Relativistic Luttinger fermions as building blocks for UV complete QFTs

UV Complete Quantum Field Theories for Particle Physics

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Marta Picciau

Theoretisch-Physikalisches-Institut Jena

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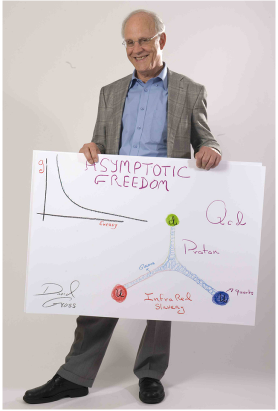
Looking for UV complete QFTs

- Quantum Field Theories can be valid up to arbitrarily high energy scale through asymptotic freedom/safety: UV complete.
- E.g. QCD is asymptotically free in d=4.
- We don't know any theory based on pure fermionic matter that is also UV complete in 4 dimensions.
- Luttinger fermions can serve as new ingredient for building UV complete QFTs.



Picture: David Gross who shared the Nobel Prize in 2004 with D. Politzer and F. Wilczek for discovering asymptotic freedom.

What is a Luttinger fermion?

Discovered by J.M. Luttinger while looking for the most general form of the Hamiltonian of a semiconductor excitation in a magnetic field:

PHYSICAL REVIEW VOLUME 102, NUMBER 4 MAY 15, 1956

Quantum Theory of Cyclotron Resonance in Semiconductors: General Theory*

J. M. LUTTINGER
University of Michigan, Ann Arbor, Michigan
(Received November 17, 1955)

The most general form of the Hamiltonian of an electron or hole in a semiconductor such as Si or Ge, in the presence of an external homogeneous magnetic field, is given. Two methods of obtaining the corresponding energy levels are discussed. The first should yield very accurate values for the magnetic field in the (111) direction for either Si or Ge. The second is a perturbation method and is expected to give good results only for Ge.

$$H = G_{ij}(\partial_i - ieA_i)(\partial_j - ieA_j)$$

Applications to solid-state physics

Effective models in low energy physics:
description of Quadratic Band Touching/Crossing (QBT/QBC) points [1,2]

$$H = \sum_{i,j=1}^{d} G_{ij} p_i p_j , \qquad H^2 = p^4 \mathbb{1}$$

Clifford algebra

$$\{G_{ij}, G_{kl}\} = -\frac{2}{d-1}\delta_{ij}\delta_{kl} + \frac{d}{d-1}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

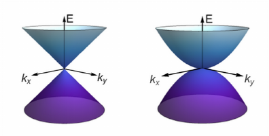


Figure: doi.org/10.1103/PhysRevB.101.161111

Some other applications include: Luttinger semimetals [3], nematic quantum criticality[2], Bernal-stacked bilayer honeycomb lattices[4], quantum spin liquids[5]...

[1] A. A. Abrikosov, Sov. Phys. JETP 39, 709 (1974)
[2] L. Janssen and I. F. Herbut, Phys. Rev. B 92, 045117 (2015)
[3] J. M. Luttinger, Phys. Rev. 102, 1030 (1956)
[4] S. Ray, M. Vojta and L. Janssen, Phys. Rev. B 98, 245128 (2018)
[5] S. Dey and J. Maciejko, Phys. Rev. B 106, 035140 (2022)

Relativistic Luttinger fermions

Kinetic action for relativistic theory of massless Luttinger fermions

$$S = \int d^d x [\bar{\psi} G_{\mu\nu} (i\partial^\mu)(i\partial^\nu)\psi]$$

By imposing the following condition on the kinetic operator

$$G_{\mu\nu}(i\partial^\mu)(i\partial^\nu) G_{\kappa\lambda}(i\partial^\kappa)(i\partial^\lambda) = (\partial^2)^2$$

we arrive at the Clifford algebra

$$\{G_{\mu\nu}, G_{\kappa\lambda}\} = -\frac{2}{d-1} g_{\mu\nu} g_{\kappa\lambda} + \frac{d}{d-1}(g_{\mu\kappa} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\kappa})$$

Properties of the algebra

- $G_{\mu\nu}$ is a $d_\gamma \times d_\gamma$ matrix for each pair $\mu, \nu = 0, \ldots, (d-1)$
- symmetric in Lorentz indices: $G_{\mu\nu} = G_{\nu\mu}$
- traceless in Lorentz indices: $G^\mu_\mu = g^{\mu\nu} G_{\mu\nu} = 0$

number of linearly independent elements to span the algebra	$d_e = \frac{1}{2} d(d-1) + d - 1$
dimension of the Clifford algebra	$d_{\gamma,irr} = 2^{\lfloor d_e/2 \rfloor}$

- $d_e \overset{\text{d=4}}{=} 9, \; d_{\gamma,irr} \overset{\text{d=4}}{=} 16$

Representation of the $G_{\mu\nu}$ matrices

$$G_{0i}^2 = -\frac{1}{2}\frac{d}{d-1}, \quad G_{ij\neq i}^2 = \frac{1}{2}\frac{d}{d-1}, \quad G_{\underline{\mu\mu}}^2 = 1 \qquad \text{metric}: g = \text{diag}(1,-1,-1,\dots)$$

$\rightarrow G_{0i}$ are anti-unitary, the others are unitary

Span $G_{\mu\nu}$ by an Euclidean Clifford algebra

$$G_{\mu\nu} = a_{\mu\nu}^A \gamma_A, \quad \{\gamma_A, \gamma_B\} = 2\delta_{AB} \qquad A, B = 1\dots d_e$$

In $d = 4$ dimensions you need 9 Dirac matrices.

Spin-base formalism

$$\bar{\psi} = \psi^\dagger h \qquad h \text{ spin metric}$$

Reality of the kinetic action implies

$$\{h, G_{0i}\} = 0, \quad [h, G_{ij}] = 0, \quad [h, G_{\underline{\mu\mu}}] = 0$$

However, there exists no solution for h in the $d_{\gamma,irr} = 16$ dimensional representation of the γ_A

invariance under Lorentz transformations	$G_{\mu\nu} \to G_{\kappa\lambda}\Lambda^\kappa_\mu \Lambda^\lambda{}_\nu$ $\Lambda \in \mathrm{SO}(1, d-1)$
invariance under spin-base transformations [6]	$G_{\mu\nu} \to \mathcal{S}\, G_{\mu\nu}\mathcal{S}^{-1}$ $\mathcal{S} \in \mathrm{SL}(d_\gamma, \mathbb{C})$

The action is spin-base invariant $\psi \to \mathcal{S}\psi$, provided $h \to (\mathcal{S}^\dagger)^{-1} h\mathcal{S}$

[6] H. Gies and S. Lippoldt, Phys. Rev. D 89, 064040 (2014)

A reducible representation

A solution for *h* exists in the reducible representation with $d_\gamma = 32$

- The number of γ_A matrices grows $9 \to 11$
- $\mathrm{SL}(32, \mathbb{C})$ spin-base symmetry
- A possible solution is $h = \gamma_1\gamma_2\gamma_3\gamma_{10}$

Quantum Electrodynamics with Luttinger fermions

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} G_{\mu\nu} (i\,\mathrm{D}^\mu)(i\,\mathrm{D}^\nu) \psi - m^2 \bar{\psi}\psi \right], \qquad \text{covariant derivative} \quad \mathrm{D}^\mu = \partial^\mu - ieA^\mu$$

One-loop effective action

$$\Gamma_{\text{1-loop}} = -i \ln \mathrm{Det}[G_{\mu\nu} (i\,\mathrm{D}^\mu)(i\,\mathrm{D}^\nu) - m^2] = -\frac{i}{2} \ln \mathrm{Det}[-G_{\mu\nu}\,\mathrm{D}^\mu\,\mathrm{D}^\nu\,G_{\kappa\lambda}\,\mathrm{D}^\kappa\,\mathrm{D}^\lambda + m^4]$$

The spin-field coupling for $F^{\mu\nu} = const$, analogue to the Pauli term $\sim -\frac{e}{2}\sigma_{\mu\nu}F^{\mu\nu}$, reads

$$\sim \frac{ie}{2} [G_{\mu\nu}, G_{\kappa\lambda}] F^{\nu\lambda} \{\mathrm{D}^\mu, \mathrm{D}^\kappa\}$$

Luttinger QED β function

Choose a constant homogeneous background magnetic field

$$A_{cl.\mu} = \frac{1}{2} B \begin{pmatrix} 0 \\ -y \\ x \\ 0 \end{pmatrix} \rightarrow \vec{B} = B\hat{e}_z$$

The one-loop β function is computed by expanding the determinant in powers of the field strength. At leading order $\sim F_{\mu\nu}F^{\mu\nu}$, it reads

$$\beta_{e^2} = \frac{4}{9\pi^2} 19 e^4 = \frac{4}{9\pi^2} \left(22|_{\text{para}} - 3|_{\text{dia}}\right) e^4$$

The spin-field coupling contributes to an enhancement of the "paramagnetic" interactions[7], yielding a positive β function as in standard QED.

[7] A. Nink and M. Reuter, Int. J. Mod. Phys. D22, 1330008 (2013)

Quantum Chromodynamics with Luttinger fermions

Generalize the action to a non-abelian SU(N_c) gauge group with N_c colors

$$S = \int d^4x \left[-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \bar{\psi}^i G_{\mu\nu} (i\,\mathrm{D}^\mu)^{ij} (i\,\mathrm{D}^\nu)^{jk} \psi^k - m^2 \bar{\psi}^i \psi^i \right], \qquad i = 1 \dots N_C \quad a = 1 \dots N_C^2 - 1$$

The covariant derivative is given by

$$\mathrm{D}^\mu_{ij} = \partial^\mu - i g \tau^a_{ij} A^{\mu,a} \xrightarrow{A^a_\mu = n^a \tilde{A}_\mu} \partial^\mu - i g (\tau^a n^a)_{ij} \tilde{A}^\mu$$

Luttinger QCD β function

The one-loop β function for QCD with N_f relativistic Luttinger quarks is

$$\beta_{g^2} = -\frac{1}{3\pi^2}\left(\frac{11}{8}N_c - \frac{2\cdot 19}{3}N_f\right)g^4.$$

Asymptotic freedom is sustained if

$$N_c > \frac{304}{33}N_f \simeq 9.21 N_f$$

$\rightarrow$ large non-abelian gauge group SU$(N_c \geq 10)$ for $N_f = 1$ Luttinger flavour.

Self-interacting Luttinger fields

Kinetic term $\sim G_{\mu\nu}\partial^\mu\partial^\nu \overset{\text{d=4}}{\rightarrow}$ canonical mass dimension of the fermions $[\psi] = 1$

Quartic self interactions $\sim (\bar{\psi}\psi)^2$ are perturbatively renormalizable (RG marginal)

1024 fermion bilinears $\bar{\psi}\Gamma\psi$ (Γ element of the Clifford algebra)

Nambu–Jona-Lasinino Luttinger fermions model

With our choice of h, the element γ_{10} anticommutes with $G_{\mu\nu}$ & with h: define axial transformations

$$\psi \to e^{i\theta\gamma_{10}}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\theta\gamma_{10}}$$

Self-interacting model with $\mathrm{U}(1)$ axial symmetry

$$S = \int d^4x \left[-\bar{\psi}G_{\mu\nu}\partial^\mu\partial^\nu\psi + \frac{\lambda}{2}[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_{10}\psi)^2\right]$$

$$\partial_t\lambda = -\frac{1}{(12\pi)^2}72\lambda^2$$

Asymptotically free $\to$ UV complete

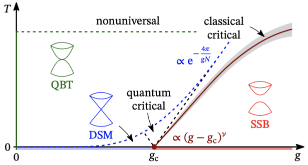
Relation with Dirac Fermions

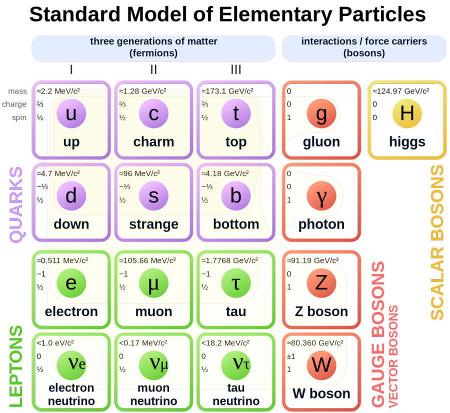
32-component Luttinger spinor contains eight 4-component Dirac spinors
UV completion of the Standard Model?

$$\mathrm{SL}(32,\mathbb{C}) \to \mathrm{SL}(4,\mathbb{C})$$

Explicit breaking term $\zeta_D \int_x \bar{\psi} i \partial\!\!\!/ P_D \psi$

Condensed matter: transition from QBT to Dirac cones (bilayer graphene)[4]





Recap and outlook

- Generalized Luttinger fermions to the relativistic case
- Constructed self-interacting QFT with Luttinger fermions that support asymptotic freedom: UV complete QFT based on pure matter degrees of freedom
- Gauge theories with Luttinger fermions exibit strong paramagnetic dominance and are asymptotically free for large gauge groups

- Transition from Luttinger to Dirac fermions: breaking the symmetry SL$(32, \mathbb{C}) \rightarrow$ SL$(4, \mathbb{C})$
- Massive Luttinger fermions come with tachyons $[(p^4 - m^4)\psi = 0]$.
  Solution: Dirac kinetic term to break the symmetry?
- Investigation of the IR phenomena (strongly interacting theories)
- Classification of the 1024 spinor bilinears

Thank you for your attention