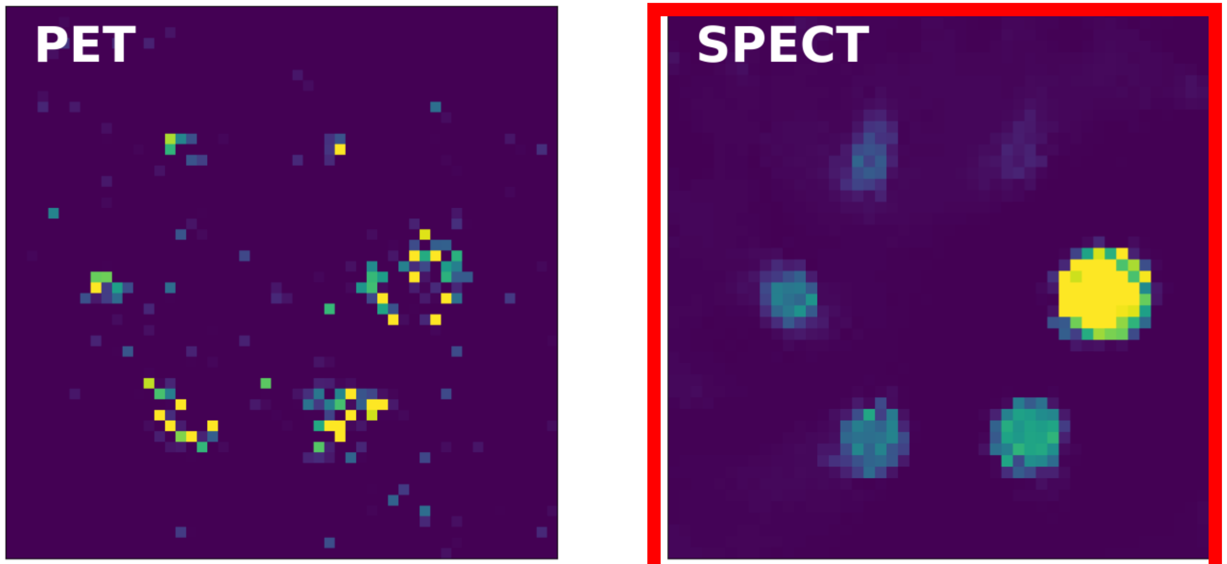


Results - Visual

$\alpha = 32, \beta = 0.0625$



$\alpha = 32, \beta = 2$



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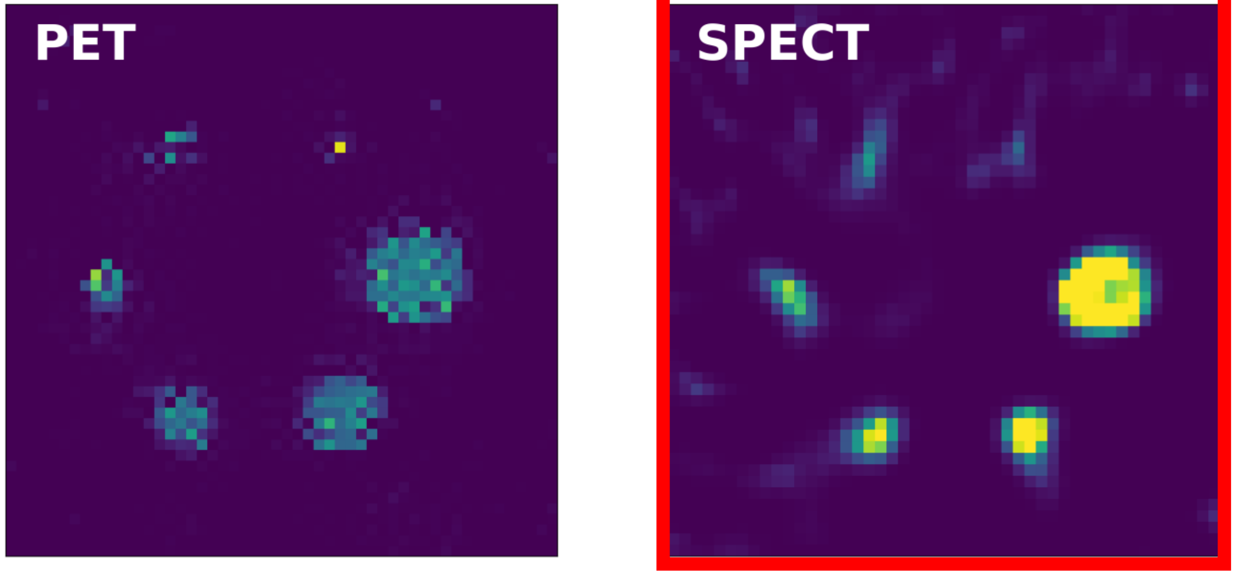
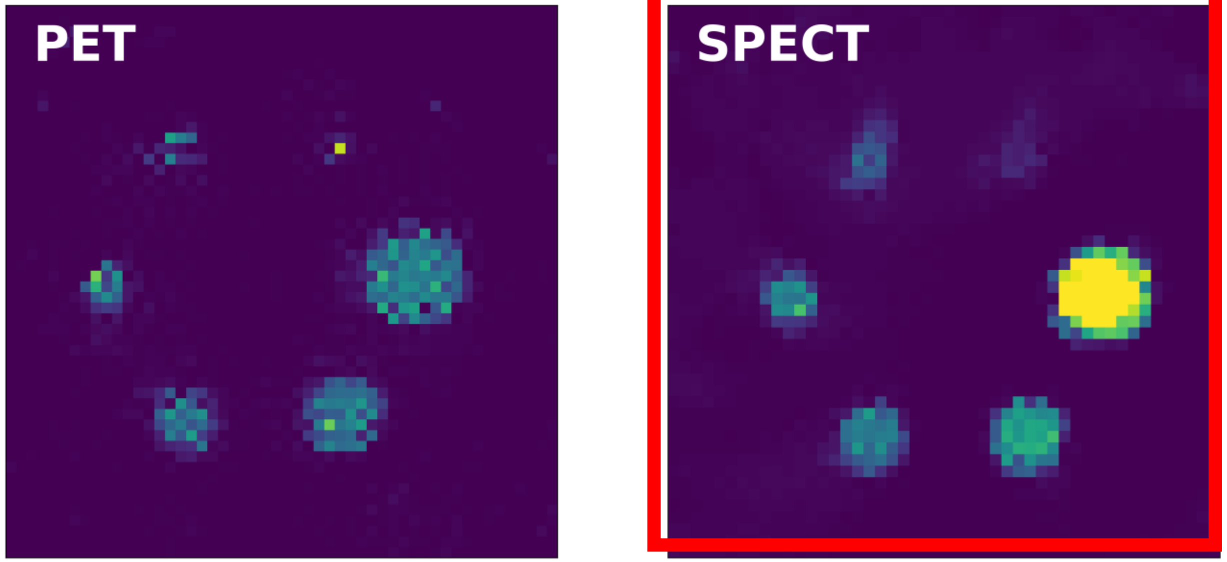
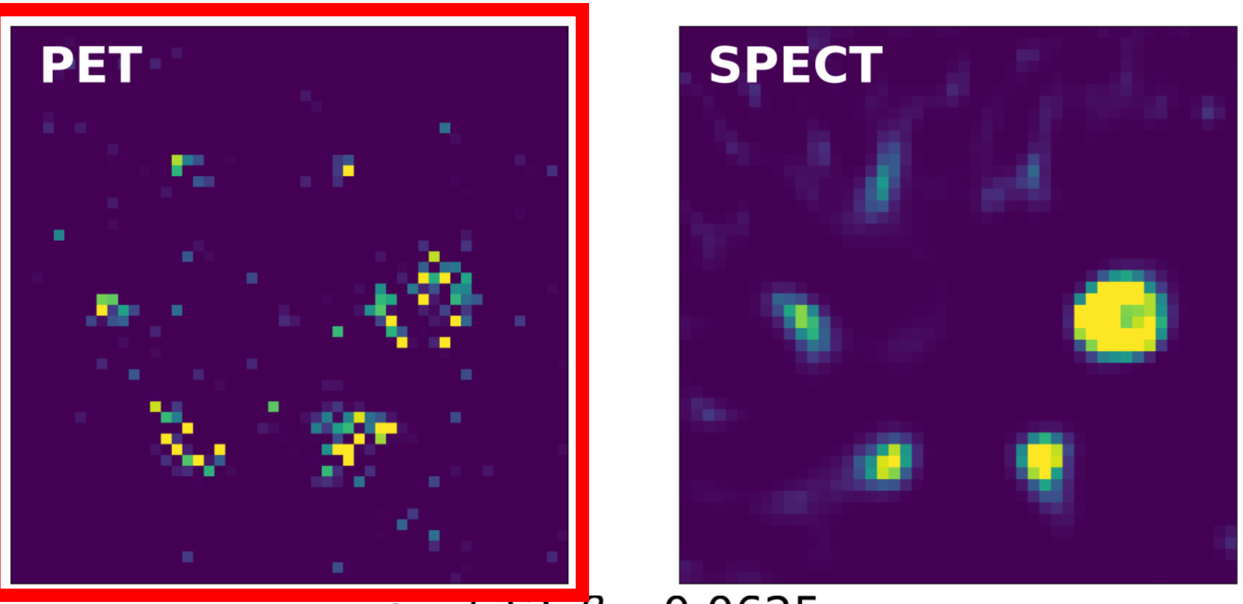


$\alpha = 512, \beta = 2$

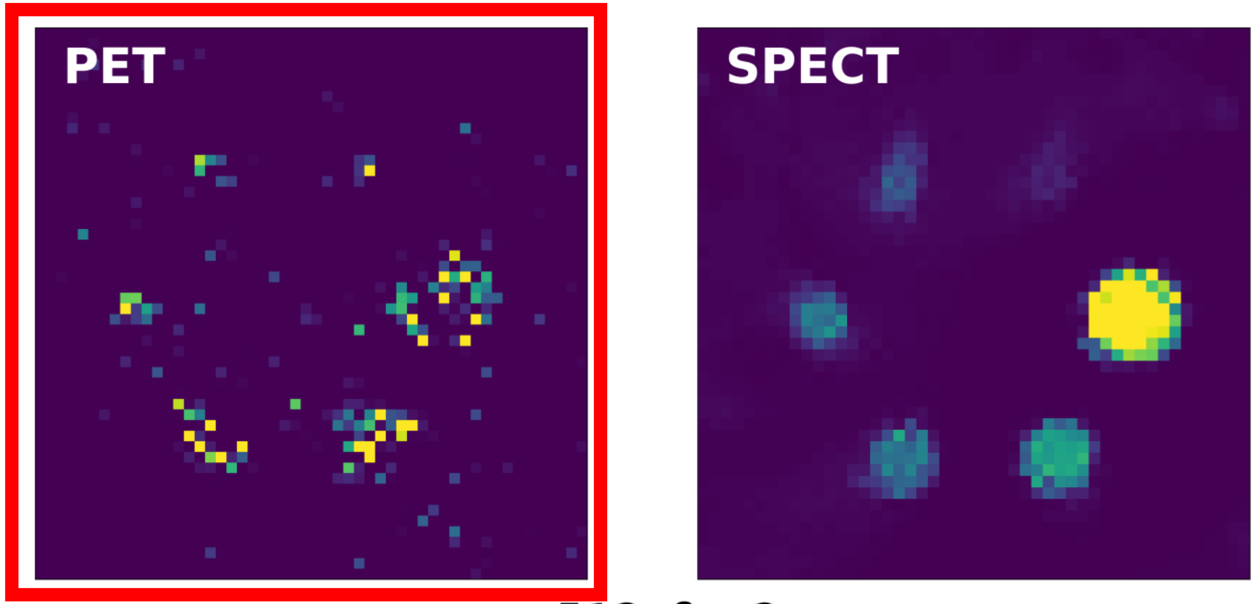


Results - Visual

$\alpha = 32, \beta = 0.0625$



$\alpha = 32, \beta = 2$



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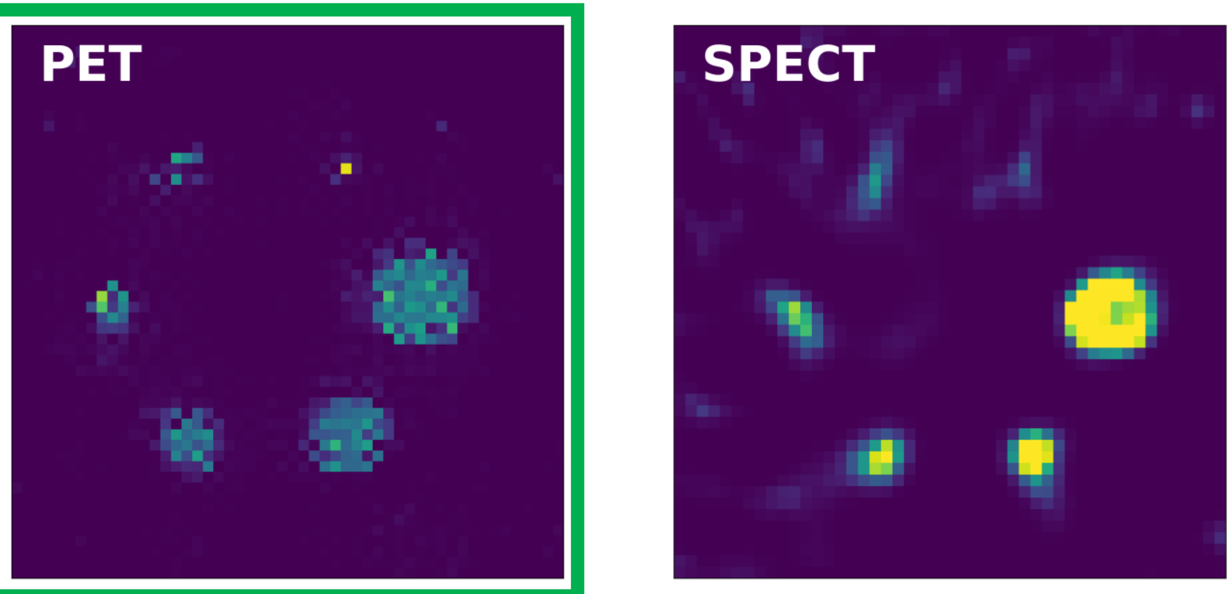
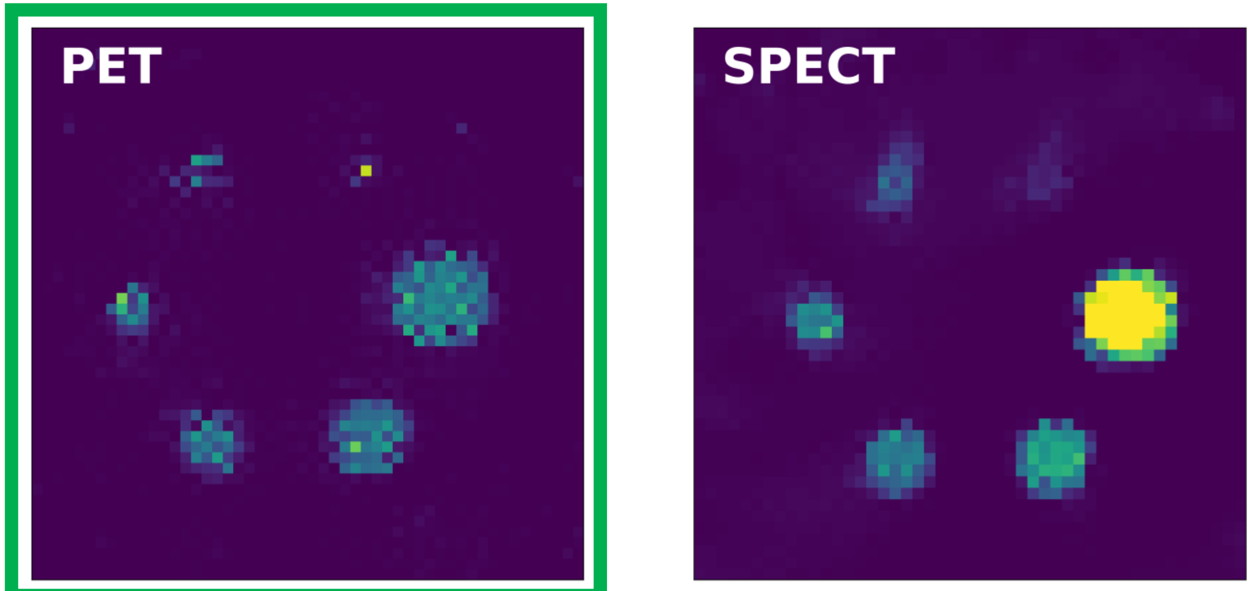


$\alpha = 512, \beta = 2$



Evaluation

- Recovery versus noise:
  - Recovery Coefficient:

$$RC = \frac{\mu_{\text{sphere}}}{E}$$

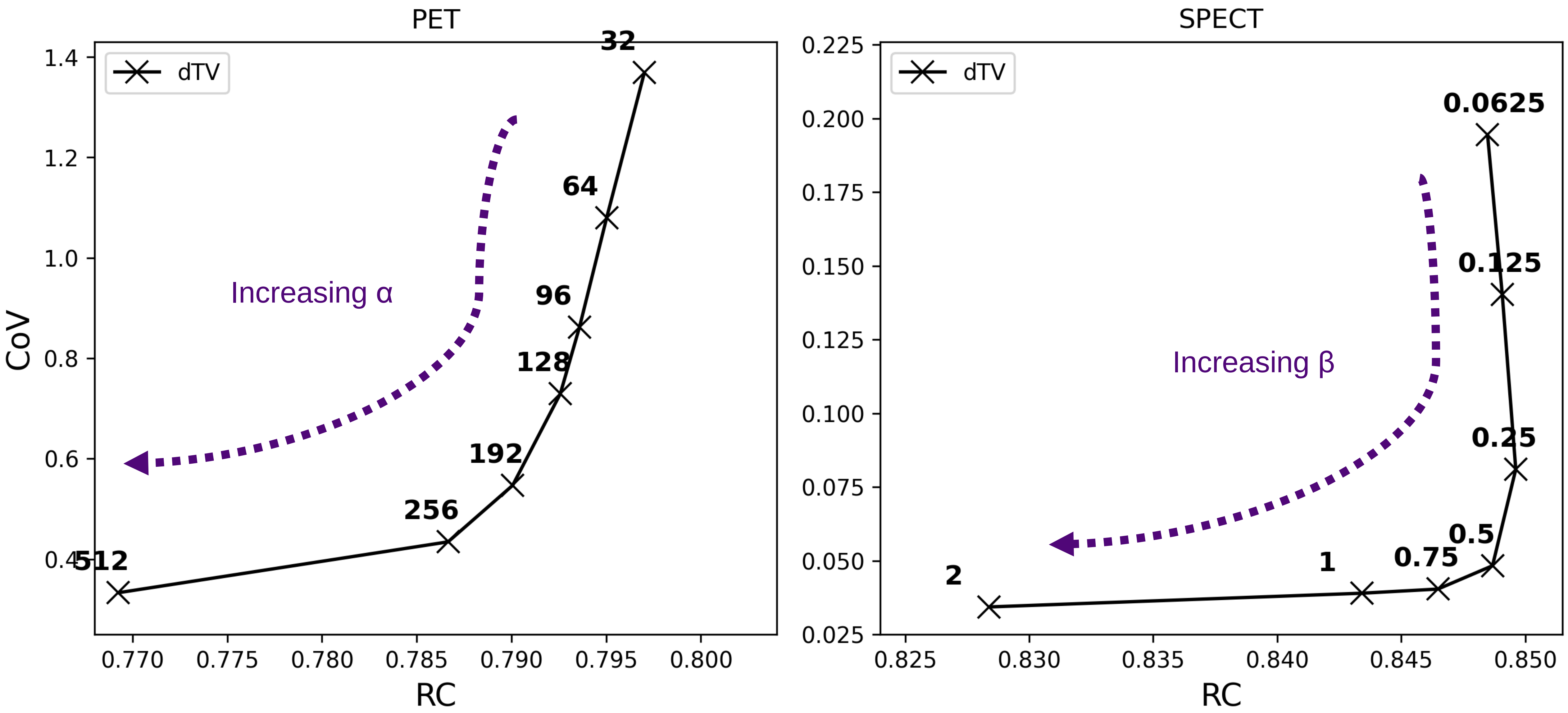
  - Coefficient of Variation (Cov):

$$CoV = \frac{\sigma_{\text{inner}}}{\mu_{\text{inner}}}$$

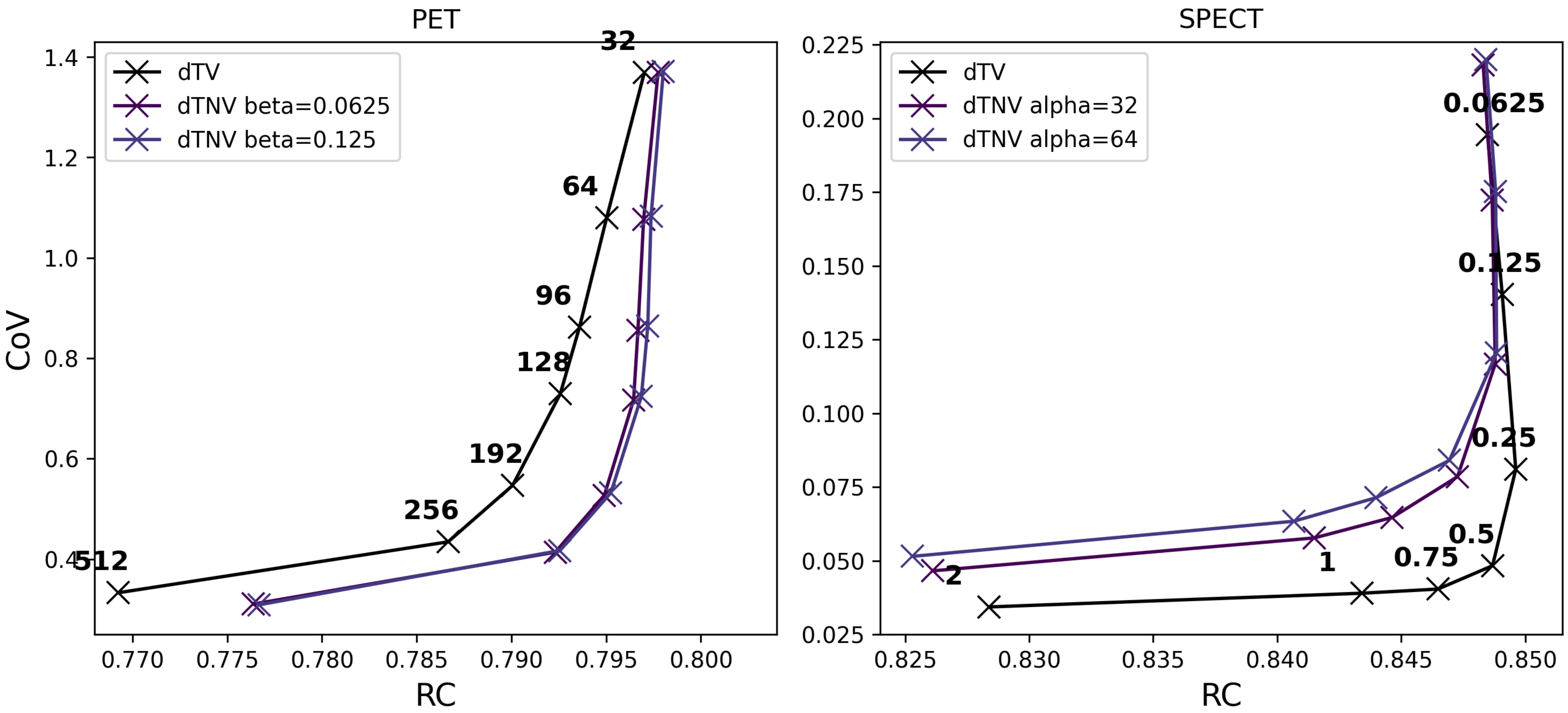
- Reconstruction accuracy:
  - Normalised root mean squared error (NRMSE):

$$NRMSE = \frac{\sqrt{\sum_j (u_j - w_j)^2}}{E}$$

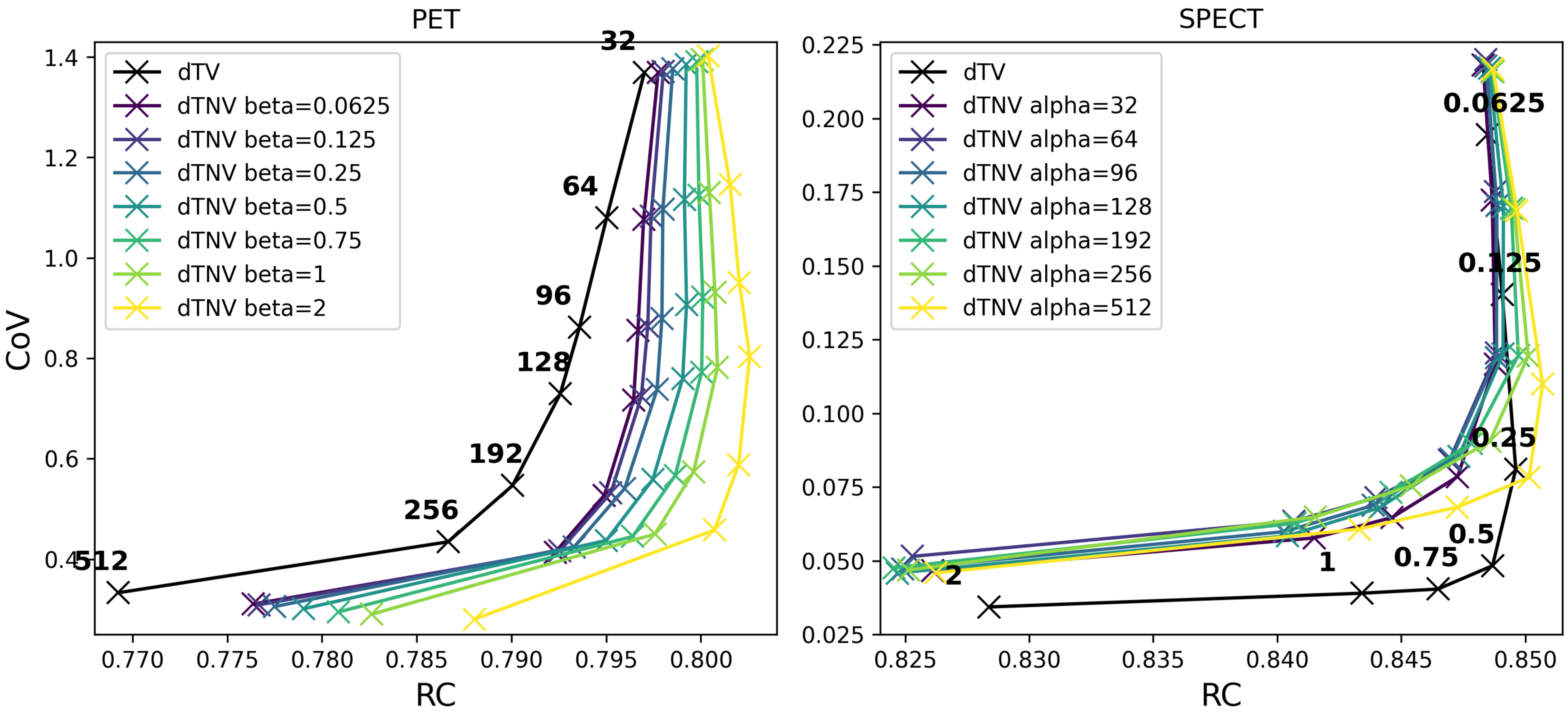
37 *mm* Sphere


PET
SPECT
dTV
dTV
32
64
96
128
192
256
512
0.0625
0.125
0.25
0.5
0.75
1
2
Increasing α
Increasing β
CoV
RC
RC
1.4
1.2
1.0
0.8
0.6
0.4
0.770
0.775
0.780
0.785
0.790
0.795
0.800
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.825
0.830
0.835
0.840
0.845
0.850

37 *mm* Sphere


PET
dTV
dTNV beta=0.0625
dTNV beta=0.125
32
64
96
128
192
256
512
CoV
RC
1.4
1.2
1.0
0.8
0.6
0.4
0.770
0.775
0.780
0.785
0.790
0.795
0.800
SPECT
dTV
dTNV alpha=32
dTNV alpha=64
0.0625
0.125
0.25
0.5
0.75
1
2
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.825
0.830
0.835
0.840
0.845
0.850
RC

37 *mm* Sphere


PET
CoV
RC
dTV
dTNV beta=0.0625
dTNV beta=0.125
dTNV beta=0.25
dTNV beta=0.5
dTNV beta=0.75
dTNV beta=1
dTNV beta=2
32
64
96
128
192
256
512
SPECT
dTV
dTNV alpha=32
dTNV alpha=64
dTNV alpha=96
dTNV alpha=128
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dTNV alpha=256
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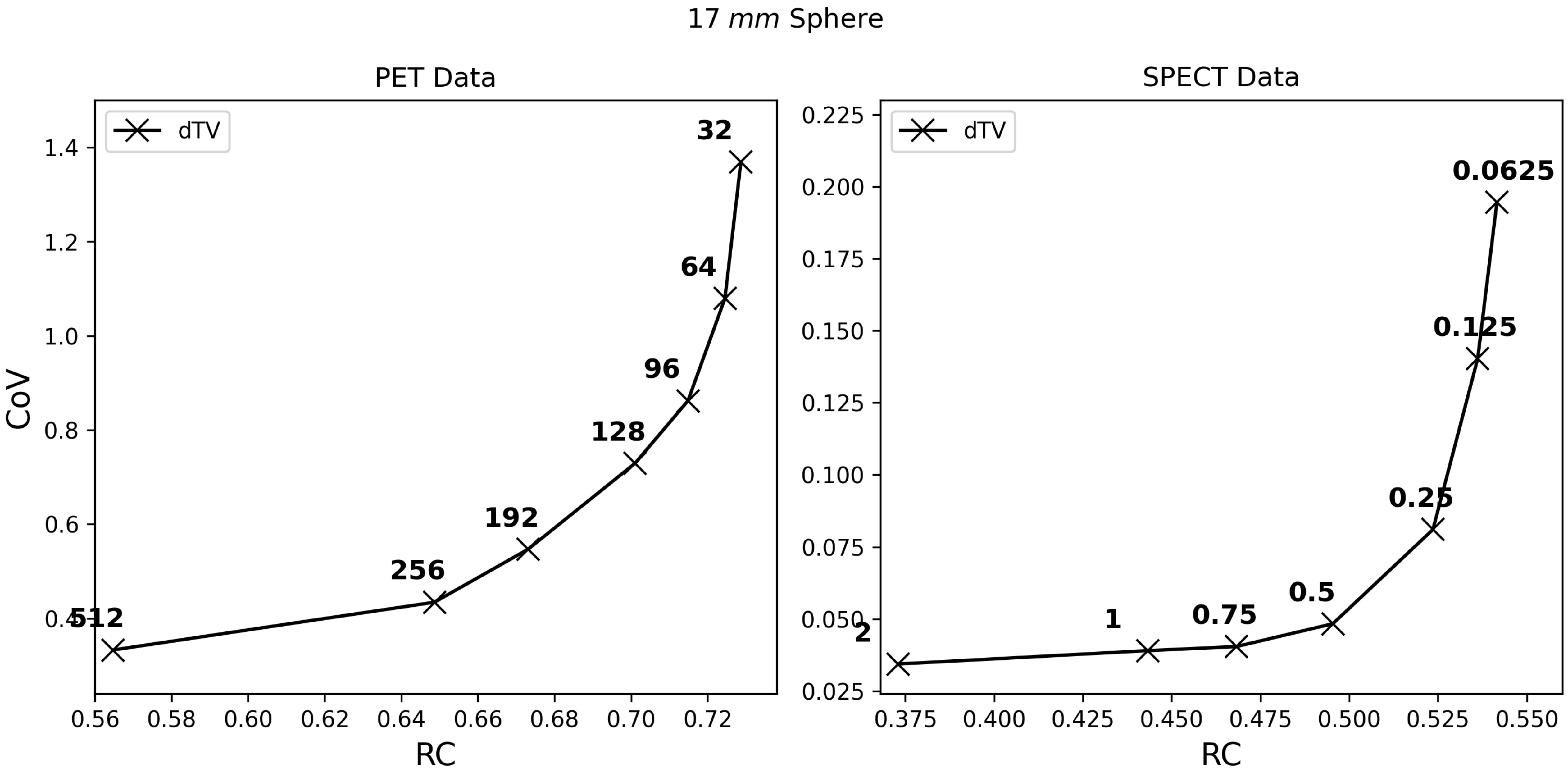
0.0625
0.125
0.25
0.5
0.75
1
2


17 mm Sphere
PET Data
dTV
CoV
RC
32
64
96
128
192
256
512
1.4
1.2
1.0
0.8
0.6
0.4
0.56
0.58
0.60
0.62
0.64
0.66
0.68
0.70
0.72
SPECT Data
dTV
RC
0.0625
0.125
0.25
0.5
0.75
1
2
0.225
0.200
0.175
0.150
0.125
0.100
0.075
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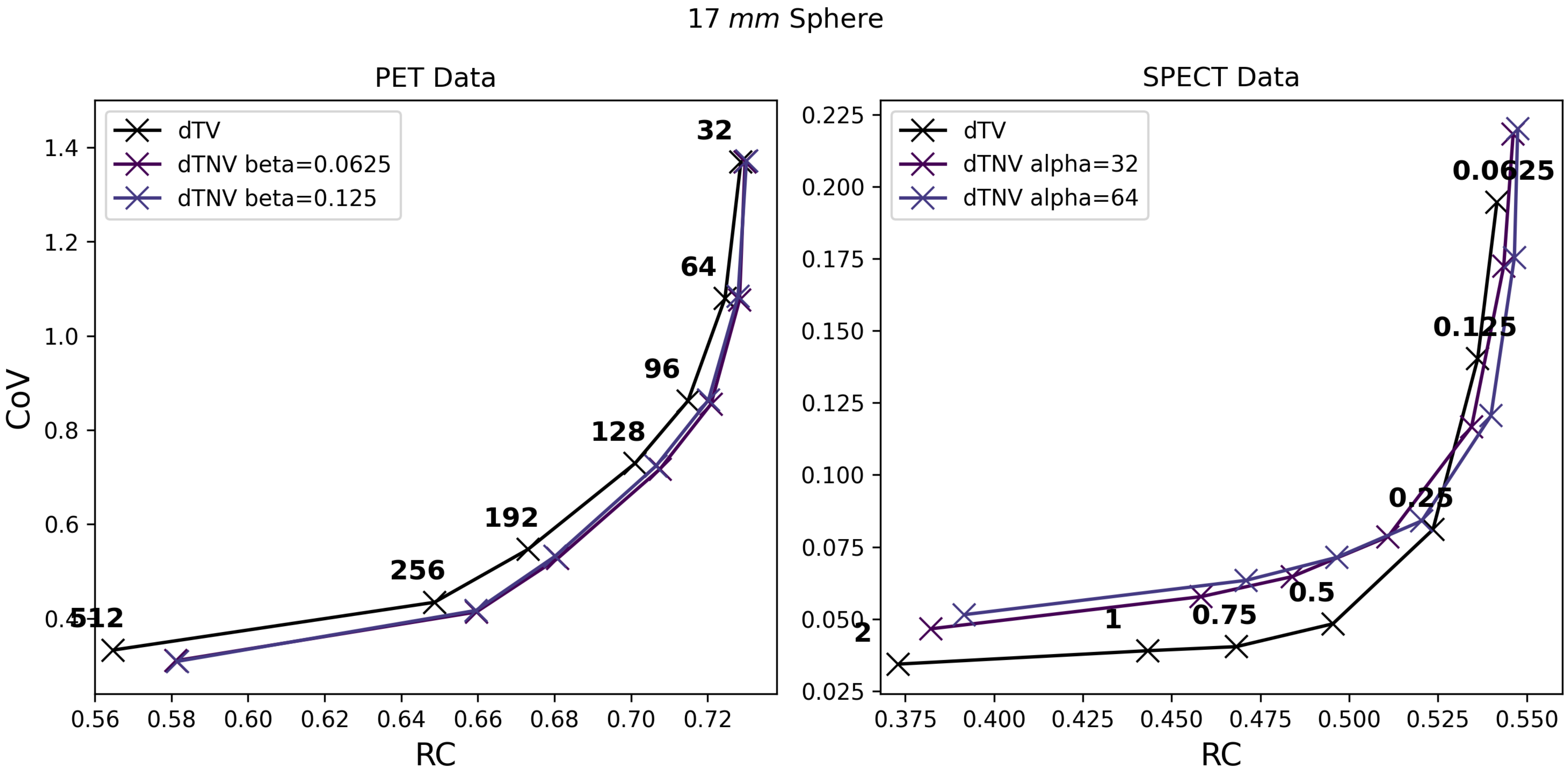
0.025
0.375
0.400
0.425
0.450
0.475
0.500
0.525
0.550


17 mm Sphere
PET Data
dTV
dTNV beta=0.0625
dTNV beta=0.125
32
64
96
128
192
256
512
CoV
RC
SPECT Data
dTV
dTNV alpha=32
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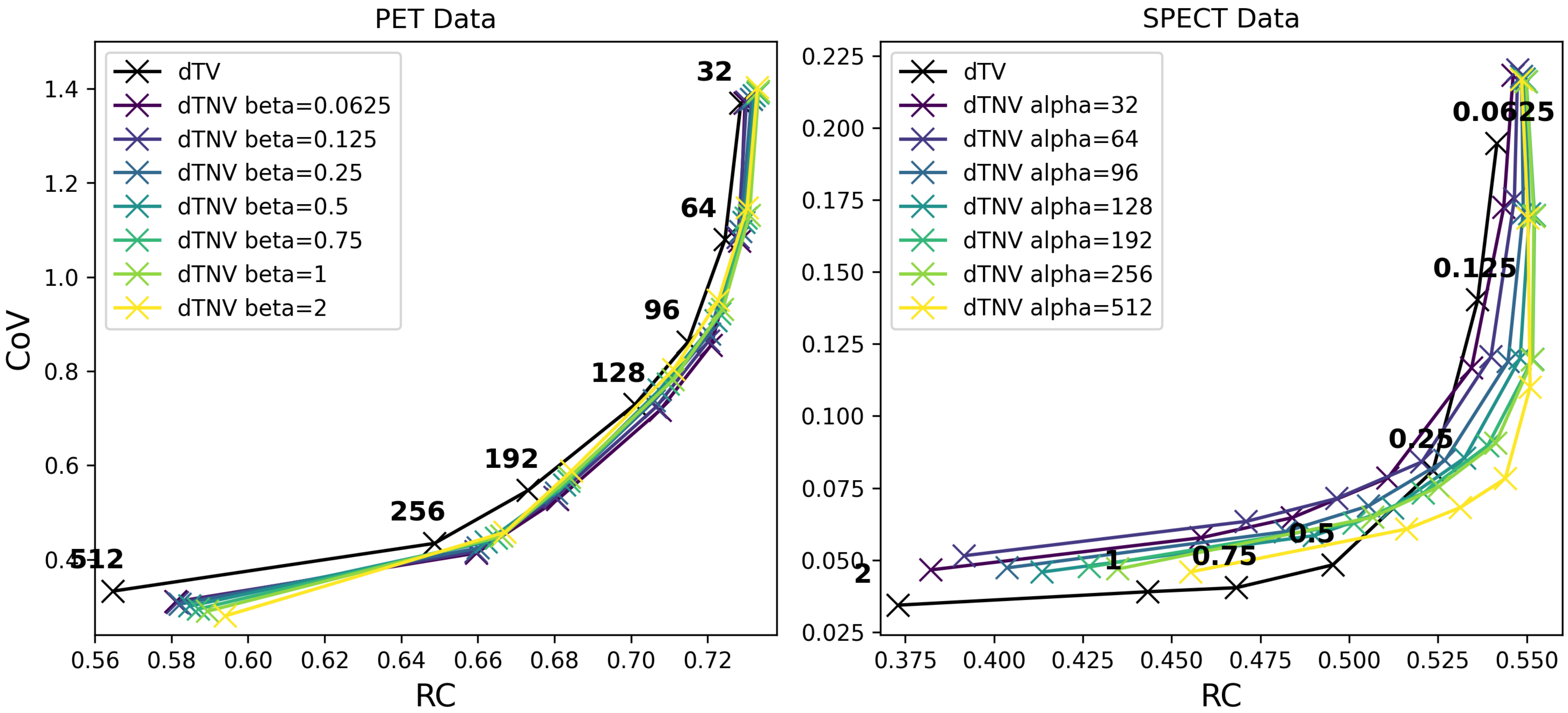
0.0625
0.125
0.25
0.5
0.75
1
2
RC

17 *mm* Sphere


PET Data
SPECT Data
dTV
dTNV beta=0.0625
dTNV beta=0.125
dTNV beta=0.25
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dTNV beta=1
dTNV beta=2
dTNV alpha=32
dTNV alpha=64
dTNV alpha=96
dTNV alpha=128
dTNV alpha=192
dTNV alpha=256
dTNV alpha=512
32
64
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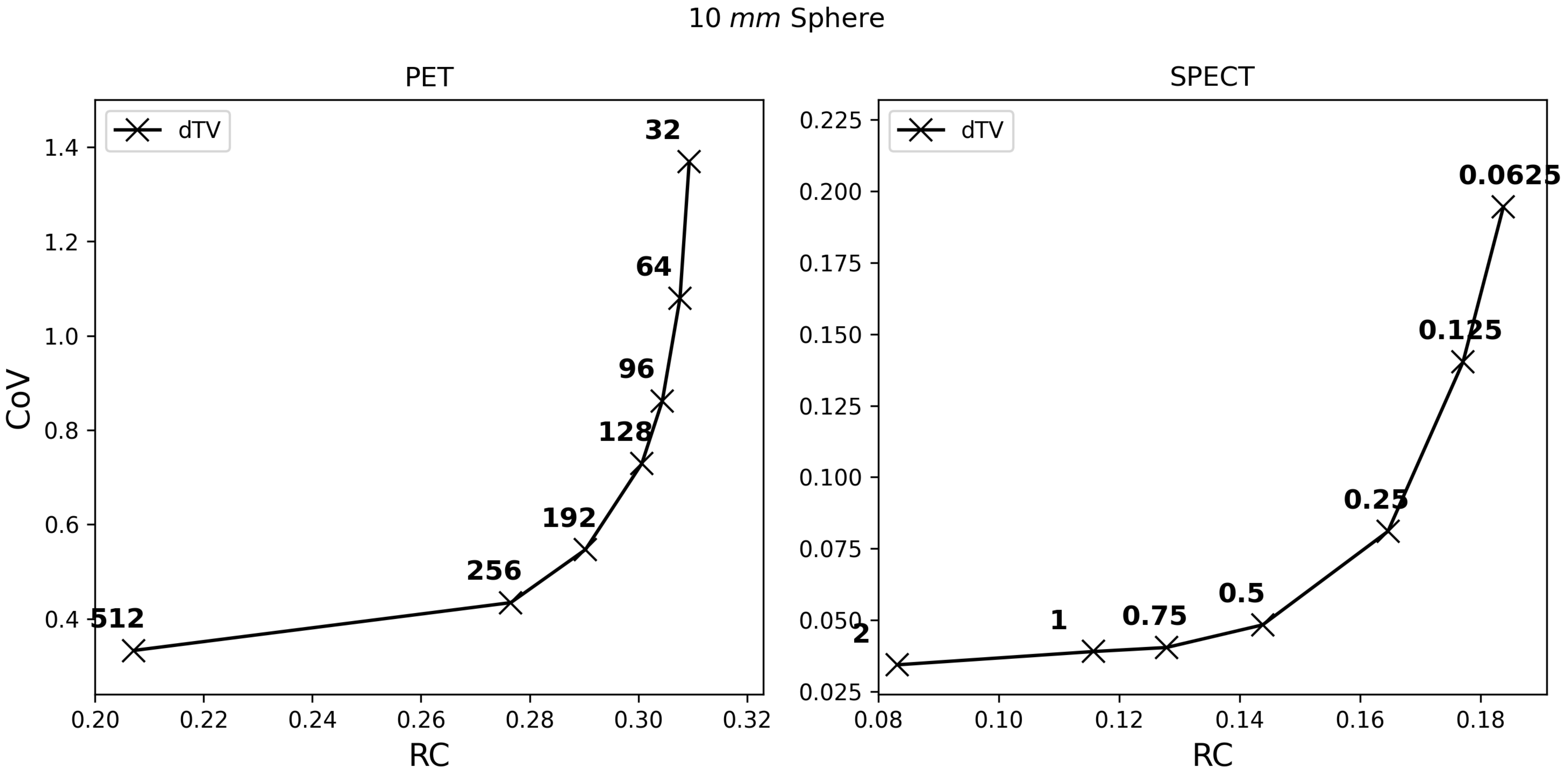
128
192
256
512
0.0625
0.125
0.25
0.5
0.75
1
2
CoV
RC
RC


10 mm Sphere
PET
dTV
CoV
RC
1.4
1.2
1.0
0.8
0.6
0.4
0.20
0.22
0.24
0.26
0.28
0.30
0.32
32
64
96
128
192
256
512
SPECT
dTV
RC
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
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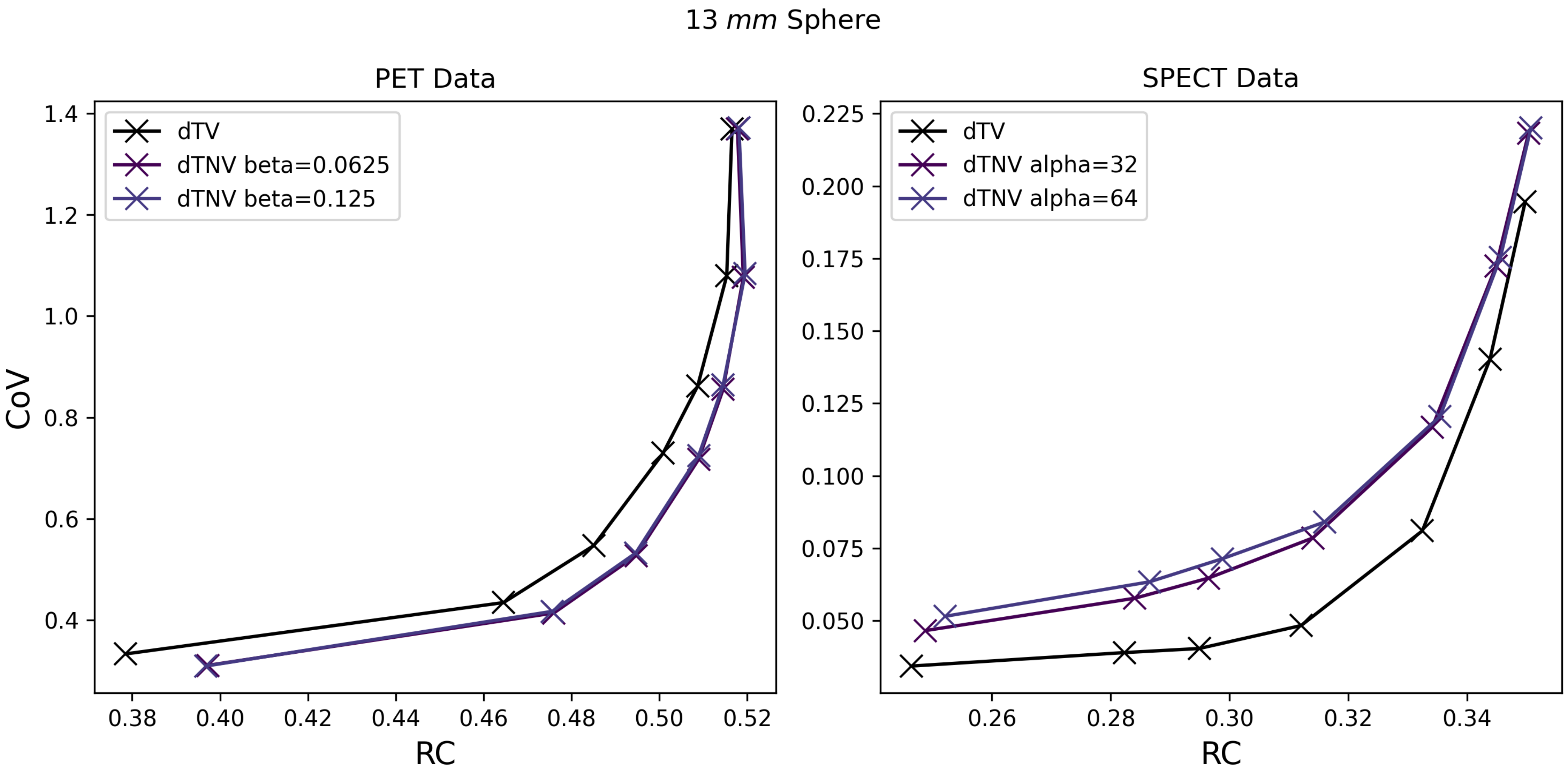
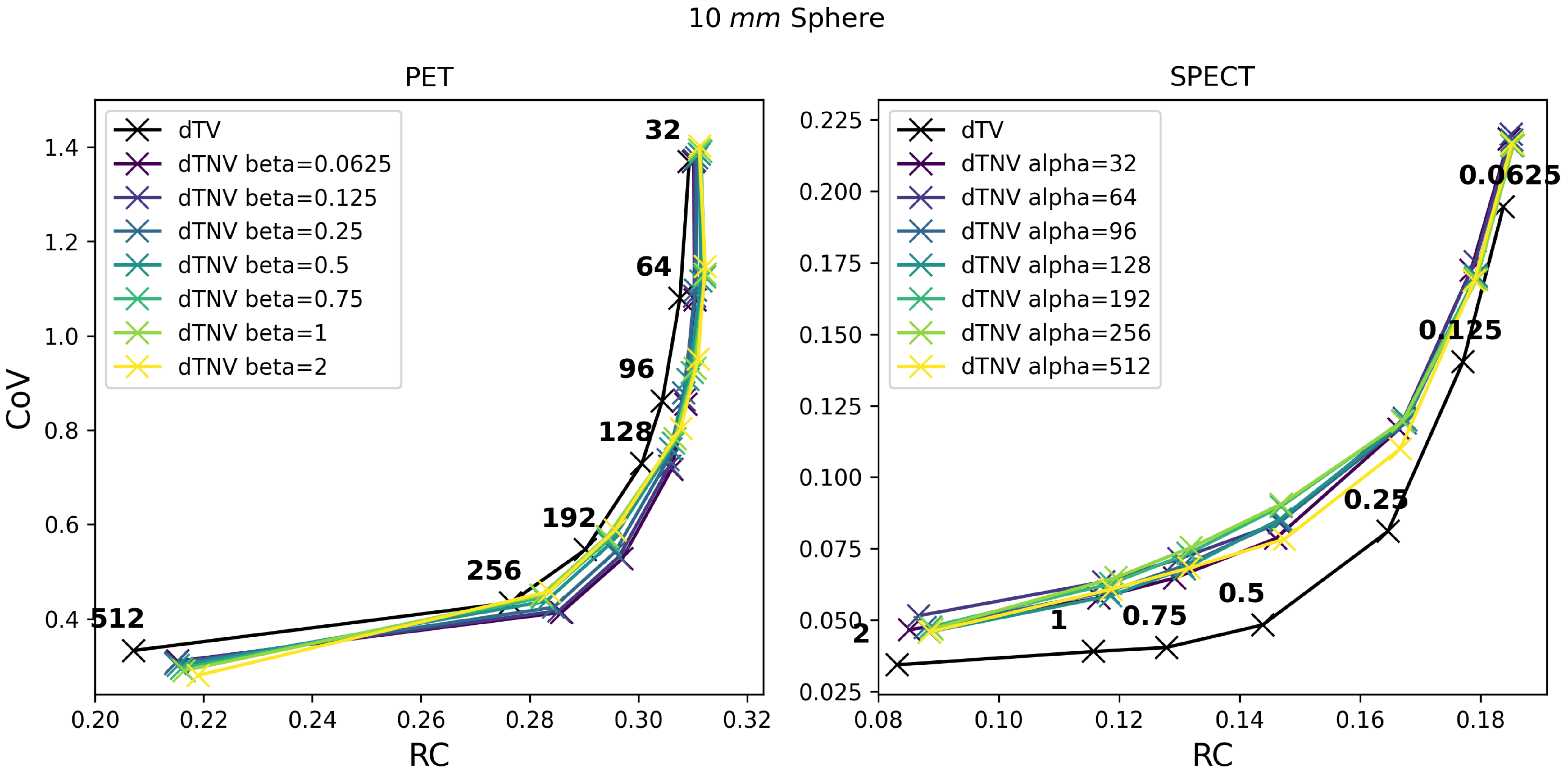
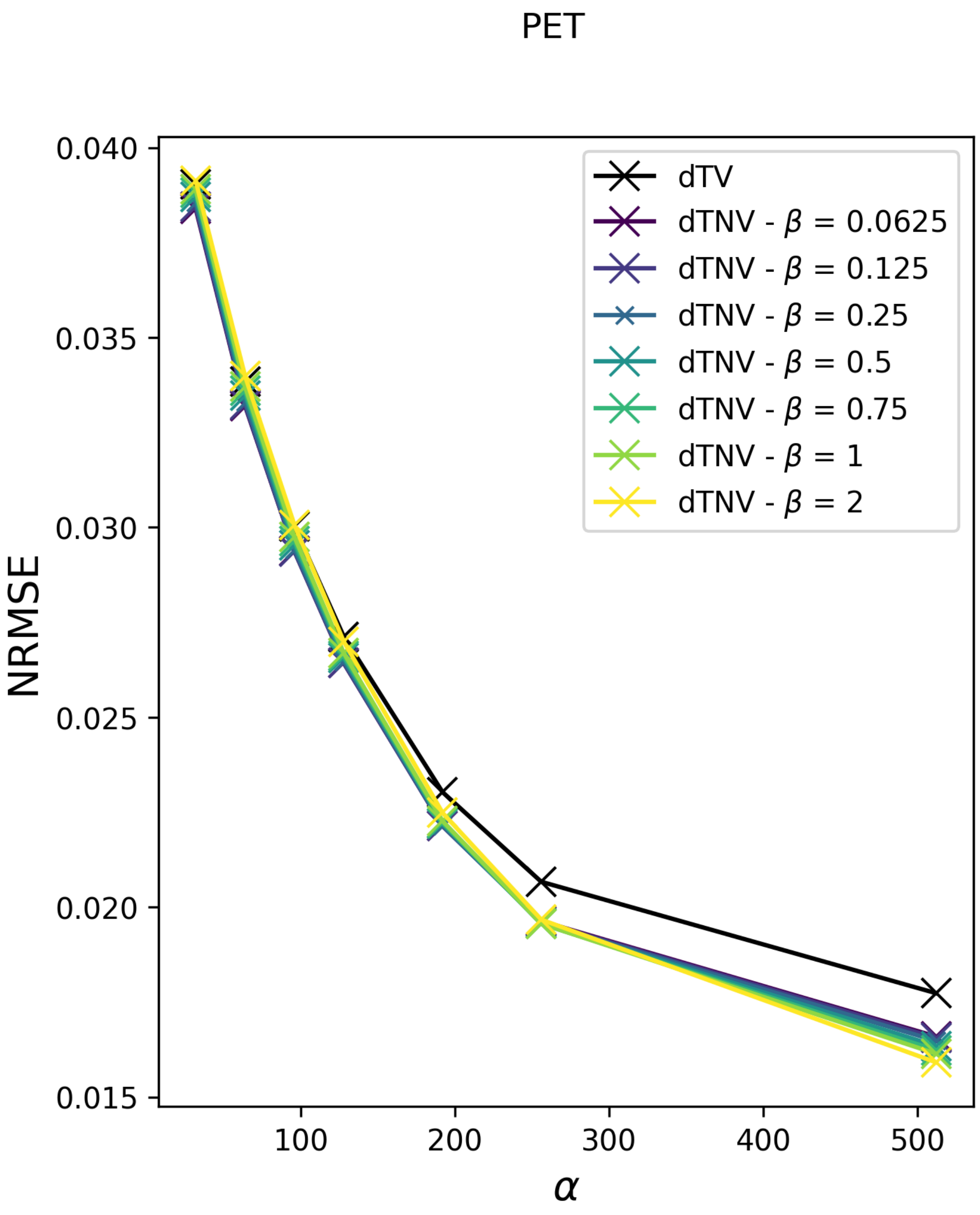
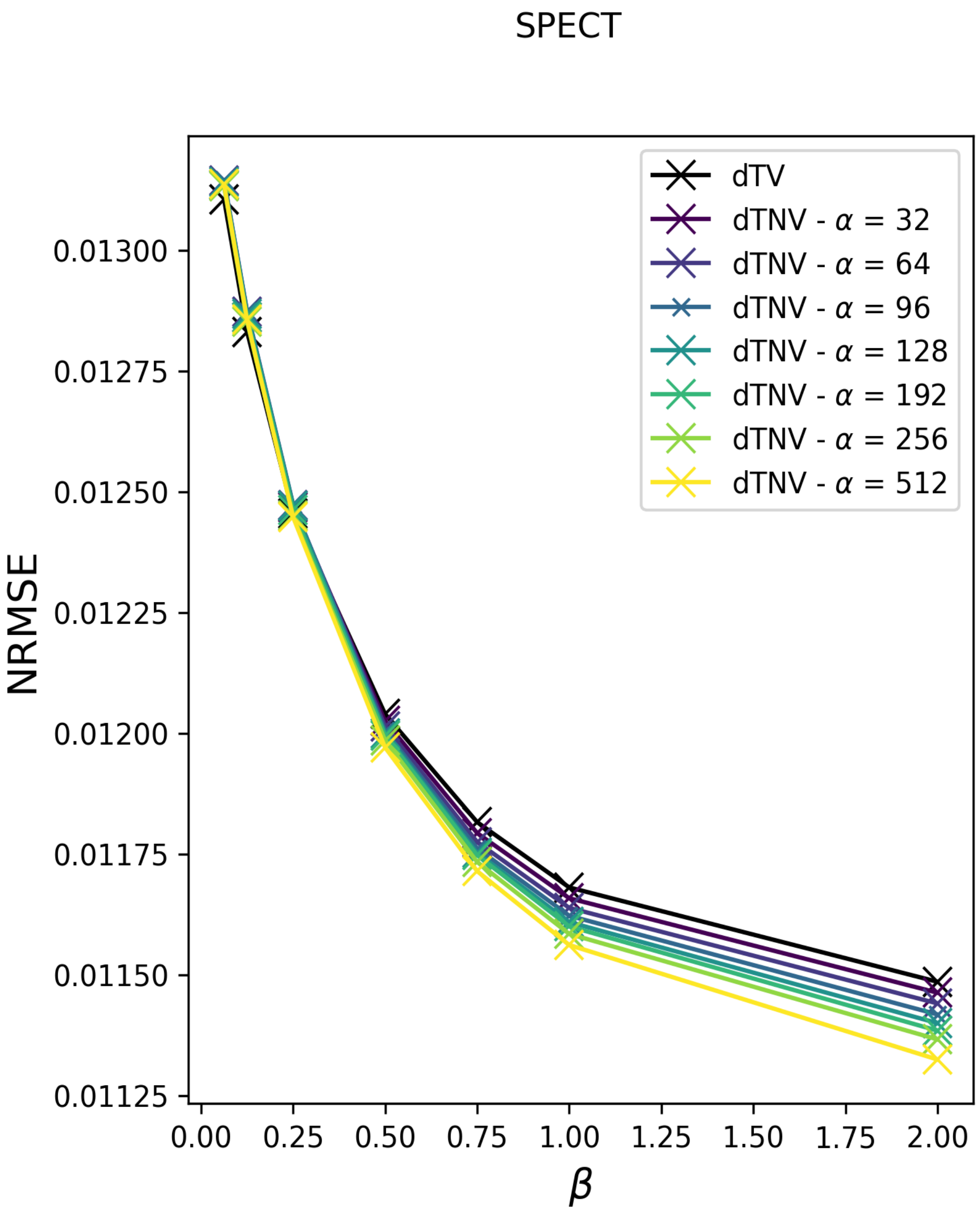
0.14
0.16
0.18
0.0625
0.125
0.25
0.5
0.75
1
2


13 mm Sphere
PET Data
CoV
1.4
1.2
1.0
0.8
0.6
0.4
0.38
0.40
0.42
0.44
0.46
0.48
0.50
0.52
RC
dTV
dTNV beta=0.0625
dTNV beta=0.125
SPECT Data
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.26
0.28
0.30
0.32
0.34
RC
dTV
dTNV alpha=32
dTNV alpha=64


10 mm Sphere
PET
SPECT
dTV
dTNV beta=0.0625
dTNV beta=0.125
dTNV beta=0.25
dTNV beta=0.5
dTNV beta=0.75
dTNV beta=1
dTNV beta=2
32
64
96
128
192
256
512
dTNV alpha=32
dTNV alpha=64
dTNV alpha=96
dTNV alpha=128
dTNV alpha=192
dTNV alpha=256
dTNV alpha=512
0.0625
0.125
0.25
0.5
0.75
1
2
CoV
RC
RC

Results - RMSE





What can we take away?

PET:

- Synergy increased RC, reduced variance for NEMA spheres.
- Synergy decreased NRMSE

SPECT:

- Synergy has potential to increase RC at the expense of CoV
- Synergy decreased NRMSE

What's Next?

More penalisation values:
- Improved RC for smaller SPECT features with higher alpha?
- How much can we increase alpha/beta before we decrease NRMSE?

More data:
- Anthropomorphic phantom data with liver compartments & tumour
- Patient data with Y-90 microspheres

More priors:
- How does this prior compare to other synergistic priors / methods?
- Modalities have very different features - can modality-specific priors in addition to joint priors further improve reconstruction?

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- For their invaluable help and support

•**The Team at NPL**

- For the phantom measurements

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•EPSRC studentship 21icas0856.

Any Questions?

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Images:
Wright, C.L., Zhang, J., Tweedle, M.F., Knopp, M.V. and Hall, N.C. (2015). Theranostic Imaging of Yttrium-90. *BioMed Research International*, 2015, pp.1–11. doi:https://doi.org/10.1155/2015/481279.

Directional Total Nuclear Variation

$$\Gamma_{dTNV} = \sum_j \sum_i \Lambda(s(\mathbf{J}_j)_i, \delta)$$

dTNV is sum of the smoothed singular values of the Jacobian Matrix

$$\mathbf{J}_j = \begin{bmatrix} \alpha(D_{v,j}\nabla u_{1,j})^\top \\ \beta(D_{v,j}\nabla u_{2,j})^\top \end{bmatrix}$$

Jacobian Matrix is the weighted & stacked directional gradients of the images

$$D_{v,j} = \mathbf{1} - \frac{\nabla v_j}{|\nabla v_j|^2 + \eta^2}(\nabla v_j)^\top$$

Directional gradient is an anatomically weighted finite difference operator

$$\Lambda(s;\delta) = \delta\left(\frac{|s|}{\delta} - \ln\left(1 + \frac{|s|}{\delta}\right)\right)$$

Smoothed by applying the Fair potential function

Directional Total Variation

$$\Gamma_{m,dTV} = \gamma_m \sum_j \Lambda(\|D_{v,j} \nabla u_m\|_2, \delta)$$

dTV is the smoothed l2 norm of the vector of gradient images

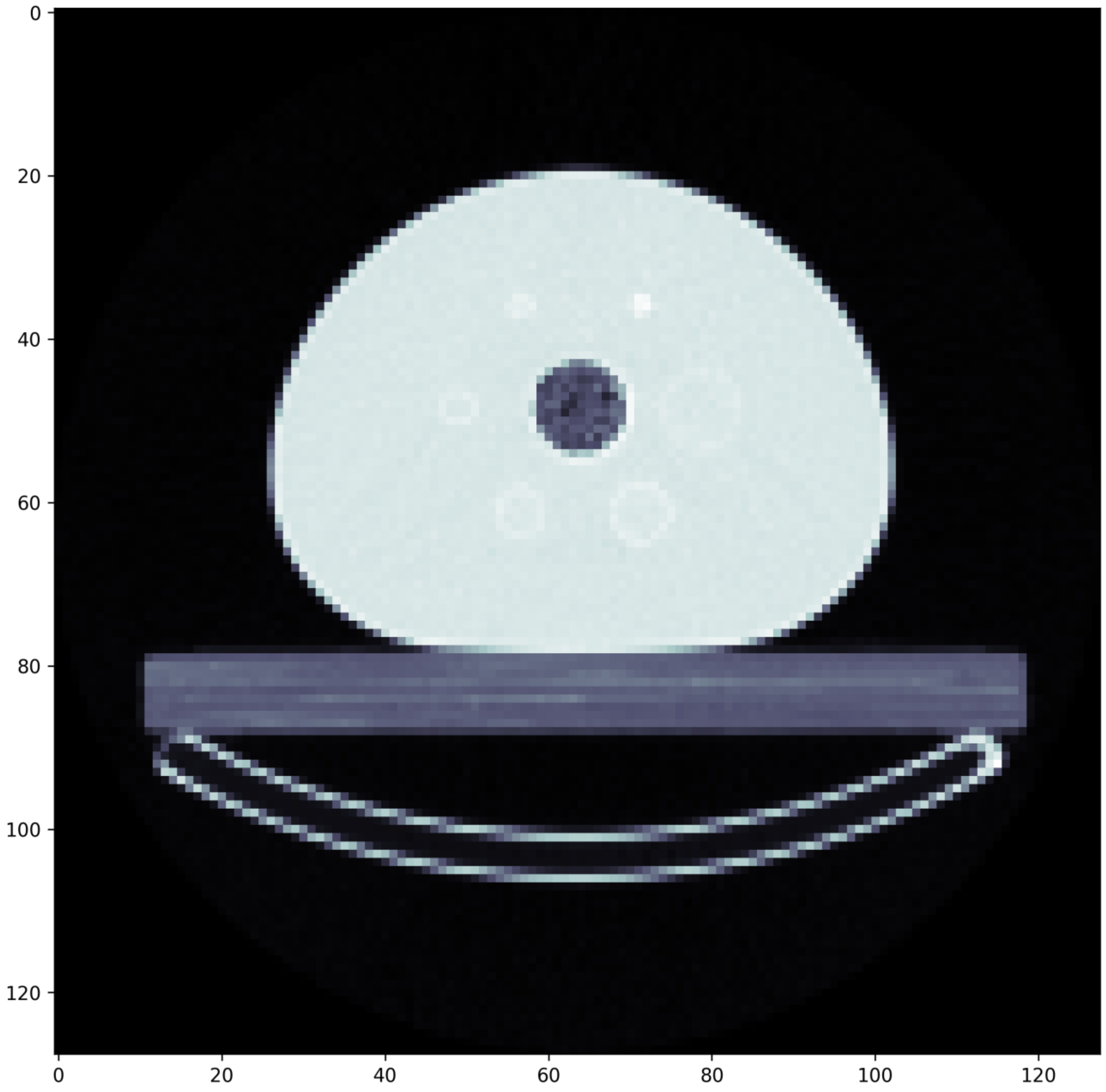
$$\gamma_m = \alpha \ \text{ or } \ \beta$$

$$D_{v,j} = \mathbf{1} - \frac{\nabla v_j}{|\nabla v_j|^2 + \eta^2} (\nabla v_j)^\top$$

Directional gradient is an anatomically weighted finite difference operator

$$\Lambda(s;\delta) = \delta \left(\frac{|s|}{\delta} - \ln \left(1 + \frac{|s|}{\delta} \right) \right)$$

Smoothed by applying the Fair potential function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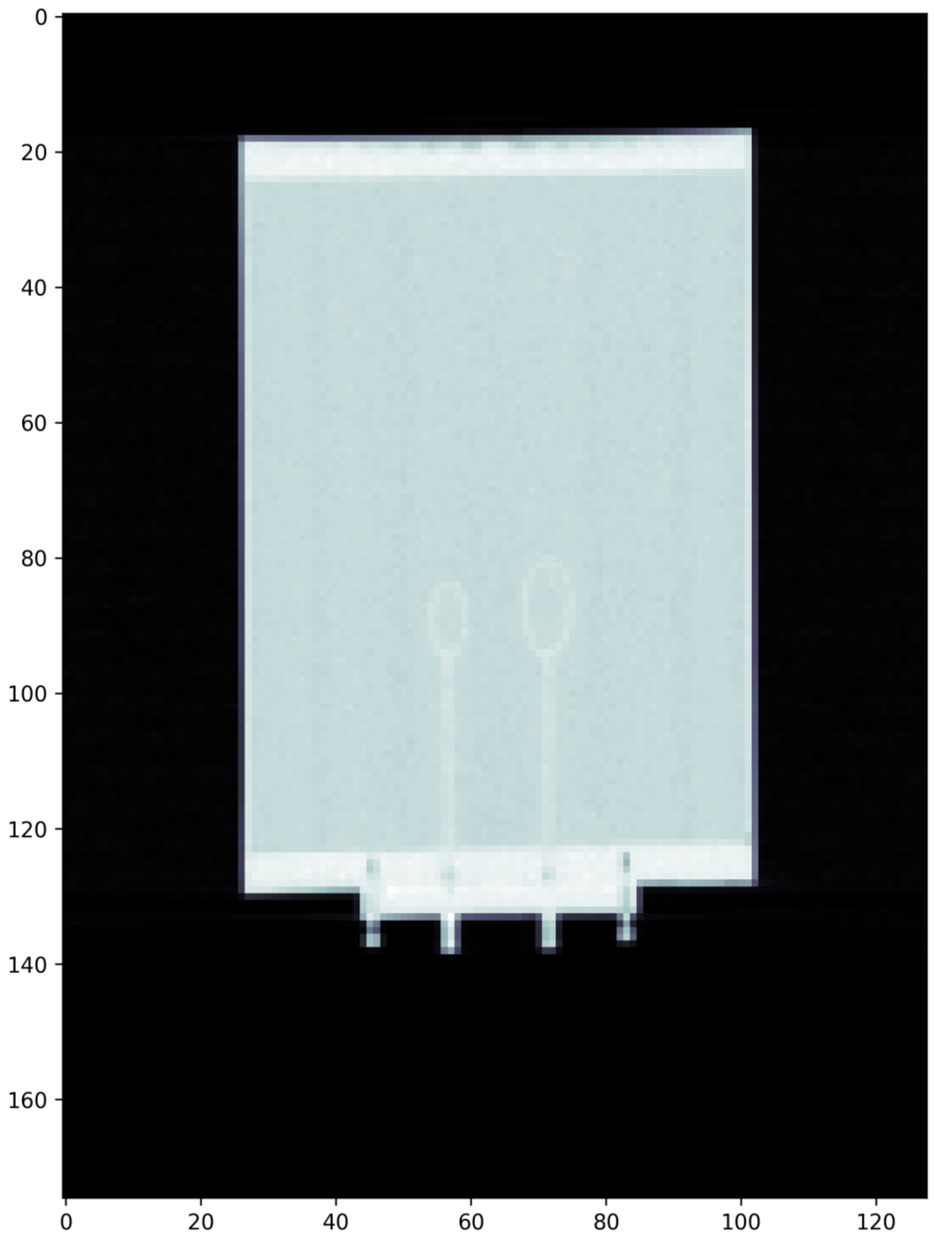
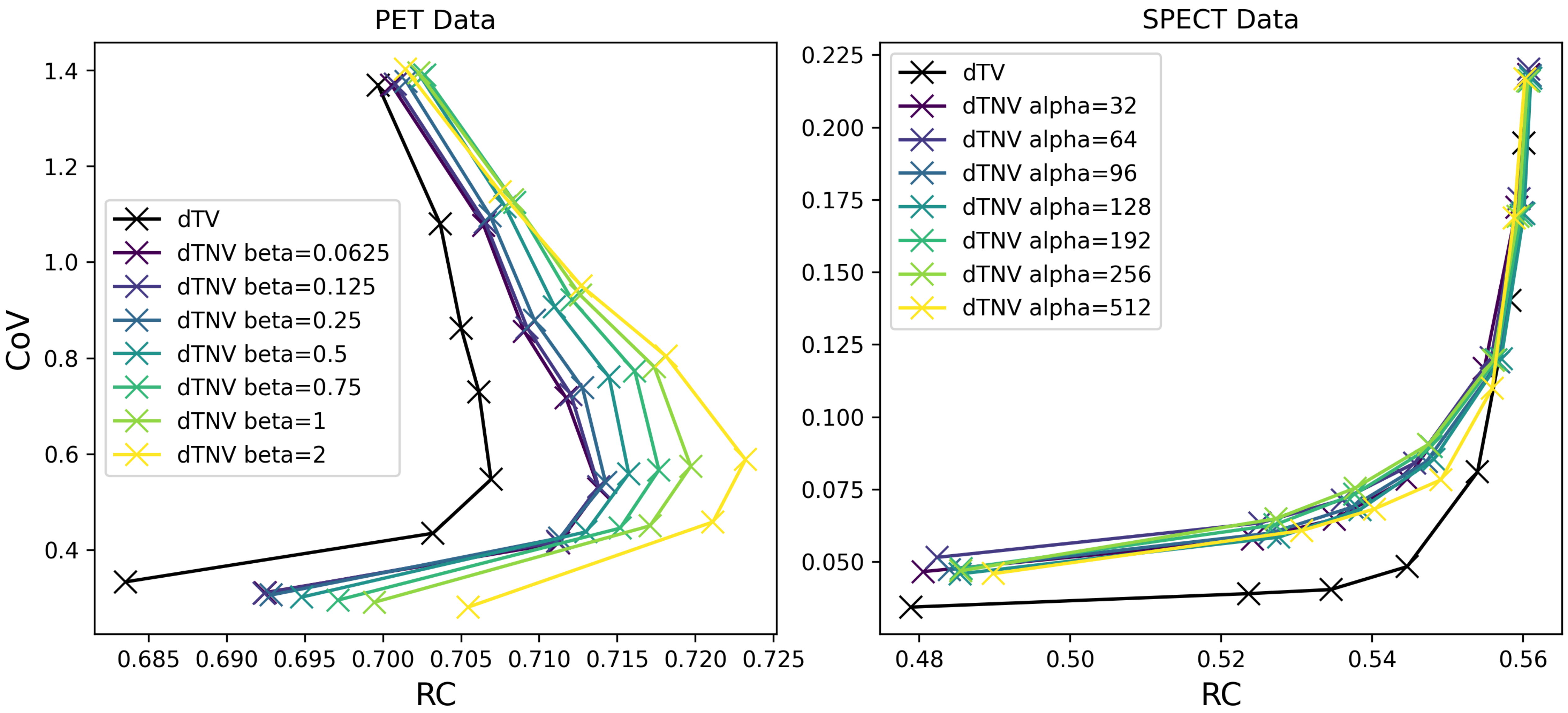
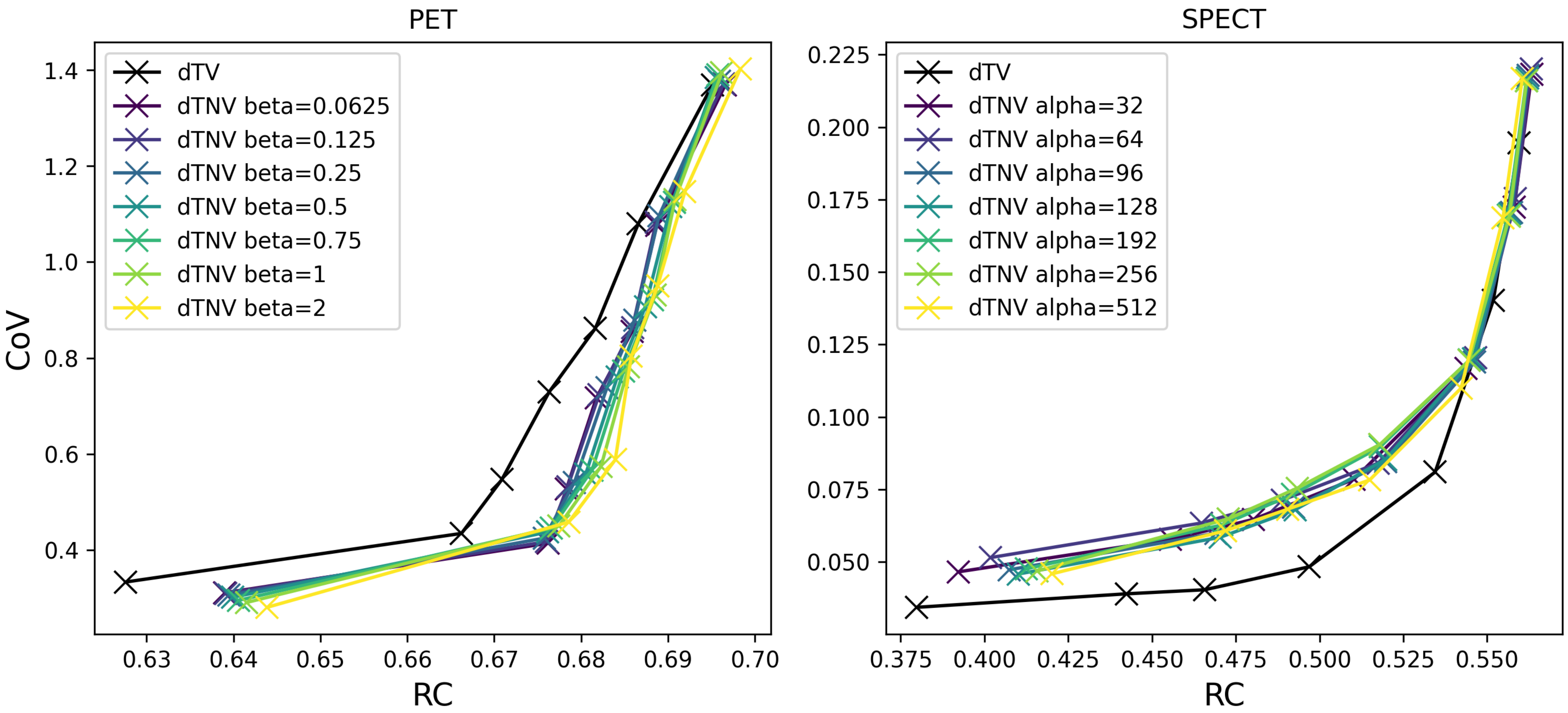
20
40
60
80
100
120
0
20
40
60
80
100
120


0
20
40
60
80
100
120
140
160
0
20
40
60
80
100
120

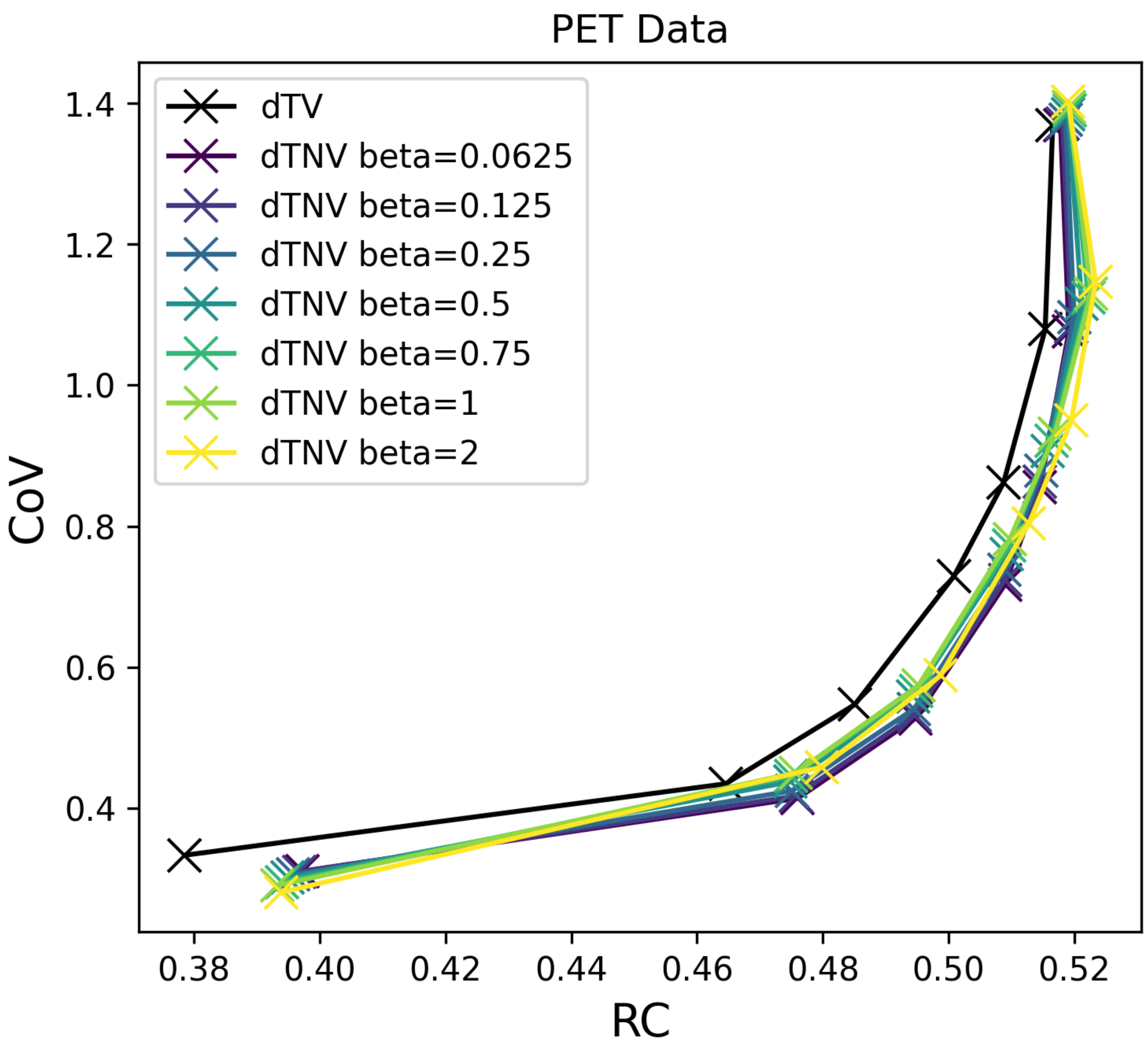
28 *mm* Sphere

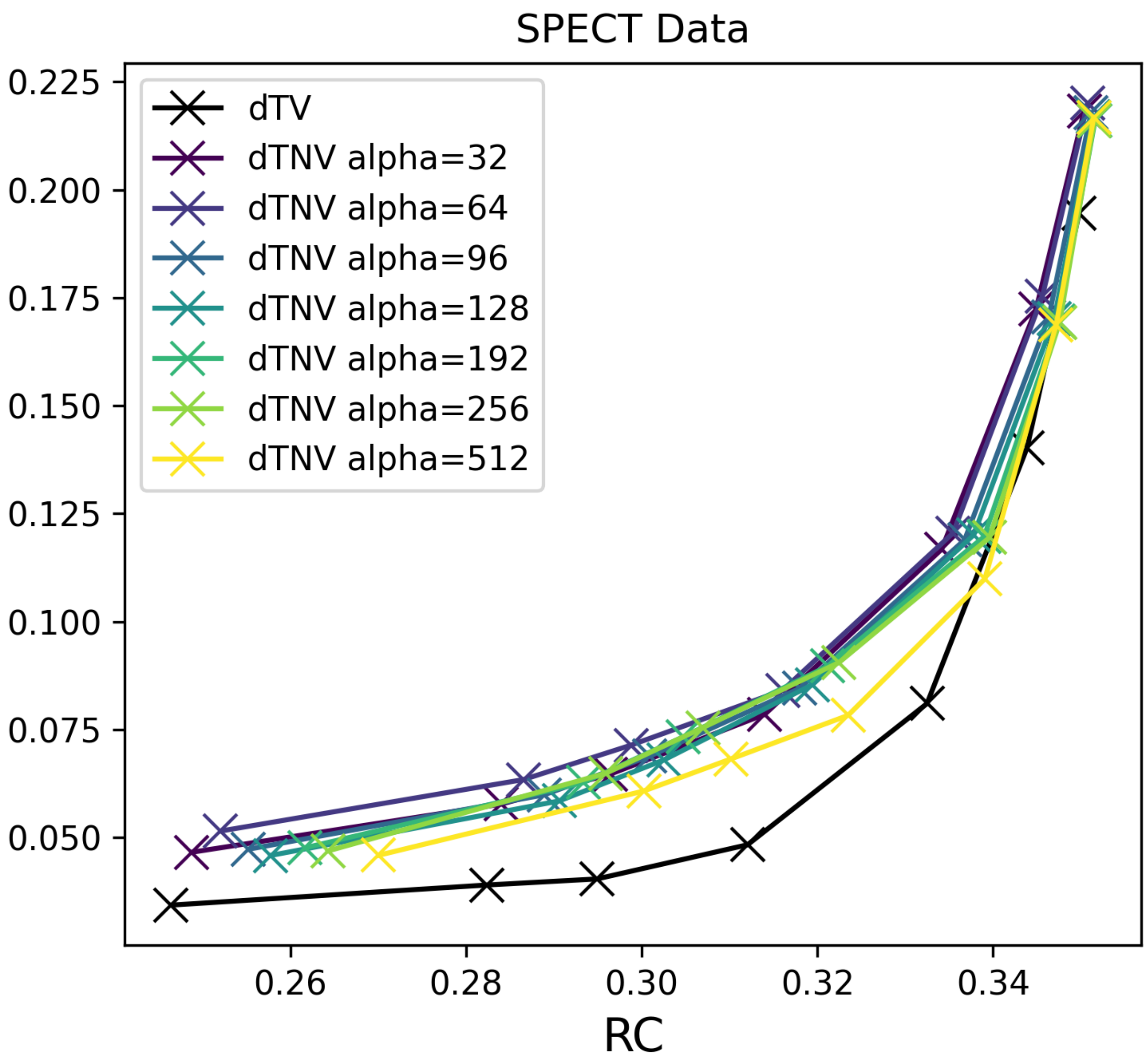

PET Data
SPECT Data
CoV
RC
RC
1.4
1.2
1.0
0.8
0.6
0.4
0.685
0.690
0.695
0.700
0.705
0.710
0.715
0.720
0.725
dTV
dTNV beta=0.0625
dTNV beta=0.125
dTNV beta=0.25
dTNV beta=0.5
dTNV beta=0.75
dTNV beta=1
dTNV beta=2
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.48
0.50
0.52
0.54
0.56
dTV
dTNV alpha=32
dTNV alpha=64
dTNV alpha=96
dTNV alpha=128
dTNV alpha=192
dTNV alpha=256
dTNV alpha=512

21 *mm* Sphere


PET
CoV
RC
dTV
dTNV beta=0.0625
dTNV beta=0.125
dTNV beta=0.25
dTNV beta=0.5
dTNV beta=0.75
dTNV beta=1
dTNV beta=2
1.4
1.2
1.0
0.8
0.6
0.4
0.63
0.64
0.65
0.66
0.67
0.68
0.69
0.70
SPECT
RC
dTV
dTNV alpha=32
dTNV alpha=64
dTNV alpha=96
dTNV alpha=128
dTNV alpha=192
dTNV alpha=256
dTNV alpha=512
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.375
0.400
0.425
0.450
0.475
0.500
0.525
0.550

13 *mm* Sphere


PET Data
dTV
dTNV beta=0.0625
dTNV beta=0.125
dTNV beta=0.25
dTNV beta=0.5
dTNV beta=0.75
dTNV beta=1
dTNV beta=2
CoV
RC
1.4
1.2
1.0
0.8
0.6
0.4
0.38
0.40
0.42
0.44
0.46
0.48
0.50
0.52


SPECT Data
dTV
dTNV alpha=32
dTNV alpha=64
dTNV alpha=96
dTNV alpha=128
dTNV alpha=192
dTNV alpha=256
dTNV alpha=512
RC
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.26
0.28
0.30
0.32
0.34