# Dimensional Transmutation in Particle Physics and Gravity

**We discuss models with classical scale invariance (CSI): *all* masses are generated by quantum effects, through dimensional transmutation**

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Università di Roma Tor Vergata

4 September 2023

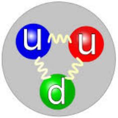
*International workshop*
*"UV Complete Quantum Field Theories for Particle Physics"*

# Some motivations for models with CSI

## Motivation 1: origin of mass and EW symmetry breaking

*Most of the mass of the matter we see has a dynamical origin*

**Example**: the proton mass $\longrightarrow$

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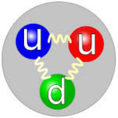


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## Motivation 2: predictivity

*CSI gives a strong restriction on the possible theories, which can lead to testable predictions. E.g. a realistic theory of this type can only be formulated in 4D: a gauge theory in higher dimensions necessarily includes dimensionful parameters in the action* $\Longrightarrow$ *explanation of the observed number of dimensions.*

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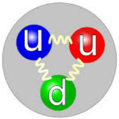


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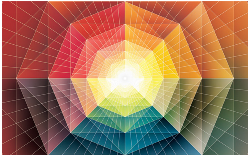
### Motivation 3: first-order phase transitions

*If symmetries are broken and masses are generated radiatively one always has first-order phase transition with corresponding observable gravitational waves and primordial black holes.*

# Classical Scale Invariance in the press

For an outreach article on CSI see
*[Natalie Wolchover for Quanta Magazine (2014)]*



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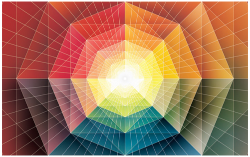

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The quantum breaking of scale invariance we need is similar to the Coleman-Weinberg (CW) mechanism, but here we need a gravitational generalization

# The Coleman-Weinberg mechanism

- The *CW mechanism (1973)* is a perturbative way to generate a mass through quantum corrections in the absence of gravity
- These effects can be captured by the effective potential $V_{\rm eff}$
- One requires the existence of an (approximately) flat direction in the tree-level potential: if such direction did not exist we would expect the radiative corrections to be negligible with respect to the slope of the tree-level potential
- Essentially this means that a quartic coupling $\lambda_{\rm CW}$ of a scalar, $\varphi_{\rm CW}$, has to vanish at some scale $\mu_{\rm CW}$: then $\varphi_{\rm CW}$ is an (approximate) flat direction:

$$V_{\rm eff}^{\rm CW} = \lambda_{\rm CW}(\varphi_{\rm CW})\varphi_{\rm CW}^4 + \text{constant} \simeq \text{constant} \quad \text{for} \quad \varphi_{\rm CW} \approx \mu_{\rm CW}$$

- The CW idea was later extended to generic theories (but still without gravity) by *[Gildener, S. Weinberg (1976)]*:

$$\mathscr{L}_{\rm matter}^{\rm ns} = -\frac{1}{4}F_{\mu\nu}^A F^{A\mu\nu} + \frac{D_\mu\phi_a\, D^\mu\phi_a}{2} + \bar{\psi}_j i\not{D}\psi_j - \frac{1}{2}(Y_{ij}^a\psi_i\psi_j\phi_a + \text{h.c.}) - V_{\rm ns}(\phi),$$

with

$$V_{\rm ns}(\phi) = \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d$$

# Gravitational generalizations of the CW mechanism (Agravity)

The general Lagrangian including gravity and a generic matter sector is

$$\mathscr{L} = \frac{R^2}{6f_0^2} - \frac{W^2}{2f_2^2} - \frac{\xi_{ab}}{2}\phi_a\phi_b R + \mathscr{L}_{\rm matter}^{\rm ns}$$

(we call this theory *agravity*)

**These gravitational terms should be added:**
**If not added to the classical Lagrangian they are generated by quantum effects.**

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**Non-gravitational sector:** $\mathscr{L}_{\rm matter}^{\rm ns} - \frac{\xi_{ab}}{2}\phi_a\phi_b R = \mathscr{L}_4^{\rm SM} + \mathscr{L}_4^{\rm BSM}$

- $\mathscr{L}_4^{\rm SM}$ is the Standard Model (SM) $\mathscr{L}$ (without $\mu_H^2|H|^2/2$ plus $-\xi_H|H|^2R$):
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**Agravity sector**

$\langle s\rangle$ generates $\bar M_{\rm Pl}$: $\quad \xi_S s^2 R \to \bar M_{\rm Pl}^2 = \xi_S\langle s\rangle^2$

# Conditions for the gravitational CW mechanism

Agravity successfully generates the Planck scale if

$$\begin{cases} \lambda_s(s) & \simeq & 0 & \leftrightarrow & \text{nearly vanishing cosmological constant (dark energy)} \\ \lambda'_s(s) & = & 0 & \leftrightarrow & \text{minimum condition} \\ \xi_s(s) & > & 0 & & \text{then we identify} \qquad \xi_s(s)s^2 = \bar{M}_{\rm Pl}^2 \end{cases}$$

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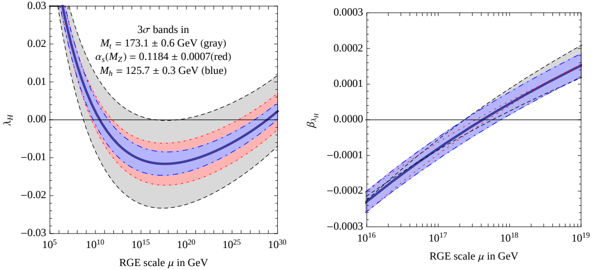
It is possible to satisfy these conditions as they are realized in the physics we know (the SM)!

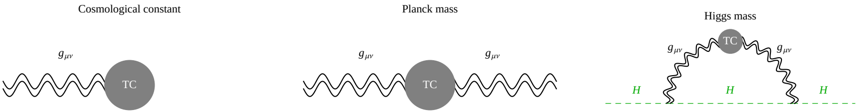
↘




[Salvio, Strumia (2014)]

# Non-perturbative generations of scales

Alternatively all scales can be induced by a new gauge group $G_{\mathrm{TC}}$ that becomes non-perturbative around the Planck scale, such that condensates are generated. *[Adler (1982)]*, *[Salvio, Strumia (2017)]*, *[Donoghue, Menezes (2017)]*, *[Kubo, Lindner, Schmitz, Yamada (2018)]*



# The Higgs mass $M_h$ can be naturally much smaller than $\bar{M}_{\mathrm{Pl}}$

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This ensures that quantum corrections respect the smallness of the physical quantity

The theory is renormalizable

$\Longrightarrow$ one can consistently compute the quantum corrections $\delta M_h$ to the Higgs mass:

$$\delta M_h^2 \sim \frac{\bar{M}_{\rm Pl}^2 f_i^4}{(4\pi)^2}, \qquad \delta M_h^2 \lesssim M_h^2 \ \to \ f_2 \lesssim \sqrt{\frac{4\pi M_h}{\bar{M}_{\rm Pl}}} \sim 10^{-8}$$

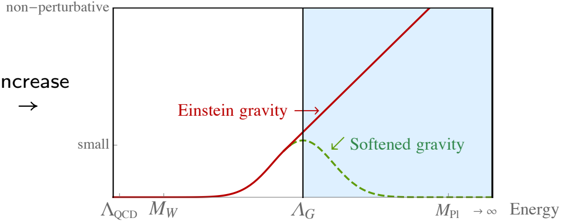
*[Salvio, Strumia (2014)]*

# Agravity is a realization of "softened gravity"

(Einstein) gravitational interactions increase with energy

Idea (*softened gravity*):

consider theories where the increase of the gravitational coupling $\rightarrow$ *stops at some* $\Lambda_G \ll M_{\rm Pl}$.



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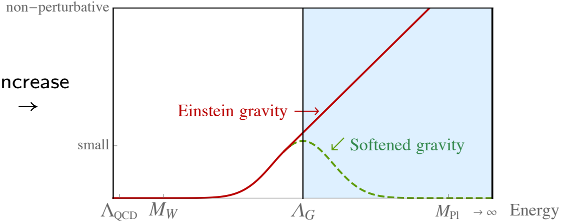


The gravitational contribution to the Higgs mass is

$$\delta M_h^2 \sim \frac{G_N \Lambda_G^4}{(4\pi)^2}$$

Requiring $\delta M_h \sim M_h \to \Lambda_G \lesssim 10^{11}$ GeV *[Giudice, Isidori, Salvio, Strumia (2014)]*

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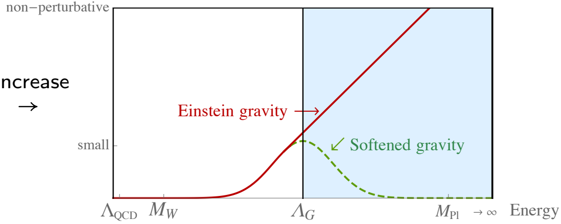


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Still the cosmological constant $\Lambda$ is not naturally equal to the observed value.
One should look for another independent mechanism to explain its value.

# What happens above $\bar{M}_{\rm Pl}$? Clash between asymptotic freedom and stability

$$
\begin{aligned}
(4\pi)^2 \frac{df_2^2}{d\ln\bar\mu} &= -f_2^4\left(\frac{133}{10}+\frac{N_V}{5}+\frac{N_f}{20}+\frac{N_s}{60}\right) \\
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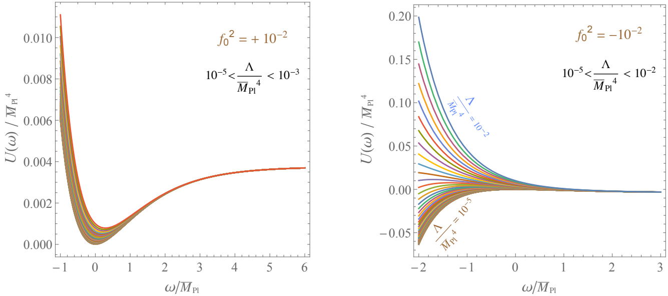
$f_0^2<0$ corresponds to a tachyonic scalar $\omega$ with squared mass $M_0^2 \sim f_0^2\bar{M}_{\rm Pl}^2$

**The potential of the effective scalar $\omega$ that corresponds to the term $R^2/6f_0^2$ :**



# What happens above $\bar{M}_{\rm Pl}$? Asymptotic safety can save us

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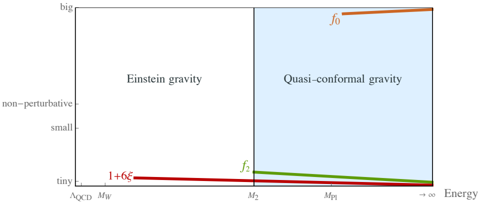


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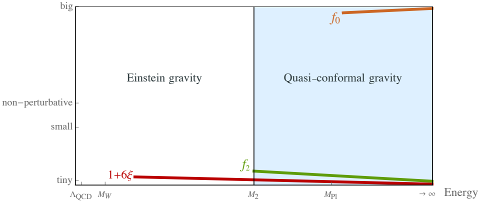


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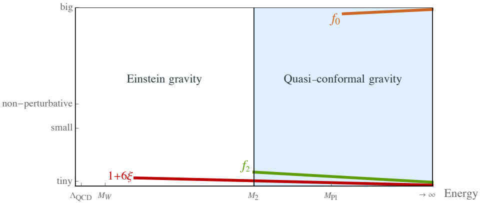


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Since agravity flows to conformal gravity in the UV and the FRW metric is conformally flat, the initial-time cosmological singularity of GR is avoided

# UV behavior and spectrum of agravity

- Agravity is renormalizable (clear from the absence of fundamental scales) and rigorously proved by *Stelle (1977)* in the presence of $\bar{M}_{\rm Pl}$ (see also a more recent proof of *Barvinsky, Blas, Herrero-Valea, Sibiryakov and Steinwachs (2017)*)
- Furthermore, it can be extended up to infinite energy if there is a UV fixed point *[Salvio, Strumia (2017)]*, predicting transplanckian physics *[Salvio, Strumia, Veermae (2018)]*, and can address the hierarchy problem as we have seen

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However, looking at the classical spectrum *[Stelle (1977)]*:

**(i)** massless graviton

**(ii)** scalar $z$ with mass $M_0^2 \sim \frac{1}{2} f_0^2 \bar{M}_{\rm Pl}^2$

**(iii)** massive spin-2 field with an abnormal-sign kinetic term (ghost) and squared mass $M_2^2 = \frac{1}{2} f_2^2 \bar{M}_{\rm Pl}^2$

This abnormal graviton is associated with $\frac{W^2}{2 f_2^2}$.

(iii) is the manifestation of the *Ostrogradsky theorem (1848)*: classical Lagrangians that depend non-degenerately on the second derivatives have Hamiltonians unbounded from below

# Ghost problem: proceeding perturbatively

Let us split the metric $g_{\mu\nu}$ as follows:

$$g_{\mu\nu} = g^{\text{cl}}_{\mu\nu} + \hat{h}_{\mu\nu}$$

- $g^{\text{cl}}_{\mu\nu}$ is a classical background that solves the classical EOMs
- $\hat{h}_{\mu\nu}$ is a *quantum* fluctuation

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- Recall that in the *free-field limit*

$$H_{\text{ghost}} = -\sum_{\lambda=\pm 2,\pm 1,0} \int d^3q \left[\tilde{P}_\lambda^2 + (q^2 + M_2^2)\tilde{Q}_\lambda^2\right]$$

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This argument can be made precise in classical quadratic gravity (the theory with quadratic-in-curvature terms we have). The whole cosmology can only involve energies below this threshold and avoid runaways

→ "metastability in quadratic gravity"

*[Salvio (2019)]* see also *[dos Reis, Chapiro, Shapiro (2019)]* and *[Gross, Strumia, Teresi, Zerilli (2020)]*

# This can be shown with a two-derivative formulation

## One separates the two-derivative d.o.f.: $\omega$, ordinary and abnormal gravitons

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First perform the field redefinition $\quad g_{\mu\nu} \to \frac{\bar{M}_{\rm Pl}^2}{f} g_{\mu\nu}, \qquad f \equiv \bar{M}_{\rm Pl}^2 - \frac{2R}{3f_0^2} > 0,$

(where the Ricci scalar above is computed in the Jordan frame metric) that gives

$$S = \int d^4x \sqrt{-g}\left(-\frac{W^2}{2f_2^2} - \frac{\bar{M}_{\rm Pl}^2}{2} R + \mathscr{L}_m^E\right) \qquad \text{“Einstein frame action”}$$

The Einstein-frame matter Lagrangian, $\mathscr{L}_m^E$, also contains an effective scalar $\omega$ (a.k.a. the scalaron), which corresponds to the $R^2$ term in the Jordan frame:

$$\mathscr{L}_\omega^E = \frac{(\partial\omega)^2}{2} - U(\omega), \qquad U(\omega) = \frac{3f_0^2 \bar{M}_{\rm Pl}^4}{8}\left(1 - e^{-2\omega/\sqrt{6}\bar{M}_{\rm Pl}}\right)^2$$

---

To make the abnormal graviton explicit consider an auxiliary field $\gamma_{\mu\nu}$:

$$S = \int d^4x \sqrt{-g}\left[\frac{M_2^2 \bar{M}_{\rm Pl}^2}{8}\left(\gamma_{\mu\nu}\gamma^{\mu\nu} - \gamma^2\right) - \frac{\bar{M}_{\rm Pl}^2}{2} G_{\mu\nu}\gamma^{\mu\nu} - \frac{\bar{M}_{\rm Pl}^2}{2} R + \mathscr{L}_m^E\right]$$

where $G_{\mu\nu}$ is the Einstein tensor and $\gamma \equiv \gamma_{\mu\nu} g^{\mu\nu}$.

One has a mixing between $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ and $\gamma_{\mu\nu}$. The tensors $\bar{h}_{\mu\nu} = h_{\mu\nu} + \gamma_{\mu\nu}$ and $\gamma_{\mu\nu}$ represent the ordinary and abnormal gravitons

# Interactions of the abnormal graviton and energy thresholds

The two-derivative formulation is good to understand the abnormal graviton interactions. First, one can easily see that they are suppressed by $f_2$

Next, $\frac{M_2^2}{8}\left(\gamma_{\mu\nu}\gamma^{\mu\nu} - \gamma^2\right)$ leads to mass and interaction terms of the schematic form

$$\frac{M_2^2}{2}\left(\phi_2^2 + \frac{\phi_2^3}{\bar{M}_{\rm Pl}} + \frac{\phi_2^4}{\bar{M}_{\rm Pl}^2} + \dots\right),$$

($\phi_2$ represents the canonically normalized spin-2 fields)

The mass term has the same order of magnitude of the interactions for $\phi_2 \sim \bar{M}_{\rm Pl}$, which gives $M_2^2\phi_2^2/2 = M_2^4/f_2^2 \equiv E_2^4$, where

$$E_2 \equiv \frac{M_2}{\sqrt{f_2}} = \sqrt{\frac{f_2}{2}}\bar{M}_{\rm Pl}$$

*For energies* $\boxed{E \ll E_2}$ *the Ostrogradsky instabilities are avoided*

This bound applies to the boundary conditions (BCs) of derivatives of the spin-2 fields.

Analogously, one can show that the energy $E$ in the matter sector must satisfy

$$\boxed{E \ll E_m} \qquad E_m \equiv \sqrt[4]{f_2}\,\bar{M}_{\rm Pl} \qquad \text{(matter sector)}$$

(one has to impose it on the BCs)

# Relations with chaotic inflation *[Linde (1983)]*

For a natural Higgs mass ($f_2 \sim 10^{-8}$, $M_2 \sim 10^{10}$ GeV)

$$E_2 \sim 10^{-4}\bar{M}_{\rm Pl}, \qquad E_m \sim 10^{-2}\bar{M}_{\rm Pl}$$

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But we live in one of those patches where the energy scales of inhomogeneities ($E_i$) and anisotropies ($E_a$) were small enough:

$$E_i \ll |U_I'/I|^{1/2}, \qquad E_i, E_a \ll H_I$$

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The chaotic theory automatically ensures that the conditions to avoid runaway solutions are satisfied. The fatal runaways above the energy thresholds give an (anthropic) rationale for a homogeneous and isotropic universe
(verified for Starobinsky, hilltop, natural, Higgs inflation and other models)

Let us go back to the the following metric splitting

$$g_{\mu\nu} = g^{\text{cl}}_{\mu\nu} + \hat{h}_{\mu\nu}$$

- $g^{\text{cl}}_{\mu\nu}$ is a classical background that solves the classical EOMs.
- $\hat{h}_{\mu\nu}$ is a *quantum* deviation

# Can a different quantization help?

Recall that the classical Dirac theory of fermions has arbitrarily negative energies and the problem is solved by a different quantization

Can we hope that something similar happens for gravitons?

Yes, renormalizability implies that the *quantum* Hamiltonian governing $\hat{h}_{\mu\nu}$ is bounded from below *[Stelle (1977)]*

However, the space of states must be endowed with an indefinite metric (with respect to which the "position" $q$ and "momentum" $p$ operators are self-adjoint).

Then the presence of an indefinite metric leads to the question:

How can we define probabilities consistently?

# A derivation of probability

- Define observable any operator $A$ with complete eigenstates $\{|a\rangle\}$ *[Salvio (2018)]*: for any state $|\psi\rangle$ there is a decomposition

$$|\psi\rangle = \sum_a c_a |a\rangle$$

  One can show that the basic operators $q, p$ and the Hamiltonian have complete eigenstates at *any* order in perturbation theory

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Experimentalists prepare a large number $N$ of times the same state, so consider

$$|\Psi_N\rangle \equiv \nu^N \underbrace{|\psi\rangle ... |\psi\rangle}_{N \text{ times}} = \sum_{a_1 ... a_N} \nu^N c_{a_1} ... c_{a_N} |a_1\rangle ... |a_N\rangle, \qquad \nu \equiv \frac{1}{\sqrt{\sum_b |c_b|^2}}$$

Define a frequency operator $F_a$ which counts the number $N_a$ of times there is the value $a$ in the state $|a_1\rangle ... |a_N\rangle$:

$$F_a |a_1\rangle ... |a_N\rangle \equiv \frac{N_a}{N} |a_1\rangle ... |a_N\rangle$$

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One can show that $$\lim_{N\to\infty} F_a |\Psi_N\rangle = B_a |\Psi_N\rangle, \qquad B_a \equiv \frac{|c_a|^2}{\sum_b |c_b|^2}$$

(all coefficients in the basis $|a_1\rangle...|a_N\rangle$ converge to the same quantities)

The probabilities are positive and sum up to one at any time (the theory is unitary)

# The emergent norms to compute probabilities

$\{|a\rangle\}$ is complete so we can ***define*** a "norm" operator $P_A$:

$$\langle a'|P_A|a\rangle \equiv \delta_{aa'}$$

where for any pair of states $|\psi_1\rangle$, $|\psi_2\rangle$, we denote the indefinite metric with $\langle\psi_2|\psi_1\rangle$.
The definition above provides a positive metric (a norm):

$$\langle\psi_2|\psi_1\rangle_A \equiv \langle\psi_2|P_A|\psi_1\rangle = \sum_a c^*_{a2} c_{a1}$$

(which is positive for $|\psi_1\rangle = |\psi_2\rangle$)

$$B_a \equiv \frac{|c_a^2|}{\sum_b |c_b^2|} = \frac{|\langle a|\psi\rangle_A|^2}{\langle\psi|\psi\rangle_A}$$

We recover the full probabilistic Born rule, but expressed in terms of the ***positive norm*** not in terms of the indefinite one

# Dirac-Pauli (DP) quantization of canonical variables

*[Dirac (1941); Pauli (1943); Salvio, Strumia (2015); Salvio (2020)]*

$A$ is normal with respect to the $A$-norm $\Rightarrow A = A_h + A_a$, where $A_h$ ($A_a$) is an (anti)Hermitian operator with respect to the $A$-norm and $[A_h, A_a] = 0$.

So we restrict to

$$A|a\rangle = \lambda_a |a\rangle, \qquad \lambda_a = \alpha_a \quad \text{or} \quad \lambda_a = i\alpha_a \quad (\text{with } \alpha_a \text{ real})$$

In quadratic gravity there are also observables that realize the second possibility: the canonical coordinates $Q, P$ (with $[Q, P] = i$) of the abnormal graviton.

This is the only option which allows a Hamiltonian of the form

$$\hat{H} = -\frac{1}{2}\left(P^2 + \omega_Q^2 Q^2\right)$$

to have a spectrum bounded from below *and normalizable eigenfunctions*

$$Q|x\rangle = ix|x\rangle, \qquad P|x\rangle = \frac{d}{dx}|x\rangle$$

# Phase transitions (PTs), gravitational waves (GWs) and primordial black holes (PBHs) in CSI theories

## Phase transitions (PTs), gravitational waves (GWs) and primordial black holes (PBHs) in CSI theories

All CSI theories where symmetries are broken (and masses are then generated) radiatively feature strong and long first-order PTs, which lead to

- GWs
- PBHs

# Radiative symmetry breaking (RSB) mechanism

To illustrate this general result we consider the general $\mathscr{L}^{\rm ns}_{\rm matter}$

In the RSB mechanism masses emerge radiatively: there is an energy $\tilde{\mu}$ at which $V_{\rm ns}$ develops a flat direction, $\phi_a = \nu_a \chi$, with $\nu_a \nu_a = 1$, and $\chi$ a single scalar field
$\Longrightarrow$ RG-improved potential $V$ along $\nu_a$ reads

$$V(\chi) = \frac{\lambda_\chi(\mu)}{4}\chi^4, \qquad \left(\lambda_\chi(\mu) \equiv \frac{1}{3!}\lambda_{abcd}(\mu)\nu_a\nu_b\nu_c\nu_d, \quad \lambda_\chi(\tilde{\mu}) = 0\right)$$

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Including the one-loop correction the quantum effective potential can always be written

$$V_q(\chi) = \frac{\bar{\beta}}{4}\left(\log\frac{\chi}{\chi_0} - \frac{1}{4}\right)\chi^4, \qquad \begin{cases} \lambda_\chi(\tilde{\mu}) = 0 & \text{(flat direction)}, \\ \bar{\beta} \equiv \left[\mu \frac{d\lambda_\chi}{d\mu}\right]_{\mu=\tilde{\mu}} > 0 & \text{(minimum condition)}, \end{cases}$$

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The fluctuations of $\chi$ around $\chi_0$ have mass

$$m_\chi = \sqrt{\bar{\beta}}\,\chi_0$$

$\chi_0 \neq 0$ can break global and/or local symmetries and generate the particle masses.
E.g. a term in $\mathscr{L}$ of the form

$$\mathscr{L}_{\chi h} = \frac{1}{2}\lambda_{\chi h}(\tilde{\mu})\chi^2 |H|^2$$

can contribute to electroweak (EW) symmetry breaking

# Thermal effective potential and PT

$$V_{\text{eff}}(\chi,T) = V_q(\chi) + \frac{T^4}{2\pi^2}\left(\sum_b n_b J_B(m_b^2(\chi)/T^2) - 2\sum_f J_F(m_f^2(\chi)/T^2)\right) + \Lambda_0$$

The thermal functions $J_B$ and $J_F$ are

$$J_B(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 - e^{-\sqrt{p^2+x}}\right) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2} - \frac{x^2}{32}\log\left(\frac{x}{a_B}\right) + O(x^3),$$

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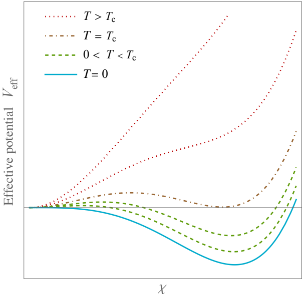
The PT associated with a RSB always turns out to be of first order! *[Salvio (2023)]*



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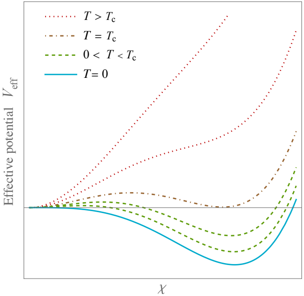


The decay rate per unit of spacetime volume, $\Gamma$, of the false vacuum into the true vacuum can be computed with the formalism of *[Coleman (1977); Callan, Coleman (1980); Linde (1981); Linde (1983)]*

# Supercooling and model-independent approach

As long as perturbation theory holds, for all CSI theories, when $T < T_c$ the scalar field $\chi$ is trapped in the false vacuum $\langle \chi \rangle = 0$ until $T$ is much below $T_c$, in other words the universe features a phase of supercooling *[Witten (1981); Salvio (2023)]*

**Explanation:** If the theory is scale invariant $\Gamma$ must scale as $T^4$ and, therefore, the smaller $T$, the smaller $\Gamma$. At quantum level scale invariance is broken by perturbative loop corrections, which introduce another dependence of $T$ in the bounce action. This dependence, however, is logarithmic and can become large only when $T$ is very small compared to the other scale of the problem, $\chi_0$.

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If enough supercooling occured a **model-independent approach** is possible! *[Salvio (2023) I; Salvio (2023) II]*

The amount of supercooling needed is quantified by

$$\epsilon \equiv \frac{g^4}{6\bar{\beta}\log\frac{\chi_0}{T}},$$

with

$$g^2 \equiv \sum_b n_b m_b^2(\chi)/\chi^2 + \sum_f m_f^2(\chi)/\chi^2$$

## Small $\epsilon$ case *[Salvio (2023) I]*

$$\bar{V}_{\rm eff}(\chi,T) \equiv V_{\rm eff}(\chi,T) - V_{\rm eff}(0,T) \approx \frac{m^2(T)}{2}\chi^2 - \frac{\lambda(T)}{4}\chi^4$$

$$m^2(T) \equiv \frac{g^2T^2}{12}, \qquad \lambda(T) \equiv \bar{\beta}\log\frac{\chi_0}{T}$$

$$\Gamma \approx T^4\left(\frac{S_3}{2\pi T}\right)^{3/2}\exp(-S_3/T), \quad \text{with} \quad S_3 = -8\pi\int_0^\infty dr\, r^2 \bar{V}_{\rm eff}(\chi,T)$$

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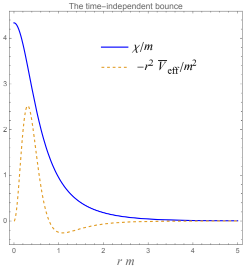
$$\chi'' + \frac{2}{r}\chi' = \frac{d\bar{V}_{\rm eff}}{d\chi}, \qquad \chi'(0) = 0, \quad \lim_{r\to\infty}\chi(r) = 0$$

one finds $S_3 \approx c_3\frac{m}{\lambda}$ with $c_3 = 18.9...$ and for $\lambda = 1$ $\longrightarrow$

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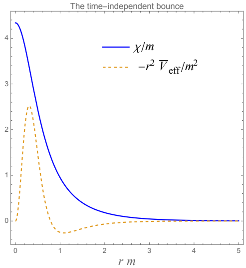


The nucleation temperature defined as the solution of $\Gamma = H_I$ is

$$T_n \approx \chi_0\exp\left(\frac{\sqrt{c^2-16a}-c}{8}\right), \quad \text{with} \quad a \equiv \frac{c_3 g}{\sqrt{12\bar{\beta}}}, \quad c \equiv 4\log\frac{4\sqrt{3}\bar{M}_{\rm Pl}}{\sqrt{\bar{\beta}}\,\chi_0}$$

One always has a very strong PT and a small inverse duration $\beta$: $\frac{\beta}{H_n} \approx \frac{a}{\log^2(\chi_0/T_n)} - 4$

**Corrections are easily computable in a small-$\epsilon$ expansion**

$\epsilon \sim 1$

## $\epsilon \sim 1$. Simple case: several d.o.f. with dominant couplings to $\chi$

**The formulæ we have seen in the small $\epsilon$ case still hold**

## $\epsilon \sim 1$. General case *[Salvio (2023) II]*

$$\bar{V}_{\rm eff}(\chi, T) \approx \frac{m^2(T)}{2}\chi^2 - \frac{k(T)}{3}\chi^3 - \frac{\lambda(T)}{4}\chi^4, \quad \text{with} \quad k(T) \equiv \frac{\tilde{g}^3 T}{4\pi}$$

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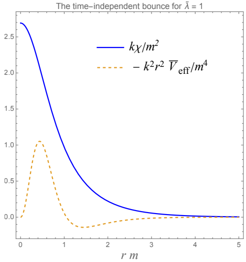
The relation between $\Gamma$ and $S_3$ we have seen still holds, but

$$S_3 = -\frac{8\pi m^3}{k^2} \int_0^\infty d\rho\, \rho^2 \left(\frac{1}{2}\varphi^2 - \frac{1}{3}\varphi^3 - \frac{\tilde{\lambda}}{4}\varphi^4\right)$$

where

$$\varphi \equiv \frac{k\chi}{m^2} \quad \text{and} \quad \tilde{\lambda} \equiv \frac{\lambda m^2}{k^2} > 0$$



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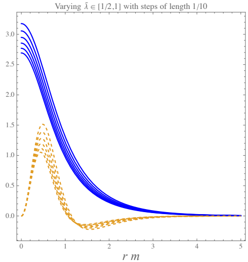
$$S_3 = -\frac{8\pi m^3}{k^2}\int_0^\infty d\rho\,\rho^2\left(\frac{1}{2}\varphi^2 - \frac{1}{3}\varphi^3 - \frac{\tilde{\lambda}}{4}\varphi^4\right)$$

where

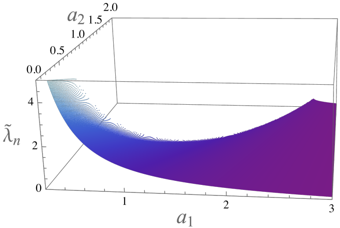
$$\varphi \equiv \frac{k\chi}{m^2} \quad \text{and} \quad \tilde{\lambda} \equiv \frac{\lambda m^2}{k^2} > 0$$

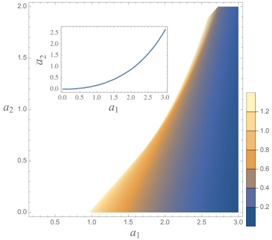


# $\epsilon \sim 1$. General case: nucleation temperature $T_n$

$T_n$ can be numerically computed once and for all as the solution $\tilde{\lambda}_n$ of

$$a_1 - a_2\tilde{\lambda} = F(\tilde{\lambda}) \equiv \frac{1 + \exp(-1/\sqrt{\tilde{\lambda}})}{2/9 + \tilde{\lambda}}, \quad \text{where} \quad a_1 \equiv \frac{c\, c_3 k^2}{3\pi a\, \bar{\beta}\, m^2}, \qquad a_2 \equiv \frac{4 c_3 k^4}{3\pi a\, \bar{\beta}^2 m^4}$$



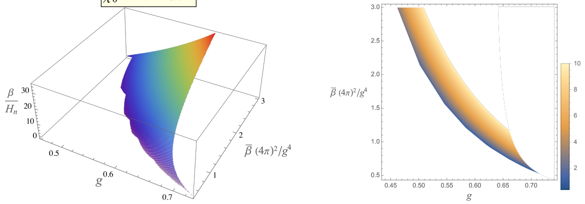

The inset in the right plot gives the maximal value of $a_2$ for a given $a_1$ such that $\tilde{\lambda}_n$ exists

# $\epsilon \sim 1$. General case: inverse duration $\beta$.

$$\frac{\beta}{H_n} \approx \frac{\pi^3 g^5}{6\sqrt{3}\tilde{g}^8} \frac{(4\pi)^2 \bar{\beta}}{\tilde{g}^4} (-F'(\tilde{\lambda}_n)) - 4$$

# $\epsilon \sim 1$. General case: inverse duration $\beta$. Imposing $\tilde{g} = g$ and $\epsilon < 3$

$\chi_0 = 10^{-1}$ GeV

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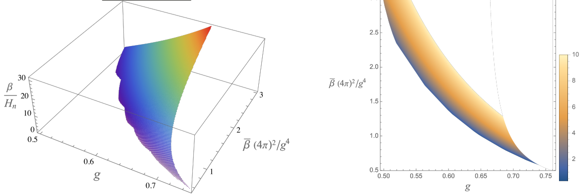
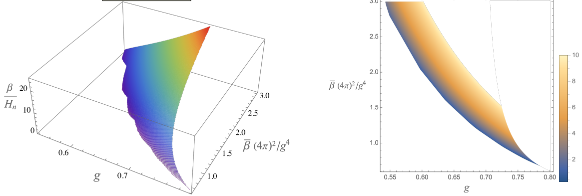
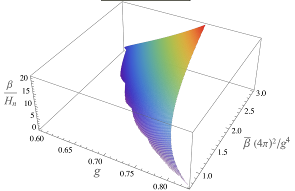


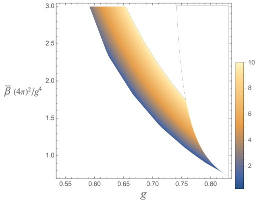

$\chi_0 = 10^{4}$ GeV



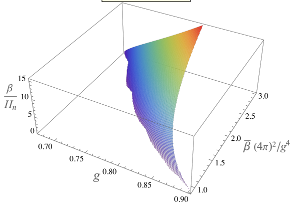
# $\epsilon \sim 1$. General case: inverse duration $\beta$. Imposing $\tilde{g} = g$ and $\epsilon < 3$

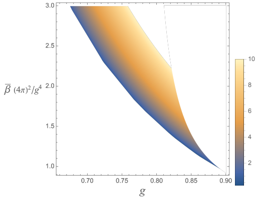
$\chi_0 = 10^6$ GeV



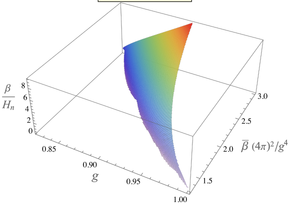



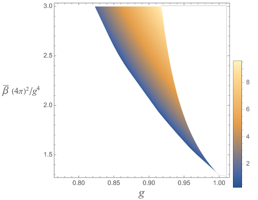

$\chi_0 = 10^9$ GeV




$\chi_0 = 10^{12}$ GeV



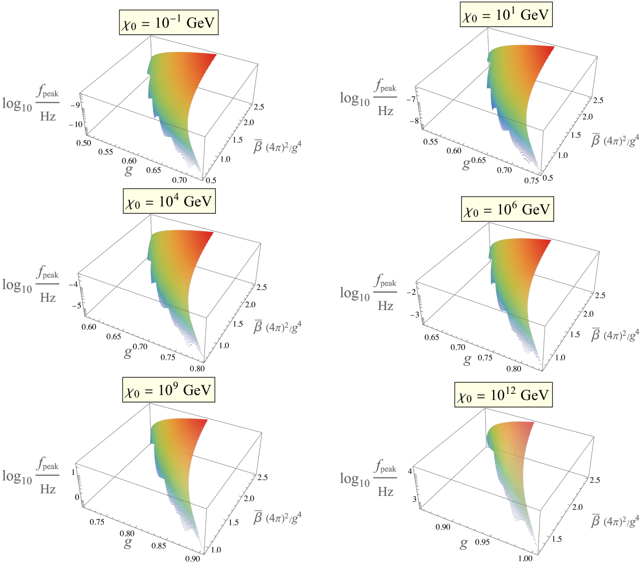



# Gravitational waves

$$h^2\Omega_{\rm GW}(f) \approx 1.29\times 10^{-6}\left(\frac{H_r}{\beta}\right)^2\left(\frac{100}{g_*(T_r)}\right)^{1/3}\frac{3.8(f/f_{\rm peak})^{2.8}}{1+2.8(f/f_{\rm peak})^{3.8}}$$

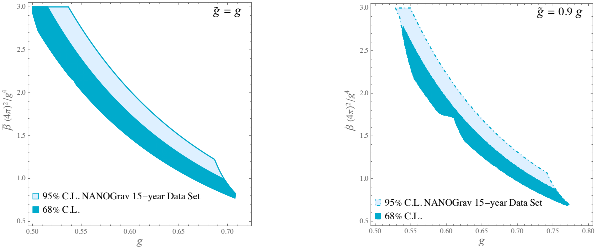
$$f_{\rm peak} \approx 3.79\,\frac{\beta}{H_r}\left(\frac{g_*(T_r)}{100}\right)^{1/6}\frac{T_r}{10^8{\rm GeV}}\,{\rm Hz}$$

# Gravitational waves: peak frequency

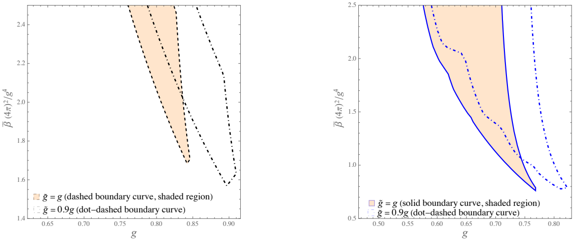



*The peak frequency as a function of $g$ and $\bar{\beta}$ in the case of fast reheating and fixing $g_*(T_r) = 110$. Also, $\tilde{g} = g$ and $\epsilon < 3$ has been imposed.*

# Gravitational waves: comparison with experiments




*Regions corresponding to the GW background detected by pulsar timing arrays. In both plots $\chi_0 = 10$ GeV, $g_*(T_r) = 110$ and fast reheating is assumed. Here $\epsilon < 3$ has been imposed.*




*Regions where $\Omega_{\rm GW}(f_{\rm peak})$ is above the sensitivities of LIGO-VIRGO O3 (left plot, where $\chi_0 = 2 \times 10^9$ GeV) and LISA (right plot, where $\chi_0 = 10^4$ GeV). In both plots $g_*(T_r) = 110$ and fast reheating is assumed. Here $\epsilon < 3$ has been imposed.*

# Primordial black holes

**Late-blooming mechanism:** Since the bubble formation process is statistical for both quantum and thermal reasons, distinct causal patches percolate at different times. Patches that percolate the latest undergo the longest vacuum-dominated stage and, therefore, develop large over-densities triggering their collapse into PBHs (see e.g. *[Gouttenoire, Volansky (2023)]*)

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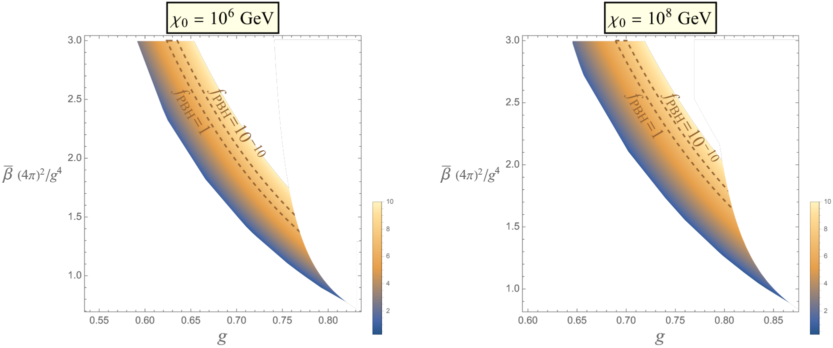


*Density plots giving the values of $\beta/H_n$ varying $g$ and $\bar{\beta}$. On the lower dashed line the whole dark matter is due to PBHs generated through the late-blooming mechanism ($f_{\rm PBH} = 1$); the upper dashed line corresponds instead to $f_{\rm PBH} = 10^{-10}$. Here $\tilde{g} = g$ and $\epsilon < 3$ has been imposed.*

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- Naturalness imply that the ghost is below $10^{10}$ GeV and can, therefore, be tested with cosmological data
- All CSI theories where symmetries are broken (and masses are then generated) radiatively feature strong and long first-order PTs, which lead to
  - observable GWs
  - PBHs that can account for a fraction or the entire dark matter

**Thank you very much for your attention!**