

# *A dipolar quantum gas with supersolid properties*

**Giovanni Modugno**

Dipartimento di Fisica e Astronomia and LENS,  
Università di Firenze,  
and CNR-INO, sezione di Pisa



*Quantum Simulations of  
Insulators and Conductors*



**INO-CNR**  
ISTITUTO  
NAZIONALE DI  
OTTICA

# First proposals

## *QUANTUM THEORY OF DEFECTS IN CRYSTALS*

A. F. ANDREEV and I. M. LIFSHITZ

Institute of Physical Problems, U.S.S.R. Academy of Sciences

Submitted January 15, 1969

Zh. Eksp. Teor. Fiz. **56**, 2057–2068 (June, 1969)

At sufficiently low temperatures localized defects or impurities change into excitations that move practically freely through a crystal. As a result instead of the ordinary diffusion of defects, there arises a flow of a liquid consisting of “defectons” and “impuritons.” It is shown that at absolute zero in crystals with a large amplitude of the zero-point oscillations (for example, in crystals of the solid helium type) zero-point defectons may exist, as a result of which the number of sites of an ideal crystal lattice may not coincide with the number of atoms. The thermodynamic and acoustic properties of crystals containing zero-point defectons are discussed. Such a crystal is neither a solid nor a liquid. Two kinds of motion are possible in it; one possesses the properties of motion in an elastic solid, the second possesses the properties of motion in a liquid. **Under certain conditions the “liquid” type of crystal motion possesses the property of superfluidity.** Similar effects should also be observed in quasiequilibrium states containing a given number of defectons.

Also:

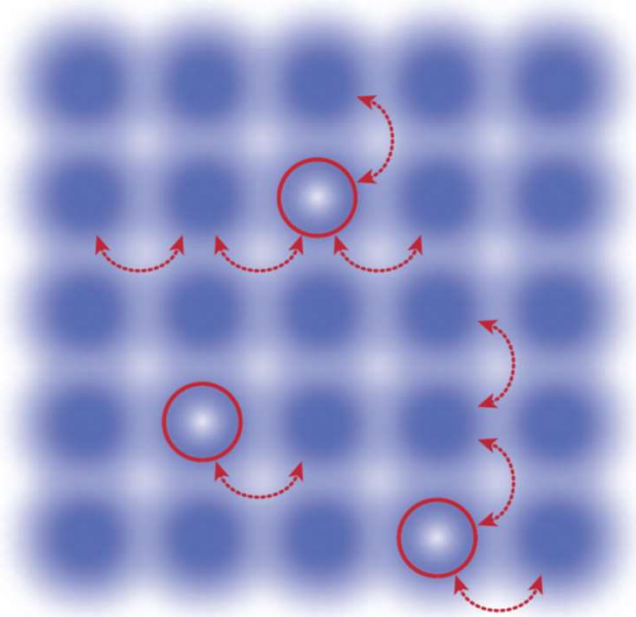
E. P. Gross, Phys. Rev. 106, 161 (1957)

D.J. Thouless, Ann. Phys, 52, 403 (1969)

G.V. Chester, Phys. Rev. 2, 161 (1970)

A.J. Leggett, Phys. Rev. Lett. 25, 1543 (1970)

# First proposals



The large **zero-point motion** in solid He allows the atoms to exchange their positions:

$$\Lambda = \frac{h}{a\sqrt{m\epsilon}} > 1$$

$a$ : interatomic distance  
 $\epsilon$ : interaction energy

Atom vacancies in  $^4\text{He}$  (defectons) can move, form a Bose-Einstein condensate and give rise to a superfluid mass transport.

The energy cost of creating a vacancy is fully compensated by the decrease of kinetic energy due to delocalization.

## Can a Solid Be “Superfluid”?

A. J. Leggett

*School of Mathematical and Physical Sciences, University of Sussex, Falmer, Brighton, Sussex, England*

(Received 15 September 1970)

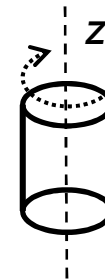
It is suggested that the property of nonclassical rotational inertia possessed by superfluid liquid helium may be shared by some solids. In particular, nonclassical rotational inertia very probably occurs if the solid is Bose-condensed as recently proposed by Chester. Anomalous macroscopic effects are then predicted. However, the associated superfluid fraction is shown to be very small (probably  $\lesssim 10^{-4}$ ) even at  $T=0$ , so that these effects could well have been missed. Direct tests are proposed.

Superfluids have a single wavefunction:

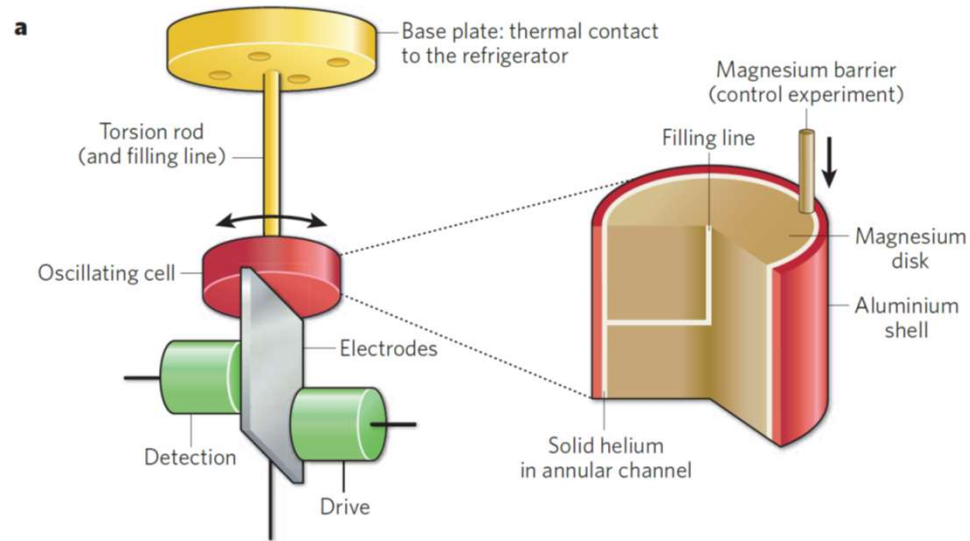
$$\Psi_0(r) = |\Psi_0(r)|e^{i\varphi(r)}$$

and the velocity is the gradient of the phase,  $v = (\hbar/m)\nabla\varphi$ , so it is irrotational ( $\nabla \times v=0$ ). As a consequence, the moment of inertia of a cylindrically symmetric superfluid is zero:

$$I = I_{rig} \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$



# The search in solid helium



Nature 427, 225 (2004)

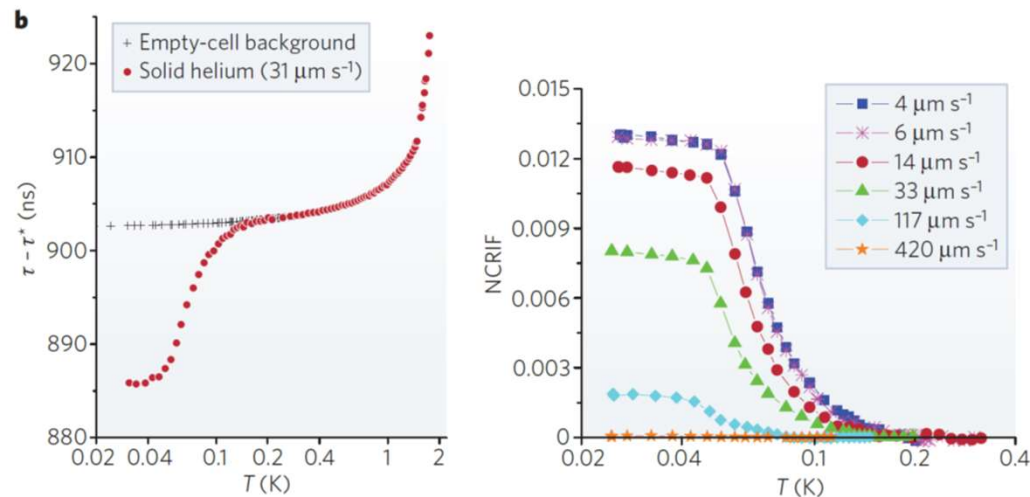
## Probable observation of a supersolid helium phase

E. Kim & M. H. W. Chan

Torsion oscillator:

$$\tau = 2\pi\sqrt{I/K}$$

$\tau$ : oscillation period  
 $I$ : moment of inertia  
 $K$ : elastic constant



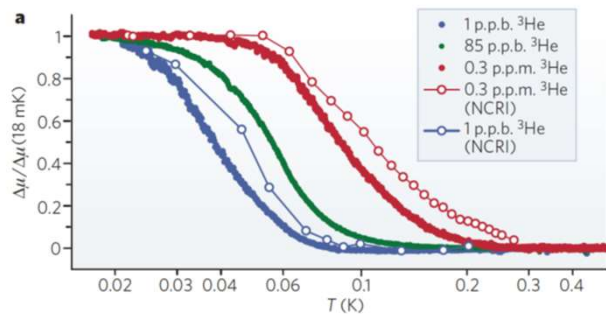
Review: S. Balibar, Nature 464, 176 (2010).

# The search in solid helium

Problem: the energy cost in creating vacancies is very large (10 K):  
the fraction of vacancies at 100 mK must be practically zero!

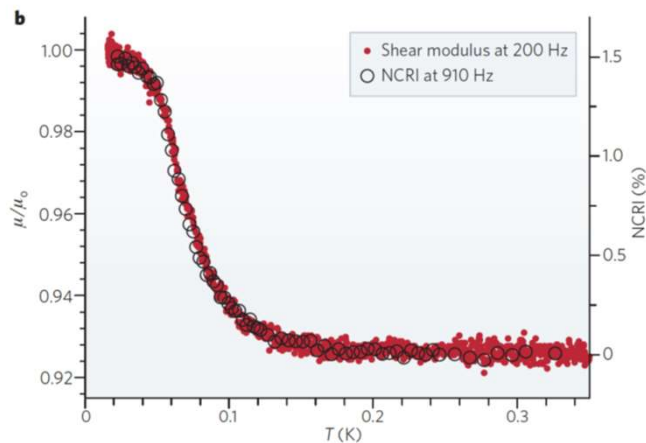
Other possible explanations for supersolidity in He:

- Lattice dislocations
- $^3\text{He}$  impurities (naturally  $10^{-7}$ )

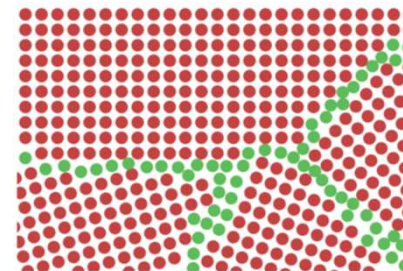


Problem: the change of period might be explained with a change of the elastic constant!

$$\tau = 2\pi\sqrt{I/K}$$



Dislocations change state when lowering the temperature and the crystal stiffens.



Review: S. Balibar, Nature 464, 176 (2010)



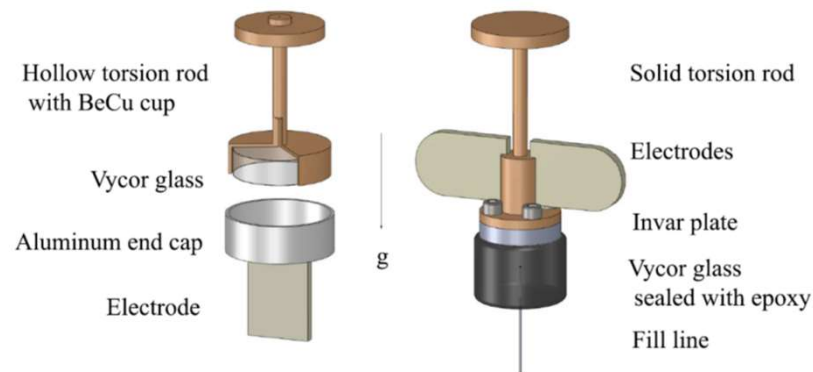
# The search in solid helium

Phys. Rev. Lett. 109, 155301 (2012)

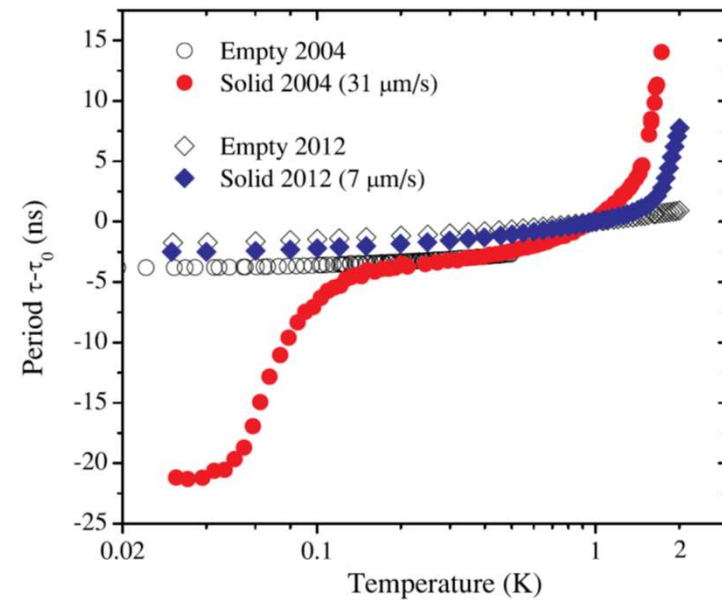
## Absence of Supersolidity in Solid Helium in Porous Vycor Glass

Duk Y. Kim and Moses H. W. Chan\*

Department of Physics, Pennsylvania State University, University Park, Pennsylvania 16802, USA  
(Received 24 July 2012; published 8 October 2012)



No reduction of the moment of inertia if bulk solid He is excluded. This contradicts the original experiment!



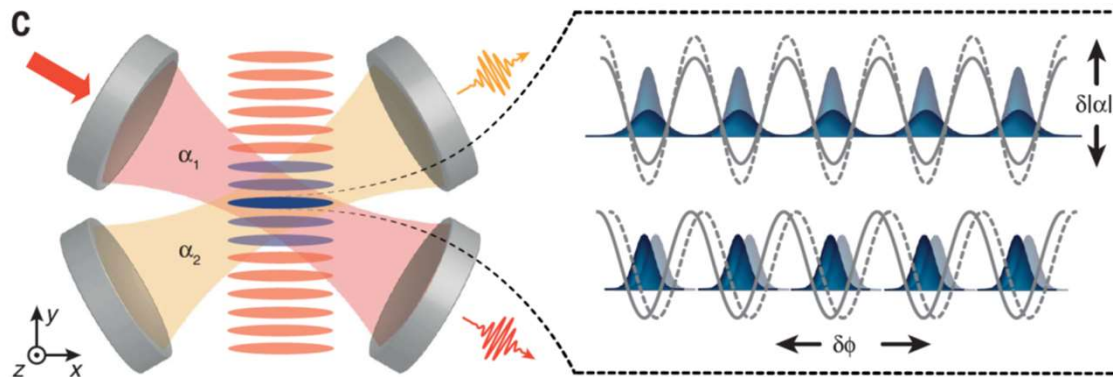
More refined experiments still do not exclude supersolidity: J. D. Reppy et al., PNAS 3203 (2016).

# Supersolids in quantum gases

gaseous Bose-Einstein condensates (**superfluidity**)  
+  
long-range interactions (**density modulation**)

Proposals for:

- Rydberg atoms with soft-core interactions
- strongly dipolar atoms
- spin-orbit coupled atoms (J.R. Li et al., Nature 543 (2017))
- atoms in optical cavities (J. Leonard et al., Nature 543 (2017))

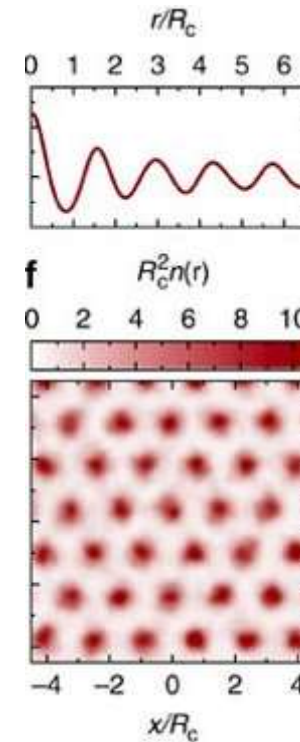
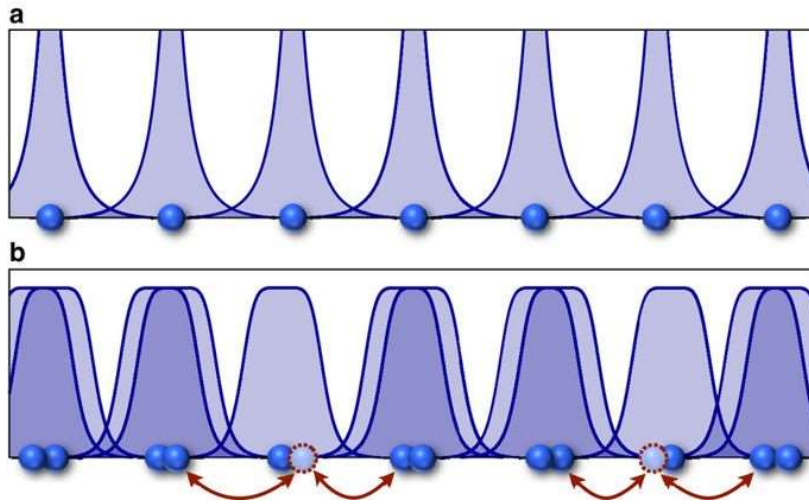


Light-coupled supersolids are perfectly stiff: the supersolid is not compressible.



# Supersolids in quantum gases

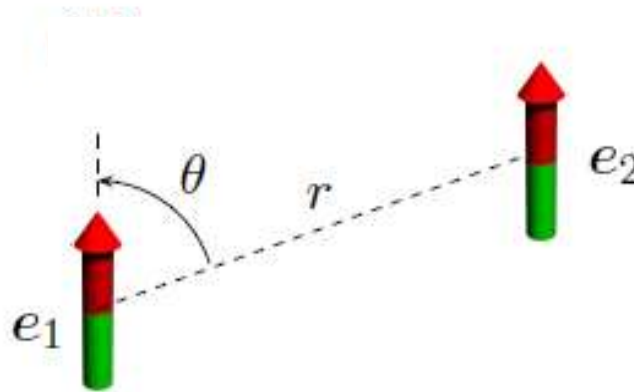
## Bosons with soft-core interactions



The key ingredient to establish superfluidity is to employ N-atom clusters and not single atoms:

Bosonic enhancement increases the strength of the phase links by  $(N+1)$ .

# Dipolar quantum gases



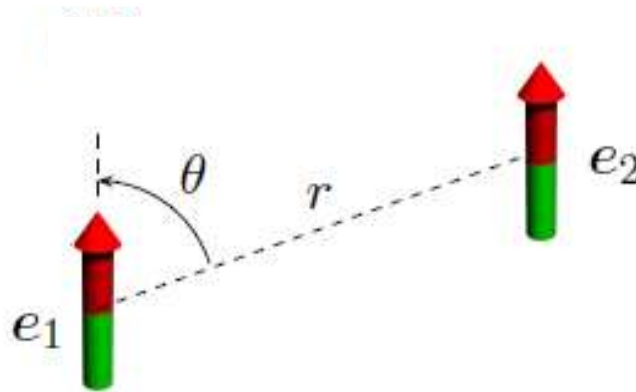
$$U(r) = \frac{4\pi\hbar^2}{m} a\delta(r) + \frac{\mu_0\mu^2}{4\pi} \frac{1 - 3\cos^2\vartheta}{r^3}$$

**Contact interaction:**  
Short-range  
Isotropic

**Dipole-dipole interaction**  
Long-range  
Anisotropic



# Dipolar quantum gases



$$U(r) = \frac{4\pi\hbar^2}{m} a \delta(r) + \frac{\mu_0 \mu^2}{4\pi} \frac{1 - 3 \cos^2 \vartheta}{r^3}$$

**Contact interaction:**  
Short-range  
Isotropic

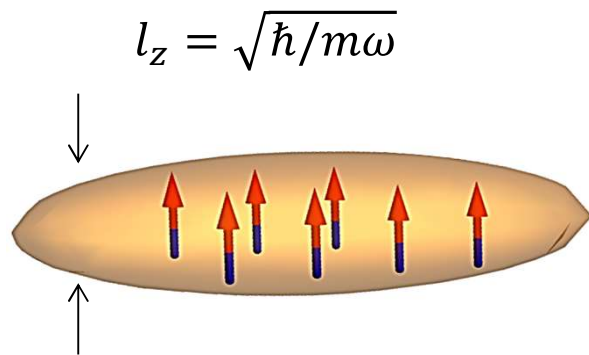
**Dipole-dipole interaction**  
Long-range  
Anisotropic

Dy atoms:  $\mu = 10 \mu_B$

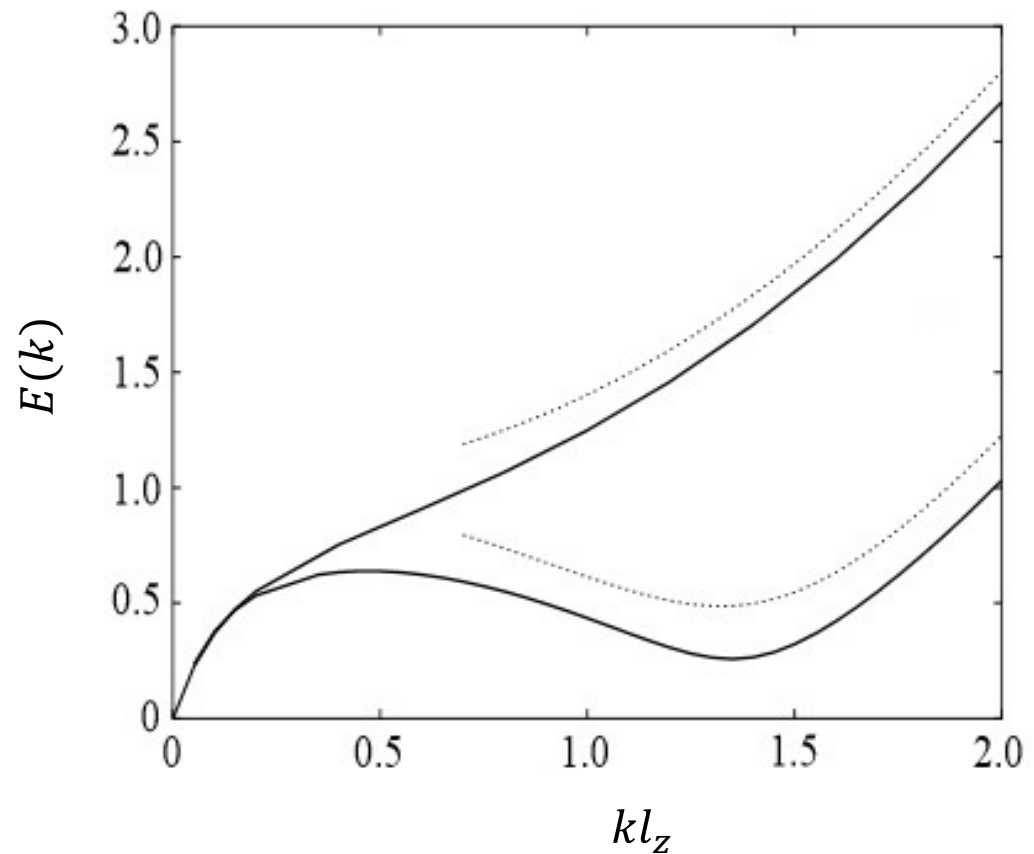
$$a_s \approx 160 a_0$$

$$a_{dd} = \frac{m \mu_0 \mu^2}{12 \pi \hbar^2} \approx 130 a_0$$

# Excitation spectrum in an elongated trap

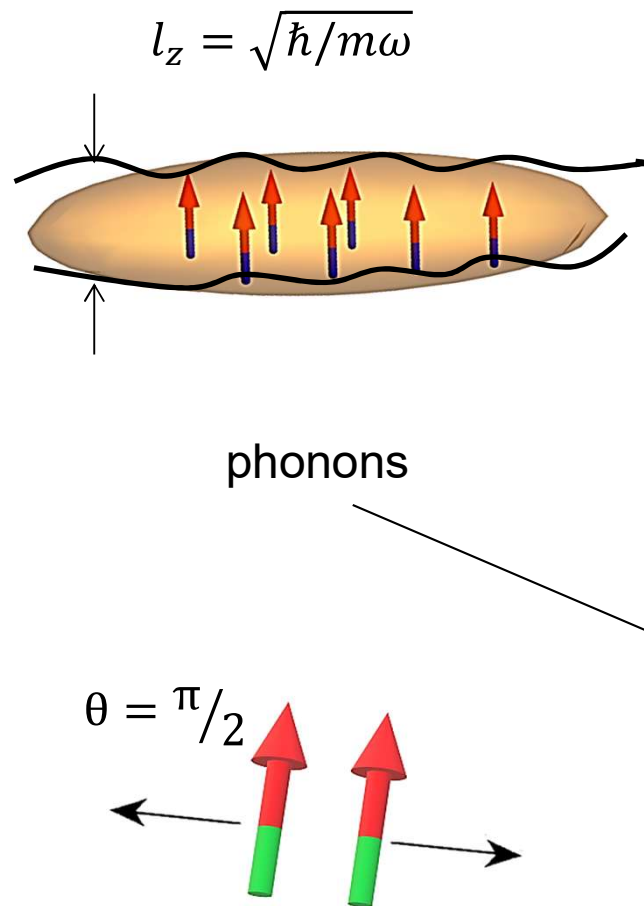


Mean-field picture (quantum fluctuations are neglected)

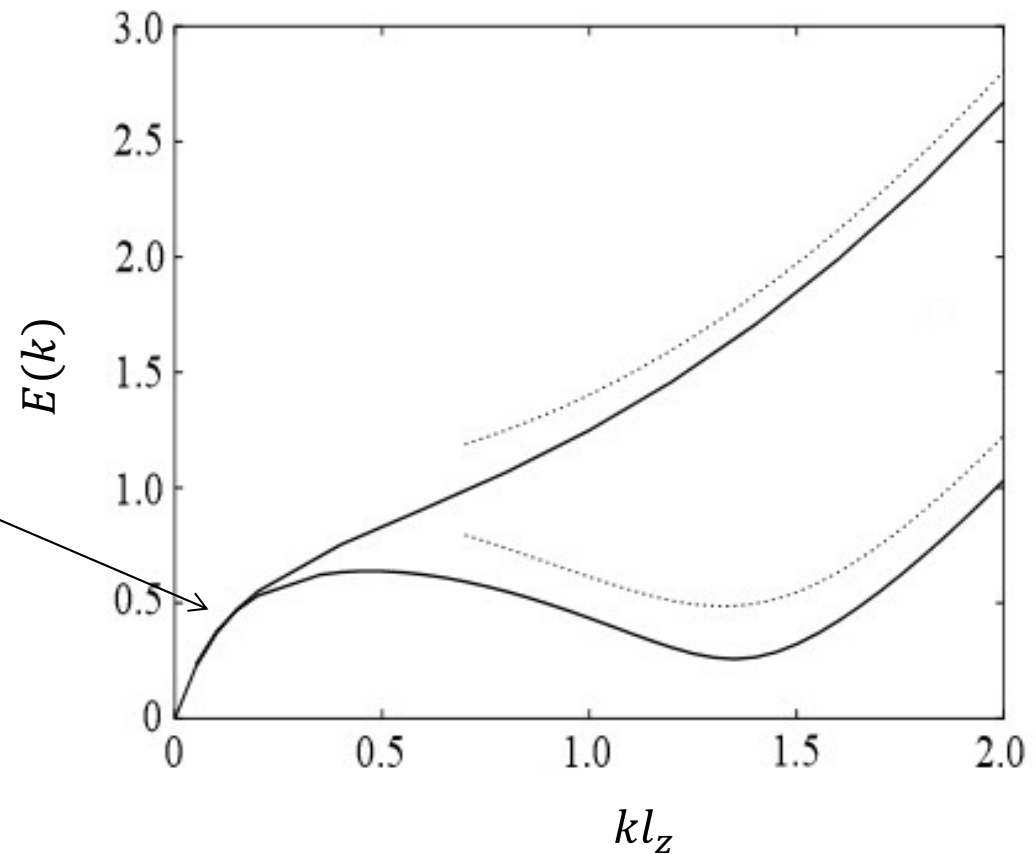


L. Santos, G. V. Shlyapnikov, and M. Lewenstein, Roton-Maxon spectrum and stability of trapped dipolar Bose-Einstein condensates, Phys. Rev. Lett. 90, 250403 (2003).

# Excitation spectrum in an elongated trap

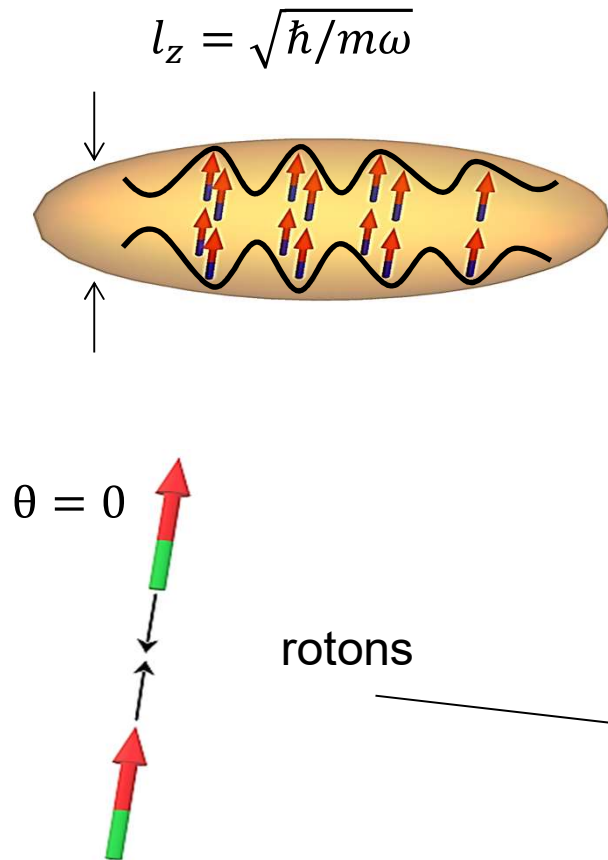


Mean-field picture (quantum fluctuations are neglected)



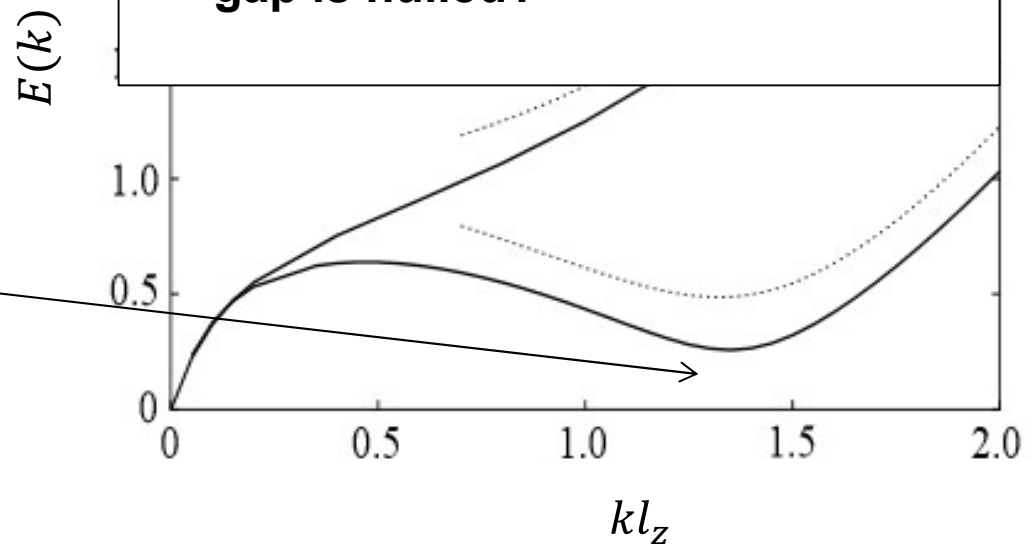
L. Santos, G. V. Shlyapnikov, and M. Lewenstein, Roton-Maxon spectrum and stability of trapped dipolar Bose-Einstein condensates, Phys. Rev. Lett. 90, 250403 (2003).

# Excitation spectrum in an elongated trap



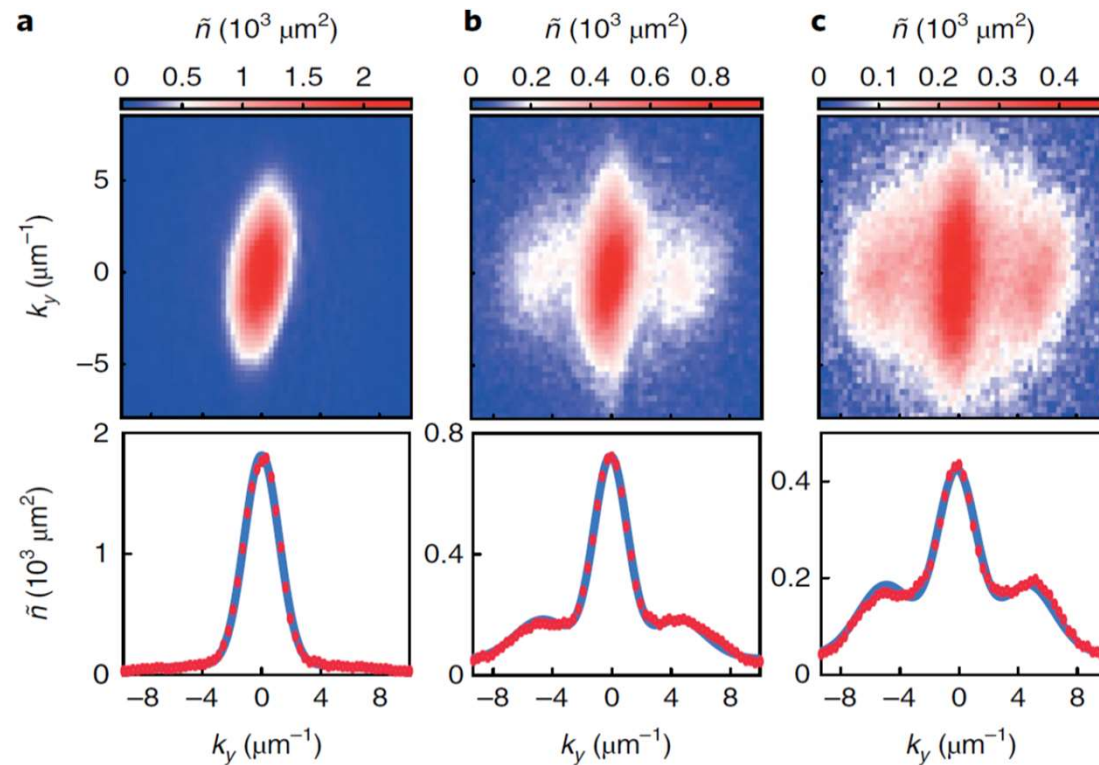
The roton gap can be changed by simply tuning the scattering length (no need to change pressure as in He).

**What happens when the roton gap is nulled?**





# Rotonic instability



Spontaneous population of the roton mode: transient density modulation (picture shows the momentum distribution), unclear what happens at long times.

**What happens then? Does the system collapse?**

Innsbruck, Er atoms ( $7 \mu_B$ ): L. Chomaz et al., Nat. Phys. 14, 442 (2018).

Attempt in Firenze, K atoms ( $1 \mu_B$ ): M. Fattori, et al., Phys. Rev. Lett. 101, 190405 (2008).

# Quantum fluctuations

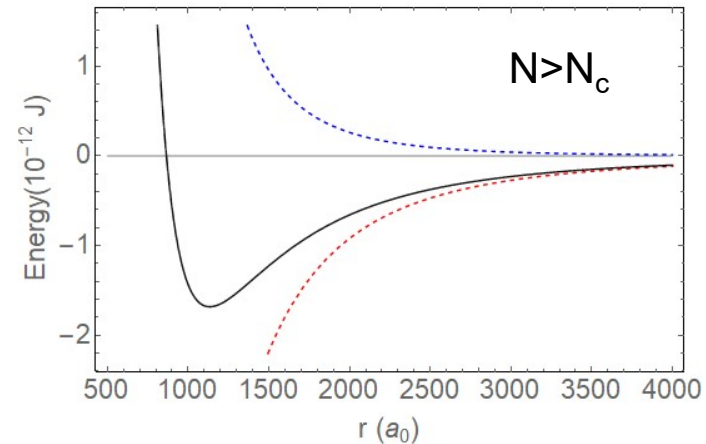
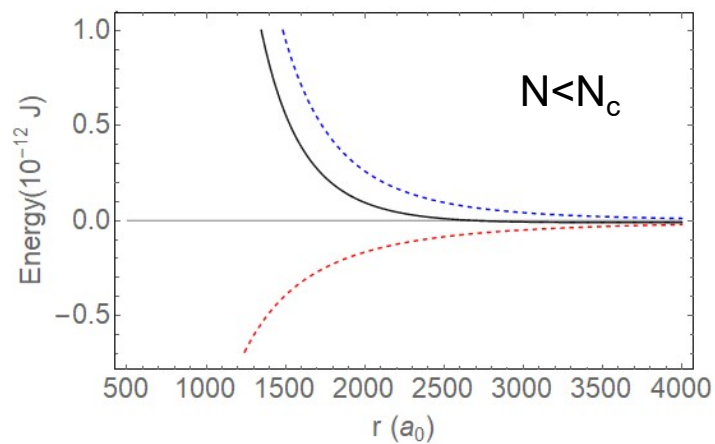
Quantum fluctuations lead to quantum depletion of the BEC and to an energy shift, due to the Lee-Huang-Yang energy term:

$$\frac{E_{\text{int}}}{V} = \frac{gn^2}{2} + E_{dd} + \frac{32ga^{3/2}}{3\sqrt{\pi}} \left(1 + \frac{3a_{dd}^2}{2a^2}\right) n^{5/2}$$

MF < 0

LHY > 0

Energy vs interatomic distance



## Ultradilute Quantum Droplets

**A new class of quantum mechanical liquids is stabilized by an elegant mechanism that allows them to exist despite being orders of magnitude thinner than air.**

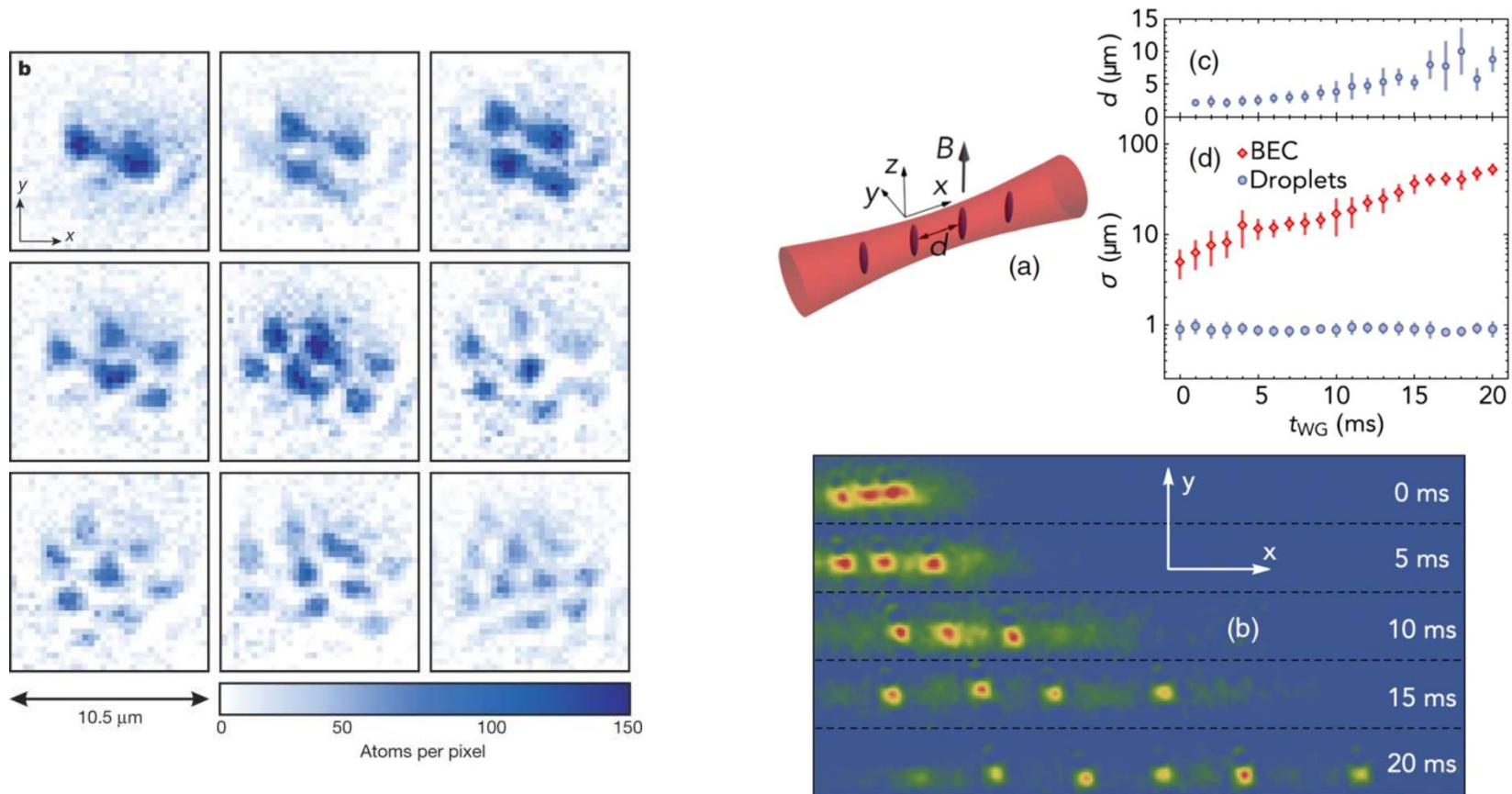
Igor Ferrier-Barbut

In his PhD thesis from 1873, Johannes van der Waals devised a theoretical framework to describe the gas and liquid phases of a molecular ensemble and the phase transition from one to the other. That work resulted in the celebrated equation of state bearing his name. To this day, the van der Waals theory is still the prevailing picture in most physicists' minds to explain the emergence of the liquid state. It asserts that the liquid state arises at high densities from an equilibrium between attractive interatomic forces and short-range repulsion. Now, a new type of liquid has emerged in ultracold, extremely dilute atomic systems for which the van der Waals model does not predict a liquid phase.

I. Ferrier-Barbut et al., *Physics Today*, April 2019.  
Theoretical proposal by D. S. Petrov – *Phys. Rev. Lett.* 115, 155302 (2015).

# Quantum droplets

Dipolar repulsion: periodic arrays of small, strongly-bound droplets.

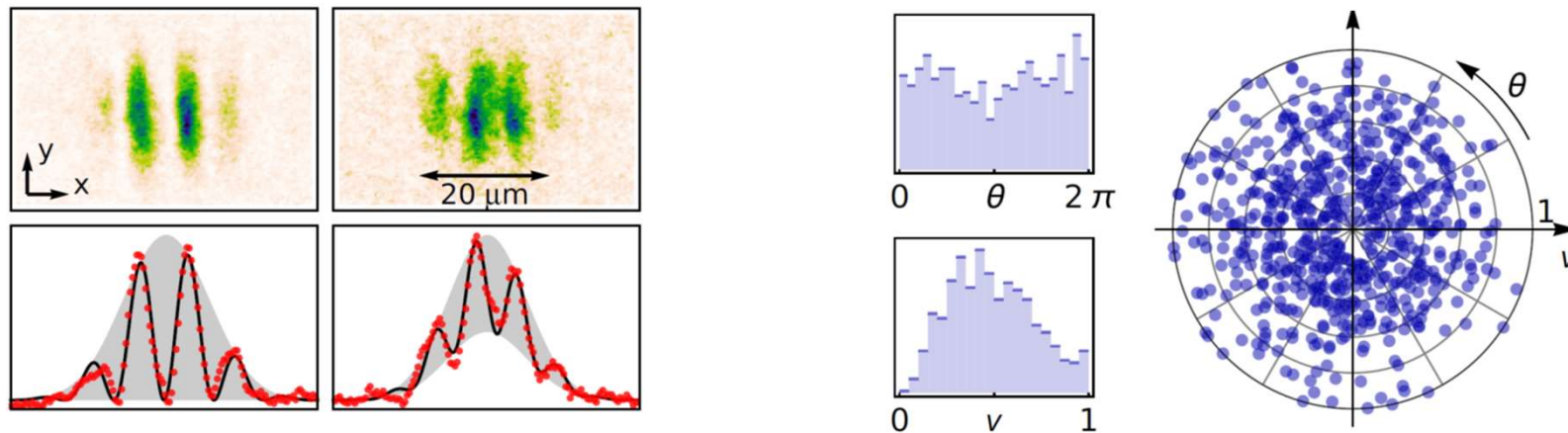


Stuttgart, Dy atoms ( $10 \mu_B$ ): H. Kadau et al., Nature 530, 194 (2016); I. Ferrier-Barbut et al., Phys. Rev. Lett. 116, 215301 (2016).

# Quantum droplets

Unfortunately, the tunneling between droplets is very small, due to the strong repulsion between droplets.

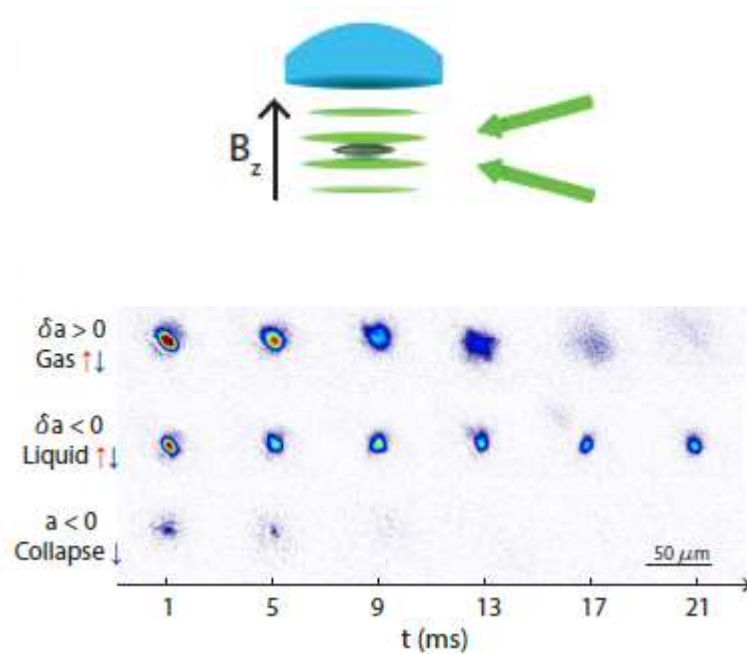
**No coherence!**



Interference of matter waves.

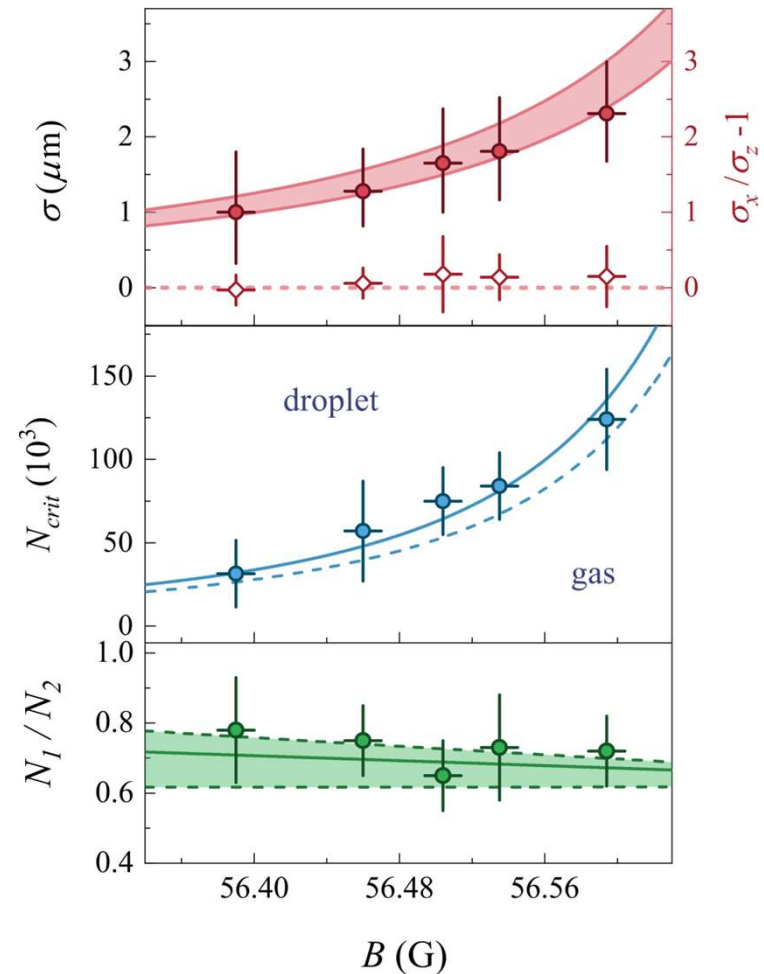
# Quantum droplets in Bose-Bose mixtures

Barcelona, K atoms



C. R. Cabrera, L. Tanzi, J. Sanz, B. Naylor, P. Thomas, P. Cheiney, and L. Tarruell, *Science* 359, 301 (2018)

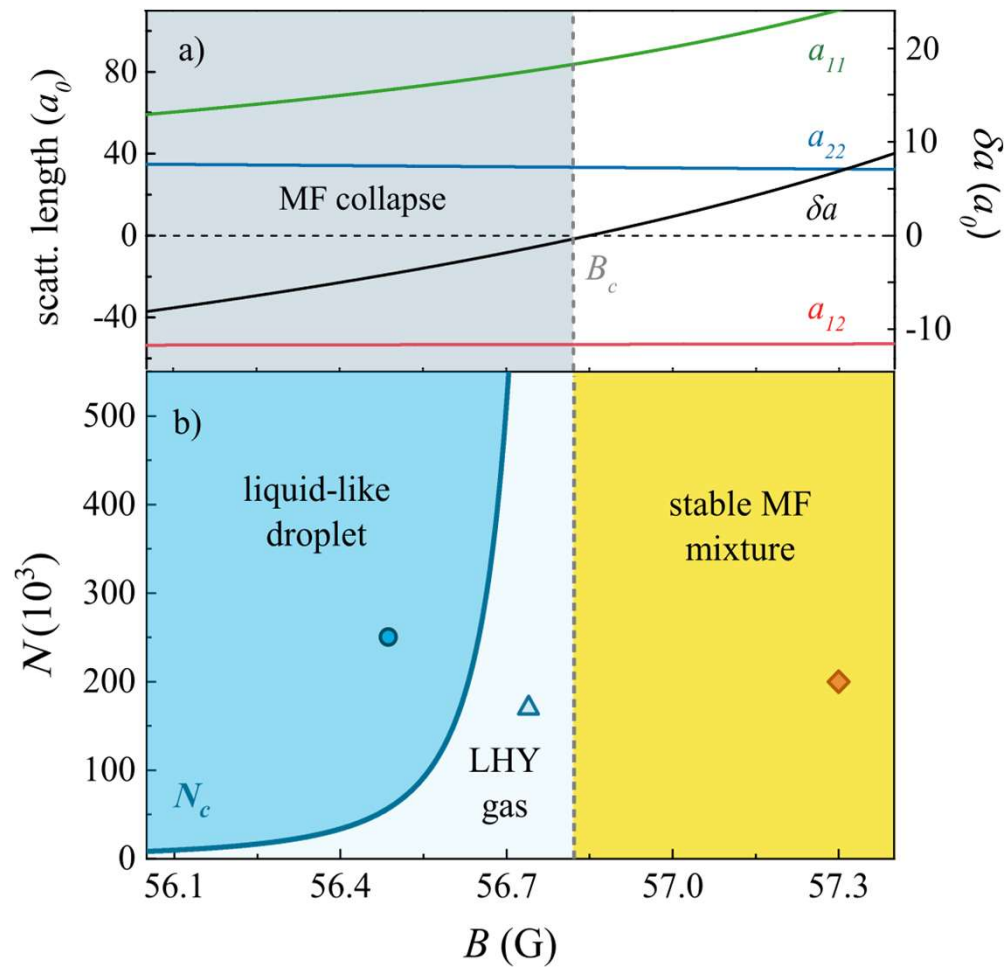
Firenze, K atoms



G. Semeghini et al., M. Fattori, *Phys. Rev. Lett.* 120, 235301 (2018)

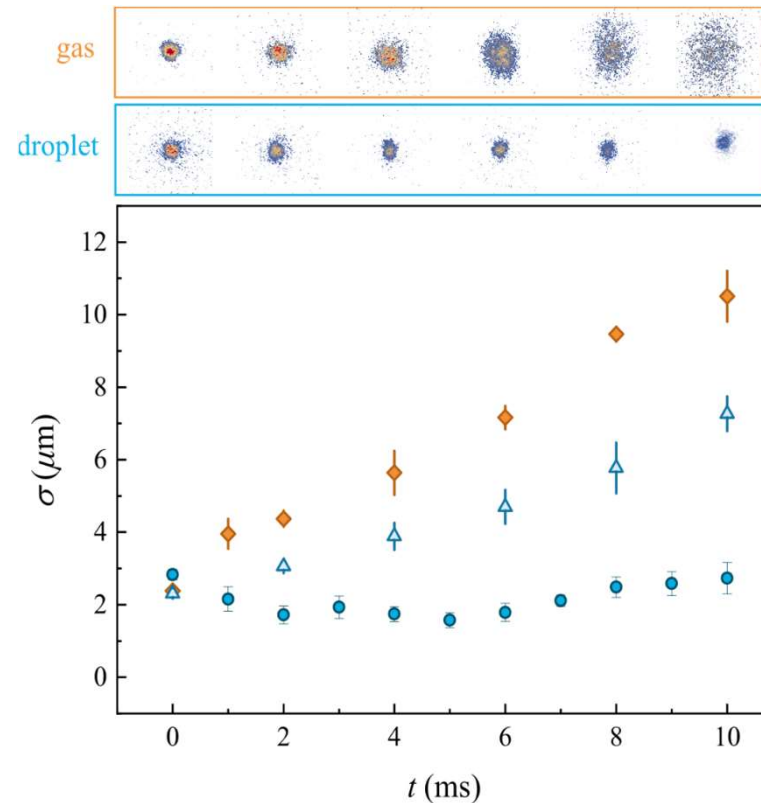


# Quantum droplets in Bose-Bose mixtures



Broad regime of weak binding

Firenze, K atoms



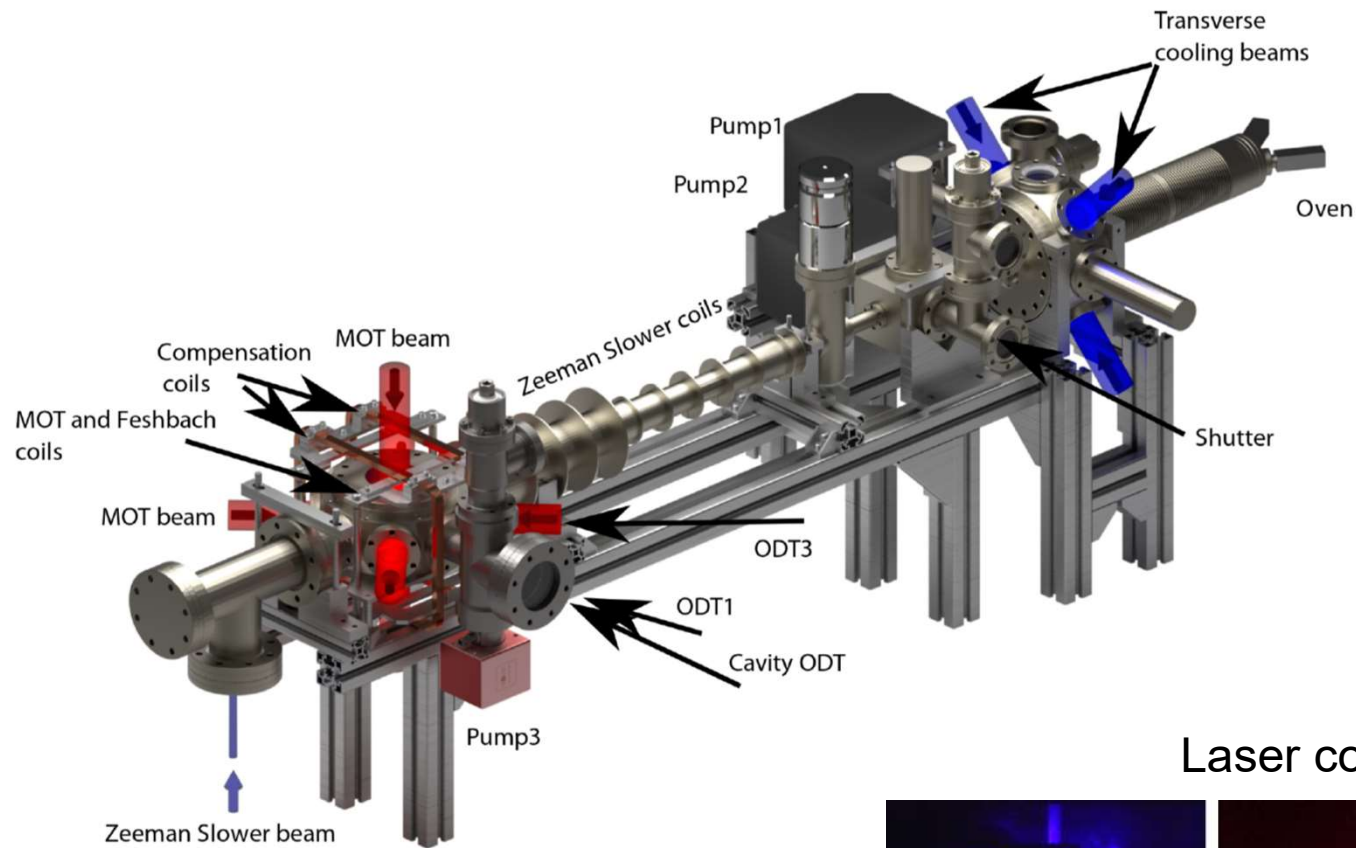
G. Semeghini et al., M. Fattori,  
Phys. Rev. Lett. 120, 235301 (2018)

## Our strategy

Combining the roton instability with a strongly dipolar system (Dy), can we reach a regime of **overlapping weakly bound droplets**?

Does that realize a **supersolid** system?

# The experiment



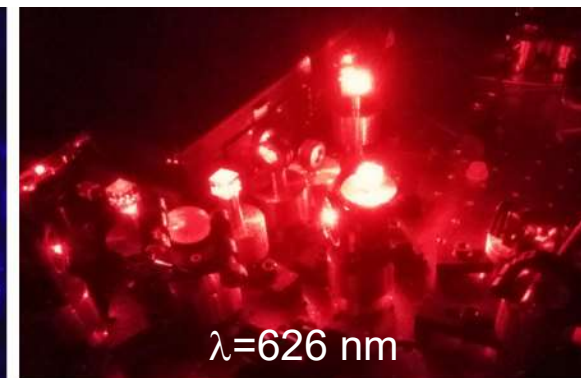
Dysprosium:  
 $\mu=10 \mu_B$



Melting point:  
1680 K

Condensation  
temperature:  
100 nK

Laser cooling:

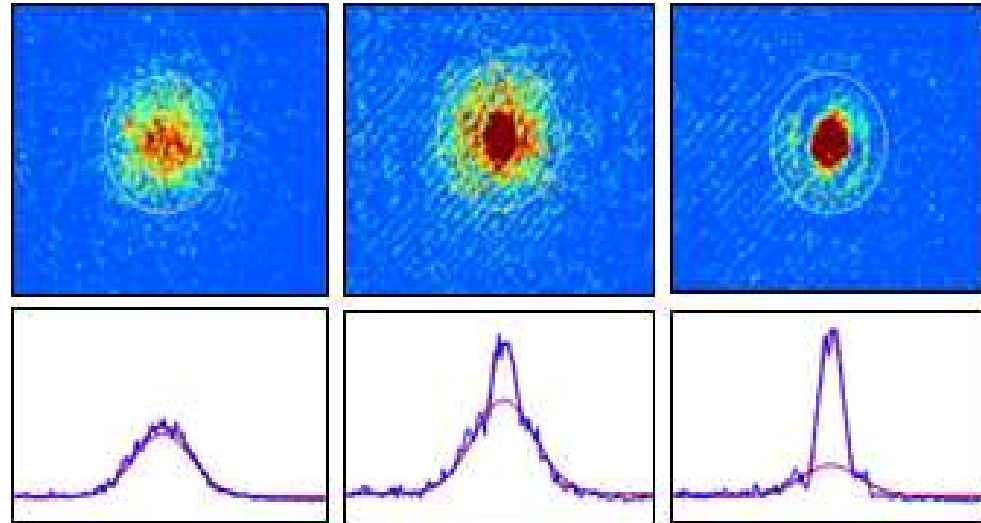


# Dy Bose-Einstein condensate



Typical condensates:

$N = 5 \times 10^4$ ,  $T < 50$  nK



BEC transition: breaking of gauge symmetry.

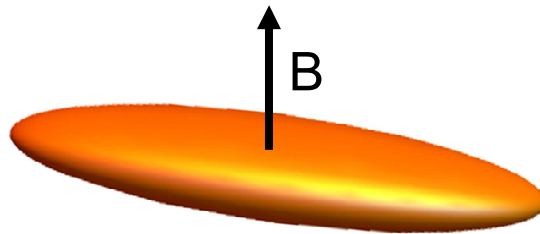
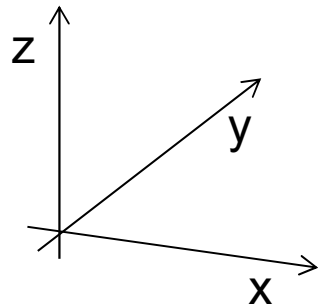
Order parameter:

$$\Psi_0(r) = |\Psi_0(r)|e^{i\varphi(r)}$$

E. Lucioni, et al. Phys. Rev. A 97, 060701(R) (2018).

# The experiment

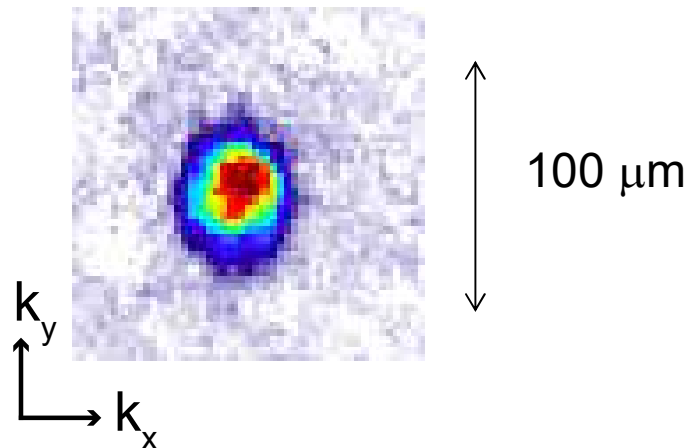
## Geometry of the BEC in the harmonic trap



$$\omega_{x,y,z} = (18, 53, 81) \text{ Hz}$$

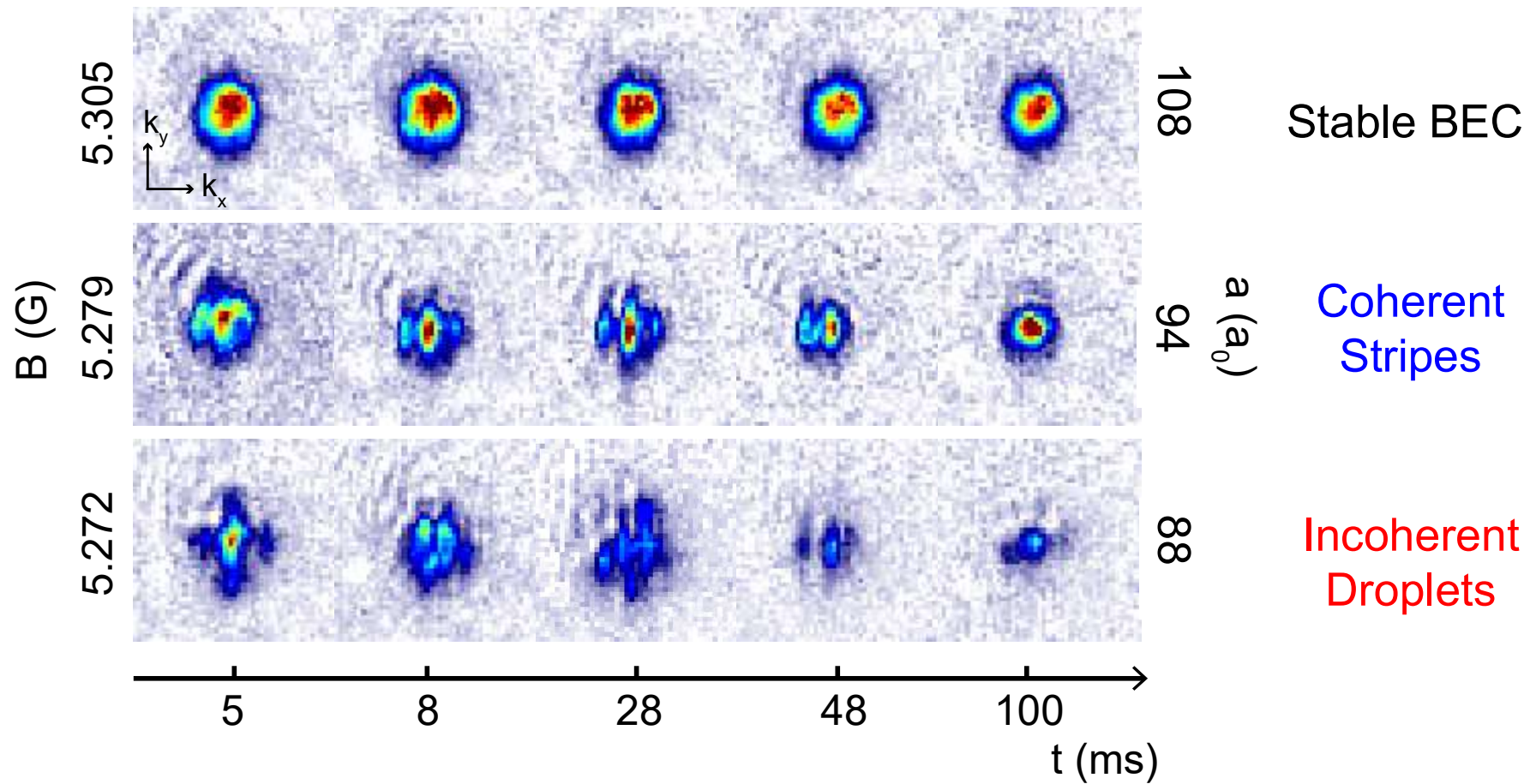
in-trap sizes: 1-10  $\mu\text{m}$

Detection in momentum space (60 ms of free fall).



# Experimental observations

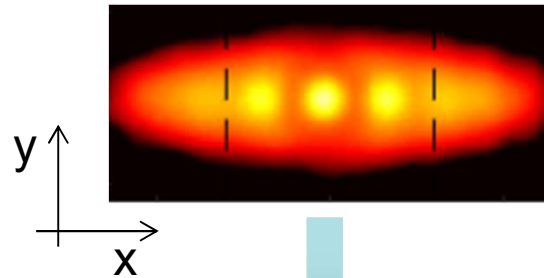
Slow tuning of the contact scattering length:



TOF pictures - momentum distribution

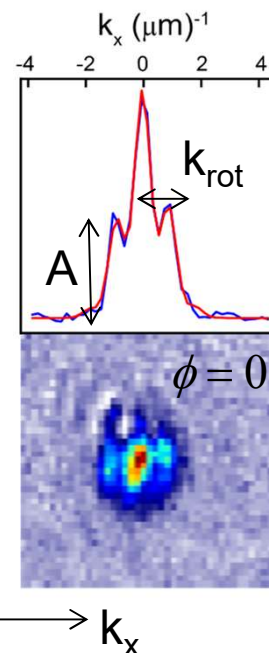


# Coherent stripe phase



In-situ density distribution

60 ms of free fall



Double slit model:

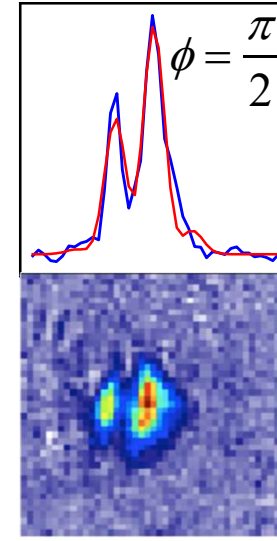
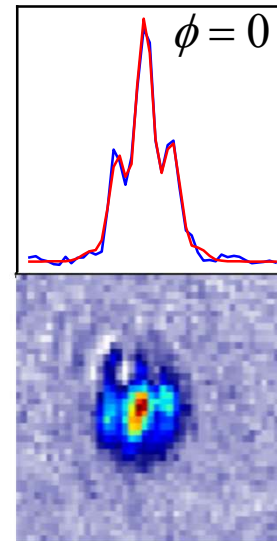
$$\tilde{n}(k) = A e^{\frac{-k^2}{2\sigma^2}} \left[ 1 + A_1 \cos^2 \left( \frac{\pi k}{k_{\text{rot}}} + \phi \right) \right]$$

Momentum distribution:  
Matter-wave interference

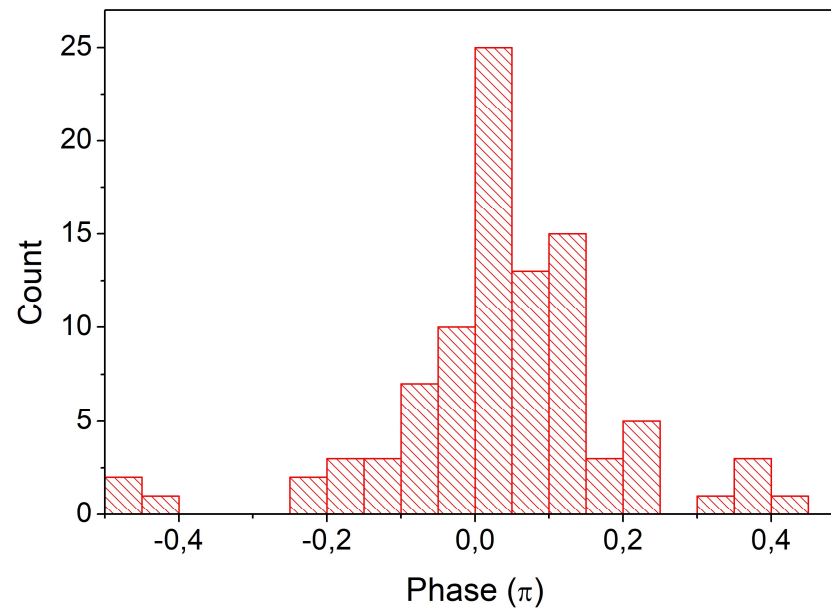
Info about: amplitude, periodicity  
and phase of the modulation

# Coherent stripe phase

$$\tilde{n}(k) = A_0 e^{\frac{-k^2}{2\sigma^2}} \left[ 1 + A_1 \cos^2 \left( \frac{\pi k}{k_{rot}} + \phi \right) \right]$$



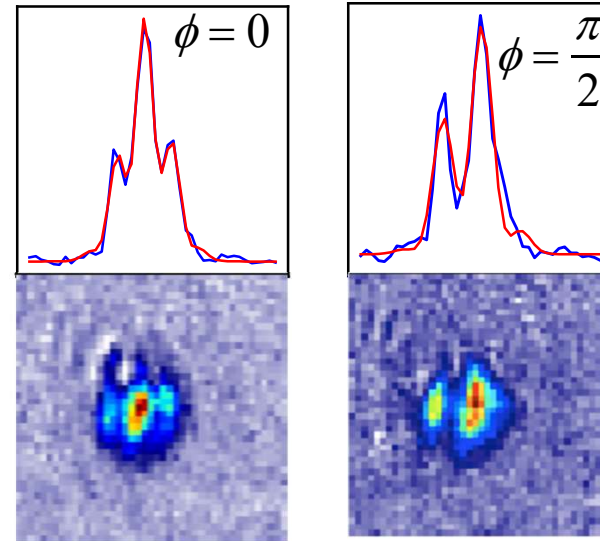
Large statistics  
(> 50 images)



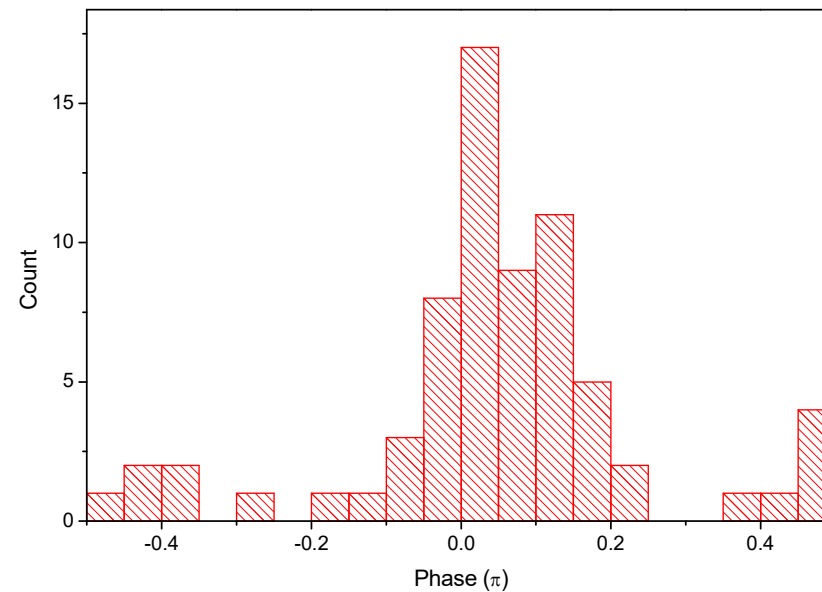
$t = 18$  ms

# Coherent stripe phase

$$\tilde{n}(k) = A_0 e^{\frac{-k^2}{2\sigma^2}} \left[ 1 + A_1 \cos^2 \left( \frac{\pi k}{k_{rot}} + \phi \right) \right]$$



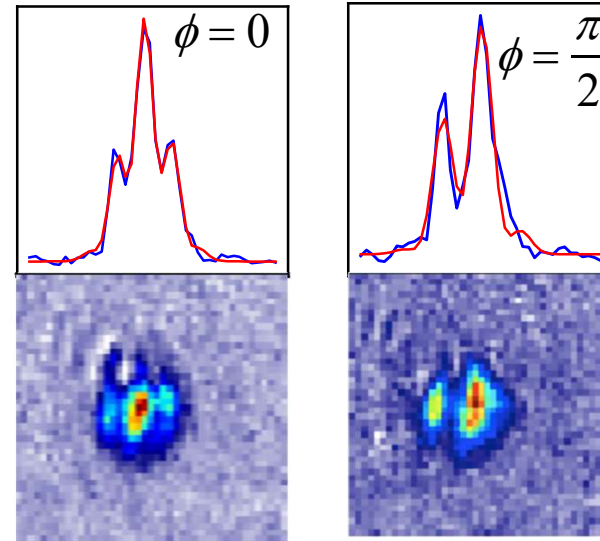
Large statistics  
(> 50 images)



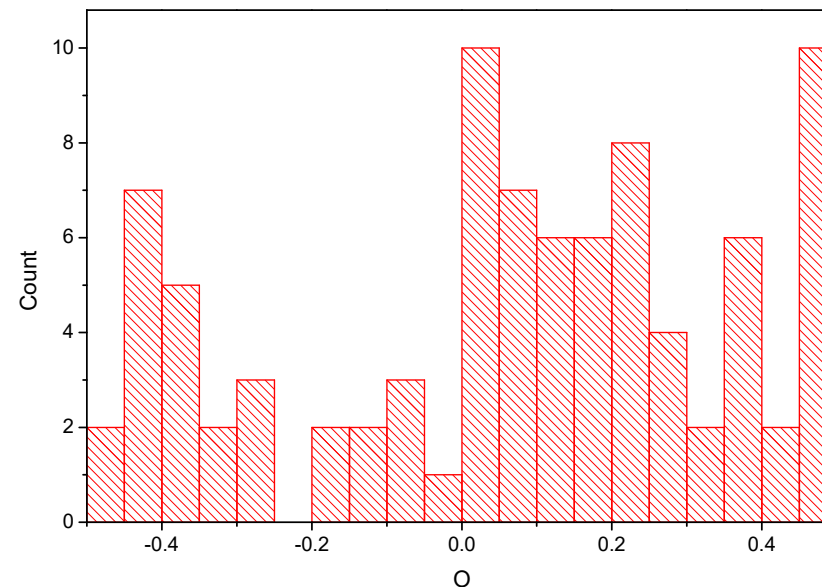
$t=28$  ms

# Coherent stripe phase

$$\tilde{n}(k) = A_0 e^{\frac{-k^2}{2\sigma^2}} \left[ 1 + A_1 \cos^2 \left( \frac{\pi k}{k_{rot}} + \phi \right) \right]$$

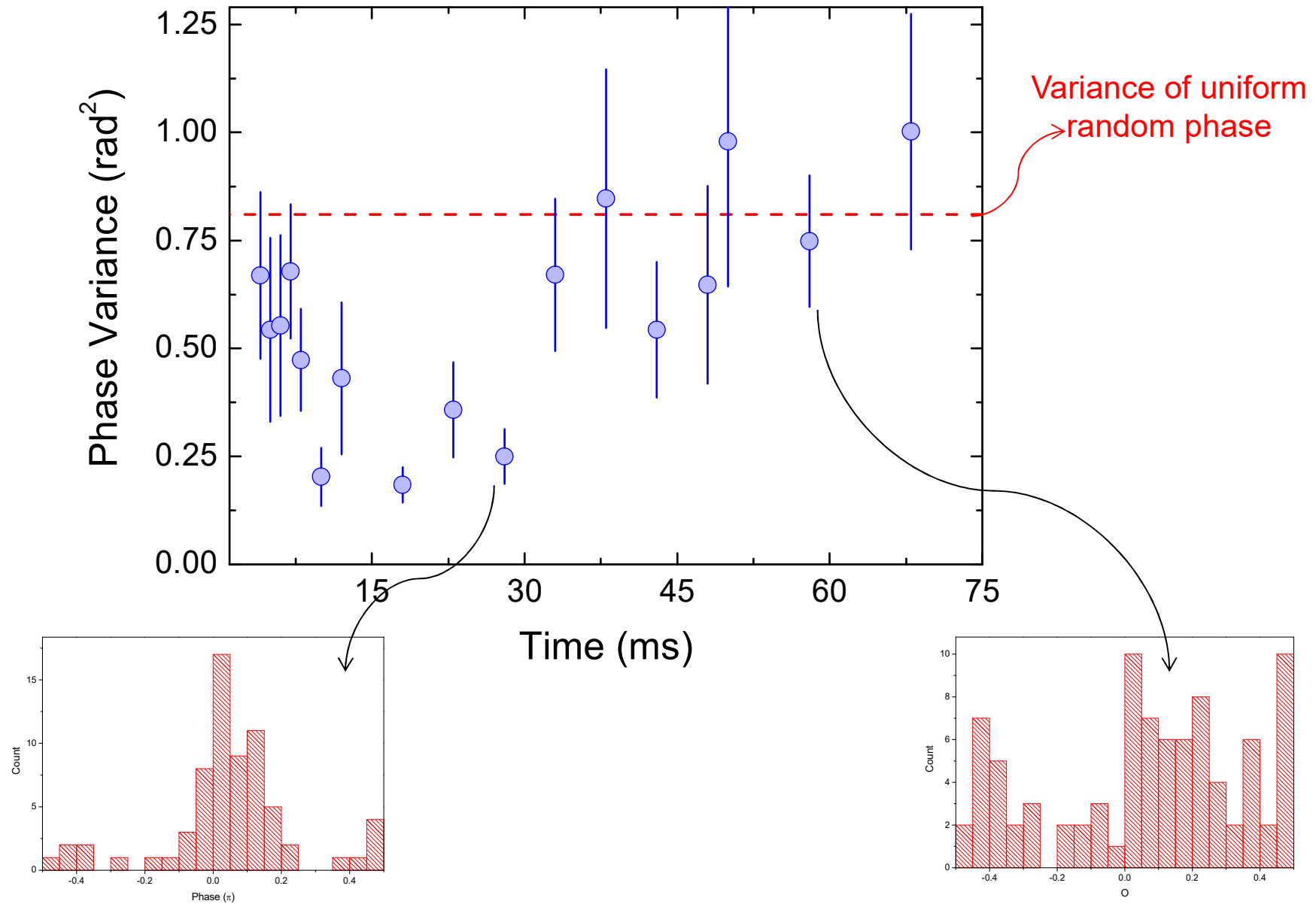


Large statistics  
(> 50 images)



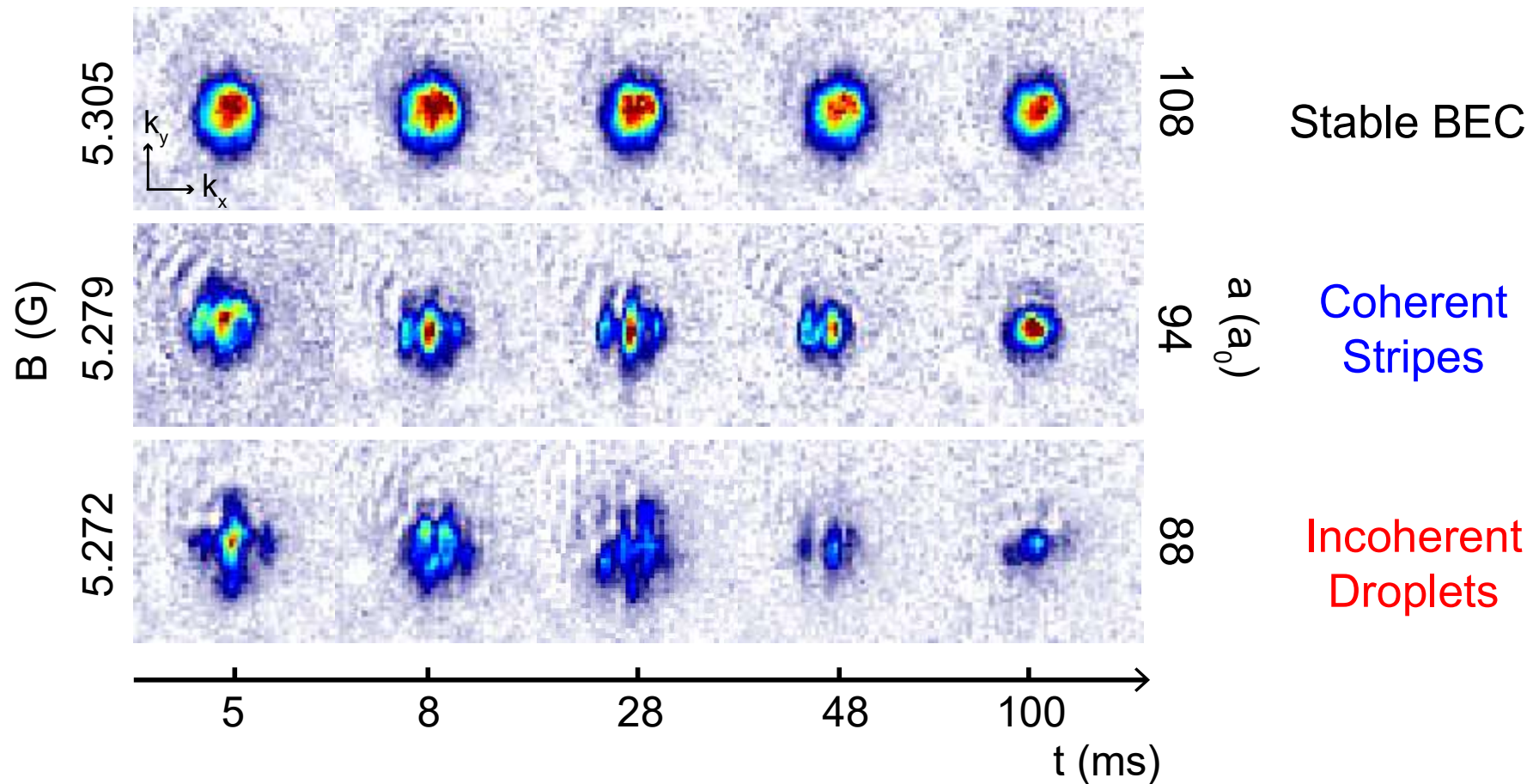
t=68 ms

# Coherent stripe phase



# Experimental observations

Slow tuning of the contact scattering length:

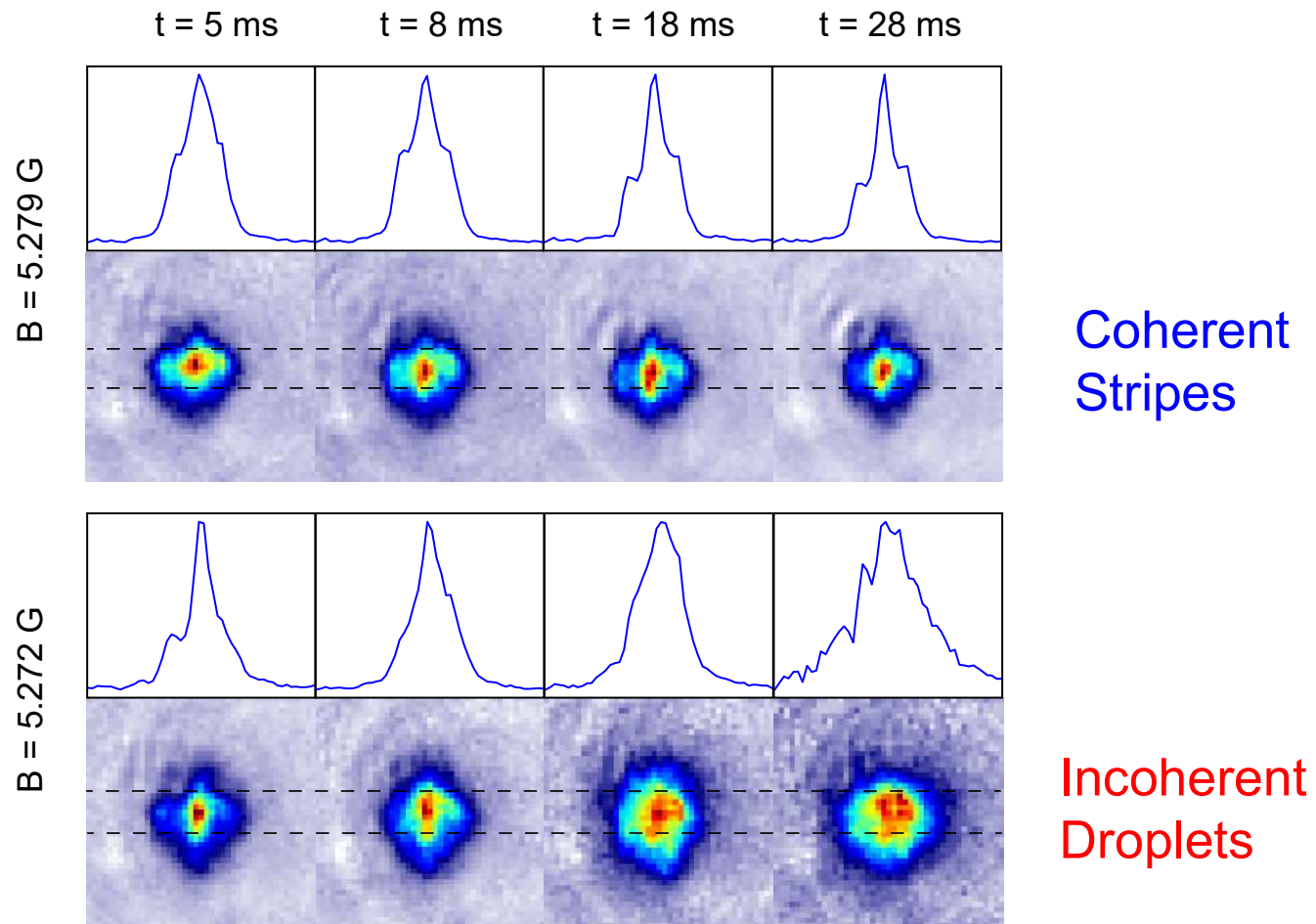


TOF pictures - momentum distribution



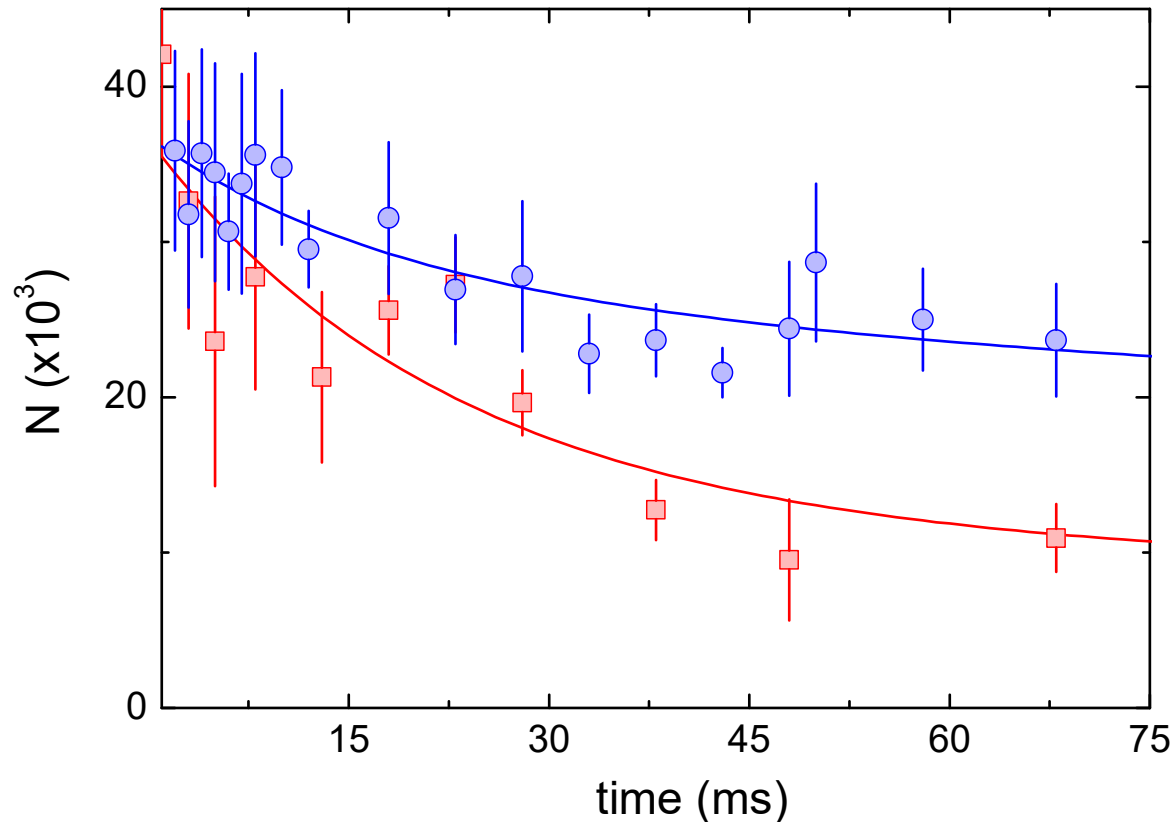
# Coherent stripe phase vs droplets

Averaged  $n(k_x, k_y)$  over 40-70 images



# Three-body recombination

The atomic gas is in a metastable state, and tends to decay towards a real solid.



Recombination rate:

$$\frac{\dot{N}}{N} = -K_3 \langle n^2 \rangle$$

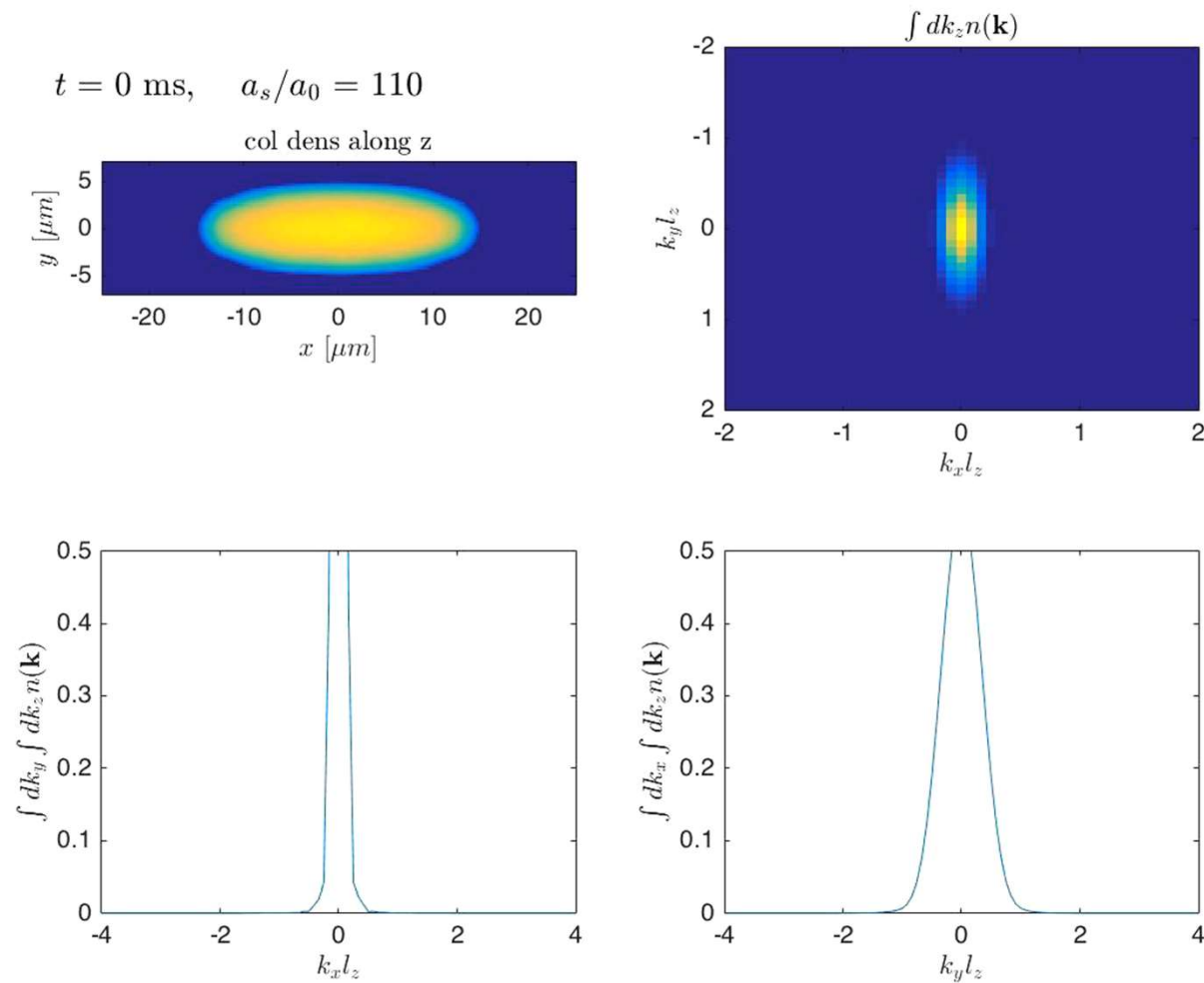
Stripes:  $\tau \approx 18$  ms

Droplets:  $\tau \approx 20$  ms

Same density:  $n \approx 4 \times 10^{14} \text{ cm}^{-3}$

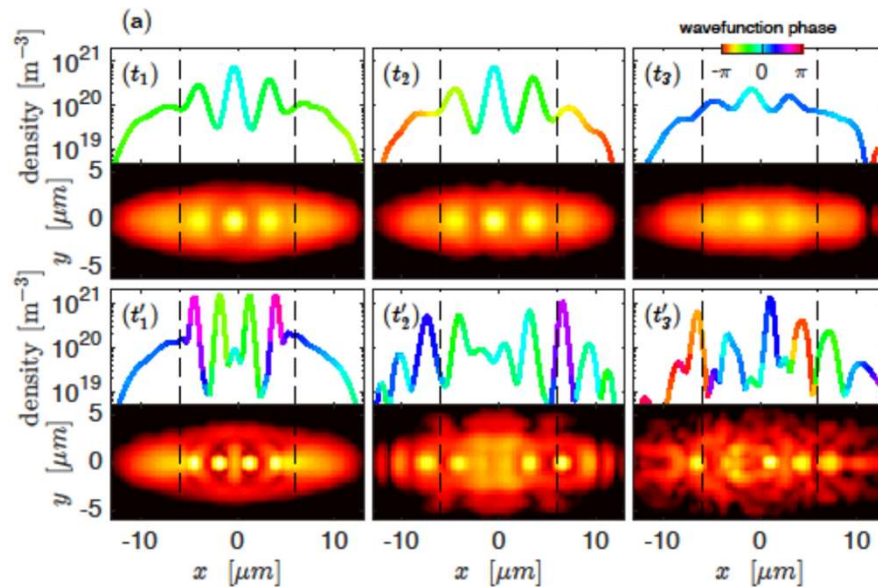
**Also stripes are stabilized by quantum fluctuations.**

# Coherent stripe phase: theory



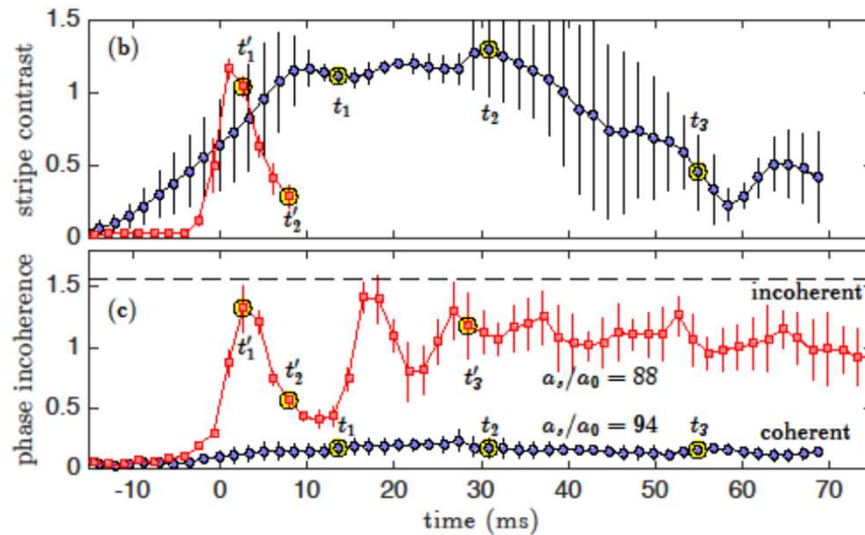
Numerical simulations by: R.N. Bisset and L. Santos, University of Hannover

# Coherent stripe phase: theory



Coherent Stripes

Incoherent Droplets



**The theory confirms that the system is supersolid!**

(transient, at finite temperature)

Numerical simulations by: R.N. Bisset and L. Santos, University of Hannover

# Strong interest by the scientific community

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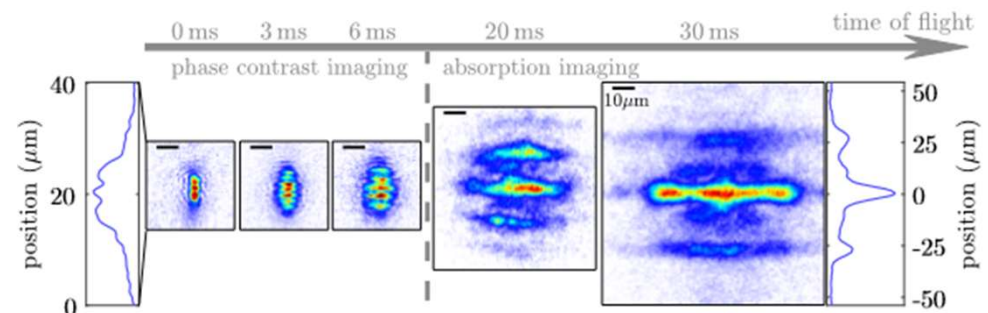
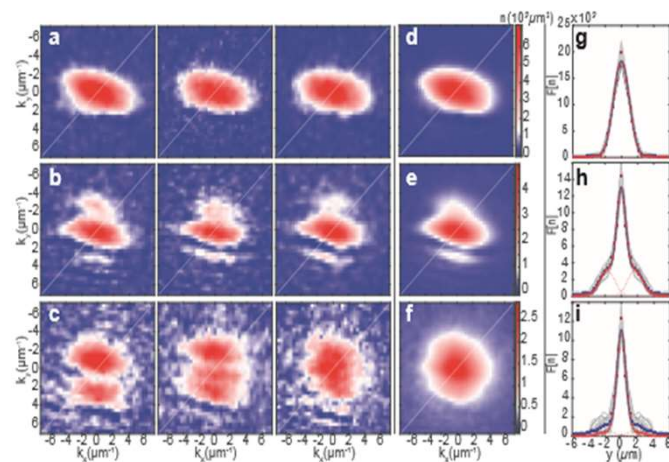
Featured in Physics

Editors' Suggestion

### Observation of a Dipolar Quantum Gas with Metastable Supersolid Properties

L. Tanzi, E. Lucioni, F. Famà, J. Catani, A. Fioretti, C. Gabbanini, R. N. Bisset, L. Santos, and G. Modugno  
Phys. Rev. Lett. **122**, 130405 – Published 3 April 2019

Physics See Viewpoint: [Dipolar Quantum Gases go Supersolid](#)



F. Böttcher et al, *Transient supersolid properties in an array of dipolar quantum droplets*, Phys. Rev. X 9, 011051 (2019)

L. Chomaz et al., *Long-lived and transient supersolid behaviors in dipolar quantum gases*, Phys. Rev. X 9, 021012 (2019).

# Strong interest by the scientific community

physicsworld  Magazine | Latest | People | Impa

states of matter



STATES OF MATTER | RESEARCH UPDATE

## Supersolid behaviour spotted in dipolar quantum gases

20 Apr 2019



# le Scienze

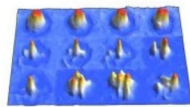
EDIZIONE ITALIANA DI SCIENTIFIC AMERICAN



le Scienze MIND alimentazione cosmologia emergenza Xylella CRISP

16 aprile 2019

## Il supersolido, nuovo stato quantistico della materia



✉ Mail Stampa

Comunicato stampa - Un team di ricercatori del Cnr e dell'Università di Firenze ha osservato nel laboratorio dell'Istituto nazionale di ottica di Pisa (Cnr-Io) un nuovo stato della materia: il supersolido. Esso ha la struttura di un solido, le proprietà di un superfluido e si comporta secondo le leggi della meccanica quantistica. Alla ricerca, pubblicata su *Physical Review Letters*, hanno collaborato anche ricercatori dell'Università di Hannover *CNR/Università di Firenze*

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## Viewpoint: Dipolar Quantum Gases go Supersolid

Tobias Donner, Institute for Quantum Electronics, ETH Zurich, Zurich, Switzerland

April 3, 2019 • *Physics* 12, 38

Three research teams observe that gases of magnetic atoms have the properties of a supersolid—a material whose atoms are crystallized yet flow without friction.

MENU

nature  
International journal of science

NEWS AND VIEWS • 20 MAY 2019

ATOMIC PHYSICS

## Quantum gases show flashes of a supersolid

Supersolids are highly sought-after structures whose atoms can simultaneously support frictionless flow and form a crystal. Hallmarks of a supersolid have now been observed in three experiments that involve quantum gases of dipolar atoms.

LODE POLLET

Sixty years ago, the theoretical physicist Eugene Gross suggested that a substance could have properties of both a solid and

initial excitement<sup>10</sup>, pure supersolidity is not observed in solid helium-4. However, in this substance, related phenomena such as giant quantum plasticity<sup>1</sup> are measurable and there is mounting evidence of frictionless flow along



# Symmetry breaking in a supersolid

SOVIET PHYSICS JETP

VOLUME 29, NUMBER 6

DECEMBER 1969

## *QUANTUM THEORY OF DEFECTS IN CRYSTALS*

A. F. ANDREEV and I. M. LIFSHITZ

Institute of Physical Problems, U.S.S.R. Academy of Sciences

Submitted January 15, 1969

Zh. Eksp. Teor. Fiz. 56, 2057–2068 (June, 1969)

At sufficiently low temperatures localized defects or impurities change into excitations that move practically freely through a crystal. As a result instead of the ordinary diffusion of defects, there arises a flow of a liquid consisting of “defectons” and “impuritons.” It is shown that at absolute zero in crystals with a large amplitude of the zero-point oscillations (for example, in crystals of the solid helium type) zero-point defectons may exist, as a result of which the number of sites of an ideal crystal lattice may not coincide with the number of atoms. The thermodynamic and acoustic properties of crystals containing zero-point defectons are discussed. Such a crystal is neither a solid nor a liquid. Two kinds of motion are possible in it; one possesses the properties of motion in an elastic solid, the second possesses the properties of motion in a liquid. Under certain conditions the “liquid” type of crystal motion possesses the property of superfluidity. Similar effects should also be observed in quasiequilibrium states containing a given number of defectons.

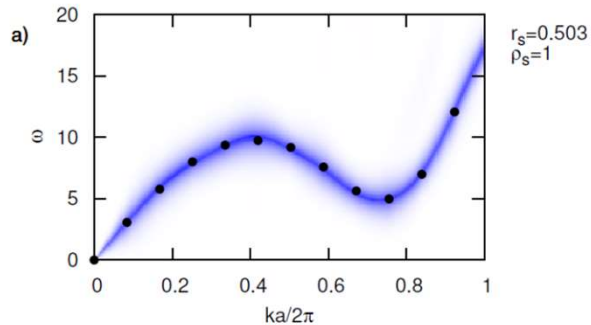
Coupled liquid and solid, both are compressible:

two sound velocities are expected, even at zero temperature.

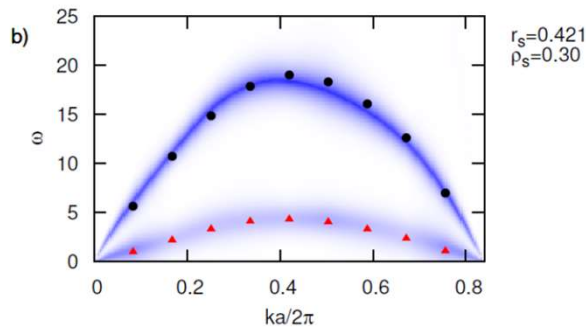


# Symmetry breaking in a supersolid

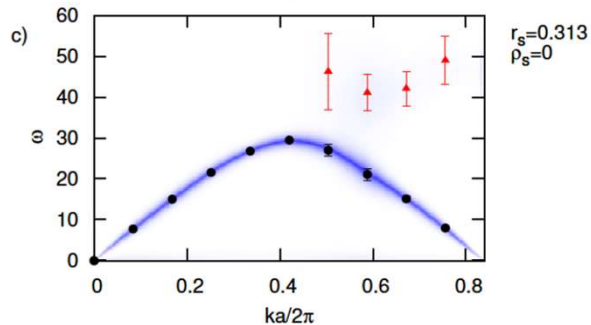
Modern treatment: a **gapless Goldstone mode** arises each time that an underlying **symmetry** is **spontaneously broken**.



Superfluid: gauge symmetry



Supersolid: gauge symmetry  
and translational symmetry



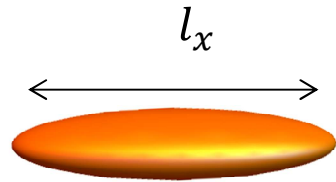
Solid: translational symmetry

# Normal modes

Question: How to observe symmetry breaking in a trapped gas?

Phonon wavelengths are bound by the system size.

Phonons are no longer defined in a non-homogeneous system.



$$k_{min} = 1/l_x$$

Answer: phonons can be mapped to the normal compressional modes of the system.

# Normal modes

The Gross-Pitaevskii equation for a BEC is equivalent to the hydrodynamic equations for an ideal liquid (zero viscosity).

$$\psi_0 = |\psi_0|e^{iS(t)} \quad i\hbar\frac{\partial}{\partial t}\psi_0(r,t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{ext}} + g|\psi_0|^2\right)\psi_0(r,t)$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0 \quad \frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{mn}\nabla p - \nabla\left(\frac{v^2}{2}\right) + \frac{1}{m}\nabla\left(\frac{\hbar^2}{2m\sqrt{n}}\nabla^2\sqrt{n}\right) - \frac{1}{m}\nabla V.$$

Normal modes are a direct consequence of the locking of the condensate phase (gauge symmetry breaking).

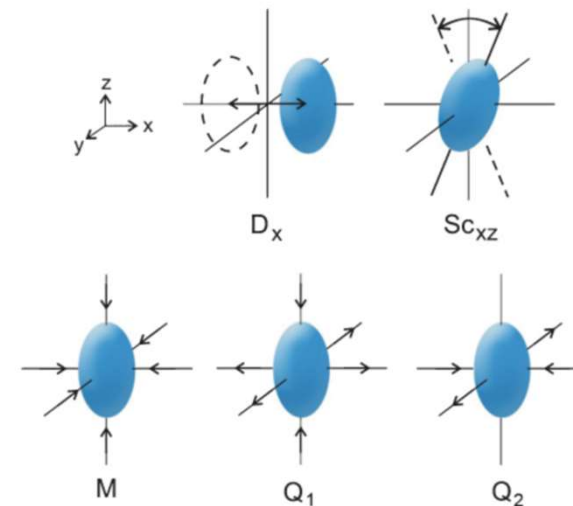


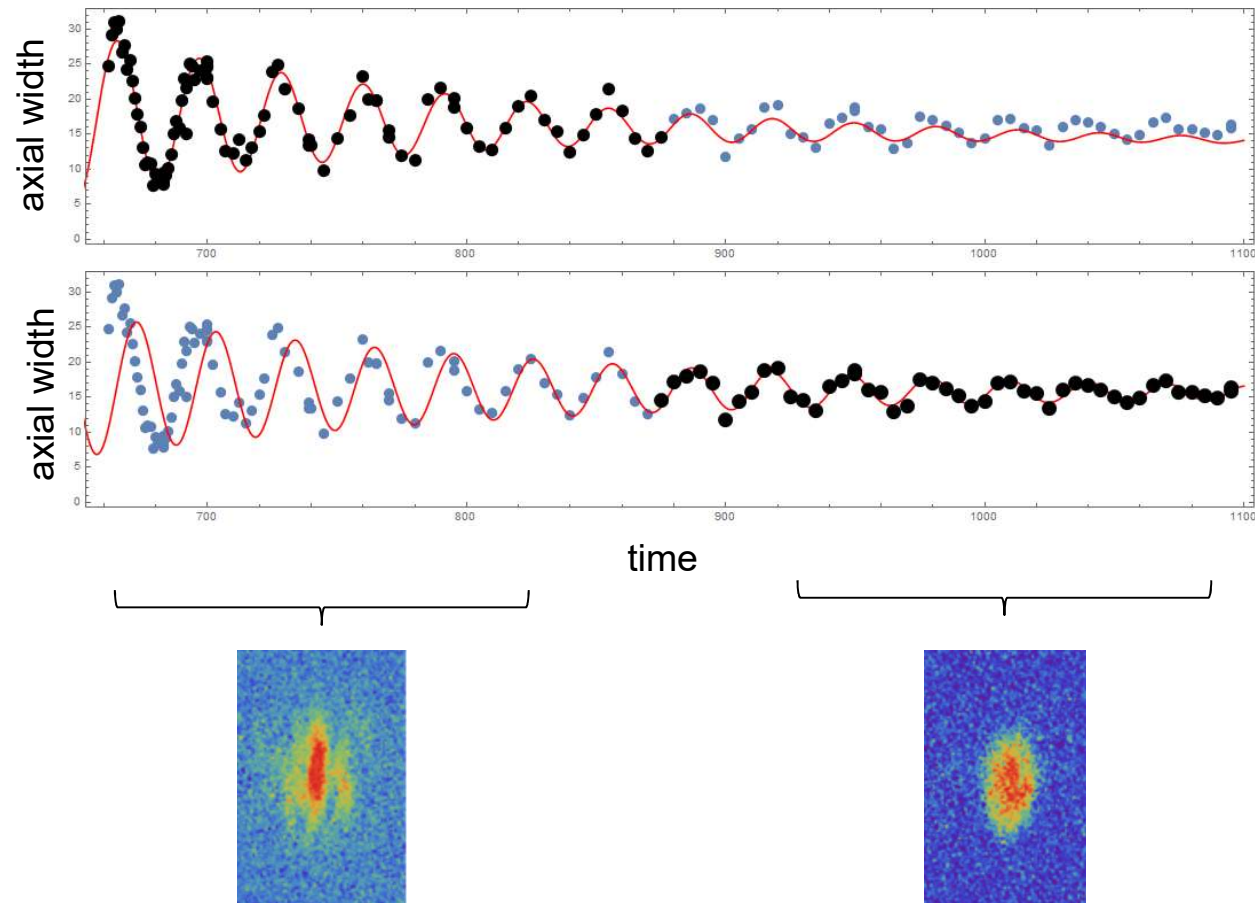
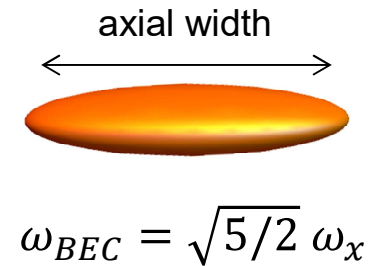
FIG. 1. (Color online) Schematic illustration of the basic collective modes under consideration: the dipole mode  $D$  (shown here in the  $x$  direction  $D_x$ ), scissors mode  $Sc$  (shown here in  $x$ - $z$  plane  $Sc_{xz}$ ), the monopole mode  $M$ , and the quadrupole modes  $Q_1$  and  $Q_2$ . These modes are discussed in more detail in Sec. III.

S. Stringari, Phys. Rev. Lett. 77, 2360 (1997).

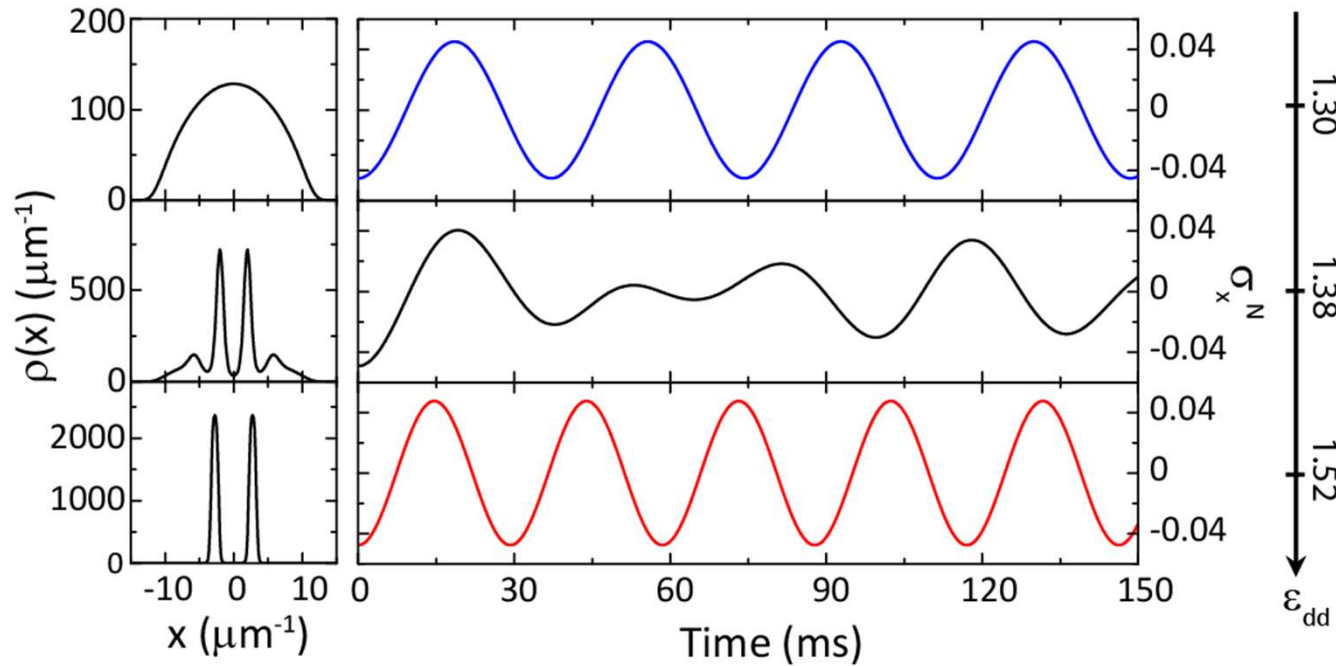
# Normal modes

We excite the lowest normal mode (axial breathing mode) by quenching the scattering length.

Supersolid regime: **frequency shift** when the stripes are present!



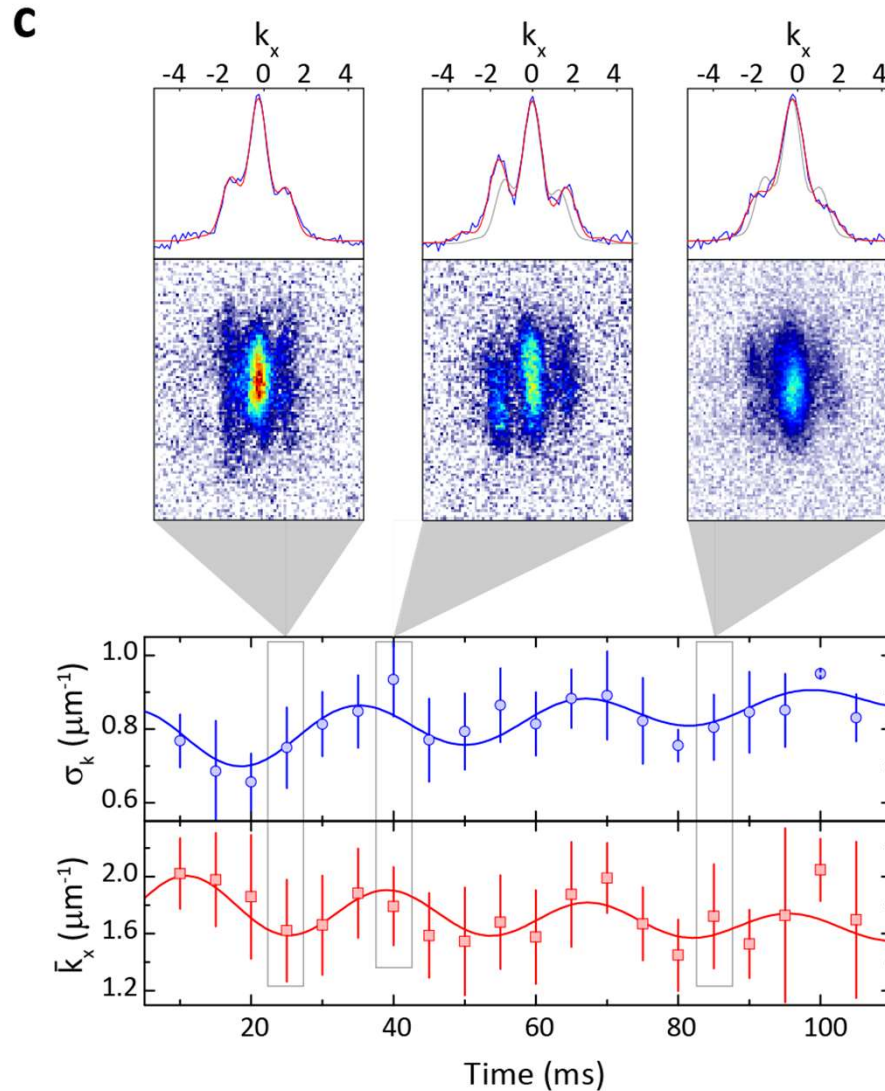
# Symmetry breaking in a supersolid



The theory predicts two coupled oscillation modes of the in-trap density.

Theory by S. Roccuzzo, A. Recati and S. Stringari.

# Symmetry breaking in a supersolid



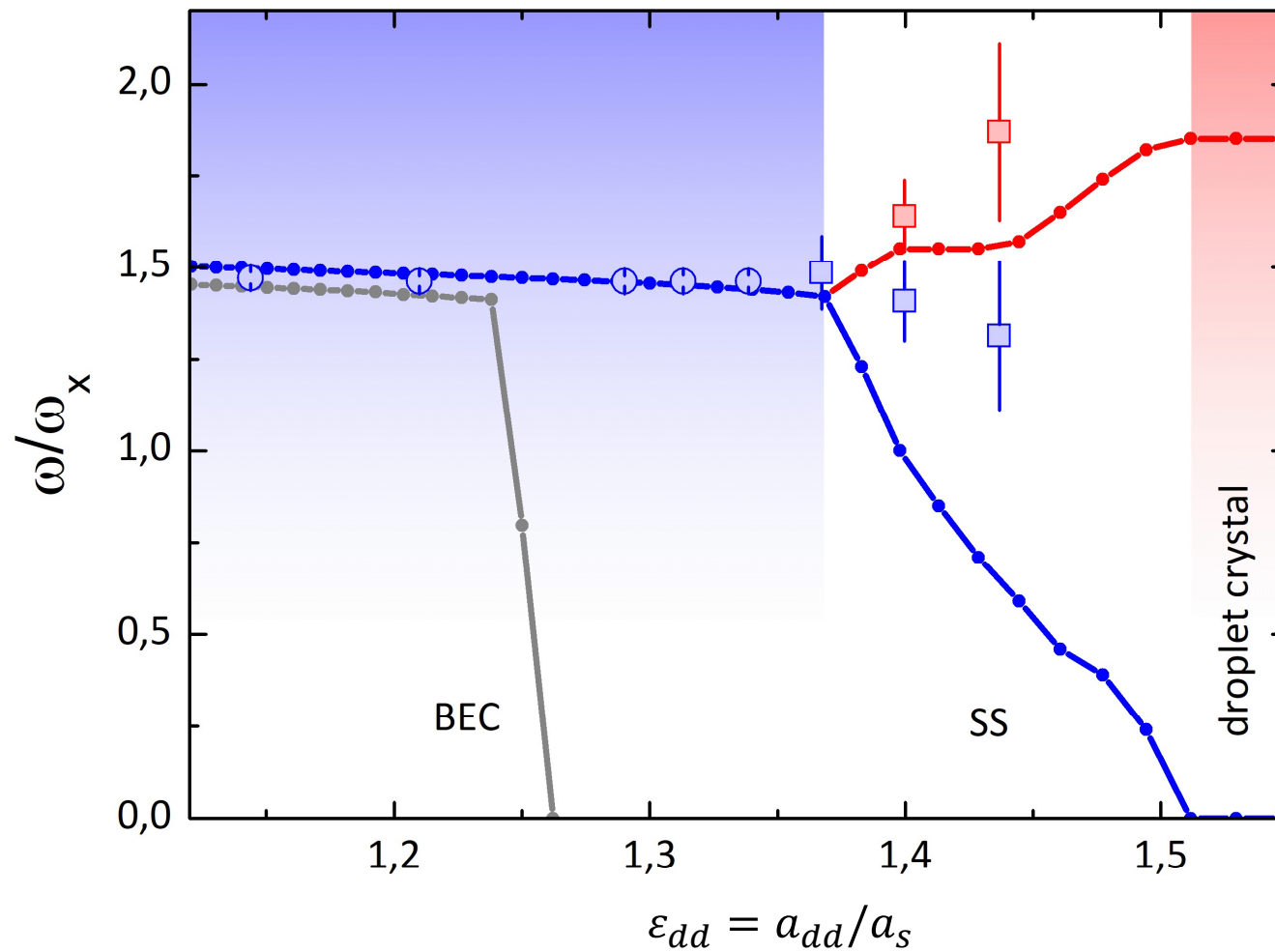
Two different frequencies for the peak spacing and the peak heights.

**Two normal modes!**

**We can study the compression mode of the solid part!!**

# Symmetry breaking in a supersolid

Phase diagram of the dipolar supersolid from normal modes:





# Conclusions and outlook

Finally, we have a compressible supersolid in the laboratory, available for investigations.

It displays a rich phase diagram. We can use the proven tools of quantum gases to explore its properties.

Quick questions:

- Non-classical moment of inertia: can we study the phenomena searched in solid He?
- Larger systems with smaller periodicity: what is the limit?
- Two-dimensional systems: how does the roton instability and the crystallization develop?
- Two quantum phase transitions: first or second order?
- How do the critical temperature and superfluid fraction of the supersolid evolve across the phase diagram?

Long term dream: use the «atomic quantum simulator» to understand supersolids, and perhaps engineer similar phases in real materials.

# The team

Luca Tanzi  
Francesca Famà  
Andrea Fioretti  
Julian Maloberti

Eleonora Lucioni  
Jacopo Catani  
Carlo Gabbanini



Theory by: Russell Bisset, Luis Santos (Hannover)  
Santo Rocuzzo, Alessio Recati, Sandro Stringari (Trento)



*Quantum Simulations of  
Insulators and Conductors*



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