Does the cosmological constant really indicate the existence of a dark dimension?

Arcangelo Pernace

University of Catania and INFN - Italy

06/10/2023

Annual meeting QGSKY

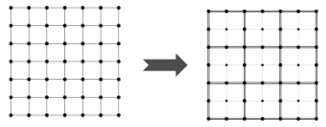
C. Branchina , V. Branchina , F. Contino , Phys.Rev.D 108 (2023) 4, 045007 , *arXiv:2304.08040*
C. Branchina , V. Branchina , F. Contino , A. Pernace, *arXiv: 2308.16548*

Plan of the talk

- Wilson's lesson.
- Higher dimensional theories. KK modes
- Scherk-Schwarz. Non-trivial boundary conditions
- Higgs Effective Potential
- Vacuum Energy
- Dark Dimension

Wilson's Lesson

What is the Wilson's lesson all about?



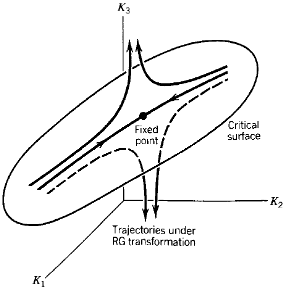
Theory at Λ $\rightarrow$ Theory at $\Lambda/2$ $\rightarrow$ ...

$$S_\Lambda \rightarrow S_{\Lambda/2} \rightarrow \ldots$$

Effective Field Theory paradigm

Any QFT is an Effective Field Theory

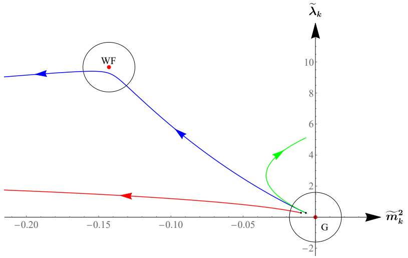
RG flow



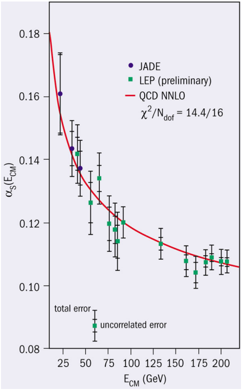
Renormalized theory: defined around a fixed point (critical surface)

... For theories in any dimension: ..., $d = 3$, $d = 4$, ...

$d = 3$ dimensions : Wilson-Fisher



$d = 4$ dimensions : AF



... Wilson's Lesson ...

EFT paradigm is physical and unavoidable

Unless we are considering the TOE

There is no cutoff in the sense that somebody finds disturbing ...

... but rather a (Wilsonian) physical running scale ...

$$\Lambda \quad \to \quad \Lambda/2 \quad \to \quad \Lambda/4 \quad \to \quad \Lambda/8 \quad \to \quad \ldots$$

Λ is the highest scale ... "UV physical cutoff"

Also for theories in $d > 4$ dimensions

in particular

Theories with compact extra dimensions: $d = 4 + n$

Field Theories with compact extra dimensions :

$$d = 4 + n$$

- Typically approached as 4D theories with infinite towers of states:

$$m_n = f_n \, m_{\mathrm{tow}}$$

- Surprising UV-softness :

Towers contribute $\sim m_{\mathrm{tow}}^4$

to Vacuum Energy / Effective Potential

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How is it possible?

Example : Scherk-Schwarz

5D SUSY theory defined on the multiply connected spacetime $\mathcal{M}^4 \times S^1$

- Different R-charges for superpartners ($i = b, f$)

$$\Psi_i(x, z + 2\pi R) = e^{2i\pi R q_i} \Psi_i(x, z) \;\Rightarrow\; \Psi_i(x, z) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{+\infty} \psi_{i,n}(x) e^{i(\frac{n}{R} + q_i) z}$$

$\int dz\, \mathcal{L}_{(5)} \rightarrow \mathcal{L}_{(4)} \leftarrow$ infinite towers of 4D KK fields, $m^2_{i,n} \propto \left(\frac{n}{R} + q_i\right)^2$

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- 4D mismatch in the masses of the superpartners :
  effective 4D non-local soft SUSY breaking

Higgs field ϕ : ϕ_0 , or 4D brane field , or ...

Effective 4D quadratic operator

$$M^2_{i,n}(\phi) = m^2(\phi) + \left(\frac{n}{R} + q_i\right)^2 , \qquad i = b, f$$

One-loop Higgs Effective Potential (4D calculation)

$$V_{1l}^{(4)}(\phi) = \frac{1}{2} \sum_a \sum_{i_a} (-1)^{\delta_{i_a, f_a}} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \log \left(p^2 + m_a^2(\phi) + \left(\frac{n}{R} + q_{i_a} \right)^2 \right)$$

One way of doing the calculation (not the only one):
Perform (first) the infinite sum; (then) integrate in $d^4 p$ with a cutoff Λ

Delgado, Pomarol, Quiros

Each tower contributes :

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$\implies$ Finite Higgs potential

~ 2000

- Finite Higgs Effective Potential (no fine-tuning)
- Finite Higgs mass (no fine-tuning)

Criticism : sum $[-L, L] \rightarrow$ UV-sensitive terms — Ghilencea, Nilles/Kim

... Heated debate ...

Calculations done in a different setup, proper time, thick brane, Pauli-Villars, dimensional regularization all seem(ed) to confirm UV-insensitive result

Antoniadis, Quiros / Delgado, v.Gersdoff, John, Quiros / Contino, Pilo / Barbieri, Hall, Nomura / Masiero, Scrucca, Silvestrini

Debate closed in favour of UV-insensitiveness ... but ...

4D Higgs Effective Potential from the 5D side

$$\mathcal{S}_{(5)} = \int dz\, d^4x \left(\frac{1}{2}\,\partial_a \widehat{\Phi}\,\partial^a \widehat{\Phi} + \partial_a \widehat{\chi}\,\partial^a \widehat{\chi}^\dagger + \frac{m_\Phi^2}{2}\,\widehat{\Phi}^2 + m_\chi^2\,\widehat{\chi}\widehat{\chi}^\dagger + \frac{\widehat{\lambda}}{4!}\,\widehat{\Phi}^4 + \frac{\widehat{g}}{2}\,\widehat{\Phi}^2 \widehat{\chi}\widehat{\chi}^\dagger \right)$$

$$\widehat{\Phi}(x, z + 2\pi R) = \widehat{\Phi}(x, z) \quad ; \quad \widehat{\chi}(x, z + 2\pi R) = e^{2i\pi R\, q}\,\widehat{\chi}(x, z)$$

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$$V_{1l}^{(5)}(\widehat{\Phi}) = \frac{1}{2}\mathrm{Tr}_5 \log \frac{p^2 + \frac{n^2}{R^2} + m_\phi^2 + \frac{\widehat{\lambda}}{2}\,\widehat{\Phi}^2}{p^2 + \frac{n^2}{R^2}} + \frac{1}{2}\mathrm{Tr}_5 \log \frac{p^2 + \left(\frac{n}{R} + q\right)^2 + m_\chi^2 + \frac{\widehat{g}}{2}\,\widehat{\Phi}^2}{p^2 + \frac{n^2}{R^2}}$$

$$\widehat{p} = (p_1, p_2, p_3, p_4, p_5 = \tfrac{n}{R}) = (p, p_5 = \tfrac{n}{R}) \to \mathrm{Tr}_5 = \tfrac{1}{2\pi R} \textstyle\sum_n \int \tfrac{d^4 p}{(2\pi)^4}$$

$$\mathrm{Tr}_5 \equiv \left(\sum_n \int \frac{d^4 p}{(2\pi)^5 R} \right)' \equiv \frac{1}{2\pi R} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \int^{C_\Lambda^n} \frac{d^4 p}{(2\pi)^4} \quad ; \quad C_\Lambda^n \equiv \sqrt{\Lambda^2 - \frac{n^2}{R^2}}$$

We cannot introduce any hierarchy between the different components of the loop momentum when calculating the Higgs Effective Potential

4D Effective Potential from the 5D Effective Potential

Fourier expansion of $\widehat{\chi}(x,z)$ (similarly for $\widehat{\Phi}$)

$$\widehat{\chi}(x,z) = \left(\sum_n \int \frac{d^4p}{(2\pi)^5 R}\right)' \widehat{\chi}_{n,p}\, e^{i\left(p\cdot x + \left(\frac{n}{R}+q\right)z\right)}$$

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$$\lambda \equiv \frac{\widehat{\lambda}}{2\pi R} \quad ; \quad g \equiv \frac{\widehat{g}}{2\pi R} \quad ; \quad \widehat{\Phi} = \frac{1}{\sqrt{2\pi R}}\phi$$

$$\boxed{V_{1l}^{(4)}(\phi) = 2\pi R\, V_{1l}^{(5)}(\widehat{\Phi})}$$

UV-sensitivity and non-trivial topology

Euler-McLaurin $\Rightarrow$

$$V_{1l}(\phi) = \frac{5m^2+3q^2}{180\pi^2} R\Lambda^3 - \frac{35m^4+14m^2q^2+3q^4}{840\pi^2} R\Lambda + \frac{m^5 R}{60\pi} - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R+3)+3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

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New q-dependent UV-sensitive terms:

- Not canceled by SUSY! $\propto \boxed{(q_b^2 - q_f^2)}\; m^2(\phi)\,\Lambda$
- Topological origin
- Absent for $q = 0$ and for $q_b = q_f$. But : (1) $q \neq 0$ in multiply connected space ; (2) $q_b \neq q_f$ for SUSY breaking

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UV-sensitive terms solely due to the non-trivial topology of space

Alternatively : Infinite sum & Smooth cut

Typical argument: cut on sum → spurious "divergences" ... But ...

$$V_{1l}(\phi) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\int \frac{d^4p}{(2\pi)^4}\log\left(\frac{p^2+m^2+\left(\frac{n}{R}+q\right)^2}{p^2+\frac{n^2}{R^2}}\right) e^{-\frac{p^2+\frac{n^2}{R^2}}{\Lambda^2}}$$

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UV-sensitive terms are NOT due to the sharp cut of the sum!
They come from a correct treatment of $\widehat{p}$ asymptotics

And now ... Cosmological Constant / Dark Dimension ...

Vacuum Energy and Dark Dimension

String theory (quantum gravity) Emergence Proposal Lee, Lerche, Weigand

Asymptotics moduli field ⇒ towers of two kinds: $\mu_{tow} = M_s$ or m_{KK}

(A)dS Distance Conjecture

$\mu_{tow} \sim |\Lambda_{cc}|^{\alpha}$ with Λ_{cc} cosmological constant (times M_P^2)

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Energy scale associated to Λ_{cc} is of the same order : $\Lambda_{cc}^{1/4} \sim 2.31$ meV

$\Rightarrow$ $\alpha = 1/4$, $\mu_{tow}^{exp} \sim 2.31\,\mathrm{meV}$ (order the neutrino scale)

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"In principle $\mu_{tow} = M_s$ or m_{KK}, but from the above equation: $\mu_{tow} = M_s$ ruled out by experiments since we know that physics around and above the neutrino scale is well described by effective field theories, and no sign of string excitations observed at these scales". Montero, Vafa, Valenzuela

Vacuum Energy and Dark Dimension

Their conclusion: "the only possibility left" is an

"Effective Field Theory decompactification scenario"

with Kaluza-Klein mass scale

$$m_{KK} \sim \mu_{tow}^{exp} \sim 2.31\,\mathrm{meV}$$

Vacuum Energy and Dark Dimension

Consider a 5D theory coupled to gravity

$$\widehat{g}_{MN} = \begin{pmatrix} e^{2\alpha\phi} g_{\mu\nu} - e^{2\beta\phi} A_\mu A_\nu & e^{2\beta\phi} A_\mu \\ e^{2\beta\phi} A_\nu & -e^{2\beta\phi} \end{pmatrix}$$

Background configuration: $g_{\mu\nu} = \eta_{\mu\nu}$, $A_\mu = 0$, $\phi = \phi_0$ (hereafter ϕ)

One-loop fermionic / bosonic contribution to the vacuum energy:

$$\rho_4 \sim (-1)^{\delta_{if}} \sum_n \int \frac{d^4 p}{(2\pi)^4} \log \frac{p^2 + e^{6\alpha\phi} \left(\frac{n}{R} + q\right)^2 + e^{2\alpha\phi} m^2}{\mu^2}, \quad i = b, f$$

Vacuum Energy and Dark Dimension

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- Correct treatment of $\widehat{p}$ asymptotics (EML):

$$\rho_4 = \frac{5\log\frac{\Lambda^2 e^{2\alpha\phi}}{\mu^2} - 2}{300\pi^2} e^{2\alpha\phi} R\Lambda^5 + \frac{5m^2 + 3q^2 e^{4\alpha\phi}}{180\pi^2} e^{2\alpha\phi} R\Lambda^3 - \frac{35m^4 + 14m^2q^2e^{4\alpha\phi} + 3q^4e^{8\alpha\phi}}{840\pi^2} e^{2\alpha\phi} R\Lambda + \frac{m^5}{60\pi} e^{2\alpha\phi} R + \frac{3\log\frac{\Lambda^2 e^{2\alpha\phi}}{\mu^2} + 2}{2880\pi^2 R^4} e^{10\alpha\phi} R\Lambda + R_4 + \mathcal{O}(\Lambda^{-1}) = 2\pi R e^{2\alpha\phi}\rho_5$$

$$R_4 = -\frac{x^2 \mathrm{Li}_3\left(r_b e^{-x}\right) + 3x\mathrm{Li}_4\left(r_b e^{-x}\right) + 3\mathrm{Li}_5\left(r_b e^{-x}\right) + 6\zeta(5)}{128\pi^6} \frac{e^{12\alpha\phi}}{R^4} + h.c.$$

$$r \equiv e^{2\pi i q R} \quad , \quad x \equiv 2\pi e^{-2\alpha\phi} R\sqrt{m^2} \implies R_4 \propto \frac{e^{12\alpha\phi}}{R^4} = m_{KK}^4$$

Dark Dimension

SUSY case : dominant contribution $\rho_4 \sim (q_b^2 - q_f^2)\, e^{6\alpha\phi} R\Lambda^3 = m_{KK}^2 R\Lambda^3$

Non-SUSY case : dominant contribution $\rho_4 \sim e^{2\alpha\phi} R\Lambda^5 = m_{KK}^{\frac{2}{3}} \left(R^{\frac{1}{3}}\Lambda\right)^5$

The UV-insensitive term R_4 cannot overthrow these contributions.
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But $\rho_4 \sim m_{KK}^4$ crucial for Dark Dimension Proposal

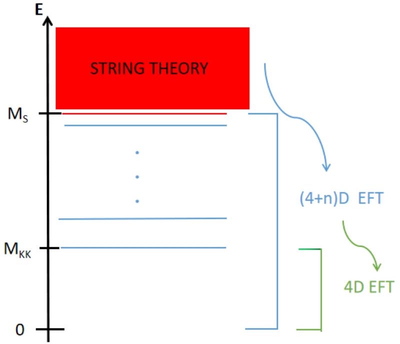
Conclusion:

Dark Dimension Proposal : Untenable

From String Theory to 4D Effective Field Theory (SM)

String theory $\Rightarrow$ EFT : takes over at M_s

from M_s down to the "physical scales" : EFT heavy artilery



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- Usual calculations mistreat the asymptotics of the loop momenta
- Correct treatment of the loop momenta asymptotics unveils the presence of UV-sensitive terms , missed in the usual calculations
- Interpretation of the $(4 + n)$ D theory with n compact extra dimensions as a 4D theory with an infinite number of fields needs to be taken with a grain of salt
- There are UV-sensitive terms of topological origin

Summary & Conclusions

- Usual calculations mistreat the asymptotics of the loop momenta
- Correct treatment of the loop momenta asymptotics unveils the presence of UV-sensitive terms , missed in the usual calculations
- Interpretation of the $(4+n)$ D theory with n compact extra dimensions as a 4D theory with an infinite number of fields needs to be taken with a grain of salt
- There are UV-sensitive terms of topological origin
- The idea that $\Lambda_{cc}^{1/4} \sim 2.31$ meV $\Rightarrow$ 5^{th} dimension (compact) of size $\sim \mu m$ is untenable

THANKS FOR ATTENTION

ADDITIONAL SLIDES

Global picture: EFTs with compact dimensions

- Start: $\mathcal{S}_\Lambda^{(5)}$ w/ "Wilsonian" mode expansion $\hat{p} \in [0, \Lambda]$
- Integrating out modes in $[k, \Lambda] \to \mathcal{S}_k^{(5)}$ $\quad$ k Wilsonian running scale

Due to $p_5 = n/R$ discreteness, p_5 eigenmodes contribution is stepwise

- For $k < 1/R$ no p_5 eigenmodes anymore: RG evolution becomes effectively of 4D type

It is **only in this sense** that the 4D theory emerges from the 4D one: **by no means it has an** *infinite* **tower of states**

Cutting the tower with Λ_{sp}

Cut in tower typical in Swampland: Species scale Λ_{sp} (e.g. emergence proposal)

Grimm, Palti, Valenzuela

Calculation of the vacuum energy ρ_4 using the species scale cutoff $\Lambda_{\rm sp}$

4D theory with N particle states, $\Lambda_{\rm sp} = M_p/\sqrt{N}$

4D theory with one compact dimension: $\Lambda_{\rm sp}$ identified by counting the number of KK states such that $m_n^2 \leq \Lambda_{\rm sp}^2$

Inequality is saturated when

$$\Lambda_{\rm sp}^2 = \frac{M_p^{4/3}}{(2R_\phi)^{2/3}} - \frac{M_p^{2/3}}{3\,(2R_\phi^4)^{1/3}} + \frac{m_\phi^2 + \frac{1}{4R_\phi^2}}{3} + \mathcal{O}(M_P^{-2/3}). \qquad (1)$$

Cutting the tower with Λ_{sp}

$$\rho_4 = \frac{20\log\left(\frac{4M_p^2}{5\mu^3 R_\phi}\right) + 12\pi - 57}{2^{-1/3}\cdot 3840\pi^2 R_\phi^{2/3}} M_p^{10/3} + \frac{-4\log\left(\frac{4M_p^2}{\mu^3 R_\phi}\right) - 6\pi + 27}{2^{-2/3}\cdot 2304\pi^2 R_\phi^{4/3}} M_p^{8/3} + \frac{12\pi - 35}{4608\pi^2 R_\phi^2} M_p^2$$

$$+ \frac{\left(4\, m_\phi^2 R_\phi^2 + 1\right)\log\left(\frac{M_p^2}{2\mu^3 R_\phi}\right) - 3(5-4\pi) m_\phi^2 R_\phi^2}{1152\pi^2 R_\phi^2} M_p^2$$

$$+ \frac{-20\log\left(\frac{M_p^2}{\mu^3 R_\phi}\right) - 120\pi + 309 + 104\log 2}{2^{-1/3}\cdot 124416\pi^2 R_\phi^{8/3}} M_p^{4/3} + \frac{3(19-8\pi) - 4\log\left(\frac{4M_p^2}{\mu^3 R_\phi}\right)}{2^{-1/3}\cdot 3456\pi^2 R_\phi^{8/3}} \left(m_\phi R_\phi\right)^2 M_p^{4/3}$$

$$+ \frac{525\pi + 367\log 2 - 1953 + 35\log\left(\frac{M_p^2}{\mu^3 R_\phi}\right)}{2^{-2/3} 1866240\pi^2 R_\phi^{10/3}} M_p^{2/3} + \frac{9\log\left(\frac{M_p^2}{\mu^3 R_\phi}\right) + 135\pi - 432 + 99\log 2}{2^{-2/3} 46656\pi^2 R_\phi^{10/3}} m_\phi^2 R_\phi^2 M_p^{\frac{2}{3}}$$

$$+ \frac{61 - 18\pi + 40(17-6\pi) m_\phi^2 R_\phi^2 + 80(33-9\pi) m_\phi^4 R_\phi^4}{138240\pi^2 R_\phi^4} + \frac{m_\phi^5 R_\phi}{60\pi} + R_4 + \mathcal{O}(M_P^{-2/3})$$

Cutting the tower with Λ_{sp}

This result does not contain UV-sensitive terms proportional to q

This comes from a physically illegitimate operation: rather than a cut on $p_5^2 = e^{-2\beta\phi}\, n^2/R^2$, $\Lambda_{\rm sp}$ implements a cut on $m_n^2 = m_\phi^2 + (n+q)^2/R_\phi^2$

$\Rightarrow$ Λ_{sp} inapplicable in theories with compact extra dimensions

These warnings do not apply to the case of a bona fide 4D theory with a large number N of fields coupled to gravity.

In this case $\Lambda_{\rm sp}$ truly is the quantum gravity physical cutoff

Secret liaison between proper time , thick brane & PV

Thick brane: $\sum_{n=-\infty}^{\infty} \int^{(\Lambda)} \frac{d^4p}{(2\pi)^4} \frac{e^{-\frac{\left(\frac{n}{R}+q\right)^2}{\Lambda^2}}}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$ Delgado, von Gersdor, John, Quiros

Pauli-Villars: $\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{(\Lambda R)^4}{(\Lambda R)^4+p^2+(\frac{n}{R}+q)^2} \frac{1}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$ Contino, Pilo

Proper Time: Antoniadis, Benakli

$$V_{1l}^{(4)}(\phi) = -\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{ds}{s} e^{-s\left(p^2+m^2+\left(\frac{n}{R}+q\right)^2\right)}$$

$$= -\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \Gamma\left(0, \frac{p^2+m^2+\left(\frac{n}{R}+q\right)^2}{\Lambda^2}\right)$$

In all cases a cut function of (p_5+q) instead of $p_5(=\frac{n}{R})$

Equivalent to introduce a hierarchy between (p_1, p_2, p_3, p_4) and p_5

$\Rightarrow$ **Again : artificial wash-out of UV-sensitive terms**

Weinberg - Laws of Progress in Theoretical Physics

This paper (Gellman-Low) is one of the most important ever published in quantum field theory ... This paper has a strange quality. It gives conclusions which are enormously powerful ... The input seems incommensurate with the output. The paper seems to violate

First Law of Progress in Theoretical Physics : Conservation of Information. Another way of expressing this law is : You will get nowhere by churning equations.

Second Law of Progress in Theoretical Physics : Do not trust arguments based on the lowest order of perturbation theory