SIGNATURES OF THE YANG-MILLS DECONFINEMENT TRANSITION: GAUGE FIXING AND CENTER SYMMETRY

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arXiv:2206.03841, 2304.00756 [hep-ph]

In collaboration with Urko Reinosa

CONTENT

INTRODUCTION

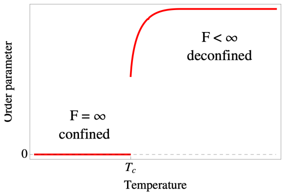
- In general: insight in the low-energy regime of QCD, especially in the confinement/deconfinement transition.
- Most results are coming from non-perturbative, numerical (lattice) or semi-analytical (FRG and SD) methods.
- From this, we know that:
  - At some very high temperature T_c, hadrons become free quarks and gluons $\rightarrow$ quark-gluon plasma.
  - This transition is related to the breaking of the center symmetry of a gauge group when using pure Yang-Mills theory (infinitely heavy quarks):

$$\mathcal{L} = \frac{1}{4}(F_{\mu\nu}^a)^2$$

CENTER SYMMETRY

- An order parameter for the confinement/deconfinement transition at finite temperature is the Polyakov loop:

$$\ell = \frac{1}{N} tr \left[\langle P e^{i \int_0^\beta d\tau A_0(\tau,x)} \rangle \right] \propto e^{-\beta F}$$



- In the confined phase, F is infinite $\rightarrow \ell = 0$. In the deconfined phase, F is finite $\rightarrow \ell \neq 0$.
- The symmetry connected to this phase transition is the center symmetry.

CENTER SYMMETRY

- Center symmetry is a symmetry present in $SU(N)$ Yang-Mills theories at finite temperature, where periodic boundary conditions restrict the gauge fields as

$$A_\mu(\tau+\beta,x) = A_\mu(\tau,x)$$

- This invokes a weaker restriction on the transformation field

$$U(\tau+\beta,x) = e^{i2\pi k/N}U(\tau,x)$$

  with $k = 0, ..., N-1$.
- Of these transformations, only U_0, corresponding to $k = 0$ are genuine gauge-transformations.

CENTER SYMMETRY

- The Polyakov loop (an observable) is not invariant

$$\ell \to tr\left[U(\beta,x)\langle Pe^{i\int_0^\beta d\tau A_0(\tau,x)}\rangle U^\dagger(0,x)\right] = e^{i2\pi k/N}\ell,$$

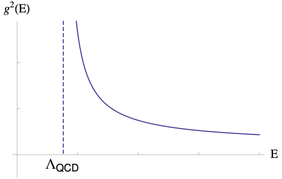
unless $\ell = 0$ (confinement region).
- The breaking of the center symmetry at high temperatures was confirmed by lattice data: second order transition for $SU(2)$, first order for $SU(3)$.
- We want to probe the breaking of center symmetry in an analytical model at finite temperature.

ANALYTICAL RESULTS

- At high energies, gluon dynamics are well described by an $SU(3)$ pure Yang-Mills action with a Faddeev-Popov gauge fixing:

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \partial_\mu \bar{c}^a (D_\mu c)^a + ib^a \partial_\mu A^a_\mu$$

- At high energies, the coupling constant decreases: Asymptotic freedom.
- At low energies, the coupling constant increases, and diverges: Landau pole.



- Does this mean we have an infinite coupling at low energies? Probably not!

GRIBOV PROBLEM

- Gribov: for high values of the coupling constant, the FP gauge fixing does not uniquely fix the gauge field

$$A'^a_\mu = A^a_\mu - D^{ab}_\mu \alpha^b \qquad \partial_\mu A^a_\mu = \partial_\mu A^{a\prime}{}_\mu = 0$$

so that

$$\partial_\mu D^{ab}_\mu \alpha^b = (\partial^2 \delta^{ab} - g f^{abc} \partial_\mu A^c_\mu)\alpha^b = 0$$

- An analytic model of the IR regime should restrict the number of Gribov copies:
  - Gribov-Zwanziger model:

$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b + i b^a \partial_\mu A^a_\mu - \bar{\omega}^{ae}_\nu \partial_\mu D^{ab}_\mu \omega^{be}_\nu$$
$$+ \bar{\varphi}^{ae}_\nu \partial_\mu D^{ab}_\mu \varphi^{be}_\nu - g\gamma^{1/2} f^{abc} A^a_\mu \left(\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu \right) - \gamma d d_G$$

  - Curci-Ferrari model:

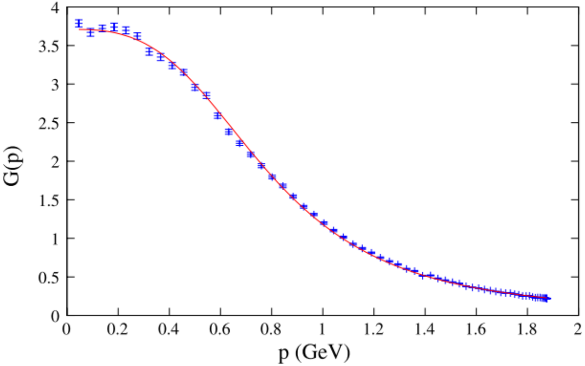
$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b + i b^a \partial_\mu A^a_\mu + m^2 A^a_\mu A^a_\mu$$

CURCI-FERRARI MODEL

- $\mathcal{L} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{c}^a\partial_\mu D^{ab}_\mu c^b + ib^a\partial_\mu A^a_\mu + m^2 A^a_\mu A^a_\mu$
- Gluons do not have a mass perturbatively
- BRST-symmetry that defines the physical space is broken
- However, for $p \gg m$, $m \approx 0$
- Confined gluons do not have a physical interpretation, so BRST symmetry might be broken non-perturbatively

CURCI-FERRARI MODEL

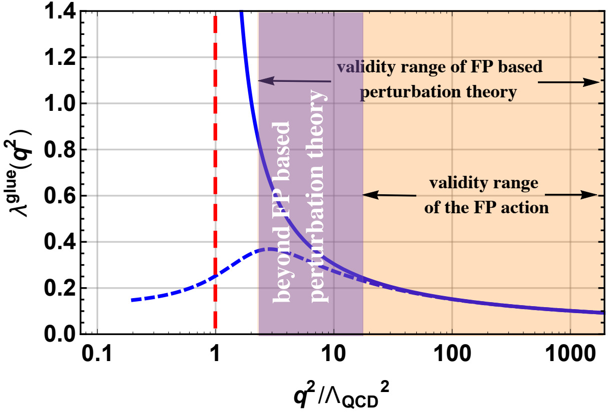
- $\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b + i b^a \partial_\mu A^a_\mu + m^2 A^a_\mu A^a_\mu$
- Lattice results show that the gluon propagator saturates in the IR
- The CF model could be an effective model that accounts for the lattice results



m=0.68 GeV, μ=1 GeV, g=7.5

*M. Tissier and N. Wschebor, PRD, 84, 045018 (2011).

CURCI-FERRARI MODEL



- The CF model is IR safe and might give a perturbative window into the low-energy regime.
- Can we use the CF (or GZ) model to describe the confinement/deconfinement transition? In principle: Yes. In practice: Maybe.

ENCODING OF THE TRANSITION

Polyakov loop:

$$\ell \sim \langle P e^{i \int_0^\beta d\tau A_0(\tau,x)} \rangle \sim e^{-\beta F}.$$

Because the Polyakov loop is related to A_0, it is expected that the transition is encoded in (the tower of)

$$\langle A_0 \rangle, \langle A_0 A_0 \rangle, \ldots, \langle A_0^n \rangle.$$

For the appropriate choice of gauge, can the transition be reflected in the lowest order correlators?

LANDAU GAUGE CORRELATOR

In principle:

- $\langle A \rangle$ is found by minimizing the effective action $\Gamma[A]$. It represents the state of the system . $\langle A_0 \rangle \to$ order parameter.
- The two-point correlator derives from the effective action

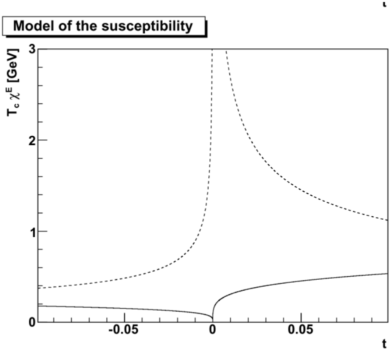
$$1 \Big/ \left. \frac{\partial^2 \Gamma}{\partial A^2} \right|_{A=\langle A \rangle} = \langle AA \rangle_c,$$

so for $SU(2)$, $\langle A_0 A_0 \rangle$ should diverge at T_c.

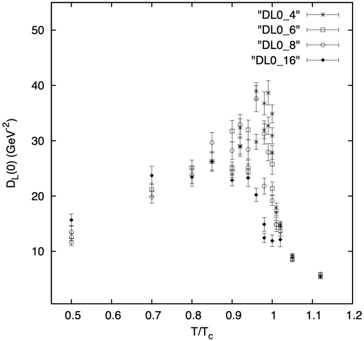
In practice:

- In the Landau gauge, $\partial_\mu A_\mu = 0$, then $\langle A_0 \rangle = 0$. $\to$ no order parameter.
- No evidence of divergence of $\langle A_0 A_0 \rangle$ was found on the (gauge-fixed) lattice and in the continuum.

$SU(2)$ LANDAU GAUGE CORRELATORS

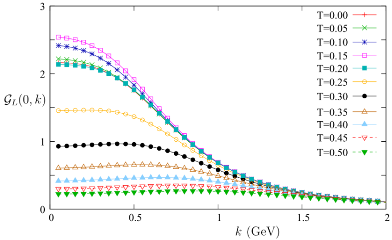


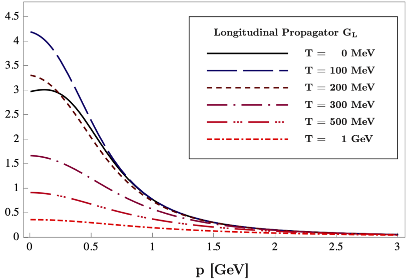
Electric susceptibility (zero momentum longitudinal propagator)



*T. Mendes and A. Cucchieri, PoS LATTICE2014, 183 (2015).

$SU(2)$ LANDAU GAUGE CORRELATORS





Longitudinal gluon propagator

*U. Reinosa, J. Serreau, M. Tissier, N. Wschebor, Phys. Rev. D, (2015)

*L. Fister and J. M. Pawlowski, [arXiv:1112.5440 [hep- ph]](2012).

BACKGROUND FIELD GAUGES

- A priori there is no reason to believe that the Landau gauge will provide the right environment to keep track of the centersymmetry breaking.
- In the Landau gauge the effective action is not explicitly center symmetric since $\Gamma[A] \neq \Gamma[A^U]$.
- This should not alter the physical results in principle, but it can lead to problems when approximations (loop calculations) are involved.
- To regain gauge invariance, one solution is to work with the Background Field Gauges.

BACKGROUND FIELD GAUGES

To retain the center symmetry, we introduce the the Landau-DeWitt gauge

$$\overline{D}_\mu (A_\mu - \overline{A}_\mu) = 0,$$

with $\overline{A}_\mu$ an arbitrary background field and $\bar{D}_\mu \equiv \partial_\mu - \left[\bar{A}_\mu, \cdot\right]$. We consider the action

$$S[A, \overline{A}] = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{D}_\mu \overline{c}^a D_\mu c^a + i b^a \overline{D}_\mu (A^a_\mu - \overline{A}^a_\mu) + m^2 (A^a_\mu - \overline{A}^a_\mu) \right\},$$

which is invariant under the simultaneous $SU(N)$ transformation of the fields A^a_μ and $\overline{A}^a_\mu$

$$S[A^U, \overline{A}^U] = S[A, \overline{A}],$$

and gauge invariance is preserved (in a way).

BACKGROUND FIELD GAUGES

$$\begin{aligned} S[A^U, \bar{A}^U] &= S[A, \bar{A}] \\ \Gamma[A^U, \bar{A}^U] &= \Gamma[A, \bar{A}] \end{aligned}$$

Checking the invariance under center transformations of $A_{\text{min}}[\bar{A}]$ is ambiguous:

$$\Gamma[A_{min}(\bar{A}), \bar{A}] = \Gamma[A^U_{min}(\bar{A}), \bar{A}^U].$$

Transforming the minimizing state transforms it into the minimizing function of another potential, with another gauge fixing. Therefore, we cannot identify the center-symmetric states.

Background effective action

In the method of the BG effective action[1], we define a new object

$$\tilde{\Gamma}[\overline{A}] \equiv \Gamma[A = \overline{A}, \overline{A}],$$

and we can show that $\overline{A}_{min}$ such that

$$\tilde{\Gamma}[A_{min}(\overline{A})] \leq \tilde{\Gamma}[\overline{A}], \qquad \forall \overline{A}$$

are alternative order parameters for center symmetry. There is no ambiguity anymore since:

$$\tilde{\Gamma}[\overline{A}] = \tilde{\Gamma}[\overline{A}^U].$$

The drawback of the BG method is that $\tilde{\Gamma}[\overline{A}]$ is a formal object an does not relate directly to gauge-fixed quantities such as propagators and vertices.

[1] J. Braun, H. Gies and J.M. Pawlowski, Phys. Lett. B 684,262-267 (2010)

- Asking for a gauge-fixing which preserves all the gauge transformations is asking too much. But we are only interested in preserving center-symmetry, not full gauge invariance!
- To find a genuine gauge-fixing which preserves center symmetry, let us go back to the transformation

$$U(\tau+\beta,x) = e^{i2\pi k/N}U(\tau,x)$$

with $k = 0,...,N-1$. We can divide all the transformations into N subsets $\mathcal{U}_k$. Different $U_k's$ within $\mathcal{U}_k$ are connected through $U_k = U_k' U_0$.

- The Polyakov loop only "feels" $\mathcal{U}_k$, so we need a gauge such that

$$\exists U_0 \text{ such that } \Gamma[A,\overline{A}] = \Gamma[A^{UU_0},\overline{A}^{UU_0}]$$

Center-symmetric effective action

In the solution of the CS effective action, we define the center symmetric state A_c as

$$\exists U_0 \text{ such that } A_c = A_c^{UU_0}$$

and make the gauge choice $\overline{A} = A_c$.
Now

$$\Gamma[A, A_c] \neq \Gamma_c[A^U, A_c], \qquad \forall U$$

but

$$\exists U_0 \ \Gamma[A, A_c] = \Gamma[A^{UU_0}, A_c].$$

There is no ambiguity since the background field $\overline{A}$ is fixed.

Since $\Gamma_c[A]$ is nothing but a particular gauge choice of $\Gamma[A, \overline{A}]$, we can directly reach gauge-fixed quantities such as propagators and vertices.

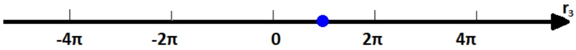
OUR SETUP

- We work in the Landau-deWitt gauge with a background field $\bar{A}_\mu$:

$$\bar{D}_\mu(A_\mu - \bar{A}_\mu) = 0, \text{ with } \bar{D}_\mu \equiv \partial_\mu - \left[\bar{A}_\mu, \cdot\right].$$

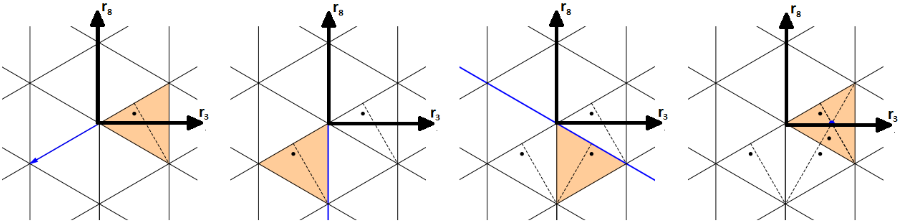
- We take $\bar{A}$ and $\langle A \rangle$ in the temporal direction, $\propto \delta_{\mu 0}$
- We take $\bar{A}$ and $\langle A \rangle$ along the diagonal color directions t^i (σ^3 for $SU(2)$, (λ^3, λ^8) for $SU(3)$), so that $\Gamma[A, \bar{A}] \propto V(A, \bar{A})$.
- We write $\langle A \rangle = \delta_{\mu 0} \frac{T}{g} r^i t^i$ and $\bar{A} = \delta_{\mu 0} \frac{T}{g} \bar{r}^i t^i$.

$SU(2)$ CENTER SYMMETRY



- $A^U : r \to r + 2\pi$
- A^{U_0} : reflection of r in $2\pi n$
- Centersymmetric point at $r_c = \pi$
- We fix $\bar{r} = r_c$: Center-symmetric Landau gauge. Center-symmetric phase when $r = r_c$, $\to$ order parameter.

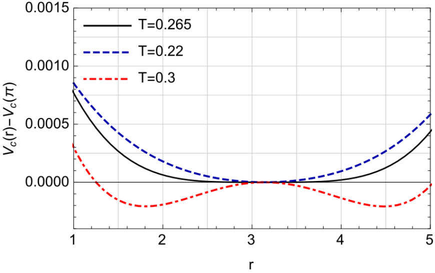
$SU(3)$ CENTER SYMMETRY

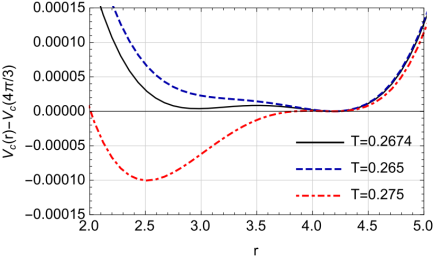


- A^U: translation of $\{r_3, r_8\}$
- A^{U_0} : reflection of $\{r_3, r_8\}$
- Centersymmetric point at $r = \{4\pi/3, 0\}$
- We fix $\bar{r} = r_c$: Center-symmetric Landau gauge. Center-symmetric phase when $r = r_c$, $\rightarrow$ order parameter.

CURCI-FERRARI MODEL

We looked at the one-loop potential to find $\langle A \rangle$ and T_c





RESULTS - $T_c(MeV)$

	Lattice	FRG-BG[2]	CF-BG, 1-lp[3]	CF-BG, 2-lp[4]	CF-CS, 1-lp[5]
SU(2)	295	230	238	284	**265**
SU(3)	270	275	185	254	**267**

BG: Background effective action
CS: Centersymmetric Landau gauge
.

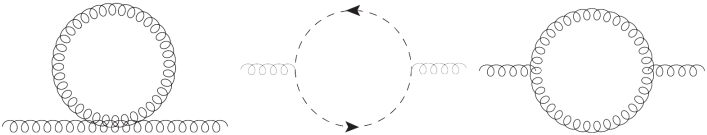
[2] L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010

[3] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys.Lett. B742 (2015) 61-68.

[4] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys.Rev. D93 (2016) 105002.

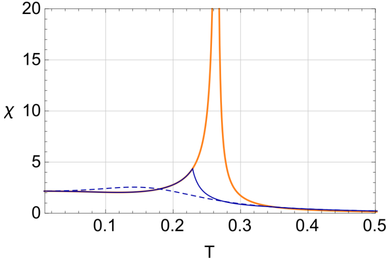
[5] DvE, U. Reinosa, J. Serreau and M. Tissier, SciPost Phys. 12, 087 (2022)

FEYNMAN DIAGRAMS GLUON PROPAGATOR



- Calculated in finite temperature trough Matsubara techniques: $\int d^d Q \to T \sum_q \int d^{d-1} q$
- We calculated the spatial integral with Feynman techniques, the Matsubara sums numerically.

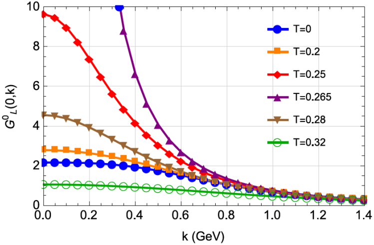
RESULTS: $SU(2)$ GLUON PROPAGATOR

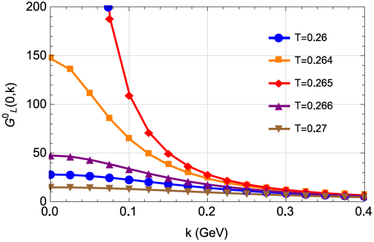


Landau gauge

Background field effective action

Centersymmetric Landau gauge

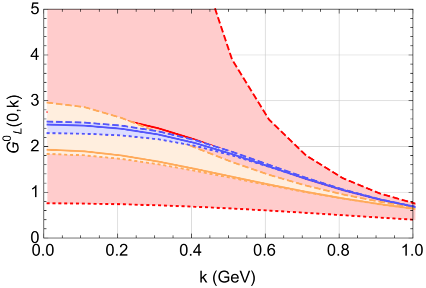




m=0.68 GeV, μ=1 GeV, g=7.5

*DvE, U. Reinosa, J. Serreau and M. Tissier, SciPost Phys. 12, 087 (2022).

RESULTS: $SU(2)$ GLUON PROPAGATOR

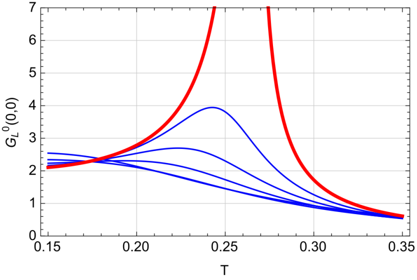


Landau gauge

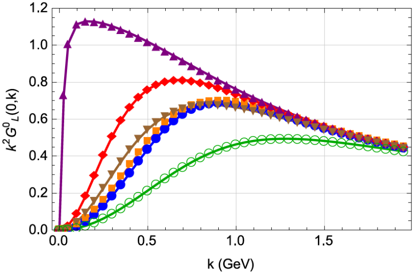
Background Field Effective action

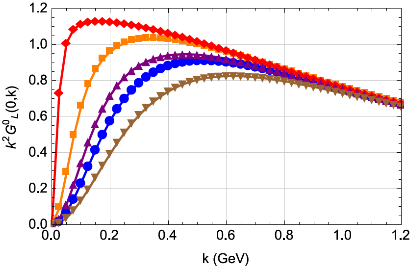
Center-symmetric Landau Gauge

RESULTS: $SU(2)$ GLUON PROPAGATOR

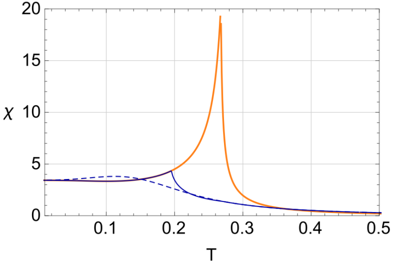


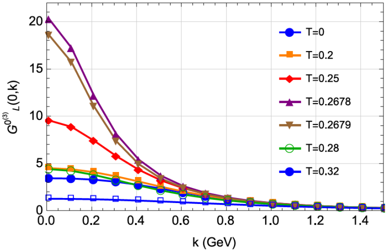
$SU(2)$ DRESSING FUNCTION

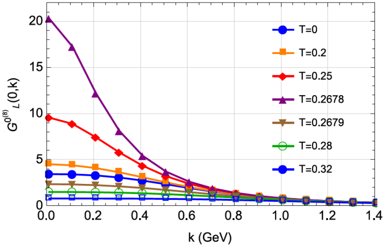




RESULTS: $SU(3)$ GLUON PROPAGATOR





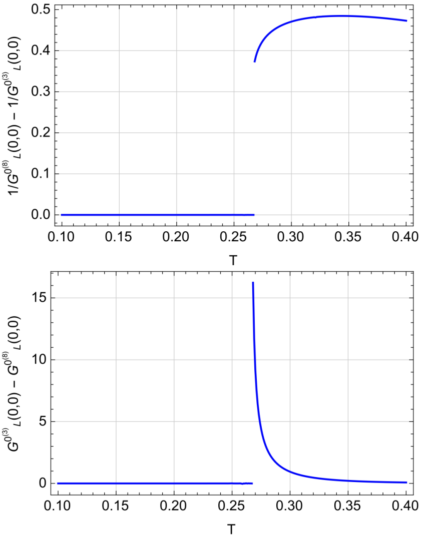


Landau gauge
Background field effective action
Centersymmetric Landau gauge

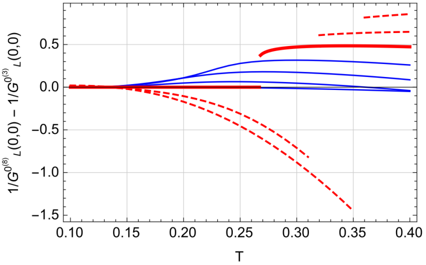
m=0.54 GeV, μ=1 GeV, g=4.9

*DvE, U. Reinosa, J. Serreau and M. Tissier, SciPost Phys. 12, 087 (2022).

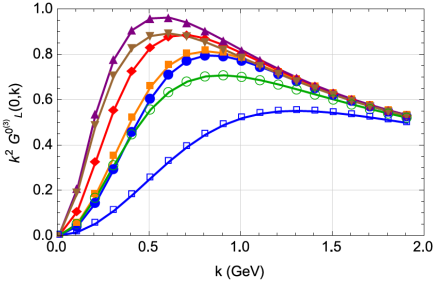
$SU(3)$ PROPAGATOR DIFFERENCE

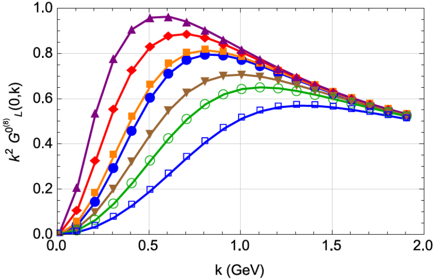


$SU(3)$ PROPAGATOR DIFFERENCE

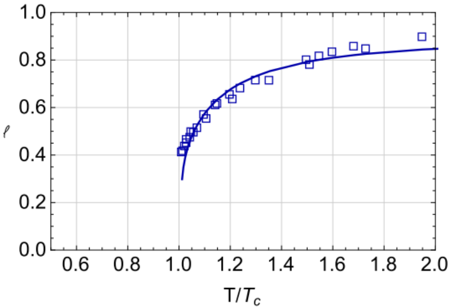


$SU(3)$ DRESSING FUNCTION





$SU(3)$ POLYAKOV LOOP



CONCLUSION AND OUTLOOK

- We have performed, for the first time, calculations of the gluon one-and two-point correlator in the centersymmetric Landau gauge.
- We find a good agreement with lattice data for T_c.
- We find that for $SU(2)$, the deconfinement transition is signaled by a divergence of the longitudinal gluon propagator for $k \to 0$.
- For $SU(3)$, the difference between the propagators in the neutral color mode is an order parameter for the transition.
- This model can be tested on the lattice by changing the boundary conditions in the Landau gauge [with O. Oliveira and P. Silva].
- Ideas for future works: RG improvement, transversal propagator and dynamically generated mass [with D. Dudal and D. Vercauteren].