Spectral functions:
from perturbative fixed points to quantum gravity

Manuel Reichert

University of Sussex, Brighton, UK

UV Complete Quantum Field Theories for Particle Physics
San Miniato, 05. September 2023

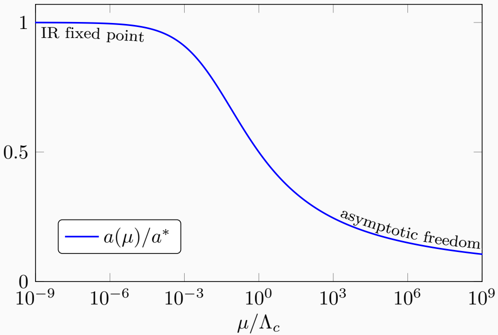
Fehre, Litim, Pawlowski, MR: PRL, 2111.13232

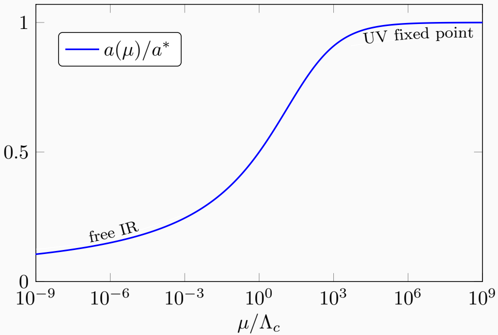
Kluth, Litim, MR: PRD, 2207:14510

Kluth, Litim, MR: 230x:xxxxx

UNIVERSITY OF SUSSEX

Asymptotic freedom vs safety



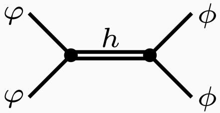


- Banks-Zaks
- Wilson-Fisher

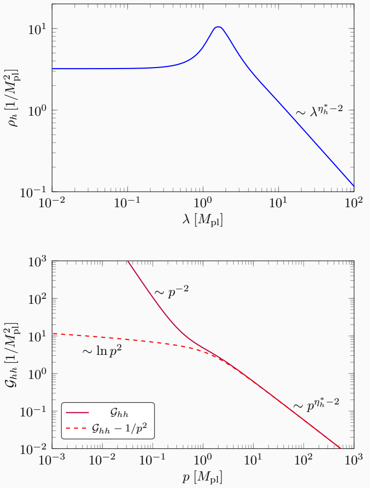
- Litim-Sannino
- Quantum gravity

Correlation functions of the quantum effective action

- Want access to full quantum 1PI effective action Γ
- Special importance: propagator $G \sim \Gamma^{(2)\,-1}$
- Important input for scattering



- Understand quantum-gravity through comparison with perturbative theories

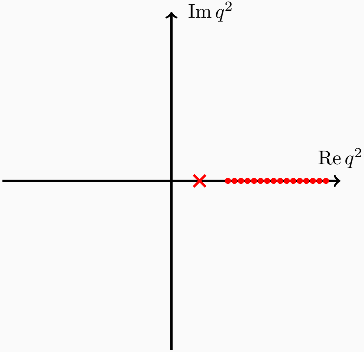


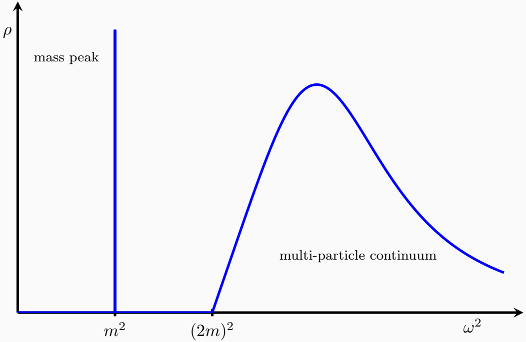
[Fehre, Litim, Pawlowski, MR '21]

Källén-Lehmann spectral representation

[Källén '52; Lehmann '54]

$$G(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2} \qquad \text{with} \qquad \rho(\omega^2) = -\lim_{\varepsilon \to 0} \operatorname{Im} G(\omega^2 + i\varepsilon)$$



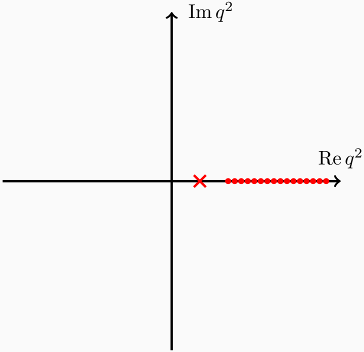


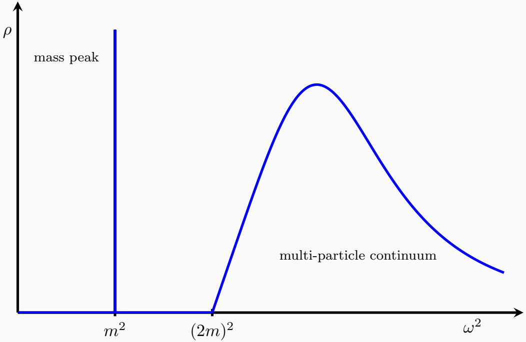
with $\rho(\omega^2) > 0$ and $\int \rho(\lambda^2)\mathrm{d}\lambda^2 = 1$

Källén-Lehmann spectral representation

[Källén '52; Lehmann '54]

$$G(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2} \qquad \text{with} \qquad \rho(\omega^2) = -\lim_{\varepsilon \to 0} \operatorname{Im} G(\omega^2 + i\varepsilon)$$





with $\rho(\omega^2) > 0$ and $\int \rho(\lambda^2)\mathrm{d}\lambda^2 = 1$?

Banks-Zaks fixed point

[Belavin, Migdal '74; Caswell '74; Banks, Zaks '82]

$SU(N_c)$ gauge theory with N_f quarks

$$\beta(a) = \beta_1 a^2 + \beta_2 a^3 + \dots$$

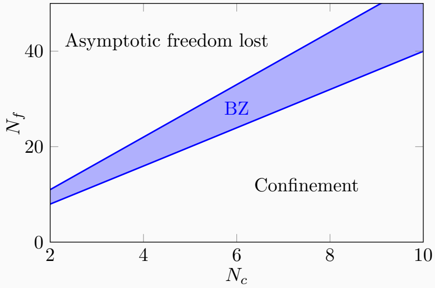
Veneziano limit gives $\beta_1 \sim -\varepsilon$ with

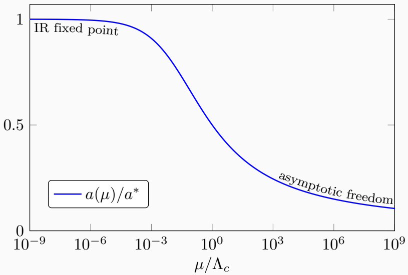
$$\varepsilon = \frac{11}{2} - \frac{N_f}{N_c} > 0$$

$$N_c \ \& \ N_f \to \infty$$

Perturbative IR fixed point

$$a_* = -\frac{\beta_1}{\beta_2} + \mathcal{O}(\beta_1^2)$$





Litim-Sannino model – asymptotically safe

[Litim, Sannino '14]

$SU(N_c)$ with N_f quarks and uncharged $N_f \times N_f$ matrix scalar

$$L \sim L_{\mathrm{YM}} + L_{\mathrm{kin}} - y\,\mathrm{Tr}\bar{\psi}_L H \psi_R - u\,\mathrm{Tr} H^\dagger H H^\dagger H - v\,(\mathrm{Tr} H^\dagger H)^2$$

Veneziano limit N_c & $N_f \to \infty$ with

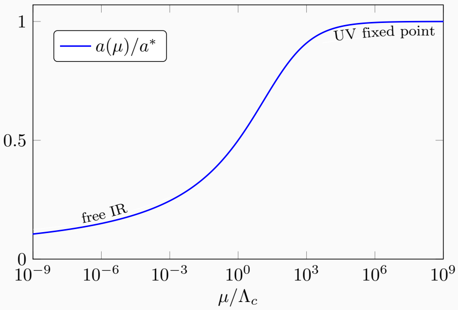
$$\varepsilon = \frac{11}{2} - \frac{N_f}{N_c} < 0$$

Perturbative UV fixed point

$$a_g^*,\ a_y^*,\ a_u^*,\ a_v^* \sim \varepsilon$$

One relevant direction

$$a_y(a_g(\mu)),\ a_u(a_g(\mu)),\ a_v(a_g(\mu))$$



Litim-Sannino model – asymptotically free

$SO(2N_c)$ with N_f quarks and uncharged $N_f \times N_f$ matrix scalar

$$L \sim L_{\rm YM} + L_{\rm kin} - y\,{\rm Tr}\bar{\psi}_L H \psi_R - u\,{\rm Tr} H^\dagger H H^\dagger H - v\,({\rm Tr} H^\dagger H)^2$$

Veneziano limit N_c & $N_f \to \infty$ with

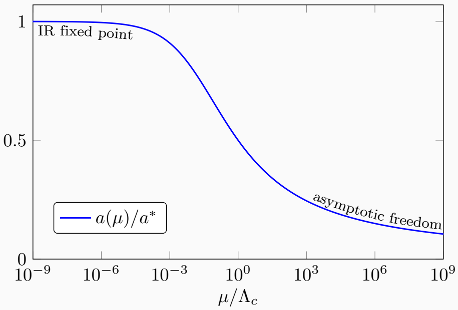
$$\varepsilon = \frac{11}{2} - \frac{N_f}{N_c} > 0$$

Perturbative UV fixed point

$$a_g^*,\ a_y^*,\ a_u^*,\ a_v^* \sim \varepsilon$$

One IR attractive direction

$$a_y(a_g(\mu)),\ a_u(a_g(\mu)),\ a_v(a_g(\mu))$$



Callan-Symanzik Resummation

[Kluth, Litim, Reichert '22]

Propagator with self-energy loop corrections

$$G_\phi(p^2,\mu^2) = \frac{1}{-p^2}\frac{1}{1+\Pi_\phi(p^2,\mu^2)}$$

Callan-Symanzik Resummation

[Kluth, Litim, Reichert '22]

Propagator with self-energy loop corrections

$$G_\phi(p^2, \mu^2) = \frac{1}{-p^2} \frac{1}{1 + \Pi_\phi(p^2, \mu^2)}$$

Callan-Symanzik equation to resum large logarithms $\log(p^2/\mu^2)$

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}(Z_\phi G_\phi) = \left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(a) \frac{\partial}{\partial a} - \gamma_\phi\right) G_\phi = 0$$

Callan-Symanzik Resummation

[Kluth, Litim, Reichert '22]

Propagator with self-energy loop corrections

$$G_\phi(p^2,\mu^2) = \frac{1}{-p^2}\frac{1}{1+\Pi_\phi(p^2,\mu^2)}$$

Callan-Symanzik equation to resum large logarithms $\log(p^2/\mu^2)$

$$\mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2}(Z_\phi G_\phi) = \left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(a)\frac{\partial}{\partial a} - \gamma_\phi\right) G_\phi = 0$$

Resummed Callan-Symanzik propagator with $\bar{a} \equiv \bar{a}(p^2)$ & $a \equiv a(\mu^2)$

$$G_\phi = \frac{1}{p^2}\frac{\mathcal{N}_\phi}{1+\sum_n \Pi_\phi^{(n)}(p^2=-\mu^2)\bar{a}(p^2)^n}\left(\frac{a(\mu^2)}{\bar{a}(p^2)}\right)^{\gamma_\phi^{(1)}/\beta_1}\prod_i\left(\frac{a(\mu^2)-a_{*,i}}{\bar{a}(p^2)-a_{*,i}}\right)^{\gamma_\phi(a_{*,i})/\theta_i}$$

Callan-Symanzik Resummation – multiple couplings

[Kluth, Litim, Reichert (in prep)]

Callan-Symanzik equation with multiple couplings

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \sum_i \beta_i \frac{\partial}{\partial a_i} - \gamma_\phi\right) G_\phi = 0$$

Assumption: one monotonic coupling to re-express all others $a_i(\mu^2) = a_i(a(\mu^2))$, leads to same CS equation

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}(Z_\phi G_\phi) = \left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta_{\text{eff}} \frac{\partial}{\partial a} - \gamma_\phi\right) G_\phi = 0$$

Callan-Symanzik Resummation – discussion

Resummed Callan-Symanzik propagator with $\bar{a} \equiv \bar{a}(p^2)$ & $a \equiv a(\mu^2)$

$$G_\phi = \frac{1}{p^2} \frac{\mathcal{N}_\phi}{1 + \sum_n \Pi_\phi^{(n)} \bar{a}^n} \left(\frac{a}{\bar{a}}\right)^{\gamma_\phi^{(1)}/\beta_1} \prod_i \left(\frac{a - a_{*,i}}{\bar{a} - a_{*,i}}\right)^{\gamma_\phi(a_{*,i})/\theta_i}$$

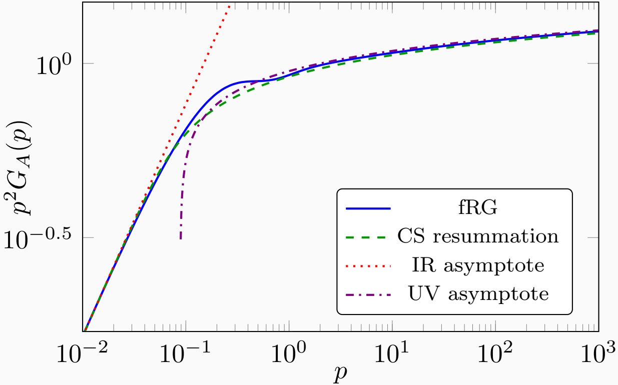
- All fixed points including complex and negative ones contribute
- Bound states can appear through the self-energies
- Self-energies and anomalous dimensions are gauge dependent if particle is charged under a gauge group
- Banks-Zaks: 5-loop β-function & 4-loop self-energies

  [Ruijl, Ueda, Vermaseren, Vogt '17; Herzog, Ruijl, Ueda, Vermaseren, Vogt '17; Chetyrkin, Falcioni, Herzog, Vermaseren '17]

  Litim-Sannino: 433-loop β-functions & no self-energies

  [Bond, Litim, Vazquez, Steudtner '18; Litim, Riyaz, Stamou, Steudtner '23]

Intermezzo – comparison to fRG compuation



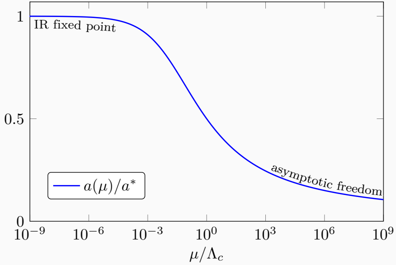
[Kluth, Litim, Reichert '22]

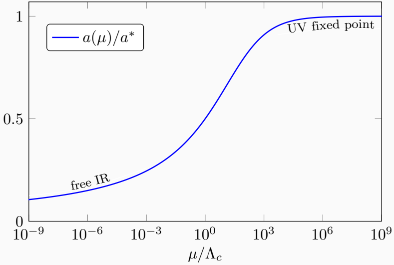
Very good agreement between fRG and CS resummation

Gauge coupling in the complex plane

- Beta function $\mu^2 \partial a / \partial \mu^2 = \beta(a) = \beta_1 a^2 + \beta_2 a^3 + \ldots$
- Analytic 2-loop solution via W-Lambert function [Corless et al '96; Gardi et al '98]

$$a(\mu^2) = \frac{a_*}{1 + W(z)} \qquad z = \omega_0 e^{\omega_0} \left(\frac{\mu^2}{\mu_0^2} \right)^{\theta} \qquad \omega_0 = \frac{a_* - a_0}{a_0}$$



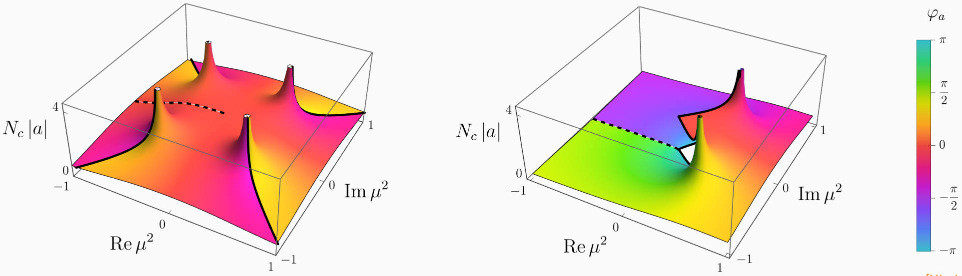


Gauge coupling in the complex plane

- Beta function $\mu^2 \partial a / \partial \mu^2 = \beta(a) = \beta_1 a^2 + \beta_2 a^3 + \dots$
- Analytic 2-loop solution via W-Lambert function [Corless et al '96; Gardi et al '98]

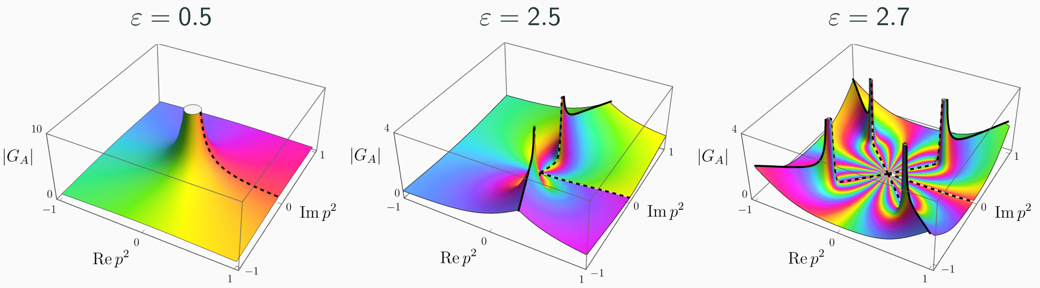
$$a(\mu^2) = \frac{a_*}{1 + W(z)} \qquad z = \omega_0 e^{\omega_0} \left(\frac{\mu^2}{\mu_0^2}\right)^\theta \qquad \omega_0 = \frac{a_* - a_0}{a_0}$$

- Branch cuts emerge for large $\varepsilon \longrightarrow$ complex coupling plane is not unique



[Kluth, Litim, MR '22]

Propagator in the complex plane (Banks-Zaks)



[Kluth, Litim, MR '22]

- Analytic, perturbative computation of propagator in entire complex momentum plane
- Propagators inherit the branch cuts of the coupling
- Violation of KL spectral representation for large ε

Existence of branch cuts

[Kluth, Litim, MR '22]

Perturbative n-loop beta function

$$\beta(a) = d_a a + \beta_1 a^2 + \beta_2 a^3 + \cdots + \beta_n a^{n+1}$$

Absence of branch cuts guaranteed if (a_0 initial value)

$$\left| \sum_i^{a_{i,*} > a_0} \frac{1}{\theta_i} \right| > 1$$

- Derived via implicit function theorem
- Depends on universal critical exponents $\theta_i = \partial_a \beta(a)\big|_{a_{i,*}}$
- Only real and positive fixed points contribute
- Absence of branch cuts guaranteed for $\varepsilon \to 0$ since $\theta_{\mathrm{BZ/LS}} \to 0$
- Generalisation to Padé type beta functions possible

Size of conformal Banks-Zaks window

[Kluth, Litim, MR '22]

$N_c = 3$	$\varepsilon_{\text{branch-cut}}$	ε_{max}
2-loop	2.2723	2.8158
3-loop	2.6798	3.5520
4-loop	2.6817	3.0538
5-loop	–	1.2019
5-loop Padé [1,3]	–	2.2183
5-loop Padé [2,2]	–	1.6993
5-loop Padé [3,1]	–	0.7304

Existence of branch cuts does not post a tighter constrain on BZ window

Disappearance of fixed point via merger is essential

Existence and normalisation of spectral functions

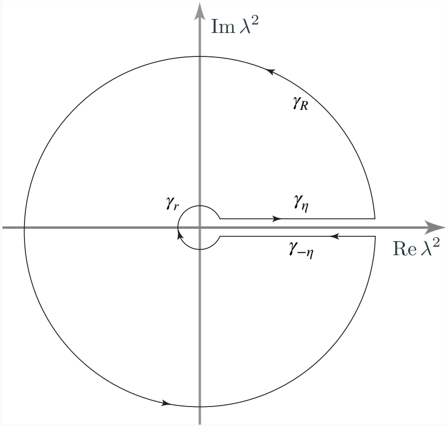
$$G_\phi = \frac{1}{p^2}\frac{\mathcal{N}_\phi}{1+\sum_n \Pi_\phi^{(n)}\bar{a}^n}\left(\frac{a}{\bar{a}}\right)^{\gamma_\phi^{(1)}/\beta_1}\prod_i\left(\frac{a-a_{*,i}}{\bar{a}-a_{*,i}}\right)^{\gamma_\phi(a_{*,i})/\theta_i}$$

When can we use

$$G(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi}\frac{\rho(\lambda^2)}{q^2-\lambda^2}$$

with

$$\rho(\omega^2) = -\lim_{\varepsilon\to 0}\mathrm{Im}\, G(\omega^2+i\varepsilon)$$



Existence and normalisation of spectral functions

$$G_\phi = \frac{1}{p^2} \frac{\mathcal{N}_\phi}{1 + \sum_n \Pi_\phi^{(n)} \bar{a}^n} \left(\frac{a}{\bar{a}}\right)^{\gamma_\phi^{(1)}/\beta_1} \prod_i \left(\frac{a - a_{*,i}}{\bar{a} - a_{*,i}}\right)^{\gamma_\phi(a_{*,i})/\theta_i}$$

Existence in the IR

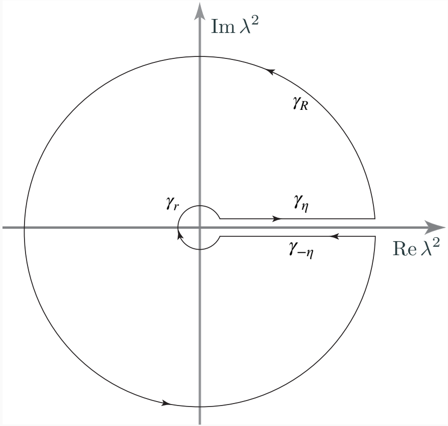
- At most singular with $G_\phi \sim 1/p^2$ for $p \to 0$
- Exception: $G_\phi \sim 1/p^{2n}$

Existence in the UV

- Decay for $p \to \infty$: $G_\phi <$ const

Normalisation determined by $p \to \infty$ behaviour

$$\int \rho_\phi \mathrm{d}\lambda^2 = \begin{cases} 0 & G_\phi < 1/p^2 \\ 1 \quad \text{if} & G_\phi = 1/p^2 \\ \infty & G_\phi > 1/p^2 \end{cases}$$



Existence and normalisation of spectral functions

[Kluth, Litim, Reichert (in prep)]

IR safe & UV free

Existence IR:

$$\gamma_\phi^* \leq 0$$

Existence UV:

trivial

Normalisable:

$$\gamma_\phi^{(1)} = 0$$

UV safe & IR free

Existence IR

$$\gamma_\phi^{(1)}/\beta_1 \leq 0$$

Existence UV

$$\gamma_\phi^* > -1$$

Normalisable

$$\gamma_\phi^* = 0$$

Existence and normalisation of spectral functions

[Kluth, Litim, Reichert (in prep)]

IR safe & UV free

Existence IR:

$$\gamma_\phi^* \leq 0$$

Existence UV:

trivial

Normalisable:

$$\gamma_\phi^{(1)} = 0$$

UV safe & IR free

Existence IR

$$\gamma_\phi^{(1)}/\beta_1 \leq 0$$

Existence UV

$$\gamma_\phi^* > -1$$

Normalisable

$$\gamma_\phi^* = 0$$

γ_ϕ is gauge dependent if ϕ carries gauge charge

Confronting the existence and normalisation conditions

[Kluth, Litim, Reichert (in prep)]

- Scalar fields have positive anomalous dimensions $\gamma_\varphi > 0$ & $\gamma_\varphi^{(1)} > 0$
- If IR free $\longrightarrow \beta_1 > 0$

Confronting the existence and normalisation conditions

[Kluth, Litim, Reichert (in prep)]

- Scalar fields have positive anomalous dimensions $\gamma_\varphi > 0$ & $\gamma_\varphi^{(1)} > 0$
- If IR free $\longrightarrow \beta_1 > 0$

IR safe & UV free	UV safe & IR free
Existence IR: $\gamma_\phi^* \leq 0$	Existence IR $\gamma_\phi^{(1)}/\beta_1 \leq 0$
Existence UV: trivial	Existence UV $\gamma_\phi^* > -1$
Normalisable: $\gamma_\phi^{(1)} = 0$	Normalisable $\gamma_\phi^* = 0$

Confronting the existence and normalisation conditions

[Kluth, Litim, Reichert (in prep)]

- Scalar fields have positive anomalous dimensions $\gamma_\varphi > 0$ & $\gamma_\varphi^{(1)} > 0$
- If IR free $\longrightarrow \beta_1 > 0$

IR safe & UV free	**UV safe & IR free**
Existence IR:	Existence IR
$\gamma_\phi^* \leq 0$	$\gamma_\phi^{(1)}/\beta_1 \leq 0$
Existence UV:	Existence UV
trivial	$\gamma_\phi^* > -1$
Normalisable:	Normalisable
$\gamma_\phi^{(1)} = 0$	$\gamma_\phi^* = 0$

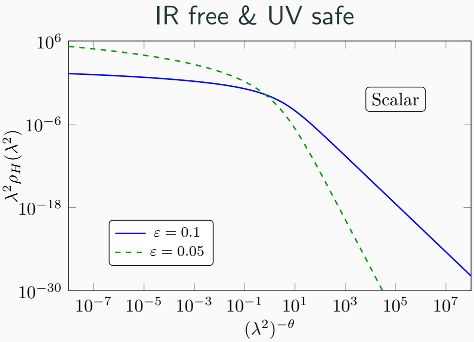
The scalar field of a most ordinary QFT does have a KL spectral representation

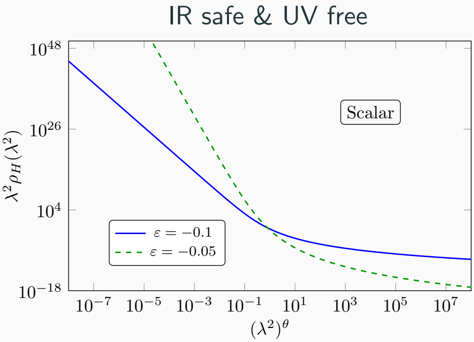
Avoiding the IR problem – masses

- Scalar mass term $\longrightarrow$ Confinement in non-abelian sector

Avoiding the IR problem – masses

- Scalar mass term $\longrightarrow$ Confinement in non-abelian sector





[Kluth, Litim, Reichert (in prep)]

(Positive) & non-normalisable spectral function

Note: $\gamma_\varphi < 0$ leads to a guaranteed partially negative spectral function since $\int \rho_\varphi \mathrm{d}\lambda = 0$

Modified spectral representations

Rescale propagator with analytic 'wave-function renormalisation' Z_ϕ

$$\tilde{G}_\phi(p^2) = Z_\phi(p^2) G_\phi(p^2)$$

- Trivial propagator $Z_\phi(p^2) = \frac{1}{p^2 G_\phi(p^2)}$
- Fixed point rescaling $Z_\phi = (-p^2/\mu^2)^{\gamma_\phi^*}$: new existence conditions & non-trivial propagator

Modified spectral representations

Rescale propagator with analytic 'wave-function renormalisation' Z_ϕ

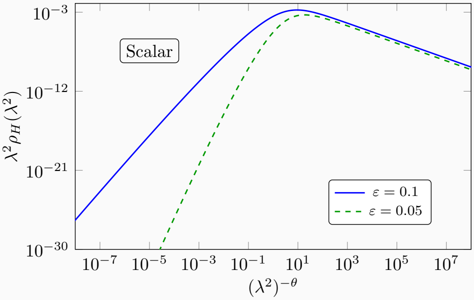
$$\tilde{G}_\phi(p^2) = Z_\phi(p^2) G_\phi(p^2)$$

- Trivial propagator $Z_\phi(p^2) = \frac{1}{p^2 G_\phi(p^2)}$
- Fixed point rescaling $Z_\phi = (-p^2/\mu^2)^{\gamma_\phi^*}$: new existence conditions & non-trivial propagator

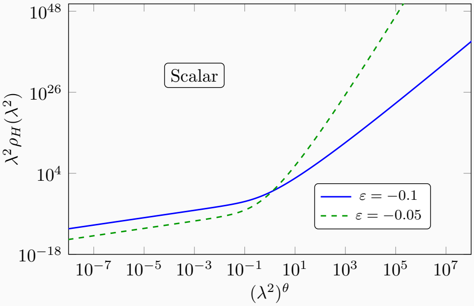
Fixed Point Free Scaling	IR	UV	Existence	Normalisability
UV Free and IR Safe	p^{-2}	$p^{-2(1-\gamma^*)} \log(-p^2)^{\gamma^{(1)}/\beta_1}$	$\gamma^* < 1$ or $(\gamma^* = 1$ & $\gamma^{(1)} \leq 0)$	$\gamma^* = 0$ & $\gamma^{(1)} = 0$
UV Safe and IR Free	$p^{-2(1-\gamma^*)} \log(-p^2)^{\gamma^{(1)}/\beta_1}$	p^{-2}	$\gamma^* > 0$ or $(\gamma^* = 0$ & $\gamma^{(1)} \leq 0)$	Yes

Modified spectral representations

IR free & UV safe



IR safe & UV free



[Kluth, Litim, Reichert (in prep)]

Positive spectral representations useful to encode propagator information

Comparison to massless particles in nature – QED

One-loop effective action of QED

$$\Gamma_{\text{1-loop,QED}} = -\frac{1}{4}\int \mathrm{d}^4x F_{\mu\nu}\left(1 + \#\log(\frac{m_e^2 + \Box}{m_e^2})\right) F_{\mu\nu} + \dots$$

Photon spectral function

$$\rho_\gamma(\lambda^2) \sim \delta(\lambda^2) + \text{multi-particle continuum} \cdot \theta(\lambda^2 - m_e^2) + \dots$$

Electron mass and lack of photon-self interactions safe the IR behaviour of the photon

Comparison to massless particles in nature – QG

One-loop effective action of QG

$$\Gamma_{\text{1-loop,QG}} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\text{Pl}}^2}{2}R + a\,R\log(\frac{\Box}{\mu^2})R + b\,C_{\mu\nu\rho\sigma}\log(\frac{\Box}{\mu^2})C^{\mu\nu\rho\sigma} + \dots\right)$$

Graviton propagator

$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2 + \#G_{\text{N}}\ln(p^2/\mu^2)p^4}$$

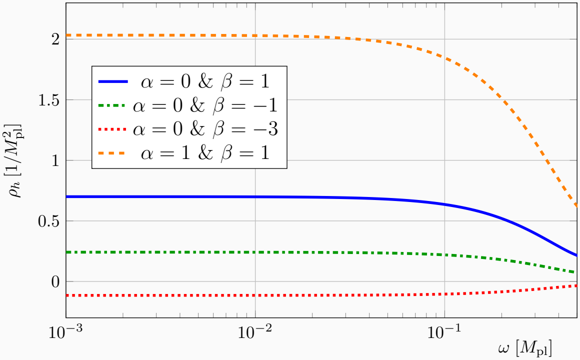
Graviton spectral function

$$\rho_h(\lambda^2) \sim \delta(\lambda^2) + \#G_{\text{N}}2\pi + \dots$$

The mass dimension of G_{N} (the non-renormalisability) safe the IR behaviour of the graviton

Gauge dependence of graviton spectral function

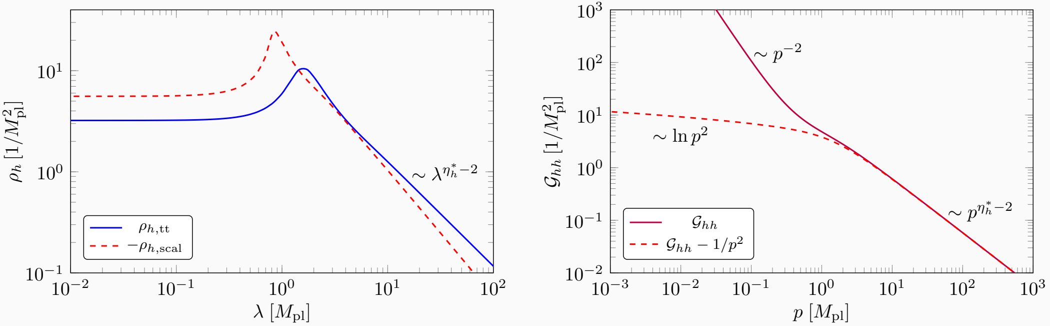
Gauge-fixing $\quad S_{\text{gf}} = \frac{1}{\alpha} \int_x F_\mu^2 \quad$ with $\quad F_\mu = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1+\beta}{4} \bar{\nabla}_\mu h^\nu{}_\nu$



[Pawlowski, MR '23]

Propagator is gauge-dependent but pole structure is typically not

Quantum Gravity results from spectral RG



[Fehre, Litim, Pawlowski, MR '21; Assant, Litim, MR (in prep)]

- Massless graviton delta-peak with multi-graviton continuum
- No ghosts and no tachyons $\longrightarrow$ no indications for unitarity violation
- Good agreement with reconstruction result and EFT [Bonanno, Denz, Pawlowski, MR '21]
- Direct relation to form factors $C f_C(\Box) C$ and $R f_R(\Box) R$

Summary

Banks-Zaks & Litim-Sannino

- Analytic and perturbatively controlled approach
- Gauge-invariant branch cuts linked to critical exponents
- Scalar field in most ordinary QFT does not have a spectral representation
- Need mass term or modified spectral representation

Asymptotically safe quantum gravity

- Direct computation of graviton spectral function with spectral fRG
- Well-behaved spectral function without cuts in the complex plane
- Key step towards scattering processes and unitarity

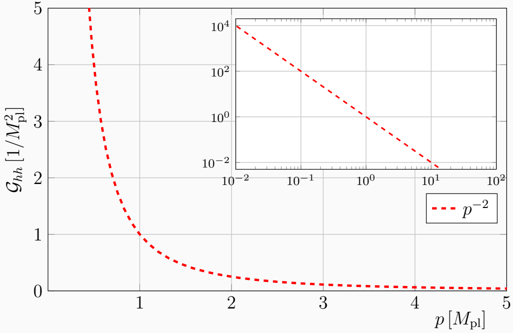
Thank you for your attention!

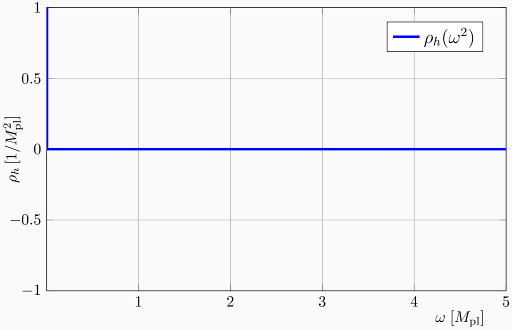
Back-up slides

Classical graviton spectral function

Einstein-Hilbert action: $S_{\text{EH}} = \frac{1}{16\pi G_{\text{N}}} \int_x \sqrt{g}\,(2\Lambda - R)$

Flat Minkowski background: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$



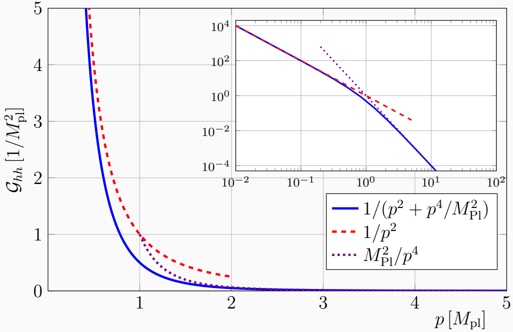


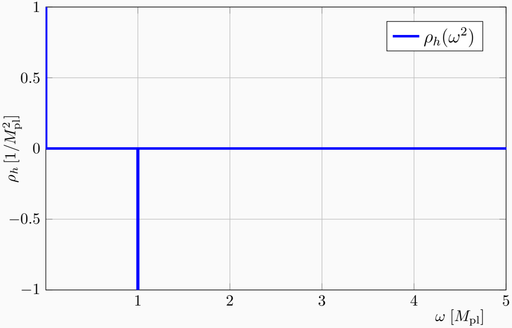
$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2}$$

$$\rho_h(\omega^2) \sim \delta(\omega^2)$$

Classical graviton spectral function

Higher-derivative action: $S_{\text{HD}} = S_{\text{EH}} + \int_x \sqrt{g}\,(aR^2 + bC^2_{\mu\nu\rho\sigma})$





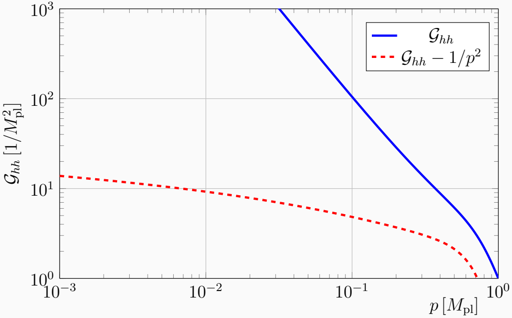
$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2} - \frac{1}{M_{\text{Pl}}^2 + p^2} \qquad \rho_h(\omega^2) \sim \delta(\omega^2) - \delta(\omega^2 - M_{\text{Pl}}^2)$$

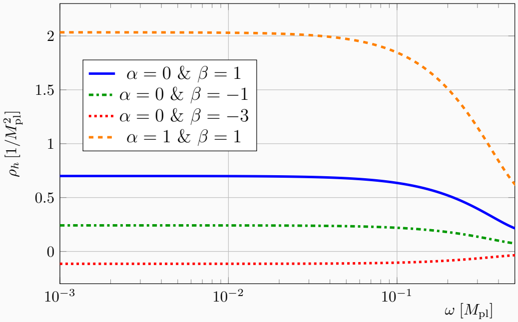
EFT graviton spectral function

One-loop effective action:

$$\Gamma_{\text{1-loop}} = S_{\text{EH}} + \int_x \sqrt{g}\,\left(c_1 R \ln(\Box) R + c_2 C_{\mu\nu\rho\sigma} \ln(\Box) C^{\mu\nu\rho\sigma}\right) + \dots$$

Gauge-fixing $\quad S_{\text{gf}} = \frac{1}{\alpha} \int_x F_\mu^2 \quad$ with $\quad F_\mu = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1+\beta}{4} \bar{\nabla}_\mu h^\nu{}_\nu$





$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2 + \ln(p^2)p^4} \qquad \rho_h(\omega^2) \sim \delta(\omega^2) + 2\pi + 4\pi w^2 \ln(w^2) + \dots$$

The functional renormalisation group

Non-perturbative renormalisation group equation [Wetterich '93]

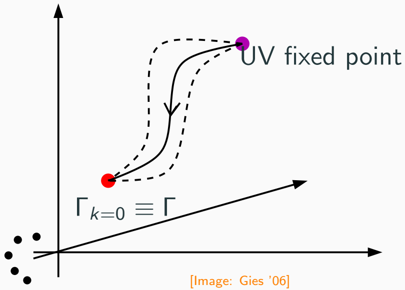
$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k} k\partial_k R_k\right] =$$

R_k = regulator

Γ_k = scale-dependent effective action

Interpolation between

- bare action / UV FP
- quantum effective action Γ



[Image: Gies '06]

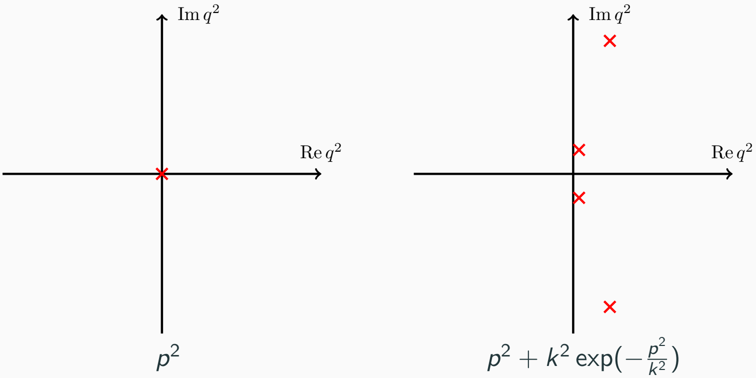
New technical development: spectral FRG on Lorentzian background

[Fehre, Litim, Pawlowski, MR '21; Braun, MR, et al '22]

Standard Euclidean formulations

- Modified dispersion $p^2 \to p^2 + R_k(p^2)$ introduces poles and cuts
- Can not use spectral representation at finite k
- Analytic continuation possible at $k = 0$

[Bonanno, Denz, Pawlowski, MR '21]



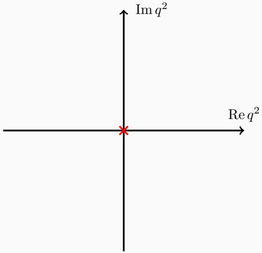
New Lorentzian formulation

[Fehre, Litim, Pawlowski, MR '21]

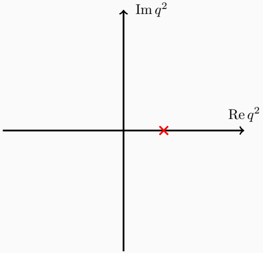
- Callan-Symanzik cutoff $R_k \sim k^2$ allows use of spectral representation
- Dimensional regularisation of UV divergences in $d = 4 - \varepsilon$

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\, \mathcal{G}_k\, \partial_t R_k - \partial_t S_{\mathrm{ct},k}$$

[Braun, Chen, Fu, Geißel, Horak, Huang, Ihssen, Pawlowski, MR, Rennecke, Tan, Töpfel, Wessely, Wink '22]



p^2



$p^2 + k^2$

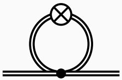
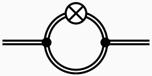
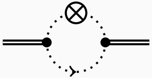
Lorentzian setup

- Einstein-Hilbert action expanded about flat Minkowski background

- Directly compute flow of spectral function

$$\partial_t \rho_h = -2\,\mathrm{Im}\,\mathcal{G}_{hh}^2 \left(\partial_t \Gamma_{\mathrm{TT}}^{(hh)} + \partial_t R_k\right)$$

- Flow of transverse-traceless graviton two point function

$\partial_t \Gamma_k^{(hh)} = -\frac{1}{2}$  $+$  $-\,2$  $-\,\partial_t S_{\mathrm{ct},k}^{(hh)}$

- Schematically

 $= \prod_{i=1}^{3} \int_0^\infty \frac{\mathrm{d}\lambda_i}{\pi} \lambda_i\, \rho_h(\lambda_i) \int \frac{\mathrm{d}^d q}{(2\pi)^d} \frac{V_{\text{3-point}}(p,q)}{(q^2+\lambda_1^2)(q^2+\lambda_2^2)\,((p+q)^2+\lambda_3^2)}$

Flow equations

Parameterisation of ρ_h with $m_h^2 = k^2(1+\mu)$ and $Z_h = Z_h(p^2 = -m_h^2)$

$$\rho_h = \frac{1}{Z_h}\Big[2\pi\delta(\lambda^2 - m_h^2) + \theta(\lambda^2 - 4m_h^2) f_h(\lambda)\Big]$$

Regularised flow of δ-terms with $\tilde{p} = p/m_h$

$$\Big|_{\text{reg},\ \delta\text{-terms}} = \frac{g\, m_h^2(2-\eta_h)}{18\pi^2}\left(\frac{3\left(11\tilde{p}^4 - 8\tilde{p}^2 + 56\right)\operatorname{arcosh}(1+\tilde{p}^2/2)}{\tilde{p}\sqrt{\tilde{p}^2+4}} - 84 + 26\tilde{p}^2\right)$$

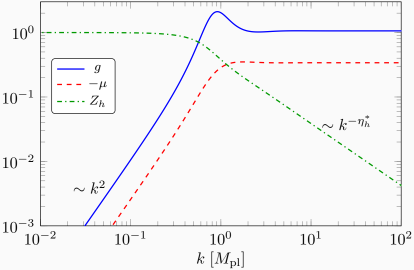
- Flow equations for μ, f_h, and on-shell Z_h
- Flow of $g = G_\mathrm{N} k^2$ from three-graviton vertex at $p = 0$

[Christiansen, Knorr, Meibohm, Pawlowski, MR ' 15; Denz, Pawlowski, MR '16]

UV-IR trajectories

$$(g,\ \eta_h,\ \mu)\big|_* = (1.06,\ 0.96,\ -0.34)$$

$$\theta = 2.49 \pm 3.17\, i$$

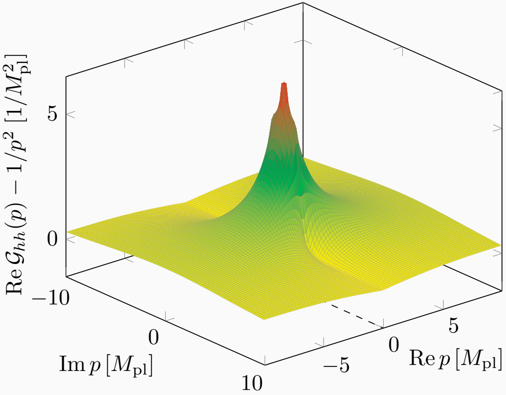


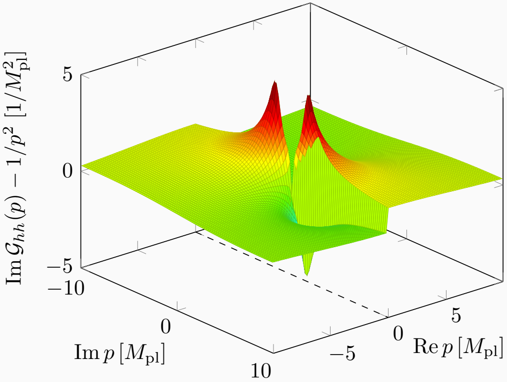
[Fehre, Litim, Pawlowski, MR '21]

$$G_{\rm N}(k) = g(k)/k^2 \xrightarrow{k\to 0} G_{\rm N}$$

$$-2\Lambda(k) = k^2\mu(k) \xrightarrow{k\to 0} -2\Lambda = 0$$

Graviton propagator in the complex plane





[Fehre, Litim, Pawlowski, MR '21]