QGSKY - Highlights on Catania Unit research work

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Vacuum Stability (2013 - 2019) ... role of Gravity ? ...

Physical Tuning - Physical Swampland (Naturalness: M_H & Λ_{cc})

EFT - Wilson - Dimensional Regularization

Kaluza-Klein and Λ_{cc} - Dark Dimension?

Quantum Scale Invariance ... So cheap? ...

Vacuum Stability

V. Branchina, E. Messina, PRL 111, 241801 (2013)
V. Branchina, E. Messina, A. Platania JHEP 1409 (2014) 182
V. Branchina, E. Messina, M. Sher, PRD 91 (2015) 1, 013003
V. Branchina, E. Messina, D. Zappalà, EPL 116 (2016)
V. Branchina, E. Messina, EPL 117 (2017) 61002
E. Bentivegna, V. Branchina, F. Contino, D. Zappalà, JHEP 1712 (2017) 100
V. Branchina, F. Contino, A. Pilaftsis, PRD 98 (2018) 7, 075001
V. Branchina, F. Contino, P. M. Ferreira, JHEP 1811 (2018) 107

V. Branchina, E. Bentivegna, F. Contino, D. Zappalà, PRD 99 (2019) 9, 096029

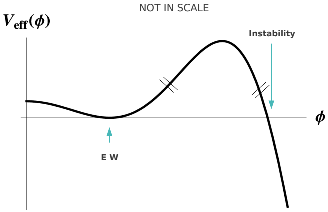
One-loop Higgs Effective Potential - RG improved Potential

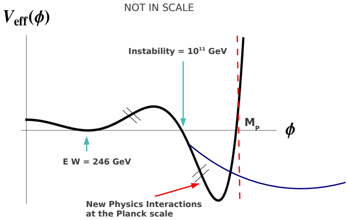
$$V_{1loop}(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{1}{64\pi^2}\left[\left(m^2+\frac{\lambda}{2}\phi^2\right)^2\left(\ln\left(\frac{m^2+\frac{\lambda}{2}\phi^2}{\mu^2}\right)-\frac{3}{2}\right)+\right.$$

$$+3\left(m^2+\frac{\lambda}{6}\phi^2\right)^2\left(\ln\left(\frac{m^2+\frac{\lambda}{6}\phi^2}{\mu^2}\right)-\frac{3}{2}\right)+\frac{6g^4}{16}\phi^4\left(\ln\left(\frac{\frac{g^2}{4}\phi^2}{\mu^2}\right)-\frac{5}{6}\right)+$$

$$\left.+3\frac{(g^2+g'^2)^2}{16}\phi^4\left(\ln\left(\frac{\frac{1}{4}(g^2+g'^2)\phi^2}{\mu^2}\right)-\frac{5}{6}\right)-12g_t^4\phi^4\left(\ln\frac{g_t^2\phi^2}{\mu^2}-\frac{3}{2}\right)\right]$$

$$V_{\text{eff}}(\phi) \sim \frac{1}{4}\lambda_{\text{eff}}(\phi)\phi^4$$



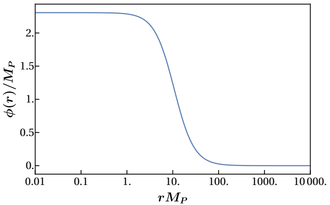


Bounce - flat spacetime - S. Coleman

$S[\phi] = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)\right]$; *false vacuum* $\phi_{\rm fv} \to$ *true vacuum* $\phi_{\rm tv}$.

Bounce $\phi_b(r)$: solution to EOM w/ $O(4)$ symmetry.

$$\ddot{\phi}(r) + \frac{3}{r}\dot{\phi}(r) = \frac{dV}{d\phi} \qquad b.c.: \ \phi(\infty) = \phi_{\rm fv} \ ; \ \dot{\phi}(0) = 0$$



Decay rate : $\Gamma = \frac{1}{\tau} = T_U^3 \frac{B^2}{4\pi^2} \left| \frac{\det'\left[-\partial^2 + V''(\phi_b):\right]}{\det\left[-\partial^2 + V''(v)\right]} \right|^{-\frac{1}{2}} e^{-B} \equiv D\, e^{-B}$

Bounce - Gravity - Coleman-de Luccia

$S[\phi, g_{\mu\nu}] = \int d^4x \sqrt{g} \left[-\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi + V(\phi) \right]$

$O(4)$ symmetry $\rightarrow ds^2 = dr^2 + \rho^2(r) d\Omega_3^2$

Bounce : $\phi_b(r)$ and $\rho_b(r)$ solution of EOMs :

$$\ddot{\phi} + 3 \frac{\dot{\rho}}{\rho} \dot{\phi} = \frac{dV}{d\phi} \qquad \dot{\rho}^2 = 1 + \frac{\kappa}{3} \rho^2 \left(\frac{1}{2} \dot{\phi}^2 - V(\phi) \right)$$

with boundary conditions : $\phi(\infty) = \phi_{\rm fv} \qquad \dot{\phi}(0) = 0 \qquad \rho(0) = 0$

Flat versus Curved Spacetime

$$\tau_{\rm flat} \sim 10^{639} \, T_U \qquad ; \qquad \tau_{\rm grav} \sim 10^{661} \, T_U$$

Gravity tends to **stabilize the vacuum**

All this is nice ... but ... there is a surprise around the corner $\Rightarrow$

Add New Physics at the Planck scale

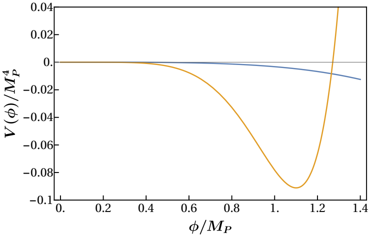
$$V(\phi) = V_{\text{eff}}(\phi) + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}$$

Decoupling: $\phi_{\text{inst}} \sim 10^{11}\,\text{GeV} << M_P \Rightarrow$ suppression $\frac{\phi_{\text{inst}}}{M_P}$ expected (!?!?)

But ... decoupling applies to **perturbative phenomena** ... false vacuum decay **non-perturbative phenomenon!** $\Rightarrow$ No suppression ϕ_{inst}/M_P

New Physics at M_P can have a strong impact on τ

consider $\boldsymbol{\lambda_6 < 0}$ and $\boldsymbol{\lambda_8 > 0}$ ...



Impact of New Physics on τ

E. Bentivegna, V. Branchina, F. Contino, D. Zappalà, JHEP 1712 (2017) 100

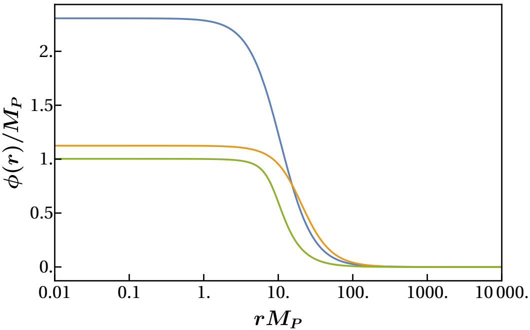
λ_6	λ_8	$\tau_{\rm flat}/T_U$	$\tau_{\rm grav}/T_U$
0	0	10^{639}	10^{661}
−0.15	0.25	10^{186}	10^{512}
−0.3	0.3	10^{-52}	10^{287}
−0.45	0.5	10^{-93}	10^{173}
−0.7	0.6	10^{-162}	10^{47}
−1.2	1.0	10^{-195}	10^{-58}
−1.7	1.5	10^{-206}	10^{-106}

Gravity tends to **stabilize the EW vacuum** ($\tau_{\rm grav}$ always higher than $\tau_{\rm flat}$)
However, New Physics has **strong impact** ...

How comes???

Bounces & New Physics - flat spacetime case

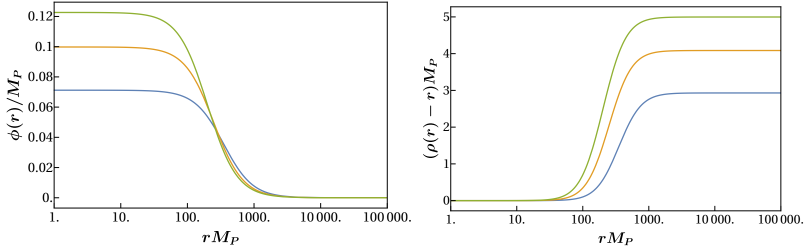
New bounce $\phi_b^{NP}(r) \Rightarrow$ New action $S[\phi_b^{NP}(r)] \Rightarrow$ New $\tau \sim e^{S[\phi_b^{NP}(r)]}$



- Blue curve: bounce profile with $\lambda_6 = \lambda_8 = 0$, i.e. with SM alone.
- Yellow curve: bounce profile with $\lambda_6 = -0.3$ and $\lambda_8 = 0.3$.
- Green curve: bounce profile with $\lambda_6 = -0.01$ and $\lambda_8 = 0.01$.

Bounce solution with New Physics - curved spacetime

E. Bentivegna, V. Branchina, F. Contino, D. Zappalà, JHEP 1712 (2017) 100



- Blue curve: bounce profile with $\lambda_6 = \lambda_8 = 0$, i.e. with SM alone.
- Yellow curve: bounce profile with $\lambda_6 = -0.03$ and $\lambda_8 = 0.03$.
- Green curve: bounce profile with $\lambda_6 = -0.04$ and $\lambda_8 = 0.04$.

Protections ???

Gravity with Non-Minimal Coupling

$$S[\phi, g_{\mu\nu}] = \int d^4x\sqrt{-g}\ \left[-\frac{R}{16\pi G} + \frac{1}{2}\ g^{\mu\nu}\nabla_\mu\phi\ \nabla_\nu\phi + V(\phi) + \frac{1}{2}\xi\phi^2 R\right]$$

Again $O(4)$ symmetry:

$$\ddot{\phi} + 3\,\frac{\dot{\rho}}{\rho}\,\dot{\phi} = \frac{dV}{d\phi} + \xi\phi R \qquad \dot{\rho}^2 = 1 - \frac{\kappa}{3}\rho^2\,\frac{-\frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\xi\frac{\dot{\rho}}{\rho}\phi\dot{\phi}}{1-\kappa\xi\phi^2}$$

with R given by:

$$R = \kappa\frac{\dot{\phi}^2(1-6\xi) + 4V(\phi) - 6\xi\phi dV/d\phi}{1-\kappa\xi(1-6\xi)\phi^2}$$

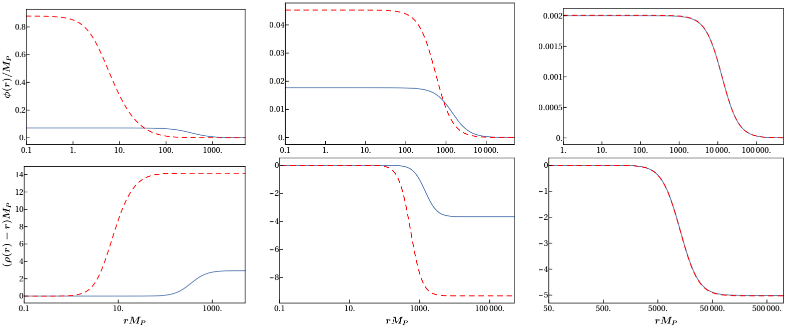
$\xi = 0 \quad \Rightarrow \quad$ back to the old minimal coupling EOMs

New Physics & Non-Minimal Coupling

V. Branchina, E. Bentivegna, F. Contino, D. Zappalà, PRD 99 (2019) 9, 096029

Adding New Physics: $V_{NP} = \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}$, with $\lambda_6 = -1.2$ and $\lambda_8 = 1$

$\xi = 0$ $\qquad\qquad$ $\xi = 1$ $\qquad\qquad$ $\xi = 10$



For $\xi > 1$: $\quad (\phi_{NP}(r), \rho_{NP}(r)) \rightarrow (\phi_{SM}(r), \rho_{SM}(r))$

New Physics & Non-Minimal Coupling

V. Branchina, E. Bentivegna, F. Contino, D. Zappalà, PRD 99 (2019) 9, 096029

Adding New Physics: $V_{NP} = \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}$, with $\lambda_6 = -1.2$ and $\lambda_8 = 1$

ξ	$(\tau/T_U)_{SM}$	$(\tau/T_U)_{NP}$
-15	10^{736}	10^{736}
-10	10^{726}	10^{726}
-5	10^{710}	10^{710}
-1	10^{684}	10^{680}
-0.5	10^{677}	10^{600}
-0.3	10^{672}	10^{358}
-0.1	10^{666}	10^{65}
0	10^{661}	10^{-58}

ξ	$(\tau/T_U)_{SM}$	$(\tau/T_U)_{NP}$
0.3	10^{660}	10^{-167}
0.5	10^{668}	10^{23}
0.7	10^{674}	10^{346}
0.8	10^{676}	10^{512}
1	10^{679}	10^{666}
5	10^{709}	10^{709}
10	10^{725}	10^{725}
15	10^{735}	10^{735}

For $\xi > 1 \quad \Rightarrow \quad$ washing out of New Physics destabilization: $\tau_{\rm NP} \simeq \tau_{\rm SM}$.

What else in ... Vacuum stability ? ...

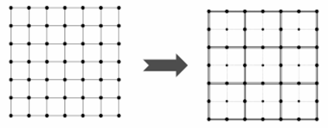
- Supergravity embedding — Branchina, Contino, Pfilatsis
- Two Higgs Doublet Model (2HDM) — Branchina, Contino , Ferreira

Physical Tuning

... Naturalness ... Higgs mass ... Cosmological Constant ...

Wilson's Lesson

What is the Wilson's lesson all about?



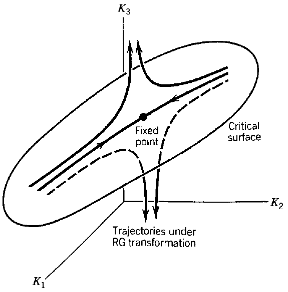
Theory at Λ $\rightarrow$ Theory at $\Lambda/2$ $\rightarrow$ ...

$$S_{\Lambda} \rightarrow S_{\Lambda/2} \rightarrow \ldots$$

Effective Field Theory paradigm

Any QFT is an Effective Field Theory

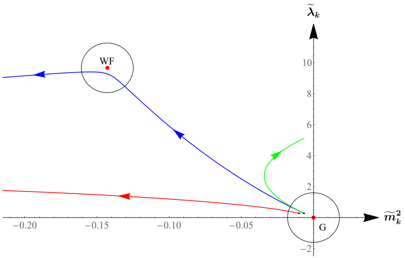
RG flow



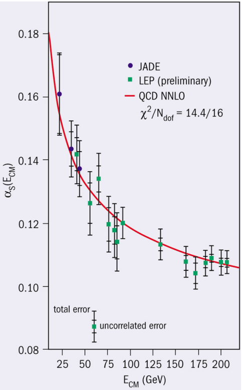
Renormalized theory: defined around a fixed point (critical surface)

... For theories in any dimesion: ..., $d = 3$, $d = 4$, ...

$d = 3$ dimensions : Wilson-Fisher



$d = 4$ dimensions : AF



... Wilson's Lesson ...

EFT paradigm is physical and unavoidable

Unless we are considering the TOE

There is no cutoff in the sense that somebody finds disturbing ...

... but rather a (Wilsonian) physical running scale ...

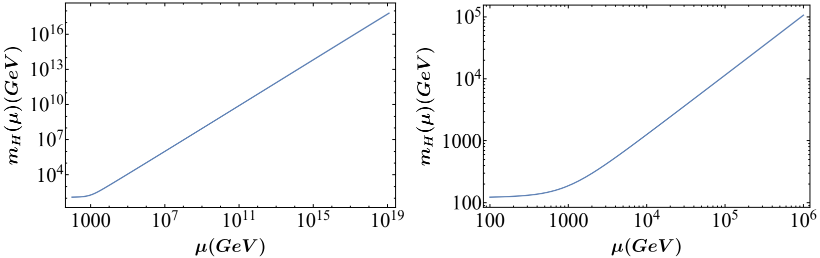
$$\Lambda \quad \to \quad \Lambda/2 \quad \to \quad \Lambda/4 \quad \to \quad \Lambda/8 \quad \to \quad \ldots$$

Λ is the highest scale ... "UV physical cutoff"

... and then? ... Filippo Contino talk ...

... and also ... Arcangelo Pernace talk ...

Numerical sol. to RG eq.(1) and **Analytical approx.** (2): **indistinguishable**



$$\mu \frac{d}{d\mu} m_H^2 = \frac{\alpha(\mu)}{16\pi^2}\mu^2 + \gamma(\mu)\, m_H^2 \qquad (1)$$

$$m_H^2(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\gamma}\left(m_H^2(\Lambda) - \frac{\alpha\,\Lambda^2}{2-\gamma}\right) + \frac{\alpha\,\mu^2}{2-\gamma} \qquad (2)$$

... Filippo Contino

Dimensional Regularization & Wilson

Textbook - Dim Reg : powerful technique ... no clear physical meaning (!?!?)

Could be of some help trying to understand ?

I mean ... Could we gain something ?

When we do not understand ... we might force technique(s) to make ...

... any sort of amazing prediction ...

... when we gain control we can make maybe OPPOSITE but robust predictions ...

Carlo Branchina, Vincenzo Branchina, Filippo Contino, Neda Darvishi, Dimensional regularization, Wilsonian RG, and the naturalness and hierarchy problem, **Phys.Rev.D 106 (2022) 6, 065007**. ArXiv: 2204.10582

One-loop effective potential ϕ^4

$d =$ integer dimension (no dim reg)

$$V_{1l}(\phi) = V_0(\phi) + \underbrace{\frac{1}{2}\int^{(\Lambda)} \frac{d^d k}{(2\pi)^d} \ln\left(1 + \frac{m_0^2 + \frac{1}{2}\mu^{4-d}\lambda_0\,\phi^2}{k^2}\right)}_{\delta V(\phi)}$$

$$\delta V(\phi) = \frac{1}{2}\int^{(\Lambda)} \frac{d^d k}{(2\pi)^d} \ln\left(1 + \frac{M^2(\phi)}{k^2}\right) \equiv \delta V_1(\phi) + \delta V_2(\phi)$$

where

$$M^2(\phi) \equiv m_0^2 + \tfrac{1}{2}\mu^{4-d}\lambda_0\,\phi^2$$

$$\delta V_1(\phi) \equiv \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)}\left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{d}{2}} \int_{\frac{M^2}{M^2+\Lambda^2}}^{1} dt\,(1-t)^{\frac{d}{2}-1}\,t^{-\frac{d}{2}}$$

$$\delta V_2(\phi) \equiv \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)}\left(\frac{\Lambda}{\mu}\right)^{d} \ln\left(1 + \frac{M^2(\phi)}{\Lambda^2}\right)$$

Calculating $\delta V(\phi)$

For any **integer** d:

$$\delta V_1(\phi) = \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{d}{2}} \int_{\frac{M^2}{M^2+\Lambda^2}}^{1} dt\, t^{-\frac{d}{2}}(1-t)^{\frac{d}{2}-1} =$$

$$= \lim_{z\to d}\left[A_1(z) - A_2(z)\right]$$

where z is **complex**, and

$$A_1(z) \equiv F(z)\cdot \overline{B}\left(1-\frac{z}{2},\frac{z}{2}\right) \qquad A_2(z) \equiv F(z)\cdot \overline{B}_i\left(1-\frac{z}{2},\frac{z}{2};\frac{M^2(\phi)}{M^2(\phi)+\Lambda^2}\right)$$

$$F(z) \equiv \frac{\mu^z}{z(4\pi)^{\frac{z}{2}}\Gamma\left(\frac{z}{2}\right)}\left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{z}{2}}$$

$\overline{B}$ and $\overline{B}_i$ are (the analytic extensions of) the Beta functions

Both $\overline{B}$ and $\overline{B}_i$ have poles in $z = 2, 4, 6, \ldots$

$\delta V_1(\phi)$ finite $\Rightarrow$ the poles of A_1 and A_2 have to cancel each other

Example: $\delta V(\phi)$ in $d = 4$ dimensions

$z \equiv 4 - \epsilon$. Expanding in powers of ϵ and M^2/Λ^2

$$A_1(4-\epsilon) = \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{M^2(\phi)}{4\pi\mu^2} - \frac{3}{2}\right) + \cancel{\mathcal{O}(\epsilon)}$$

$$A_2(4-\epsilon) = \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{M^2(\phi)}{4\pi\mu^2} - \frac{3}{2}\right) + \cancel{\mathcal{O}(\epsilon)} + \cancel{\mathcal{O}\left(\frac{M^2}{\Lambda^2}\right)}$$

$$- \frac{\mu^{-\epsilon}}{64\pi^2}\left[M^2(\phi)\right]^2\left(\frac{\Lambda^2}{M^2(\phi)} - \log\frac{\Lambda^2}{M^2(\phi)}\right)$$

Remember: $\delta V_1(\phi) = \lim_{\epsilon\to 0}\left[A_1(4-\epsilon) - A_2(4-\epsilon)\right]$. Adding $\delta V_2(\phi)$

$$\delta V(\phi) = \delta V_1 + \delta V_2 = \frac{\Lambda^2 M^2(\phi)}{32\pi^2} - \frac{\left[M^2(\phi)\right]^2}{64\pi^2}\left(\ln\frac{\Lambda^2}{M^2(\phi)} + \frac{1}{2}\right) + \cancel{\mathcal{O}\left(\frac{\phi^6}{\Lambda^2}\right)}$$

$$\Rightarrow \quad V_{1l}(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4 + \frac{\Lambda^2 M^2}{32\pi^2} - \frac{\left(M^2\right)^2}{64\pi^2}\left(\ln\frac{\Lambda^2}{M^2} + \frac{1}{2}\right)$$

No reference whatsoever to ϵ (of course!) ...

$\Leftarrow$ Critical region ... Critical Surface ...

Kaluza Klein

quotation: “che ci azzecca?”

Wilson also for theories with $d > 4$ dimesions

in particular

Theories with compact extra dimensions: $d = 4 + n$

- Typically approached as 4D theories with infinite towers of states:

$$m_n = f_n \, m_{\rm tow}$$

- Surprising UV-softness :

Towers contribute $\sim m_{\rm tow}^4$

to Vacuum Energy / Effective Potential

How is it possible?

One-loop Higgs Effective Potential (4D calculation)

$$V_{1l}^{(4)}(\phi) \sim \frac{1}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \log \left(p^2 + m_a^2(\phi) + \left(\frac{n}{R} + q_{i_a} \right)^2 \right)$$

One way of doing the calculation (not the only one):
Perform (first) the infinite sum; (then) integrate in $d^4 p$ with a cutoff Λ

Delgado, Pomarol, Quiros

Each tower contributes :

$$V_{1l}^{(4)}(\phi) = R \left(\frac{m^2 \Lambda^3}{48\pi} - \frac{m^4 \Lambda}{64\pi} + \frac{m^5}{60\pi} \right) - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3) \cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

- Power UV-sensitivity through $m \implies$ canceled by SUSY
- No UV-sensitivity through q

$\implies$ Finite Higgs potential !!!!!!!!!!!

... Arcangelo Pernace talk

Scale Invariance

quotation: “che ci azzecca?”

Naturalness/Hierarchy problem ... again ...

$m_\phi^2 \sim \Lambda^2 \quad \Rightarrow$ Fine-tuning   Physical mechanism ??

SUSY, composite models ... No sign of new physics at LHC

Eureka (?!?!): Conformal extension of SM & "Scale-invariant" DR-like reg.

Classically scale invariant $d = 4$ theory:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma + V(\phi,\sigma)$$

$$V(\phi,\sigma) = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_m}{2}\phi^2\sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4$$

At quantum level ... breaking of Scale Invariance ... unless ... you are a wizard

Scale-invariant DR $\quad \lambda_i \to \mu(\phi,\sigma)^{4-d}\lambda_i \quad$ After SSB $\mu = \mu(\langle\phi\rangle,\langle\sigma\rangle)$

Equivalent to **change the theory** in $d \neq 4$

$$V(\phi,\sigma) \to \tilde{V}^{(d)}(\phi,\sigma) = \mu(\phi,\sigma)^{4-d}V(\phi,\sigma) = \mu(\phi,\sigma)^{4-d}\left(\frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_m}{2}\phi^2\sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4\right)$$

Scale Invariant-regularized 1-loop correction

Generic d dimensions: 1-loop correction to the effective potential

$$\tilde{V}_{1L}^{(d)} = \frac{1}{2}\int \frac{d^d k}{(2\pi)^d} \,\mathrm{Tr}\,\log\left(k^2 + \tilde{M}_{(d)}^2\right) = -\frac{\mu(\phi,\sigma)^d}{2}\,\Gamma\left(-\frac{d}{2}\right)\,\mathrm{Tr}\left(\frac{\tilde{M}_{(d)}^2}{4\pi\mu(\phi,\sigma)^2}\right)^{\frac{d}{2}}$$

$$\tilde{M}_{(d),\,\alpha\beta}^2 = \frac{\partial^2 \tilde{V}_0^{(d)}(\phi,\sigma)}{\partial\alpha\partial\beta} = \mu^{4-d}\left[M_{\alpha\beta}^2 + (4-d)N_{\alpha\beta}\right]$$

$$N_{\alpha\beta} = \left[(3-d)\frac{\mu_\alpha\mu_\beta}{\mu^2} + \frac{\mu_{\alpha\beta}}{\mu}\right] V + \left[\frac{\mu_\alpha}{\mu}V_\beta + \frac{\mu_\beta}{\mu}V_\alpha\right]$$

$$\tilde{V}_{1L}^{(4-\epsilon)} = -\frac{\mu(\phi,\sigma)^\epsilon}{64\pi^2}\sum_{s=1,2}\left\{M_s^4\left(\frac{2}{\epsilon} - \gamma + \log 4\pi\right) - M_s^4\left[\log\frac{M_s^2}{\mu^{2-\epsilon}} - \frac{3}{2}\right] + 4M_s^2 N_s\right\} + \mathcal{O}(\epsilon)$$

Renormalized Scale Invariant Effective Potential

Counterterms

$$\delta\lambda_\phi = \frac{1}{8\pi^2\,\bar\epsilon}\left(9\lambda_\phi^2 + \lambda_m^2\right) \quad \delta\lambda_m = \frac{1}{8\pi^2\,\bar\epsilon}\left(4\lambda_m^2 + 3\lambda_m\lambda_\phi + 3\lambda_m\lambda_\sigma\right) \quad \delta\lambda_\sigma = \dots$$

After cancelling the divergences, taking the limit $\epsilon \to 0$ we obtain:

$$V_{eff,SI} = V(\phi,\sigma) + \frac{1}{64\pi^2}\mathrm{Tr}\,M^4\left(\log\frac{M^2}{\mu(\phi,\sigma)^2} - \frac{3}{2}\right) + \Delta V_{SI}$$

where (we magically have new terms ...)

$$\Delta V_{SI} = -\frac{1}{16\pi^2}\mathrm{Tr}\,M^2 N = -\frac{1}{16\pi^2}\left\{2\,V_{\phi\sigma}\left[V\left(\frac{\mu_{\phi\sigma}}{\mu} - \frac{\mu_\phi\mu_\sigma}{\mu^2}\right) + V_\phi\frac{\mu_\sigma}{\mu} + V_\sigma\frac{\mu_\phi}{\mu}\right]\right.$$
$$\left. + V_{\phi\phi}\left[V\left(\frac{\mu_{\phi\phi}}{\mu} - \frac{\mu_\phi^2}{\mu^2}\right) + 2\,V_\phi\frac{\mu_\phi}{\mu}\right] + V_{\sigma\sigma}\left[V\left(\frac{\mu_{\sigma\sigma}}{\mu} - \frac{\mu_\sigma^2}{\mu^2}\right) + 2\,V_\sigma\frac{\mu_\sigma}{\mu}\right]\right\}$$

... but if we make the calculation according to ...

... these terms magically disappear ...