Combined extraction of pion and proton TMDs

Patrick Barry (JLab) – GFI 1st miniworkshop - Improving TMD factorization for the next generation of hadron physics experiments

June 29th, 2023

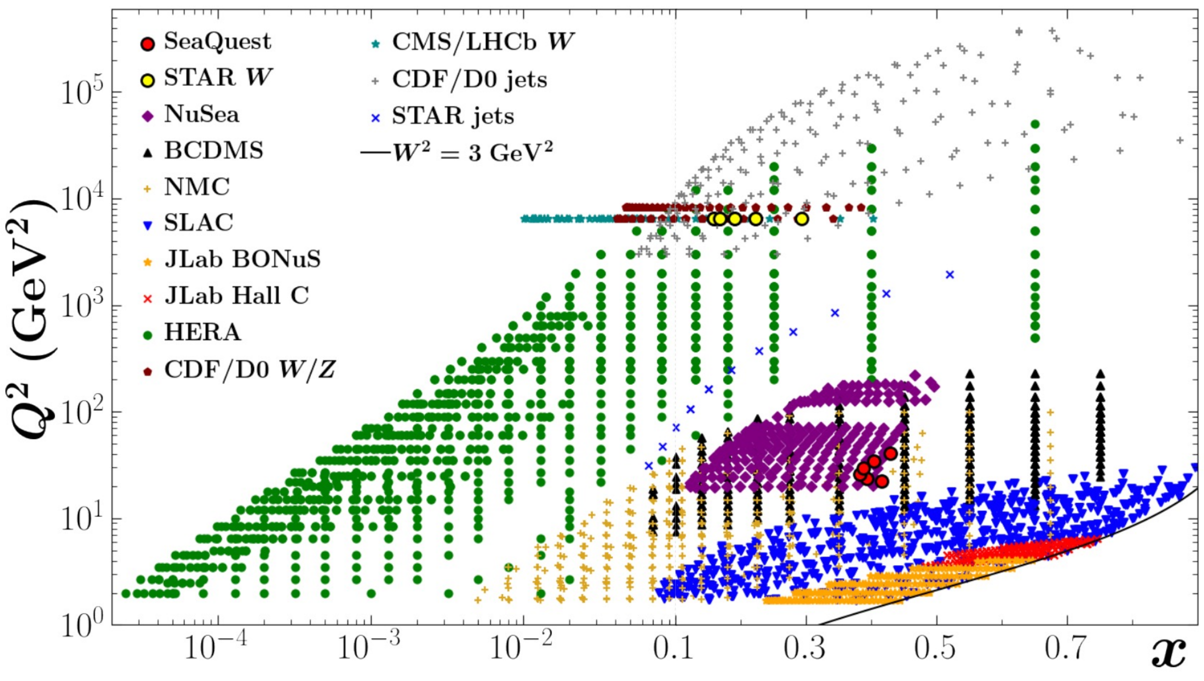
In collaboration with: Leonard Gamberg, Wally Melnitchouk, Eric Moffat, Daniel Pitonyak, Alexei Prokudin, Nobuo Sato

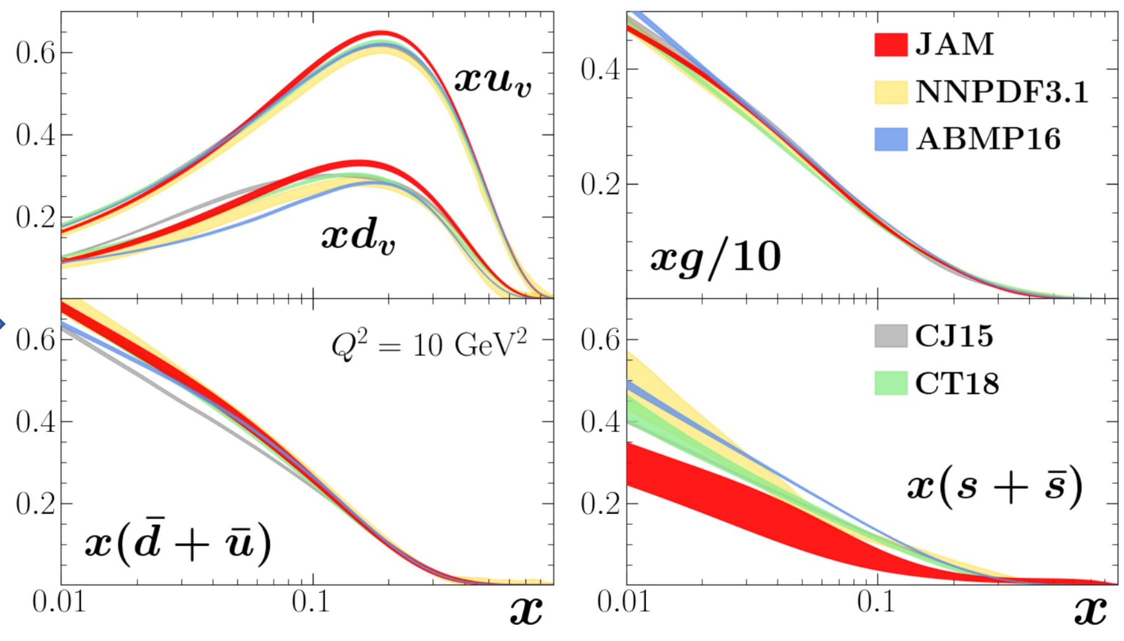
Based on: arXiv:2302.01192

Jefferson Lab
Exploring the Nature of Matter

What do we know about structures?

- Most well-known structure is through longitudinal structure of hadrons, particularly protons

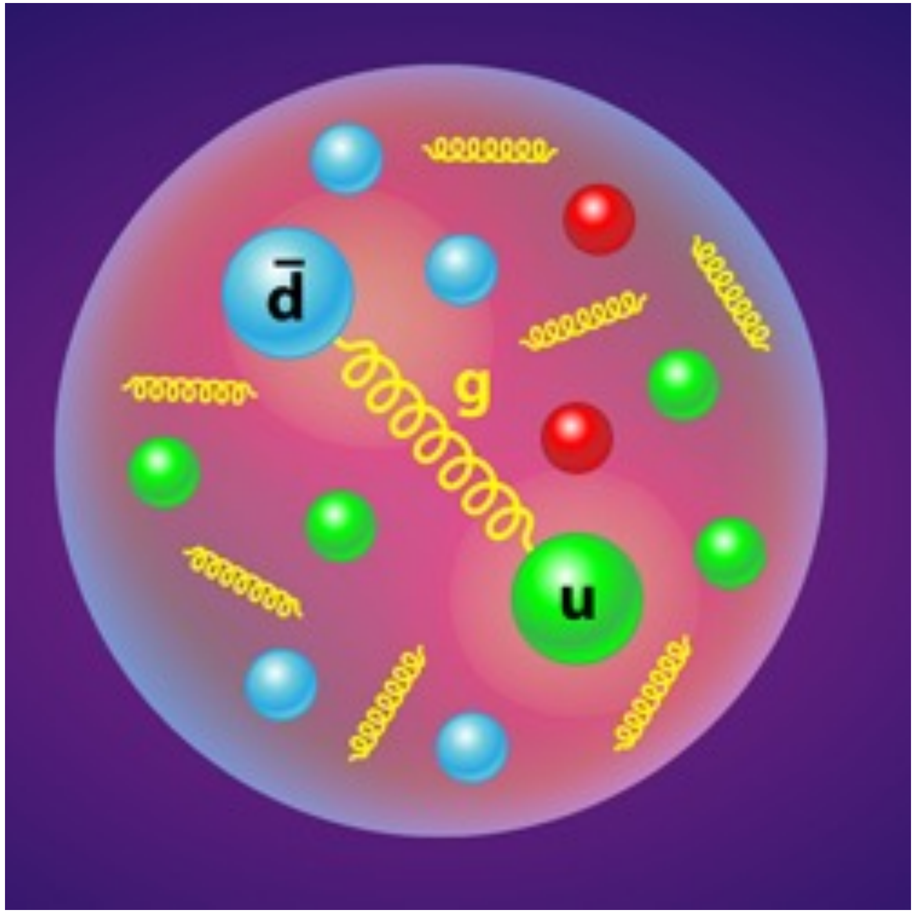




C. Cocuzza, et al., Phys. Rev. D **104**, 074031 (2021)

Other structures?

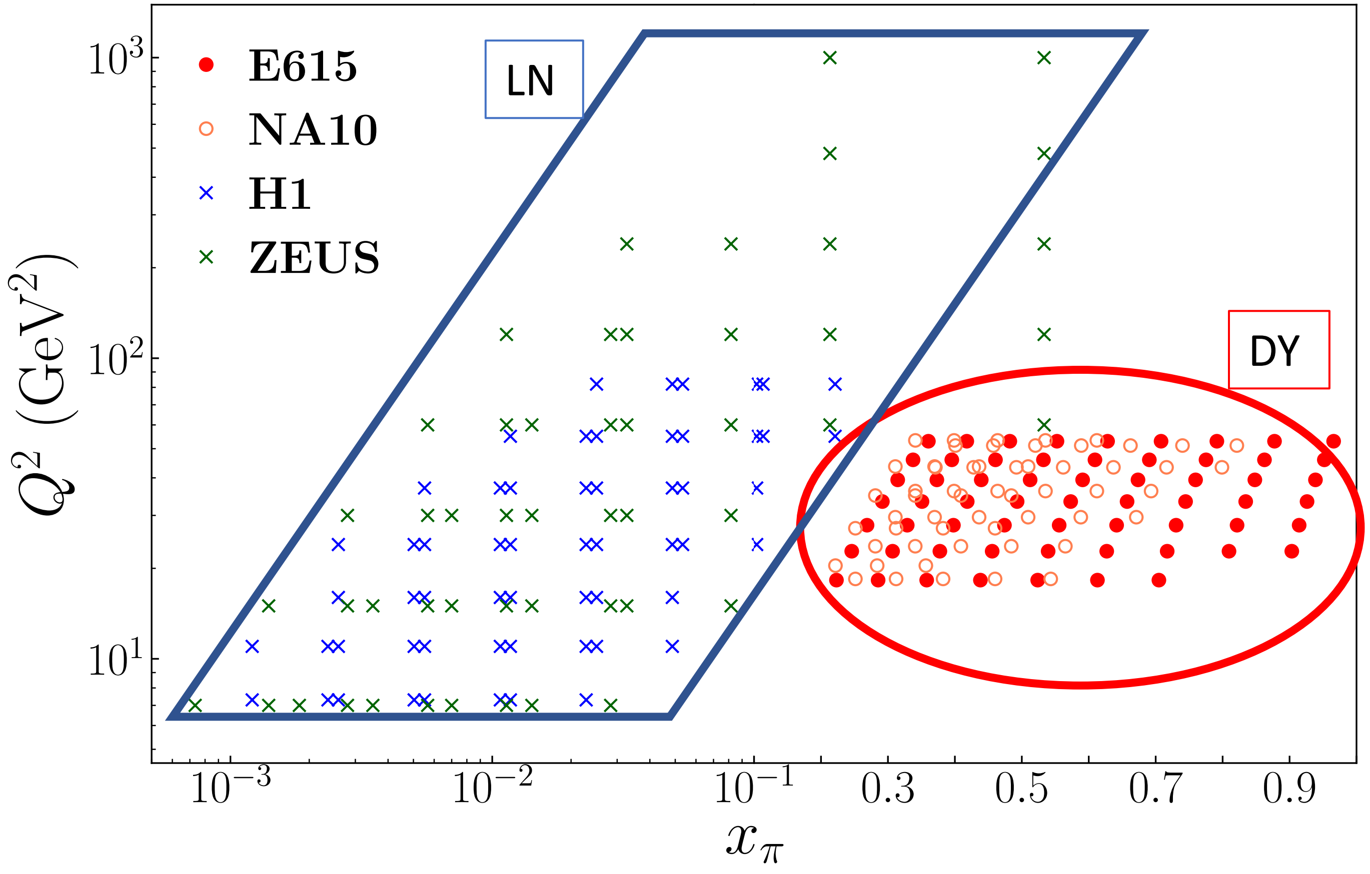
- To give deeper insights into color confined systems, we shouldn't limit ourselves to proton structures
- Pions are also important because of their Goldstone-boson nature while also being made up of quarks and gluons



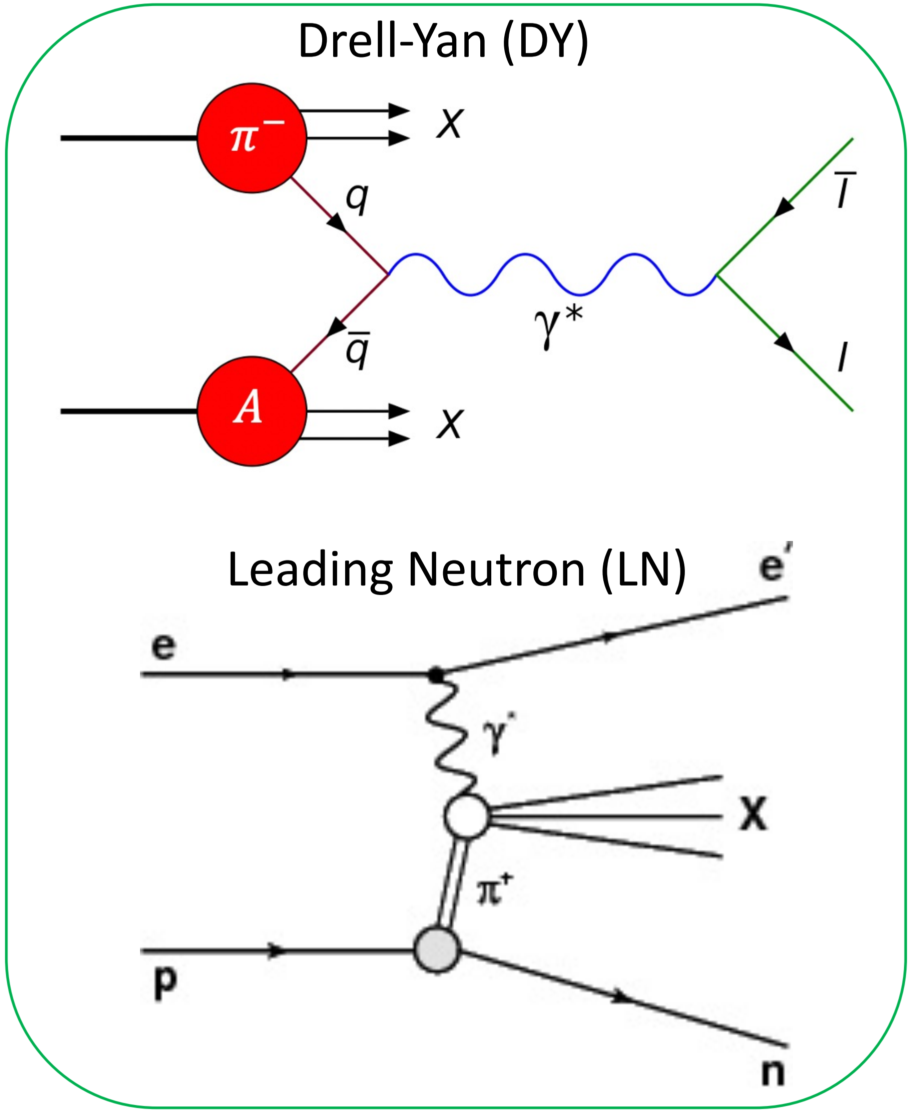
Available datasets for pion structures

- Much less available data than in the proton case

- Still valuable to study



Pion PDFs in JAM

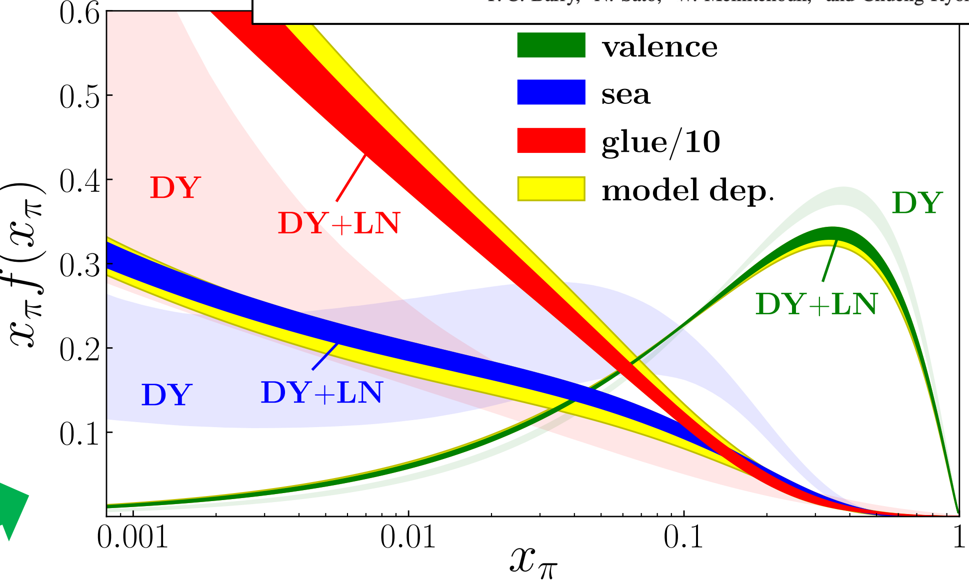


PHYSICAL REVIEW LETTERS 121, 152001 (2018)

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

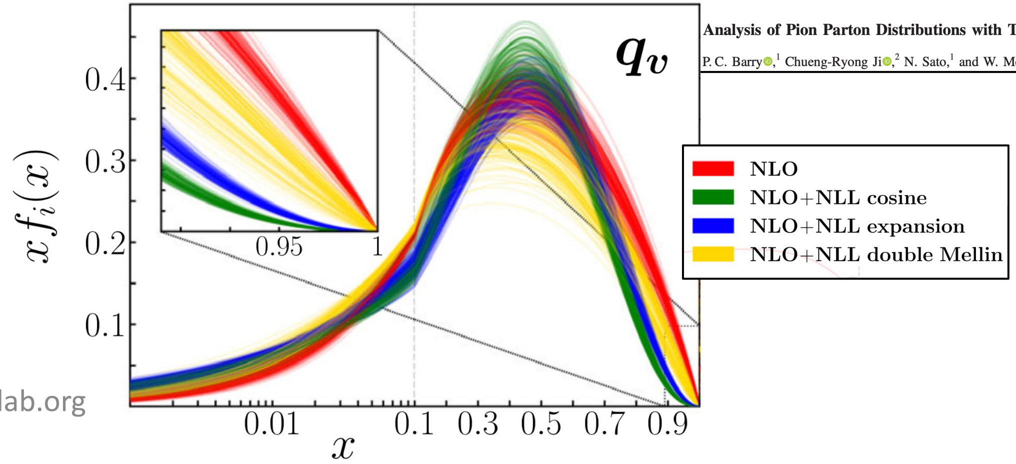


Threshold resummation in DY

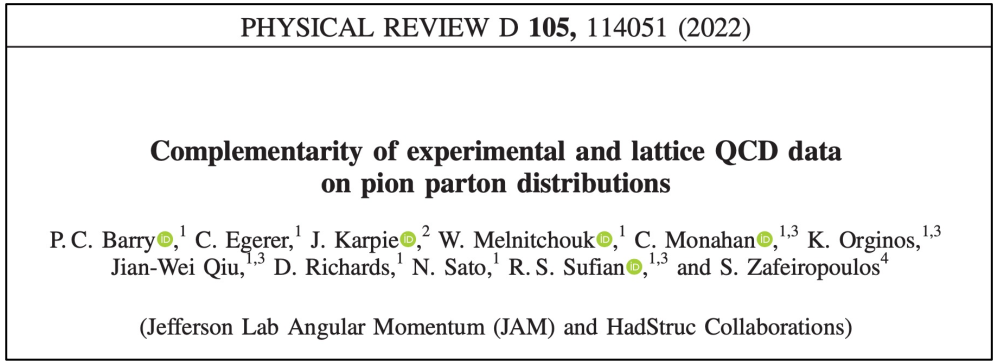
PHYSICAL REVIEW LETTERS 127, 232001 (2021)

Analysis of Pion Parton Distributions with Threshold Resummation

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]



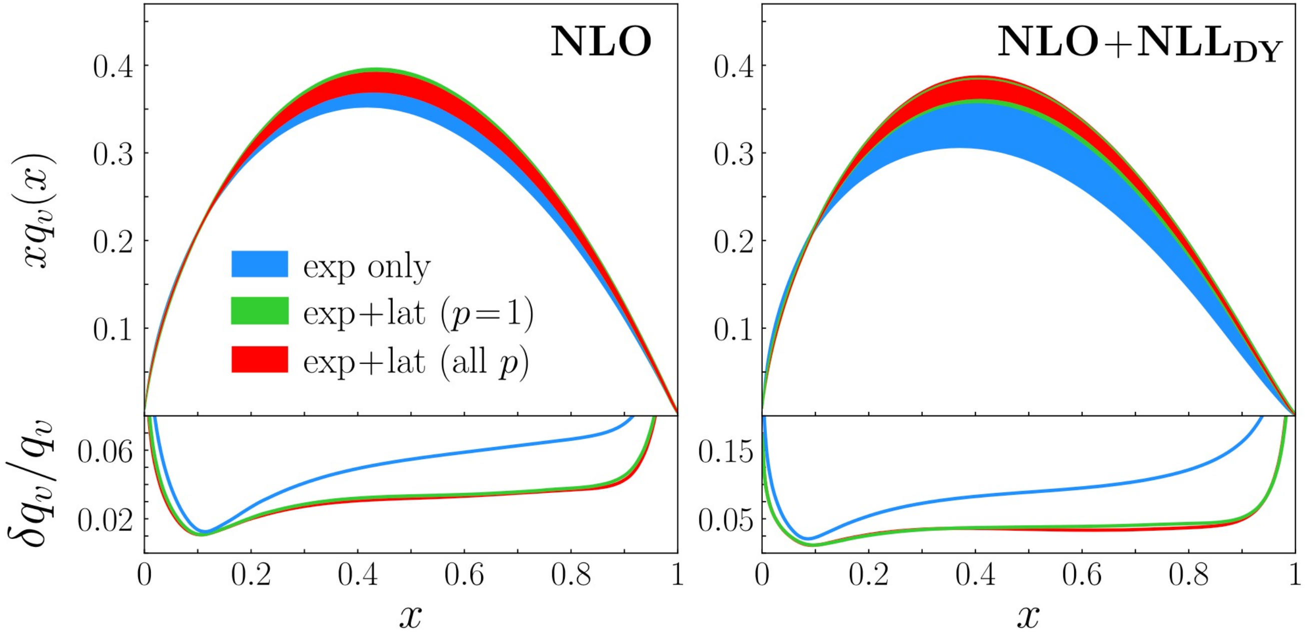
Pion PDFs from lattice + experimental data


PHYSICAL REVIEW D **105,** 114051 (2022)

Complementarity of experimental and lattice QCD data on pion parton distributions

P. C. Barry,[1] C. Egerer,[1] J. Karpie,[2] W. Melnitchouk,[1] C. Monahan,[1,3] K. Orginos,[1,3] Jian-Wei Qiu,[1,3] D. Richards,[1] N. Sato,[1] R. S. Sufian,[1,3] and S. Zafeiropoulos[4]

(Jefferson Lab Angular Momentum (JAM) and HadStruc Collaborations)

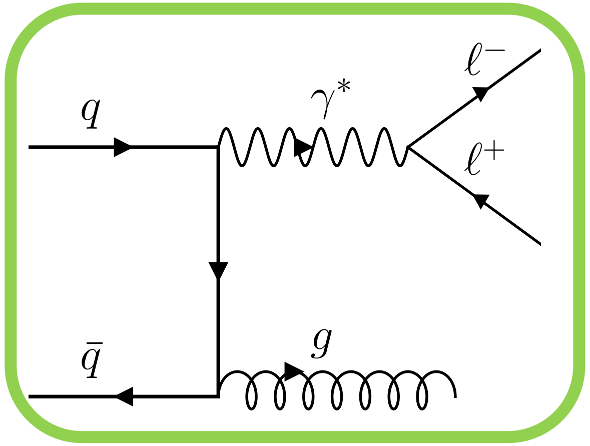


- The inclusion of lattice QCD data along with experimental data can also help us to reveal pion structure

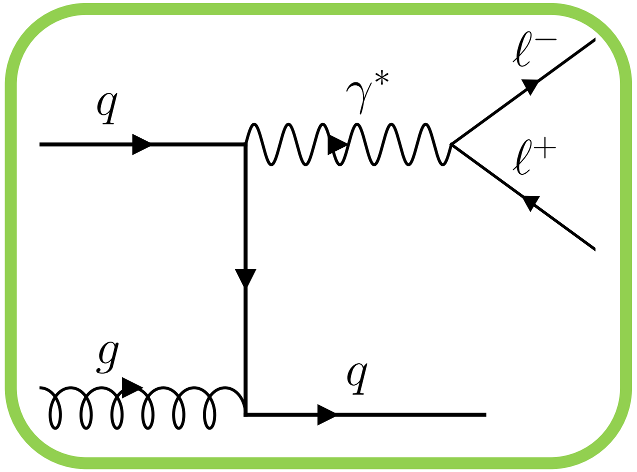
Large transverse momentum

- p_T dependent DY in **collinear factorization**

$$\frac{d\sigma}{dQ^2 dY dq_T^2} = \frac{4\pi\alpha^2}{9Q^2 S} \sum_{i,j} e_q^2 \int_{x_\pi^0}^1 dx_\pi f_i^\pi(x_\pi,\mu) f_j^A(x_A,\mu) \times \frac{d\hat{\sigma}_{i,j}}{dQ^2 d\hat{t}}$$



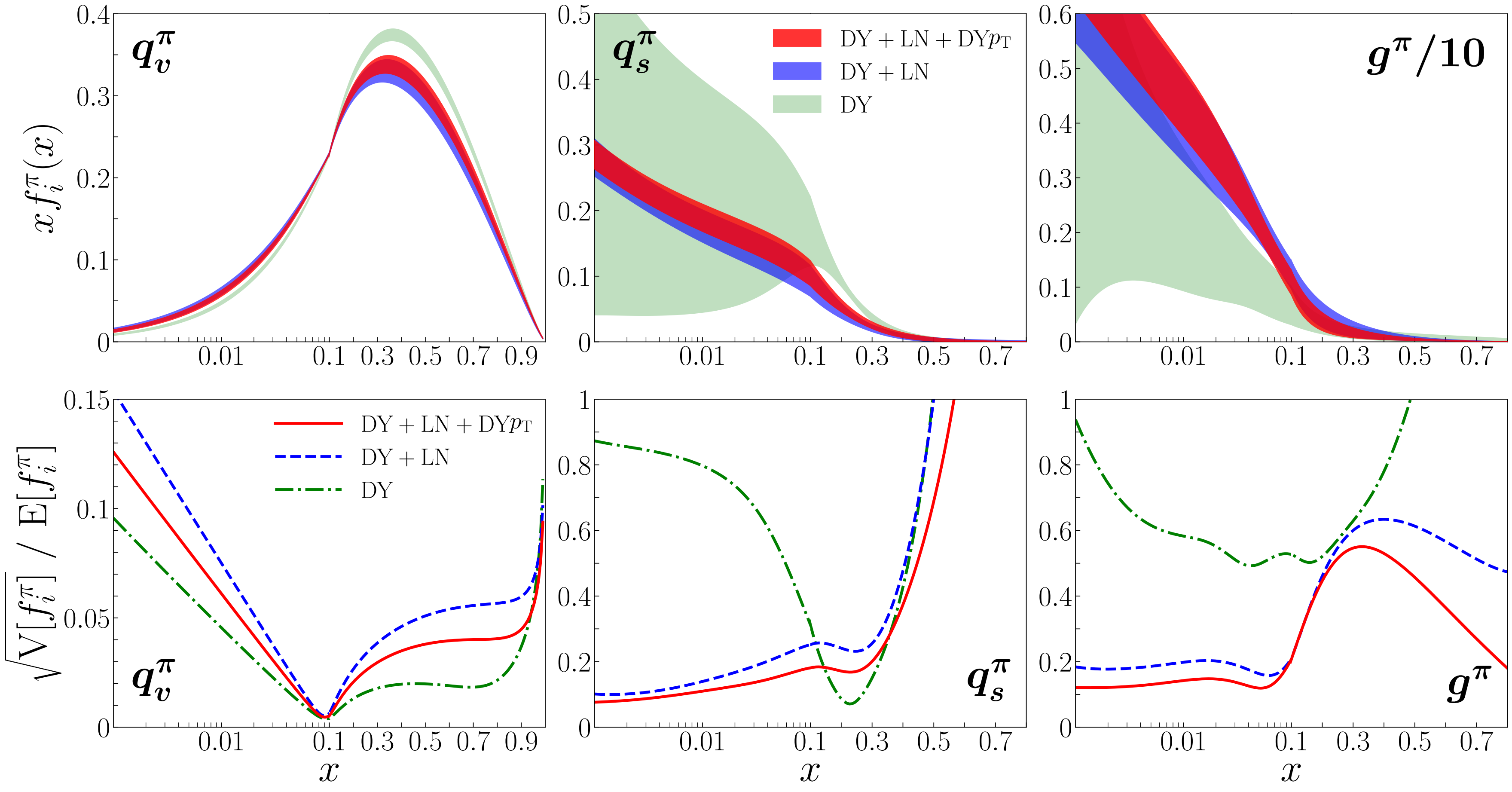
$q\bar{q}$ channel example



qg channel example

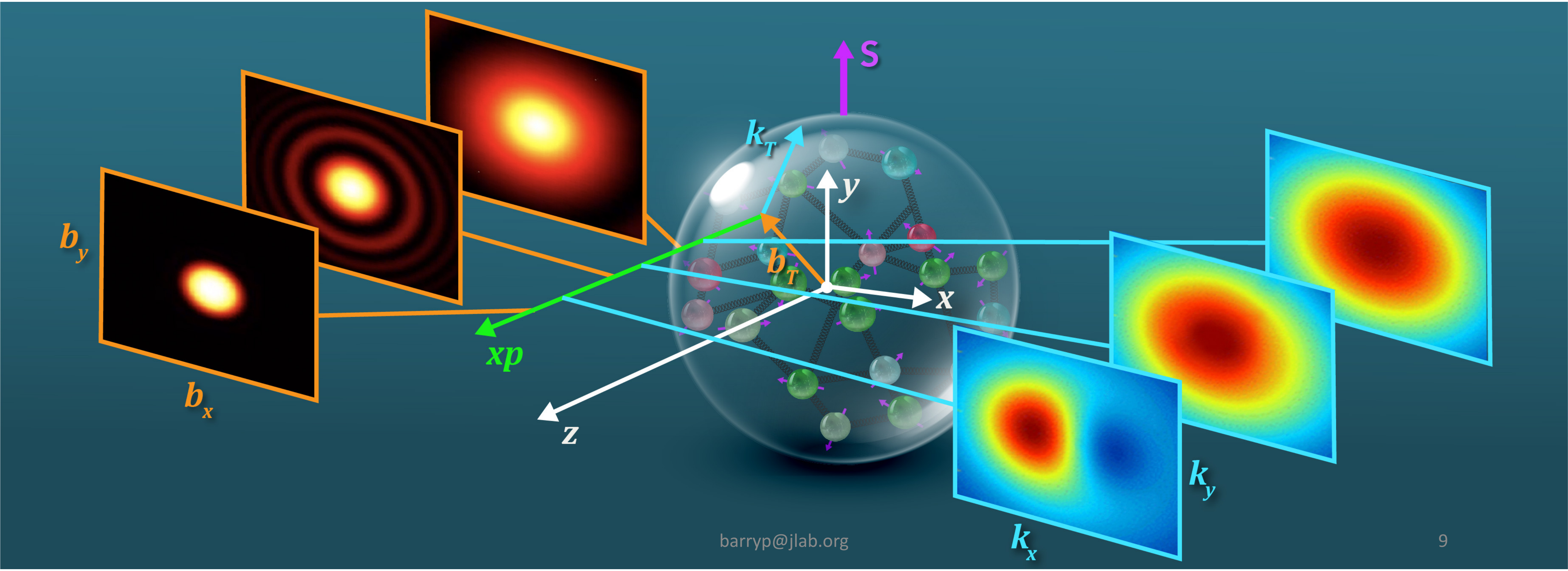
Effects of Each Dataset

- Not much impact from the transverse-momentum dependent DY data



3D structures of hadrons

- Even more challenging is the 3d structure through GPDs and TMDs



Unpolarized TMD PDF

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \mathrm{Tr}\left[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N} \rangle\right]$$

$$b \equiv (b^-, 0^+, \boldsymbol{b_T})$$

- $\boldsymbol{b_T}$ is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, $\boldsymbol{k_T}$
- We can learn about the coordinate space correlations of quark fields in hadrons
- Modification needed for UV and rapidity divergences; acquire regulators: $\tilde{f}_{q/\mathcal{N}}(x, b_T) \to \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$

Factorization for low-q_T Drell-Yan

- Like collinear observable, a hard part with two functions that describe structure of beam and target
- So called "W"-term, valid only at low-q_T

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\tau \mathrm{d}Y \mathrm{d}q_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2, \mu) \int \mathrm{d}^2 b_T\, e^{i b_T \cdot q_T} \times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2)\, \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2),$$

TMD PDF within the b_* prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1+b_T^2/b_{\max}^2}}.$$

Low-b_T: perturbative
high-b_T: non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = \boxed{(C \otimes f)_{q/\mathcal{N}(A)}(x; b_*)}$$
$$\times \exp\left\{\boxed{-g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T)\ln\frac{Q}{Q_0}} - \boxed{S(b_*, Q_0, Q, \mu_Q)}\right\}$$

Relates the TMD at small-b_T to the **collinear** PDF
$\Rightarrow$ TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative structure of the TMD
g_K: universal non-perturbative Collins-Soper kernel

Controls the perturbative evolution of the TMD

Collins, Soper, Sterman, NPB **250**, 199 (1985).

TMD factorization in Drell-Yan

- In small-q_{T} region, use the Collins-Soper-Sterman (CSS) formalism and b_* prescription

Can these data constrain the pion collinear PDF?

$$\frac{d\sigma}{dQ^2\,dy\,dq_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} H_{j\bar{j}}^{\mathrm{DY}}(Q,\mu_Q,a_s(\mu_Q))\int\frac{d^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2}e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}$$

$$\times\, e^{-g_{j/A}(x_A,b_{\mathrm{T}};b_{\max})}\int_{x_A}^{1}\frac{d\xi_A}{\xi_A}\boxed{f_{j_A/A}(\xi_A;\mu_{b_*})}\,\tilde{C}_{j/j_A}^{\mathrm{PDF}}\left(\frac{x_A}{\xi_A},b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times\, e^{-g_{\bar{j}/B}(x_B,b_{\mathrm{T}};b_{\max})}\int_{x_B}^{1}\frac{d\xi_B}{\xi_B}f_{j_B/B}(\xi_B;\mu_{b_*})\,\tilde{C}_{\bar{j}/j_B}^{\mathrm{PDF}}\left(\frac{x_B}{\xi_B},b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)$$

$$\times\exp\left\{-\boxed{g_K}(b_{\mathrm{T}};b_{\max})\ln\frac{Q^2}{Q_0^2}+\tilde{K}(b_*;\mu_{b_*})\ln\frac{Q^2}{\mu_{b_*}^2}+\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma_j(a_s(\mu'))-\ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Non-perturbative pieces – high b_T

Perturbative pieces – low b_T

Non-perturbative piece of the CS kernel

Nuclear TMD PDFs

- The TMD factorization allows for the description of a quark inside a nucleus to be $\tilde{f}_{q/A}$
- However, the intrinsic non-perturbative structure will in-principle change from nucleus-to-nucleus
- Want to model these in terms of protons and neutrons as we don't have enough observables to separately parametrize different nuclei

Nuclear TMD PDFs – working hypothesis

- We must model the nuclear TMD PDF from proton

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A} \tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A} \tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

- Each object on the right side independently obeys the CSS equation
  - **Assumption** that the bound proton and bound neutron follow TMD factorization
- Make use of approximate isospin symmetry in that $u/p/A \leftrightarrow d/n/A$, etc.

Building of the nuclear TMD PDF

- Then taking into account the intrinsic non-perturbative, we model the flavor-dependent pieces of the TMD PDF as

$$(C \otimes f)_{u/A}(x) e^{-g_{u/A}(x,b_T)} \to \frac{Z}{A} (C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x,b_T)} + \frac{A-Z}{A} (C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x,b_T)}$$

and

$$(C \otimes f)_{d/A}(x) e^{-g_{d/A}(x,b_T)} \to \frac{Z}{A} (C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x,b_T)} + \frac{A-Z}{A} (C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x,b_T)}.$$

Nuclear TMD parametrization

- Specifically, we include a parametrization similar to Alrashed, et al., Phys. Rev. Lett **129**, 242001 (2022).

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}} \left(1 - a_{\mathcal{N}} \left(A^{1/3} - 1\right)\right)$$

- Where $a_{\mathcal{N}}$ is an additional parameter to be fit

Datasets in the q_T-dependent analysis

Expt.	$\sqrt{s}$ (GeV)	Reaction	Observable	Q (GeV)	x_F or y	$N_{\text{pts.}}$
E288 [39]	19.4	$p + Pt \to \ell^+\ell^- X$	$Ed^3\sigma/d^3\mathbf{q}$	$4 - 9$	$y = 0.4$	38
E288 [39]	23.8	$p + Pt \to \ell^+\ell^- X$	$Ed^3\sigma/d^3\mathbf{q}$	$4 - 12$	$y = 0.21$	48
E288 [39]	24.7	$p + Pt \to \ell^+\ell^- X$	$Ed^3\sigma/d^3\mathbf{q}$	$4 - 14$	$y = 0.03$	74
E605 [40]	38.8	$p + Cu \to \ell^+\ell^- X$	$Ed^3\sigma/d^3\mathbf{q}$	$7 - 18$	$x_F = 0.1$	49
E772 [41]	38.8	$p + D \to \ell^+\ell^- X$	$Ed^3\sigma/d^3\mathbf{q}$	$5 - 15$	$0.1 \le x_F \le 0.3$	61
E866 [50]	38.8	$p + Fe \to \ell^+\ell^- X$	R_{FeBe}	$4 - 8$	$0.13 \le x_F \le 0.93$	10
E866 [50]	38.8	$p + W \to \ell^+\ell^- X$	R_{WBe}	$4 - 8$	$0.13 \le x_F \le 0.93$	10
E537 [38]	15.3	$\pi^- + W \to \ell^+\ell^- X$	$d^2\sigma/dx_F dq_T$	$4 - 9$	$0 < x_F < 0.8$	48
E615 [4]	21.8	$\pi^- + W \to \ell^+\ell^- X$	$d^2\sigma/dx_F dq_T^2$	$4.05 - 8.55$	$0 < x_F < 0.8$	45

- Total of 383 number of points
- All fixed target, low-energy data
- We perform a cut of $q_T^{\max} < 0.25\, Q$

Parametrizations of the TMDs

- First perform single fits of these data to explore various aspects
- Many types of parametrizations have been used in the past
- For the "intrinsic" non-perturbative TMD, we perform fits with each of the following

Gaussian

$$\exp(-g_{q/N}(x, b_T)) = \exp\left(-g_q(x, A)\, b_T^2\right) ,$$

Exponential

$$\exp(-g_{q/N}(x, b_T)) = \exp\left(-g_q(x, A)\, b_T\right) ,$$

Gaussian-to-Exponential

$$\exp(-g_{q/N}(x, b_T)) = \exp\left(-g_q(x, A)\, \frac{b_T^2}{\sqrt{1 + B_{NP}(x) b_T^2}}\right) ,$$

Parametrizations

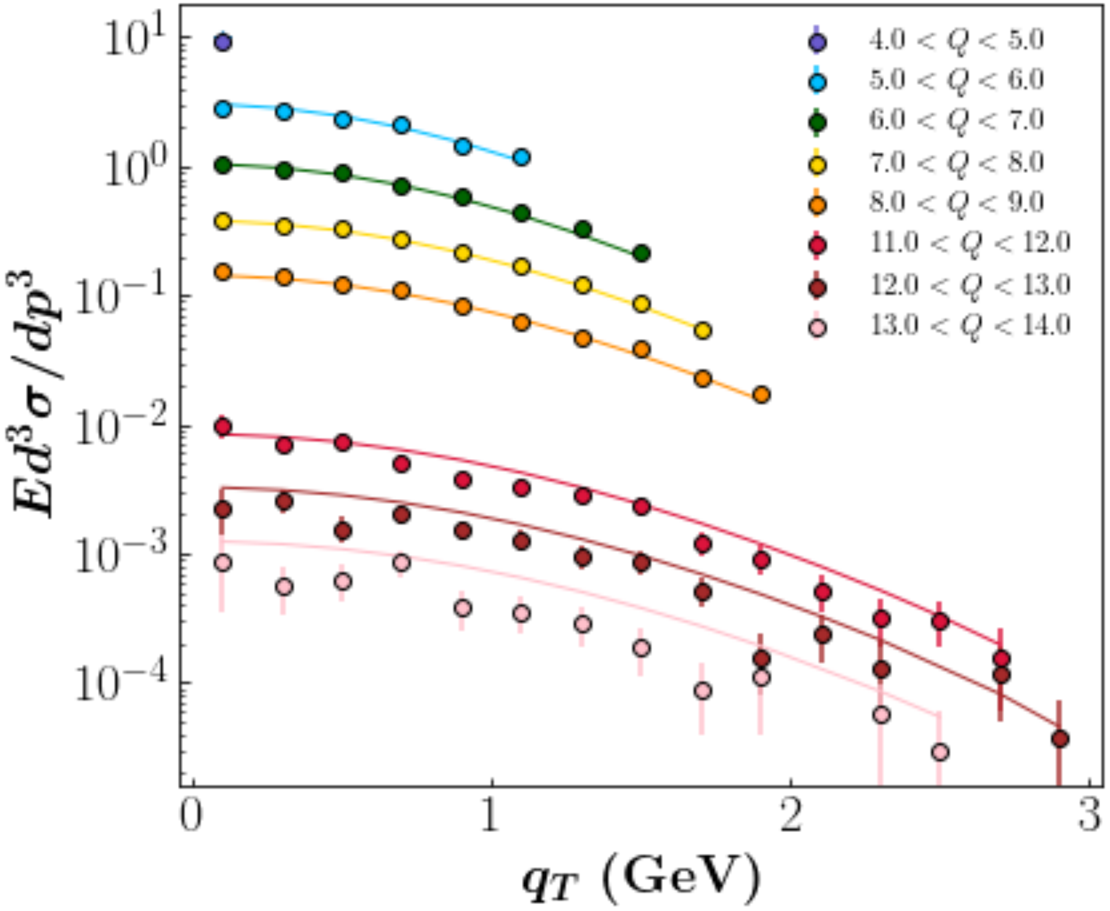
- We can test whether or not the x-dependence is important for these functions (it is!)
- For these g_q functions, we have the following

$$g_q(x, A) = |g^q + g_2^q x + g_3^q (1-x)^2|(1 + g_1(A^{1/3} - 1)) ,$$
$$B_{NP}(x) = b_{NP} x^2 ,$$

- 4 free parameters for each scheme (5 for Gaussian-to-Exponential)
- We may also open up these for each flavor in the proton (u, d, and sea) and for the pion (val, sea)

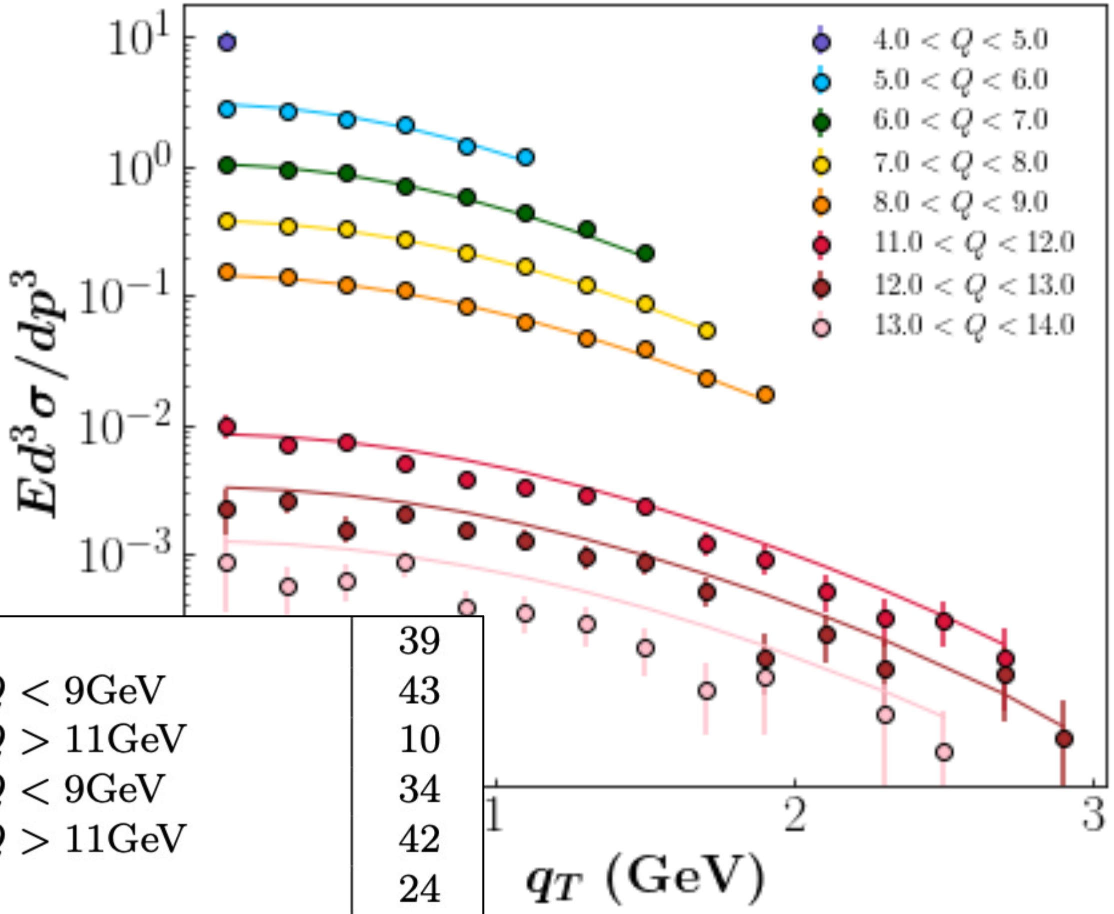
Problem describing data

- The E288 400 GeV data are difficult to describe the same above and below the Υ resonance
- Theory overpredicts data when $Q > 11\text{GeV}$



Problem describing data

- The E288 400 GeV data are difficult to describe the same above and below the Υ resonance
- Theory overpredicts data when $Q > 11\text{GeV}$
- Could treat as separate datasets – separate normalizations:



E228-200	39
E228-300 $Q < 9\text{GeV}$	43
E228-300 $Q > 11\text{GeV}$	10
E228-400 $Q < 9\text{GeV}$	34
E228-400 $Q > 11\text{GeV}$	42
E772	24
E605 $Q < 9\text{GeV}$	21
E605 $Q > 11\text{GeV}$	32

MAP parametrization

- A recent work from the MAP collaboration (Phys. Rev. D **107**, 014014 (2023).) used a complicated form for the non-perturbative function

$$f_{1\,NP}(x, \boldsymbol{b}_T^2; \zeta, Q_0) = \frac{g_1(x)\, e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\, g_{1B}^2(x) \left[1 - g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right] e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\, g_{1C}(x)\, e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x) + \lambda^2\, g_{1B}^2(x) + \lambda_2^2\, g_{1C}(x)} \left[\frac{\zeta}{Q_0^2}\right]^{\boxed{g_K(\boldsymbol{b}_T^2)/2}}, \tag{38}$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}}(1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}}(1-\hat{x})^{\alpha^2_{\{1,2,3\}}}},$$

$$\boxed{g_K(\boldsymbol{b}_T^2) = -g_2^2\,\frac{\boldsymbol{b}_T^2}{2}}$$

Universal CS kernel

- 11 free parameters; 12 to include the nuclear TMD parameter

MAP parametrization in the pion

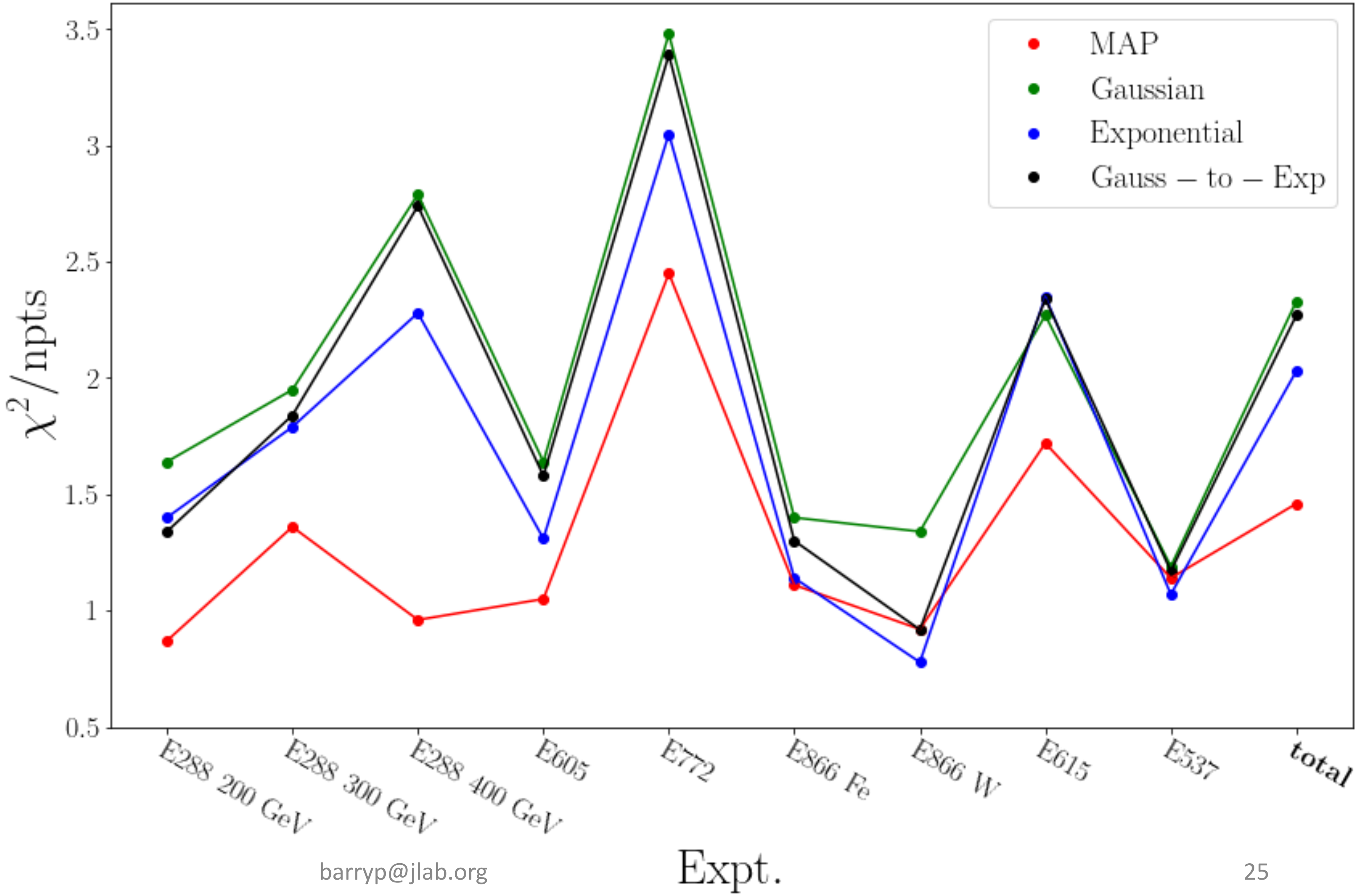
- As was shown in MAP's pion paper, Phys. Rev. D **107**, 014014 (2023), the pion does not require multiple weighted Gaussians

$$f_{1NP}^{\pi}(x, \boldsymbol{b}_T^2; \zeta) = e^{-g_{1\pi}(x)\frac{\boldsymbol{b}_T^2}{4}}$$

- We find the same result in our analysis
  - Original fits using 11 free parameters showed that the added Gaussians had no support in the kinematic region of the data

Resulting χ^2 for each parametrization

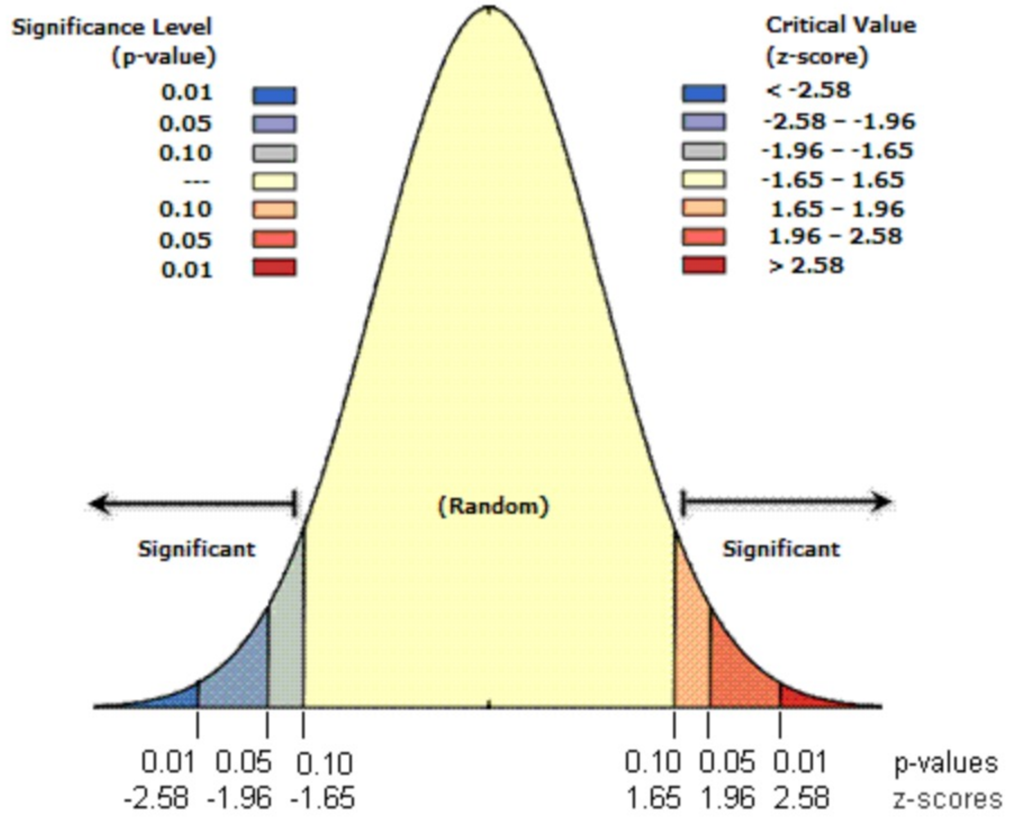
- MAP gives best overall
- How significant?



Z-scores

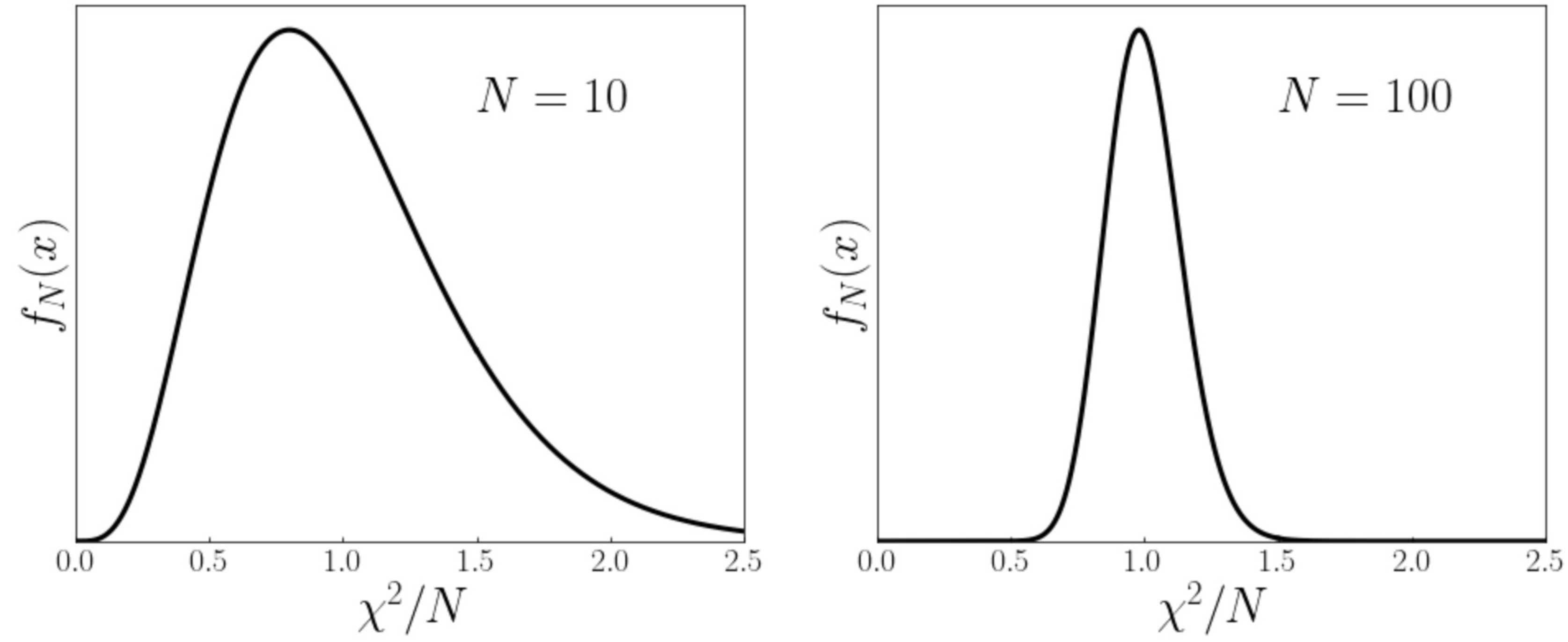
- A measure of significance with respect to the normal distribution
- Null hypothesis is the expected χ^2 distribution
- Alternative hypothesis is the resulting χ^2 from the fit

$$Z = \Phi^{-1}(p) \equiv \sqrt{2}\mathrm{erf}^{-1}(2p - 1).$$



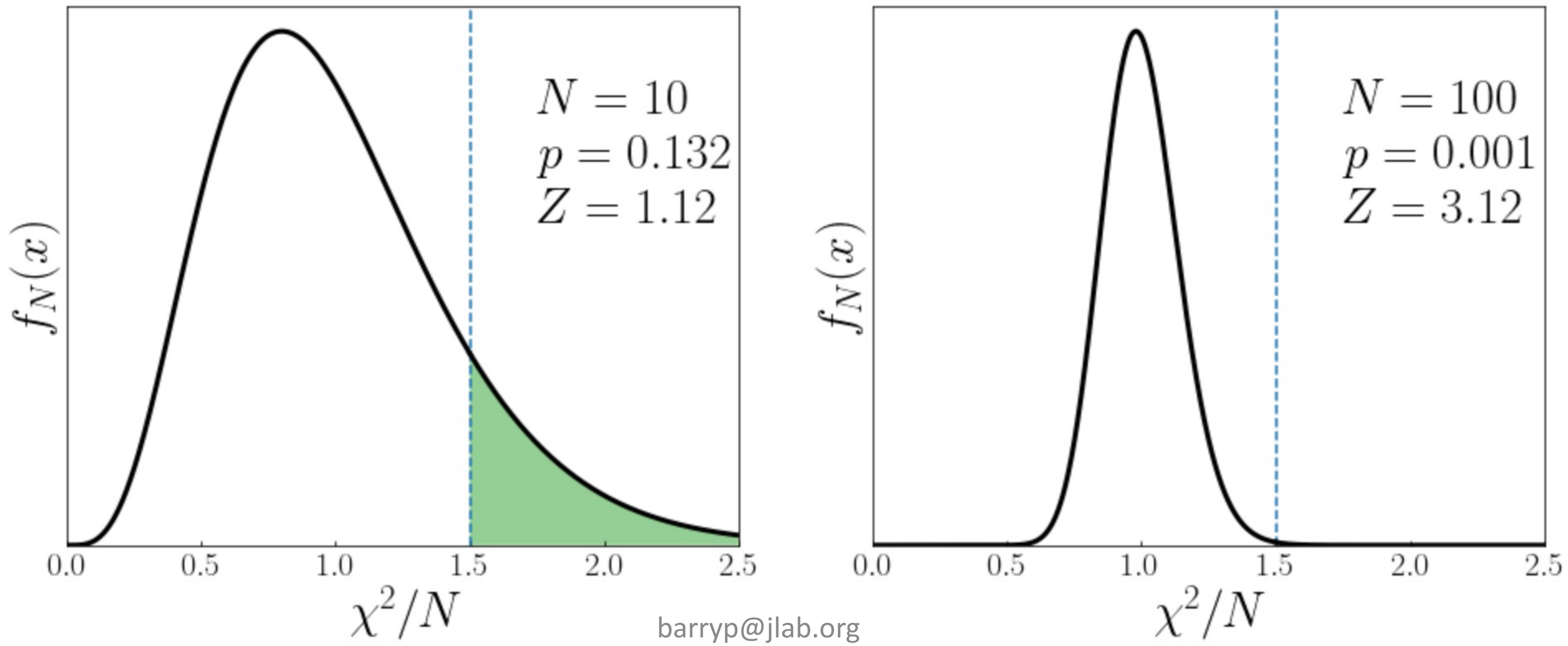
Visualizing Z-scores

- First show the $\chi^2(/N)$ distributions – null hypothesis
- N is the number of degrees of freedom



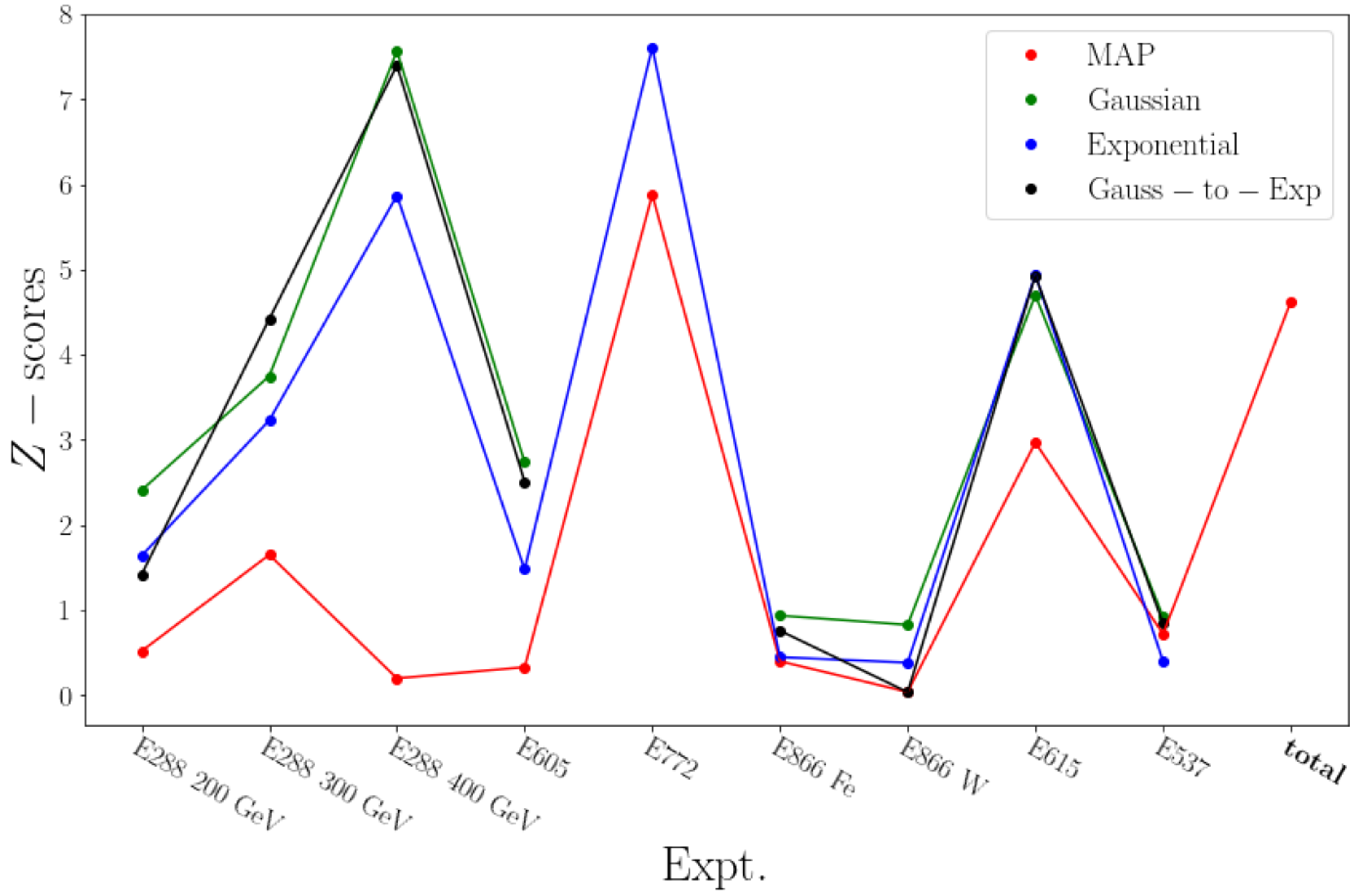
Visualizing Z-scores

- Now, introduce the alternative hypothesis, let's say $\chi^2/N = 1.5$ for both cases
- Compute area under the curve in green – this is the p-value



Z-scores

- Example of significance of the χ^2 values with respect to the expected χ^2 distribution



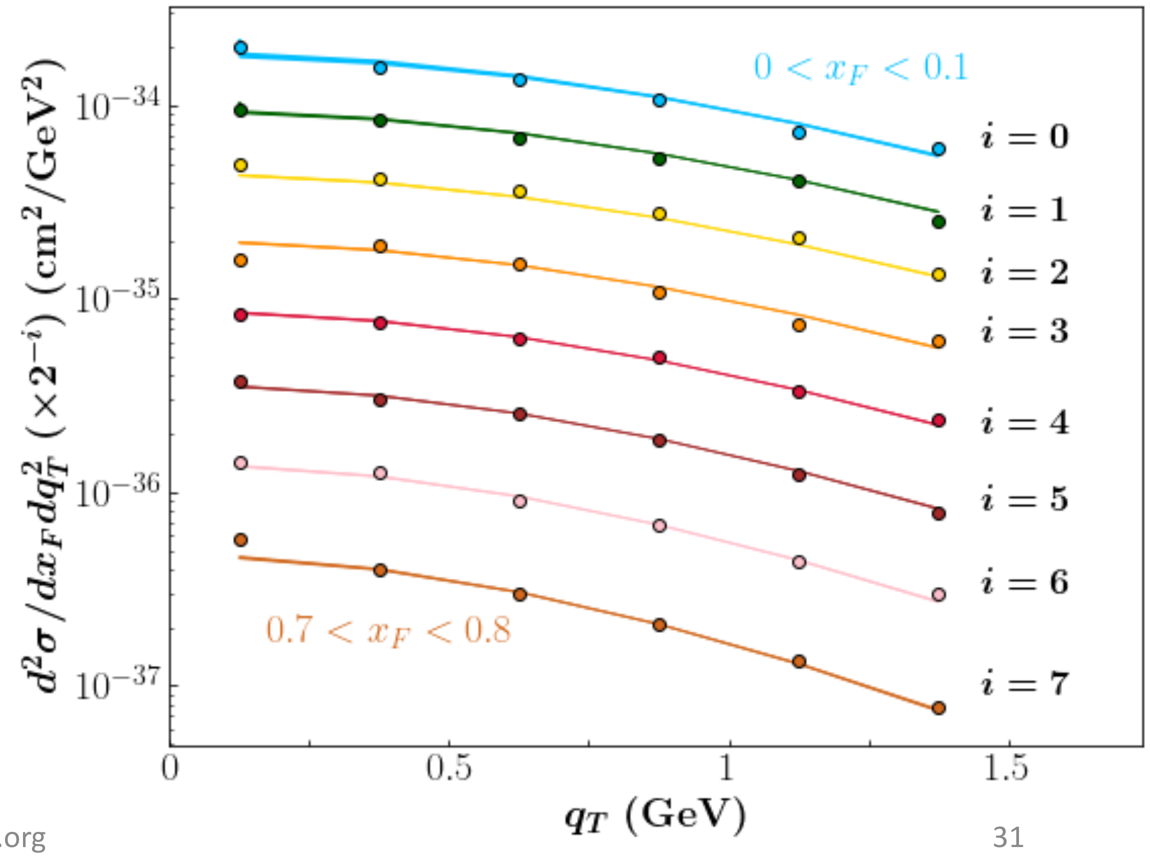
Perform the Monte Carlo

- We use the MAP parametrization
- Now, we can include the pion collinear PDF and its collinear datasets
- Include an additional 225 collinear data points
- Simultaneously extract
    1. Pion TMD PDFs
    2. Pion collinear PDFs
    3. Proton TMD PDFs
    4. Nuclear dependence
    5. Non-perturbative CS kernel

Data and theory agreement

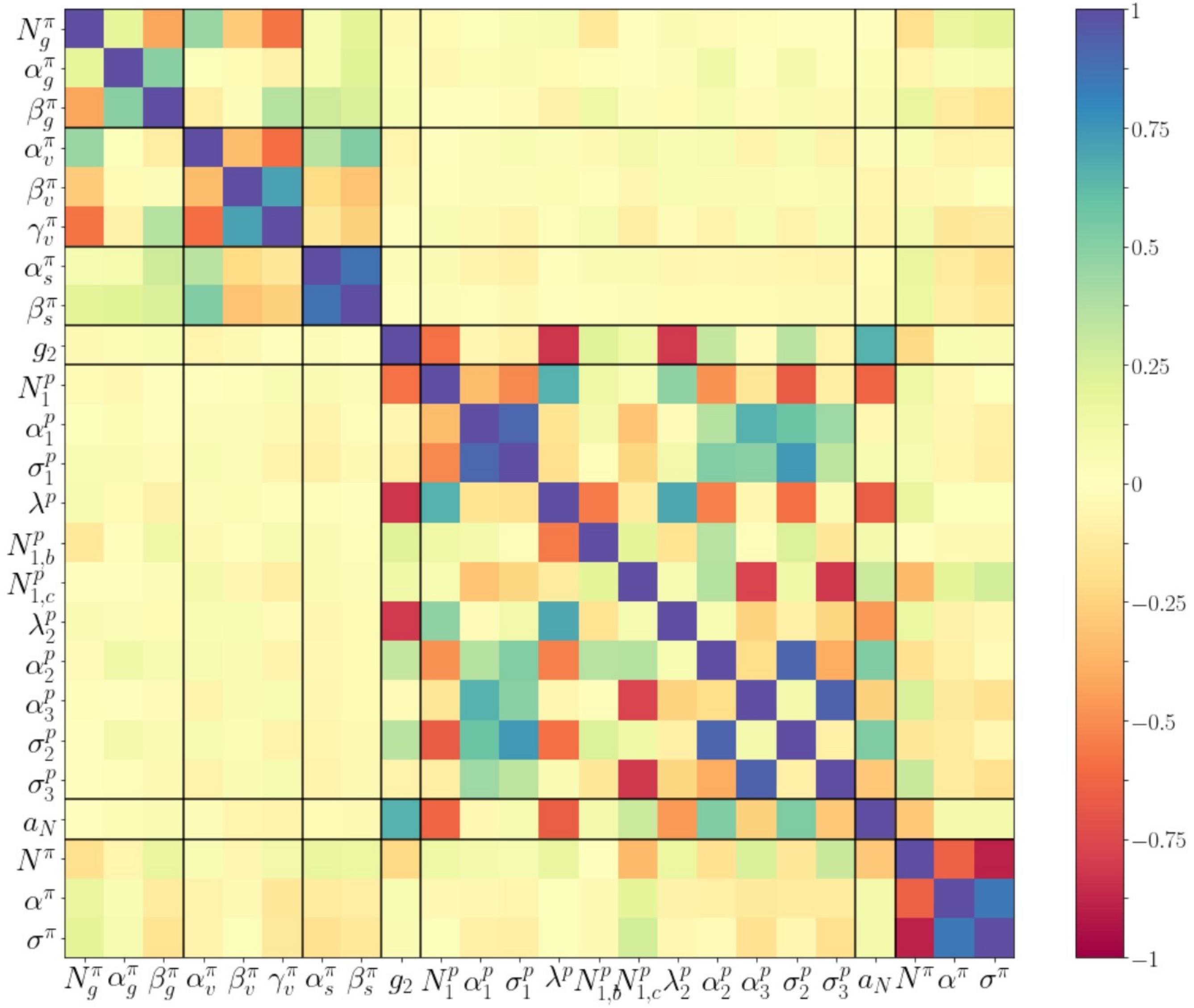
- Fit both pA and πA DY data and achieve good agreement to both

Process	Experiment	$\sqrt{s}$ GeV	χ^2/np	Z-score
q_T-*integr. DY* $\pi W \to \mu^+\mu^- X$	E615 [37]	21.8	0.86	0.76
	NA10 [38]	19.1	0.54	2.27
	NA10 [38]	23.2	0.91	0.18
Leading neutron $ep \to e'nX$	H1 [73]	318.7	0.36	4.61
	ZEUS [74]	300.3	1.48	2.16
q_T-*dep.* pA *DY* $pA \to \mu^+\mu^- X$	E288 [67]	19.4	0.93	0.25
	E288 [67]	23.8	1.33	1.54
	E288 [67]	24.7	0.95	0.23
	E605 [68]	38.8	1.07	0.39
	E772 [69]	38.8	2.41	5.74
	E866 (Fe/Be) [70]	38.8	1.07	0.29
	E866 (W/Be) [70]	38.8	0.89	0.11
q_T-*dep.* πA *DY* $\pi W \to \mu^+\mu^- X$	E615 [37]	21.8	1.61	2.58
	E537 [71]	15.3	1.11	0.57
Total			1.15	2.55

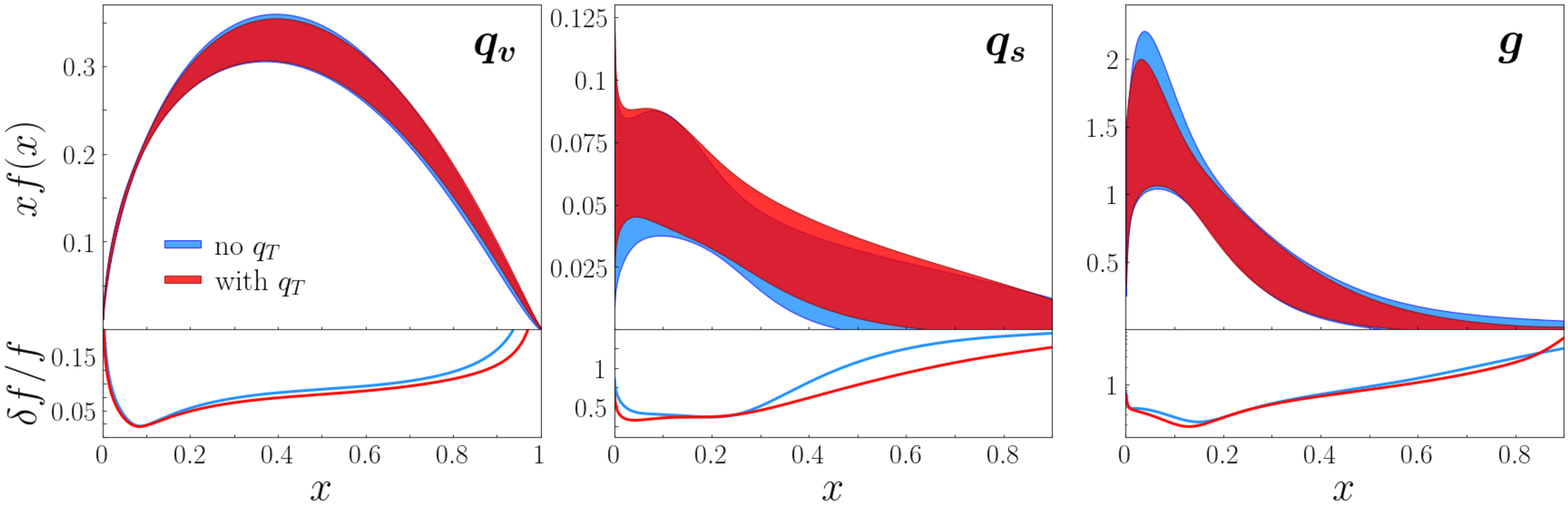


Correlations

- Level at which the distributions are correlated with each other
- Different distributions are largely correlated only within themselves



Extracted pion PDFs



- The small-q_T data do not constrain much the PDFs

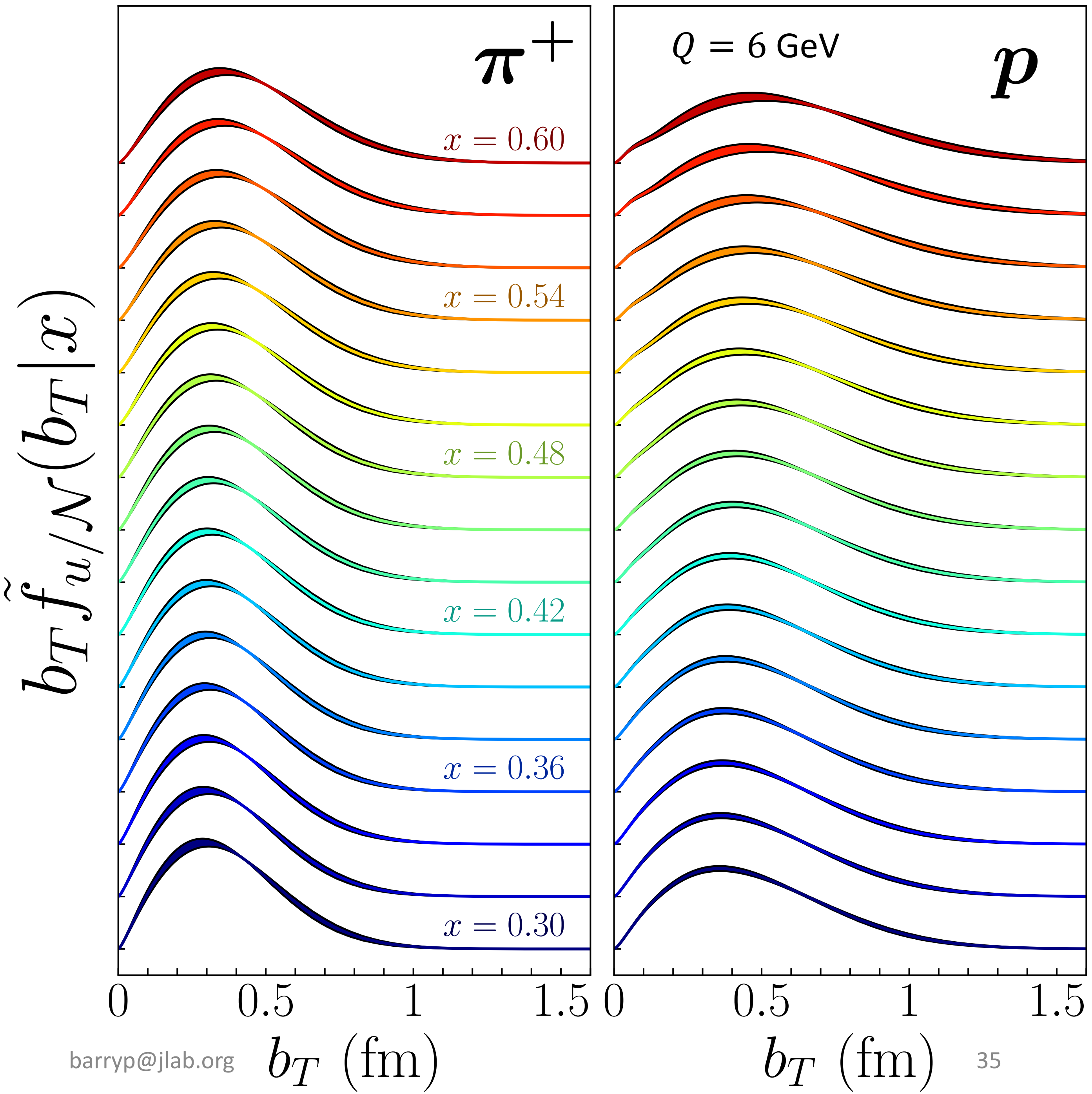
Conditional density

- We define a quantity in which describes the ratio of the 2-dimensional density to the integrated, b_T-independent number density, dependent on "b_T given x"

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T\,\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}\,.$$

Resulting TMD PDFs of proton and pion

- Shown in the range where pion and proton are both constrained
- Broadening appearing as x increases
- Up quark in pion is narrower than up quark in proton



Average b_T

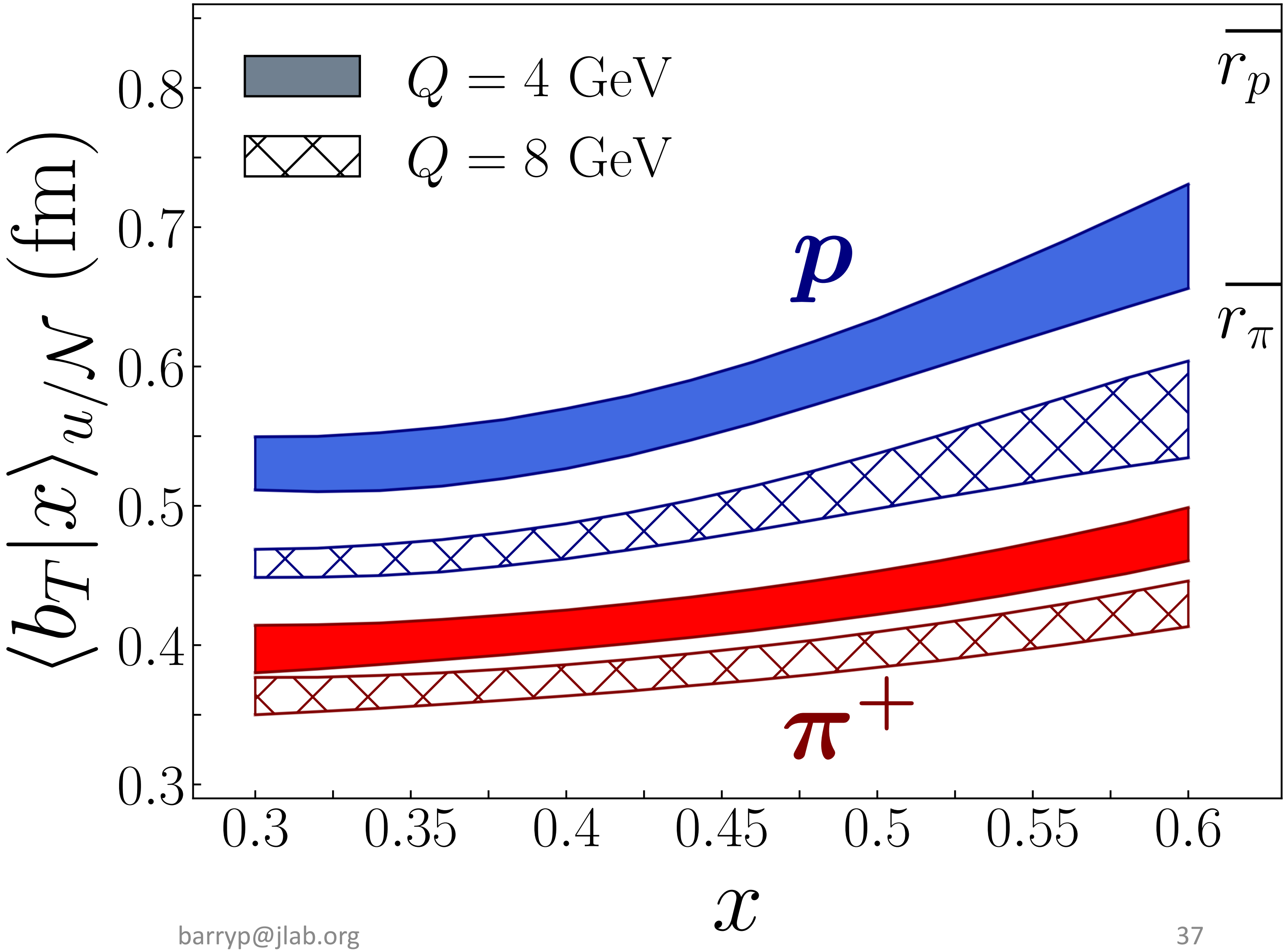
- The conditional expectation value of b_T for a given x

$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int \mathrm{d}^2 \boldsymbol{b}_T \, b_T \, \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$

- Shows a measure of the transverse correlation in coordinate space of the quark in a hadron for a given x

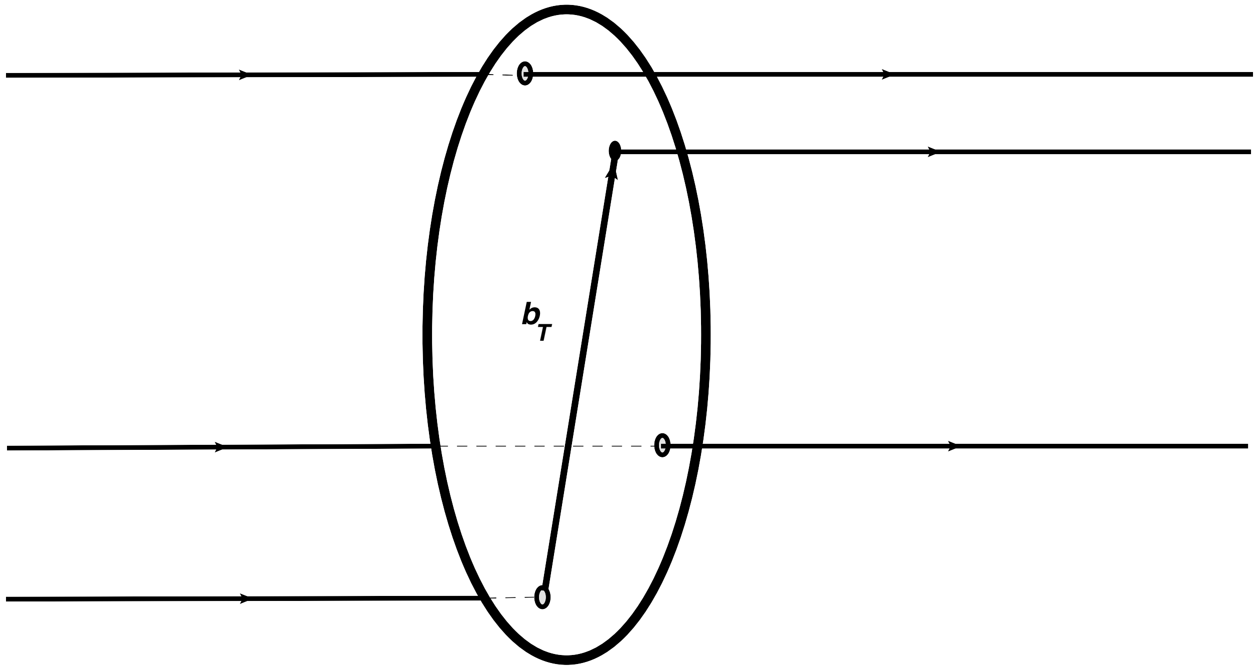
Resulting average $\boldsymbol{b_T}$

- Pion's $\langle b_T | x \rangle$ is $5.3 - 7.5\sigma$ smaller than proton in this range
- Decreases as x decreases



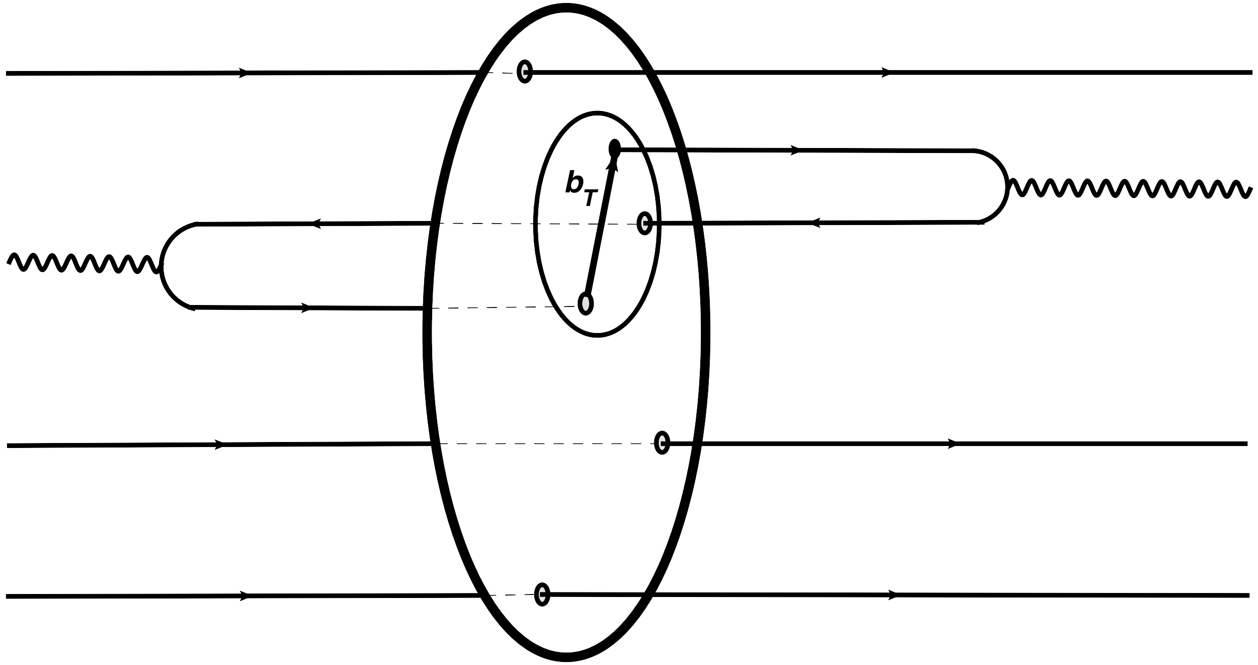
Possible explanation

- At large x, we are in a valence region, where only the valence quarks are populating the momentum dependence of the hadron



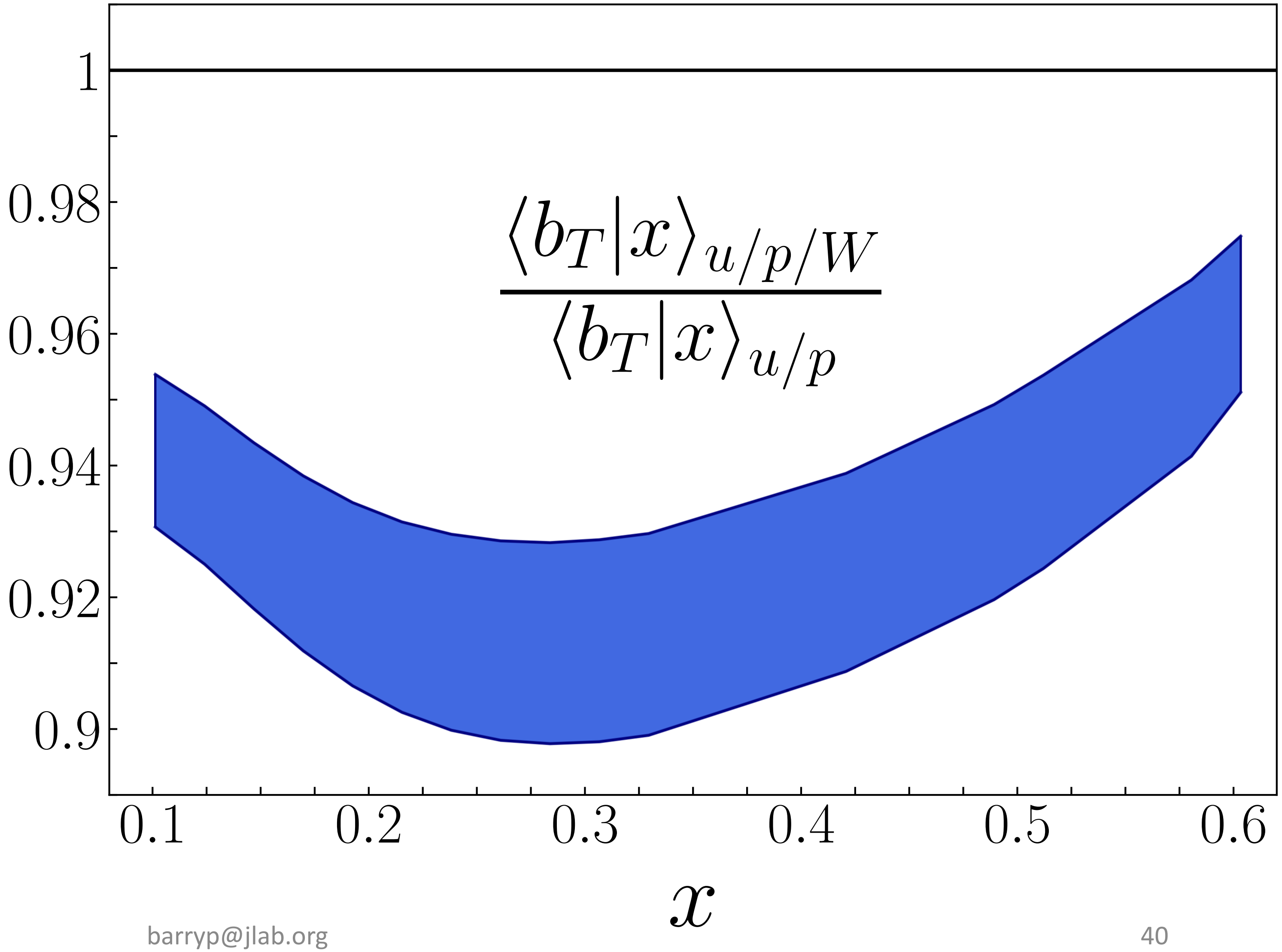
Possible explanation

- At small x, sea quarks and potential $q\bar{q}$ bound states allowing only for a smaller bound system



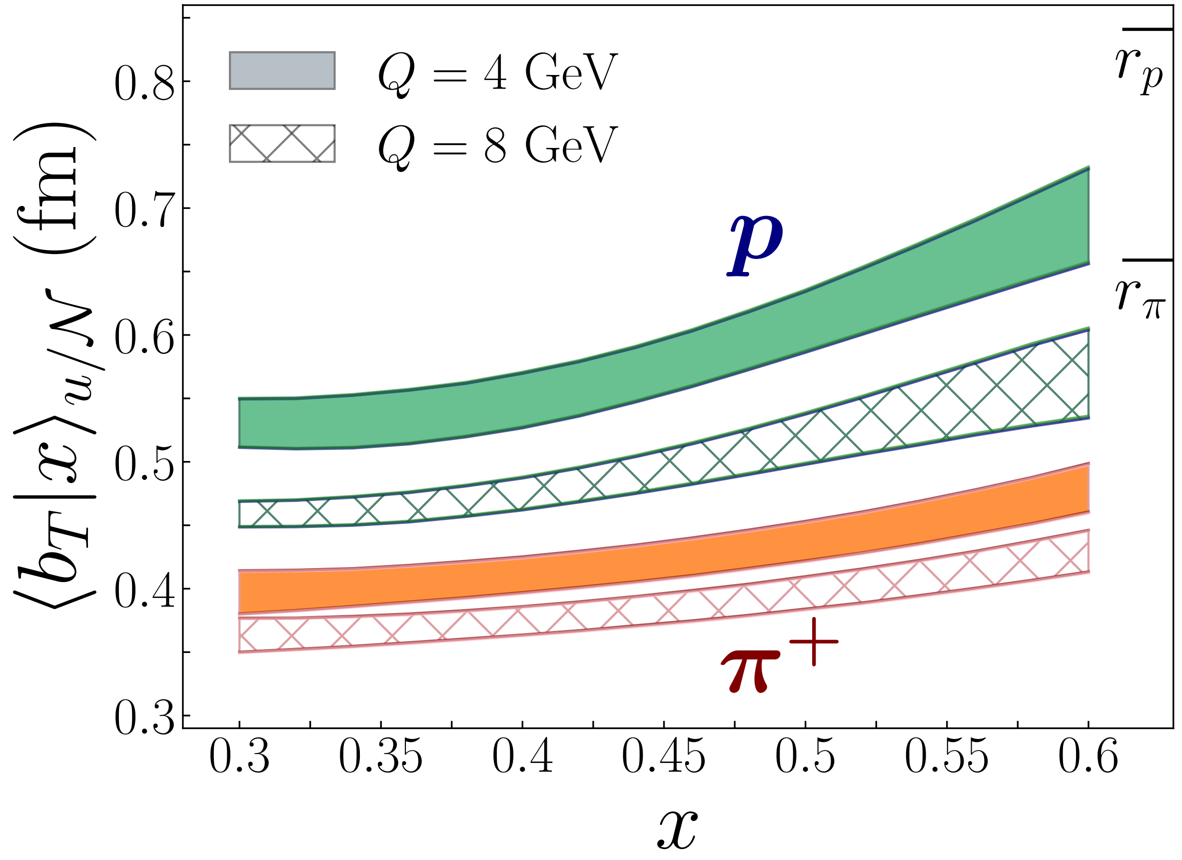
Transverse EMC effect

- Compare the average b_T given x for the up quark in the bound proton to that of the free proton
- Less than 1 by $\sim 5-10\%$ over the x range



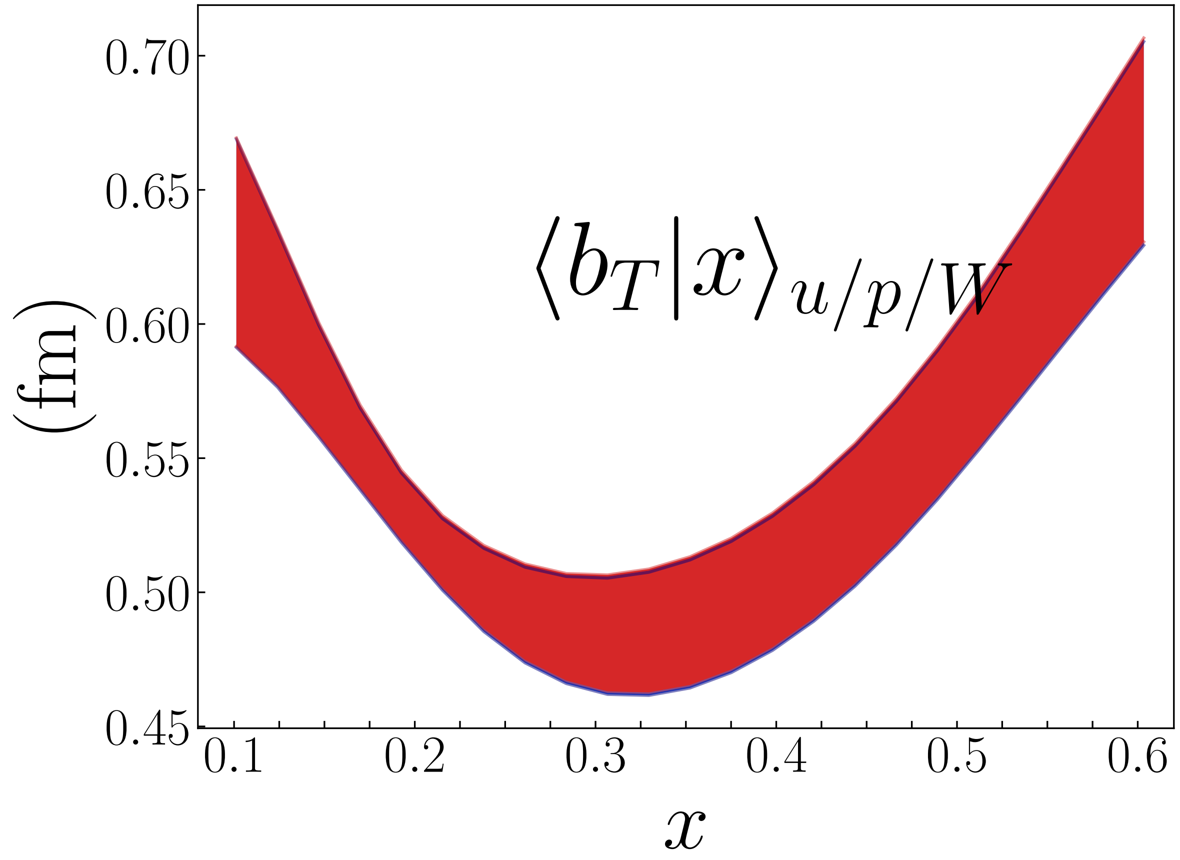
How sensitive are results to collinear PDFs?

- Blue (background) is proton with CT14nlo collinear
- Green - proton with MMHT
- Red (background) is pion with JAM collinear
- Orange - pion with xFitter



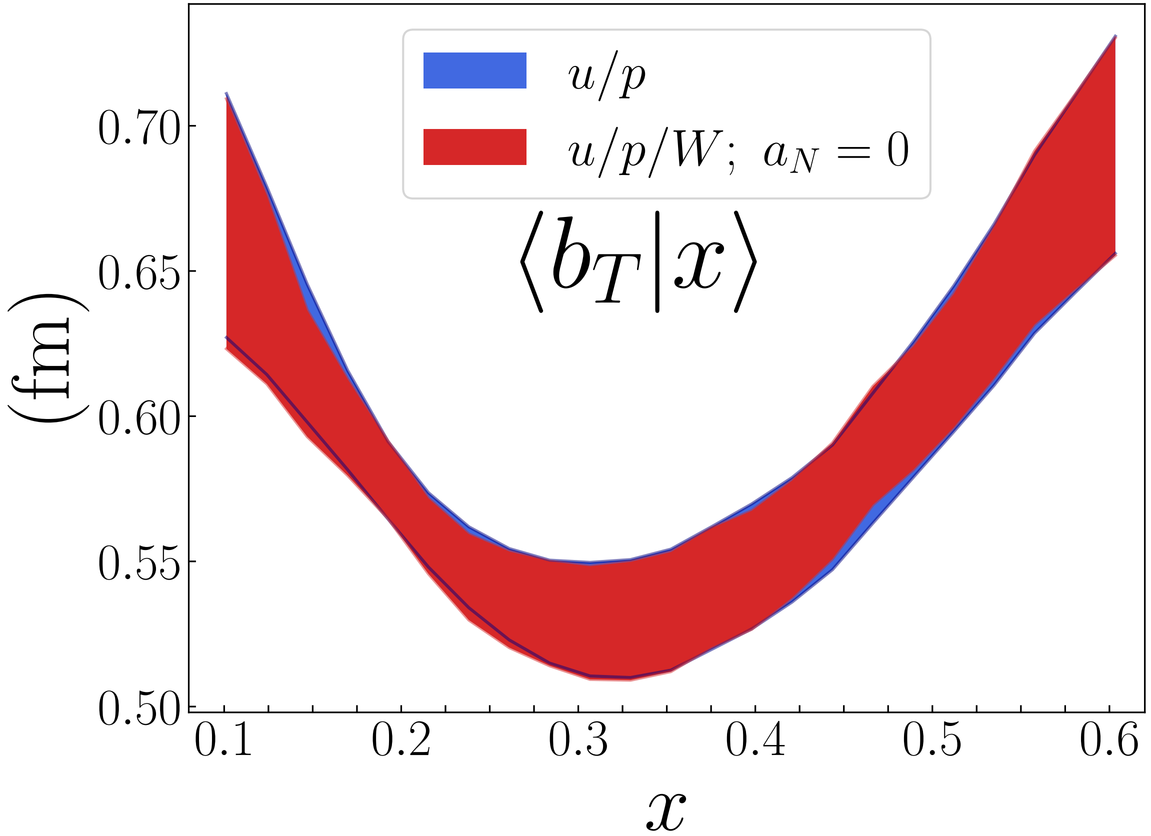
How sensitive are results to collinear PDFs?

- Change the nuclear PDF
- Blue: usual EPPS16
- Red: nCTEQ
- Almost identical!

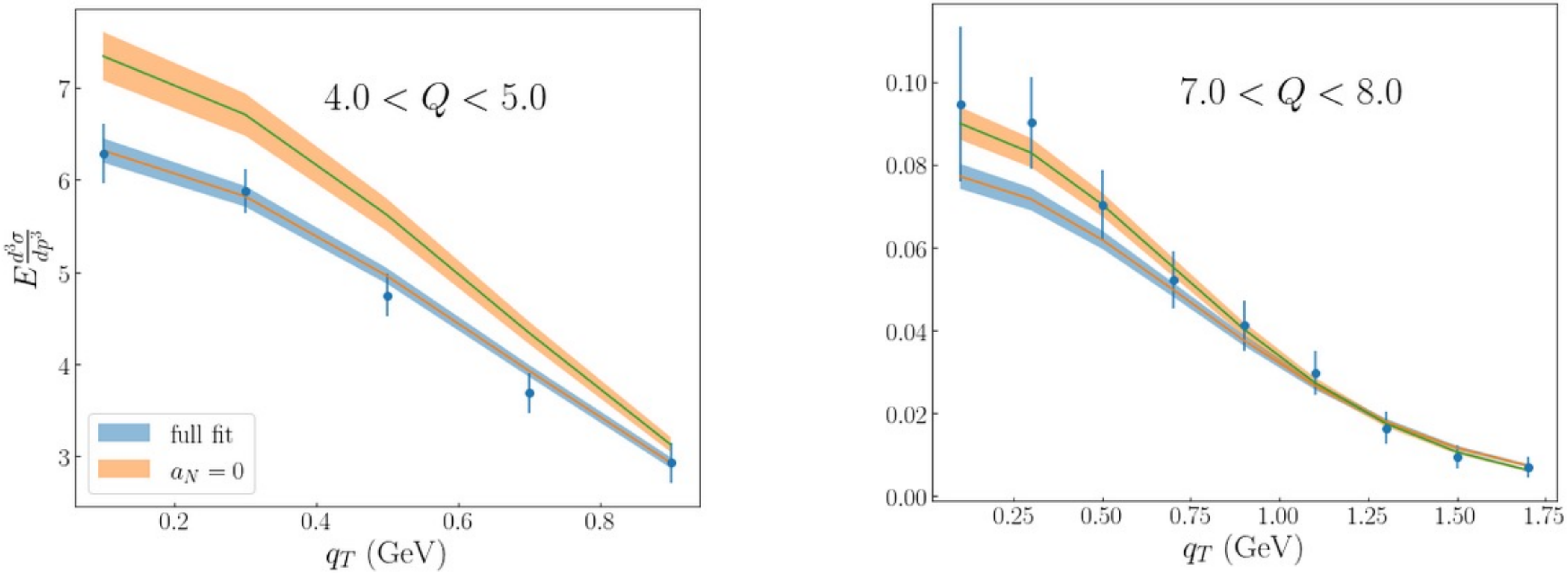


Importance of the nuclear parameter

- Let's compare the **free proton** with the proton bound in a nucleus, but **setting $\boldsymbol{a_N = 0}$**
- No discernable difference between them
- Collinear PDF has little to no impact on $\langle b_T|x\rangle$
- The corresponding ratio is consistent with 1



Compare theory bands for $a_N = 0$



- Blue band – fitted value from MC
- Orange band – prediction from fit, but setting $a_N = 0$

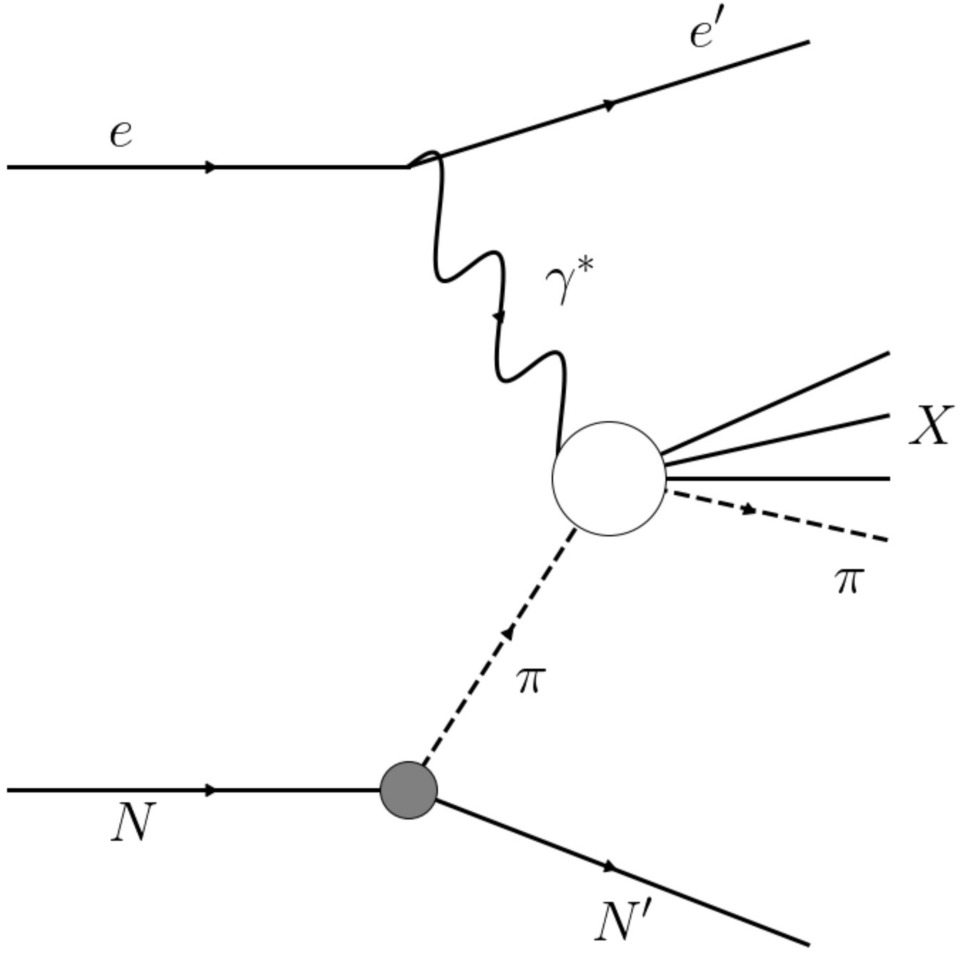
Outlook/Summary

- Implement a theoretical uncertainty due to the nuclear corrections in a fit – can we capture the nuclear corrections there?
- Important to study various hadronic systems to provide a more complete picture of strongly interacting quark-gluon systems emerging from QCD
- Lattice QCD can in principle calculate any hadronic state – look to kaons, rho mesons, etc.
- Future tagged experiments such as at EIC and JLab 22 GeV can provide measurements for neutrons, pions, and kaons

Future experiment – pion SIDIS

$$eN \to e'N'\pi X$$

- Measure an outgoing pion in the TDIS experiment
- Gives us another observable sensitive to pion TMDs
  - Needed for tests of universality



Backup

Reduced Ioffe time pseudo-distribution (Rp-ITD)

- Lorentz-invariant Ioffe time pseudo-distribution:

$$\mathcal{M}(\nu, z^2) = \frac{1}{2p^0} \langle p | \boxed{\bar{\psi}(0)} \gamma^0 \boxed{\mathcal{W}(z;0)} \boxed{\psi(z)} | p \rangle$$

Quark and antiquark fields

Gauge link

"Ioffe time"

$$\boxed{\nu = p \cdot z}$$

$$z = (0,0,0,z_3)$$

Observable is the *reduced* Ioffe time pseudo-distribution (Rp-ITD)

$$\boxed{\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}}$$

Ratio cancels UV divergences

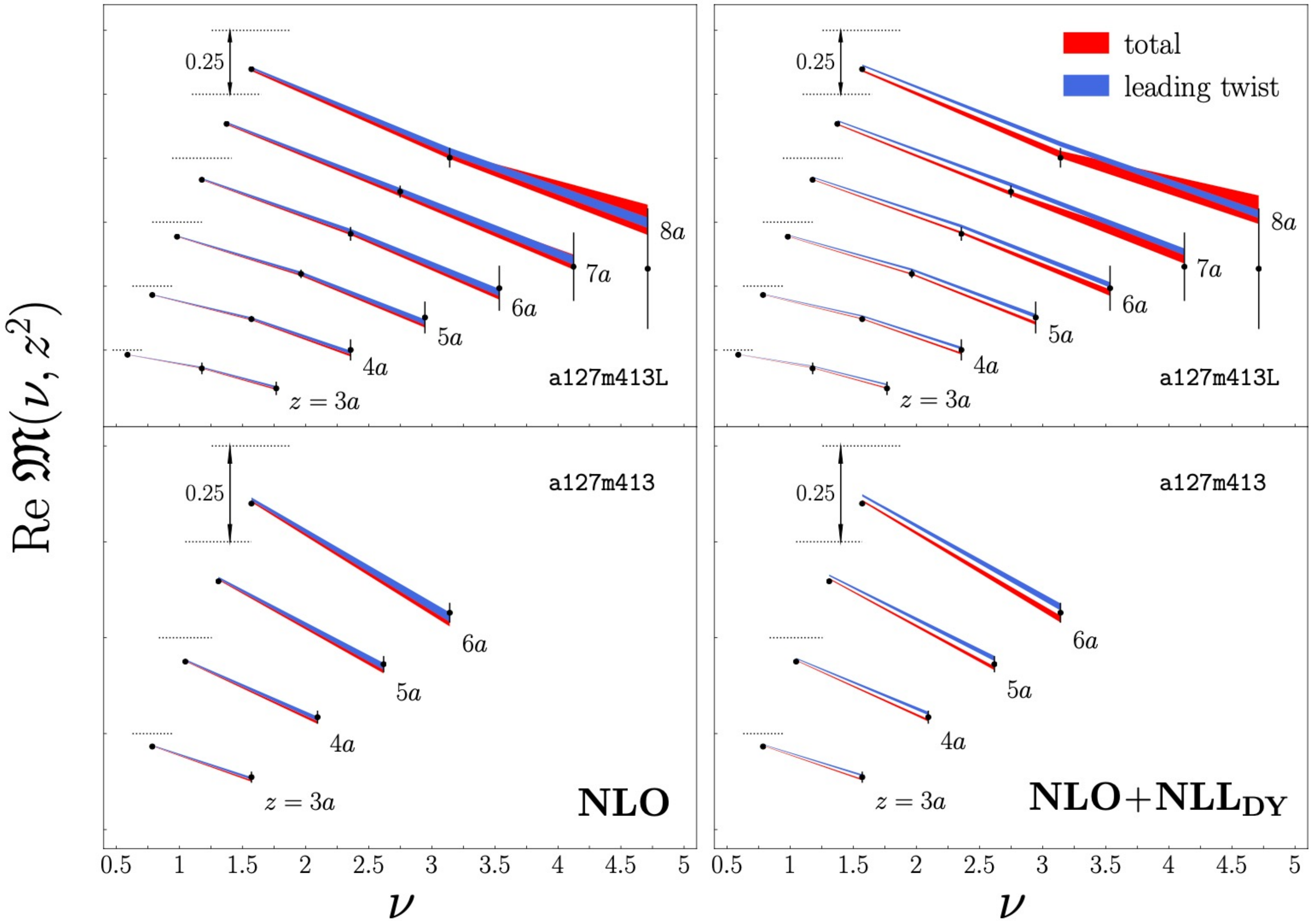
Goodness of fit

- Scenario A: experimental data alone
- Scenario B: experimental + lattice, no systematics
- Scenario C: experimental + lattice, with systematics

			Scenario A		Scenario B		**Scenario C**	
			NLO	+NLL$_{\mathrm{DY}}$	NLO	+NLL$_{\mathrm{DY}}$	NLO	+NLL$_{\mathrm{DY}}$
Process	Experiment	N_{dat}	$\overline{\chi}^2$		$\overline{\chi}^2$		$\overline{\chi}^2$	
DY	E615	61	0.84	0.82	0.83	0.82	**0.84**	**0.82**
	NA10 (194 GeV)	36	0.53	0.53	0.52	0.54	**0.52**	**0.55**
	NA10 (286 GeV)	20	0.80	0.81	0.78	0.79	**0.78**	**0.87**
LN	H1	58	0.36	0.35	0.39	0.39	**0.37**	**0.37**
	ZEUS	50	1.56	1.48	1.62	1.69	**1.58**	**1.60**
Rp-ITD	`a127m413L`	18	–	–	1.04	1.06	**1.04**	**1.06**
	`a127m413`	8	–	–	1.98	2.63	**1.14**	**1.42**
Total		**251**	**0.82**	**0.80**	**0.89**	**0.92**	**0.85**	**0.87**

Agreement with the data

- Results from the full fit and isolating the leading twist term
- Difference between bands is the systematic correction



Evolution equations for the TMD PDF

$$\frac{\partial \ln \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\boxed{\zeta}}} = \boxed{\tilde{K}(b_T; \mu)}$$

Collins-Soper (CS) kernel

Rapidity scale

Has its own renormalization group equation

$$\frac{\mathrm{d}\tilde{K}(b_T; \mu)}{\mathrm{d}\ln\mu} = \boxed{-\gamma_K\left(\alpha_S(\mu)\right)}$$

Anomalous dimension of CS kernel

$$\frac{\mathrm{d}\ln \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\mathrm{d}\ln \boxed{\mu}} = \boxed{\gamma_f(\alpha_S(\mu))} - \frac{1}{2}\boxed{\gamma_K(\alpha_S(\mu))}\ln\frac{\zeta}{\mu^2}$$

Renormalization scale

Anomalous dimension of TMDPDF

Small b_T operator product expansion

- At small b_T, the TMD PDF can be described in terms of its OPE:

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{\mathcal{C}}_{q/j}(x/\xi, b_T; \mu, \zeta_F) f_{q/\mathcal{N}}(\xi; \mu) + \mathcal{O}((\Lambda_{\mathrm{QCD}} b_T)^a)$$

- where $\tilde{C}$ are the Wilson coefficients, and $f_{q/\mathcal{N}}$ is the collinear PDF
- Breaks down when b_T gets large

b_* prescription

- A common approach to regulating large b_T behavior

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1+b_T^2/b_{\max}^2}}.$$

Must choose an appropriate value; a transition from perturbative to non-perturbative physics

- At small b_T, $b_*(b_T) = b_T$
- At large b_T, $b_*(b_T) = b_{\max}$

Introduction of non-perturbative functions

- Because $b_* \neq b_T$, have to non-perturbatively describe large b_T behavior

Completely general – independent of quark, hadron, PDF or FF

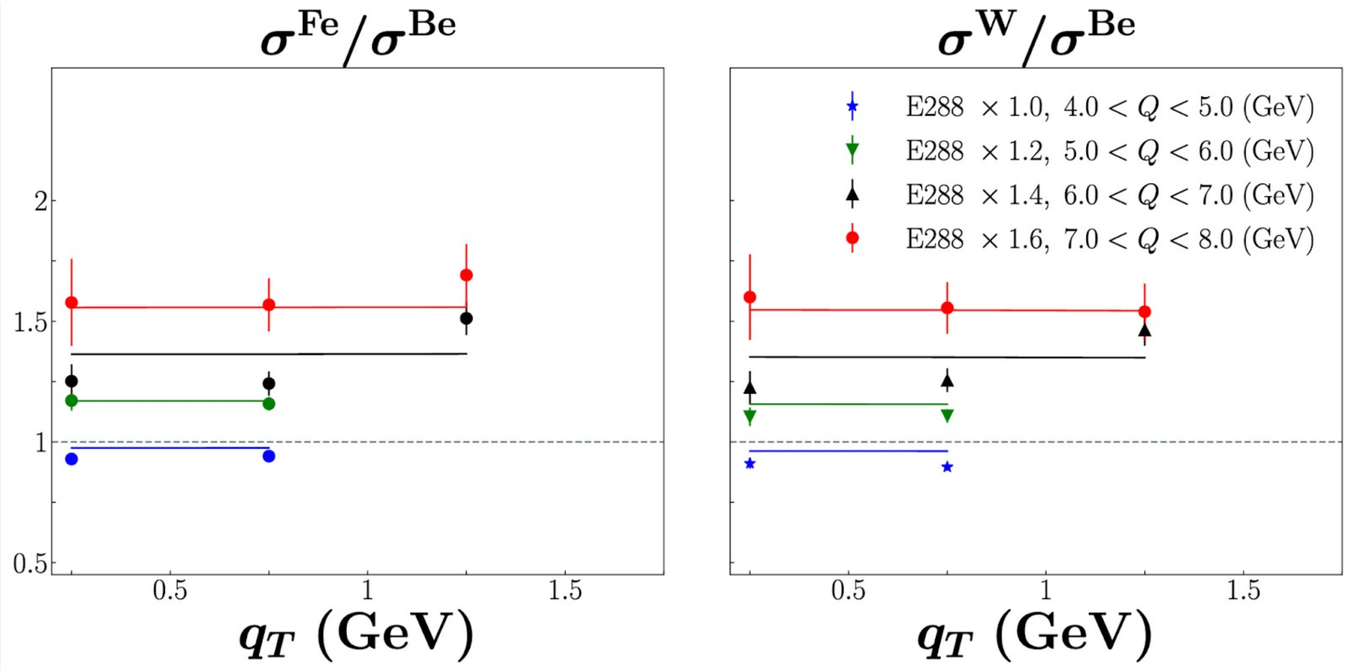
$$g_K(b_T; b_{\max}) = -\tilde{K}(b_T, \mu) + \tilde{K}(b_*, \mu)$$

Non-perturbative function dependent in principle on flavor, hadron, etc.

$$e^{-g_{q/\mathcal{N}(A)}(x,b_T)} = \frac{\tilde{f}_{q/\mathcal{N}(A)}(x, b_T; \mu, \zeta)}{\tilde{f}_{q/\mathcal{N}(A)}(x, b_*; \mu, \zeta)} e^{g_K(b_T; b_{\max}) \log\left(\sqrt{\zeta}/Q_0\right)}$$

A few words on nuclear dependence

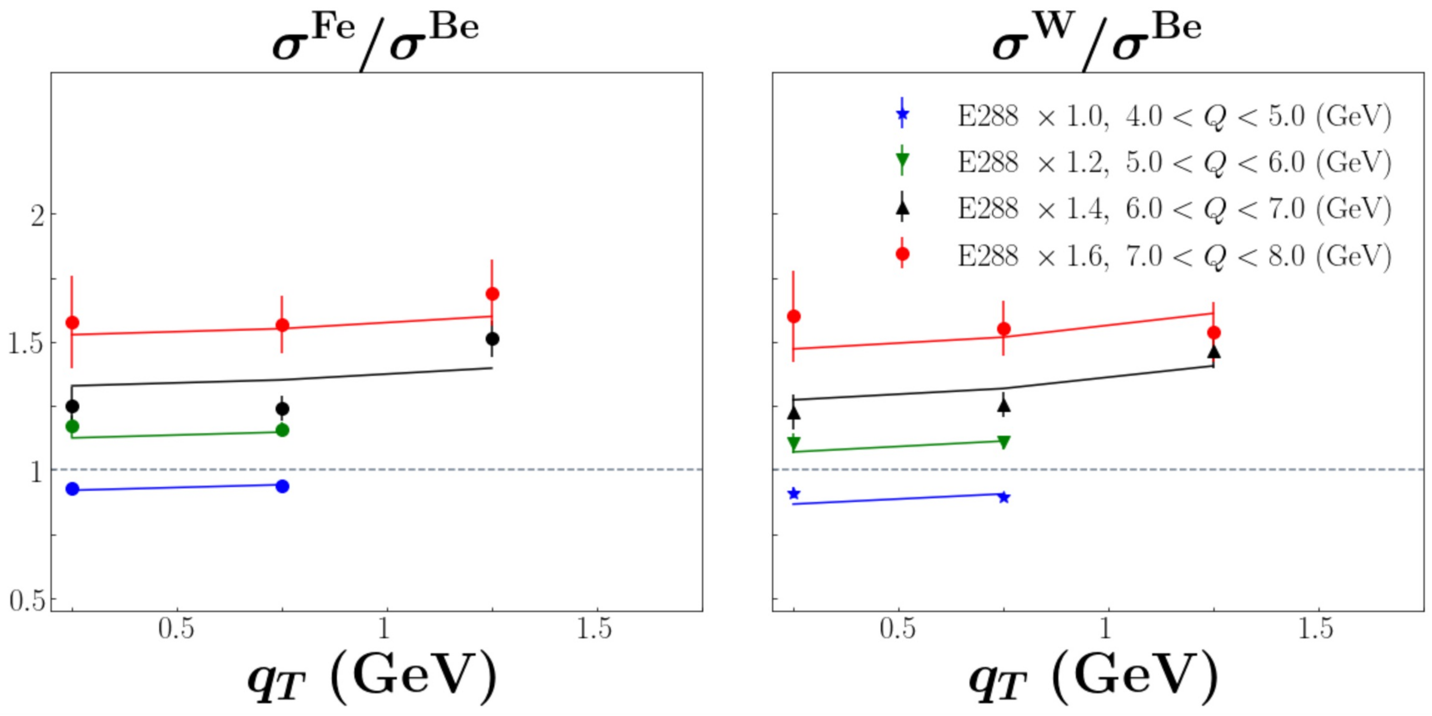
- The ratios from the E866 experiment provided a look to nuclear effects in TMDs as well as the importance of nuclear collinear effects
- Ignoring any nuclear corrections in TMDs and collinear PDFs



col	obs	tar	npts	chi2/npts	Z-score
E866	ratio	Fe/Be	10	2.2	2.16
E866	ratio	W/Be	10	3.51	3.67

Including nuclear dependence

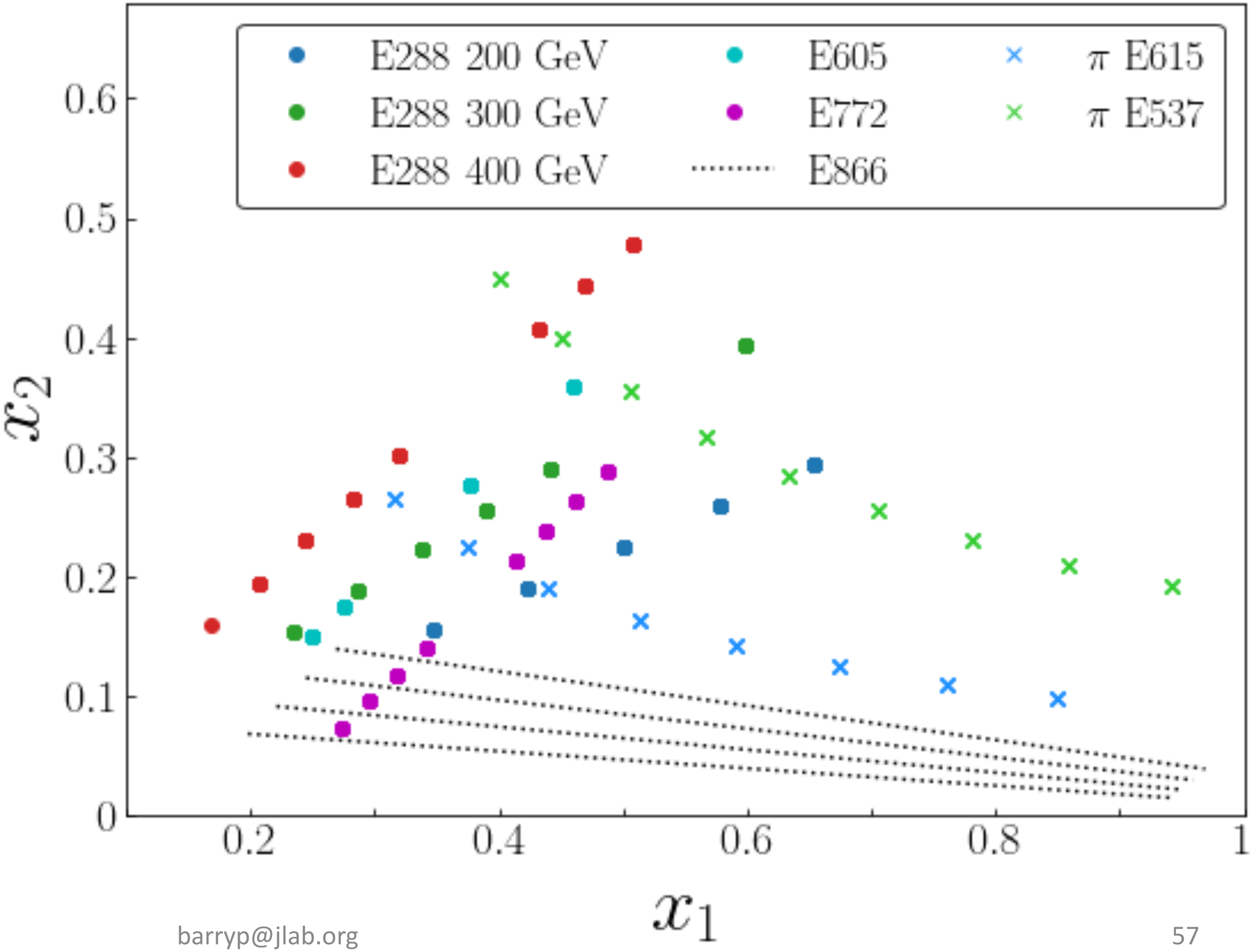
- Better description when including the nuclear dependence in the collinear PDF and TMD



col	obs	tar	npts	chi2/npts	Z-score
E866	ratio	Fe/Be	10	1.11	0.4
E866	ratio	W/Be	10	0.92	0.04

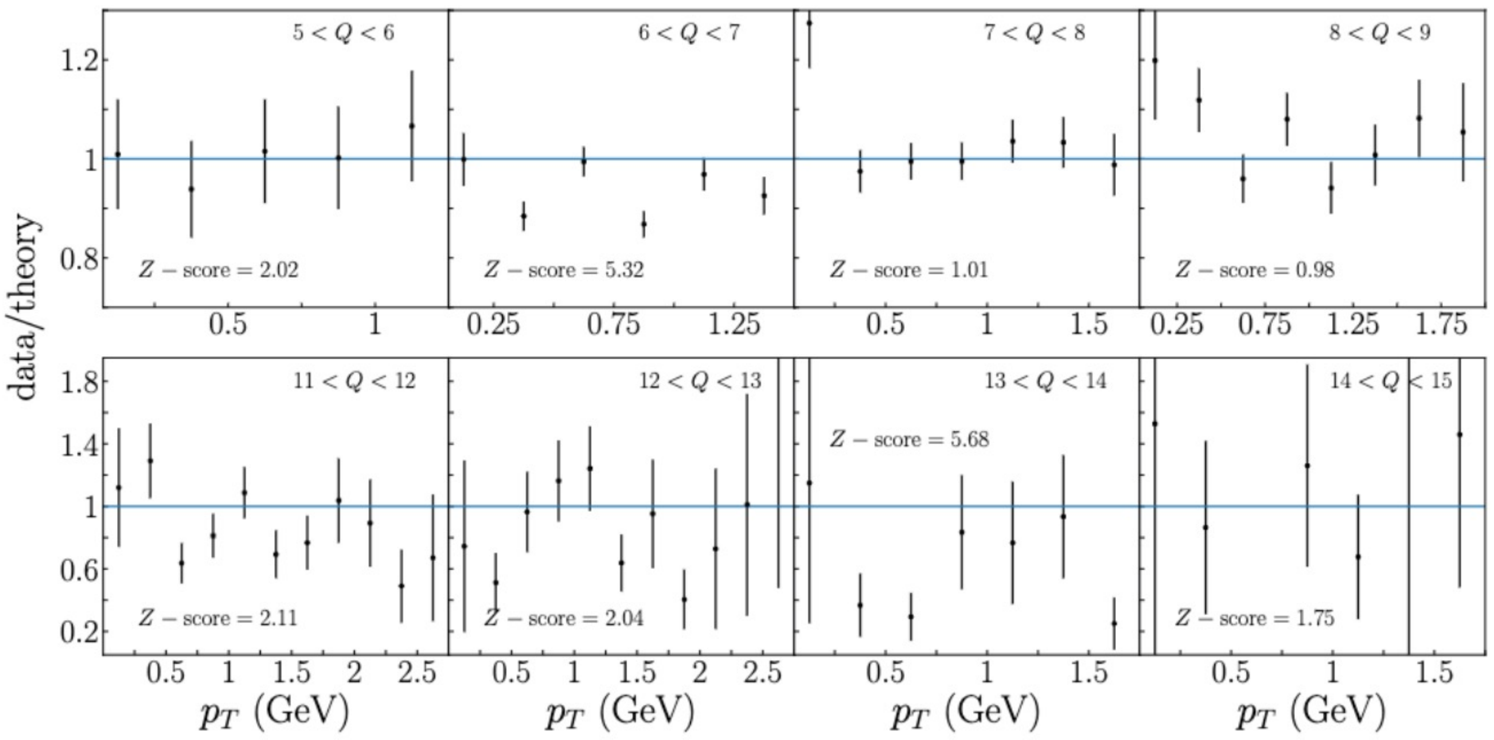
Kinematics in x_1, x_2

- Using the kinematic mid-point from each of the bins, we show the range in x_1 and x_2



E772 data

- Let's take a look at the data and theory agreement
- Data do not always follow the general trend and uncertainties appear underestimated



The Collins-Soper (CS) kernel

- From the simultaneous πA and pA analysis, which uses the same CS kernel, we compare with the lattice-generated data