Pathways to UV Complete Quantum Field Theories

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Perfect Theories?

- e.g., classical mechanics of point particles

$$m\mathbf{a} = \mathbf{F}$$

I. Newton, L. Euler

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▷ e.g., classical mechanics of point particles

$$m\,\mathbf{a} = \mathbf{F}$$

I. Newton, L. Euler

BUT: no intrinsic knowledge about validity regimes,

e.g., $|\mathbf{v}| \ll c$, $S \gg \hbar$

A “superpower” of quantum field theories

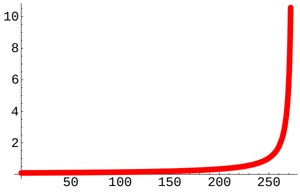
▷ e.g., QED: perturbation theory predicts its own failure (LANDAU’55)

▷ β function:

$$\beta_{e^2} = \partial_t e^2 = +\beta_0 e^4$$

▷ integration:

$$\frac{1}{e_R^2} - \frac{1}{e_\Lambda^2} = \beta_0 \ln \frac{\Lambda}{m_{\mathrm{R}}}, \qquad \beta_0 = \frac{N_{\mathrm{f}}}{6\pi^2}$$



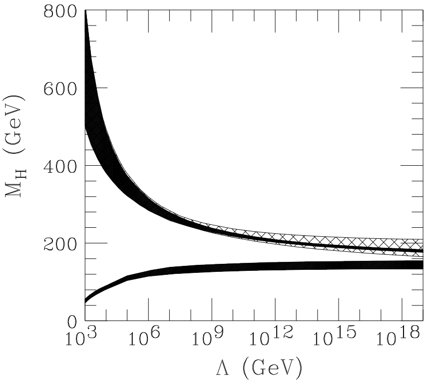
▷ e_R^2 and m_R fixed: $\Longrightarrow \Lambda \to \mu_L \simeq m_R \exp\left(\frac{1}{\beta_0\, e_R^2}\right) \simeq 10^{272}\mathrm{GeV}$ (2 loop)

Landau pole singularity

$\Longrightarrow$ QED: world’s best tested theory & maximum validity scale

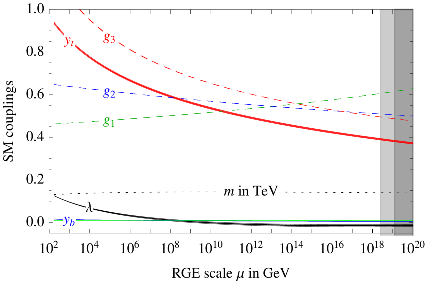
A purely academic “superpower” ...?

▷ Higgs mass bounds in the Standard Model from Landau pole position



(Hambye,Riesselmann'97)

▷ Landau Pole in Standard Model



$$\mu_{\text{L, SM}} \simeq 10^{40}\ \text{GeV}$$

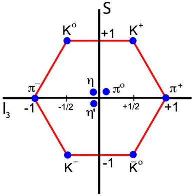
▷ BSM:

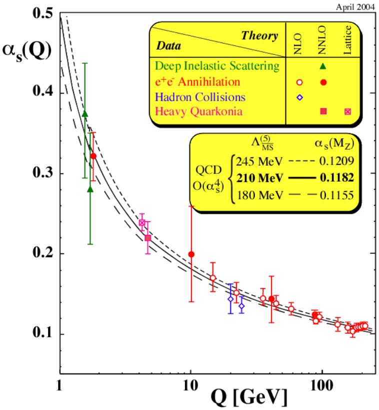
$$\mu_{\text{L, MSSM}} \simeq 10^{20}\ \text{GeV}$$

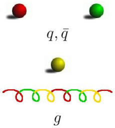
$$\mu_{\text{L, MSSM+4H}} \simeq 10^{17}\ \text{GeV}$$

A Truly Perfect Theory

▷ e.g., Quantum Chromodynamics







(Gross,Wilczek'73;Politzer'73)

$\Longrightarrow$ predicts the long-range/low-energy physics & could be valid on any scale

Classification of (perturbatively) renormalizable QFTs

▷ assumptions: Lorentz invariance, locality, unitarity, (perturbative) renormalizability ($\hat{=}$ predictivity) + implicit assumptions $D = 3 + 1$

spin	S_{int}	particle physics	UV complete?
0	ϕ^4	Higgs scattering	
1/2	$\bar{\psi}\psi\phi$	fermion-Higgs interactions	
1/2	$\bar{\psi}\gamma_\mu A_\mu \psi$	gauge interactions	
1	$f^{abc} A^a_\mu A^b_\nu \partial_\mu A^c_\nu$ $f^{abc} f^{ade} A^a_\mu A^b_\nu A^d_\mu A^e_\nu$	gluon self-interactions	
3/2	–	–	
2	–	graviton ?	

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3/2	–	–	✗
2	–	graviton ?	✗

(Implicit) Assumptions

▷ (perturbative) renormalizability

beyond: asymptotic safety

▷ (conventional) canonical scaling

beyond: higher-derivative theories

▷ (simple) boundary conditions for generating functionals

beyond: global potential flows, general b.c.'s, "asymptotic flatness"

Asymptotic Safety

A U(1) example: the Pauli-QED universality class

Necessity of Renormalizability

- IR physics well separated from UV physics

(…no/mild cutoff Λ dependence)

- # of physical parameters $\Delta < \infty$ …or countably ∞

(…predictive power)

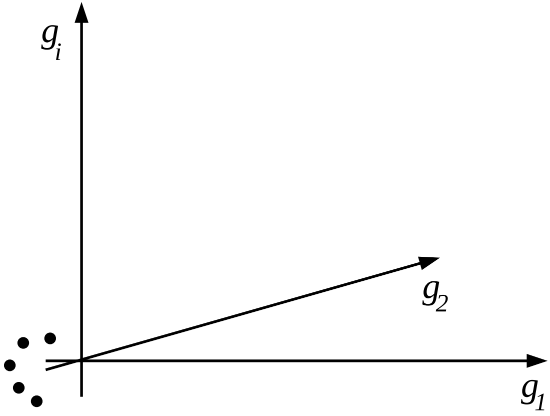
$\Longrightarrow$ realized by perturbative RG …

$\Longrightarrow$ …and by "Asymptotic Safety"

(Weinberg'76)

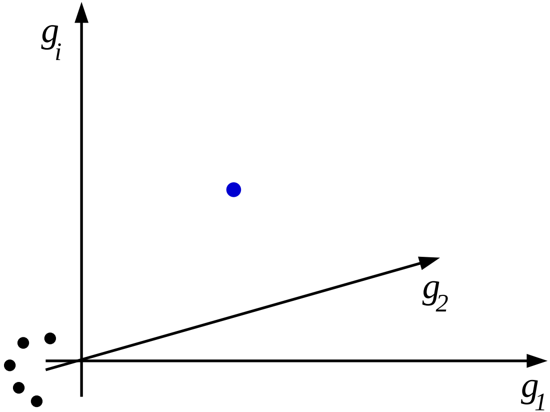
(Gell-Mann, Low'54)

Asymptotic Safety



"Theory Space"

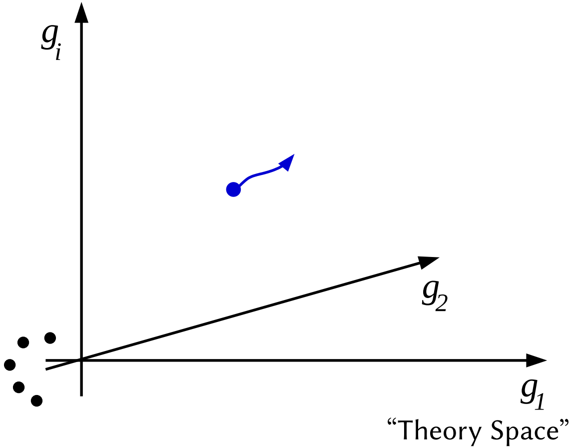
Asymptotic Safety



"Theory Space"

effective action Γ_k

Asymptotic Safety



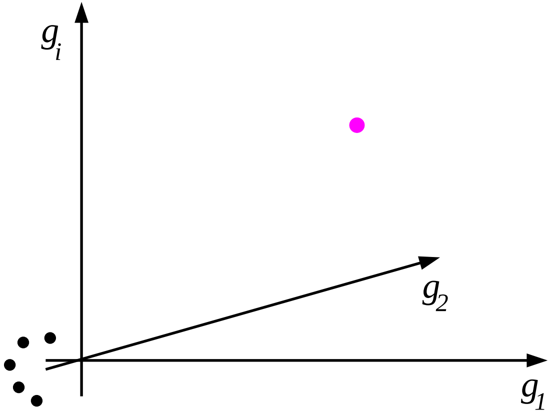
RG step

functional RG: (Wetterich'93)

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

(Wilson'71; Wegner,Houghton'73; Polchinski'84)

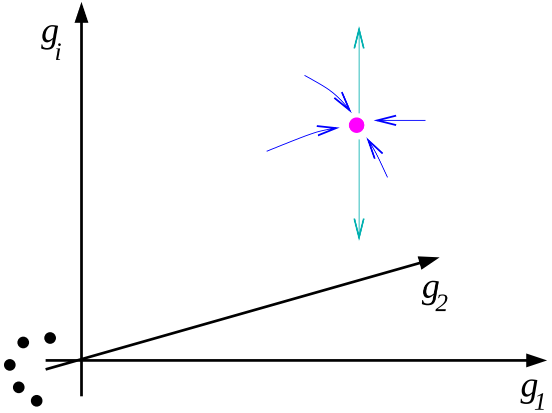
Asymptotic Safety



"Theory Space"

Fixed Point: g_*

Asymptotic Safety



"Theory Space"

linearized RG flow:

$$\partial_t g = -\Theta(g - g_*) + \ldots$$

UV repulsive, "irrelevant"

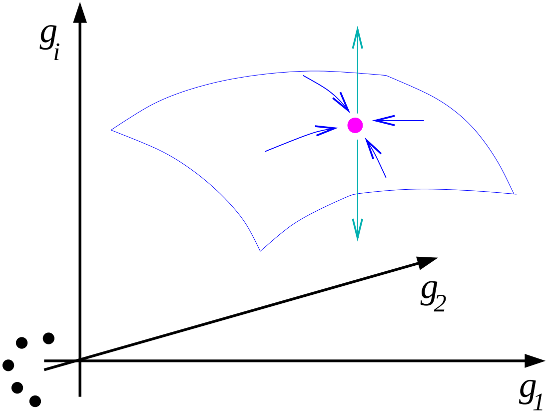
$\Theta < 0$

UV attractive, "relevant"

$\Theta > 0$

Asymptotic Safety

critical surface $\mathcal{S}$



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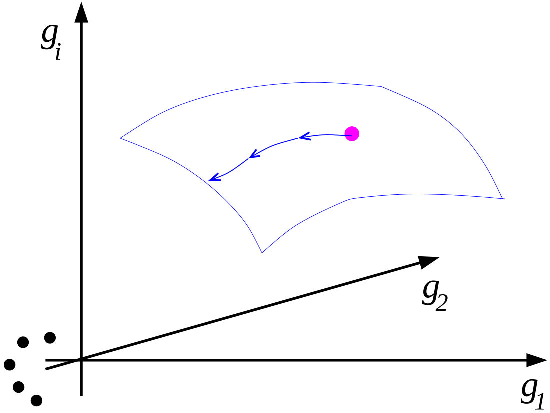
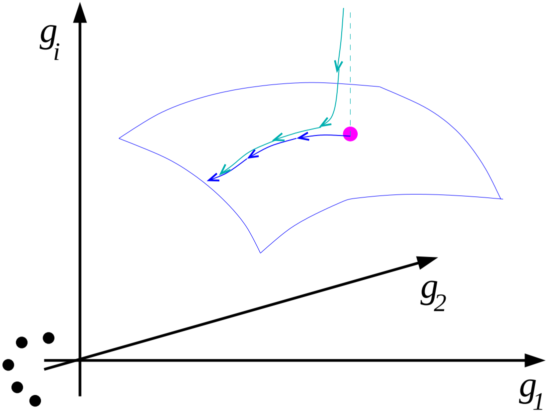
Asymptotic Safety



"Theory Space"

$\Delta = \dim \mathcal{S}$ = # Θ's>0

Asymptotic Safety



"Theory Space"

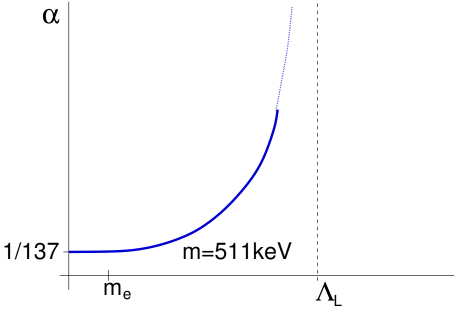
(Λ independence ✓)

(# phys. parameters $< \infty$ ✓)

(universality & predictivity ✓)

Strong QED?

- Could asymptotic safety make QED UV complete?



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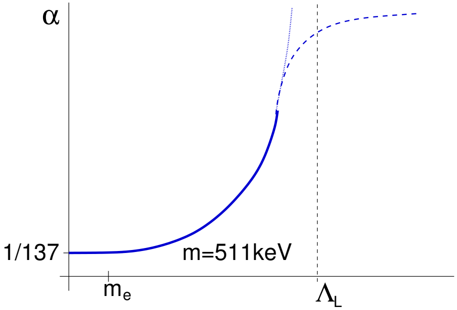
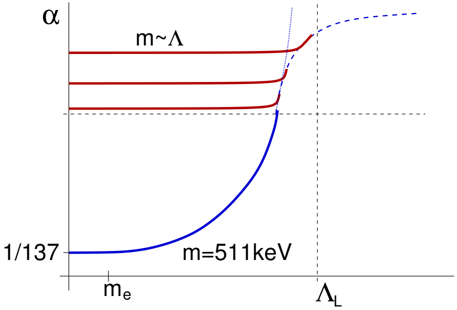


▷ Lattice+FRG: no UV fixed point in pure QED, but even if ...

Strong QED?

▷ Could asymptotic safety make QED UV complete?



▷ χSB leads to heavy fermions $m \sim \Lambda$ if $\alpha > \alpha_{\rm cr} \simeq 3.06$

$\Longrightarrow$ Strong QED and "real" QED not on a line of constant physics

(Gockeler et al.'98; HG,Jaeckel'04)

Other universality classes?

▷ observation in EFT: (Djukanovic,Gegelia,Meißner'18)

$$\Gamma \sim \int \bar{\kappa}\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi$$

$\Longrightarrow$ finite Pauli coupling $\bar{\kappa}$ can screen the Landau pole

Can Pauli coupling be UV stabilized?

New universal class including finite Pauli coupling?

Status of chiral symmetry?

QED RG flow with Pauli term

▷ effective action to lowest order derivative expansion (HG,Ziebell'20)

$$\Gamma_k = \int_x \left[\bar{\psi}\left(iZ_\psi \partial\!\!\!/ + \bar{e} A\!\!\!/ - i\bar{m} + i\bar{\kappa}\sigma_{\mu\nu}F^{\mu\nu}\right)\psi + \frac{1}{4}Z_A F_{\mu\nu}F^{\mu\nu}\right]$$

▷ Pauli coupling in QED:

unique lowest-derivative dimension-5 operator

QED RG flow with Pauli term

▷ effective action to lowest order derivative expansion (HG,ZIEBELL'20)

$$\Gamma_k = \int_x \left[\bar{\psi} \left(i Z_\psi \partial\!\!\!/ + \bar{e} A\!\!\!/ - i \bar{m} + i \bar{\kappa} \sigma_{\mu\nu} F^{\mu\nu} \right) \psi + \frac{1}{4} Z_A F_{\mu\nu} F^{\mu\nu} \right]$$

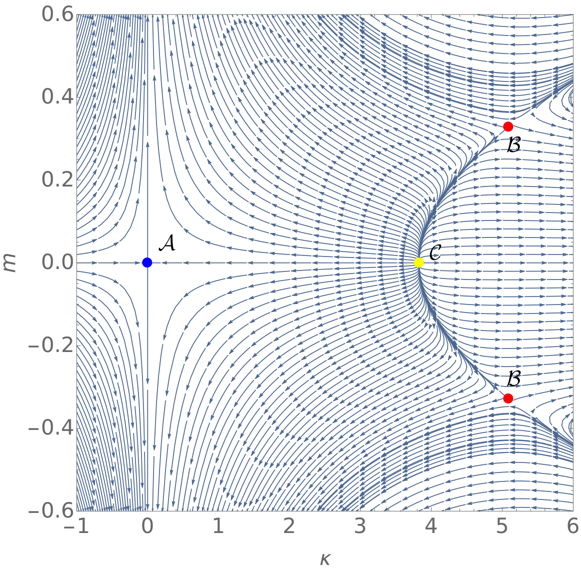
▷ RG flow for all dim'less couplings $e, m \sim \frac{\bar{m}}{k}, \kappa \sim \bar{\kappa} k, Z_\psi, Z_A$

e.g.

$$\begin{aligned} \partial_t e \;=\; & e\left(\frac{d}{2} - 2 + \eta_\psi + \frac{\eta_A}{2}\right) - 4 v_d \frac{(d-4)(d-1)}{d} e^3\, l_d^{(1,\mathrm{B},\tilde{\mathrm{F}}^2)}(0, m^2) \\ & - 16 v_d \frac{(d-2)(d-1)}{d} e\kappa^2\, l_d^{(2,\mathrm{B},\tilde{\mathrm{F}}^2)}(0, m^2) - 16 v_d \frac{(d-4)(d-1)}{d} e\kappa^2 m^2\, l_d^{(2,\mathrm{B},\mathrm{F}^2)}(0, m^2) \\ & - 32 v_d \frac{d-1}{d} e^2 \kappa m\, l_d^{(1,\mathrm{B},\mathrm{F},\tilde{\mathrm{F}})}(0, m^2, m^2) - 4 v_d \frac{(d-2)(d-1)}{d} e^3 m^2\, l_d^{(\mathrm{B},\mathrm{F}^2)}(0, m^2) \end{aligned}$$

▷ similarly for $\partial_t \kappa = \ldots, \partial_t m = \ldots, \eta_A = \ldots, \eta_\psi = \ldots$

Phase diagram of QED



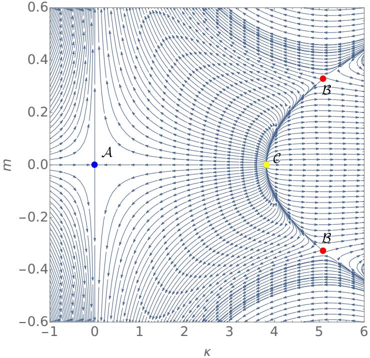
Universality classes of QED

▷ fixed points:

$\mathcal{A}$: Gaussian fixed point

$\mathcal{C}$: non-Gaussian "Pauli" fixed point
(deep Euclidean region)

$\mathcal{B}$: non-Gaussian "non-chiral" fixed point
(no asymptotic symmetry!)



	e	κ	m	multiplicity	Δ	Θ_{max}	η_ψ	η_A
$\mathcal{A}$:	0	0	0	–	1	1.00	0.00	0.00
$\mathcal{B}$:	0	5.09	0.328	$\mathbb{Z}_2 \times \mathbb{Z}_2$	2	3.10	−1.38	0.53
$\mathcal{C}$:	0	3.82	0	$\mathbb{Z}_2$	3	2.25	−1.00	0.00

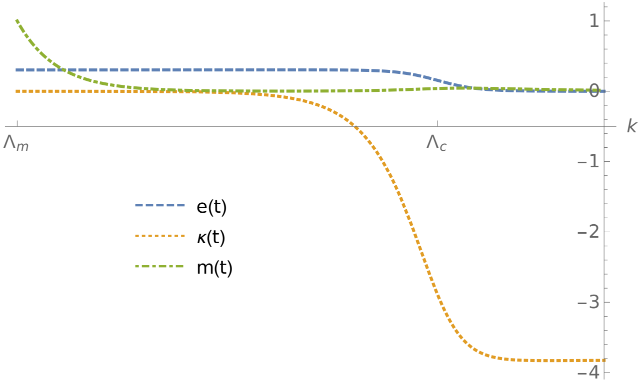
“Pauli” universality class $\mathcal{C}$

▷ $\Delta = 3$ physical parameters

also a_e needs to be tuned

▷ Do physical RG trajectories (compatible with Nature) exist?

Physical trajectories in Pauli universality class?



$\Longrightarrow$ Yes!

$$\text{for } k \to 0: \qquad \alpha \simeq \frac{1}{137}, \quad m_e \simeq 511\,\text{keV}, \quad a_e \simeq \frac{\alpha}{2\pi}$$

Check: dim-6 operators irrelevant (HG,Tam'23)

Crossover scale in Pauli universality class

▷ crossover scale:

scale where RG flow deviates from perturbative flow

$$\frac{\Lambda_\mathrm{c}}{m_e} \simeq 4.6 \times 10^5, \qquad \Lambda_c \simeq 23.7\,\mathrm{GeV}$$

$\sim$ scale where electroweak contributions start to dominate

embedding into standard model?

Pauli coupling $\to$ dimension-6 operator

Higher derivative theories

Relativistic Luttinger fermions and asymptotic freedom

An inspiration from solid state physics

- ▷ Luttinger'56:

  Quantum theory of cyclotron resonance in semiconductors: general theory

"The most general form of the Hamiltonian …"

$$H\psi = G_{ij}\left(\partial_i - ieA_i\right)\left(\partial_j - ieA_j\right)\psi = E\psi$$

generalization of covariant Laplacian to symmetric inter-band couplings

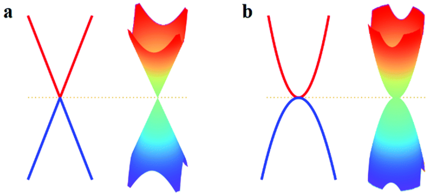
Luttinger fermions in nonrelativistic field theory

▷ Effective field theory of low energy degrees of freedom near quadratic band touching/crossing points (QBT/QBC): (Abrikosov,Beneslavskii'71) (Janssen,Herbut'15)

$$\mathcal{L} = \psi^\dagger \left(\partial_\tau - G_{ij}\, \partial_i \partial_j\right) \psi + \mathcal{L}_{\text{int}}$$

▷ For $(-G_{ij}\, \partial_i \partial_j)^2 = (-\partial^2)^2$, the G_{ij} satisfy a Clifford algebra (Abrikosov'74)

$$\{G_{ij}, G_{kl}\} = -\frac{2}{d-1}\delta_{ij}\delta_{kl} + \frac{d}{d-1}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$$



[figure: doi.org/10.1039/C8TC04813D]

cond-mat applications:
- $j = \frac{3}{2}$ Luttinger semi-metals (Luttinger'56)
- nematic quantum criticality (Janssen,Herbut'15)
- Bernal-stacked honeycomb bilayers (Ray,Vojta,Janssen'18)
- gapless spin liquids (Dey,Maciejko'22)

Dirac vs. Luttinger fermions

Dirac algebra:

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij}\mathbb{1}$$

Luttinger algebra:

$$\{G_{ij}, G_{kl}\} = \left[-\frac{2}{d-1}\delta_{ij}\delta_{kl} + \frac{d}{d-1}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)\right]\mathbb{1}$$

▷ A simple example in (Euclidean) 2d:

Dirac operator:

$$\partial\!\!\!/ = \gamma_i \partial_i$$

representation $\gamma_1 = \sigma_1, \gamma_2 = \sigma_3$:

$$\partial\!\!\!/ = \begin{pmatrix} \partial_2 & \partial_1 \\ \partial_1 & -\partial_2 \end{pmatrix}$$

Luttinger operator:

$$G_{ij}\partial_i\partial_j$$

representation $G_{12} = G_{21} = \sigma_1$, $G_{11} = -G_{22} = \sigma_3$:

$$G_{ij}\partial_i\partial_j = \begin{pmatrix} \partial_1^2 - \partial_2^2 & \partial_1\partial_2 \\ \partial_1\partial_2 & -\partial_1^2 + \partial_2^2 \end{pmatrix}$$

Relativistic Luttinger fermions?

(HG, Heinzel, Laufkötter, Piccau to appear)

- Clifford algebra

$$\{G_{\mu\nu}, G_{\kappa\lambda}\} = -\frac{2}{d-1} g_{\mu\nu} g_{\kappa\lambda} + \frac{d}{d-1}\left(g_{\mu\kappa} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\kappa}\right), \quad g = \mathrm{diag}(+,-,-,-,\dots)$$

- kinetic action:

$$S = -\int d^d x\, \bar{\psi}\, G_{\mu\nu} \partial^\mu \partial^\nu\, \psi$$

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▷ kinetic action:

$$S = -\int d^d x\, \bar\psi\, G_{\mu\nu} \partial^\mu \partial^\nu\, \psi$$

▷ BUT what is $\bar\psi$?

$$\Big(\text{Dirac:} \quad \bar\psi = \psi^\dagger \gamma_0\Big)$$

▷ Also: cond-mat generalization to $d = 4$ Euclidean dimensions: (Janssen,Herbut'15)

time reversal: $\quad T^2 = +1 \quad$ spinless fermions

Properties of the Luttinger algebra

- ▷ $G_{\mu\nu}$: representation in terms of $d_\gamma \times d_\gamma$ matrices for each fixed $\mu, \nu = 0, 1, 2, \ldots, d-1$
- ▷ $G_{\mu\nu} = G_{\nu\mu}$ symmetric in Lorentz indices
- ▷ Lorentz-traceless: $\mathrm{tr}_\mathrm{L}\, G_{\mu\nu} \equiv G^\mu{}_\mu = 0$

…excludes the trivial Klein-Gordon part $\sim \partial^2$

$\Longrightarrow$ number of independent elements d_e:

d	2	3	4	5	6
d_e	2	5	9	14	20
d_γ	2	4	16	128	1024

$\Big($Dirac: $d_\mathrm{e} = d\Big)$

Construction of the conjugate spinor

▷ Unitary time evolution requires a real action, suggesting:

$$\bar{\psi} = \psi^\dagger\, h$$

▷ Spin metric h acts as a “hermitizer”, i.e. action is spin-base invariant (Schrödinger’32;Bargman’32)

$$\psi \to \mathcal{S}\psi, \quad G_{\mu\nu} \to \mathcal{S} G_{\mu\nu} \mathcal{S}^{-1}, \quad \mathcal{S} \in \mathrm{SL}(d_\gamma, \mathbb{C}),$$

provided that spin metric transforms as (Weldon’00; HG,Lippoldt’13)

$$h \to (\mathcal{S}^\dagger)^{-1}\, h\, \mathcal{S}^{-1}$$

$\Longrightarrow$ for a given set of $G_{\mu\nu}$’s, spin metric must be constructed from the Clifford algebra

$\Big($Dirac: $\quad h = \gamma_0\Big)$

Construction of the conjugate spinor

▷ action $\in \mathbb{R}$ (unitarity) requires:

$$\{h, G_{0i}\} = 0, \quad [h, G_{ij}] = 0, \quad [h, G_{\underline{\mu\mu}}] = 0$$

$\Longrightarrow$ no solution in irreducible $d_\gamma = 16$ dimensional representation!

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▷ BUT: spin metric exists in $d_\gamma = 32$ dimensional representation, e.g.

$$h = \gamma_1\gamma_2\gamma_3\gamma_{10}$$

$\Longrightarrow$ action is Lorentz invariant as

$$\mathcal{S}_{\text{Lor}} \in \text{SO}(1,3) \subset \text{SL}(32, \mathbb{C})$$

$\mathcal{S}_{\text{Lor}}$ together with $\Lambda^{\mu}{}_{\nu}$ maps $G_{\mu\nu} \to G_{\mu\nu}$

Free theory of relativistic Luttinger fermions

▷ action

$$S = \int d^4x \left(\bar{\psi}\, G_{\mu\nu}\, (i\partial^\mu)(i\partial^\nu)\, \psi - m^2\, \bar{\psi}\psi \right)$$

$\Longrightarrow$ Lorentz invariant and spin-base $\mathrm{SL}(32, \mathbb{C})$ invariant

▷ canonical mass dimension

$$[\psi] = 1$$

$\sim$ like a scalar field in $d = 3 + 1$

$\Longrightarrow$ $\sim (\bar{\psi}\psi)^2$-type self-interactions can be marginal

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▷ NOTE: there are $\binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \cdots = 1024$ independent bilinears

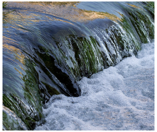
only 16 for Dirac fermions

Example: 4d self-interacting Luttinger fermionic model

▷ $\sim$ “Gross-Neveu-Thirring”-type model:

$$S[\psi,\bar\psi] = \int d^4x \left[\bar\psi G_{\mu\nu}(i\partial^\mu)(i\partial^\nu)\psi + \frac{\bar\lambda_0}{2}(\bar\psi\psi)^2 + \frac{\bar\lambda_t}{2}(\bar\psi G_{\mu\nu}\psi)^2\right]$$

▷ renormalization flow: (Wetterich'93)



$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)} + R_k}\,\partial_t R_k$$

▷ renormalized coupling flow (point-like truncation), e.g., for $N_f = 1$

$$\partial_t\lambda_0 = -\frac{1}{8\pi^2}\left[30\,\lambda_0^2 - \frac{88}{3}\,\lambda_0\,\lambda_t\right]$$

$$\partial_t\lambda_t = -\frac{1}{8\pi^2}\left[-\frac{486}{9}\lambda_t^2 + \frac{16}{9}\,\lambda_0\,\lambda_t - \frac{1}{6}\lambda_0^2\right]$$

Example: 4d self-interacting Luttinger fermionic model

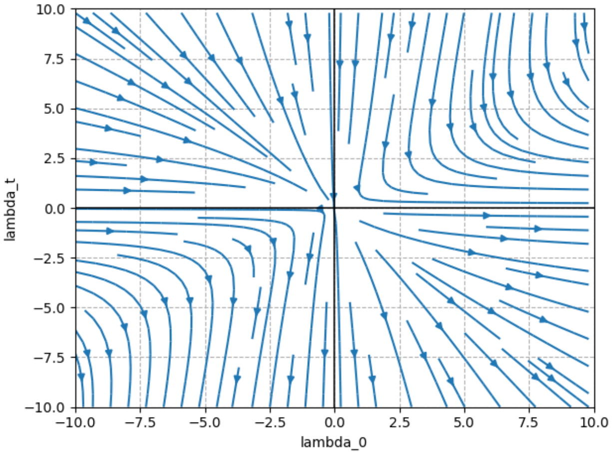
▷ $N_\mathrm{f} = 1$ phase diagram of the model:
$(\eta_\psi \sim \lambda_i^2$, LPA: $\to 0)$

$\Longrightarrow$ Gaußian fixed point:
attractive & repulsive

$\Longrightarrow$ asymptotically free for

$$\lambda_0 > 0, \quad \lambda_t < 0$$

$\Longrightarrow$ UV complete ✓
& long-range interacting



Example: 4d self-interacting Luttinger fermionic model

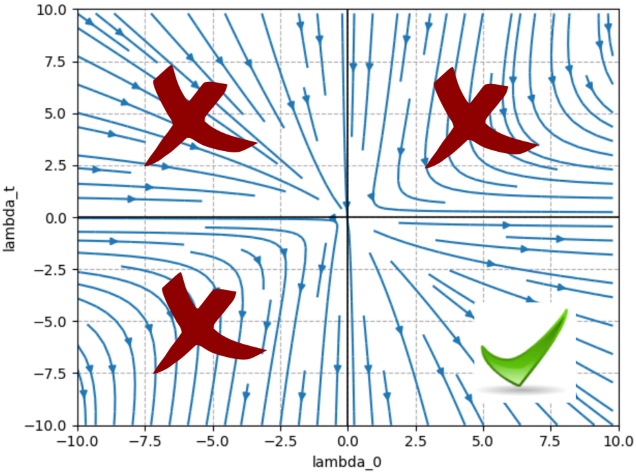
▷ $N_\mathrm{f} = 1$ phase diagram of the model:
$(\eta_\psi \sim \lambda_i^2$, LPA: $\to 0)$

$\Longrightarrow$ Gaußian fixed point:
attractive & repulsive

$\Longrightarrow$ asymptotically free for

$$\lambda_0 > 0, \quad \lambda_t < 0$$

$\Longrightarrow$ UV complete ✓
& long-range interacting



Example: 4d self-interacting Luttinger fermionic model

- ▷ initial condition

$$\tan\alpha = \frac{\lambda_0}{\lambda_t}$$

- ▷ channel competition

scalar vs. tensor channel

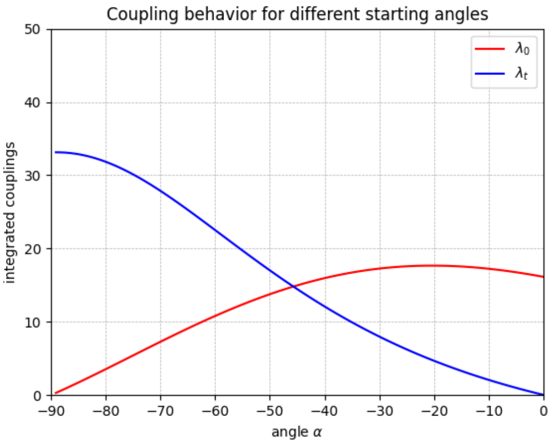
- ▷ quantum phase transition ?

$$\alpha_{\text{cr}} \simeq -46^\circ$$

- ▷ long-range order?

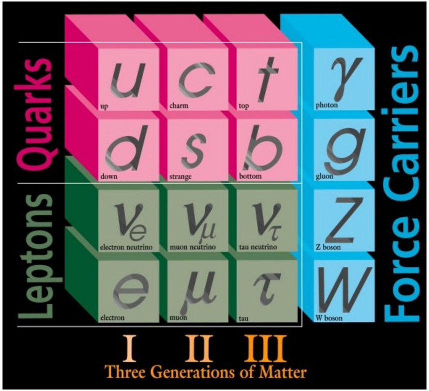
$\Longrightarrow$ Luttinger gauge theories

(Talk: M. Picciau)



Luttinger numerology

- ▷ Observation relativistic QFT with Luttinger fermions requires a 32-component spinor



- ▷ Dirac fermions in each family:

u-type :	3	(color)
d-type :	3	(color)
e-like :	1	
ν :	1	(incl. right-handed)
sum:	8	Dirac fermions

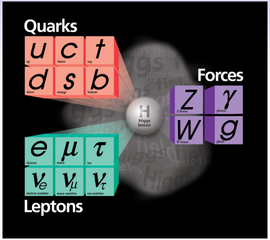
$\Longrightarrow$ 32 components

General boundary conditions, “Asymptotic Flatness”

Paving the way to asymptotically free gauge-Higgs models

Gauge Yukawa models

▷ backbone of particle physics:



UV behavior of central interest

▷ perturbative RG analysis

- deep Euclidean region (DER)

$$\frac{m^2, p_i^2}{\mu^2} \ll 1$$

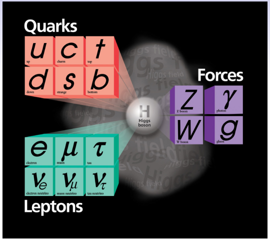
- no threshold phenomena
- "asymptotic symmetry" (Lee,Weisberger'74)
- only marginal operators

asymptotic freedom: (Gross,Wilczek'73; Cheng,Eichten,Lee'74; Chang'74;...; Guidice,Isidori,Salvio,Strumia'14,...)

asymptotic safety: (Litim,Sannino'14; Bond,Litim'16; Bond,Litim,MedinaVazquez,Steudtner'18; Held'20; ...)

Gauge Yukawa models

▷ backbone of particle physics:



UV behavior of central interest

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- only marginal operators

▷ functional RG analysis

- beyond DER
- threshold phenomena included
- any symmetry status
- functional flow: $U(\phi)$

asymptotic freedom: (Gross,Wilczek'73; Cheng,Eichten,Lee'74; Chang'74;...; Guidice,Isidori,Salvio,Strumia'14,...)

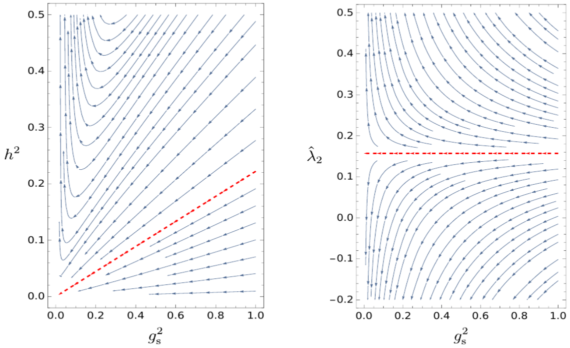
asymptotic safety: (Litim,Sannino'14; Bond,Litim'16; Bond,Litim,MedinaVazquez,Steudtner'18; Held'20; ...)

Example: $\mathbb{Z}_2$-Yukawa-QCD model

$$S = \int_x \frac{1}{4} F^i_{\mu\nu} F^i_{\mu\nu} + i\bar{\psi}\not{D}\psi + \frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{ih}{\sqrt{2}}\phi\bar{\psi}\psi + \frac{m^2}{2}\phi^2 + \frac{\lambda}{8}\phi^4$$

▷ perturbative RG flow:

(Cheng,Eichten,Lee'74)



asymptotic freedom through "gauge trapping":

$$\left.\begin{array}{l} h^2 \sim g_s^2 \\ \lambda \sim g_s^2 \end{array}\right\} \quad \text{for } g_s \to 0$$

$\Longrightarrow$ enhanced predictive power:

for fixed $\Lambda_{\rm QCD}, v_{\rm Fermi}$: $m_{\rm top}, m_{\rm Higgs}$ is a prediction (Pendleton,Ross'81)

Functional RG perspective

▷ generalize "gauge trapping" to "quasi fixed points" (QFP): (HG, Zambelli'15,'16)

$$\left.\begin{array}{l} h^2/g_s^2 \\ \lambda/g_s^2 \end{array}\right\} \overset{UV}{\to} \text{const.}, \quad x := \frac{1}{2}h\phi^2 \sim g_s\phi^2, \quad f(x) := \frac{U(\phi)}{k^4}$$

Functional RG perspective

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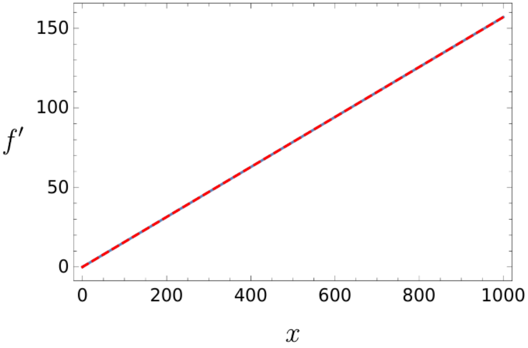
▷ global QFP solution (HG,Sondenheimer,Ugolotti,Zambelli'18)

$$\partial_t f(x) = 0$$



pseudo-spectral solver

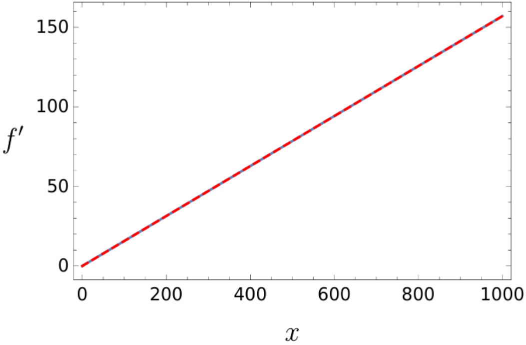
(Borchardt,Knorr'15'16)

Functional RG perspective

▷ global QFP solution

(HG, Sondenheimer, Ugolotti, Zambelli'18)

$$\partial_t f(x) = 0$$



pseudo-spectral solver

(Borchardt, Knorr'15'16)

▷ global analytic approximation:

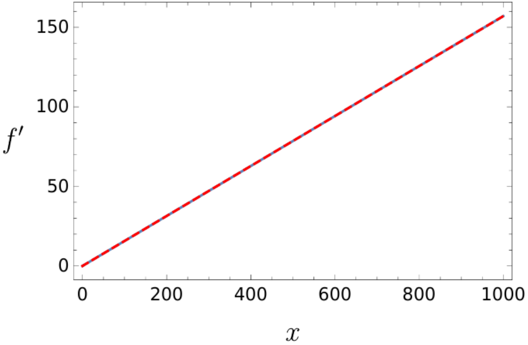
$$f(x) = \frac{1}{128\pi^2}\, {}_2F_1\left(1, -\tfrac{4}{2+\eta_x}, \tfrac{-2+\eta_x}{2+\eta_x}, -3\xi_2 h x\right) - \frac{3}{32\pi^2}\, {}_2F_1\left(1, -\tfrac{4}{2+\eta_x}, \tfrac{-2+\eta_x}{2+\eta_x}, -hx\right) + C_{\mathrm{f}}\, x^{\frac{4}{2+\eta_x}},$$

Functional RG perspective

▷ global QFP solution

(HG,Sondenheimer,Ugolotti,Zambelli'18)

$\partial_t f(x) = 0$



pseudo-spectral solver

(Borchardt,Knorr'15'16)

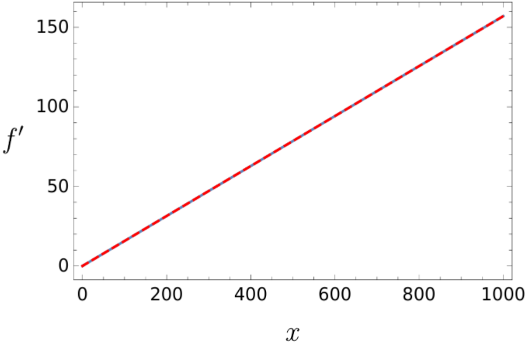
$\Longrightarrow$ Cheng-Eichten-Lee solution:

UV complete ✓ exists globally ✓ global stability ✓

Functional RG perspective

▷ global QFP solution (HG,Sondenheimer,Ugolotti,Zambelli'18)

$\partial_t f(x) = 0$



pseudo-spectral solver

(Borchardt,Knorr'15'16)

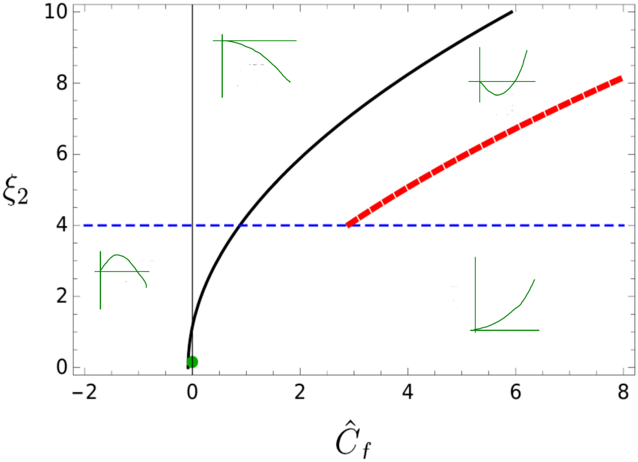
▷ functional RG: 2nd order PDE requires boundary conditions!

$$f(x) = \frac{1}{128\pi^2}\,{}_2F_1\left(1,-\tfrac{4}{2+\eta x},\tfrac{-2+\eta x}{2+\eta x},-3\xi_2 hx\right) - \frac{3}{32\pi^2}\,{}_2F_1\left(1,-\tfrac{4}{2+\eta x},\tfrac{-2+\eta x}{2+\eta x},-hx\right) + C_\mathrm{f}\, x^{\frac{4}{2+\eta x}},$$

New asymptotically free scaling solutions in $\mathbb{Z}_2$-Yukawa-QCD

(HG,Sondenheimer,Ugolotti,Zambelli'18)

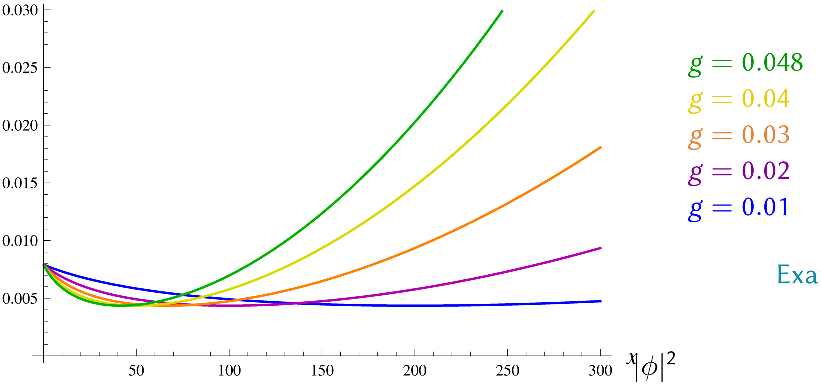
- UV complete ✓
- global existence ✓
- global stability ✓
- non-singular correlation functions above vacuum ✓



$\Longrightarrow$ cf. new scaling solution in nonlinear electrodynamics (Talk: J. Schirrmeister)

Asymptotic Flatness

▷ UV scaling towards asymptotic freedom (HG,ZAMBELLI'15'16)

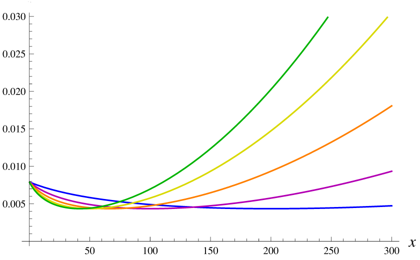


Example: non-abelian Higgs

Asymptotic Flatness

▷ UV scaling towards asymptotic freedom

(HG,Zambelli'15'16)



$|\phi|^2$

$g = 0.048$
$g = 0.04$
$g = 0.03$
$g = 0.02$
$g = 0.01$

Example: non-abelian Higgs

▷ generic approach to UV $k \to \infty$:

$$g^2 \to 0, \quad \underline{\underline{|\phi_{\rm min}|^2 \sim \frac{1}{g^{2P}} \to \infty}}, \quad \lambda \sim g^{4P} \to 0, \quad \frac{m_k^2}{k^2} \to \text{const.}$$

$\Longrightarrow$ deep Euclidean region is sidestepped

…no "asymptotic symmetry"

cf. (Pastor-Gutiérrez,Pawlowski,Reichert'22)

Conclusion

- QFT "superpower":

  prediction of maximum validity scale Λ

- For $\Lambda \to \infty$: QFT can be "perfect"

  classification of UV complete theories

- pathways to UV complete QFTs

  asymptotic freedom/safety ✓

  higher derivative theories: e.g. Luttinger fermions ✓

  boundary conditions & asymptotic flatness ✓

- UV complete theories = consistent theories

  consistency: relevant criterion (in absence of data)