Simulation results for a CPW transmission line with $L_k = 8.5$ pH/sq

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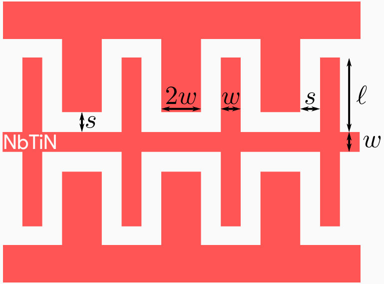
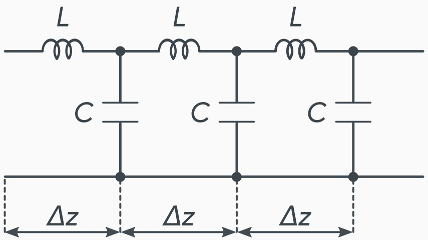


- NbTiN kinetic inductance: $L_k = 8.5$ pH/sq;
- NbTiN transmission line thickness: $t = 13$ nm;
- Line and stub width: $w = 1\,\mu$m;
- Ground stub width: $2w = 2\,\mu$m;
- Elementary cell length: $L_{\text{cell}} = 2s + w + 2w = 5\,\mu$m
- Spacing between line and ground: $s = 1\,\mu$m;
- Silicon substrate thickness: $S = 525\,\mu$m;
- Silicon substrate dielectric constant: $\varepsilon_r = 11.9$;
- Stub length in the $\ell \in [5, 120]\,\mu$m range with step of $5\,\mu$m;



$$\text{artificial line model} \xrightarrow[\Delta z \to 0]{} \text{distributed line model}$$

$$\text{artificial line model} \underset{\Delta z < \lambda_{min}/10}{\Longleftrightarrow} \text{distributed line model}$$

A physical approximation to the distributed model circuit is a circuit consisting of lumped inductances C and capacitances L;

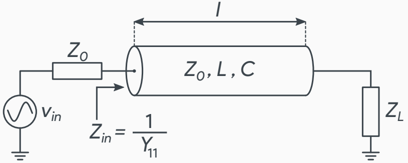
λ_{min}: smallest signal wavelength of interest

- An artificial transmission line is made up of lumped capacitance C, inductance L, resistance R, and conductance G per unit length;
- Lumped elements at RF and microwave frequencies are designed based on small sections of Transverse Electro-Magnetic (TEM) lines, such as microstrip lines, which are much smaller than the operating wavelength;
- In a lossless line ($R \to 0$ and $G \to \infty$) only capacitors and inductors give a contribution;

  Line characteristic impedance: $Z_0 = \sqrt{\frac{L}{C}}$;

  Line cut-off frequency: $\omega_c = \frac{2}{\sqrt{LC}}$;

  Phase velocity: $v_p = \frac{\Delta z}{\sqrt{LC}}$;
- L and C are the characteristic inductance and capacitance at zero frequency ($\omega \to 0$);



$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \underset{\text{loss-less line: } \alpha = 0}{\overset{=}{\uparrow}} Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

$\gamma = \alpha + j\beta = j\beta$

γ: propagation constant, α: attenuation constant, $\beta = \omega\sqrt{LC}$: phase constant

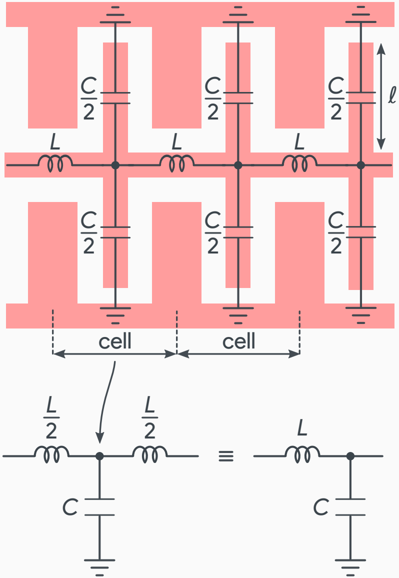
Open circuit line: $Z_L \to \infty$

$$Z_{in} = Z_0 \frac{Z_L}{jZ_L \tan(\beta l)} = jZ_0 \cot(\beta l) = j\sqrt{\frac{L}{C}} \cot\left(\omega l\sqrt{LC}\right) \underset{\omega \to 0}{\overset{\simeq}{\uparrow}} -\frac{j}{\omega l}\sqrt{\frac{L}{C}\frac{1}{LC}} = -\frac{j}{\omega l}\frac{1}{C} \Rightarrow \boxed{C = -\frac{1}{\omega l \,\mathrm{Im}\left(\frac{1}{Y_{11}}\right)}}$$

Short circuit line: $Z_L \to 0$

$$Z_{in} = Z_0 \frac{Z_0}{jZ_0 \tan(\beta l)} = jZ_0 \tan(\beta l) = j\sqrt{\frac{L}{C}} \tan\left(\omega l\sqrt{LC}\right) \underset{\omega \to 0}{\overset{\simeq}{\uparrow}} j\omega l\sqrt{\frac{L}{C}}LC = -j\omega l L \Rightarrow \boxed{L = \frac{1}{\omega l}\mathrm{Im}\left(\frac{1}{Y_{11}}\right)}$$

By simulating the line with $Z_L \to \infty$ and $Z_L \to 0$ is possibile to extrapolated L and C from Y_{11} (and S_{11})



- A stub is a length of transmission line that is connected at one end only;
- Stub-loaded coplanar waveguide (CPW) transmission line $\Rightarrow$ series of CPW open stubs are connected to the transmission line to control the characteristic impedance;
- Each stub approximates a shunt capacitance which value depends on the stub length ℓ;
- In this configuration each elementary cell composed by series inductance L flanked by two interdigitated capacitor (IDC) stubs that form the capacitance to ground C and such that $Z_0 = \sqrt{L_d/C}$;
- By varying the length ℓ of the line stubs it is possible to change the value of the capacitance per unit length (or per cell) C and, consequently, the effective line characteristic impedance Z_0;
- Goal for simulations: varying ℓ in order to find the better L, C combination for obtaining the desired Z_0;

Sonnet allows to extrapolate C and L by its Capacitance1 and Inductance1 parameters

Definition

Capacitance1 is the capacitance of a one-port or two-port circuit assuming a series RC

$\Rightarrow$ Effective Capacitance of a series RC network: sonnetsoftware.com/support/help-18/Sonnet_Suites/Capacitance1.html

Inductance1 is the inductance of a one-port or two-port circuit assuming a series RL

$\Rightarrow$ Effective Inductance of a series RL network: sonnetsoftware.com/support/help-18/Sonnet_Suites/Inductance1.html

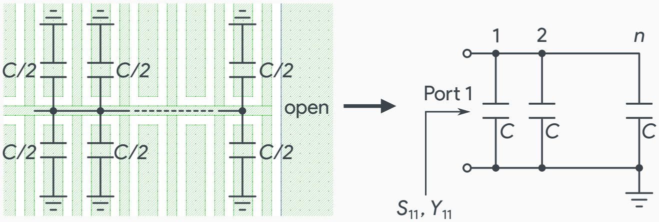
- Sonnet computes Capacitance1 and Inductance1 (but also Capacitance2 and Inductance2) and shows the related plots;
- Sonnet does not allow to save these parameters on file but allows to save S, Y, Z-Parameters
- From the S and Y parameters it is possible to compute C and L:

$$Y_{11} = \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}} \cdot \frac{1}{Z_0} \quad \Rightarrow \quad \text{1-Port measurement} \quad \Rightarrow \quad Y_{11} = \frac{1-S_{11}}{1+S_{11}} \cdot \frac{1}{Z_0}$$

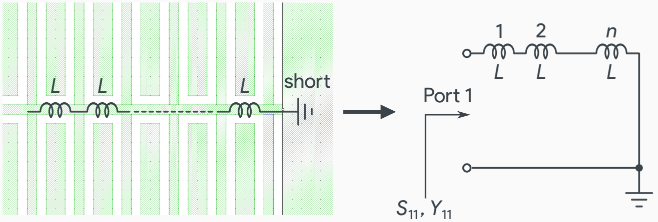
and then $\Rightarrow$

$$\boxed{C = -\frac{1}{\omega \, \mathrm{Im}\left(\frac{1}{Y_{11}}\right)}} \quad , \quad \boxed{L = \frac{1}{\omega} \mathrm{Im}\left(\frac{1}{Y_{11}}\right)}$$

Sonnet provides the values of (C) and (L) for the entire line consisting of n cells. The obtained values must be divided by n.

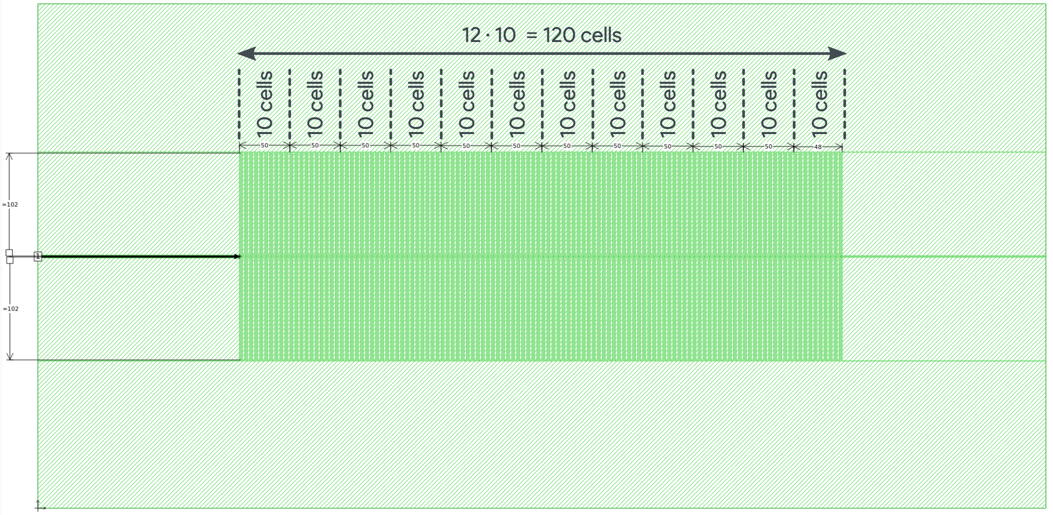


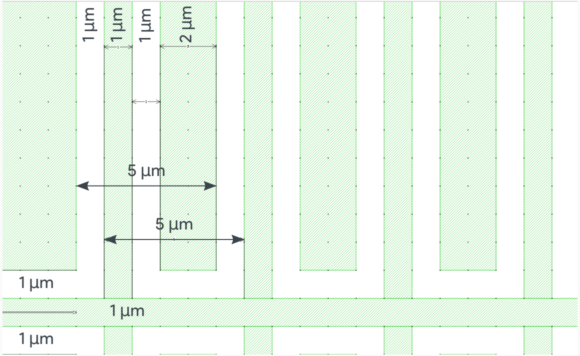
$$C = -\frac{1}{n\,\omega\,\mathrm{Im}\left(\frac{1}{Y_{11}}\right)}$$



$$L = \frac{1}{n\,\omega}\,\mathrm{Im}\left(\frac{1}{Y_{11}}\right)$$

1-port simulation with a transmission line composed by 120 elementary cells





- Line and stubs width: $w = 1\,\mu$m;
- Ground stubs width: $2w = 2\,\mu$m;
- Gap between line and ground: $s = 1\,\mu$m;
- Cell length: $L_{cell} = 5\,\mu$m;
- Number of cells: $n_{cell} = 120$;
- NbTiN sheet inductance: $L_s = 8.5$ pH/sq;

Sonnet 1-Port simulation with:

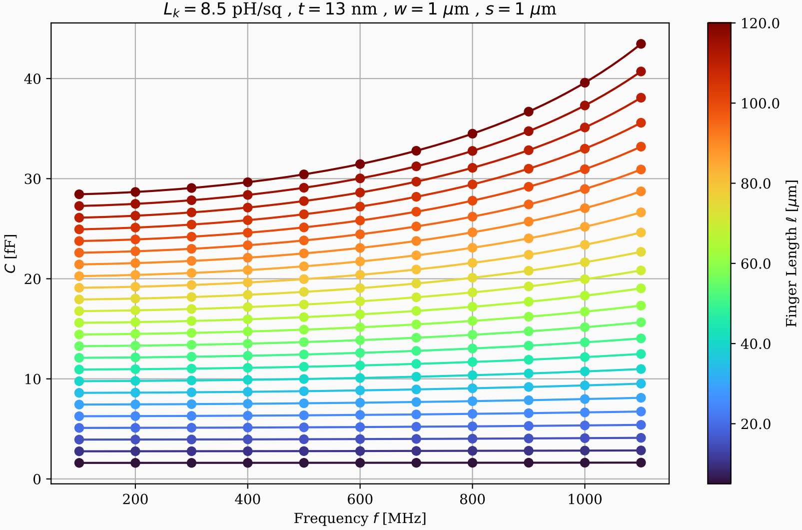
- parametric sweep in the range $\ell \in [5, 120]\,\mu$m with step of $5\,\mu$m
- frequency sweep in the range $f \in [0.1, 1.1]$ GHz with step of 0.1 GHz

Simulations for $L_k = 8.5$ pH/sq

$t = 13$ nm , $w = 1\,\mu$m , $s = 1\,\mu$m

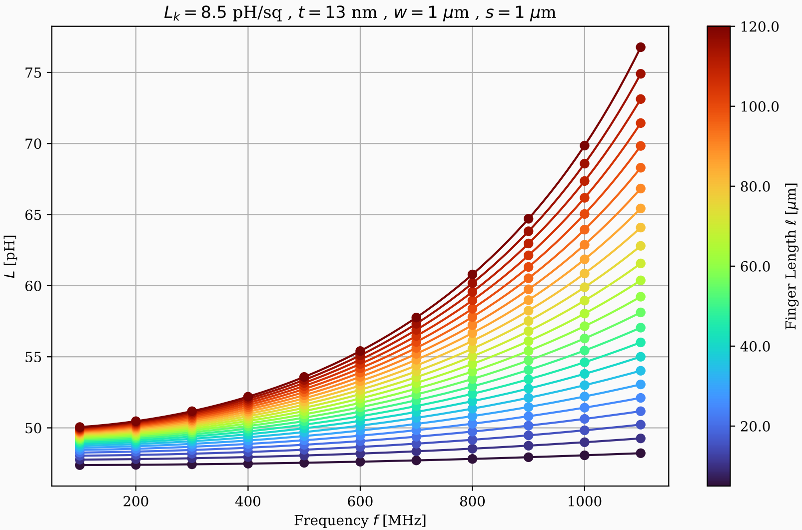
Capacitance1 estimation from Sonnet simulation

$$C = -\frac{1}{n\,\omega\ \mathrm{Im}\left(\frac{1}{Y_{11}}\right)}$$

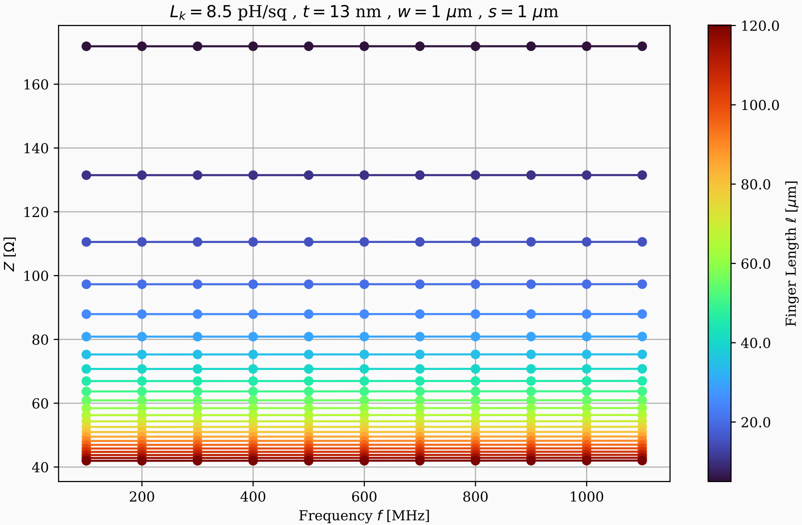


Inductance1 estimation from simulation

$$L = \frac{1}{n\,\omega}\,\mathrm{Im}\left(\frac{1}{Y_{11}}\right)$$



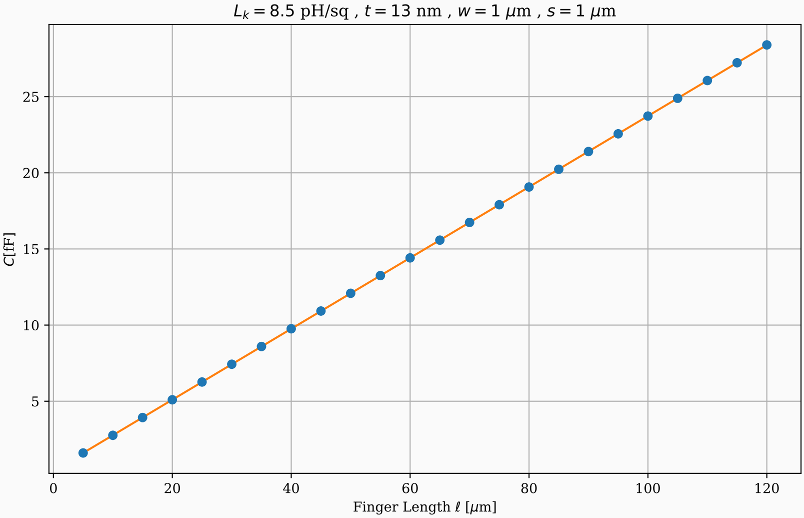
$$Z_0 = \sqrt{\frac{L}{C}}$$



Capacitance1 extrapolation for $\omega \to 0$

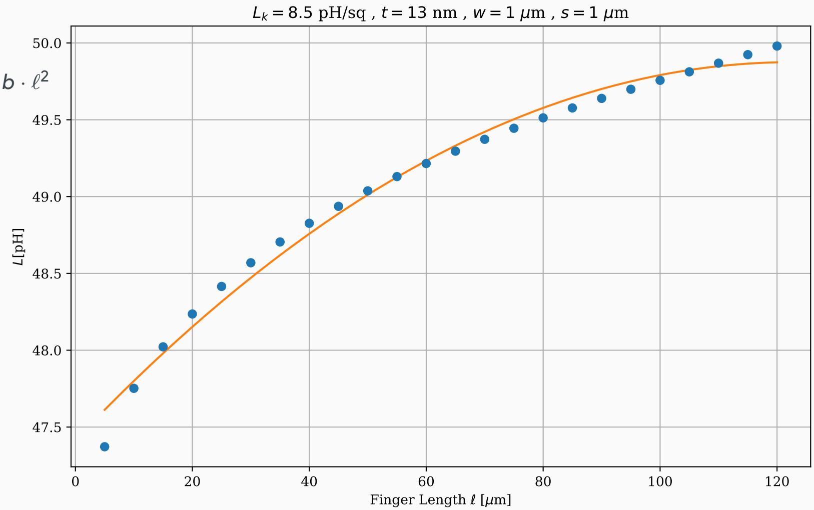
$C(\ell) = C_0 + a \cdot \ell$

$C_0 = 0.44$ fF



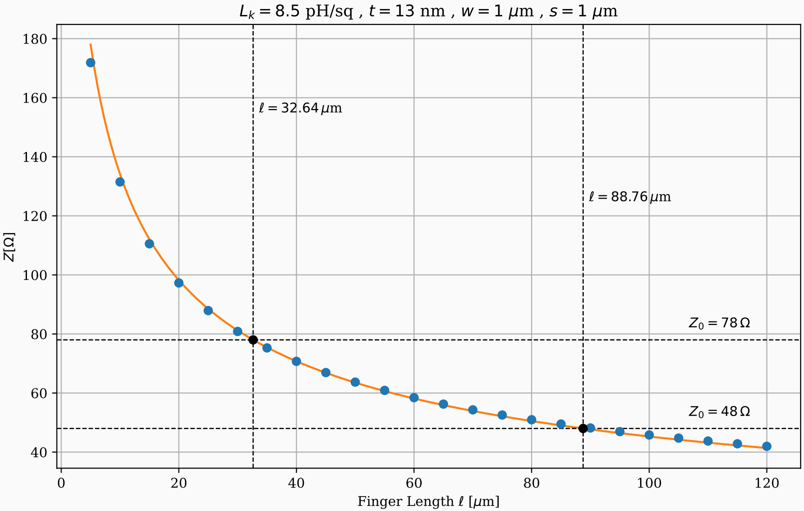
$L(\ell) \sim L_0 + a \cdot \ell + b \cdot \ell^2$

$L_0 = 47.42\,\mathrm{fF}$



Characteristic impedance Z_0 vs. finger length (for $s = 1\,\mu m$)

$$Z_0 = \sqrt{\frac{L}{C}}$$



$$Z_0 \sim \sqrt{\frac{L_0}{C_0 + a \cdot \ell}}$$

$L_0 = 70$ pH

$C_0 = 0.53$ fF

ℓ [μm]	Z_0 [Ω]	L [pH/cell]	C [fF/cell]	$v_p = 1/\sqrt{LC}$ [ns/cell]	L_k [pH/sq]	$L_f = L_k \cdot \ell w$ [nH]	$C_f = C/2$ [fF]
32.6	78	48.6	8.0	1604	8.5	0.28	4.0
88.8	48	49.7	21.1	977	8.5	0.75	10.6
33.5	77	48.6	8.2	1584	8.5	0.28	4.1
102.0	44	49.8	24.2	912	8.5	0.87	12.1
33.5	78	43.2	7.1	1806	9.6	0.32	3.6
102.0	48	43.2	18.8	1110	9.6	0.99	9.4

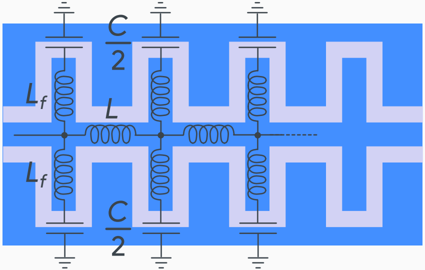
Best configuration from simulations Current configuration from simulations PRX values

- Best configuration for obtaining $Z_0 = 50\,\Omega$ and $Z_0 = 80\,\Omega$: $\ell_{80} = 32.6\,\mu$m , $\ell_{50} = 88.8\,\mu$m
- Current configuration: $\ell_{80} = 33.5\,\mu$m , $\ell_{50} = 102.0\,\mu$m

- The loaded cells nearly match: 1.3% difference for Z_0, 0% difference for L, 2.4% difference for C
- The unloaded cells are slightly further apart: 9% difference for Z_0, 0.2% difference for L, 12.8% difference for C

In the Malnou's model (PRX Quantum 2 (2021) 010302) each elementary cell is modeled as:

- a lumped element artificial transmission line of impedance $Z = \sqrt{L/C}$;
- a low-Q $\lambda/4$ resonators with resonant frequency at $f_r = 1/\sqrt{L_f C_f}$ (with $L_f = L_k \cdot \ell \cdot w$ and $C_f = C/2$);



$$ABCD_{\text{cell}} = \begin{bmatrix} 1 & j\omega L \\ \dfrac{2j\omega C}{2-\omega^2 L_f C} & 1 - \dfrac{2\omega^2 LC}{2-\omega^2 L_f C} \end{bmatrix}$$

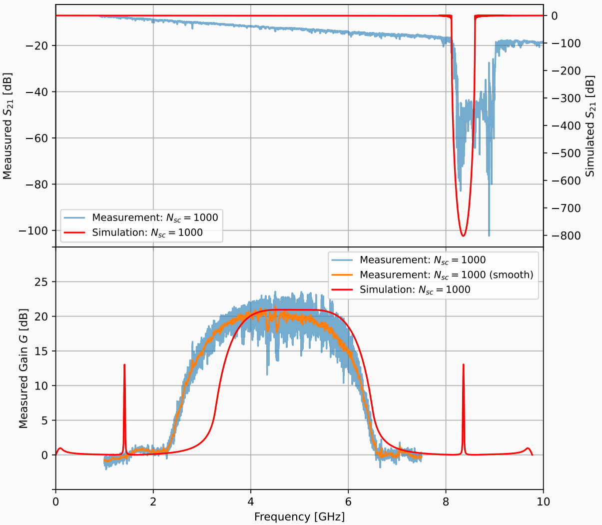
- The S_{21} stop-band is obtained by computing the *ABCD* matrix of the line starting from the *ABCD* matrix of each the single cell;
- The gain as a function of the frequency is computed by solving the CMEs equations system using the wavenumbers k computed from ABCD matrix of the line;
- The input parameters for the model are:

Cell Inductance: L (from EM simulation)
Cell Capacitance: C (from EM simulation)
Flanked Inductance: L_f (from L_k, ℓ and w)
Scaling Current: I_* (experimentally determined)
Bias Current: I_{dc} (experimentally determined)
Input Pump Current: I_{p0} (experimentally determined)
Input signal Current: I_{s0} (experimentally determined)
Number of cells : N_{sc}, N_{u} and N_{l}

Unloaded cell			Unloaded cell		
L [pH]	C [fF]	L_f [nH]	L [pH]	C [fF]	L_f [nH]
43.2	18.8	0.99	43.2	7.1	0.32

I_* [mA]	I_{dc} [mA]	I_{p0} [μA]	I_{s0} [μA]	N_u	N_l	N_{sc}	N_{tot}
7	1.5	117 -31 dBm	2 -67 dBm	60	60	1000	66000

- Simulation with detuning between signal end idler of $2 * \delta f = 1\,\text{GHz}$;
- Phase matching reached for $f_{\text{pump}} = 9.77\,\text{GHz}$;
- Experimental pump frequency: $f_{\text{pump}} = 8.886\,\text{GHz}$;
- (Likely) phase matching reached with a different detuning;
- Good matching despite all the possible differences between measurement and ideal behaviour.

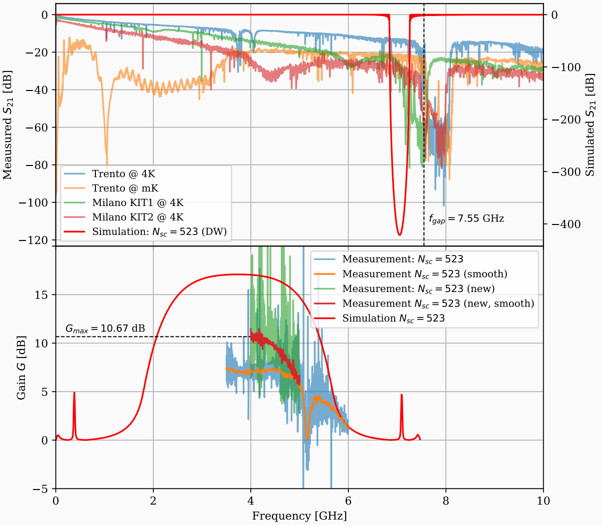


Model applied on the DARTWARS Prototype

Unloaded cell			Unloaded cell		
L [pH]	C [fF]	L_f [nH]	L [pH]	C [fF]	L_f [nH]
49.9	24.2.8	0.87	48.6	8.2	0.28

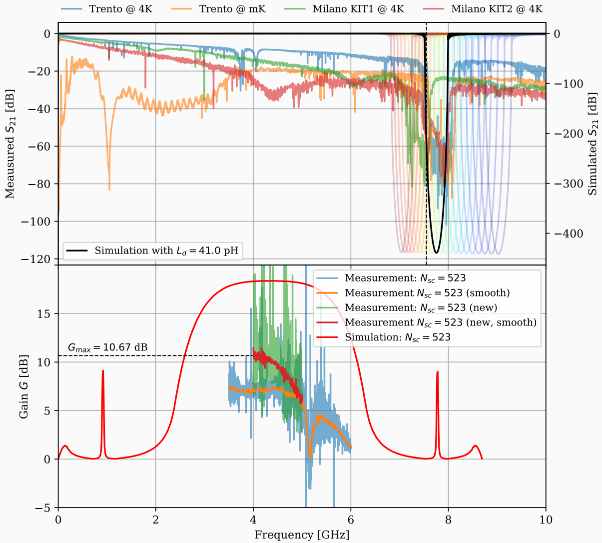
I_* [mA]	I_{dc} [mA]	I_{p0} [μA]	I_{s0} [μA]	N_u	N_l	N_{sc}	N_{tot}
6.3	1	251 -25 dBm	0.14 -90 dBm	60	60	523	34518

- Simulation with detuning between signal end idler of $2 * \delta f = 1\,\text{GHz}$;
- Phase matching reached for $f_{\text{pump}} = 7.475\,\text{GHz}$;
- Experimental pump frequency: $f_{\text{pump}} = 8.05161\,\text{GHz}$;
- Experimental stop-band at higher frequency with respect simulations $\Rightarrow$ (probably) the experimental inductance per cell L is lower;
- Lack of clarity regarding the comparison of gains $\Rightarrow$ a wide band measurement is needed;



Model variation to match the experimental data

- Adjusting the value of the cell impedance L to align the theoretical stop-band with the experimental results.
- Keeping the cell capacitance C fixed as it solely depends on the geometry of the stubs (?)
- Cell impedance varied in the $L = (30 - 50)$ pH range;
- Good match for $L = 41$ pH, but $Z_0 = 41.2\,\Omega$;
- Model computed with $L = 41$ pH and with the parameters used for the previous simulations;
- Theoretical and experimental gain probably in a different position $\Rightarrow$ a wide band measurement is needed



Current Devices:

- Measurement of the gain figure in a wide range setup (Milano 4K setup)
  Milano's team is facing challenges in finding a functional amplifier
  $\Rightarrow$ the amplifier that was measured in Trento could be transferred to Milano for further testing.
- The measured gain figure exhibits a periodic ripple
  $\Rightarrow$ performing FFT on the gain profile is possible to estimate the length of the ripple in μm
  By comparing this length with the length of the typical device lengths (port-to-port, super-cell, etc) can provide useful information;

Future Devices:

- The same design but without the SiO_2 layer. Could this be this interesting?
- Same loaded/unloaded configuration (60/6) but with the stub length suggested by the simulations;
- Different loaded/unloaded configuration (to push the stop-band at higher frequencies) and with the stub length suggested by the simulations;
- All of the configuration above-mentioned. Again half lenght? Complete lenght? Both the configurations?
- From now, every wafer containing KI-TWPA devices must include a resonator array for the purpose of check the RF characteristics of the material
  $\Rightarrow$ I am currently designing an array with 2 normal IDC resonators and 2 tunable IDC resonator fot that purpose.