Modified dispersion relations, generalized Bell nonlocality, quantum gravitational decoherence: windows to Planck-scale physics

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Work done at the Salerno Quantum Physics Group:

L. P. and F. Illuminati, *Nature Comm.* **12**, 4449 (2021);
P. Bosso, L. P., F. Wagner, and F. Illuminati, *Comm. Physics* **6**, 114 (2023);
P. Bosso, L. P., F. Wagner, and F. Illuminati, *Preprint arXiv:2309.xxxxx* (to appear).

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Outline

- Quantum gravity phenomenology: minimal length, modified dispersion relations, and deformed quantum mechanics
- Generalized spin operator and the anomalous magnetic moment of the electron
- Plank-scale effects on Bell nonlocality and generalized Tsirelson bounds
- Minimal length, Planck scale fluctuations, and quantum gravitational decoherence

Quantum gravity phenomenology and generalized dispersion relations

Quantum gravity regime: gravitational and quantum effects at the Planck length and energy scales

$$E_p \simeq 10^{19}\,\mathrm{GeV} \qquad \ell_p \simeq 10^{-35}\,\mathrm{m}$$

Phenomenology: where to look for? A common offspring of different quantum gravity theories are the modified dispersion relations (MDRs)

$$M^2 c^4 = f\,(E, k, M)$$

Searching for effects at scales reachable in the near future: lowest-order expansion in the Planck-scale parameters

$$M^2 c^4 = E^2 - k^2 c^2 + \ell_p\,\delta f\,(E, k, M)$$

MDRs and generalized Dirac equation

General quantum-gravity induced correction δf:

$$M^2 c^4 = E^2 - k^2 c^2 + \ell_p \sum_{n=0}^{2} \sum_{m=0}^{3-n} a_{n,m} (M\, c)^n k^m \left(\frac{E}{c}\right)^{3-n-m}$$

MDRs modify the Dirac spinor wave equation*.
Modified wave equation for the upper component of the Dirac spinor φ in an external field B in the non-relativistic limit:

$$i\partial_t \varphi = \left(1 - \frac{\ell_p}{\hat{k}_A^2} \sum_{n=0}^{2} \xi_n (M\, c)^n \hat{k}_{\mathcal{A}}^{3-n}\right) \frac{\hat{k}_A^2 - 2q\hat{S}^i B_i}{2M} \varphi \qquad \hat{k}_{\mathcal{A}}^i = \hat{k}_A^i - M|\mathcal{A}^i|$$

*P. Bosso, L.P., F. Wagner and F. Illuminati, arXiv:2309.xxxxx, to appear.

Anomalous magnetic moment of the electron

Modified electron magnetic moment and *g*-factor:

$$\mu^i_S = -\left(1 - \frac{\ell_p}{\hat{k}^2_A}\sum_{n=1}^{3}\xi_n\,(M\,c)^n\hat{k}^{3-n}_{\mathcal{A}}\right) g_S\,\mu_B\,B^i\,.$$

Via high-precision tests of g with Penning traps*, one can set bounds on the correction factors ξ_n. The estimates

$$\langle k^2_A\rangle \simeq (M\,v)^2 \sim \left(10^{-5}M\,c\right)^2 \qquad \langle k^n_{\mathcal{A}}\rangle \simeq (M|\mathcal{A}^i|)^n \sim \left(10^{-3}M\,c\right)^n$$

yield the bounds

$$\xi_1 \geq 10^2 \qquad \xi_2 \geq 10^5 \qquad \xi_3 \geq 10^8$$

*D. Hanneke, S. Fogwell and G. Gabrielse, Phys. Rev. A (2011).

Minimal-length quantum mechanics

MDRs with $n = 0, m = 3$ yield deformed CCRs and generalized uncertainty relations typically associated with the existence of a minimal scale of length*:

$$[\hat{x}, \hat{p}] = i\left(1 + \beta \ell_p^2 \hat{p}^2\right)$$

$$\Downarrow$$

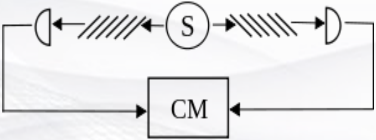
$$\Delta x \geq \frac{1}{2\Delta p} + \beta \ell_p^2 \Delta p$$

*C. A. Mead, Phys. Rev. (1964); A. Kempf, G. Mangano and R. B. Mann, Phys. Rev. D (1995); F. Scardigli, Phys. Lett. B (1999).

Phenomenological implications

Important phenomenological implications:

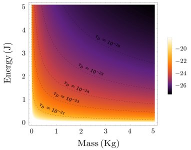
Deformed spin operator, generalized Bell nonlocality, and modified Tsirelson bound[a]



$$\mathcal{S}^{\max}_{MLQM} = 2\sqrt{2}\langle f^2(p^2)\rangle$$

[a] P. Bosso, L. P., F. Wagner, and F. Illuminati, Comm. Phys. (2023).

Universal quantum gravitational decoherence inspired by spin-foam models[a]



$$\tau_D = \hbar^6 / 16 m^2 \ell_p^4 t_p E^4$$

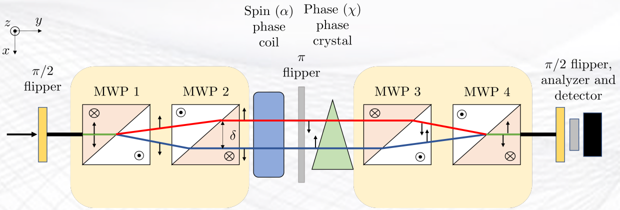
[a] L. P. and F. Illuminati, Nat. Comm. (2021).

Spin deformation and generalized Bell nonlocality

$$\left[\hat{x}^i, \hat{p}_j\right] = i\left[f\left(\hat{p}^2\right)\delta^i_j + g\left(\hat{p}^2\right)\frac{\hat{p}^i\hat{p}_j}{\hat{p}^2}\right]$$

In the non-relativistic limit of the modified Dirac equation the spin expectations are momentum-dependent*:

$$\langle\hat{s}_i\rangle = \frac{\langle f(\hat{p}^2)\rangle}{2}\langle\hat{\sigma}_i\rangle$$

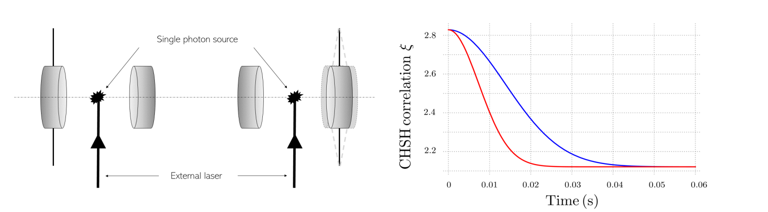


*P. Bosso, L. P., F. Wagner, and F. Illuminati, Comm. Phys. (2023).

Fluctuating deformation parameter at the Planck scale and quantum gravitational decoherence

$$\beta = \sqrt{t_p}\chi(t) \qquad \langle\chi(t)\rangle = \bar{\beta} \qquad \langle\chi(t)\chi(t')\rangle = \delta(t-t')$$

With this assumption, minimal-length quantum mechanics predicts a universal decoherence mechanism consistent with observational data and experimentally falsifiable*



*L. P. and F. Illuminati, Nat. Comm. (2021).

Take-home message

- MDRs yield potentially accessible phenomenological predictions of quantum gravity effects
- High-precision tests (e.g. anomalous magnetic moment of the electron) can put stringent constraints on MDRs
- Quantum gravity induced deformations can give rise to testable far-reaching consequences (generalized Bell nonlocality and generalized Tsirelson bound, universal quantum gravitational decoherence, etc.)


Thank you