Fast and memory-efficient reconstruction of sparse Poisson data in listmode with non-smooth priors with application to time-of-flight PET

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PAPER

Fast and memory-efficient reconstruction of sparse Poisson data in listmode with non-smooth priors with application to time-of-flight PET

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Iterative TOF PET reconstruction with non-smooth priors

$$\mathrm{argmin}_{x\geq 0} \sum_i \bar{y}_i(x) - d_i \log \bar{y}_i(x) + \beta\, R\,(\nabla x)$$

negative Poisson logL

non-smooth prior

e.g. TV, GTV, DTV

forward model

$$\bar{y} = \boxed{P}x + \boxed{s}$$

TOF PET fwd projector

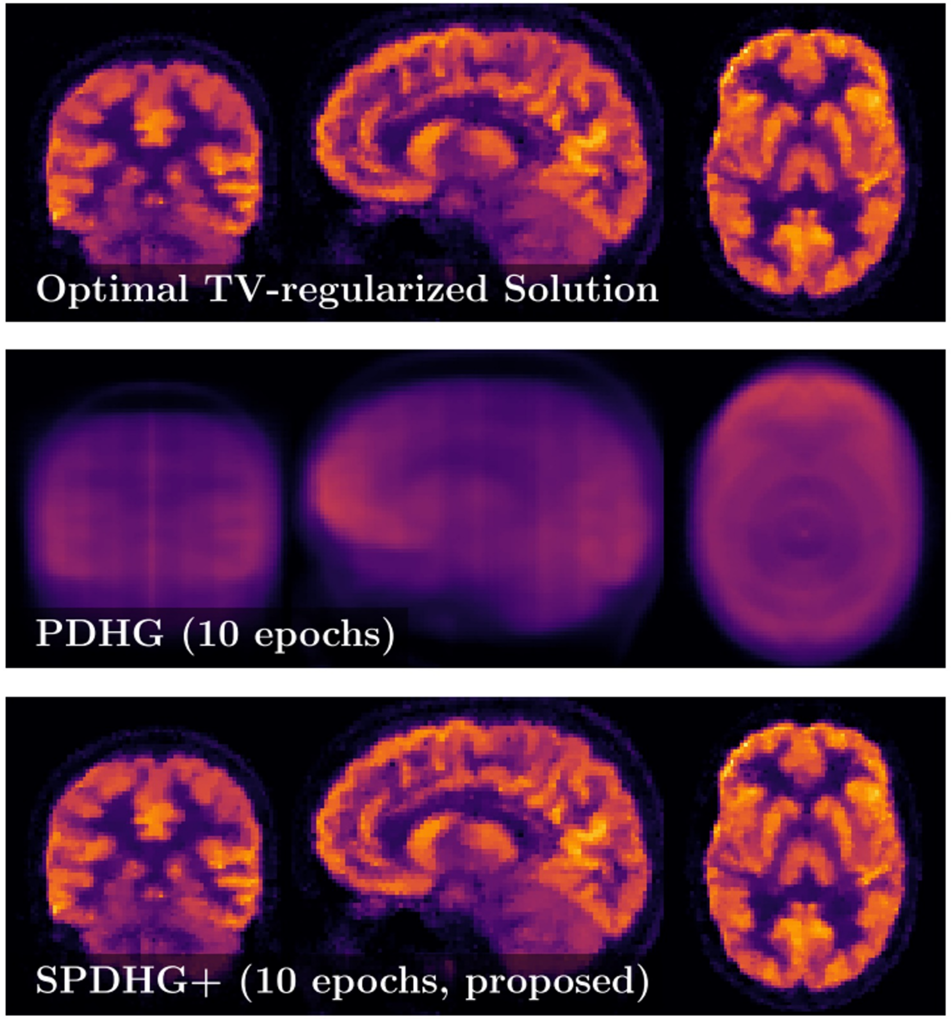
TOF contamination sinogram

Challenges

1. standard **gradient-based methods cannot be applied** if R is non-smooth
2. **computation** of complete **TOF forward model** is **slow** (20s ... several minutes)
3. TOF data **sinograms are huge**

Stochastic Primal-Dual Hybrid Gradient (SPDHG)

Initialize $x(=0), y(=0), (S_i)_i, T, (p_i)_i,$
$\bar{z} = z = P^T y$
repeat
    $x = \mathrm{proj}_{\geq 0}(x - T\bar{z})$
    Select $i \in \{1, \ldots, n+1\}$ randomly according to $(p_i)_i$
    if $i \leq n$ **then**
        $y_i^+ \leftarrow \mathrm{prox}_{D_i^*}^{S_i}(y_i + S_i(P_i x + s_i))$
        $\delta z \leftarrow P_i^T(y_i^+ - y_i)$
    else
        $y_i^+ \leftarrow \mathrm{prox}_{R^*}^{S_i}(y_i + S_i \nabla x)$
        $\delta z \leftarrow \nabla^T(y_i^+ - y_i)$
    end if
    $y_i \leftarrow y_i^+$
    $z \leftarrow z + \delta z$
    $\bar{z} \leftarrow z + (\delta z / p_i)$
until stopping criterion fulfilled
return x



M. J. Ehrhardt, P. Markiewicz, and C. B. Schönlieb. “Faster PET reconstruction with non-smooth priors by randomization and preconditioning”, PMB 2019

SPDHG - a closer look

Initialize $x(=0), y(=0), (S_i)_i, T, (p_i)_i,$
$\bar{z} = z = P^T y$
repeat
  $x = \text{proj}_{\geq 0}(x - T\bar{z})$
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  end if
  $y_i \leftarrow y_i^+$
  $z \leftarrow z + \delta z$
  $\bar{z} \leftarrow z + (\delta z / p_i)$
until stopping criterion fulfilled
return x

PROS

- guaranteed convergence (“almost surely”)
- huge number of subsets possible
  → “reasonable” convergence after
    e.g. 10 it. / 252 ss.
- applicable to many convex priors
  (TV, DTV, GTV ...) – also non-smooth

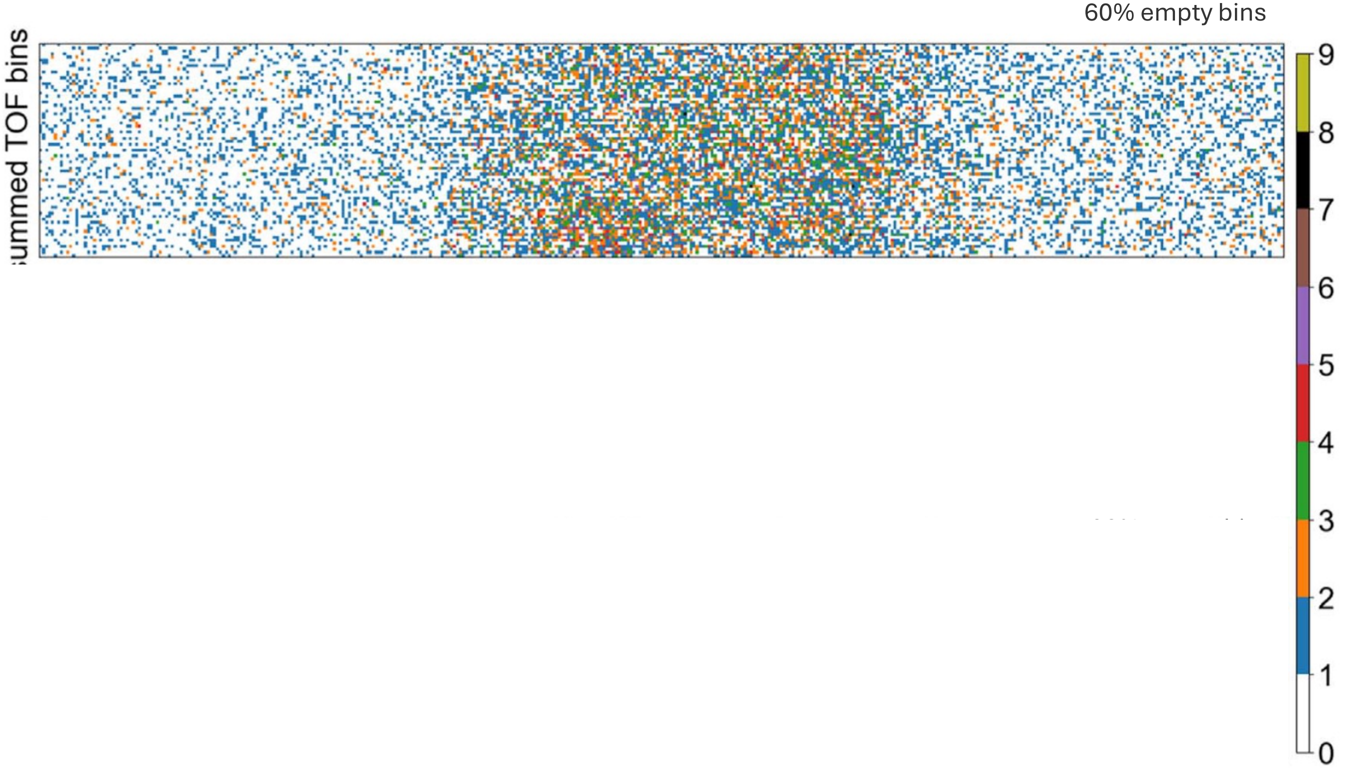
CONS

- only works in sinogram space (binned data)
- need to store 2nd complete (TOF)
  sinogram during iterations (y)
  → not efficient for sparse and huge TOF data

M. J. Ehrhardt, P. Markiewicz, and C. B. Schönlieb. “Faster PET reconstruction with non-smooth priors by randomization and preconditioning”, PMB 2019

Sparsity of TOF PET sinograms

- modern **TOF emission sinograms** are **huge**, but **very sparse**
- ***sparsity ~ 1/(n. TOF bins) ~ 1/(TOF resolution)***
  → **further increase of sparsity** in **future** with better TOF resolution
- **reconstruction** in "**sinogram**/histogram mode" very **inefficient**
  → sparse sinogram or listmode processing



emission sinogram, 80s liver bed position

323 MBq [^{18}F]FDG, 70min p.i., **$5 \cdot 10^7$ prompt counts**

4 ring GE Discovery DMI (**400ps TOF FWHM**, 169ps TOF bin width)

sinogram dim. (425, 272, 1261, 29) → **10^9 bins**

Reducing the memory requirements of SPDHG

A better initialization → no need for empty data bins during iterations

Initialize $x(=0)$, $\boxed{y(d \neq 0)}$, $(S_i)_i, T, (p_i)_i,$
$\bar{z} = z = P^T y$
repeat
  $x = \mathrm{proj}_{\geq 0}(x - T\bar{z})$
  Select $i \in \{1, \ldots, n+1\}$ randomly according to $(p_i)_i$
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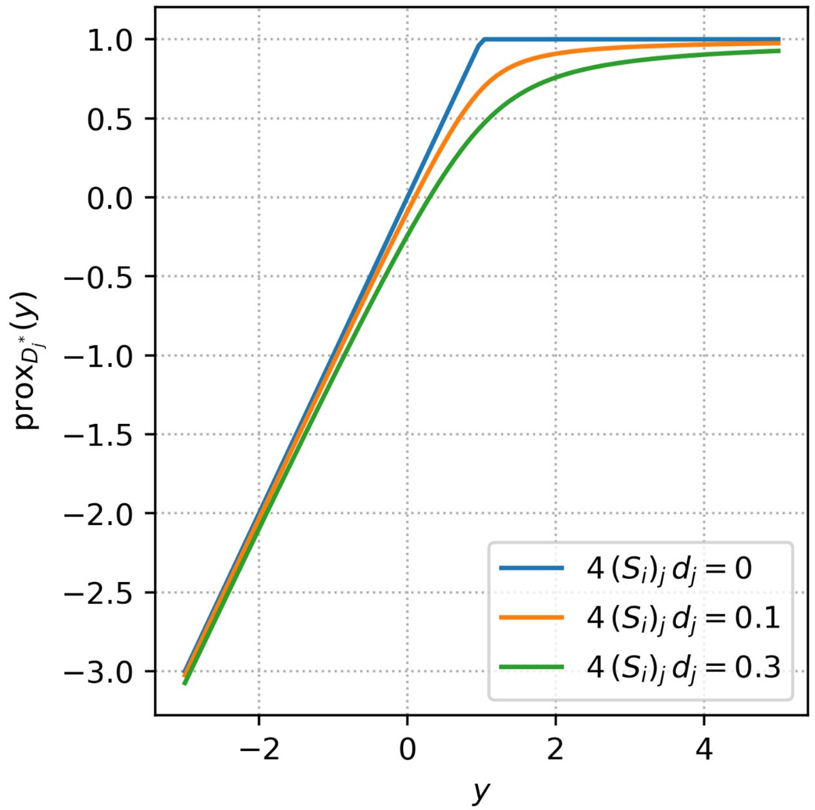
  end if
  $y_i \leftarrow y_i^+$
  $z \leftarrow z + \delta z$
  $\bar{z} \leftarrow z + (\delta z / p_i)$
until stopping criterion fulfilled
return x

$$(\mathrm{prox}_{D_j^*}^{S_i}(y))_j = \frac{1}{2}\left(y_j + 1 - \sqrt{(y_j - 1)^2 + 4(S_i)_j d_j}\right)$$



Schramm & Holler "Fast and memory-efficient reconstruction of sparse TOF PET data with non-smooth priors",
Proceedings of the 16th Virtual International Meeting on Fully 3D Image Reconstruction in Radiology and Nuclear Medicine, https://arxiv.org/abs/2110.04143

“Listmode” SPDHG

accelerate TOF fwd/back projections

LM-SPDHG

Input event list N
Calculate event counts μ_e for each e in N
Split event list N into m sublists N_i
Initialize m sub lists l_{N_i} with 0s
Initialize $x, (S_i)_i, T, (p_i)_i, g$
Preprocessing $\bar{z} = z = P^T(d \neq 0)$
repeat
  $x = \mathrm{proj}_{\geq 0}(x - T\bar{z})$
  Select $i \in \{1, \dots, m+1\}$ randomly accord. to $(p_i)_i$
  if $i \leq m$ **then**
    $l_{N_i}^+ \leftarrow \mathrm{prox}_{D^*}^{S_i}\left(l_{N_i} + S_i\left(P_{N_i}^{LM}x + s_{N_i}\right)\right)$
    $\delta z \leftarrow P_{N_i}^{LM\,T}\left(\frac{l_{N_i}^+ - l_{N_i}}{\mu_{N_i}}\right)$
    $l_{N_i} \leftarrow l_{N_i}^+$
  else
    $g^+ \leftarrow \mathrm{prox}_{||\cdot||^*}^{S_i}(g + S_i \nabla x)$
    $\delta z \leftarrow \nabla^T(g^+ - g)$
    $g \leftarrow g^+$
  end if
  $z \leftarrow z + \delta z$
  $\bar{z} \leftarrow z + (\delta z / p_i)$
until stopping criterion fulfilled
return x

Schramm and Holler: *"Fast and memory-efficient reconstruction of sparse Poisson data in listmode with non-smooth priors with application to time-of-flight PET"*
Phys Med Biol 2022

LM-SPDHG

listmode fwd / back projections
instead of sinogram projections

$$(\mathrm{prox}^{S_i}_{D_j^*}(y))_j = \frac{1}{2}\left(y_j + 1 - \sqrt{(y_j - 1)^2 + 4(S_i)_j\mu_j}\right)$$

Input event list N
Calculate event counts μ_e for each e in N
Split event list N into m sublists N_i
Initialize m sub lists l_{N_i} with 0s
Initialize $x, (S_i)_i, T, (p_i)_i, g$
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  $x = \mathrm{proj}_{\geq 0}(x - T\bar{z})$
  Select $i \in \{1, \ldots, m+1\}$ randomly accord. to $(p_i)_i$
  if $i \leq m$ **then**
    $l^+_{N_i} \leftarrow \mathrm{prox}^{S_i}_{D^*}\left(l_{N_i} + S_i\left(\boxed{P^{LM}_{N_i}}x + s_{N_i}\right)\right)$
    $\delta z \leftarrow \boxed{{P^{LM}_{N_i}}^T}\left(\frac{l^+_{N_i} - l_{N_i}}{\mu_{N_i}}\right)$
    $l_{N_i} \leftarrow l^+_{N_i}$
  else
    $g^+ \leftarrow \mathrm{prox}^{S_i}_{\|\cdot\|^*}(g + S_i\nabla x)$
    $\delta z \leftarrow \nabla^T(g^+ - g)$
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Schramm and Holler: *"Fast and memory-efficient reconstruction of sparse Poisson data in listmode with non-smooth priors with application to time-of-flight PET"*
Phys Med Biol 2022

LM-SPDHG

event num	det 1	det 2	TOF bin
1	1	3	1
2	1	4	1
3	2	4	2
4	1	3	2
5	1	3	1

Time to calc μ_e	0.23s (1e7 counts)
(single V100 GPU)	2.76s (1e8 counts)

Input event list N
$\boxed{\textbf{Calculate}\text{ event counts } \mu_e \text{ for each e in N}}$
Split event list N into m sublists N_i
Initialize m sub lists l_{N_i} with 0s
Initialize $x, (S_i)_i, T, (p_i)_i, g$
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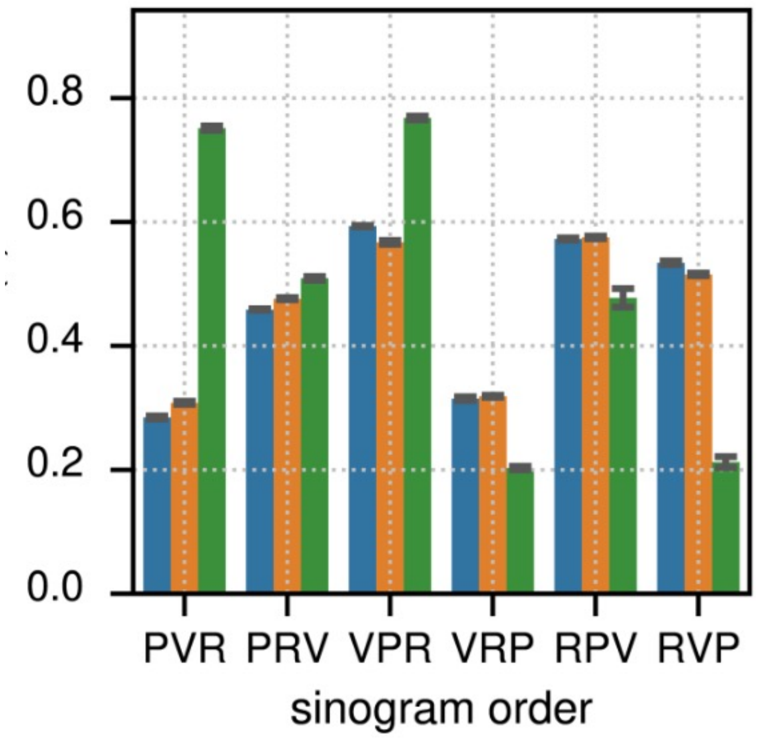
Schramm and Holler: *"Fast and memory-efficient reconstruction of sparse Poisson data in listmode with non-smooth priors with application to time-of-flight PET"*
Phys Med Biol 2022

Listmode projectors are (almost always) faster

PARALLELPROJ—an open-source framework for fast calculation of projections in tomography

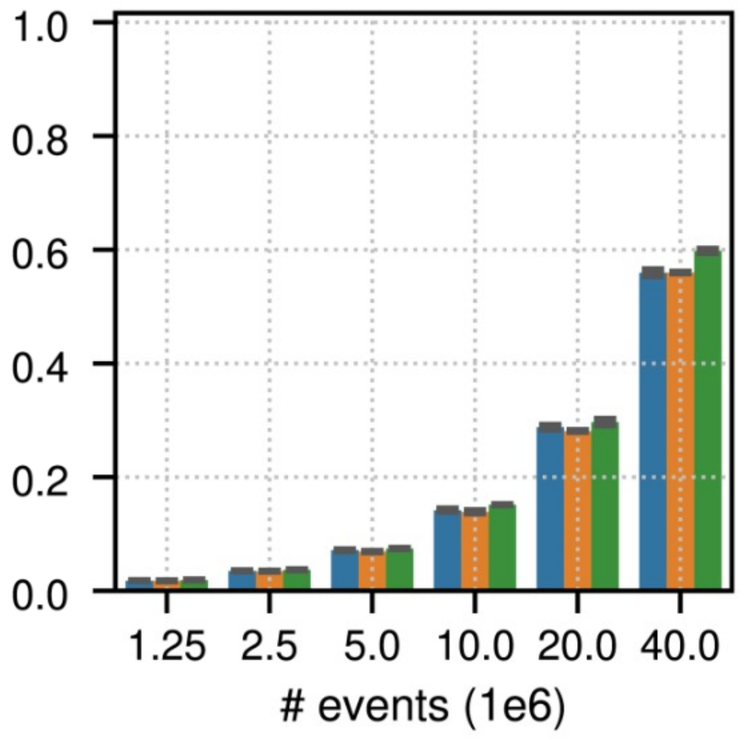
Georg Schramm[1*] Kris Thielemans[2,3]

Timing (s) for TOF sinogram fwd+back projection (1 out of 28 subsets, GE 4 ring DMI, 400ps TOF)



28*0.2s = **5.6s** needed for complete sino fwd + back projection

Timings (s) for TOF listmode fwd+back projection



4e7 events (80s liver scan) can be fwd + back projected in **0.6s**

Memory requirements of SPDHG vs LM-SPDHG

algorithm	5e8 prompts	7e7 prompts	1e7 prompts
SPDHG	60.0 GB	60.0 GB	60.0 GB
LM-SPDHG	12.5 GB	2.1 GB	0.8 GB

GE DMI-4 (20cm axial FOV) geometry – using “span 1” TOF sinograms
400ps TOF resolution, 29 TOF bins

Methods

Methods

- **reconstruction** of **simulated 2D TOF PET** data from brain phantom using
  PDHG (10000 iterations) → **reference** solution (x^*)
  SPDHG (100 iterations / diff. num. subsets)
  LM-SPDHG (100 iterations / diff. num. subsets)
  LM-EMTV (100 iterations / diff. num. subsets)

- **different count levels, prior strength and two priors**: TV and DTV (directional TV)

- 2D/3D data **simulation** including attenuation, smooth contamination, finite resolution

- reconstruction of **real 3D TOF data** from GE DMI (NEMA IQ phantom)

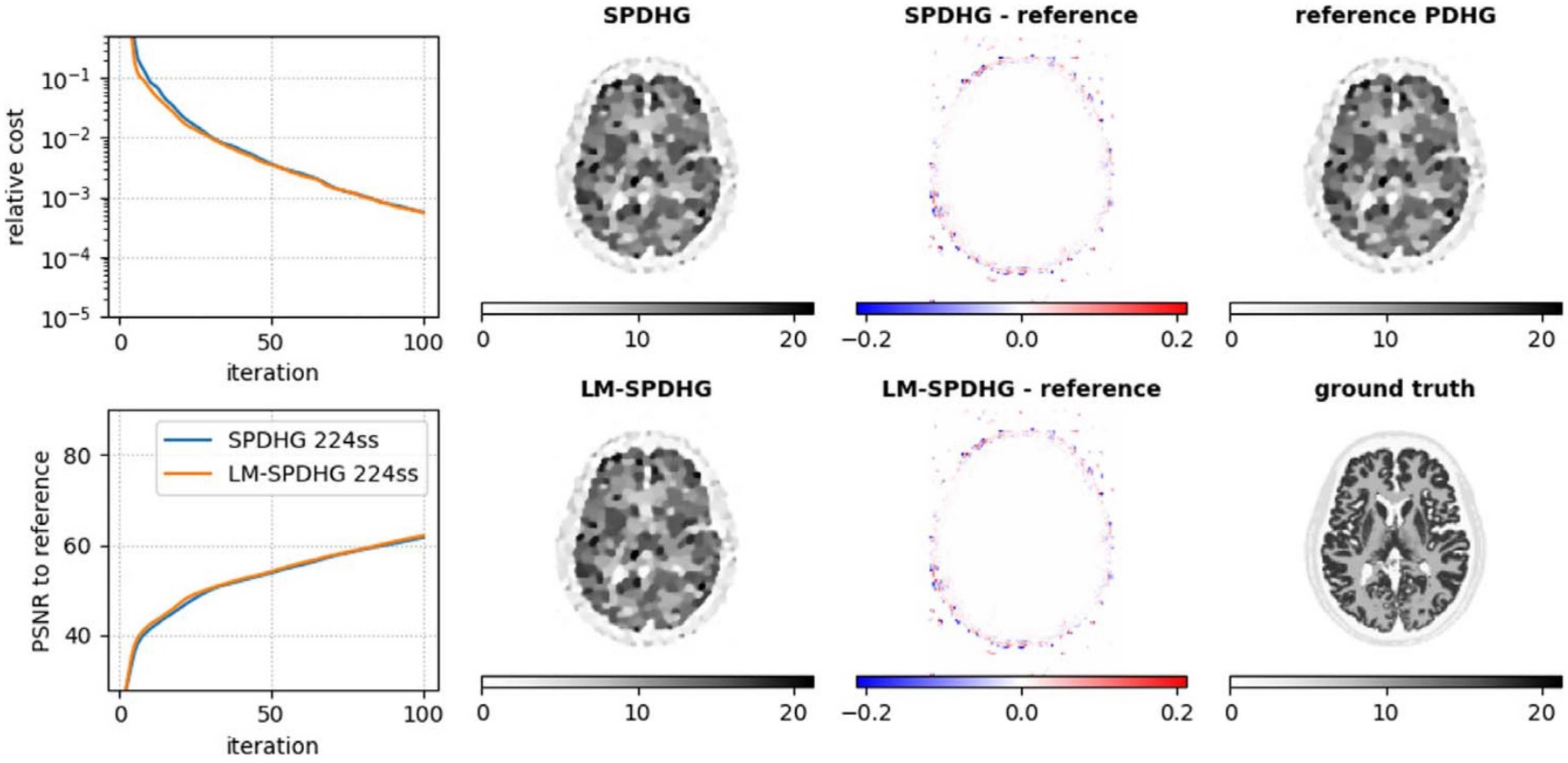
Convergence monitored via

$$\text{relative cost } c_{rel}(x) = \frac{c(x) - c(x^*)}{c(x^0) - c(x^*)}$$

$$\text{PSNR} = 20 \log \frac{|x^*|_\infty}{\text{MSE}(x, x^*)}$$

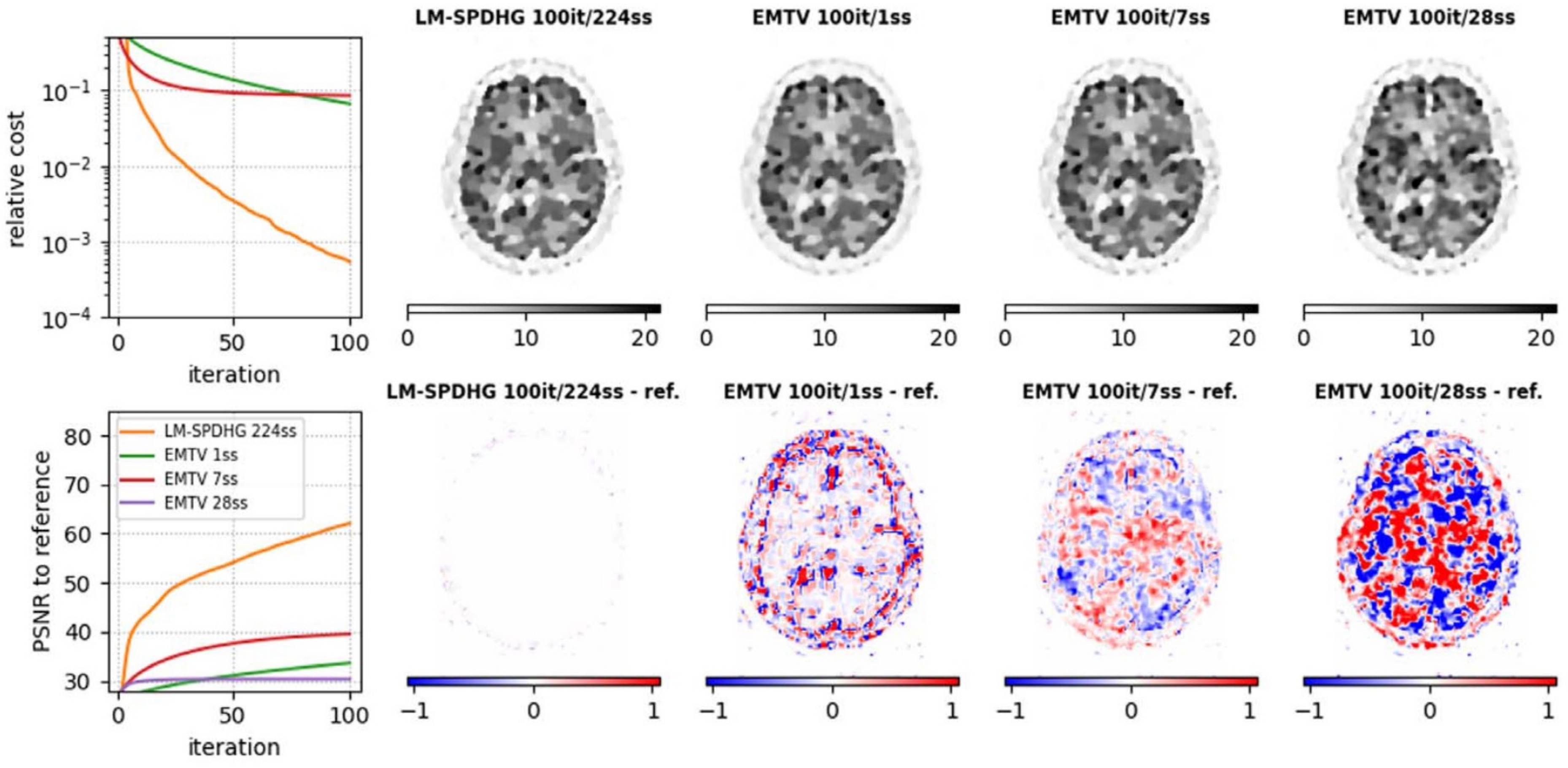
Results

LM SPDHG converges as fast as sinogram SPDHG



(a) 3e5 true (5e5 prompt) counts, TV prior, $\beta = 0.03$

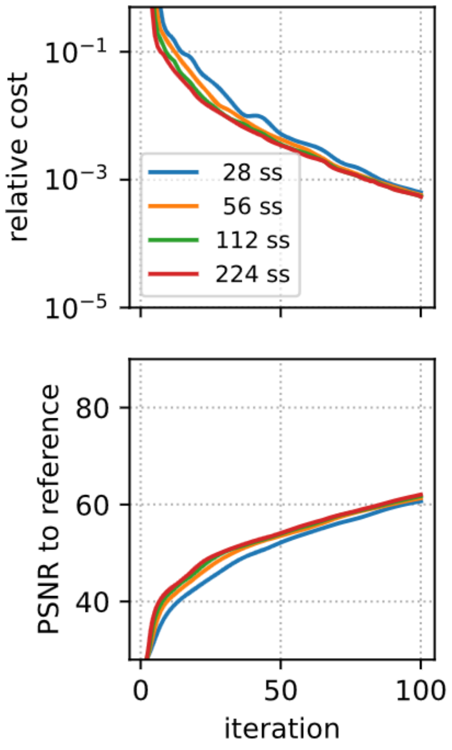
LM SPDHG vs EM-TV in 2D simulations



(a) 3e5 true (5e5 prompt) counts, TV prior, $\beta = 0.03$

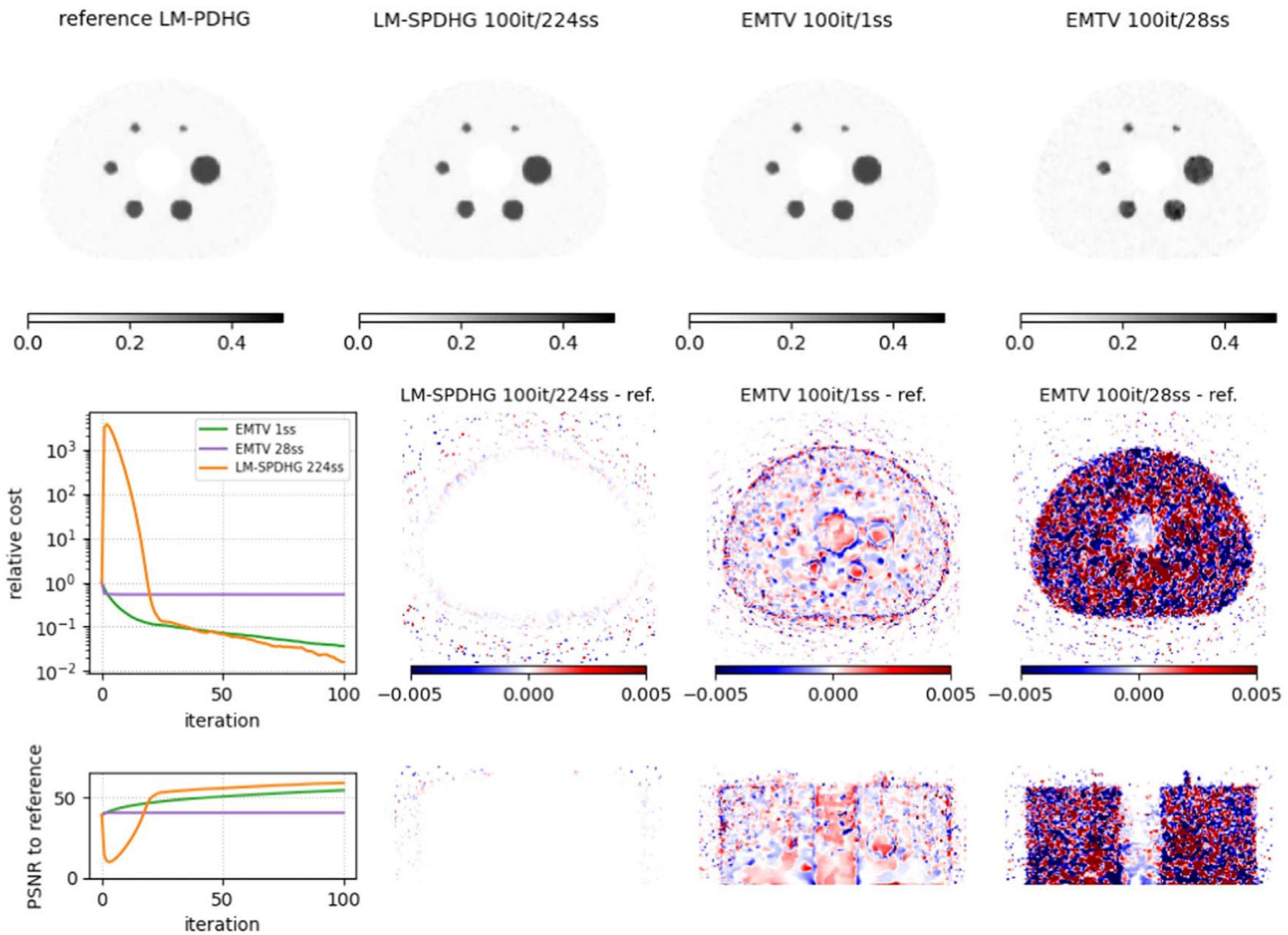
Sawatzky et al. "Accurate EM-TV algorithm in PET with low SNR", IEEE NSS/MIC 2008

Speed of Convergence vs Number of Subsets in 2D simulations



(a) 3e5 true (5e5 prompt) counts, TV prior, $\beta = 0.03$

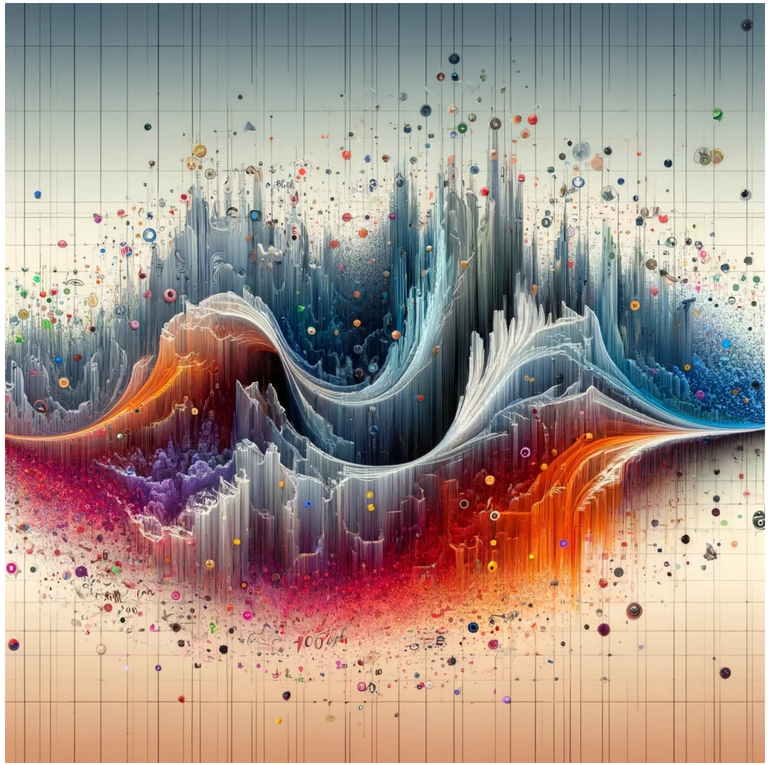
Reconstructions of NEMA IQ phantom scan



Discussion and Conclusion

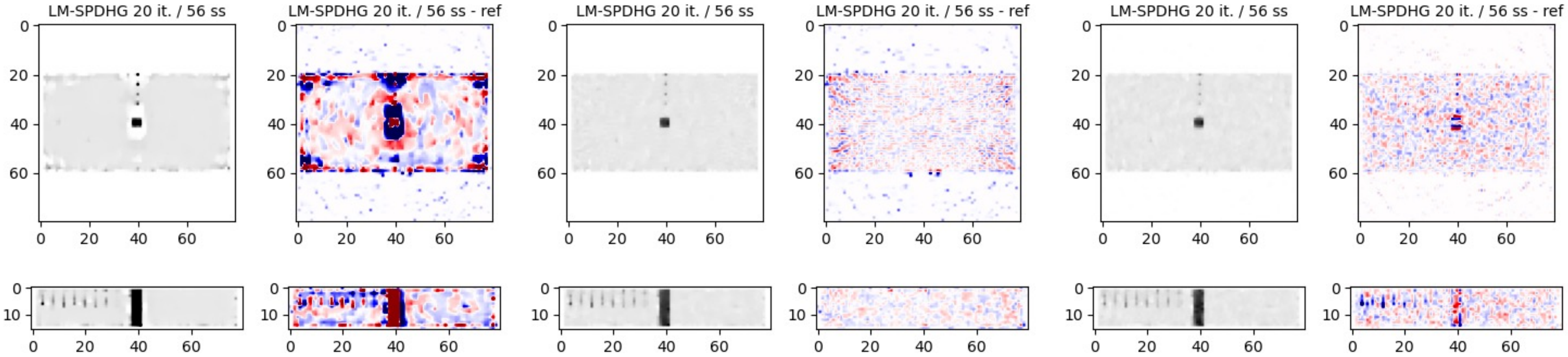
Discussion

- convergence speed **LM-SPDHG** very **similar** to (sinogram) **SPDHG**
- for "**normal count**" acquisitions @ 400ps systems:
  → **LM-SPDHG** much **faster** and **memory efficient** than SPDHG
- all **PDHG** versions are **non-monotonic**
  → stopping (very) early not recommended
- behaviour of all PDHG-variants in **early iterations very sensitive** to:
  - **initialization** of primal and dual variable
  - **step size ratio** ("S vs T")

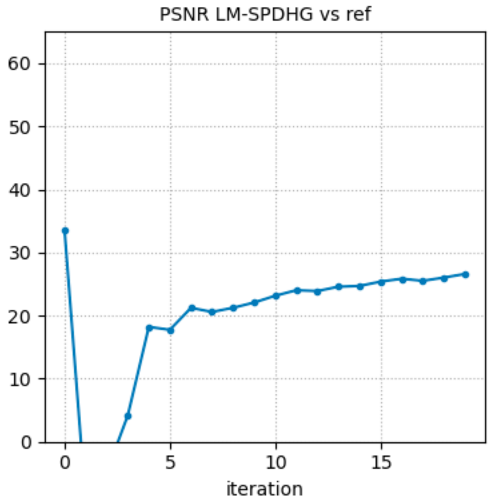


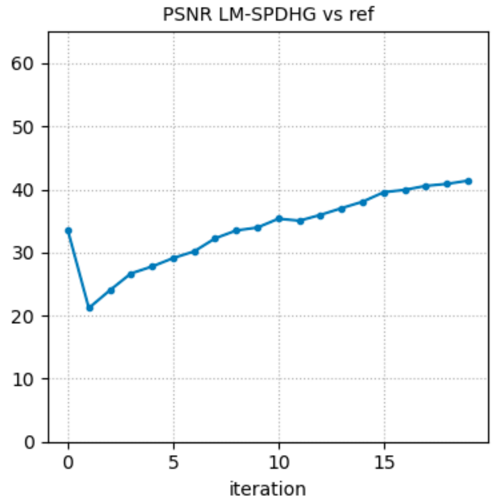
Impact of the step size ratio on (LM-S)PDHG

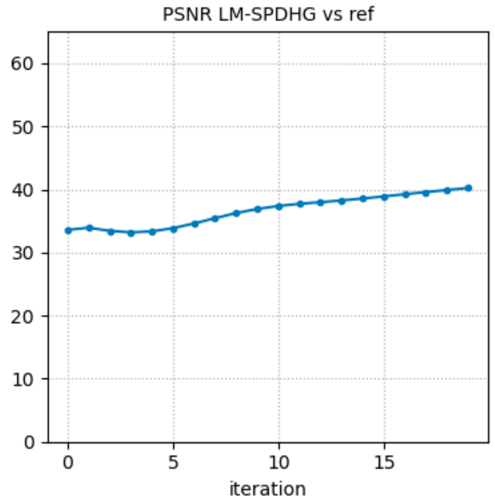
γ = 0.03 / max(img) γ = 3 / max(img) γ = 300 / max(img)

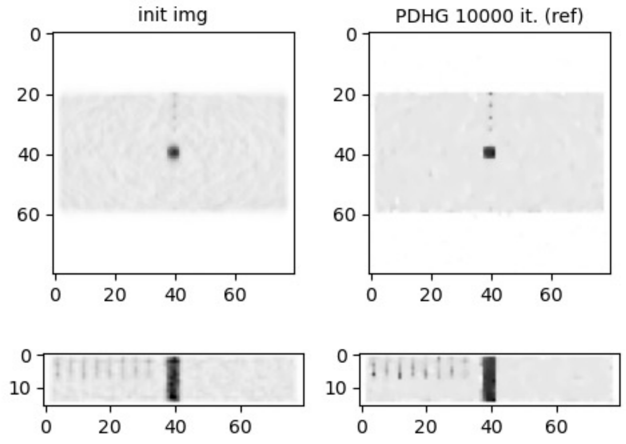


20 it / 56 ss





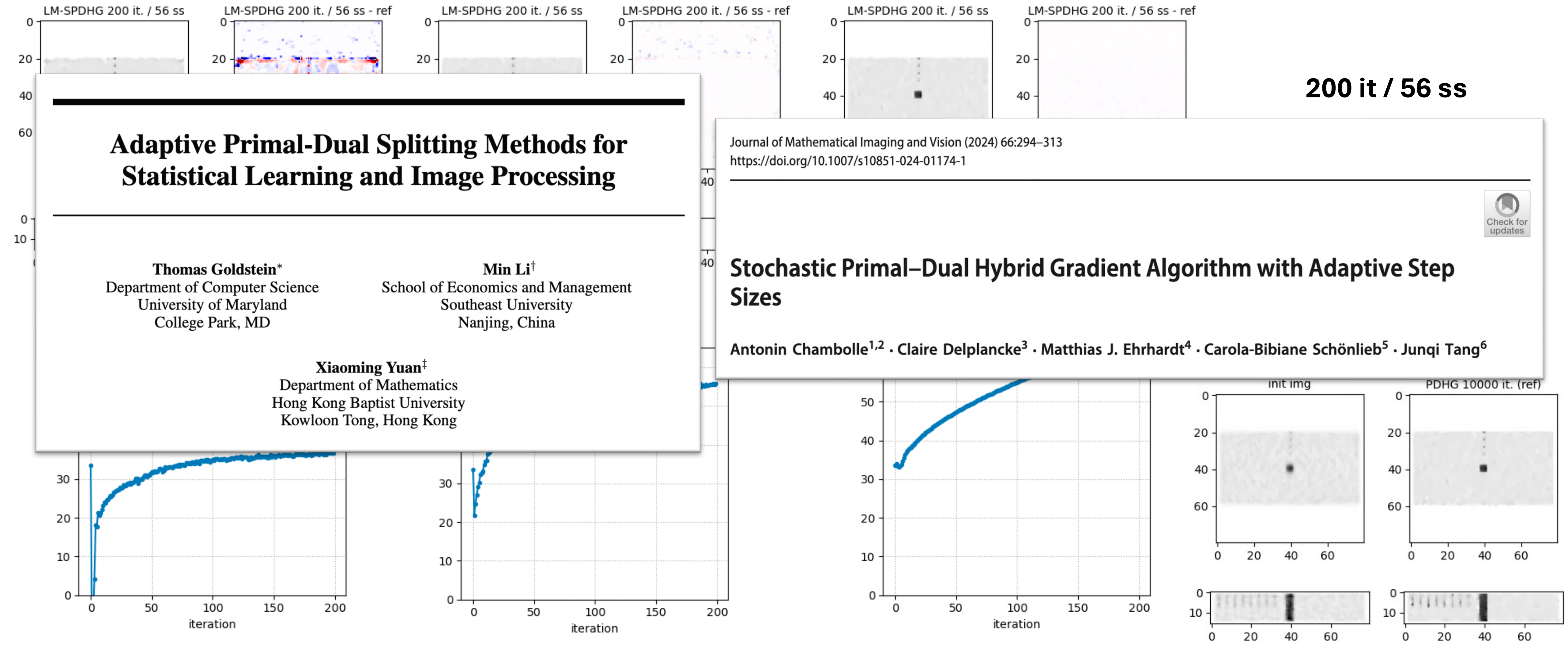




Impact of the step size ratio on (LM-S)PDHG

γ = 0.03 / max(img) | γ = 3 / max(img) | γ = 300 / max(img)

200 it / 56 ss



Adaptive Primal-Dual Splitting Methods for Statistical Learning and Image Processing

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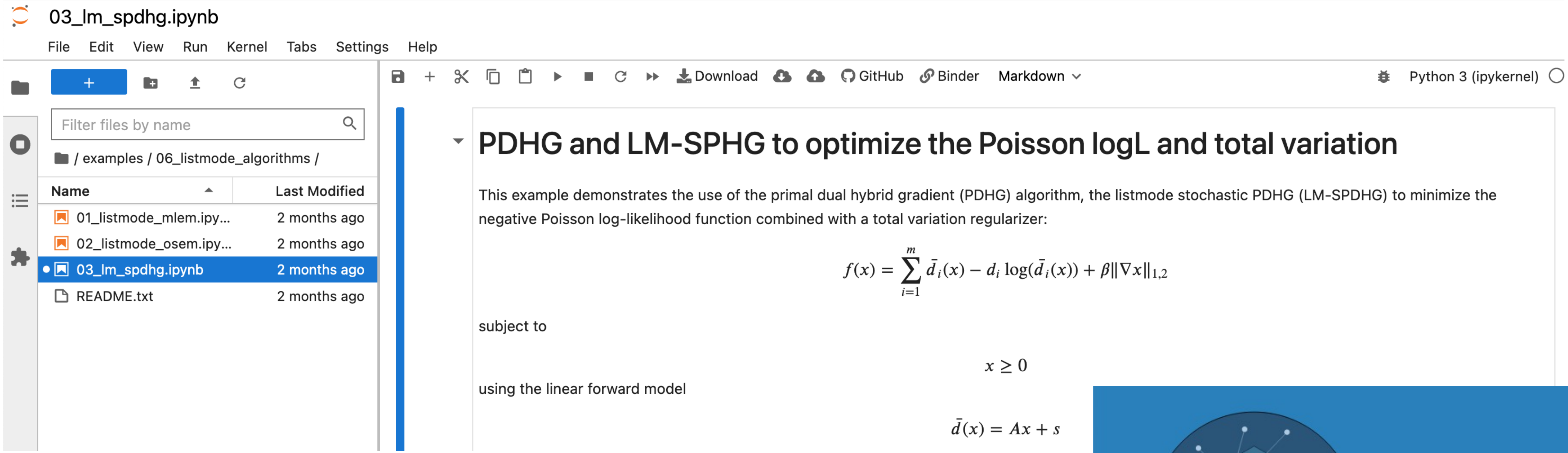
Journal of Mathematical Imaging and Vision (2024) 66:294–313
https://doi.org/10.1007/s10851-024-01174-1

Check for updates

Stochastic Primal–Dual Hybrid Gradient Algorithm with Adaptive Step Sizes

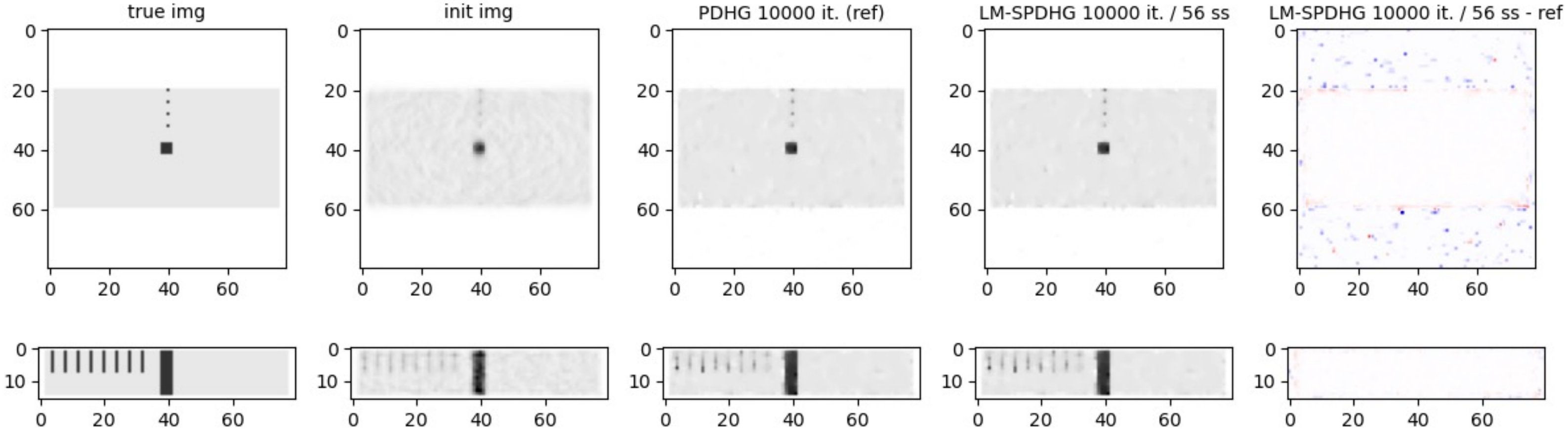
Antonin Chambolle[1,2] · Claire Delplancke[3] · Matthias J. Ehrhardt[4] · Carola-Bibiane Schönlieb[5] · Junqi Tang[6]

Try LM SPDHG yourself



https://parallelproj.readthedocs.io/en/stable/auto_examples/06_listmode_algorithms/03_lm_spdhg.html#listmode-spdhg

$\gamma = 0.03 / \max(img)$



10000 it / 56 ss