Naturalness. Higgs mass, cosmological constant and Physical Tuning

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C. Branchina, V. Branchina, FC, PRD 107 (2023) 9, 096012
see also: C. Branchina, V. Branchina, FC, N. Darvishi, PRD 106 (2022) 6, 065007

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Effective Field Theory and Standard Model

QFT contains an **ultimate scale** $\Lambda_{phys} \Rightarrow \mathcal{L}_{\Lambda_{phys}}$

(For notational convenience : $\Lambda_{phys} \rightarrow \Lambda$)

Below Λ : Effective Field Theory (EFT): ok $\mathcal{L}_{\Lambda}$

Above Λ : UV completion needed : $\not{\mathcal{L}}_{\Lambda}$

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STANDARD MODEL

- Higgs boson: **un-suppressed** quantum fluctuations $\Rightarrow m_H^2 \sim \Lambda^2$

  "**Quadratic sensitivity**" to the ultimate scale of the theory

  Note: $m_H^2 \sim \Lambda^2$ is $m_H^2(\mu)$ at $\mu = \Lambda$

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- If Λ too large $\Rightarrow$ $m_H^2(\Lambda)$ "**unnaturally**" large
  $\Rightarrow$ problem of "**hierarchy**" with Fermi scale μ_F
  where $m_H^2(\mu_F) \sim (125\ \mathrm{GeV})^2$

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- **Several attempts** to "solve" this naturalness/hierarchy (NH) problem.

  Let's focus on some of them ...

RG equation for $m_H^2(\mu)$

$$\mu \frac{d}{d\mu} m_H^2(\mu) = \gamma\, m_H^2(\mu) \tag{1}$$

1. **Quantum Gravity Miracle** - UV completion of SM provides $m_H^2(\Lambda) \ll \Lambda^2$. **Naturalness** "solved" **outside** SM. **Hierarchy inside** SM with (1). Perturbative RG: $\gamma \ll 1 \Rightarrow m_H^2(\mu_F)$ and $m_H^2(\Lambda)$ **same order**!

Giudice, PoS EPS-HEP2013, 163 (2013)

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2. **Self-organized criticality** - Assume **gravity** provides **non-perturbative** $\gamma\,(\sim 2)$. **Large hierarchy** between Fermi scale μ_F and UV scale Λ **accommodated** $\Rightarrow$ **no NH problem**

   Pawlowski, Reichert, Wetterich, Yamada, Phys. Rev. D99, 086010 (2019)

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3. **Dimensional regularization** - DR endowed with special physical properties $\Rightarrow$ NH problem **absent from the beginning**

These approaches cannot solve the NH problem

Why?

The EFT nature of the SM **not properly** and **fully** taken into account

Statement: The SM in an Effective Theory

Meaning:

(A) the parameters (masses, couplings) $g_i(\Lambda)$ in $\mathcal{L}_{SM}^{(\Lambda)}$ result from **integrating out** the **higher energy dof** related to the **UV completion** of the SM

(B) the same parameters $g_i(\mu)$ at scales $\mu < \Lambda$ result from **integrating out** the modes of the **fields that appear in** $\mathcal{L}_{SM}^{(\Lambda)}$ in the range $[\mu, \Lambda]$.

Wilson's Lesson

Wilson RG equations - One component Scalar Theory

Action $\quad \mathcal{S}_\Lambda[\Phi] = \int d^4x\, \mathcal{L}_\Lambda \quad$ with $\quad \Phi(x) = \sum_{0<|p|<\Lambda} \varphi_p e^{ipx}$

$$\Phi(x) = \varphi(x) + \varphi'(x)\ ;\ \varphi(x) = \sum_{0<|p|<k} \varphi_p e^{ipx} \quad \varphi'(x) = \sum_{k<|p|<\Lambda} \varphi_p e^{ipx}$$

Wilsonian Action at $k < \Lambda \quad S_k[\varphi] \quad \Leftarrow \quad e^{-S_k[\varphi]} \equiv \int D[\varphi']e^{-\mathcal{S}_\Lambda[\varphi+\varphi']}$

Wilsonian Action at $k-\delta k \quad S_{k-\delta k}[\varphi] \quad \Leftarrow \quad e^{-S_{k-\delta k}[\varphi]} = \int D[\varphi']e^{-S_k[\varphi+\varphi']}$

$$\varphi(x) = \sum_{0<|p|<k-\delta k} \varphi_p e^{ipx} \qquad \varphi'(x) = \sum_{k-\delta k<|p|<k} \varphi_p e^{ipx}$$

Legendre Effective Action $\quad \Gamma[\varphi] = S_{k=0}[\varphi] \quad ; \quad$ Action $\quad \mathcal{S}_\Lambda[\varphi] = S_{k=\Lambda}[\varphi]$

$$S_{k-\delta k}[\varphi] = -\ln\left(\int D[\varphi']e^{-S_k[\varphi+\varphi']}\right)$$

$$\varphi(x) = \sum_{0<|p|<k-\delta k} \varphi_p e^{ipx} \qquad \varphi'(x) = \sum_{k-\delta k<|p|<k} \varphi_p e^{ipx}$$

$$S_k[\varphi] = \int d^4x \left(U_k(\varphi) + \frac{Z_k(\varphi)}{2}\,\partial_\mu\varphi\,\partial_\mu\varphi + Y_k(\varphi)\,(\partial_\mu\varphi\,\partial_\mu\phi)^2 + W_k(\varphi)\,(\varphi\,\partial_\mu\partial_\mu\varphi)^2 + \cdots \right)$$

Local Potential Approximation $\quad Z_k(\varphi) = 1 \ , \ \ Y_k(\varphi) = W_k(\varphi) = \cdots = 0$

Homogeneous background $\varphi(x) = \varphi_0 \qquad \left(U''_k(\varphi) \equiv \frac{\partial^2 U_k(\varphi)}{\partial\varphi^2} \right)$

Limit $\delta k \to 0$: **Wegner-Houghton equation**

$$k\frac{\partial}{\partial k}U_k(\varphi_0) = -\frac{k^4}{16\pi^2}\ln\frac{k^2 + U''_k(\varphi_0)}{k^2 + U''_k(0)}$$

Non-perturbative RG equation for $U_k(\varphi_0)$. Inserting in this equation the polynomial expansion ($Z(2)$ symmetry $\varphi_0 \to -\varphi_0$ assumed)

$$U_k(\varphi_0) = \frac{1}{2} m_k^2 \varphi_0^2 + \frac{\lambda_k}{4!} \varphi_0^4 + \frac{\lambda_k^{(6)}}{6!} \varphi_0^6 + \frac{\lambda_k^{(8)}}{8!} \varphi_0^8 + \cdots$$

$\Rightarrow$ **RG Equations for the couplings** (here for $d = 4$, but more general)

$$k \frac{dm_k^2}{dk} = -\frac{k^4}{16\pi^2} \frac{\lambda_k}{k^2 + m_k^2}$$

$$k \frac{d\lambda_k}{dk} = -\frac{k^4}{16\pi^2} \left[\frac{\lambda_k^{(6)}}{k^2 + m_k^2} - 3 \frac{\lambda_k^2}{(k^2 + m_k^2)^2} \right]$$

$$k \frac{d\lambda_k^{(6)}}{dk} = -\frac{k^4}{16\pi^2} \left[\frac{\lambda_k^{(8)}}{k^2 + m_k^2} - 15 \frac{\lambda_k \lambda_k^{(6)}}{(k^2 + m_k^2)^2} + 30 \frac{\lambda_k^3}{(k^2 + m_k^2)^3} \right]$$

$$\cdots$$

Wilson RG equations for ϕ^4 theory

Wilsonian action $\quad S_k[\phi] = \int d^4x \left(\frac{1}{2}\,\partial_\mu\phi\,\partial_\mu\phi + U_k(\phi)\right)$

Truncating the potential $\quad U_k(\phi) = \frac{1}{2} m_k^2 \phi^2 + \frac{1}{4!}\lambda_k \phi^4 \quad \Rightarrow$

$$k\,\frac{dm_k^2}{dk} = -\frac{k^4}{16\pi^2}\,\frac{\lambda_k}{k^2 + m_k^2}$$
$$k\,\frac{d\lambda_k}{dk} = \frac{k^4}{16\pi^2}\,\frac{3\lambda_k^2}{(k^2 + m_k^2)^2}$$

When $m_k^2 \ll k^2$ in the whole range of integration, well approximated by

$$k\,\frac{dm_k^2}{dk} = -\frac{\lambda_k}{16\pi^2}k^2 + \frac{\lambda_k}{16\pi^2}m_k^2$$
$$k\,\frac{d\lambda_k}{dk} = \frac{3\lambda_k^2}{16\pi^2}$$

Taking "SM-like" boundaries, $m(\mu_F) = 125.7$ GeV and $\lambda(\mu_F) = 0.1272$, numerical solutions to the two systems coincide with **great accuracy** (!)

Non-perturbative, excellent approximate solution, obtained replacing $\lambda_k \to \lambda$ in the rhs of the RG equation for m_k^2

$$m^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \underbrace{\left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2 - \lambda}\right)}_{\text{UV completion fine-tuning}} - \underbrace{\frac{\lambda\mu^2}{32\pi^2 - \lambda}}_{\text{Quadratic running}}$$

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Contains several lessons

1. Shows how **fine-tuning** operates in **Wilsonian framework**. Boundary at Λ: m_Λ^2 and $\frac{\lambda\Lambda^2}{32\pi^2-\lambda}$ need to be fine-tuned if at μ_F we have $m_{\mu_F} \sim \mathcal{O}(100)$ GeV.

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2. For most of the running towards the IR, **flow dominated by the μ^2 term**. When $\left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2-\lambda}\right) \sim \frac{\lambda\mu^2}{32\pi^2-\lambda}$, **first term takes over** (perturbative running)

$$m^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2 - \lambda}\right) - \frac{\lambda\mu^2}{32\pi^2 - \lambda}$$

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3. Defining the critical mass $m_{cr}^2(\mu)$ and the combination $m_r^2(\mu)$

$$m_{cr}^2(\mu) \equiv -\frac{\lambda\,\mu^2}{32\pi^2 - \lambda} \quad \Rightarrow \quad m_r^2(\mu) \equiv m^2(\mu) - m_{cr}^2(\mu)$$

we see that $m_r^2(\mu)$ obeys the RG equation

($\gamma = \frac{\lambda}{16\pi^2}$ = mass anomalous dimension at one-loop)

$$\mu\frac{d}{d\mu} m_r^2(\mu) = \gamma\, m_r^2(\mu) \quad \Rightarrow \quad m_r^2(\mu) = \left(\frac{\mu}{\mu_0}\right)^{\frac{\lambda}{16\pi^2}} m_r^2(\mu_0)$$

$$m^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \left(m^2_\Lambda + \frac{\lambda\Lambda^2}{32\pi^2 - \lambda}\right) - \frac{\lambda\mu^2}{32\pi^2 - \lambda}$$

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$$\mu\frac{d}{d\mu}m^2_r(\mu) = \gamma\, m^2_r(\mu) \quad \Rightarrow \quad m^2_r(\mu) = \left(\frac{\mu}{\mu_0}\right)^{\frac{\lambda}{16\pi^2}} m^2_r(\mu_0)$$

$\Rightarrow$ The above equation is nothing but the one-loop improved RG equation for the renormalized running mass $\Rightarrow$ $m^2_r(\mu)$ **is the renormalized running mass**

Comments: We derived the equation

$$\mu \frac{d}{d\mu} m_r^2(\mu) = \frac{\lambda}{16\pi^2}\, m_r^2(\mu) \qquad (2)$$

in the **Wilsonian framework**, namely from the **Wilsonian RG flow equation**

$$\mu \frac{d}{d\mu} m^2(\mu) = -\frac{\lambda}{16\pi^2}\, \mu^2 + \frac{\lambda}{16\pi^2} m^2(\mu) \qquad (3)$$

after subtracting $m_{cr}^2(\mu)$. Usually Eq. (2) derived in the context of "**technical schemes**": **dimensional**, **heat kernel**, **zeta function** regularization, ...

This gives **direct access to Eq. (2)**. Info that $m_r^2(\mu)$ is **physically** obtained only after the **subtraction**: $\boldsymbol{m^2(\mu) \;\to\; m_r^2(\mu) \equiv m^2(\mu) - m_{cr}^2(\mu)}$ **lost!**

When quantum fluctuations calculated within the Wilsonian "physical scheme"

$\Rightarrow$ **we see how the renormalized mass emerges**

Questions

Question 1: Should we identify the **physical** running mass $m^2_{phys}(\mu)$ with the original (Wilsonian) $m^2(\mu)$, or with the subtracted (renormalized) $m^2_r(\mu)$?

Running couplings $g_i(\mu) \Leftarrow$ integrating out quantum fluctuations in $[\mu, \Lambda]$
$g_i(\mu)$: **effective couplings** at the scale μ. True, in particular, for the mass.

$\Rightarrow$ Identify $m^2_{phys}(\mu)$ with $m^2(\mu)$ **not with the subtracted** $m^2_r(\mu)$

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Question 2: in QFT textbooks $m^2_{phys}(\mu)$ **usually identified** with $m_r^2(\mu)$!?!?

Let us **focus on** how $m^2(\mu)$ and $m_r^2(\mu)$ **depend** on μ ... and note that:

For **sufficiently low values** of μ (IR regime) $m^2(\mu)$ and $m_r^2(\mu)$ **coincide**

$$\frac{\lambda\mu^2}{32\pi^2-\lambda} \ll \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}} \left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2-\lambda}\right)$$

The above relation shows the **limitations** of the renormalized RG equation (2). If we are interested in energy scales μ **above this region**, we must go back to the original flow equation (3), that has a **much wider range of validity**

Standard Model - RG Equation for the Higgs mass

$$\mu \frac{d}{d\mu} m_H^2 = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m_H^2$$

$\alpha(\mu)$: combination of SM couplings (gauge, Yukawa, scalar). At one-loop:

$$16\pi^2\alpha(\mu) = 12y_t^2 - 12\lambda - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2$$

$\gamma(\mu)$: mass anomalous dimension

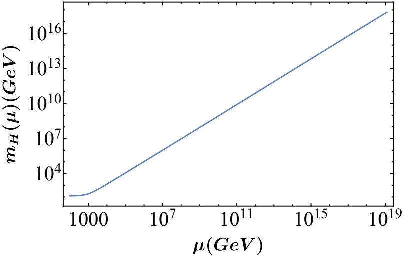
$$16\pi^2\gamma(\mu) = 6y_t^2 + 12\lambda - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2$$

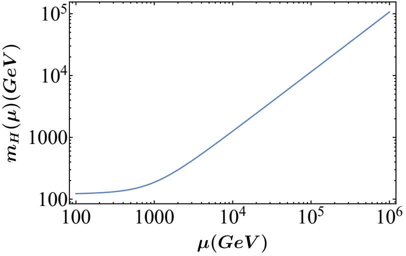
Integrating the RG equation for $m_H^2(\mu)$

$$m^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\gamma} \underbrace{\left(m_H^2(\Lambda) - \frac{\alpha\Lambda^2}{2-\gamma}\right)}_{\text{UV completion fine-tuning}} - \underbrace{\frac{\alpha\mu^2}{2-\gamma}}_{\text{Quadratic running}}$$

Very good analytical approximation to the flow

Numerical sol. to RG eq.(4) and **Analytical approx.** (5): **indistinguishable**





$$\mu \frac{d}{d\mu} m_H^2 = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m_H^2 \tag{4}$$

$$m_H^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\gamma}\left(m_H^2(\Lambda) - \frac{\alpha\,\Lambda^2}{2-\gamma}\right) + \frac{\alpha\,\mu^2}{2-\gamma} \tag{5}$$

As for Scalar Theory: **critical mass** $m^2_{H,\mathrm{cr}}(\mu)$ and **subtracted mass** $m^2_{H,r}(\mu)$

$$m^2_{H,\mathrm{cr}}(\mu) \equiv \frac{\alpha\,\mu^2}{2-\gamma} \qquad \text{and} \qquad m^2_{H,r}(\mu) \equiv m^2_H(\mu) - m^2_{H,\mathrm{cr}}(\mu)$$

From which we immediately have

$$\mu \frac{d}{d\mu} m^2_{H,r}(\mu) = \gamma\; m^2_{H,r}(\mu) \qquad \Rightarrow \qquad m^2_{H,r}(\mu) = \left(\frac{\mu}{\mu_0}\right)^{\gamma} m^2_{H,r}(\mu_0)$$

The above equation **coincides** with the one-loop improved RG equation for the renormalized running mass $\Rightarrow$ $m^2_{H,r}(\mu)$ = **renormalized running Higgs mass**

However the original equation (together with the solution) is

$$\mu \frac{d}{d\mu} m^2_H = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m^2_H \Rightarrow m^2_H(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\gamma}\left(m^2_H(\Lambda) - \frac{\alpha\,\Lambda^2}{2-\gamma}\right) + \frac{\alpha\,\mu^2}{2-\gamma}$$

Change in the usual paradigm

$$\mu\frac{d}{d\mu}m_H^2 = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m_H^2 \qquad ; \qquad \mu\frac{d}{d\mu}m_{H,r}^2(\mu) \quad = \gamma\; m_{H,r}^2(\mu)$$

The two flows **coincide** for values of μ such that

$$\frac{\alpha\,\mu^2}{2-\gamma} \ll \left(\frac{\mu}{\Lambda}\right)^{\gamma}\left(m_H^2(\Lambda) - \frac{\alpha\,\Lambda^2}{2-\gamma}\right)$$

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Physical Lessons

- **Fine-tuning** of $m_H^2(\Lambda)$ has a profound **physical meaning**: provides the boundary at the UV scale Λ for the RG flow of $m_H^2(\mu)$

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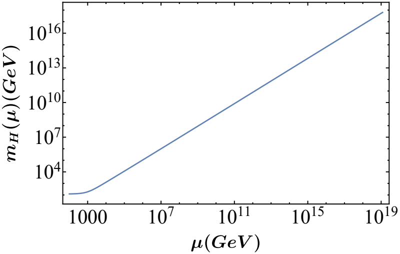
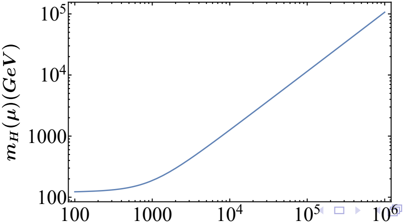
Physical Lessons

- **Fine-tuning** of $m_H^2(\Lambda)$ has a profound **physical meaning**: provides the boundary at the UV scale Λ for the RG flow of $m_H^2(\mu)$
- **Large hierarchy** between UV and IR values of m_H^2 is **physically necessary**

Change in the usual paradigm

- **Quadratic running** lasts for most of the $m_H^2(\mu)$ flow towards the IR
- **Multiplicative** renormalization **emerges** flowing towards **IR**. The "**elbow**" signals the "**transition**" **additive** $\rightarrow$ **multiplicative** renormalization





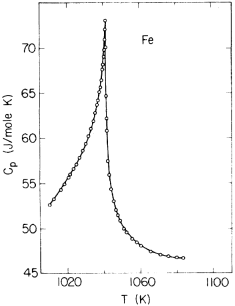
Change in the usual paradigm

Usual **connection** between **QFT** and **Statistical Physics**: correspondence between the request $\xi \gg a$ in the **Theory of Critical Phenomena** ($a =$ lattice spacing, $\xi =$ correlation length) and the request $m^2 \ll \Lambda^2$ in **QFT**

Phrased in RG language $\rightarrow$ **tuning towards the "critical surface"**, achieved through the **subtraction of the "critical mass"**: $m^2_{ren}(\mu) = m^2(\mu) - m^2_{cr}(\mu)$

However: $m^2_{ren}(\mu)$ **captures** the **IR final part** of the running of $m^2_{phys}(\mu)$

Flow **physically meaningful** even far from **critical surface** and **fixed points**



Nice Example. Landau-Ginzburg Theory - Ferromagnetic Transition

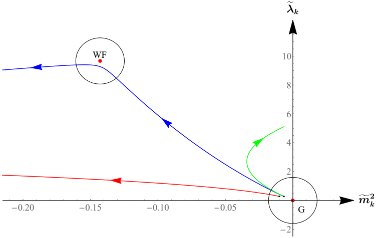
$$F_k[\phi] = \int d^3x \left(\tfrac{1}{2}(\vec{\nabla}\phi)^2 + U_k(\phi)\right) \qquad U_k(\phi) = \tfrac{1}{2} m_k^2 \phi^2 + \tfrac{\lambda_k}{4!}\phi^4$$

$$k\frac{dm_k^2}{dk} = -\frac{k^3\lambda_k}{4\pi^2\left(k^2+m_k^2\right)} \qquad k\frac{d\lambda_k}{dk} = \frac{3k^3\lambda_k^2}{4\pi^2\left(k^2+m_k^2\right)^2}$$

Dimensionless couplings $\widetilde{m}_k^2 \equiv k^{-2} m_k^2$ and $\widetilde{\lambda}_k \equiv k^{-1}\lambda_k$

$$k\frac{d\widetilde{m}_k^2}{dk} = -2\widetilde{m}_k^2 - \frac{\widetilde{\lambda}_k}{4\pi^2(1+\widetilde{m}_k^2)} \qquad k\frac{d\widetilde{\lambda}_k}{dk} = -\widetilde{\lambda}_k + \frac{3\widetilde{\lambda}_k^2}{4\pi^2(1+\widetilde{m}_k^2)^2}$$

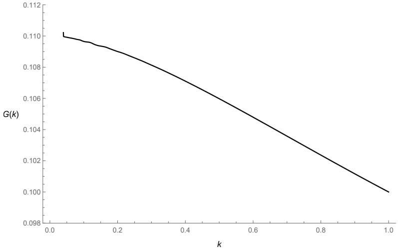
Gaussian G and **Wilson-Fisher** WF **fixed points**. G is IR repulsive (UV attractive)

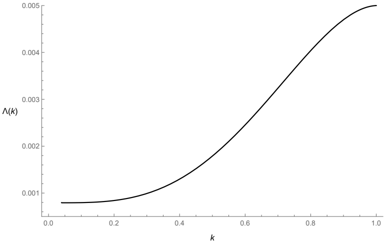


Blue and **Red** **IR** flows: **Different boundaries** in the UV region around G (Green: linearization)
UV linearly divergent boundary (d=3) crucial for physics at WF: **Ferromagnetic transition**

Fine-tuning Physically needed

Physical tuning and Cosmological Constant





$\Lambda_{cc}(\Lambda)$ needs to be **fine-tuned** to obtain $\Lambda_{cc}^{\rm exp} \sim 10^{-120} M_p^2$ at the Fermi scale

Conclusions and outlook

- Wilson mandatory $\Rightarrow$ Fine-tuning at the UV of the boundaries physically meaningful and mandatory
- IR region **dictated by the UV completion**: the **Theory of Everything**.
- In schemes as DR: direct access to the **renormalized flow**, but **no physical content**

Thank you for your attention!

Backup Slides

More Slides

Wilsonian RG

versus

Dimensional Regularization

Very useful example: Scalar Theory in d-dimensions

$d =$ integer dimension (no dim reg)

- Wilsonian Effective Action: $S_k[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + V_k(\phi) \right]$

Wilson (Polchinski) RG Equation (LPA)

$$k \frac{\partial}{\partial k} V_k(\phi) = -\frac{k^d}{(4\pi)^{\frac{d}{2}} \Gamma\left(\frac{d}{2}\right)} \ln\left(\frac{k^2 + V_k''(\phi)}{k^2} \right)$$

- UV boundary: $V_\Lambda(\phi) \equiv V_0(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\mu^{4-d}\lambda_0}{4!}\phi^4$

Approximating $V_k(\phi)$ in the rhs as $V_k(\phi) \to V_\Lambda(\phi)$

One-loop effective potential

$$V_{1l}(\phi) = V_0(\phi) + \underbrace{\frac{1}{2} \int^{(\Lambda)} \frac{d^d k}{(2\pi)^d} \ln\left(1 + \frac{m_0^2 + \frac{1}{2}\mu^{4-d}\lambda_0 \phi^2}{k^2} \right)}_{\delta V(\phi)}$$

Lesson: One-loop Effective Potential Approx. of the Wilsonian Potential

Let us focus on the Radiative Correction $\delta V(\phi)$

$$\delta V(\phi) = \frac{1}{2}\int^{(\Lambda)} \frac{d^d k}{(2\pi)^d} \ln\left(1+\frac{M^2(\phi)}{k^2}\right) \equiv \delta V_1(\phi) + \delta V_2(\phi)$$

where

$$M^2(\phi) \equiv m_0^2 + \tfrac{1}{2}\mu^{4-d}\lambda_0\,\phi^2$$

$$\delta V_1(\phi) \equiv \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)}\left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{d}{2}} \int_{\frac{M^2}{M^2+\Lambda^2}}^{1} dt\,(1-t)^{\frac{d}{2}-1}\,t^{-\frac{d}{2}}$$

$$\delta V_2(\phi) \equiv \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)}\left(\frac{\Lambda}{\mu}\right)^{d} \ln\left(1+\frac{M^2(\phi)}{\Lambda^2}\right)$$

Calculating $\delta V(\phi)$

For any **integer** d:

$$\delta V_1(\phi) = \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)}\left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{d}{2}}\int_{\frac{M^2}{M^2+\Lambda^2}}^{1} dt\, t^{-\frac{d}{2}}(1-t)^{\frac{d}{2}-1} =$$

$$= \lim_{z\to d}\left[A_1(z) - A_2(z)\right]$$

where z is **complex**, and

$$A_1(z) \equiv F(z)\cdot \overline{B}\left(1-\frac{z}{2},\frac{z}{2}\right) \qquad A_2(z) \equiv F(z)\cdot \overline{B}_i\left(1-\frac{z}{2},\frac{z}{2};\frac{M^2(\phi)}{M^2(\phi)+\Lambda^2}\right)$$

$$F(z) \equiv \frac{\mu^z}{z(4\pi)^{\frac{z}{2}}\Gamma\left(\frac{z}{2}\right)}\left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{z}{2}}$$

$\overline{B}$ and $\overline{B}_i$ are (the analytic extensions of) the Beta functions

Both $\overline{B}$ and $\overline{B}_i$ have poles in $z = 2, 4, 6, \ldots$

$\delta V_1(\phi)$ **finite** $\Rightarrow$ **the poles of** A_1 **and** A_2 **have to cancel each other**

Example: $\delta V(\phi)$ in $d = 4$ dimensions

$z \equiv 4 - \epsilon$. Expanding in powers of ϵ and M^2/Λ^2

$$A_1(4-\epsilon) = \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{M^2(\phi)}{4\pi\mu^2} - \frac{3}{2}\right) + \cancel{\mathcal{O}(\epsilon)}$$

$$A_2(4-\epsilon) = \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{M^2(\phi)}{4\pi\mu^2} - \frac{3}{2}\right) + \cancel{\mathcal{O}(\epsilon)} + \cancel{\mathcal{O}\left(\frac{M^2}{\Lambda^2}\right)}$$
$$- \frac{\mu^{-\epsilon}}{64\pi^2}\left[M^2(\phi)\right]^2\left(\frac{\Lambda^2}{M^2(\phi)} - \log\frac{\Lambda^2}{M^2(\phi)}\right)$$

Remember: $\delta V_1(\phi) = \lim_{\epsilon\to 0}\left[A_1(4-\epsilon) - A_2(4-\epsilon)\right]$. Adding $\delta V_2(\phi)$

$$\delta V(\phi) = \delta V_1 + \delta V_2 = \frac{\Lambda^2 M^2(\phi)}{32\pi^2} - \frac{\left[M^2(\phi)\right]^2}{64\pi^2}\left(\ln\frac{\Lambda^2}{M^2(\phi)} + \frac{1}{2}\right) + \cancel{\mathcal{O}\left(\frac{\phi^6}{\Lambda^2}\right)}$$

$$\Rightarrow \quad V_{1l}(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4 + \frac{\Lambda^2 M^2}{32\pi^2} - \frac{\left(M^2\right)^2}{64\pi^2}\left(\ln\frac{\Lambda^2}{M^2} + \frac{1}{2}\right)$$

No reference whatsoever to ϵ (of course!)

With $\Omega_0 = \Omega + \delta\Omega_\Lambda$, $m_0^2 = m^2 + \delta m_\Lambda^2$, $\lambda_0 = \lambda + \delta\lambda_\Lambda$

and $\delta\Omega_\Lambda = -\frac{m^2\Lambda^2}{32\pi^2} + \frac{m^4}{64\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$; $\delta m_\Lambda^2 = -\frac{\lambda\Lambda^2}{32\pi^2} + \frac{\lambda m^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$

$$\delta\lambda_\Lambda = \frac{3\lambda^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$$

... where $\delta\Omega_\Lambda$ and δm_Λ^2 realize fine-tunings (*) ...

$\Rightarrow$ **Renormalized One-Loop Effective Potential (take $\Omega = 0$)**

$$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right]$$

(*) Physically ... in the parameter space of the theory we go close to the Critical region, or Critical Surface ...

... Let's move now to Dim Reg ...

Radiative correction $\delta V(\phi)$ in Dim. Reg.

- $\delta V(\phi)$ in **Dim Reg.** $\boxed{d \to \text{complex}\,,\ d \equiv 4 - \epsilon}$

$$\delta V(\phi) \to \delta V_\epsilon(\phi) \equiv -\frac{\mu^{4-\epsilon}}{2(4\pi)^{2-\frac{\epsilon}{2}}} \left(\frac{M^2(\phi)}{\mu^2}\right)^{2-\frac{\epsilon}{2}} \overline{\Gamma}\left(\frac{\epsilon}{2} - 2\right)$$

$$= \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{M^2(\phi)}{4\pi\mu^2} - \frac{3}{2}\right) + \mathcal{O}(\epsilon)$$

$\overline{\Gamma}(-d/2)$ defined for any complex $d \neq 2, 4, 6, \ldots$

- Counterterms in $\overline{MS}$ scheme $\left(\overline{\epsilon} \equiv \epsilon\left(1 + \frac{\epsilon}{2}\ln\frac{e^\gamma}{4\pi}\right)\right)$:

$$\delta\Omega_\epsilon = \frac{m^4}{32\pi^2\overline{\epsilon}}\mu^{-\epsilon} \quad , \quad \delta m_\epsilon^2 = \frac{\lambda m^2}{16\pi^2\overline{\epsilon}} \quad , \quad \delta\lambda_\epsilon = \frac{3\lambda^2}{16\pi^2\overline{\epsilon}}$$

- **Renormalized One-loop Effective Potential (take $\Omega = 0$) as before**

$$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right]$$

Before going on with our analysis ... Let's hear “news” from the Literature

“Dim Reg” versus “Wilson” (= “successive elimination of modes”)

Views on “Dim Reg” and “Wilson”

1) Typical textbook statement . . . “Dimensional Regularization has no direct physical interpretation” (J. Zinn-Justin - **Quantum field theory of critical phenomena**)

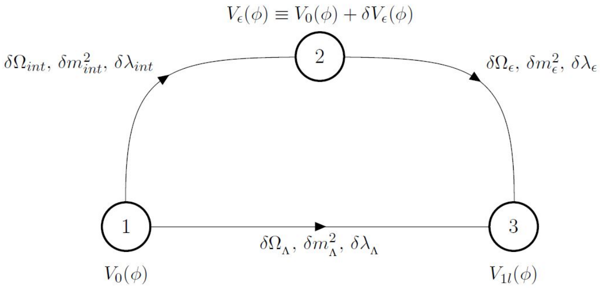
2) Recent ideas (gaining lot of followers)

“Maybe power divergences vanish because the ultimate unknown physical cut-off behaves like dimensional regularization” (M. Farina, D. Pappadopulo and A. Strumia, JHEP 08 (2013) 022)

“Wilsonian computation techniques attribute physical meaning to momentum shells of loop integrals” . . . “The naturalness problem can be more generically formulated as a problem of the **Effective Theory Ideology**” (A. Salvio and A. Strumia, JHEP 06 (2014) 080)

Accordingly **DR** should have special physical properties that make it the **correct way** to calculate the quantum fluctuations ... while **Wilson** ... **incorrect** ...

Dim Reg.: Physical Meaning? ... Special Physical Properties?



$$V_0(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4 = (\Omega + \delta\Omega_\Lambda) + \frac{1}{2}(m^2 + \delta m^2_\Lambda)\,\phi^2 + \frac{1}{4!}(\lambda + \delta\lambda_\Lambda)\phi^4$$

$$= (\Omega + \delta\Omega_{int} + \delta\Omega_\epsilon) + \frac{1}{2}(m^2 + \delta m^2_{int} + \delta m^2_\epsilon)\phi^2 + \frac{1}{4!}(\lambda + \delta\lambda_{int} + \delta\lambda_\epsilon)\phi^4$$

$$\Rightarrow \text{Ren.Pot.}: \quad V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right]$$

Dim Reg.: Physical Meaning? ... Special Physical Properties?

DR secretly realizes the fine-tuning:

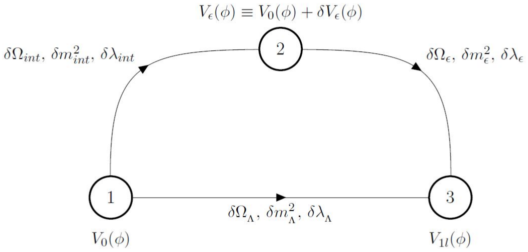
$$\delta\Omega_{int} = -\frac{m^2\Lambda^2}{32\pi^2} + \frac{m^4}{64\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{m^4}{32\pi^2\overline{\epsilon}}\mu^{-\epsilon}$$

$$\delta m^2_{int} = -\frac{\lambda\Lambda^2}{32\pi^2} + \frac{\lambda m^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{\lambda m^2}{16\pi^2\overline{\epsilon}}$$

$$\delta\lambda_{int} = \frac{3\lambda^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{3\lambda^2}{16\pi^2\overline{\epsilon}}$$

DR has a Physical Meaning but No Special Physical Properties. It implements the Wilsonian iterative elimination of modes for including the quantum fluctuations in the Effective Theory, and **secretly** realizes the fine-tuning

Summary on DR



DR setting, "Bubble (2)", obtained by introducing an intermediate step, (1) $\rightarrow$ (2), in the process of obtaining the Renormalized Potential, "Bubble (3)".

DR provides a shortcut: "Bubble (3)" is reached starting from "Bubble (2)". The fine-tuning step "Bubble (1)" $\rightarrow$ "Bubble (2)" is skipped (secretly realized)

Lesson: DR is a way to implement the Wilson's strategy in the perturbative regime, where the *fine-tuning* (in the Wilsonian language: tuning toward the critical regime, critical surface) is secretly performed.

Naturalness and Dimensional Regularization

What should we then say on those attempts to solve the Naturalness/Hierarchy problem with DR?

- **Classically Scale Invariant BSM**. The theory does not possess mass or length scales $\Rightarrow$ only dimension four operators
- Dimensional Regularization used $\Rightarrow$ Scale Invariance only softly broken $\Rightarrow$ apparently **no fine-tuning needed** . . . seems good . . .
- . . . But . . . we have just shown . . . DR **secretly realizes the fine-tuning**

$\Rightarrow$ **No way to solve the Naturalness/Hierarchy problem with DR**

Flourishing literature

- K. A. Meissner and H. Nicolai, Phys. Lett. B **648**, 312-317 (2007).
- K. A. Meissner and H. Nicolai, Phys. Lett. B **660**, 260-266 (2008).
- R. Foot, A. Kobakhidze, K. L. McDonald and R. R. Volkas, Phys. Rev. D **77**, 035006 (2008).
- F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703-706 (2008).
- M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671**, 162-166 (2009).
- L. Alexander-Nunneley and A. Pilaftsis, JHEP **09**, 021 (2010).
- L. Boyle, S. Farnsworth, J. Fitzgerald and M. Schade, arXiv:1111.0273 [hep-ph].
- C. D. Carone and R. Ramos, Phys. Rev. D **88**, 055020 (2013).
- A. Farzinnia, H. J. He and J. Ren, Phys. Lett. B **727**, 141-150 (2013).
- Y. Kawamura, PTEP **2013**, no.11, 113B04 (2013).
- I. Bars, P. Steinhardt and N. Turok, Phys. Rev. D **89**, no.4, 043515 (2014).
- M. Heikinheimo, A. Racioppi, M. Raidal, *et al*, Mod. Phys. Lett. A **29**, 1450077 (2014).
- A. Salvio and A. Strumia, JHEP **06**, 080 (2014).
- T. G. Steele, Z. W. Wang, *et al.*, Phys. Rev. Lett. **112**, no.17, 171602 (2014).
- F. Bezrukov, J. Rubio and M. Shaposhnikov, Phys. Rev. D **92**, no.8, 083512 (2015).
- J. Guo and Z. Kang, Nucl. Phys. B **898**, 415-430 (2015).
- D. M. Ghilencea, Phys. Rev. D **93**, no.10, 105006 (2016).
- D. M. Ghilencea, Z. Lalak and P. Olszewski, Eur. Phys. J. C **76**, no.12, 656 (2016).
- Z. W. Wang, T. G. Steele, T. Hanif and R. B. Mann, JHEP **08**, 065 (2016).
- D. M. Ghilencea, Z. Lalak and P. Olszewski, Phys. Rev. D **96**, no.5, 055034 (2017).
- I. Oda, Eur. Phys. J. C **78**, no.10, 798 (2018).
- S. Mooij, M. Shaposhnikov and T. Voumard, Phys. Rev. D **99**, no.8, 085013 (2019).
- M. Farina, D. Pappadopulo and A. Strumia, JHEP **08**, 022 (2013).
- I. Brivio and M. Trott, Phys. Rept. **793**, 1-98 (2019).

Consider now attempts to solve the NH problem in a RG framework

Flourishing literature

- C. Wetterich, Phys. Lett. B **104**, 269-276 (1981).
- C. Wetterich, Phys. Lett. B **140**, 215-222 (1984).
- C. Wetterich, Z. Phys. C **48**, 693-705 (1990).
- C. Wetterich, Z. Phys. C **57**, 451-470 (1993).
- S. Bornholdt and C. Wetterich, Phys. Lett. B **282**, 399-405 (1992).
- M. Shaposhnikov and C. Wetterich, Phys. Lett. B **683**, 196-200 (2010).
- C. Wetterich, Phys. Lett. B **718**, 573-576 (2012).
- M. Holthausen, K. S. Lim and M. Lindner, JHEP **02**, 037 (2012).
- G. F. Giudice, PoS **EPS-HEP2013**, 163 (2013).
- M. Holthausen, J. Kubo, K. S. Lim and M. Lindner, JHEP **12** (2013), 076.
- S. Iso and Y. Orikasa, PTEP **2013** (2013), 023B08.
- L. E. Ibanez and I. Valenzuela, JHEP **05** (2013), 064.
- I. Masina and M. Quiros, Phys. Rev. D **88** (2013), 093003.
- M. Hashimoto, S. Iso and Y. Orikasa, Phys. Rev. D **89** (2014) no.1, 016019.
- N. Haba, K. Kaneta and R. Takahashi, JHEP **04** (2014), 029.
- M. Hashimoto, S. Iso and Y. Orikasa, Phys. Rev. D **89** (2014) no.5, 056010.
- P. H. Chankowski, A. Lewandowski, K. A. Meissner and H. Nicolai, Mod. Phys. Lett. A **30**, 1550006 (2015).
- C. Wetterich and M. Yamada, Phys. Lett. B **770**, 268-271 (2017)
- J. M. Pawlowski, M. Reichert, C. Wetterich and M. Yamada, Phys. Rev. D **99**, no.8, 086010 (2019).
- S. Mooij and M. Shaposhnikov, arXiv:2110.05175.
- S. Mooij and M. Shaposhnikov, arXiv:2110.15925.

"Wilson" versus "Perturbatively-Renormalized" RG Equations

Scalar Theory : $\mathcal{L}_\Lambda = \frac{1}{2}\left(\partial_\mu \phi_\Lambda\right)^2 + \frac{1}{2} m_\Lambda^2 \phi_\Lambda^2 + \frac{\lambda_\Lambda}{4!}\phi_\Lambda^4$

Wilson-Polchinski RG Equations

$$\mu \frac{d\Omega}{d\mu} = -\frac{m^2\mu^2}{16\pi^2} + \frac{m^4}{32\pi^2} \quad ; \quad \mu \frac{dm^2}{d\mu} = -\frac{\lambda\mu^2}{16\pi^2} + \frac{\lambda m^2}{16\pi^2} \quad ; \quad \mu \frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2}$$

$\mu \in [0, \Lambda]$ is the running scale. Λ is the UV boundary (physical cut-off)

Define:

$$m_{\rm cr}^2(\mu) \equiv \frac{\lambda(\mu)}{16\pi^2}\,\mu\,\delta\mu \quad \text{and} \quad \widetilde{m}^2(\mu-\delta\mu) \equiv m^2(\mu-\delta\mu) - m_{\rm cr}^2(\mu)$$

$$\Omega_{\rm cr}(\mu) \equiv \frac{\widetilde{m}^2(\mu)}{16\pi^2}\,\mu\,\delta\mu \quad \text{and} \quad \widetilde{\Omega}(\mu-\delta\mu) \equiv \Omega(\mu-\delta\mu) - \Omega_{\rm cr}(\mu)$$

Perturbatively-Renormalized RG Equations $(\delta\mu \to 0)$

$$\mu \frac{d\widetilde{\Omega}}{d\mu} = \frac{\widetilde{m}^4}{32\pi^2} = \beta_\Omega \quad ; \quad \mu \frac{d\widetilde{m}^2}{d\mu} = \frac{\lambda \widetilde{m}^2}{16\pi^2} = \widetilde{m}^2\gamma_m \quad ; \quad \mu \frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2} = \beta_\lambda$$

The Perturbatively-Renormalized RG Equations **contain the fine-tuning**

Physically: **Tuning towards the Critical Surface**

Perturbatively-Renormalized RG equations in the Standard Model

Well-known Standard Model perturbative RG equations (*)

$$\mu \frac{d}{d\mu}\lambda_i = \beta_{\lambda_i} \qquad \mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

λ_i $(i = 1, \ldots, 5)$ are the SM couplings

$(*)$ similarly for SM extensions

Perturbatively-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 1 : Quantum Gravity "**miracle**" G.F. Giudice, PoS EPS-HEP2013, 163 (2013)

$$m_H^2(\Lambda) \ll \Lambda^2$$

With the SM perturbative γ_m $(\gamma_m \ll 1)$ $\Rightarrow$

Apparently no Hierarchy Problem : $m_H^2(\Lambda) \sim m_H^2(\mu_F)$

. . . But . . . remember . . . in the above RG Equation m_H^2 is the **tuned mass** $\Rightarrow$

Fine-tuning encoded in the RG Equation above

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbatively-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 2 : "**Self-organized criticality**"

J. M. Pawlowski, M. Reichert, C. Wetterich and M. Yamada, Phys. Rev. D **99**, 086010 (2019)

Assumes Quantum Gravity might give a non-perturbative $\gamma_m \sim 2 \;\Rightarrow$

Hierarchy can be tolerated : $m_H^2(\Lambda) \gg m_H^2(\mu_F)$

... But ... remember ... m_H^2 is the **tuned mass** $\Rightarrow$

Fine-tuning encoded in the above RG Equation

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbatively-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 3 : $m_H^2(\mu)$ from $\lambda(\mu)$ and $v(\mu)$ ...

P. H. Chankowski, A. Lewandowski, K. A. Meissner and H. Nicolai, Mod. Phys. Lett. A **30**, 1550006 (2015)

M. Holthausen, K. S. Lim and M. Lindner, JHEP **02**, 037 (2012)

Apparently no large corrections : $m_H^2(\mu_F) \sim 125\,\mathrm{GeV}$

... However ... same problem as before ... Tuning encoded in the RG equation for the vev $v(\mu)$ (equivalent to the above RG equation for $m_H^2(\mu)$)

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbative-Renormalized RG equations in the Standard Model

Attempt 4 : "Finite formulation" of QFT using RG equations *à la* Callan-Symanzik for the Green's functions . . .

S. Mooij and M. Shaposhnikov, arXiv:2110.05175

S. Mooij and M. Shaposhnikov, arXiv:2110.15925

Apparently no quadratic corrections for the mass m^2 of scalar particles

However . . . Tuning encoded in taking derivatives with respect to m^2 of the Green's functions, until they become finite

Callan has shown that this is just a way of implement the subtraction of Λ^2 and $\log \Lambda$ terms C. G. Callan, Jr., Conf. Proc. C **7507281**, 41-77 (1975)

$\Rightarrow$ **Can't solve the Hierarchy Problem**

$$k\frac{dm_k^2}{dk} = -\frac{\lambda_k}{16\pi^2}k^2 + \frac{\lambda_k}{16\pi^2}m_k^2$$
$$k\frac{d\lambda_k}{dk} = \frac{3\lambda_k^2}{16\pi^2}$$

Can be **solved analytically** with no further approximations. Second Equation:

$$\lambda(\mu) = \frac{\lambda_\Lambda}{1 - \frac{3}{16\pi^2}\lambda_\Lambda \log\left(\frac{\mu}{\Lambda}\right)}$$

Inserting in the First Equation $\Rightarrow$ **Non-perturbative** RG equation for $m^2(\mu)$

($E_{\frac{2}{3}}(x)$ is the generalized exponential integral function $E_p(x)$ with $p = \frac{2}{3}$)

$$m^2(\mu) = \frac{1}{3\cdot 2^{2/3}\left(3\lambda_\Lambda \log\left(\frac{\mu}{\Lambda}\right) - 16\pi^2\right)} \times \left[2^{2/3}\Lambda^2 e^{\frac{32\pi^2}{3\lambda_\Lambda}} \times \left(16\pi^2 - 3\lambda_\Lambda \log\left(\frac{\mu}{\Lambda}\right)\right) E_{\frac{2}{3}}\left(\frac{32\pi^2}{3\lambda_\Lambda} - 2\log\left(\frac{\mu}{\Lambda}\right)\right) + 4\lambda_\Lambda \sqrt[3]{-\frac{1}{\lambda_\Lambda}}\left(\Lambda^2 e^{\frac{32\pi^2}{3\lambda_\Lambda}} E_{\frac{2}{3}}\left(\frac{32\pi^2}{3\lambda_\Lambda}\right) + 3m_\Lambda^2\right) \times \left(3\pi\log\left(\frac{\mu}{\Lambda}\right) - \frac{16\pi^3}{\lambda_\Lambda}\right)^{2/3}\right]$$

Nice features of this **Non-perturbative** evolution equation for $m^2(\mu)$
(replace $\lambda_\Lambda \to \lambda$)

1) Expanding for $\lambda \ll 1$ and $\mu^2 \ll \Lambda^2 \quad \Rightarrow \quad$ **well-known perturbative result**

$$m_\mu^2 = m_\Lambda^2 + \frac{\lambda}{32\pi^2}\left(\Lambda^2 - m_\Lambda^2 \log\frac{\Lambda^2}{\mu^2}\right)$$

2) Also: very interesting **non-perturbative** approximation, obtained by replacing λ_k with λ in the rhs of the RG equation for $m^2(\mu)$

$$m_\mu^2 = \left(\frac{\mu}{\Lambda}\right)^{\frac{\lambda}{16\pi^2}}\left(m_\Lambda^2 + \frac{\lambda\Lambda^2}{32\pi^2 - \lambda}\right) - \frac{\lambda\mu^2}{32\pi^2 - \lambda}$$

Excellent approximation for $m^2(\mu)$ (see **ugly equation** previous page)
Important result

Wilsonian - Polchinski RG equations

- Flow of the theory parameters:

$$\Lambda\frac{d}{d\Lambda}\Omega_0 = -\frac{m_0^2\Lambda^2}{16\pi^2} + \frac{m_0^4}{32\pi^2} \qquad \Lambda\frac{d}{d\Lambda}m_0^2 = -\frac{\lambda_0\Lambda^2}{16\pi^2} + \frac{\lambda_0 m_0^2}{16\pi^2} \qquad \Lambda\frac{d}{d\Lambda}\lambda_0 = \frac{3\lambda_0^2}{16\pi^2}$$

- From the Wegner-Houghton equation for $d = 4$, inserting the expansion $U_k(\phi) = \Omega_k + \frac{1}{2}m_k^2\phi^2 + \frac{1}{4!}\lambda_k\phi^4 + \frac{1}{6!}\lambda_k^{(6)}\phi^6 + \ldots$ we have the flow equations:

$$k\frac{\partial\Omega_k}{\partial k} = -\frac{k^4}{16\pi^2}\log\left(\frac{k^2+m_k^2}{k^2}\right)$$

$$k\frac{\partial m_k^2}{\partial k} = -\frac{k^4}{16\pi^2}\frac{\lambda_k}{k^2+m_k^2}$$

$$k\frac{\partial\lambda_k}{\partial k} = \frac{k^4}{16\pi^2}\frac{3\lambda_k^2}{(k^2+m_k^2)^2}$$

- Under the condition $k^2 \gg m_k^2$, i.e. in the UV regime, they reduce to the bare parameters flow equations.

Critical term

- Finite difference RG equation for the mass:

$$m_0^2(\Lambda-\delta\Lambda)=m_0^2(\Lambda)+\frac{\delta\Lambda}{\Lambda}\frac{\lambda_0(\Lambda)}{16\pi^2}\Lambda^2-\frac{\delta\Lambda}{\Lambda}\frac{\lambda_0(\Lambda)\,m_0^2(\Lambda)}{16\pi^2}+\mathcal{O}\left(\frac{\delta\Lambda^2}{\Lambda^2}\right)$$

- Subtracted mass parameter at the scale $\Lambda-\delta\Lambda$

$$\widetilde{m}^2(\Lambda-\delta\Lambda)\equiv m_0^2(\Lambda-\delta\Lambda)-m_{\rm cr}^2(\Lambda)$$

where the *critical mass* $m_{\rm cr}^2$, and the boundary at Λ are given by

$$m_{\rm cr}^2(\Lambda)\equiv\frac{\lambda_0(\Lambda)}{16\pi^2}\Lambda\,\delta\Lambda\qquad\widetilde{m}^2(\Lambda)=m_0^2(\Lambda)$$

- In the limit $\delta\Lambda\to 0$ we recover the perturbative RG equations:

$$\beta_\Omega=\mu\frac{d\Omega}{d\mu}=\frac{m^4}{32\pi^2}\qquad\gamma_m=\frac{1}{m^2}\left(\mu\frac{dm^2}{d\mu}\right)=\frac{\lambda}{16\pi^2}\qquad\beta_\lambda=\mu\frac{d\lambda}{d\mu}=\frac{3\lambda^2}{16\pi^2}$$

- The renormalized RG equations **contain the fine-tuning**: physically, this corresponds to a *tuning towards the critical surface*.

Perturbative-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 5 : hierarchy between M_P and μ_F generated by an instanton configuration contributing to the vev of the Higgs field . . .

M. Shaposhnikov and A. Shkerin, Phys. Lett. B **783**, 253 (2018)

M. Shaposhnikov and A. Shkerin, JHEP **10**, 024 (2018)

Apparently Hierarchy explained

however . . . quantum corrections calculated with DR, and flow of the parameters studied with the perturbative RG flows . . . same problems as before

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Gauge theories

Attempts to a gauge invariant Wilsonian RG

- V. Branchina, K. Meissner and G. Veneziano, The Price of an exact, gauge invariant RG flow equation, Phys. Lett. B **574**, 319-324 (2003)
- S.P. de Alwis, Exact RG Flow Equations and Quantum Gravity, JHEP **03**, 118 (2018)