The Dilaton EFT, the Lattice and the Composite Higgs

James Ingoldby (ICTP Trieste)

September 4, 2023

UV Complete Quantum Field Theories for Particle Physics

Outline

Near–Conformal Gauge Theories I

Consider $\mathrm{SU}(N_c)$ gauge theories with N_f fermions:

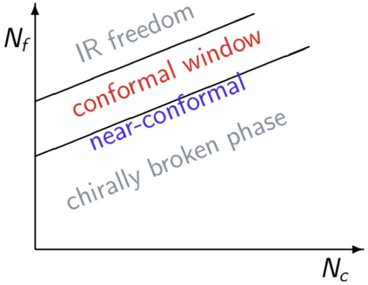


Figure: Phase diagram obtained from PoS Lattice **2018**, 006 (2019).

- $N_f > \frac{11}{2} N_c$: Not asymptotically free.
- $\frac{11}{2} N_c > N_f > N_{fc}$: Asymptotically free, but approaches conformality in IR.
- $N_{fc} > N_f$: Confinement. Low energy states are colorless composites.
- Can generalize to other gauge groups and scalar matter.

Near–Conformal Gauge Theories II

- Near–conformal gauge theories *confine*.

Near–Conformal Gauge Theories II

- Near–conformal gauge theories *confine*.
- But only just. The field content is chosen to ensure that they lie just beneath the boundary of the conformal window.

Near–Conformal Gauge Theories II

- Near–conformal gauge theories *confine*.
- But only just. The field content is chosen to ensure that they lie just beneath the boundary of the conformal window.
- There is also evidence for a light scalar composite forming in these gauge theories, unlike in QCD.

Evidence for a Light Scalar I

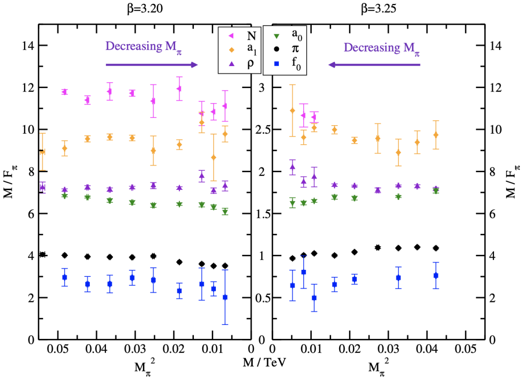


Figure: Lattice data for the masses of composites $\mathrm{SU}(3)$ gauge theories with $N_f = 2$ fermions in 2–index symmetric rep. From the LatHC collaboration: PoS LATTICE2015 (2016) 219.

Evidence for a Light Scalar II

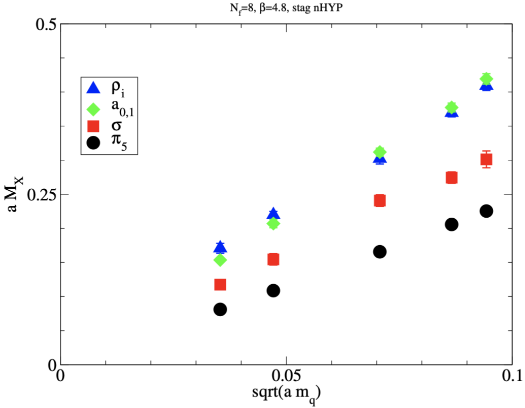


Figure: Lattice data for the masses of composites in the $\mathrm{SU}(3)$ gauge theory with $N_f = 8$ fundamental fermions from the LSD collaboration: 2306.06095

Advantages of EFT Description

- Lattice calculations are very resource intensive. The EFT can provide a fast cross check.

Advantages of EFT Description

- Lattice calculations are very resource intensive. The EFT can provide a fast cross check.
- Enables calculation of observables that are particularly hard to calculate directly on the lattice (e.g. 6 scalar scattering amplitude).

Advantages of EFT Description

- Lattice calculations are very resource intensive. The EFT can provide a fast cross check.
- Enables calculation of observables that are particularly hard to calculate directly on the lattice (e.g. 6 scalar scattering amplitude).
- Enables extrapolation of lattice data to $m \to 0$ limit.

Advantages of EFT Description

- Lattice calculations are very resource intensive. The EFT can provide a fast cross check.
- Enables calculation of observables that are particularly hard to calculate directly on the lattice (e.g. 6 scalar scattering amplitude).
- Enables extrapolation of lattice data to $m \to 0$ limit.
- Interactions with SM degrees of freedom can be more straightforwardly added. Can test whether specific nearly conformal gauge theories could yield a viable composite Higgs model.

Advantages of EFT Description

- Lattice calculations are very resource intensive. The EFT can provide a fast cross check.
- Enables calculation of observables that are particularly hard to calculate directly on the lattice (e.g. 6 scalar scattering amplitude).
- Enables extrapolation of lattice data to $m \to 0$ limit.
- Interactions with SM degrees of freedom can be more straightforwardly added. Can test whether specific nearly conformal gauge theories could yield a viable composite Higgs model.
- But you need the lattice to find the low energy constants of the EFT.

The Program in a Nutshell

- Write down an EFT to simply describe the lightest composite states. These are light scalar and the multiplet of pseudoscalars.

The Program in a Nutshell

- Write down an EFT to simply describe the lightest composite states. These are light scalar and the multiplet of pseudoscalars.
- Compare EFT predictions with lattice data for nearly conformal gauge theories.

The Program in a Nutshell

- Write down an EFT to simply describe the lightest composite states. These are light scalar and the multiplet of pseudoscalars.
- Compare EFT predictions with lattice data for nearly conformal gauge theories.
- Test whether dilaton EFT is the correct description.

The Program in a Nutshell

- Write down an EFT to simply describe the lightest composite states. These are light scalar and the multiplet of pseudoscalars.
- Compare EFT predictions with lattice data for nearly conformal gauge theories.
- Test whether dilaton EFT is the correct description.
- Extract values of free parameters from the lattice data.

The Program in a Nutshell

- Write down an EFT to simply describe the lightest composite states. These are light scalar and the multiplet of pseudoscalars.
- Compare EFT predictions with lattice data for nearly conformal gauge theories.
- Test whether dilaton EFT is the correct description.
- Extract values of free parameters from the lattice data.

Ultimate goal

Extract predictions for the low energy phenomenology of this unique class of gauge theories.

Dilaton EFT

Reviewed in Universe 9 (2023) 1, 10 with T. Appelquist and M. Piai.

Field Content

1. $N_f^2 - 1$ NGB fields π^a
   $\Sigma = \exp\{2i\pi^a T^a/F_\pi\}$
   $\langle\Sigma\rangle = \mathbb{1}$
2. Dilaton field χ
   $\langle\chi\rangle = F_d$

See also dilaton EFT of Golterman and Shamir: Phys.Rev.D **94** (2016)

Symmetries

Chiral Symmetry

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$$
$$\Sigma \to L\Sigma R^\dagger$$

Scale Invariance

Scale $\times$ Poincaré $\to$ Poincaré

$$\chi(x) \to e^\lambda \chi(e^\lambda x)$$

Leading Order Lagrangian

$\mathcal{L}_{\mathrm{LO}} = \mathcal{L}_\pi + \mathcal{L}_m + \mathcal{L}_d - V_\Delta$

Kinetic term for the NGBs

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] \quad (1)$$

- Similar to NGB kinetic term in chiral Lagrangian.
- Dependence on compensator field χ is determined by scale invariance.
- Expect $f_\pi \sim f_d$ set by confinement scale.

Leading Order Lagrangian

Chiral Symmetry Breaking Term

$$\mathcal{L}_m = \frac{m B_\pi f_\pi^2}{2} \left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right] \tag{2}$$

- Fermion mass breaks both scale and chiral symmetry.
- Parameter y has been identified with scaling dimension of $\bar{\psi}\psi$ above the confinement scale: Nucl. Phys. B **323**, 493 (1989).

$$\mathcal{L}_m = N_f m B_\pi f_\pi^2 \left(\frac{\chi}{f_d}\right)^y - m B_\pi \left(\frac{\chi}{f_d}\right)^y \pi^a \pi^a + \cdots$$

Leading Order Lagrangian

Dilaton Kinetic Term

$$\mathcal{L}_d = \frac{1}{2} \left(\partial_\mu \chi\right)^2 \quad (3)$$

- Has engineering dimension of 4, consistent with scale invariance.

Leading Order Lagrangian

Dilaton Potential I

$$V_\Delta = \frac{m_d^2 \chi^4}{4(4-\Delta) f_d^2}\left[1 - \frac{4}{\Delta}\left(\frac{f_d}{\chi}\right)^{4-\Delta}\right]. \tag{4}$$

Leading Order Lagrangian

Dilaton Potential I

$$V_\Delta = \frac{m_d^2 \chi^4}{4(4-\Delta) f_d^2} \left[1 - \frac{4}{\Delta} \left(\frac{f_d}{\chi} \right)^{4-\Delta} \right]. \tag{4}$$

- Potential contains a scale invariant term ($\sim \chi^4$) and a deformation ($\sim \chi^\Delta$), which explicitly violates scale invariance.

Leading Order Lagrangian

Dilaton Potential I

$$V_\Delta = \frac{m_d^2 \chi^4}{4(4-\Delta) f_d^2}\left[1 - \frac{4}{\Delta}\left(\frac{f_d}{\chi}\right)^{4-\Delta}\right]. \tag{4}$$

- Potential contains a scale invariant term ($\sim \chi^4$) and a deformation ($\sim \chi^\Delta$), which explicitly violates scale invariance.
- This potential has a minimum at $\chi = f_d$, and a weak curvature $m_d^2 \ll (4\pi f_d)^2$.

Leading Order Lagrangian

Dilaton Potential I

$$V_\Delta = \frac{m_d^2 \chi^4}{4(4-\Delta) f_d^2} \left[1 - \frac{4}{\Delta} \left(\frac{f_d}{\chi} \right)^{4-\Delta} \right] . \tag{4}$$

- Potential contains a scale invariant term ($\sim \chi^4$) and a deformation ($\sim \chi^\Delta$), which explicitly violates scale invariance.
- This potential has a minimum at $\chi = f_d$, and a weak curvature $m_d^2 \ll (4\pi f_d)^2$.
- For $\Delta < 4$, V_Δ grows as χ^4 for large χ.

Leading Order Lagrangian

Dilaton Potential I

$$V_\Delta = \frac{m_d^2 \chi^4}{4(4-\Delta) f_d^2} \left[1 - \frac{4}{\Delta} \left(\frac{f_d}{\chi} \right)^{4-\Delta} \right] . \tag{4}$$

- Potential contains a scale invariant term ($\sim \chi^4$) and a deformation ($\sim \chi^\Delta$), which explicitly violates scale invariance.
- This potential has a minimum at $\chi = f_d$, and a weak curvature $m_d^2 \ll (4\pi f_d)^2$.
- For $\Delta < 4$, V_Δ grows as χ^4 for large χ.
- For $\Delta > 4$, V_Δ grows as χ^Δ for large χ.

Leading Order Lagrangian

Dilaton Potential I

$$V_\Delta = \frac{m_d^2 \chi^4}{4(4-\Delta) f_d^2}\left[1 - \frac{4}{\Delta}\left(\frac{f_d}{\chi}\right)^{4-\Delta}\right]. \quad (4)$$

- Potential contains a scale invariant term ($\sim \chi^4$) and a deformation ($\sim \chi^\Delta$), which explicitly violates scale invariance.
- This potential has a minimum at $\chi = f_d$, and a weak curvature $m_d^2 \ll (4\pi f_d)^2$.
- For $\Delta < 4$, V_Δ grows as χ^4 for large χ.
- For $\Delta > 4$, V_Δ grows as χ^Δ for large χ.
- Potentials of this form are discussed in e.g: Rattazzi & Zaffaroni JHEP **0104**, 021 (2001), GGS PRL.**100** 111802, (2008), CCT PRD.**100** 095007 (2019).

Leading Order Lagrangian

Dilaton Potential II

Δ can be identified with the scaling dimension, at the confinement scale, of an operator responsible for breaking scale symmetry in the underlying theory.

Leading Order Lagrangian

Dilaton Potential II

Δ can be identified with the scaling dimension, at the confinement scale, of an operator responsible for breaking scale symmetry in the underlying theory.

Special case: The SM Higgs potential $\Delta = 2$.

$$V(\chi) = \frac{m_d^2}{8f_d^2}\left(\chi^2 - f_d^2\right)^2 \tag{5}$$

Leading Order Lagrangian

Dilaton Potential II

Δ can be identified with the scaling dimension, at the confinement scale, of an operator responsible for breaking scale symmetry in the underlying theory.

Special case: The SM Higgs potential $\Delta = 2$.

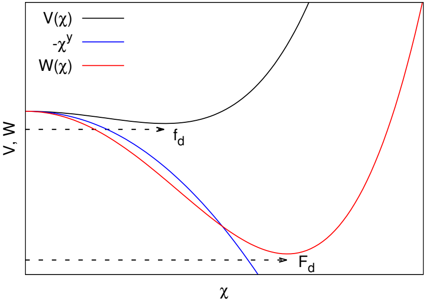
$$V(\chi) = \frac{m_d^2}{8 f_d^2} \left(\chi^2 - f_d^2\right)^2 \tag{5}$$

Special case: Near marginal deformation $\Delta \to 4$.

$$V(\chi) = \frac{m_d^2}{16 f_d^2} \chi^4 \left(4 \ln \frac{\chi}{f_d} - 1\right) \tag{6}$$

Scaling Features

$$W(\chi) = V_\Delta(\chi) - N_f m B_\pi f_\pi^2 \left(\frac{\chi}{f_d}\right)^y$$



$$\langle \chi \rangle = F_d$$

$$M_d^2 = \left.\frac{\partial^2 W}{\partial \chi^2}\right|_{F_d}$$

$$\frac{F_\pi^2}{f_\pi^2} = \frac{F_d^2}{f_d^2}$$

$$\frac{M_\pi^2}{2mB_\pi} = \left(\frac{F_d}{f_d}\right)^{y-2}$$

Fit Equations

Plugging the expression for V_Δ into W, we can derive three equations which are convenient for fitting lattice data:

$$M_\pi^2 F_\pi^{2-y} = 2B_\pi f_\pi^{2-y} m, \tag{7}$$

$$\frac{M_\pi^2}{F_\pi^2} = \frac{2m_d^2 f_d^2}{yN_f(4-\Delta)f_\pi^4}\left(1-\left(\frac{f_\pi}{F_\pi}\right)^{4-\Delta}\right), \tag{8}$$

$$\frac{M_d^2}{F_\pi^2} = \frac{m_d^2}{(4-\Delta)f_\pi^2}\left(4-y+(y-\Delta)\left(\frac{f_\pi}{F_\pi}\right)^{4-\Delta}\right). \tag{9}$$

Scalar Decay Constant

Define scalar decay constant using the matrix element

$$\langle 0| J_S(x) |\chi(p)\rangle \equiv F_S M_d^2 e^{-p\cdot x} \,, \tag{10}$$

where

$$J_S(x) \equiv m \sum_{i=1}^{N_f} \bar{\psi}_i \psi_i \,. \tag{11}$$

1. F_S can be extracted from lattice measurement of correlator $\langle J_S(x) J_S(0)\rangle$, which is used already to measure M_d.
2. It is a true decay constant: It would control the decay rate of the dilaton if there was a heavy scalar mediator coupled to $\bar{\psi}\psi$ along with light states. Analogous to f_π for the QCD pion decaying to leptons via $W^{\pm}$.

Scalar Decay Constant

This quantity can also be calculated in dilaton EFT:

$$|F_S| = \frac{yN_f M_\pi^2 F_\pi}{2M_d^2}\frac{f_\pi}{f_d}. \tag{12}$$

1. Incorporating Eq. (12) into our EFT fit provides a direct test of the coupling between the light scalar and the fermion mass, treated as an external source.
2. It is $\propto y = d - \Delta_m$, as expected for a dilaton coupling.

Lattice Calculation of Scattering Phase Shift

Phys.Rev.D **105** (2022) with LSD Collaboration

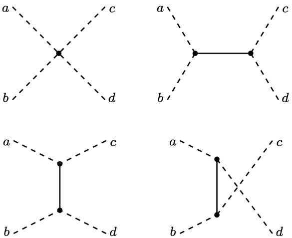
[M. Lüscher NPB 354(1991)]

$$k^2 = \frac{1}{4}E_{\pi\pi}^2 - M_\pi^2 \tag{13}$$

$$k \cot \delta(k) = \frac{2\pi}{L}\pi^{-3/2} Z_{00}\left(1, \frac{k^2L^2}{4\pi^2}\right) \tag{14}$$

- Restrict ourselves to $I = 2$ channel.
- $E_{\pi\pi}$ is the two–PNGB ground state energy.
- Measured at finite volume (L) on the lattice from a fit to a two point correlation function of two PNGB operators. Schematically: $C(t) \sim \langle \mathcal{O}^{I=2}(t)\mathcal{O}^{\dagger\, I=2}(0)\rangle$ where $\mathcal{O}^{I=2} \sim \pi\pi$.

$I = 2$ Scattering Length



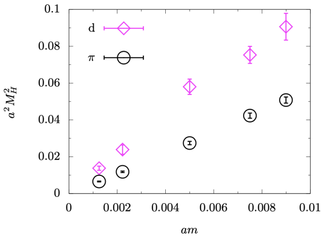
- Scattering amplitude at threshold $= M_\pi a^{I=2}$
- First diagram, same as χPT. The others only arise for light scalar (dilaton).

$$M_\pi a^{I=2} = -\frac{M_\pi^2}{16\pi F_\pi^2}\left(1 - (y-2)^2 \frac{f_\pi^2}{f_d^2}\frac{M_\pi^2}{M_d^2}\right). \quad (15)$$

Simplifies to χPT result when $y \to 2$ or $f_\pi^2/f_d^2 \to 0$.

Lattice Data

$N_c = 3,\ N_f = 8$ gauge theory



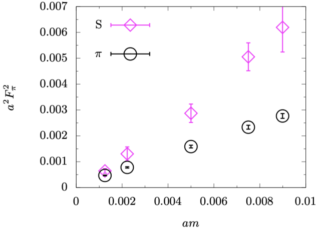


Figure: Lattice data for M_π^2, M_d^2, F_π^2 and F_S^2 from LSD 2306.06095. The lattice spacing is denoted by a.

We also include data for the π–π scattering length in the I=2, $\ell = 0$ channel from LSD PRD **105** (2022) 034505

Result Of Global Fit

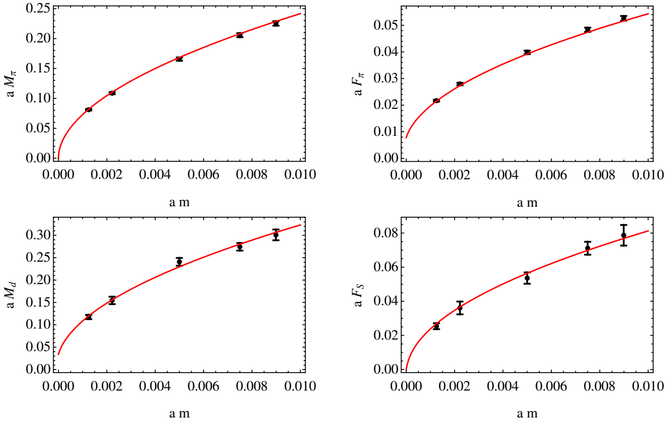
2305.03665

Parameter	Value and Uncertainty
y	2.091(32)
aB_π	2.45(13)
Δ	3.06(41)
$a^2 f_\pi^2$	$6.1(3.2) \times 10^{-5}$
f_π^2/f_d^2	0.1023(35)
m_d^2/f_d^2	1.94(65)
χ^2/dof	21.3/19=1.12

Table: Central values of fit parameters obtained in a six parameter global fit to upcoming LSD data for $M^2_{\pi,d}$, $F^2_{\pi,S}$ and scattering length using Eqs. 7, 8, 9, 12 and 15.

Result Of Global Fit

2305.03665



Simplification of Fit Equations

We see from the table that $y \sim 2$ and that $f_\pi^2 \ll F_\pi^2$. This suggests that the scaling dimension of $\bar{\psi}\psi \sim 2$.

In this limit, the fit equations simplify:

$$M_\pi^2 = 2B_\pi m, \tag{16}$$

$$\frac{M_\pi^2}{F_\pi^2} = \frac{2m_d^2 f_d^2}{yN_f(4-\Delta)f_\pi^4}, \tag{17}$$

$$\frac{M_d^2}{F_\pi^2} = \frac{m_d^2}{(4-\Delta)f_\pi^2}(4-y)\,. \tag{18}$$

Range for Δ

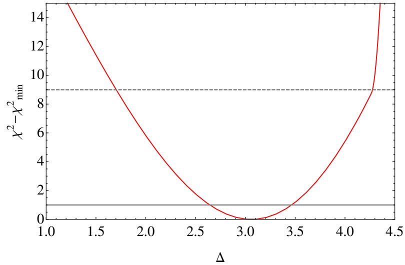


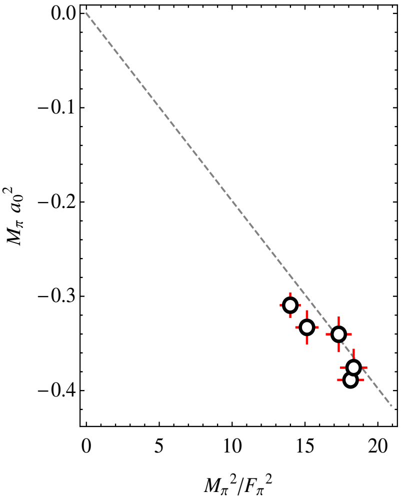
Figure: The χ^2 has been minimised w.r.t the other 5 parameters for each value of Δ.

- The 1σ range for Δ:

$$2.6 \lesssim \Delta \lesssim 3.5$$

- In particular $\Delta \to 4$ is mildly disfavored.
- Notice the kink at $\Delta = 4.2$. It arises because the form V_Δ is restricted to always have a minimum at $f_d > 0$.

$I = 2$ Scattering Length



- Compare with lattice data.
- Dashed line uses values for y etc taken from global fit

$$M_\pi a^{I=2} \approx -\frac{M_\pi^2}{16\pi F_\pi^2}.$$

- Perhaps mild evidence of tension.
- Evidence for NLO effect?

Higher Order Corrections to the Potential

- Corrections to V_Δ can be organised into a series of terms governed by the scale breaking parameter $m_d^2/(4\pi f_d)^2$:

$$V(\chi) = V_\Delta(\chi) + \chi^4 \sum_{n=2}^{\infty} a_n \left(\frac{m_d^2}{(4-\Delta) f_d^2} \right)^n \left(1 - \left(\frac{f_d}{\chi} \right)^{4-\Delta} \right)^n .$$

Higher Order Corrections to the Potential

- Corrections to V_Δ can be organised into a series of terms governed by the scale breaking parameter $m_d^2/(4\pi f_d)^2$:

$$V(\chi) = V_\Delta(\chi) + \chi^4 \sum_{n=2}^{\infty} a_n \left(\frac{m_d^2}{(4-\Delta) f_d^2} \right)^n \left(1 - \left(\frac{f_d}{\chi} \right)^{4-\Delta} \right)^n .$$

- This form can be derived from a spurion analysis or from loop expansion.

Higher Order Corrections to the Potential

- Corrections to V_Δ can be organised into a series of terms governed by the scale breaking parameter $m_d^2/(4\pi f_d)^2$:

$$V(\chi) = V_\Delta(\chi) + \chi^4 \sum_{n=2}^{\infty} a_n \left(\frac{m_d^2}{(4-\Delta) f_d^2} \right)^n \left(1 - \left(\frac{f_d}{\chi} \right)^{4-\Delta} \right)^n .$$

- This form can be derived from a spurion analysis or from loop expansion.
- Corrections organised to ensure smoothness in $\Delta \to 4$ limit.

Spurion Analysis I

Start from an exactly scale invariant EFT, possessing a moduli space of degenerate vacua. Next, introduce a spurion that transforms with scaling dimension $4 - \Delta$:

$$\mathcal{S}(x) \to e^{\rho(4-\Delta)}\mathcal{S}(e^{\rho}x) \tag{19}$$

Spurion Analysis I

Start from an exactly scale invariant EFT, possessing a moduli space of degenerate vacua. Next, introduce a spurion that transforms with scaling dimension $4 - \Delta$:

$$\mathcal{S}(x) \to e^{\rho(4-\Delta)}\mathcal{S}(e^{\rho}x) \qquad (19)$$

The potential will include all scale invariant operators built from $\mathcal{S}(x)$ and χ.

Spurion Analysis I

Start from an exactly scale invariant EFT, possessing a moduli space of degenerate vacua. Next, introduce a spurion that transforms with scaling dimension $4 - \Delta$:

$$\mathcal{S}(x) \to e^{\rho(4-\Delta)}\mathcal{S}(e^{\rho}x) \tag{19}$$

The potential will include all scale invariant operators built from $\mathcal{S}(x)$ and χ.

$$X_3 \equiv \frac{\mathcal{S}(x)}{\Lambda^2}\left(\frac{f_d}{\chi}\right)^{4-\Delta}, \qquad V(\chi) = \chi^4 \sum_{n=0}^{\infty} b_n X_3^n\,. \tag{20}$$

Potential must be analytic in $\mathcal{S}$ but nonanalytic in χ. Break scale invariance explicitly $\mathcal{S}(x) \to m_d^2$.

Spurion Analysis II

Take the previous expression and truncate after N terms:

$$V(\chi) = \chi^4 \sum_{n=0}^{N} b_n \left[\frac{m_d^2}{\Lambda^2} \left(\frac{f_d}{\chi} \right)^{4-\Delta} \right]^n ,$$

These N terms can be rearranged to yield:

$$V(\chi) = V_\Delta(\chi) + \chi^4 \sum_{n=2}^{N} a_n \left(\frac{m_d^2}{(4-\Delta) f_d^2} \right)^n \left(1 - \left(\frac{f_d}{\chi} \right)^{4-\Delta} \right)^n .$$

Spurion Analysis III

A similar spurion analysis reveals three other symmetry invariant building blocks:

$$X_1 = \left(\frac{\chi}{f_d}\right)^{-2} \frac{\partial_\mu}{\Lambda}\left(\frac{\chi}{f_d}\right), \qquad X_2 = \left(\frac{\chi}{f_d}\right)^{-1} \frac{\partial_\mu \Sigma}{\Lambda},$$
$$X_3 = \frac{m_d^2}{\Lambda^2}\left(\frac{\chi}{f_d}\right)^{\Delta-4}, \qquad X_4 = \frac{m_\pi^2}{\Lambda^2}\left(\frac{\chi}{f_d}\right)^{y-4} \mathbf{1}_{N_f}, \tag{21}$$

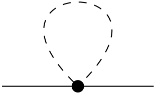
All terms in the dilaton EFT Lagrangian at higher order can be built from these building blocks. Terms with more powers of X_i are suppressed, so only a finite number need to be retained at a given order in power counting.

Next to Leading Order Theory

The NLO Lagrangian contains 18 new operators [Universe 9 (2023) 1, 10]
To give an impression, here is a sample:

$$\mathcal{L}_{\text{NLO}} = g_1 \frac{(\partial_\mu \chi)^4}{\chi^4} + g_2 \frac{(\partial_\mu \chi)^2}{\chi^2} \mathrm{Tr}\left[\partial_\nu \Sigma \partial^\nu \Sigma^\dagger\right] + g_3 \frac{\partial_\mu \chi \partial_\nu \chi}{\chi^2} \mathrm{Tr}\left[\partial^\mu \Sigma \partial^\nu \Sigma^\dagger\right] + c_2 \frac{m_d^2 m_\pi^2}{(4-\Delta)} \left(1 - \frac{4}{\Delta}\left(\frac{f_d}{\chi}\right)^{4-\Delta}\right)\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right] + \ldots \quad (22)$$

To calculate observables at NLO, you should also include one loop diagrams. Just like in χPT!

Loop correction to dilaton mass:  $+ \ldots$

Composite Higgs

Model Structure

Replace the scalar sector of the standard model (SM) with a confining gauge fermion theory.

Assume the Higgs is a pNGB

BSM Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM-h} + \mathcal{L}_{\mathrm{SD}} + \mathcal{L}_{int} \tag{23}$$

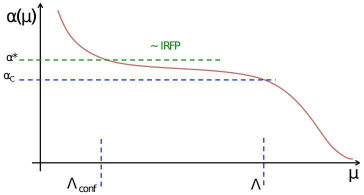
- $\mathcal{L}_{\mathrm{SD}}$: Confines around 5 - 10 TeV. Study in isolation numerically.
- $\mathcal{L}_{int}$: Irrelevant operators with scale around 10^{3-4} TeV. Couplings between the SM and new strong sector which generate mass for the standard model fermions.

$\mathcal{L}_{SD}$

Any gauge theory responsible for the composite Higgs is likely to differ markedly from QCD:

1. Exotic pattern of chiral symmetry breaking.
2. Large anomalous dimensions.

Following from 2, the bottom of the conformal window is a good place to look for large γs, as these gauge theories are strongly coupled over a large interval of scales.



An Explicit Example.

Based on PRL.**126**, no.19, (2021) with T. Appelquist and M. Piai.

We can describe the pNGBs of the $\mathrm{SU}(3)$ gauge theory with 8 light flavors in the fundamental representation well using a dilaton EFT...

$$\mathcal{L}_{SD} = \frac{1}{2}\left(\partial_\mu \chi\right)^2 + \frac{F_\pi^2}{4}\left(\frac{\chi}{F_d}\right)^2 \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] + \frac{M_\pi^2 F_\pi^2}{4}\left(\frac{\chi}{F_d}\right)^y \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right] - V(\chi) \quad (24)$$

Lattice studies constrain low energy constants

SM gauge interactions

Fermion	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$SU(3)_c$	$SU(3)$
L_α	2	0	1	3
$R_{1,2}$	1	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	1	3
T	1	2/3	3	3
S	1	0	1	3

Table: Quantum number assignments of the Dirac fermions. The fermions denoted by $R_{1,2}$ form a fundamental representation of the global $SU(2)_R$ custodial symmetry. The assignments are similar to L. Vecchi: JHEP 02, 094 (2017)

$\mathcal{L}_{int}$

Represent the effect of $\mathcal{L}_{int}$ on the low energy dynamics by perturbing the dilaton EFT with two new operators:

$\mathcal{L}_{int}$

Represent the effect of $\mathcal{L}_{int}$ on the low energy dynamics by perturbing the dilaton EFT with two new operators:

1. A Yukawa type operator for the top quark

$$\mathcal{L}_Y = y_t F_\pi \left(\frac{\chi}{F_d}\right)^z \left(\bar{Q}_L^\alpha t_R\right) \mathrm{Tr}\left[P_\alpha \Sigma\right] + \text{h.c.}\,. \tag{25}$$

$\mathcal{L}_{int}$

Represent the effect of $\mathcal{L}_{int}$ on the low energy dynamics by perturbing the dilaton EFT with two new operators:

1. A Yukawa type operator for the top quark

$$\mathcal{L}_Y = y_t F_\pi \left(\frac{\chi}{F_d}\right)^z \left(\bar{Q}_L^\alpha t_R\right) \mathrm{Tr}\left[P_\alpha \Sigma\right] + \text{h.c.}\,. \tag{25}$$

The exponent z descends from the underlying gauge theory. It can be generated from either ETC or partial compositeness.

$\mathcal{L}_{int}$

Represent the effect of $\mathcal{L}_{int}$ on the low energy dynamics by perturbing the dilaton EFT with two new operators:

1. A Yukawa type operator for the top quark

$$\mathcal{L}_Y = y_t F_\pi \left(\frac{\chi}{F_d}\right)^z \left(\bar{Q}_L^\alpha t_R\right) \mathrm{Tr}\left[P_\alpha \Sigma\right] + \text{h.c.}\,. \tag{25}$$

The exponent z descends from the underlying gauge theory. It can be generated from either ETC or partial compositeness.

The matrices P_α project out the pNGB components corresponding to the composite Higgs doublet $H_\alpha = \mathrm{Tr}\left[P_\alpha \Sigma\right]$.

$\mathcal{L}_{int}$

2. An operator which breaks chiral symmetry, generating a potential for the Composite Higgs.

$$V_t = -C_t \left(\frac{\chi}{F_d}\right)^w \sum_{\alpha=1}^{2} |\mathrm{Tr}\left[P_\alpha \Sigma\right]|^2 . \qquad (26)$$

$\mathcal{L}_{int}$

2. An operator which breaks chiral symmetry, generating a potential for the Composite Higgs.

$$V_t = -C_t \left(\frac{\chi}{F_d}\right)^w \sum_{\alpha=1}^{2} \left|\mathrm{Tr}\left[P_\alpha \Sigma\right]\right|^2 . \qquad (26)$$

The exponent w descends from the underlying gauge theory.

The Vacuum

Give one of the pNGBs (the Higgs) a vacuum expectation value:

$$\Sigma = \exp \frac{i\theta}{2} \begin{pmatrix} \mathbb{0}_{2\times 2} & -i\mathbb{1}_2 & \mathbb{0}_{2\times 4} \\ i\mathbb{1}_2 & \mathbb{0}_{2\times 2} & \mathbb{0}_{2\times 4} \\ \mathbb{0}_{4\times 2} & \mathbb{0}_{4\times 2} & \mathbb{0}_{4\times 4} \end{pmatrix}, \tag{27}$$

The “misalignment” angle θ sets the electroweak scale in terms of the strong dynamics scale:

$$\sin\theta = \frac{v}{F_\pi \sqrt{2}}$$

The Vacuum

Unlike in most composite Higgs models, the dilaton and pNGB vacuum values have to be determined simultaneously:

$$W(\chi,\theta) = V(\chi) - C_t\left(\frac{\chi}{F_d}\right)^w \sin^2\theta - 2M_\pi^2 F_\pi^2\left(\frac{\chi}{F_d}\right)^y (1+\cos\theta)\,, \quad (28)$$

The C_t term destabilizes the electroweak symmetry preserving vacuum ($\theta = 0$).

The M_π^2 term (from underlying fermion masses) helps stabilize it.

The Vacuum

At the minimum of the potential, we have:

$$\cos\theta = \frac{M_\pi^2 F_\pi^2}{C_t}, \tag{29}$$

Tuning between M_π^2 and C_t is required to get a hierarchy between v and F_π.

The Effect of the Dilaton

There is a mass matrix for χ and θ dofs.

The Effect of the Dilaton

There is a mass matrix for χ and θ dofs.

This implies mixing between the dilaton and pNGBs. The lightest eigenvalue is:

The Higgs Mass

$$\frac{m_h^2}{v^2} = \frac{M_\pi^2}{2F_\pi^2}\left(1 - \frac{2M_\pi^2 F_\pi^2 (y-w)^2}{M_d^2 F_d^2}\right), \qquad (30)$$

The Effect of the Dilaton

There is a mass matrix for χ and θ dofs.

This implies mixing between the dilaton and pNGBs. The lightest eigenvalue is:

The Higgs Mass

$$\frac{m_h^2}{v^2} = \frac{M_\pi^2}{2F_\pi^2}\left(1 - \frac{2M_\pi^2 F_\pi^2 (y-w)^2}{M_d^2 F_d^2}\right), \tag{30}$$

M_d is mass of dilaton in theory without $\mathcal{L}_{int}$.

The Effect of the Dilaton

There is a mass matrix for χ and θ dofs.

This implies mixing between the dilaton and pNGBs. The lightest eigenvalue is:

The Higgs Mass

$$\frac{m_h^2}{v^2} = \frac{M_\pi^2}{2F_\pi^2}\left(1 - \frac{2M_\pi^2 F_\pi^2 (y-w)^2}{M_d^2 F_d^2}\right), \tag{30}$$

M_d is mass of dilaton in theory without $\mathcal{L}_{int}$.

Term in brackets can be small, allowing M_π^2/F_π^2 to be big. Puts us in range of lattice data and makes other pNGBs heavier.

The Higgs Mass

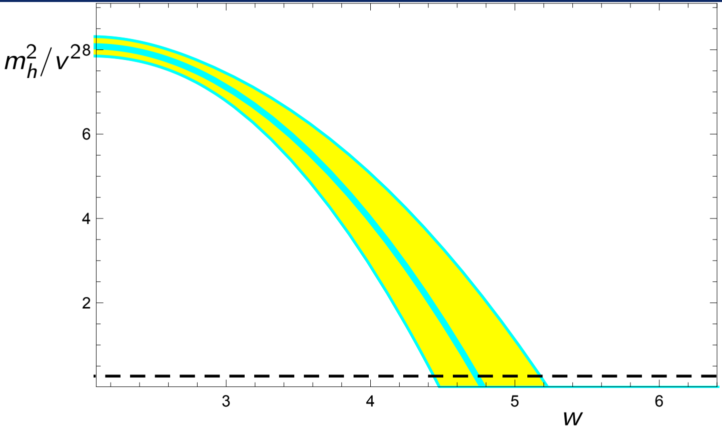


Figure: The allowed range of m_h^2/v^2 assuming different values for w. LSD lattice data (at $am = 0.00222$) has been used to fix M_π^2/F_π^2 and M_d^2/F_π^2.

The pNGBs

We have 63 pNGBs. They inherit the following quantum numbers under the global symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)$:

$$63 = (3,1,1) + (1,3,1) + (1,1,1) + (2,2,1) \\ + (1,1,15) + (2,1,4) + (1,2,4) + (1,1,1)\,.$$

Note: Vacuum misalignment spontaneously breaks $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_D$.

Spectrum

$SU(2)_D$	$SU(4)$	Mass (TeV)				
		m_{f1}	m_{f2}	m_{f3}	m_{f4}	m_{f5}
1	1	4.31	4.73	4.29	4.96	4.87
3	1	4.35	4.37	4.39	4.40	4.40
2	4	4.18	4.19	4.20	4.20	4.20
3	1	4.03	4.03	4.04	4.04	4.04
1	1	4.03	4.03	4.04	4.04	4.04
1	1	4.03	4.03	4.04	4.04	4.04
1	15	4.00	4.00	4.00	4.00	4.00
1	1	3.99	3.98	3.98	3.98	3.98
2	4	3.84	3.83	3.83	3.82	3.82
3	1	3.67	3.66	3.64	3.64	3.64
1	1	0.126	0.126	0.126	0.126	0.126
3	1	0	0	0	0	0

Table: Scale of the new composite sector is set by $M_\pi = 4$ TeV.

Tuning

Using a benchmark of $M_\pi = 4$ TeV, the lattice implies $F_\pi \sim 1$ TeV.

$$\sin\theta = \frac{v}{F_\pi\sqrt{2}} \sim 0.17 \tag{31}$$

$$\cos\theta = \frac{M_\pi^2 F_\pi^2}{C_t} \sim 0.98 \tag{32}$$

Needs 2% fine tuning between C_t and M_π^2.

Phenomenology

- Heavy pNGBs: Easiest to discover color octet, but $M_8 \sim 4$ TeV probably too heavy for LHC. Can always make pNGBs heavier by tuning θ.

Phenomenology

- Heavy pNGBs: Easiest to discover color octet, but $M_8 \sim 4$ TeV probably too heavy for LHC. Can always make pNGBs heavier by tuning θ.
- Higgs Couplings: Suppression of couplings to t, W and Z by $\cos\theta$.

Phenomenology

- Heavy pNGBs: Easiest to discover color octet, but $M_8 \sim 4$ TeV probably too heavy for LHC. Can always make pNGBs heavier by tuning θ.
- Higgs Couplings: Suppression of couplings to t, W and Z by $\cos\theta$.
- Electroweak Precision Observables: S parameter requires a more detailed study. An LSD lattice calculation is ongoing. In general, we can always reduce the new physics contribution by fine tuning.

Phenomenology

- Heavy pNGBs: Easiest to discover color octet, but $M_8 \sim 4$ TeV probably too heavy for LHC. Can always make pNGBs heavier by tuning θ.
- Higgs Couplings: Suppression of couplings to t, W and Z by $\cos\theta$.
- Electroweak Precision Observables: S parameter requires a more detailed study. An LSD lattice calculation is ongoing. In general, we can always reduce the new physics contribution by fine tuning.
- Other resonances: $M_\rho \sim 7$ TeV and $M_{a1} \sim 10$ TeV with these benchmarks.

Conclusions

- A fully consistent EFT can be built incorporating both pNGBs and a dilaton.

Conclusions

- A fully consistent EFT can be built incorporating both pNGBs and a dilaton.
- The tree level dilaton EFT fits lattice data well for the $N_f = 8$ theory. We extracted the values of some model parameters with high precision.

Conclusions

- A fully consistent EFT can be built incorporating both pNGBs and a dilaton.
- The tree level dilaton EFT fits lattice data well for the $N_f = 8$ theory. We extracted the values of some model parameters with high precision.
- Dilaton EFT is a flexible model building instrument and can form the foundation of viable composite Higgs models.

Conclusions

- A fully consistent EFT can be built incorporating both pNGBs and a dilaton.
- The tree level dilaton EFT fits lattice data well for the $N_f = 8$ theory. We extracted the values of some model parameters with high precision.
- Dilaton EFT is a flexible model building instrument and can form the foundation of viable composite Higgs models.
- In progress: Model for a hidden sector explaining DM and Gravitational waves.

Lattice Action

- LSD numerical calculations use improved nHYP smeared staggered fermions with smearing parameters $\alpha = (0.5, 0.5, 0.4)$.
- $\beta_A/\beta_F = -0.25$ where $\beta_F = 4.8$.
- After taste splitting, only $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ flavor symmetry preserved in massless theory (3 exact NGBs).
- Spectral study has revealed that the taste splitting of the 63-plet masses are on the order of 20–30%.

$I = 2$ Interpolating Operators

$$\pi^+(t) = \sum_{\vec{x}} \bar{\chi}_2(x)\epsilon(x)\chi_1(x), \text{ where } \epsilon(x) = (-1)^{x+y+z+t} \quad (33)$$

$$\mathcal{O}_{I=2}(t) = \pi^+(t)\pi^+(t+1) \quad (34)$$

$$\begin{aligned} C_{I=2}(t, t_0) &= \langle \mathcal{O}_{I=2}(t)\mathcal{O}_{I=2}(t_0)^\dagger \rangle \\ &= \sum_{\vec{x}_1,\cdots,\vec{x}_4} \langle \pi^+(t_4,\vec{x}_4)\pi^+(t_3,\vec{x}_3)\pi^+(t_2,\vec{x}_2)^\dagger \pi^+(t_1,\vec{x}_1)^\dagger \rangle \end{aligned} \quad (35)$$

Wall sources used - moving wall method.

Extrapolation Of F_π^2