

Transformation Optics

Ulf Leonhardt

Weizmann Institute of Science



ESSENTIAL

Quantum Optics

From Quantum Measurements to Black Holes

ULF LEONHARDT

CAMBRIDGE

Ulf Leonhardt and Thomas Philbin

GEOMETRY AND LIGHT

The Science of
INVISIBILITY





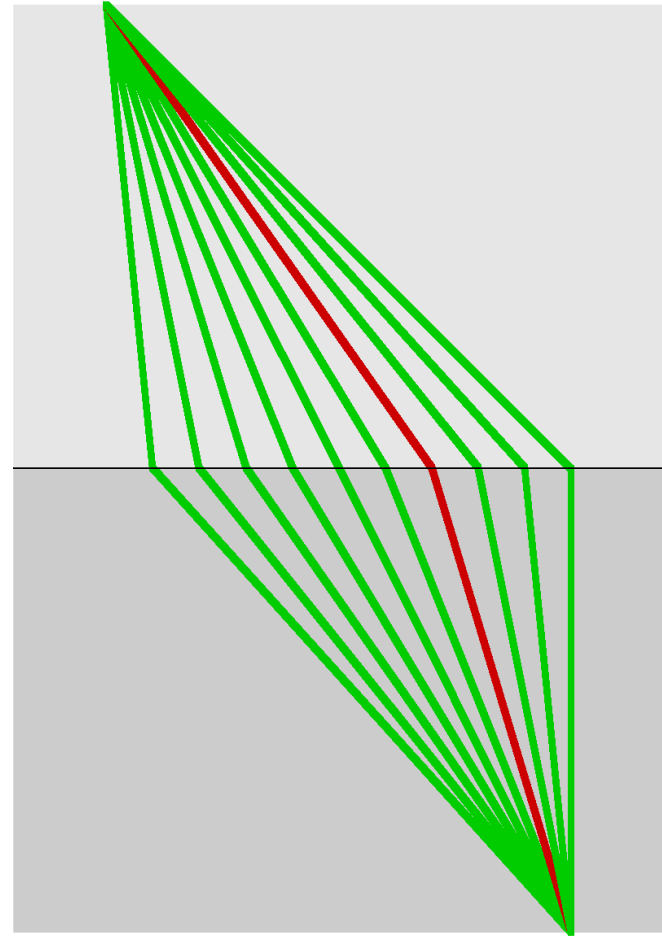
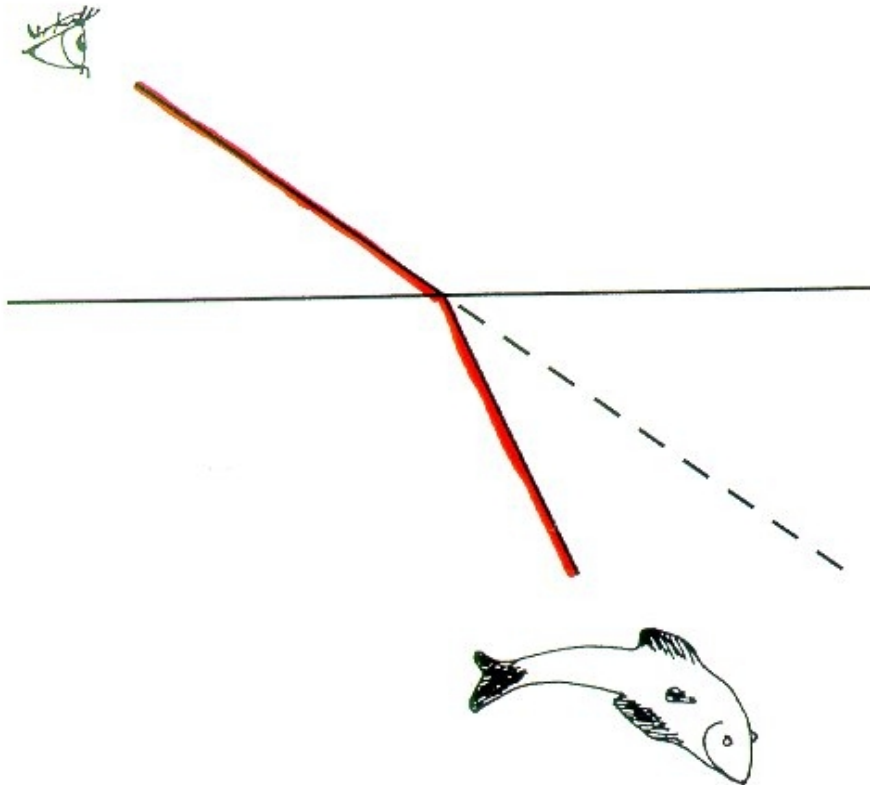


Mirage

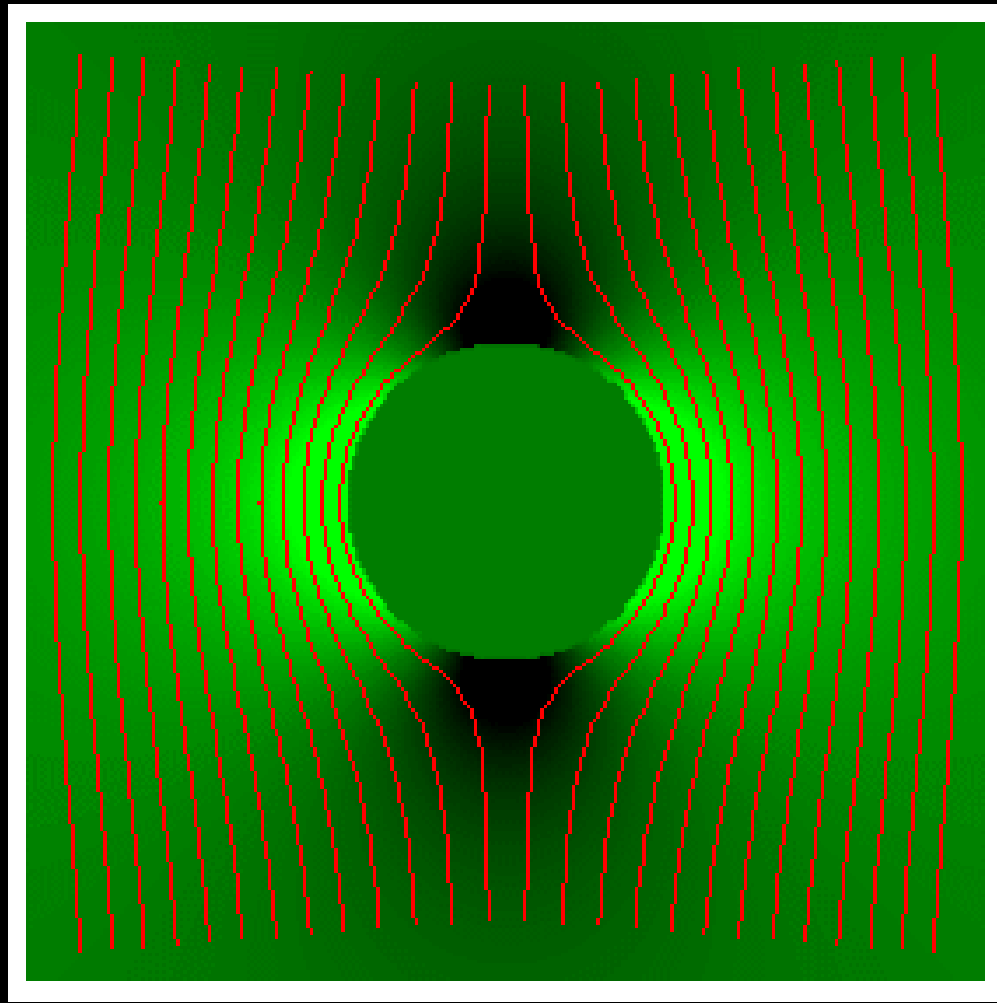


Fermat's Principle - the principle of the shortest optical path

$$s = \int n dl$$



Leonhardt 2002: Invisibility cloak?



Invisibility: Invisible Man versus Invisible Woman



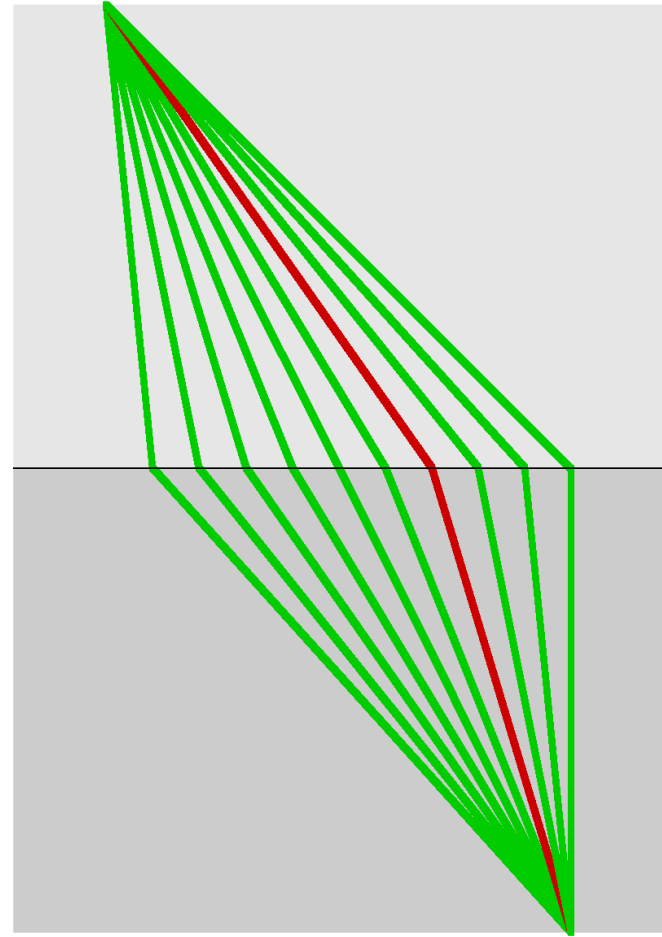
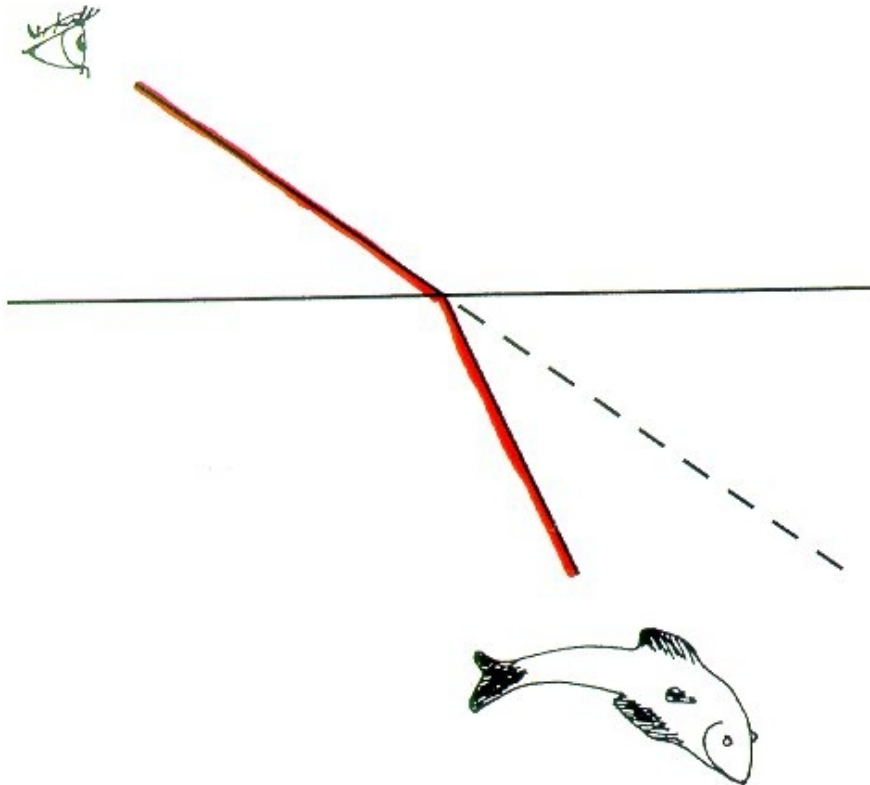
transparency



curved space

Fermat's Principle - the principle of the shortest optical path

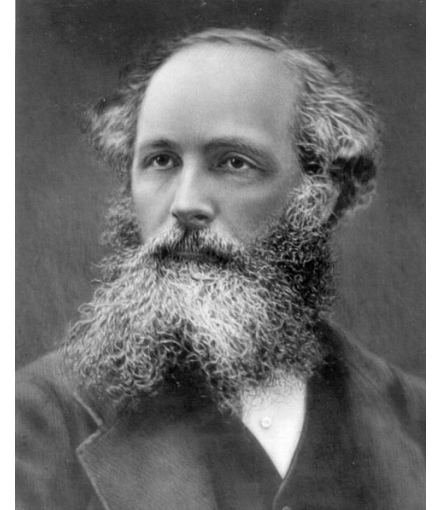
$$s = \int n dl$$



Maxwell's electromagnetism and Einstein's general relativity

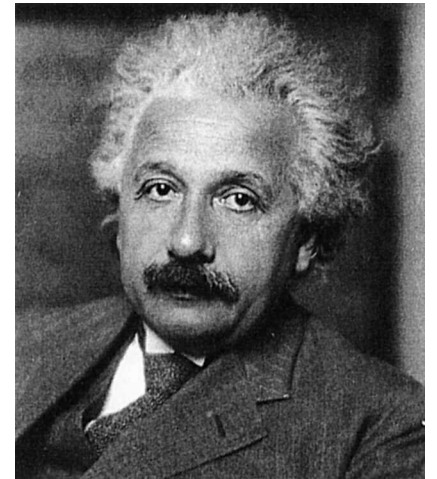
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j}, \quad \nabla \cdot \vec{D} = \rho$$

The covariant free-space Maxwell equations are equivalent to electromagnetism in a material medium (Tamm, 1924; Plebanski, 1960).

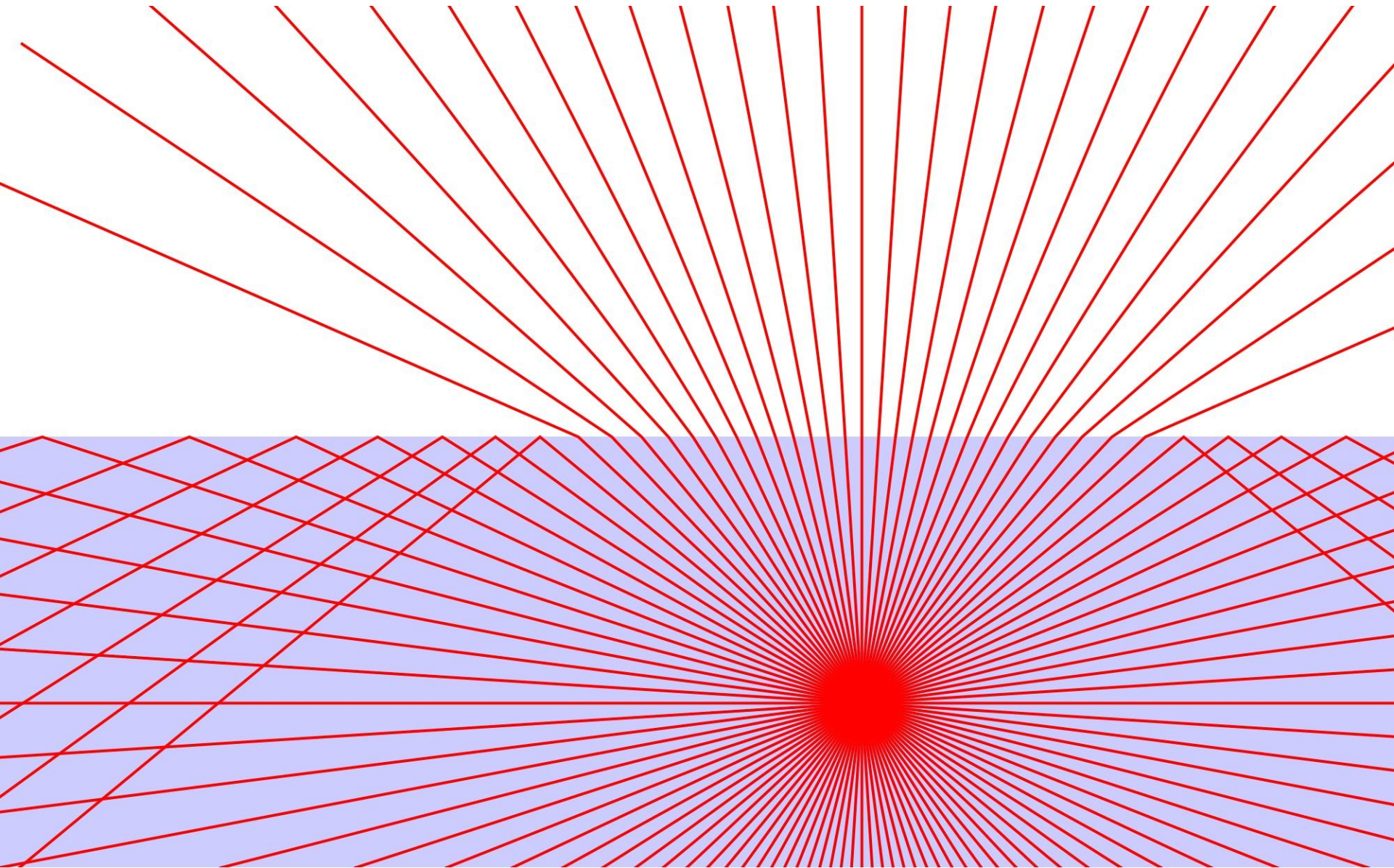


$$\vec{D} = \epsilon_0 \epsilon \vec{E} + \frac{\vec{w}}{c} \times \vec{H}, \quad \vec{B} = \frac{\mu}{\epsilon_0 c^2} \vec{H} - \frac{\vec{w}}{c} \times \vec{E}$$

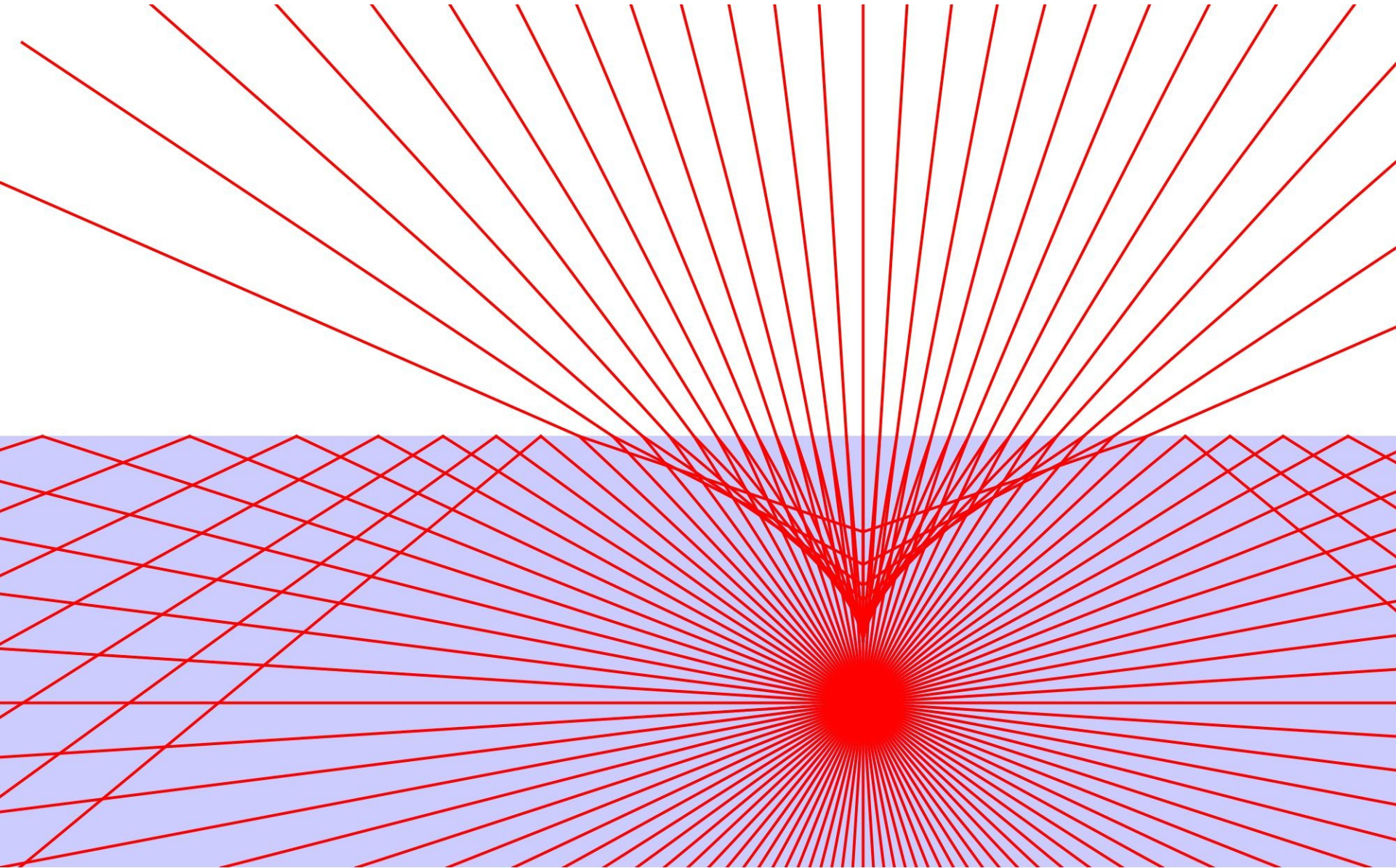
$$\epsilon^{ij} = \mu^{ij} = \mp \frac{\sqrt{-g}}{g_{00}} g^{ij}, \quad w_i = \frac{g_{0i}}{g_{00}}$$



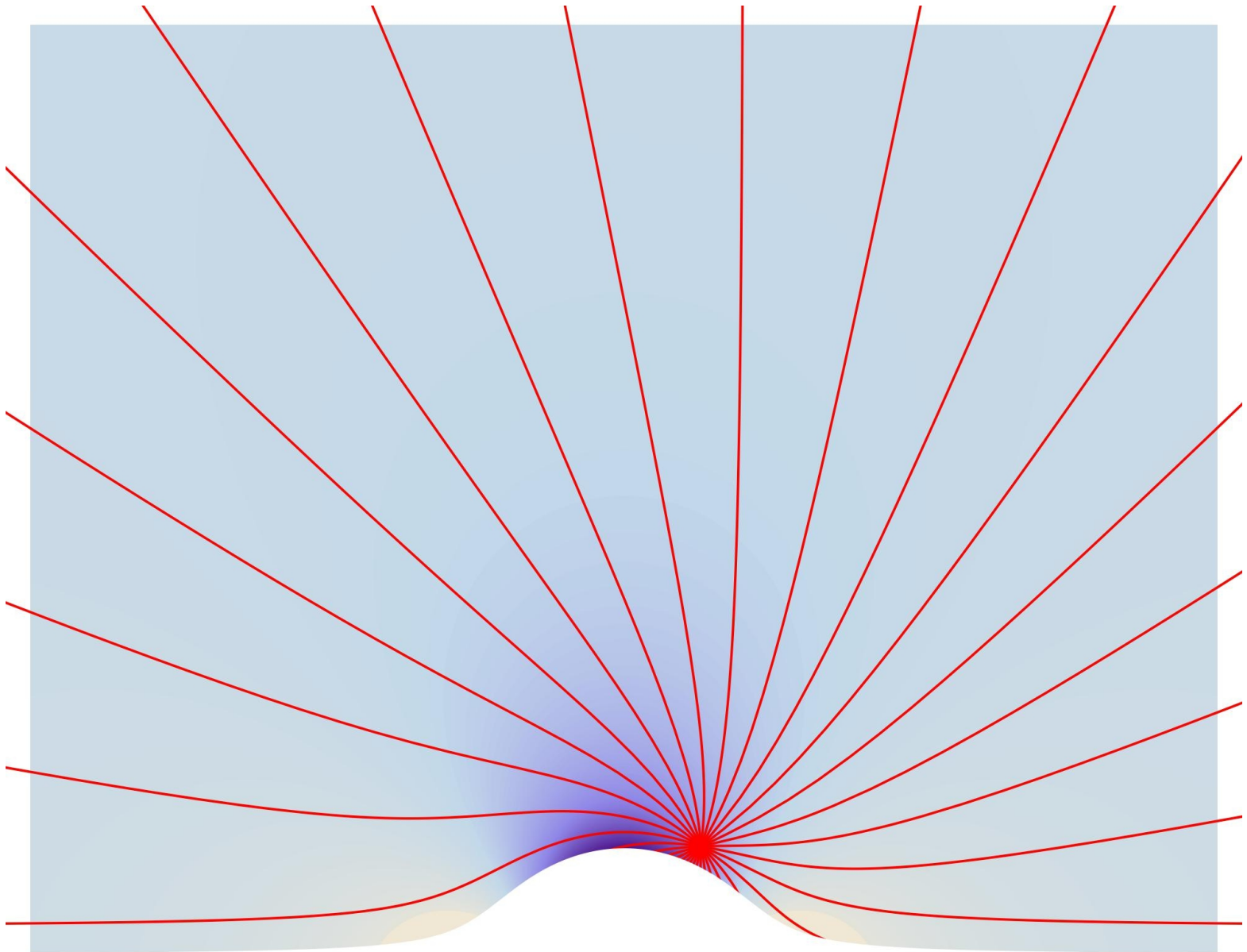
Refraction in ordinary medium



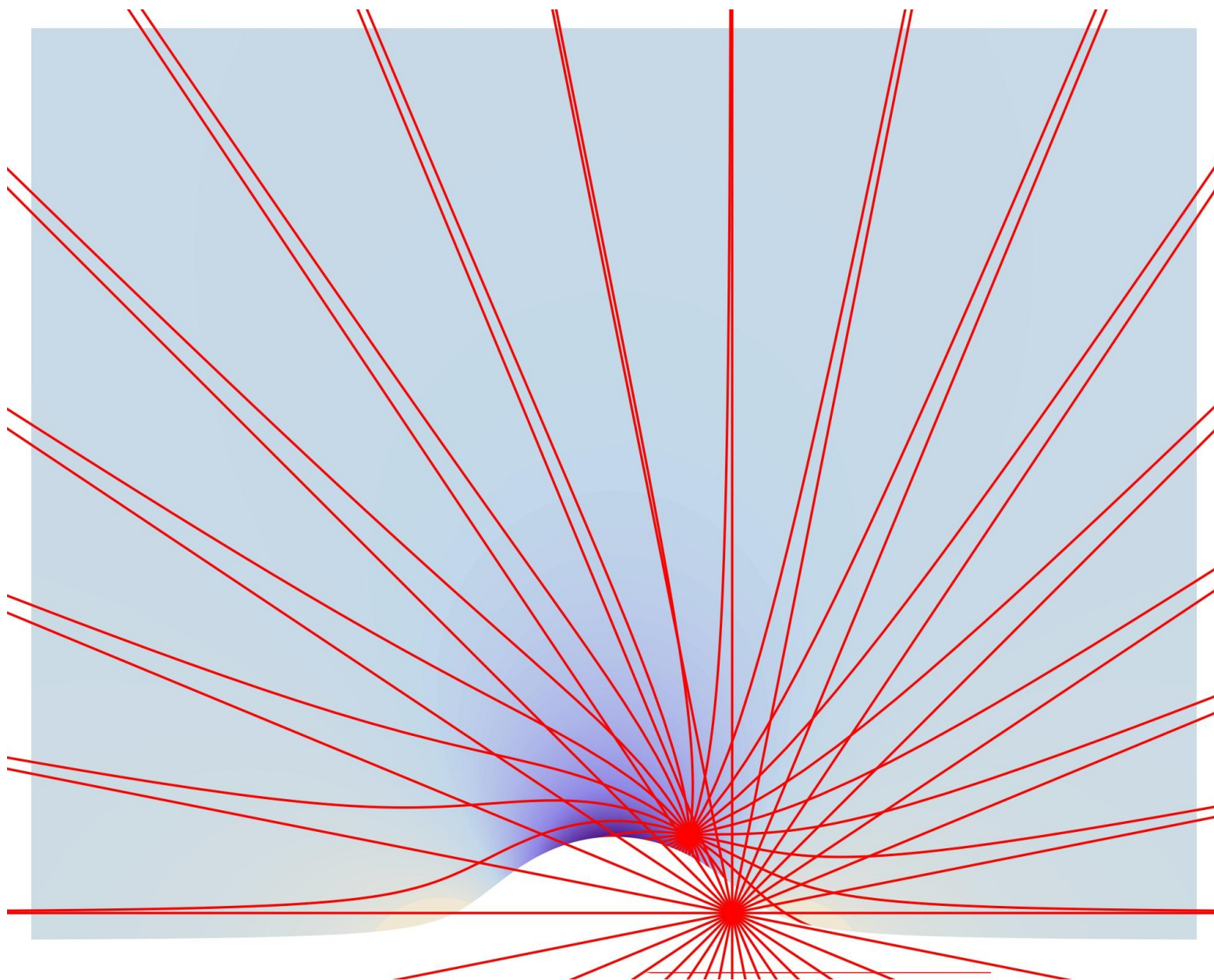
Refraction in ordinary medium:
virtual image depends on viewpoint



Transformation medium

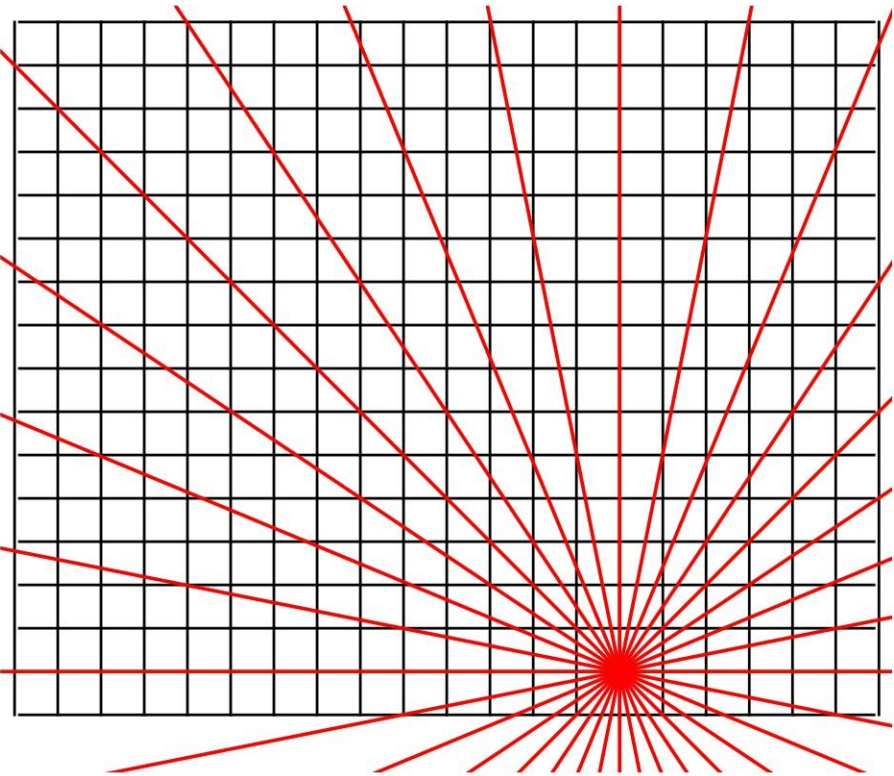


Transformation medium: definite virtual image

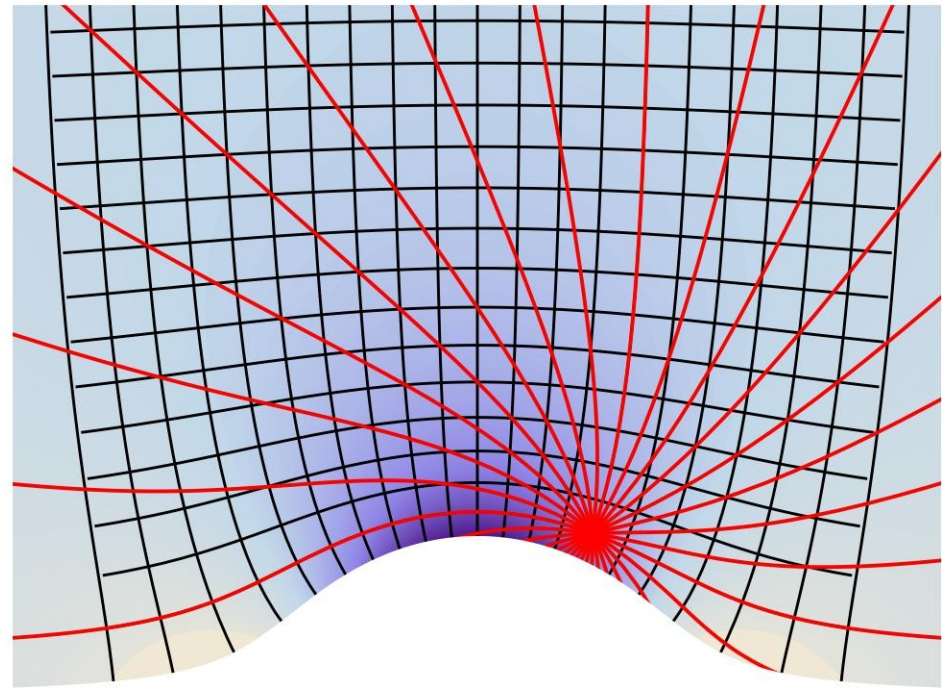


Transformation medium

Virtual space

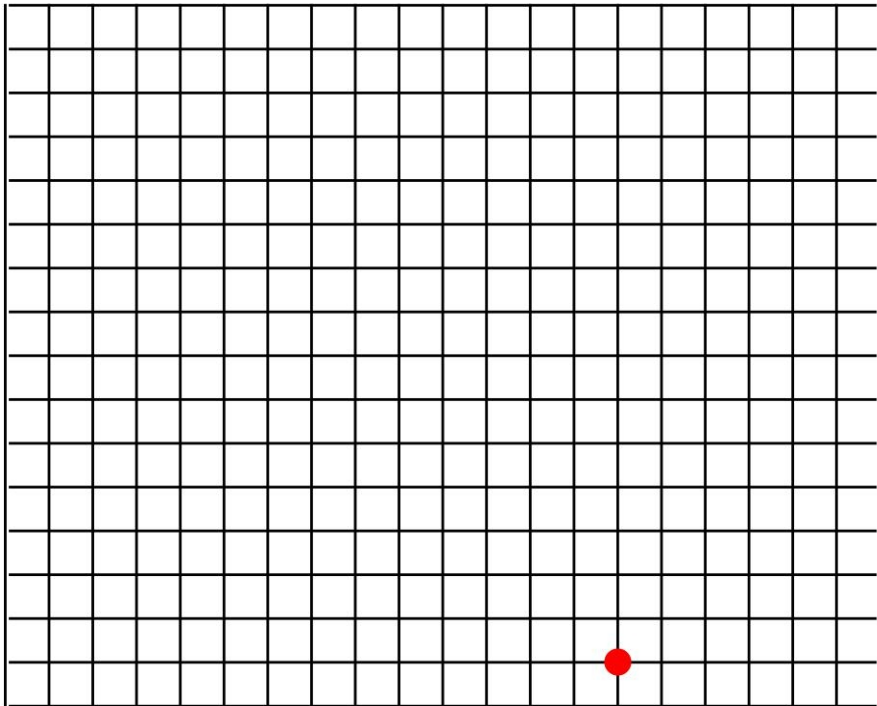


Physical space

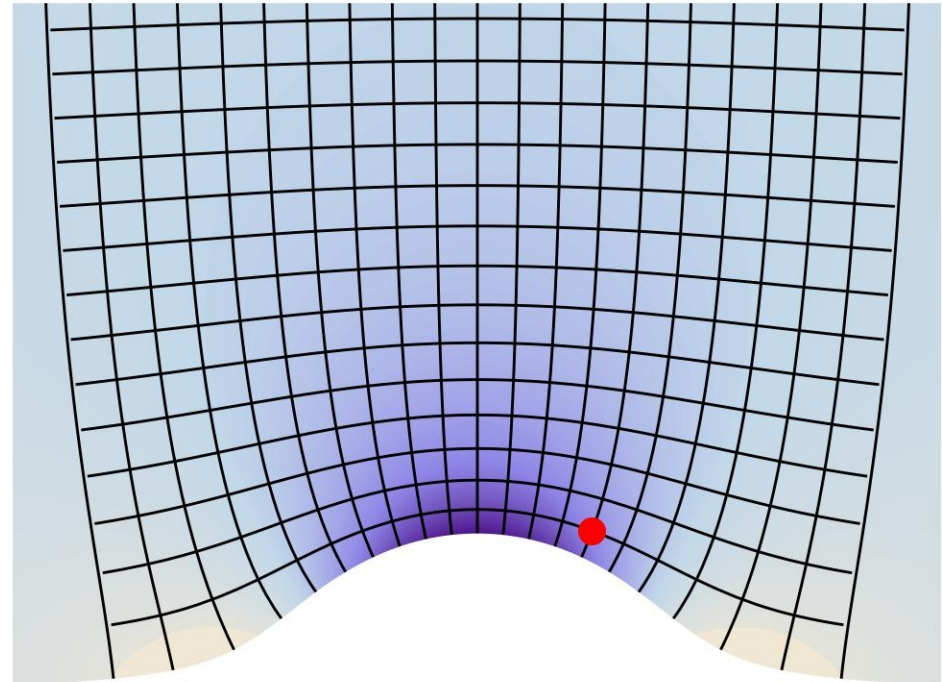


Transformation medium

Virtual space

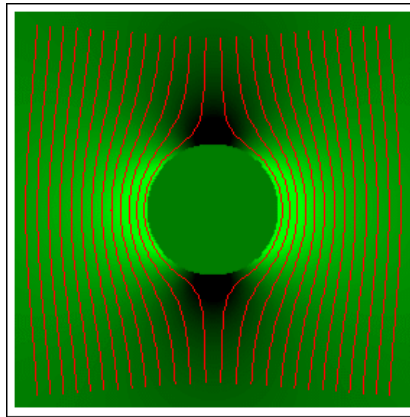


Physical space

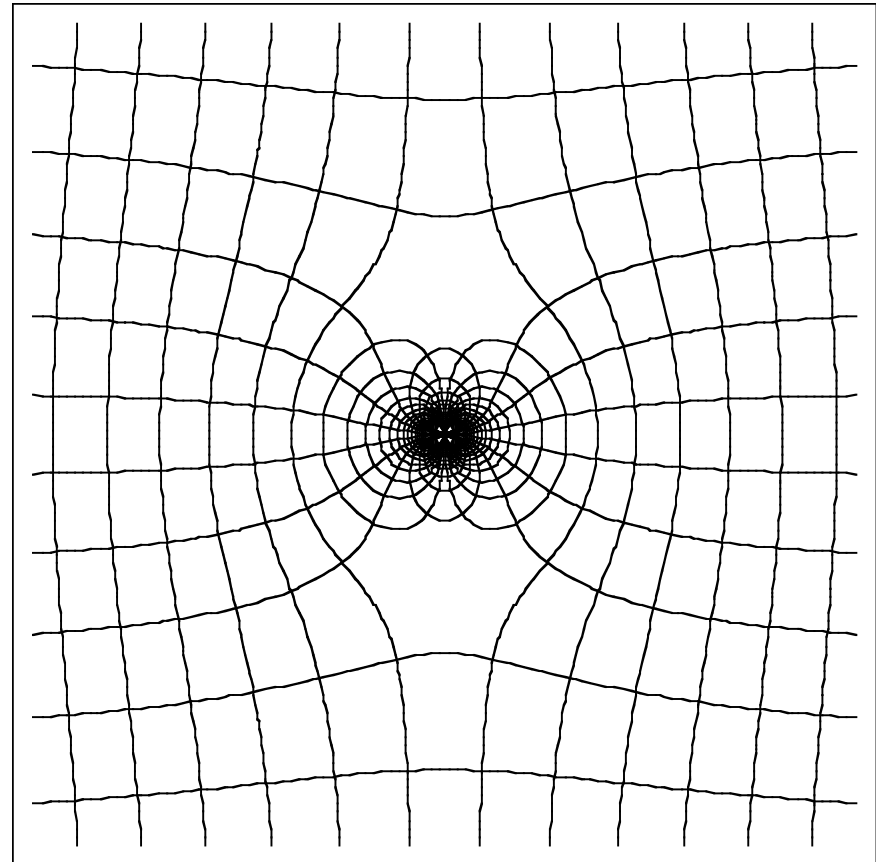
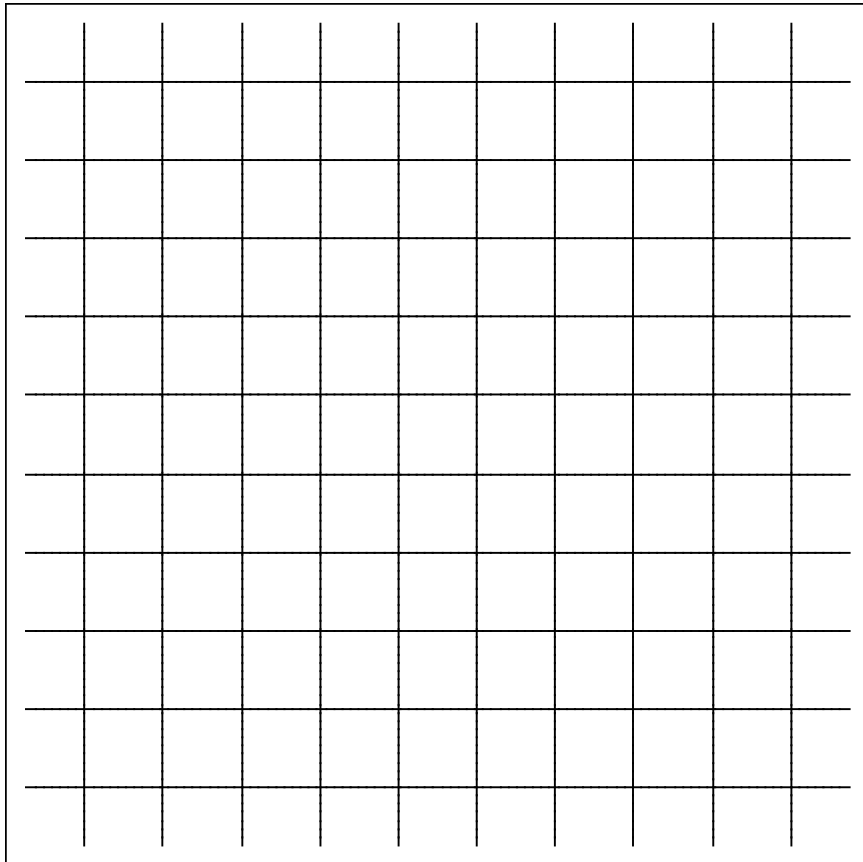


[Leonhardt, Science **312**, 1777 (2006)]

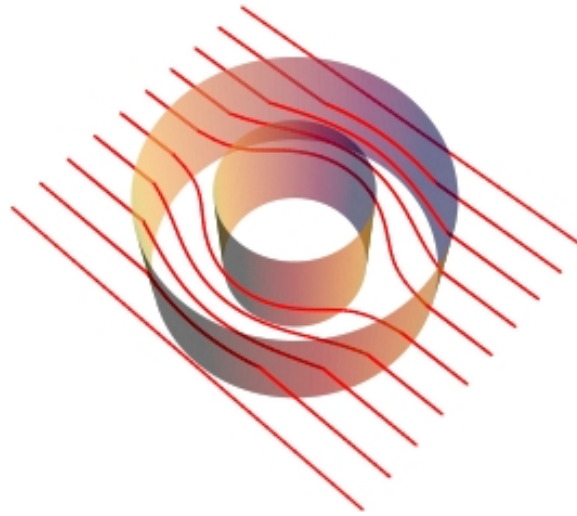
Virtual space



Physical space

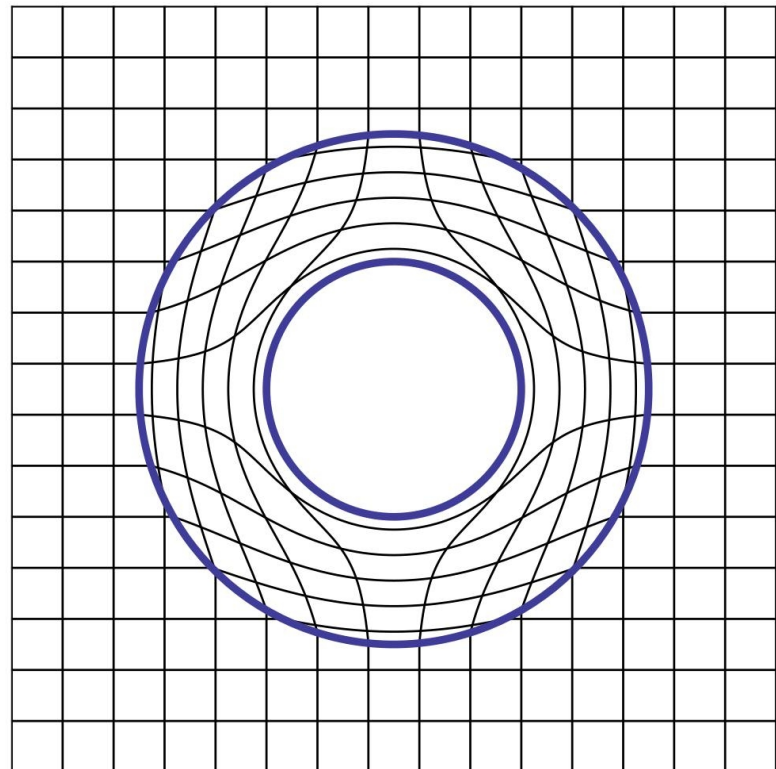
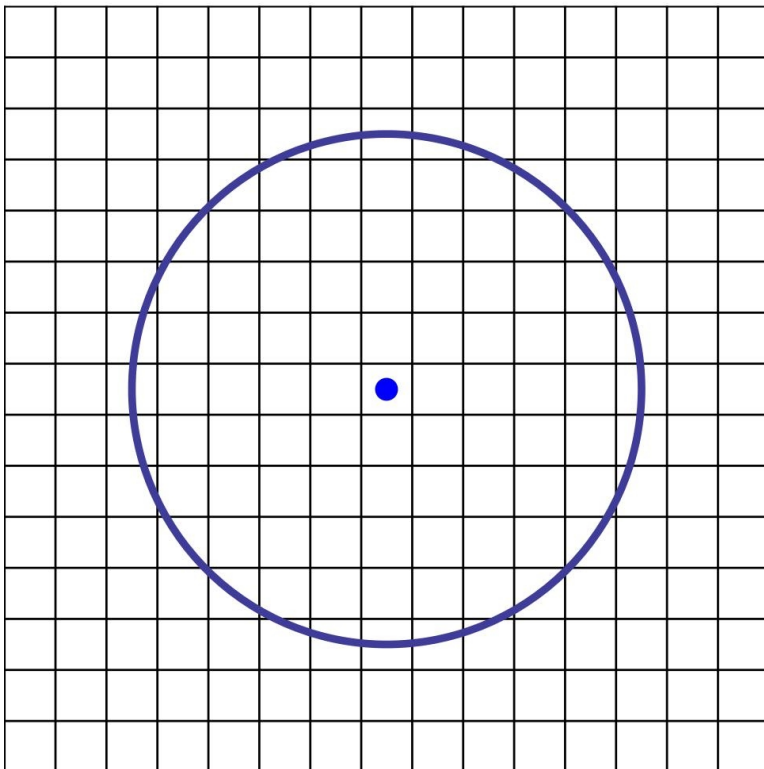


[Pendry, Schurig and Smith,
Science **312**, 1780 (2006)]



Virtual space

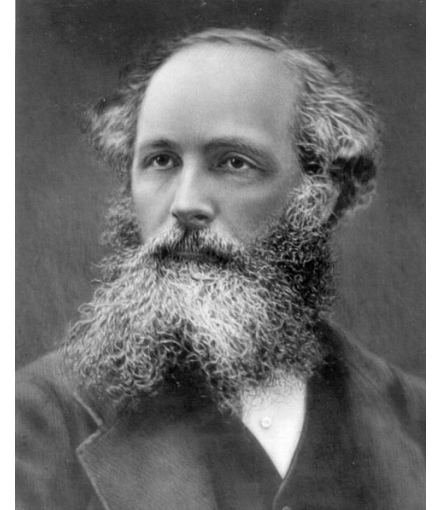
Physical space



Maxwell's electromagnetism and Einstein's general relativity

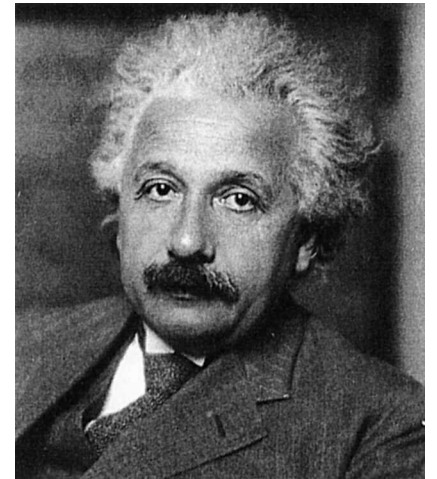
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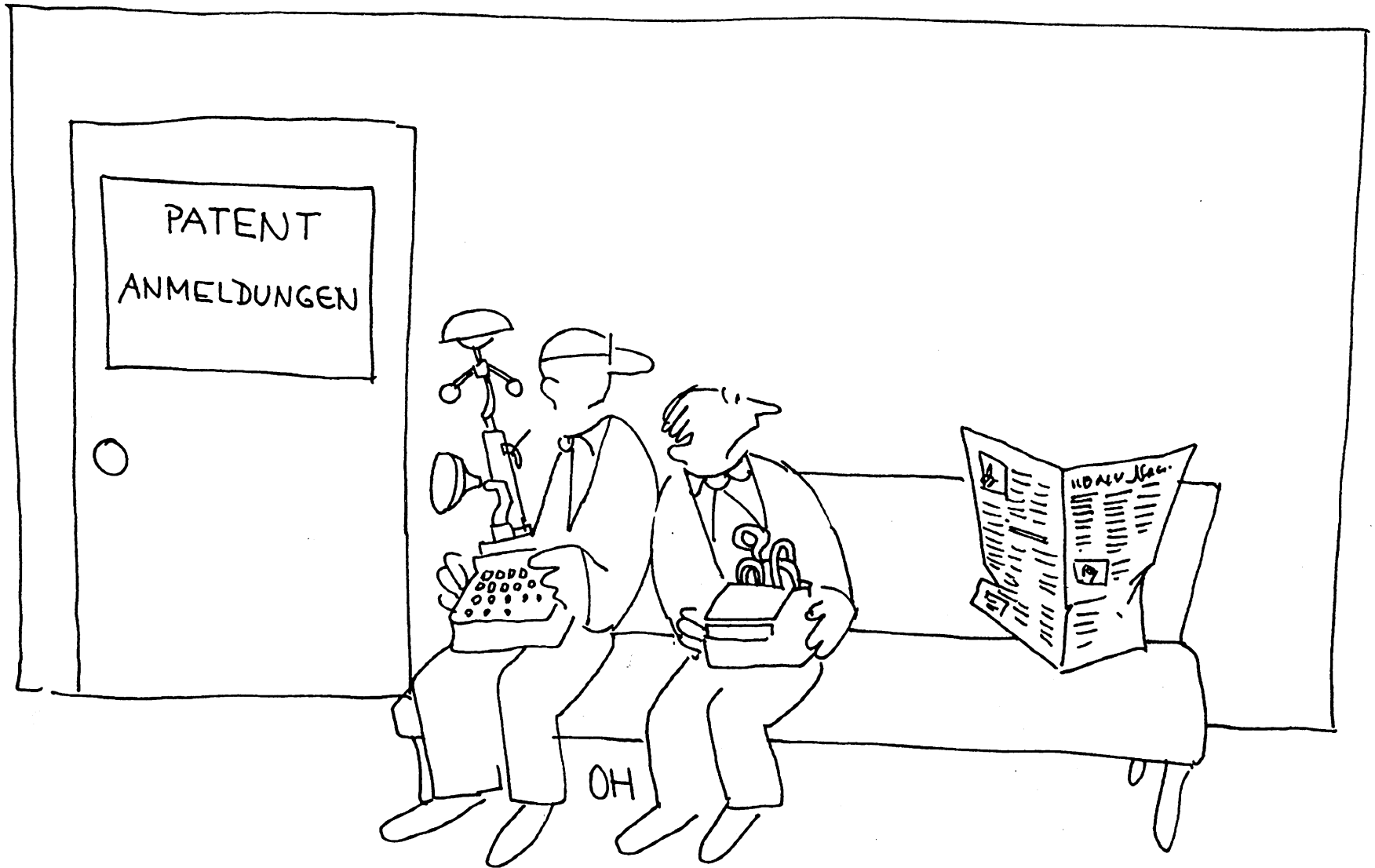


$$\vec{D} = \epsilon_0 \epsilon \vec{E} + \frac{\vec{w}}{c} \times \vec{H}, \quad \vec{B} = \frac{\mu}{\epsilon_0 c^2} \vec{H} - \frac{\vec{w}}{c} \times \vec{E}$$

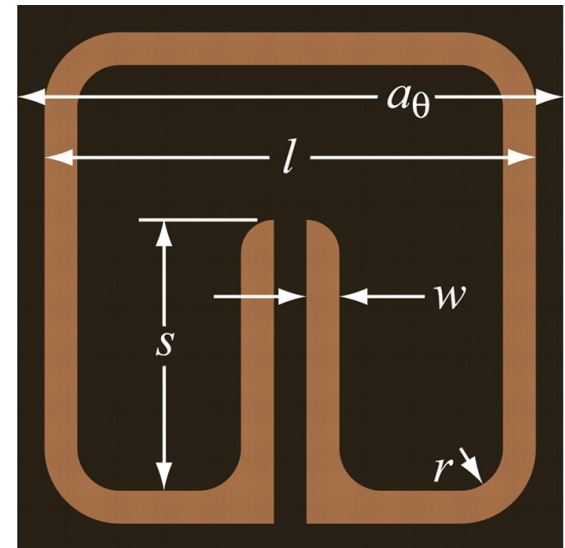
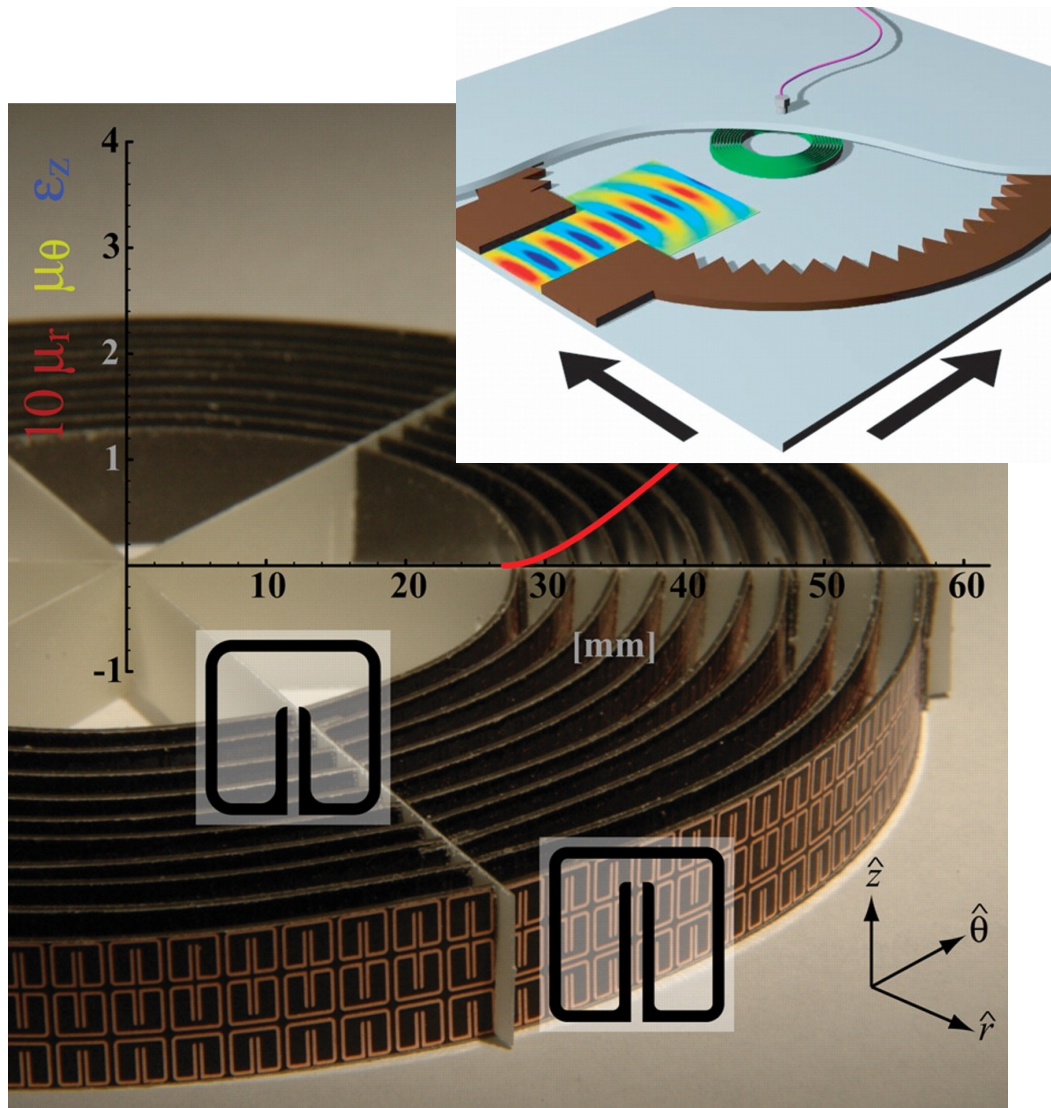
$$\epsilon^{ij} = \mu^{ij} = \mp \frac{\sqrt{-g}}{g_{00}} g^{ij}, \quad w_i = \frac{g_{0i}}{g_{00}}$$



Patent office

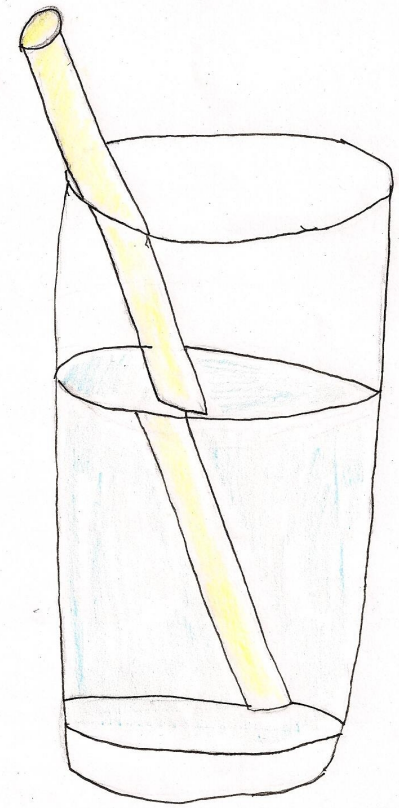
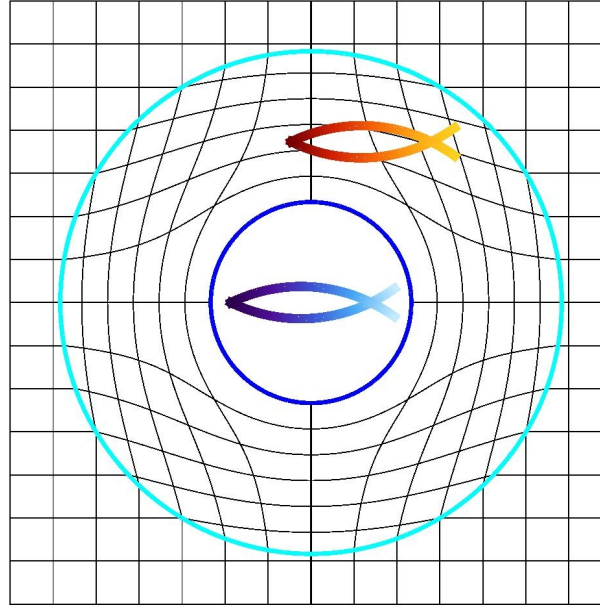
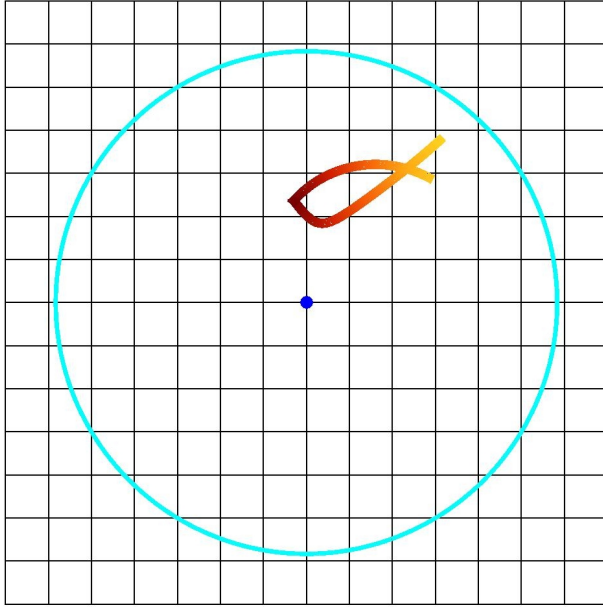


Cloaking device for electromagnetic microwaves



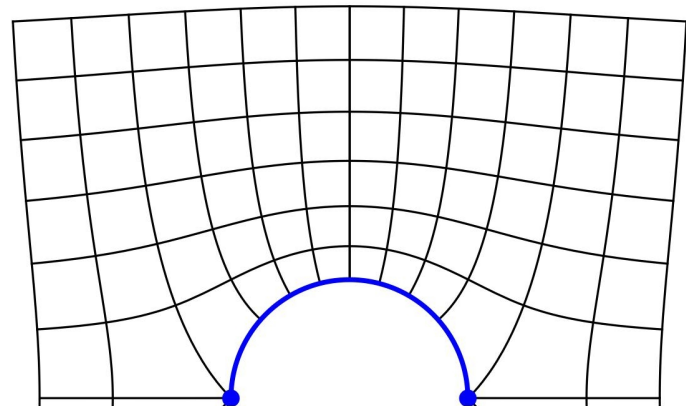
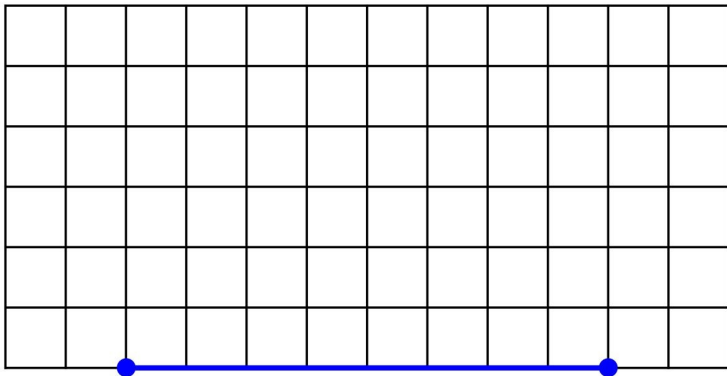
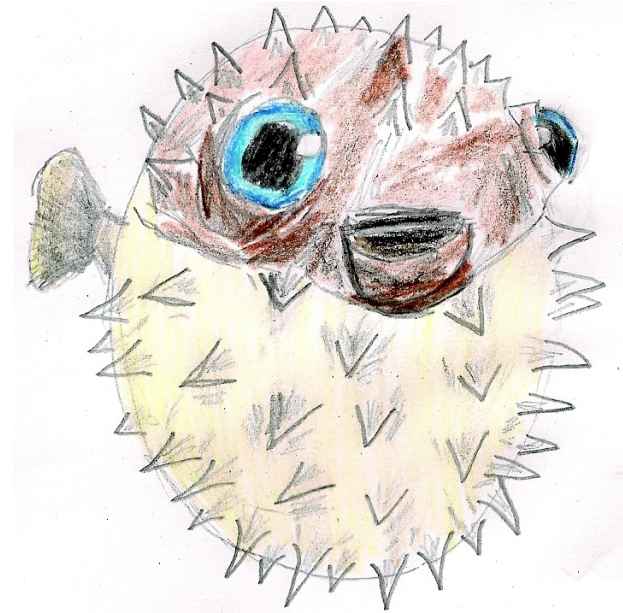
cyl.	r	s	μ_r
1	0.260	1.654	0.003
2	0.254	1.677	0.023
3	0.245	1.718	0.052
4	0.230	1.771	0.085
5	0.208	1.825	0.120
6	0.190	1.886	0.154
7	0.173	1.951	0.188
8	0.148	2.027	0.220
9	0.129	2.110	0.250
10	0.116	2.199	0.279

Transformation optics



[Greenleaf, Lassas and Uhlmann, Math. Res. Lett. **10**, 685 (2003) [electrostatics](#);
Leonhardt, Science **312**, 1777 (2006) [conformal transformations](#);
Pendry, Schurig and Smith, Science **312**, 1780 (2006) [spatial transformations](#);
Leonhardt and Philbin, NJP **8**, 247 (2006)] [space-time & negative refraction](#)]

Turning a fugu into a flatfish



Credit: Maria Leonhardt



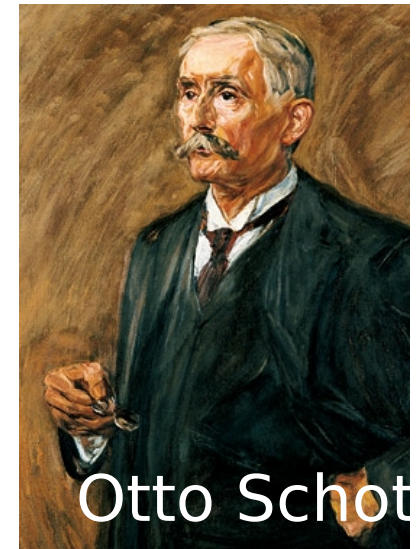
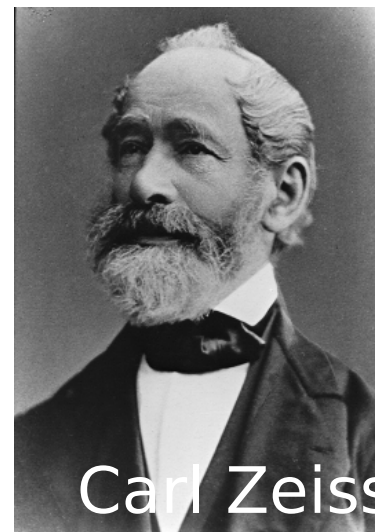
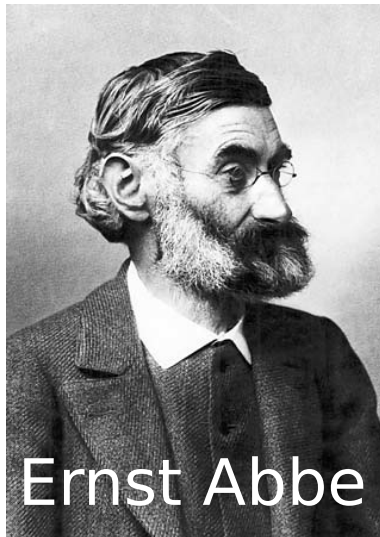
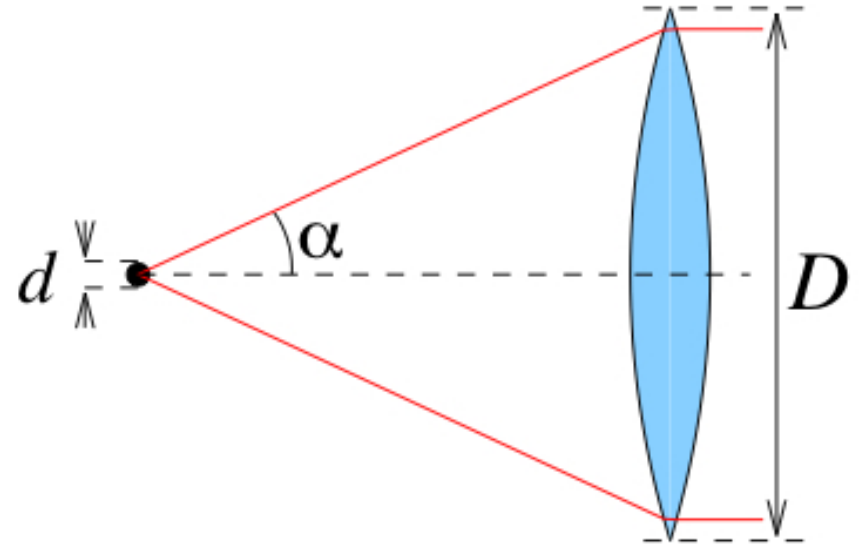
A new fish-eye lens
based on an idea of
James Clark Maxwell.

To invisibility and beyond

Combining Maxwell's equations with Einstein's general relativity promises perfect images and cloaking devices, explains **Ulf Leonhardt**.

[Leonhardt, Nature **471**, 292 (2011)]

The resolution limit of imaging, established around 1870



Negative Refraction Makes a Perfect Lens

J. B. Pendry

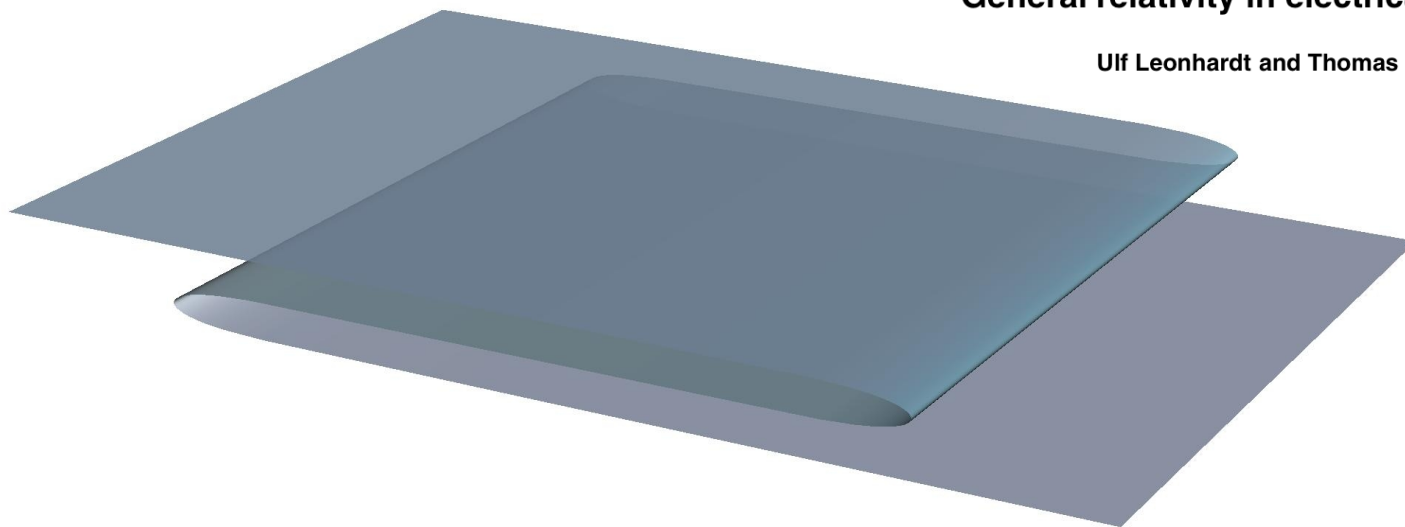
Condensed Matter Theory Group, The Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom
(Received 25 April 2000)

With a conventional lens sharpness of the image is always limited by the wavelength of light. An unconventional alternative to a lens, a slab of negative refractive index material, has the power to focus all Fourier components of a 2D image, even those that do not propagate in a radiative manner. Such “superlenses” can be realized in the microwave band with current technology. Our simulations show that a version of the lens operating at the frequency of visible light can be realized in the form of a thin slab of silver. This optical version resolves objects only a few nanometers across.

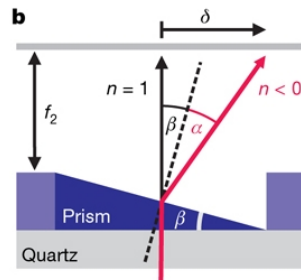
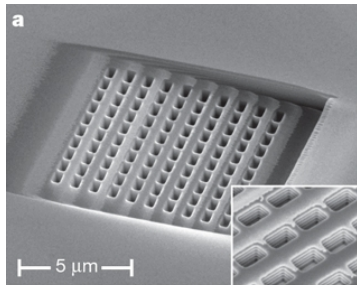
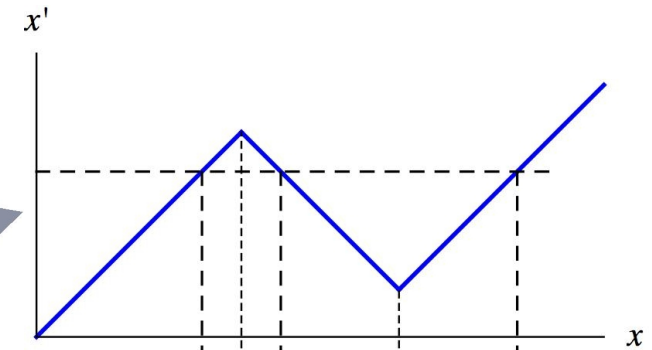
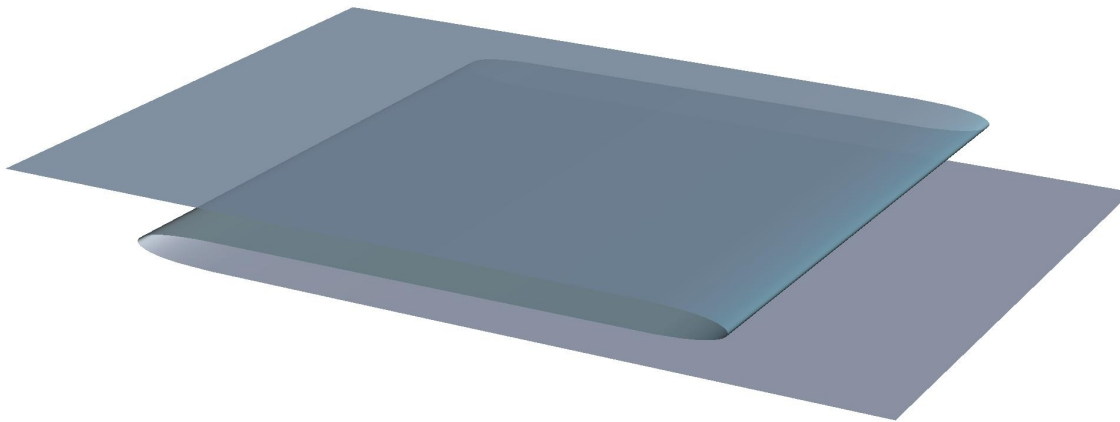
New Journal of Physics
The open-access journal for physics

General relativity in electrical engineering

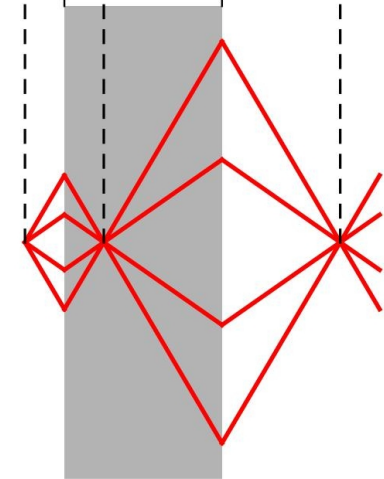
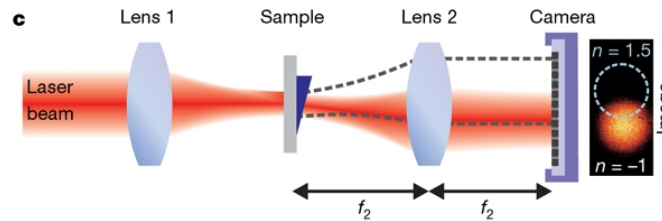
Ulf Leonhardt and Thomas G Philbin



Negative refraction and perfect lens



$$\varepsilon = \mu = -\frac{\sqrt{-g}}{g_{00}} g^{ij}$$



$$\varepsilon = \mu = 1$$

$$\varepsilon = \mu = -1$$

$$\varepsilon = \mu = 1$$

Xiang Zhang et al.
@ Berkeley

[Leonhardt and Philbin, New J. Phys. **8**, 247 (2006)]

“Poor man’s perfect lens” [Science. **308**, 534 (2005)]

REPORTS

Sub-Diffraction-Limited Optical Imaging with a Silver Superlens

Nicholas Fang, Hyesog Lee, Cheng Sun, Xiang Zhang*

Recent theory has predicted a superlens that is capable of producing sub-diffraction-limited images. This superlens would allow the recovery of evanescent waves in an image via the excitation of surface plasmons. Using silver as a natural optical superlens, we demonstrated sub-diffraction-limited imaging with 60-nanometer half-pitch resolution, or one-sixth of the illumination wavelength. By proper design of the working wavelength and the thickness of silver that allows access to a broad spectrum of subwavelength features, we also showed that arbitrary nanostructures can be imaged with good fidelity. The optical superlens promises exciting avenues to nanoscale optical imaging and ultrasmall optoelectronic devices.

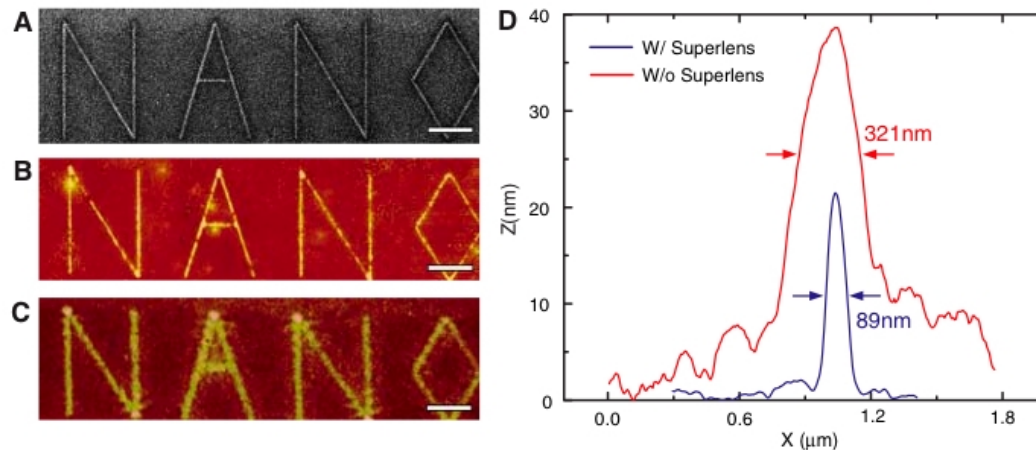
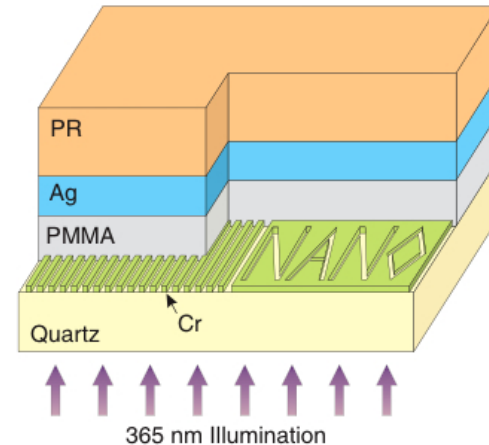


Fig. 4. An arbitrary object “NANO” was imaged by silver superlens. (A) FIB image of the object. The linewidth of the “NANO” object was 40 nm. Scale bar in (A) to (C), 2 μ m. (B) AFM of the developed image on photoresist with a silver superlens. (C) AFM of the developed image on photoresist when the 35-nm-thick layer of silver was replaced by PMMA spacer as a control experiment. (D) The averaged cross section of letter “A” shows an exposed line width of 89 nm (blue line), whereas in the control experiment, we measured a diffraction-limited full width at half-maximum line width of 321 ± 10 nm (red line).

Invisibility: Invisible Man versus Invisible Woman



transparency

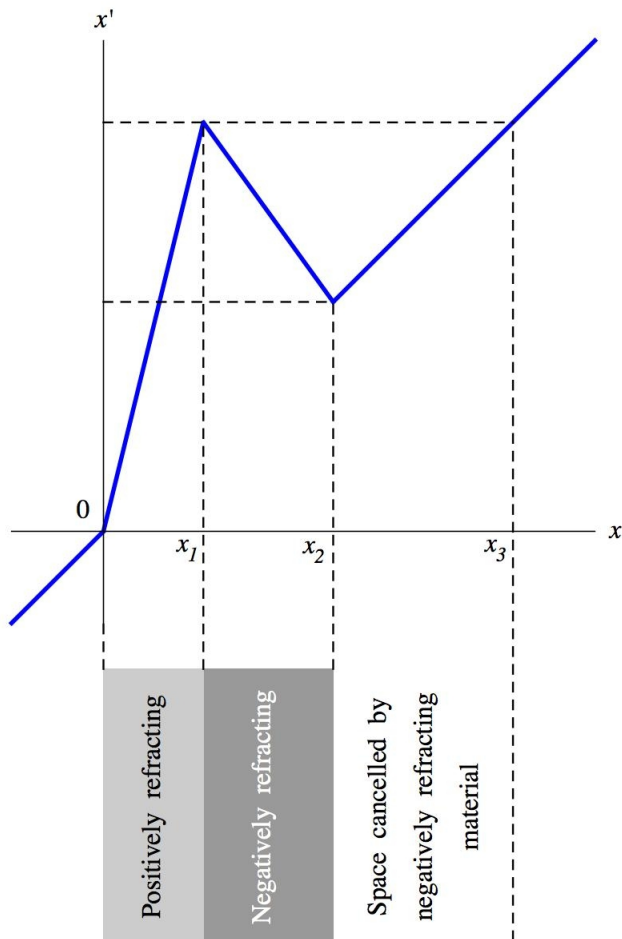


curved space

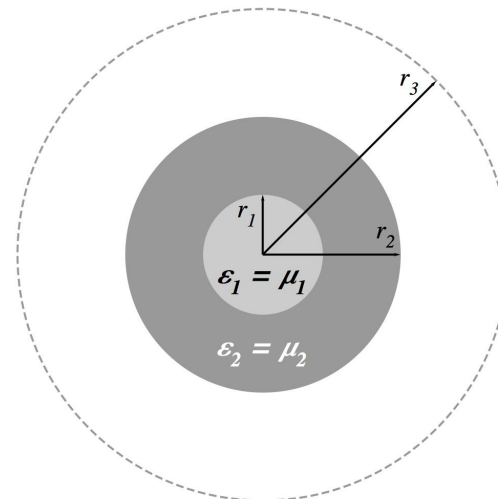
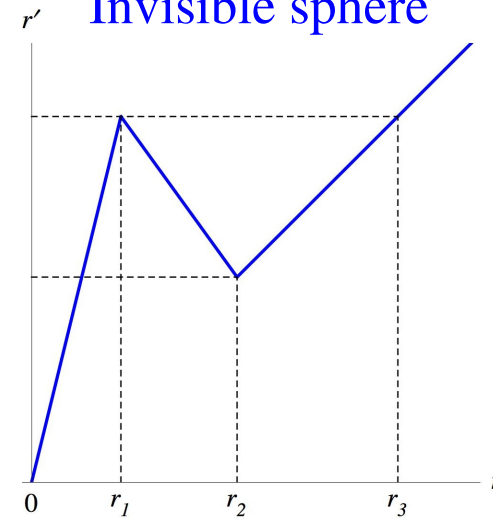
Cloaking at a distance

[Lai, Chen, Zhang and Chan, Phys. Rev. Lett. **102**, 093901 (2009)]

Invisible slab

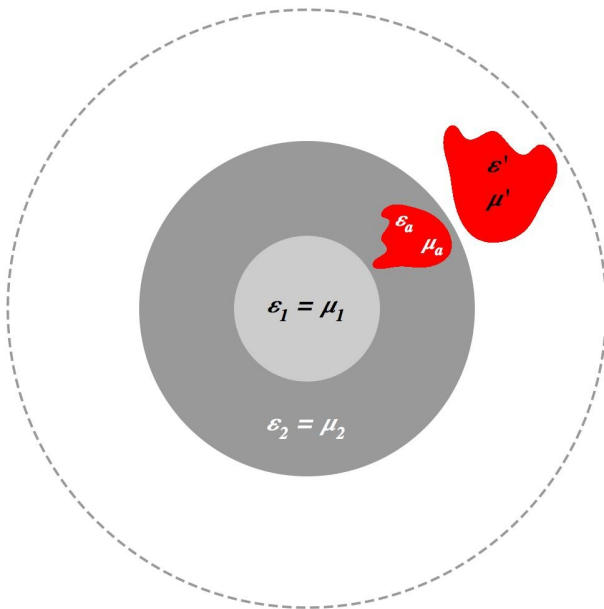
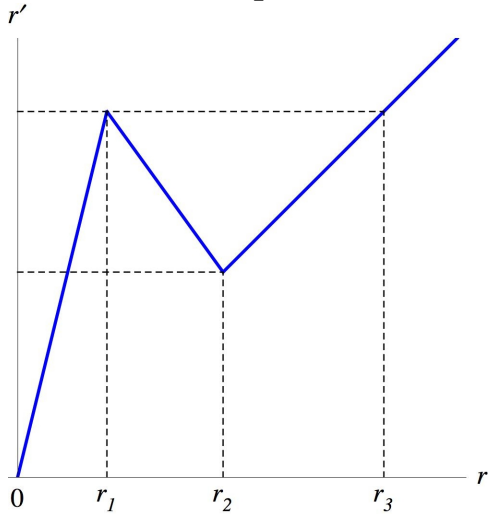


Invisible sphere



The Invisible Man - cloaking at a distance

[Lai, Chen, Zhang and Chan, Phys. Rev. Lett. **102**, 093901 (2009)]



Born and Wolf

**Principles
of Optics**

7th (expanded) edition

CAMBRIDGE

Principles of optics

*Electromagnetic theory of propagation,
interference and diffraction of light*

MAX BORN

MA, Dr Phil, FRS

Nobel Laureate

Formerly Professor at the Universities of Göttingen and Edinburgh

and

EMIL WOLF

PhD, DSc

Wilson Professor of Optical Physics, University of Rochester, NY

Section “Perfect imaging”

LUNEBURG

MATHEMATICAL THEORY OF OPTICS

CALIFORNIA

QC
355.L8

THE SCIENTIFIC PAPERS OF JAMES CLERK MAXWELL

MATHEMATICAL THEORY OF OPTICS

by
R. K. LUNEBURG

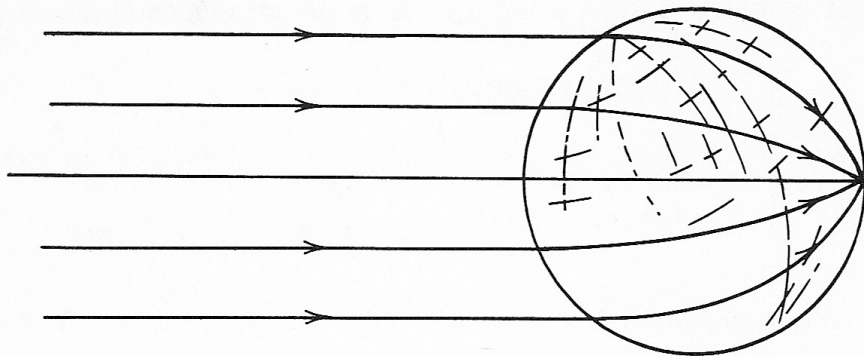
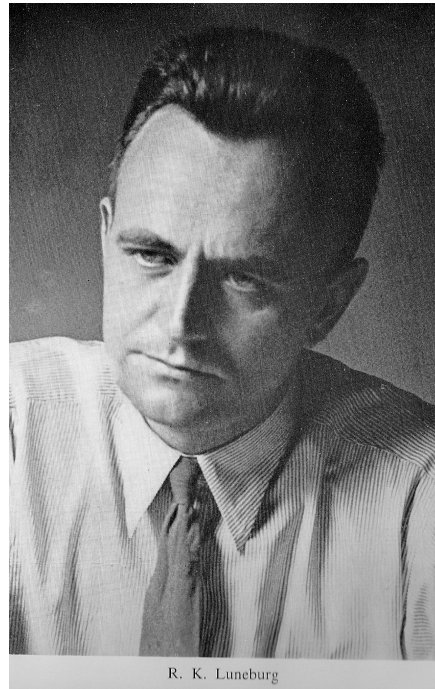
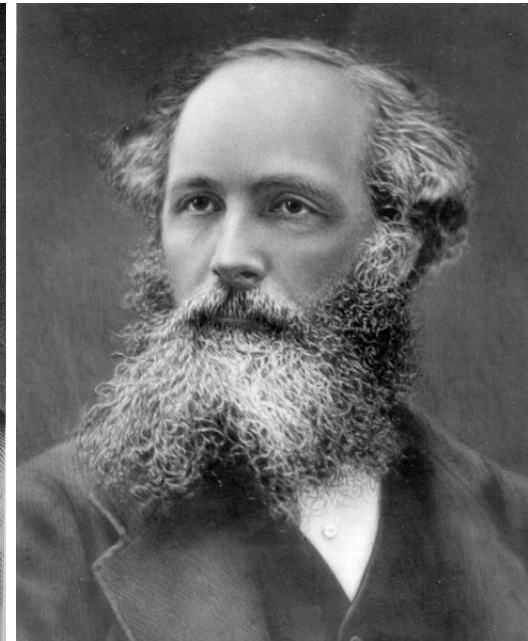


Figure 114



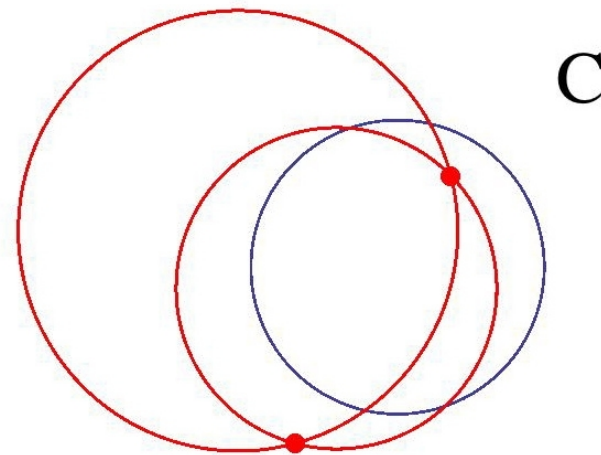
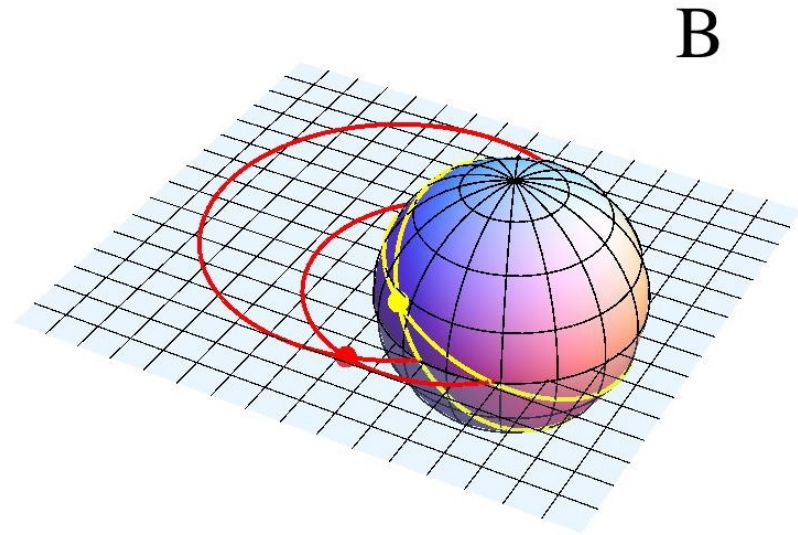
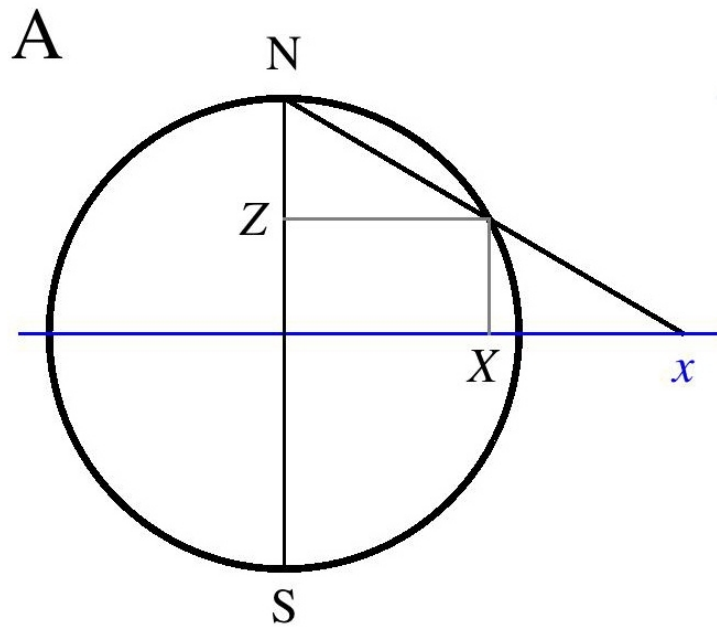
R. K. Luneburg



Maxwell's fish eye makes a perfect lens

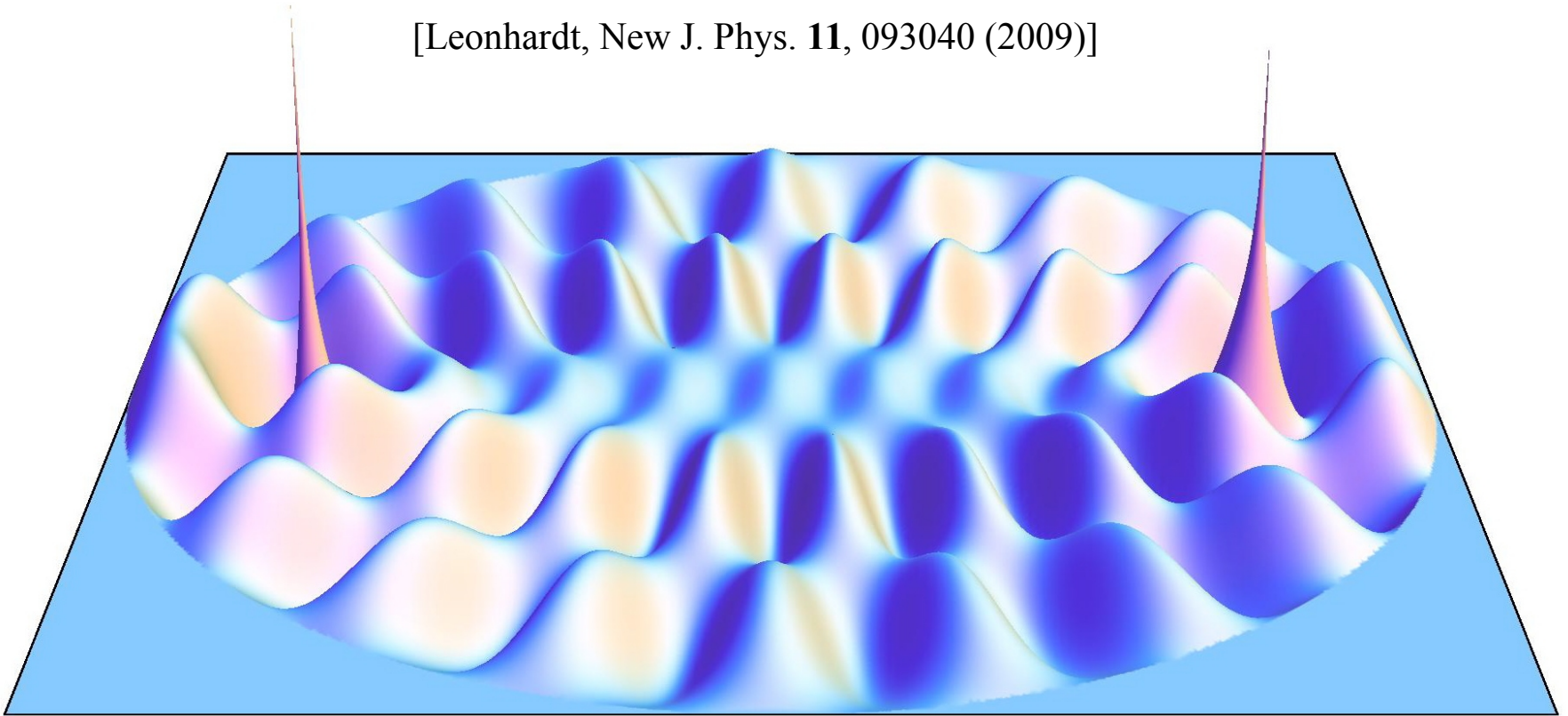
Maxwell 1854

Luneburg 1944: Stereographic projection



Perfect imaging without negative refraction

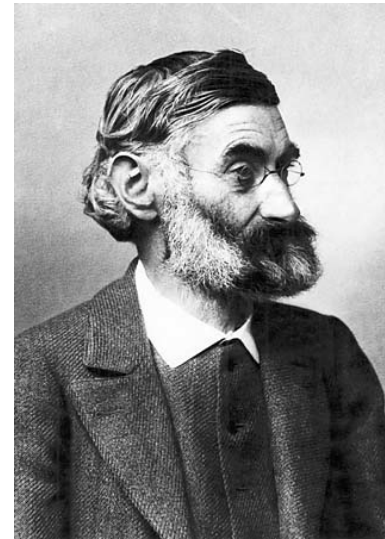
[Leonhardt, New J. Phys. **11**, 093040 (2009)]



$$n = \frac{2n_0}{1 + r^2 / r_0^2}$$

Index contrast: factor of 2

Feynman's objection to the diffraction limit

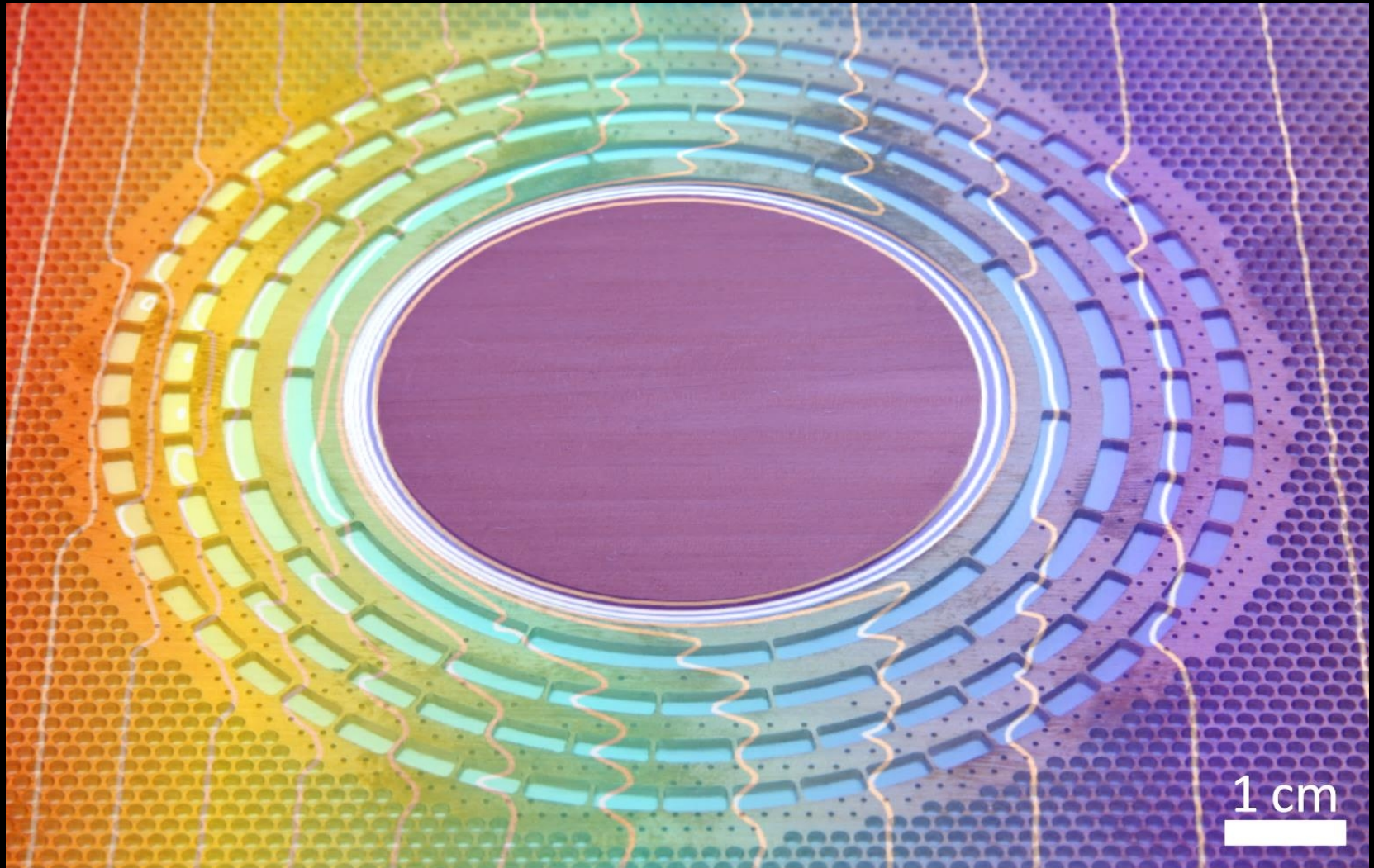


Maxwell's equations are time-reversible!
But you have to inverse the source, too.

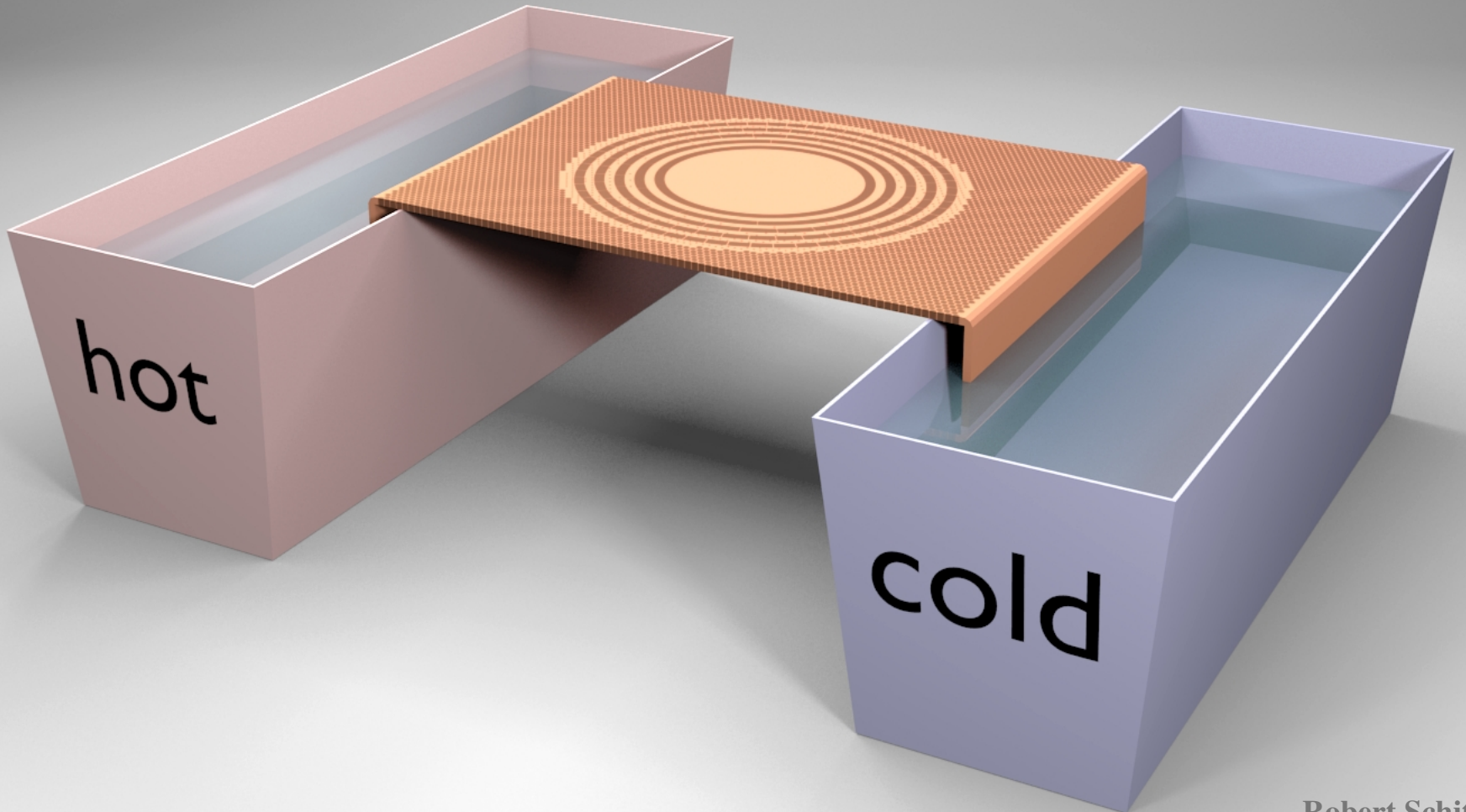
Thermal cloaking



Thermal Cloaking



IR-
camera

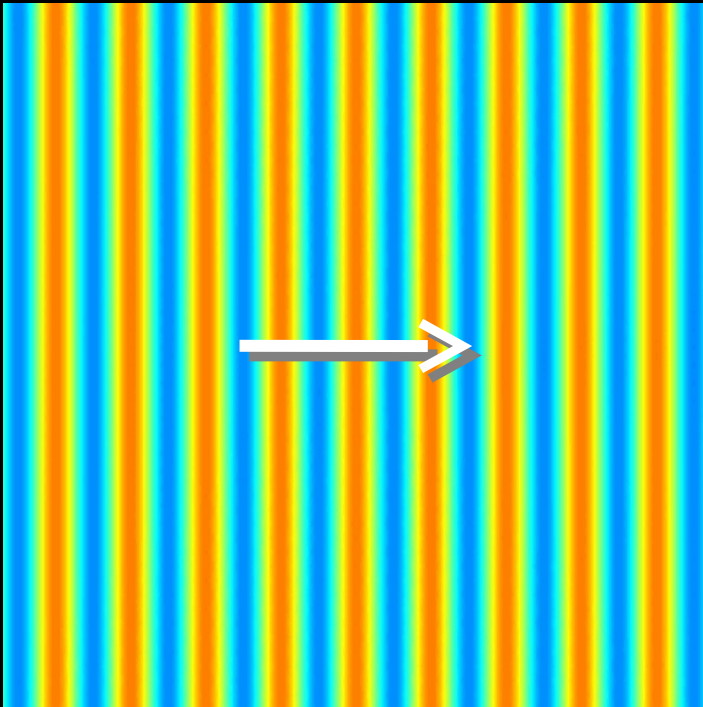


Thermal cloaking is not just thermal isolation

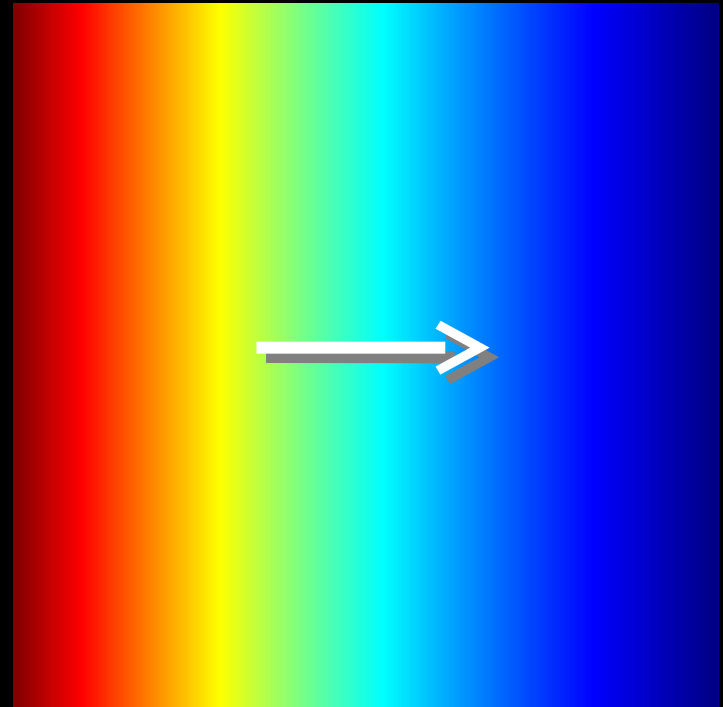


“Free Space”

optics



thermodynamics



Simple Isolation

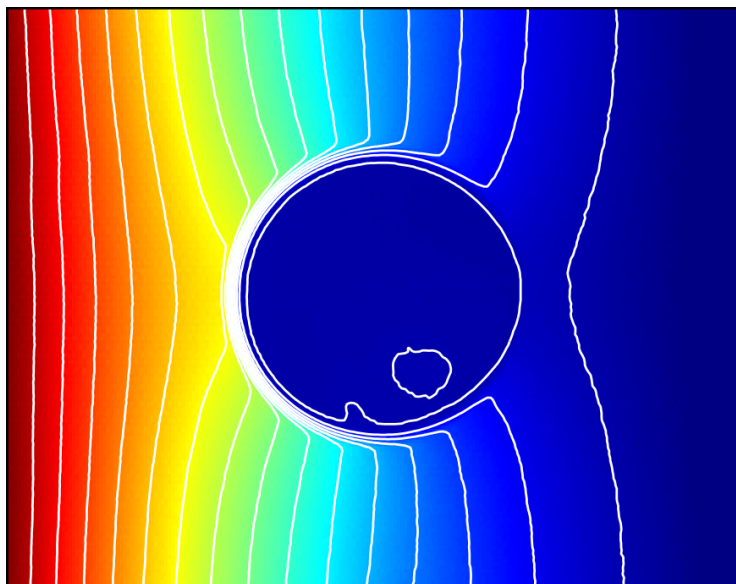
optics



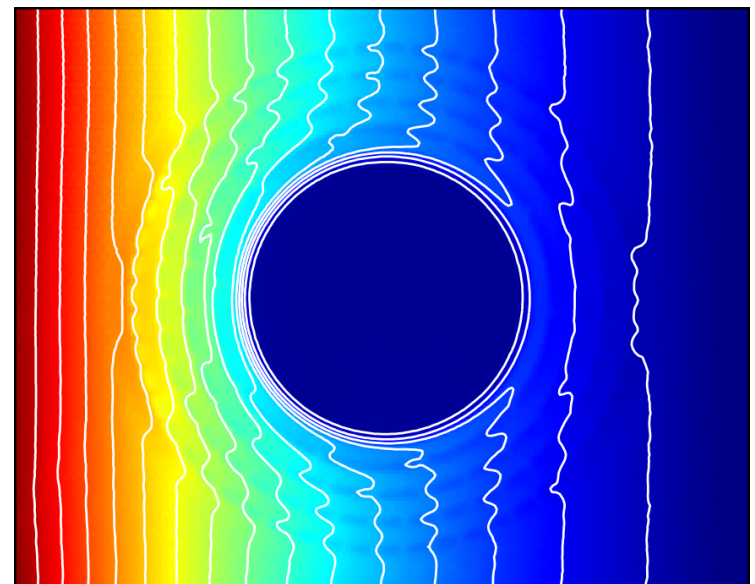
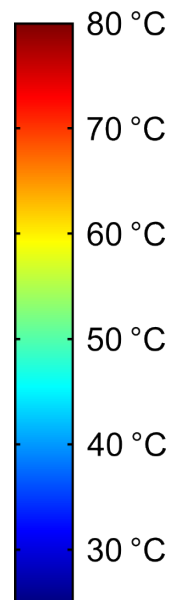
thermodynamics



simple isolation



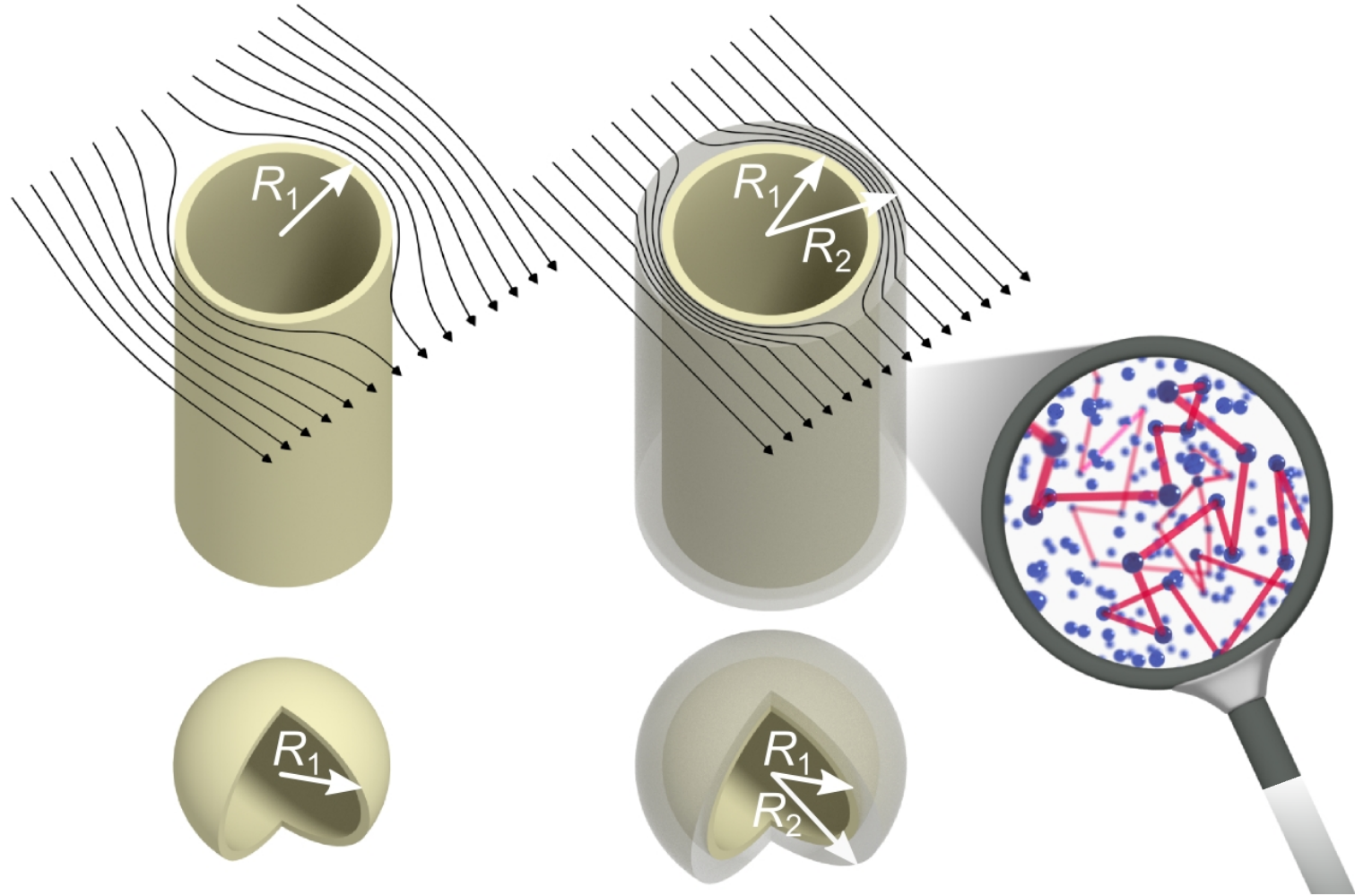
complete cloak



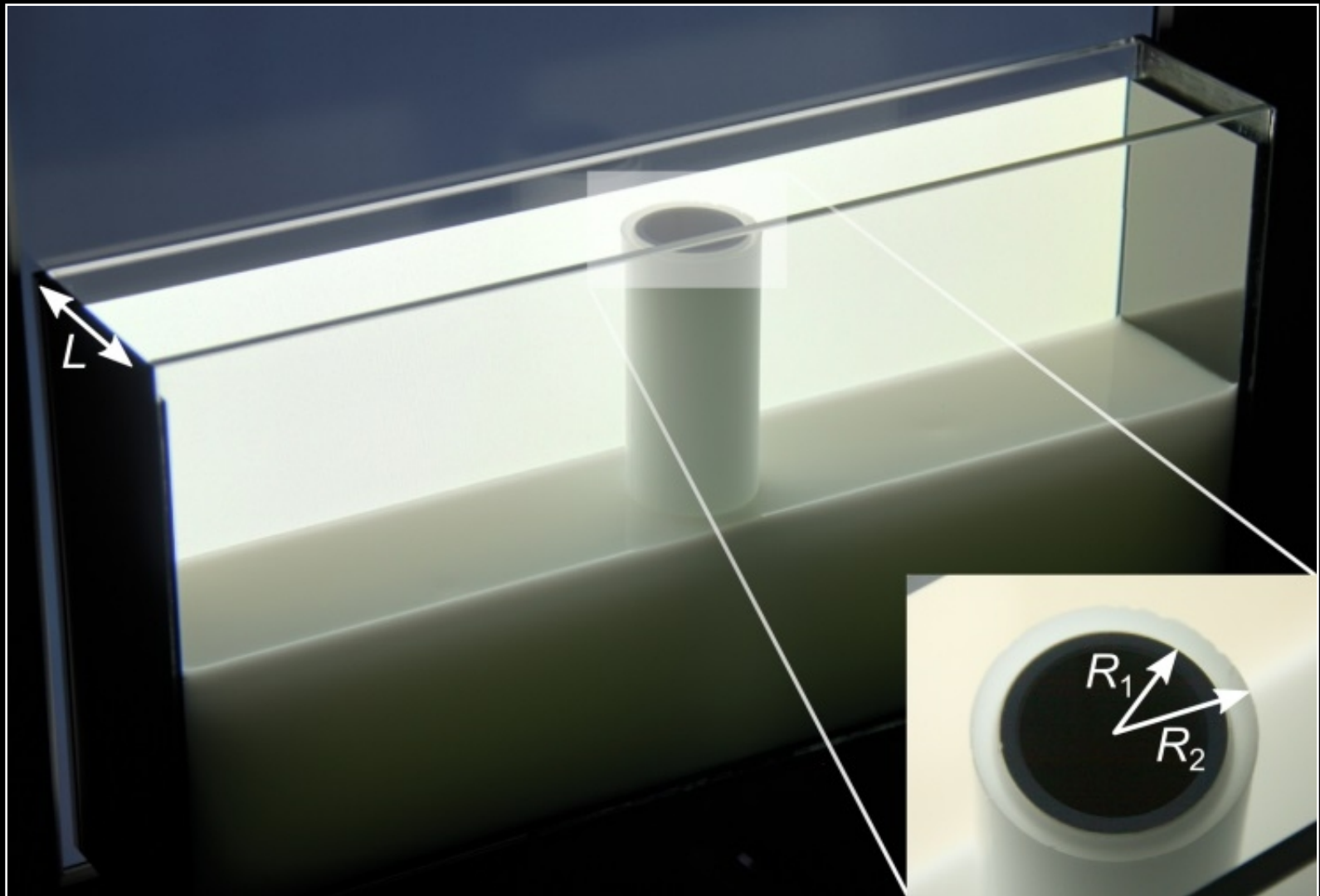




Invisible for Diffusive Light



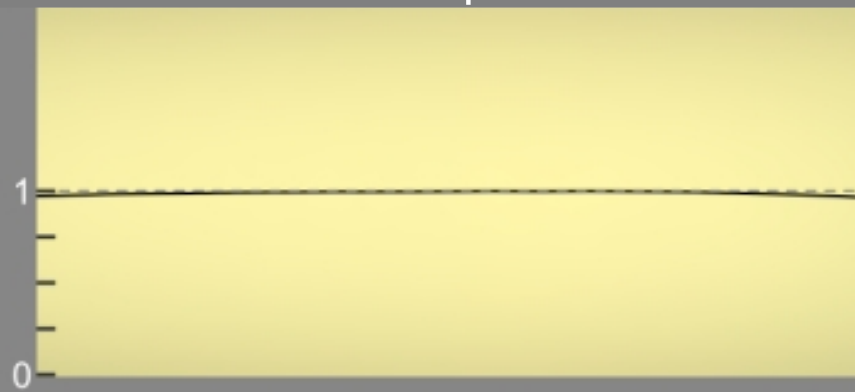
Experimental Setup



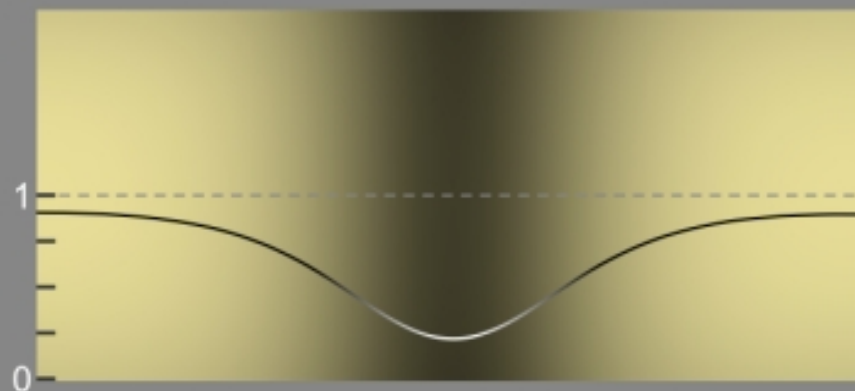
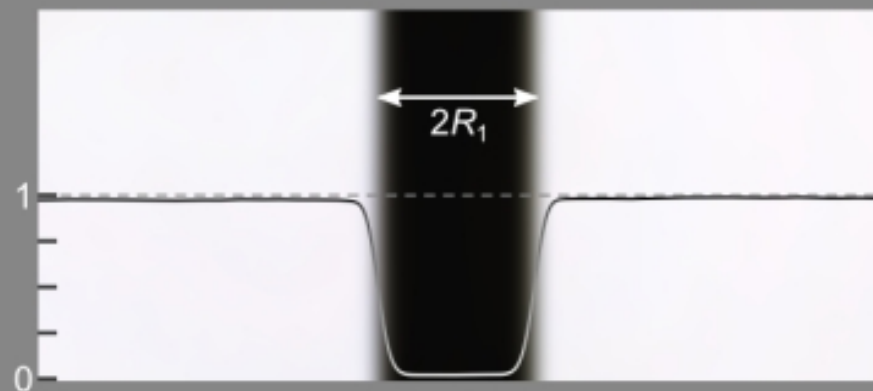
$$L = 6.0 \text{ cm}, 2R_1 = 3.2 \text{ cm}, 2R_2 = 4.0 \text{ cm}$$

air

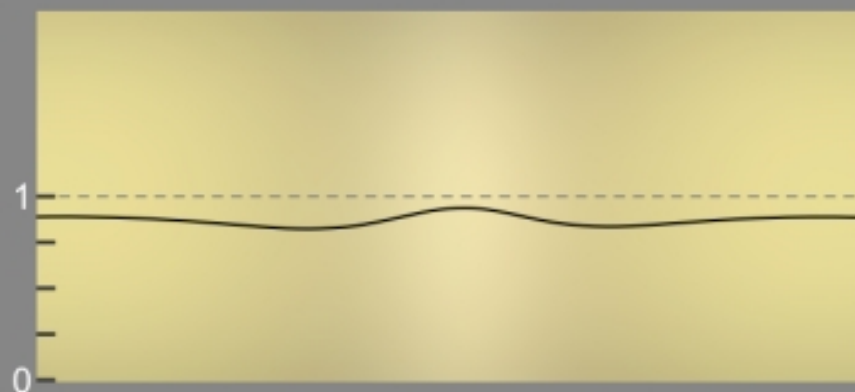
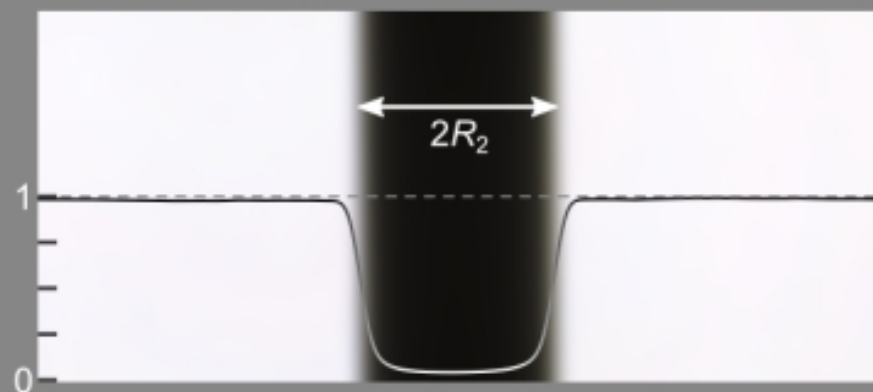
water-paint



reference



obstacle



cloak



Sound waves and sonar



Sound waves and sonar



Sound waves and sonar

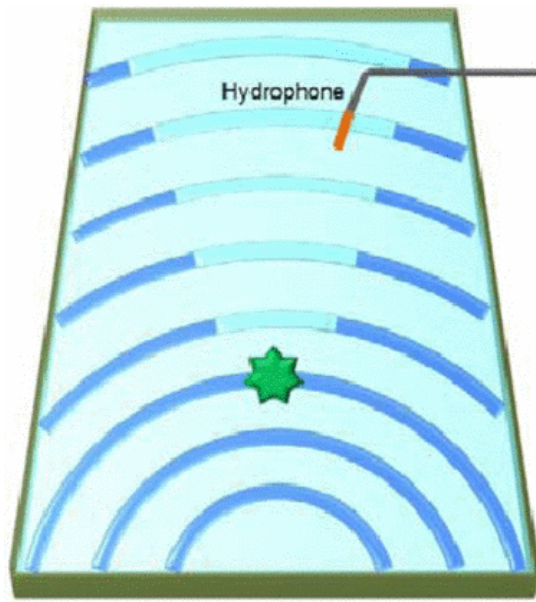
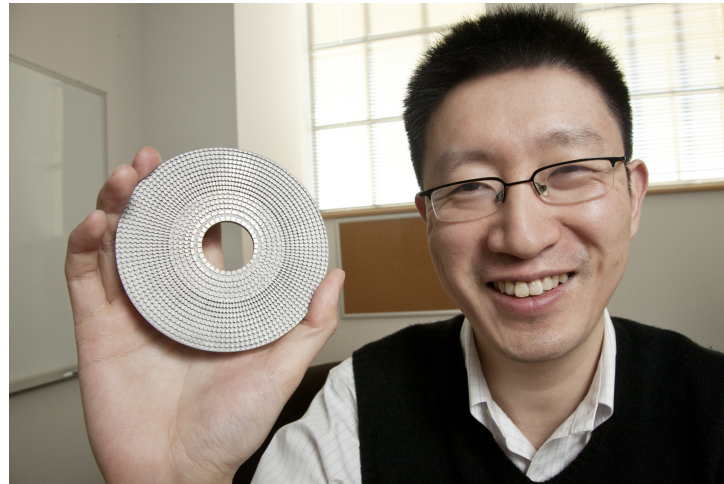


SONAR
HOW IT WORKS

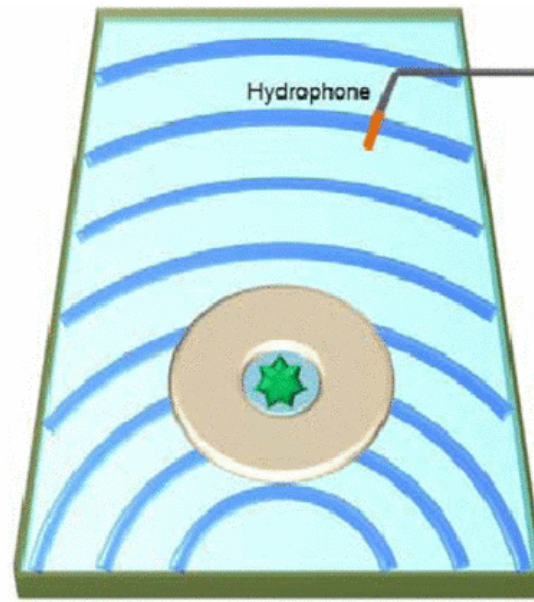
Cloaking against sound waves

Nicholas Fang,
MIT

Photo: L. Brain
Stauffer

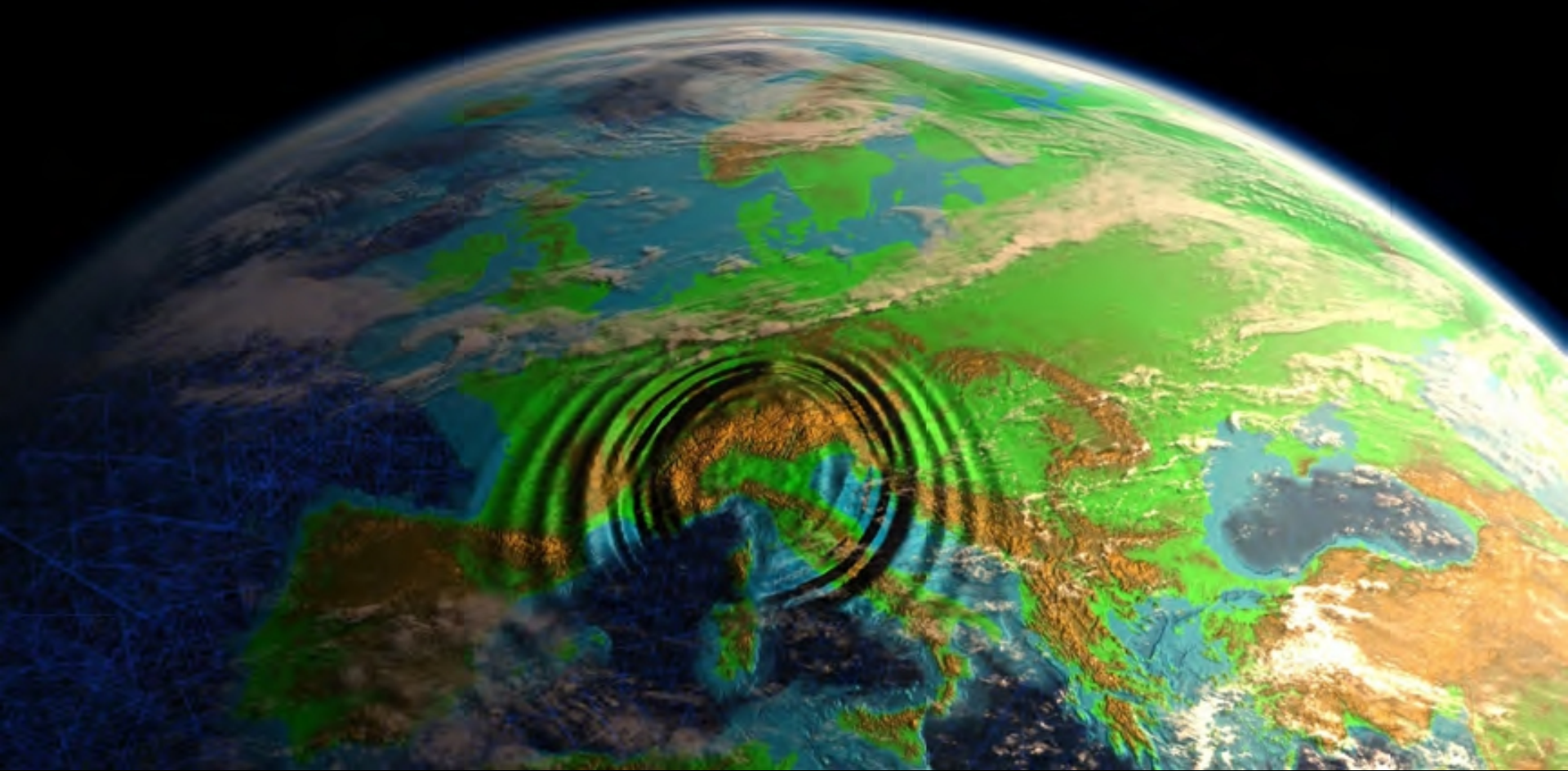


(a)



(b)

Earthquakes







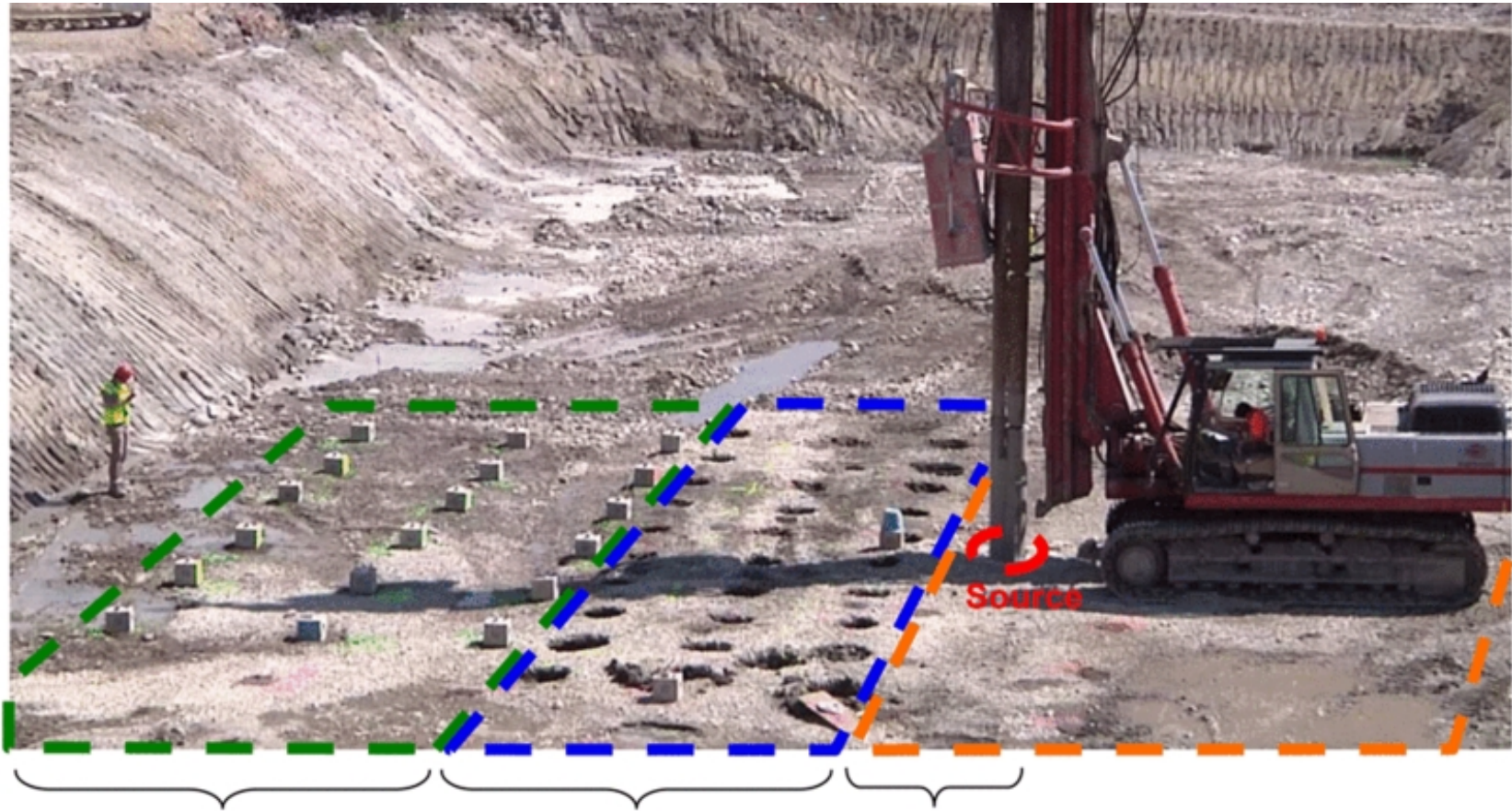
Experiments on Seismic Metamaterials: Molding Surface Waves

S. Brûlé,¹ E. H. Javelaud,¹ S. Enoch,² and S. Guenneau²

¹Ménard, 91 620 Nozay, France

²Aix-Marseille Université, CNRS, Centrale Marseille, Institut Fresnel, UMR 7249, 13013 Marseille, France

(Received 18 May 2013; published 31 March 2014)



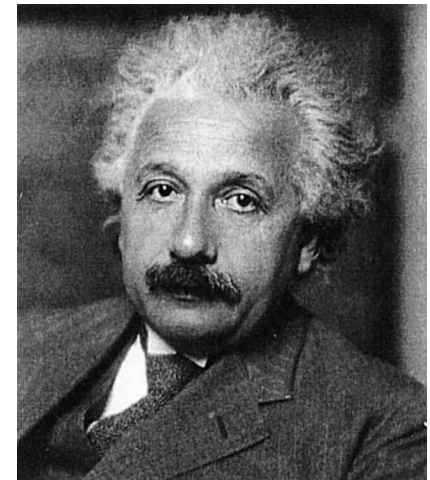
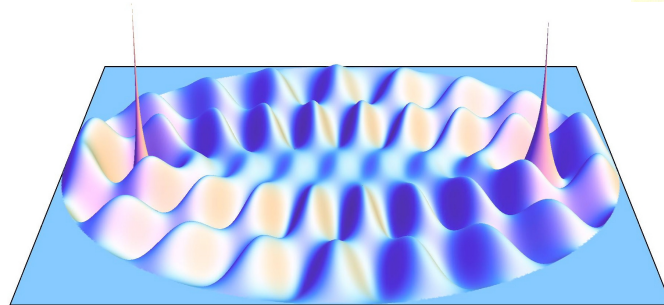
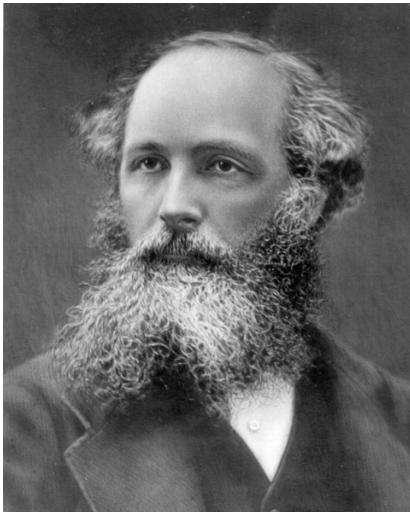
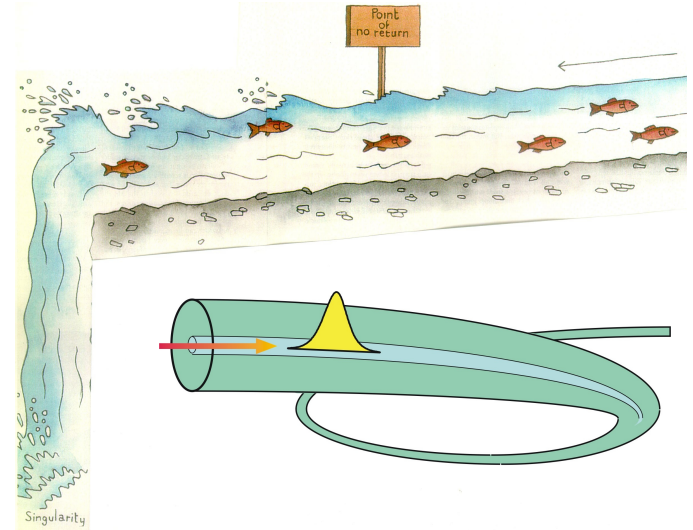
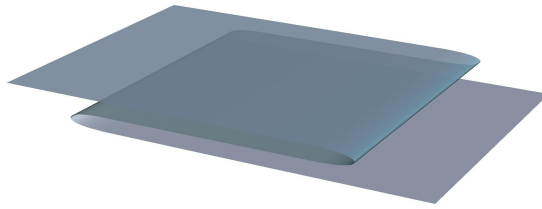
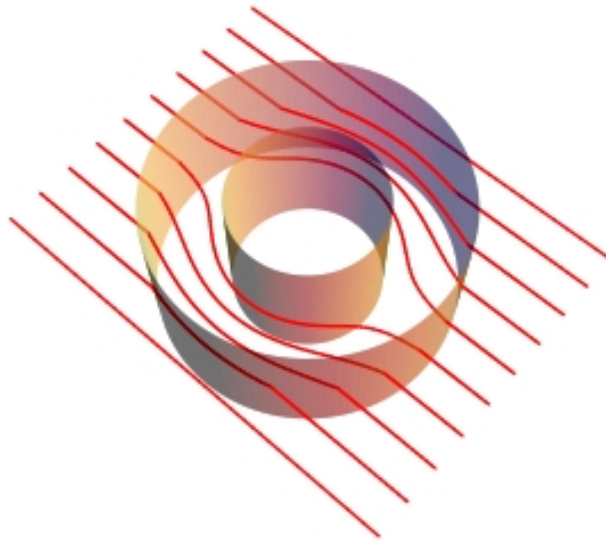
Sensitive three components
velocimeters (green grid)

Five meters deep
320 mm holes

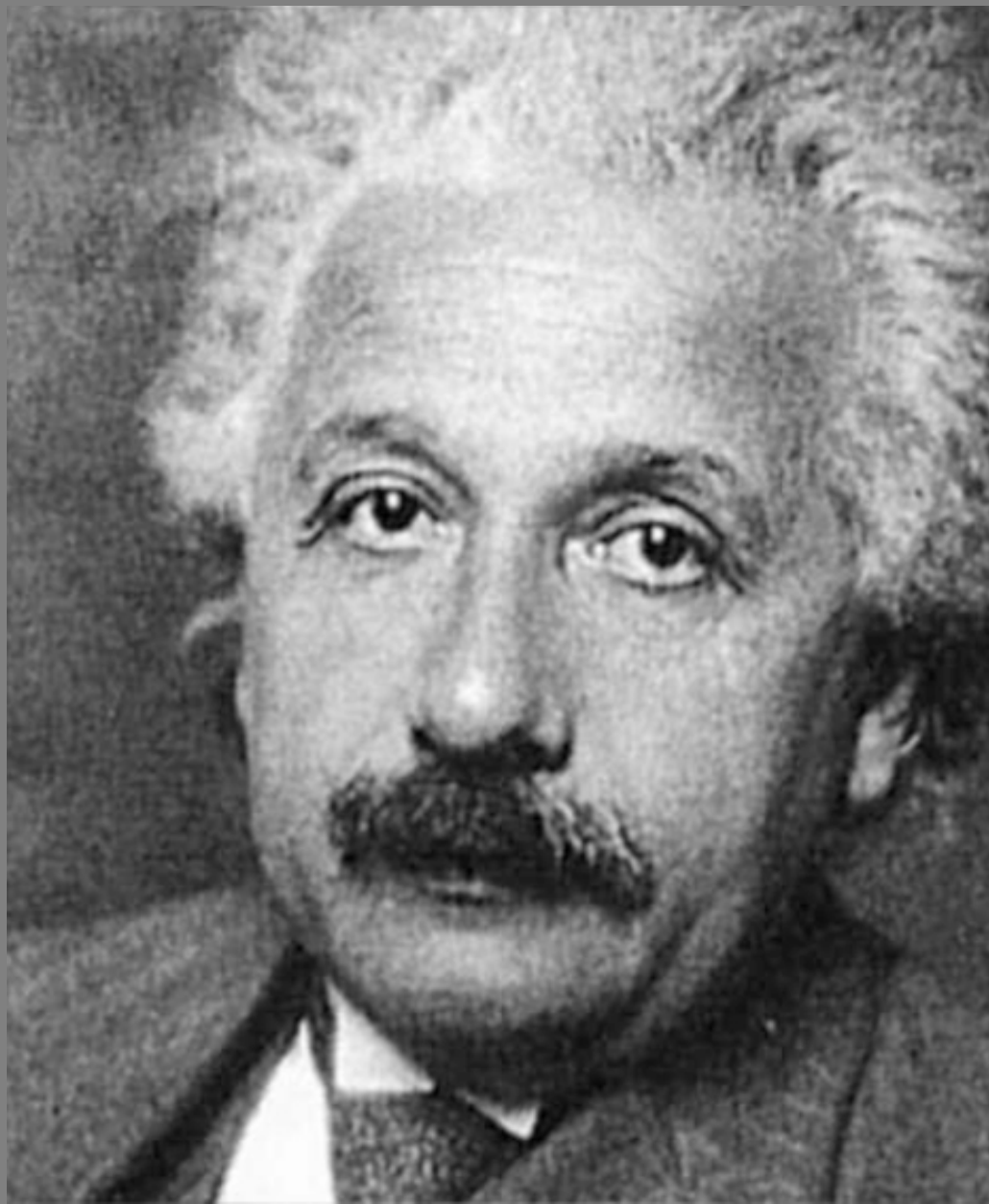
Source :
- Frequency : 50 Hz
- Horizontal displacement : 14 mm

General relativity in electrical engineering

[Leonhardt and Philbin, New J. Phys. **8**, 247 (2006)]



Einwell and Maxstein



ESSENTIAL

Quantum Optics

From Quantum Measurements to Black Holes

ULF LEONHARDT

CAMBRIDGE

Ulf Leonhardt and Thomas Philbin

GEOMETRY AND LIGHT

The Science of
INVISIBILITY

