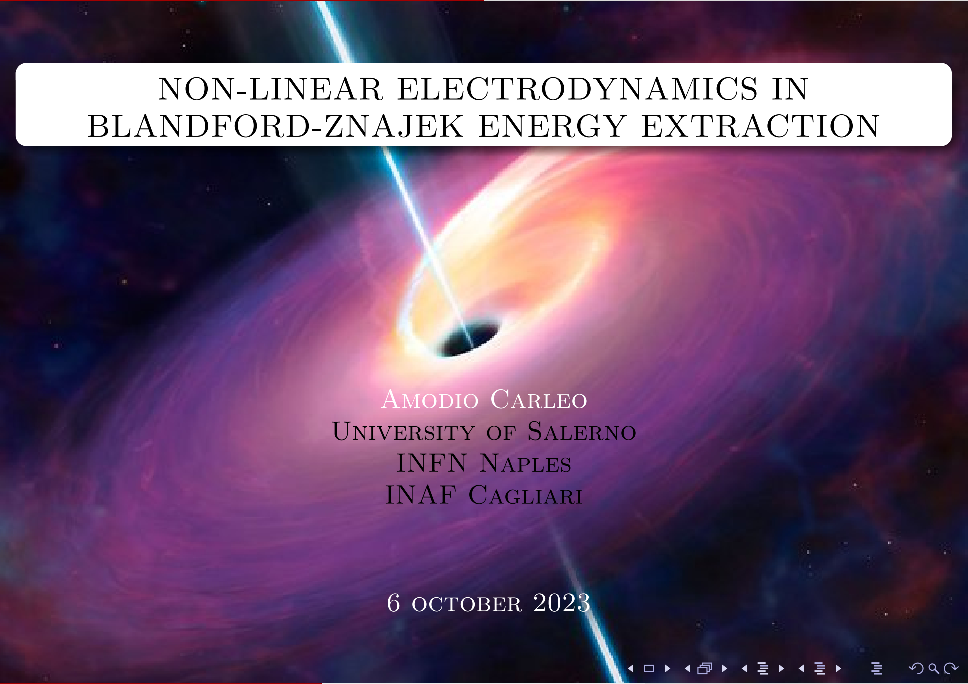


NON-LINEAR ELECTRODYNAMICS IN BLANDFORD-ZNAJEK ENERGY EXTRACTION

Amodio Carleo
University of Salerno
INFN Naples
INAF Cagliari

6 october 2023

Outline

Based on:

A. Carleo, G. Lambiase, A. Ovgun; *Non-Linear Electrodynamics in Blandford-Znajek Energy Extraction*, AdP, 2023 click here

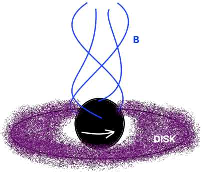
- What is the Blandford-Znajek (BZ) mechanism ?
- Why non-linear electrodynamics theories ?
- Mathematical and astrophysical results
- Conclusions and future directions

The BZ mechanism

In presence of an external magnetic field, a significant quantity of energy could be extracted from a spinning black hole

- Only one possibility among energy extraction ways (Hawking effect, Penrose process, magnetic reconnection, ...)
- Explains high energy phenomena as highly relativistic jets
- Requires general relativistic magnetohydrodynamic modeling

$$P \simeq \frac{B^2 r^4 w^2}{c}$$



Why non-linear theories ? (I)

- General electromagnetic Lagrangian governing the surrounding plasma:

$$\mathcal{L}_{NLED} = f(X), \qquad X \equiv \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}(B^2 - E^2)$$

When $\mathcal{L}_{NLED} = -X \rightarrow$ Maxwell theory

- Investigated in a wide range of contexts (optics, plasma, nuclear physics, superconductors, **astrophysics**...) [Delphenich(2006), Lundin et al.(2006)Lundin, Brodin, and Marklund, Ohnishi and Yamamoto(2014), Panotopoulos(2021), Garcia-Salcedo and Breton(2000)]

- Non-linear effects arise in QED (but very tricky to test as they require extremely high-energetic particles or extremely high magnetic field)

Why non-linear theories ? (II)

- **Cosmology (I)**: when coupled to a gravitational field, NLED theories can give the necessary negative pressure and enhance cosmic inflation, as well as avoid the Big Bang singularity

- **Cosmology (II)**: a non-linear Lagrangian brakes the conformal invariance → survival of primordial magnetic fields

There are two ways to generate large scale fields:

- allow a non-minimal coupling gravity-electromag.

- modify the EM sector and leave it as a minimal coupling

A plethora of models...

Model	Lagrangian $\mathscr{L}$	$f(F)$	Maxwell's limit
Born-Infeld model	$-\alpha^2\left(\sqrt{1+\frac{2F}{\alpha^2}}-1\right)$	$\frac{\alpha^2}{F}\left(\sqrt{1+\frac{2F}{\alpha^2}}-1\right)$	$\alpha \gg \sqrt{2\|F\|}$
De Lorenci et al. model	$-F+16\alpha F^2$	$1-16\alpha F$	$\alpha \to 0$
Novello's Toy model	$-F+16\alpha^2 F^2-\frac{\beta}{F}$	$1-16\alpha F+\frac{\beta}{F^2}$	$\alpha \to 0, \beta \to 0$
Kruglov's model A	$-F\left(1-\frac{\alpha}{2\beta F+1}\right)$	$1+\frac{1}{2\beta F}$	$\alpha \to 0$
Kruglov's model B	$-\frac{F}{\beta F+1}$	$\frac{1}{\beta F+1}$	$\beta \to 0$
Kruglov's model C	$-F\,\mathrm{sech}^2\left(\sqrt[4]{\|F\beta\|}\right)$	$\mathrm{sech}^2\left(\sqrt[4]{\|F\beta\|}\right)$	$\beta F \to 0$
Övgün's exponential correction model	$\frac{-Fe^{-\alpha F}}{\alpha F+\beta}$	$\frac{e^{-\alpha F}}{\alpha F+\beta}$	$\alpha \to 0, \beta \to 1$
Benaoum and Övgün model	$\frac{-F}{(\beta F^\alpha+1)^{1/\alpha}}$	$\frac{1}{(\beta F^\alpha+1)^{1/\alpha}}$	$\beta \to 0$

[Övgün(2017)]

An astrophysical application (I)

We apply some NLED models to the BZ mechanism, i.e. to a SMBH, in order to find out if non-linear phenomena could exists in such scenarios

Ingredients:

- EM stress-energy tensor: $T^{EM}_{\mu\nu} = -L(X)g_{\mu\nu} + L_X F_{\mu\rho}F_{\nu\sigma}g^{\rho\sigma}$
- Generalized Maxwell equations: $\frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}L_X F^{\mu\nu}\right] = -J^\nu$
- Radial energy and angular momentum fluxes:

$$F^{(r)}_E := T^r_t\,, \quad F^{(r)}_L := -T^r_\phi$$

An astrophysical application (II)

On the horizon, $r_+ = 1 + \sqrt{1-a^2}$, in presence of a force-free, magnetized, ideal plasma:

The energy flux becomes $F_E(\theta) = -2L_X^{(r_+)} B_r^2 w r_+ (\Omega_H - w) \sin^2\theta$

- It will depend not only on the radial magnetic field B_r, but also on the other two components, namely B_θ and B_ϕ
- The extracted power (rate) is:

$$P^{NLED} = 4\pi \int_0^{\pi/2} d\theta \sqrt{-g} F_E(\theta)$$

$\rightarrow$ Perturbative expansion in powers of "a"

An astrophysical application (III)

Unknowns: $\{B_r, B_\theta, B_\phi, w\} \rightarrow$ only $\{A_\phi\}$

We solve the generalized Maxwell eqs. in the limit $a << 0$ using

$$A_\phi = A_\phi^{(0)} + a^2 A_\phi^{(2)} + \mathcal{O}(a^4)$$

$$B_\phi = aB_\phi^{(1)} + \mathcal{O}(a^3)$$

$$w = a\, w_\phi^{(1)} + \mathcal{O}(a^3)$$

The other components of **B** are

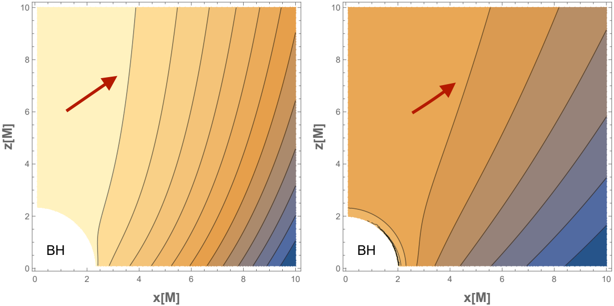
$$B_r = -\frac{1}{\sqrt{-g}}\left(\partial_\theta A_\phi^{(0)} + a^2 \partial_\theta A_\phi^{(2)}\right), \quad B_\theta = \frac{1}{\sqrt{-g}}\left(\partial_r A_\phi^{(0)} + a^2 \partial_r A_\phi^{(2)}\right).$$

$$\rightarrow \left[\frac{1}{\sin\theta}\frac{\partial}{\partial r} L_X^{(0)}\left(1-\frac{2}{r}\right)\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial_\theta}\frac{L_X^{(0)}}{\sin\theta}\frac{\partial}{\partial\theta}\right] A_\phi^0 = 0$$

Results (I)

Assuming separated solutions: $A_{\phi}^{(0)} = R(r) \cdot U(\theta)$ and $L_{X}^{(0)} = f(r) \cdot g(\theta)$

- For $L_{NLED} = -CX - \gamma X^{\delta}$: no monopole solutions and only $\delta = 2$ makes sense !



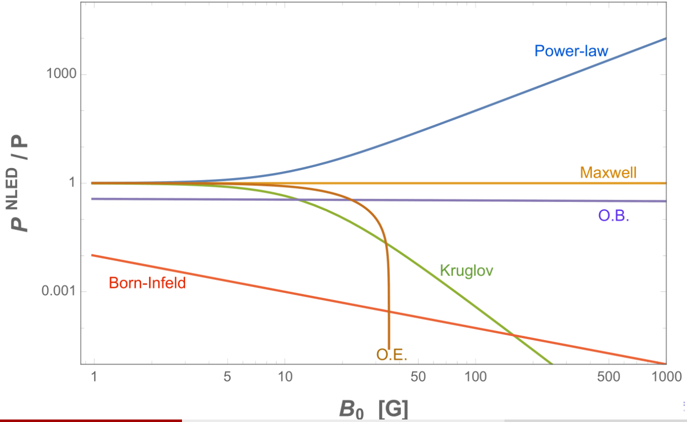
Non-linear theory — Linear theory

Results (II)

- If B_θ is negligible, a very simple estimate is possible:

$$\frac{P^{NLED}}{P} = -L_X(X_0)$$

where $X_0 := B_0^2/2$ and $B_0 \sim \sqrt{\sigma_0}$ is the magnetic strength.

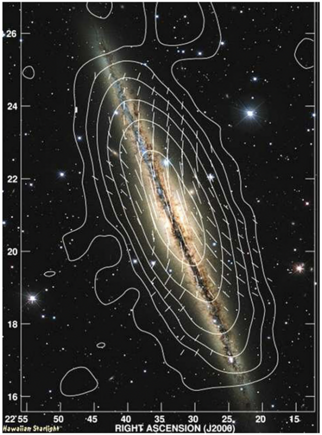


Primordial Magnetic Fields (I)

- **Galactic**
  Typical length ~ 10 kpc
  Typical strength $\sim 10^{-6}$ G

- **Intergalactic**
  Typical length ~ 1 Mpc
  Typical strength
  $10^{-15} G < B < 10^{-9} G$

NGC 891



Primordial Magnetic Field (II)

- According to GR, primordial magnetic fields (PMF) decayed adiabatically as

$$B(\tau) \sim \frac{1}{a(\tau)^2} \quad \rightarrow \quad \rho_B = \frac{|\mathbf{B}|^2}{8\pi} \propto \frac{1}{a^4}$$

- This scaling is the same for every cosmic energy density in the Universe

$$\rho_\gamma \sim \frac{1}{a^4} \quad \rightarrow \quad r \doteq \frac{\rho_B}{\rho_\gamma} \approx const$$

- Observations tell us that $r \approx 1$. Why ?

- In order to explain this value, one needs a pregalactic seed field with
$r \simeq 10^{-34}$ (dynamo)
$r \simeq 10^{-8}$ (compression)

Cosmological Setup

- Same eqs as before but source free ($J^\nu = 0$)
- Conformally flat FRW metric:

$$ds^2 = a^2(\eta)\left(d\eta^2 - d\mathbf{x}^2\right) = dt^2 - a(t)d\mathbf{x}^2 \quad (1)$$

conformal time

- The resulting field equation is

$$L_X F'' + \left(\partial_0 L_X\right) F' = 0 \quad (2)$$

- Definition of the variable $F := a^2 B$ with $B := |\mathbf{B}|$
- Long-wave approximation $\quad aL >> H^{-1} \rightarrow k\eta << 1$

Tested Models (I)

- (A) $L_{NLED} = -CX - \gamma X^{\delta}$

  Same results of [Cuesta and Lambiase(2009)], i.e. $\delta < 2$ (precisely 1.26-1.38)

- (B) $L_{NLED}(X) = -\dfrac{X}{(\beta X^{\alpha} + 1)^{1/\alpha}}$

M_{GUT}	T_{RH}	T_*	α
10^{16}	10^{9}	10^{12}	28.4
10^{17}	10^{15}	10^{15}	15.1
10^{17}	10^{17}	10^{16}	15.4

$\rightarrow$ taking $\alpha \simeq 15$, the power-law solution for the radiation era goes like

$$F \sim a^{u} \text{ with } u \simeq \{2, 63\}$$

Tested Models (II)

- (C) $L_{NLED} = -\frac{Xe^{-\alpha X}}{\alpha X + \beta}$ → DOES NOT ALLOW POWER-LAW SOLUTIONS FOR F ! IT REDUCES TO A POWER-LAW ONE WITH $\delta = 2$ ($C = 1/\beta$ AND $\gamma = -\alpha/\beta$) AS LONG AS $\beta \leq 1$ and $B \ll \sqrt{2/\alpha}$ → NOT COMPATIBLE WITH THE OBSERVATION $r \sim 1$

A Cosmological Confirmation ?

OUR (ANALYTICAL) RESULTS EMPHASIZE THAT THE EXISTENCE AND THE BEHAVIOR OF NON-LINEAR ELECTROMAGNETIC PHENOMENA STRONGLY DEPEND ON THE MODEL AND THE PHYSICAL CONTEXT, AND THAT POWER-LAW MODELS $L_{NLED} = -CX - \gamma X^{\delta}$ WITH $\delta \leq 2$ SHOULD BE FURTHER STUDIED.

CONLUSIONS

- WE FOUND THE EXTRACTED POWER UP TO SECOND ORDER IN a IN PRESENCE OF NLED
- ONLY $L_{NLED} = -CX - \gamma X^2$ IS SIGNIFICANT $\rightarrow$ PARABOLOIDAL SOLUTION
- NOTICEABLY ABLE TO EXTRACT MORE ENERGY ($\sim 10^3 \times$ MAXWELL)
- MORE 'VERTICALITY' OF FIELD LINES: $A_\phi^{(0)} \sim r^s(1 - \cos\theta)$ WITH $s > 1$
- WE ALSO FOUND AN EASY FORMULA TO ESTIMATE THE POWER (W.R.T. MAXWELL), USABLE FOR ALL MODELS !
- MAYBE A COSMOLOGICAL CONFIRMATION FOR NON-LINEAR TERMS IN THE EARLY UNIVERSE

FUTURE DIRECTIONS

- WE ASSUMED SEPARABLE SOLUTIONS $\rightarrow$ NUMERICAL SIMULATIONS
- INVESTIGATE HIGHER PERTURBATION ORDERS $\rightarrow$ FAST-SPINNING BHS

THANKS FOR YOUR ATTENTION

References I

Herman J. Mosquera Cuesta and Gaetano Lambiase.
Primordial magnetic fields and gravitational baryogenesis in nonlinear electrodynamics.
Physical Review D, 80(2), jul 2009.
doi: 10.1103/physrevd.80.023013.
URL https://doi.org/10.1103%2Fphysrevd.80.023013.

D. H. Delphenich.
Nonlinear optical analogies in quantum electrodynamics.
10 2006.

Ricardo Garcia-Salcedo and Nora Breton.
Born-Infeld cosmologies.
Int. J. Mod. Phys. A, 15:4341–4354, 2000.
doi: 10.1016/S0217-751X(00)00216-9.

J. Lundin, G. Brodin, and M. Marklund.
Short wavelength QED correction to cold plasma-wave propagation.
Phys. Plasmas, 13:102102, 2006.
doi: 10.1063/1.2356315.

Akira Ohnishi and Naoki Yamamoto.
Magnetars and the Chiral Plasma Instabilities.
2 2014.

Grigoris Panotopoulos.
Building (1+1) holographic superconductors in the presence of non-linear Electrodynamics.
Chin. J. Phys., 69:295–302, 2021.
doi: 10.1016/j.cjph.2020.11.023.

References II

A. Övgün.
Inflation and acceleration of the universe by nonlinear magnetic monopole fields.
The European Physical Journal C, 77(2), feb 2017.
doi: 10.1140/epjc/s10052-017-4673-4.
URL https://doi.org/10.1140%2Fepjc%2Fs10052-017-4673-4.