THE DIFFERENT FACES OF COSMOGRAPHY

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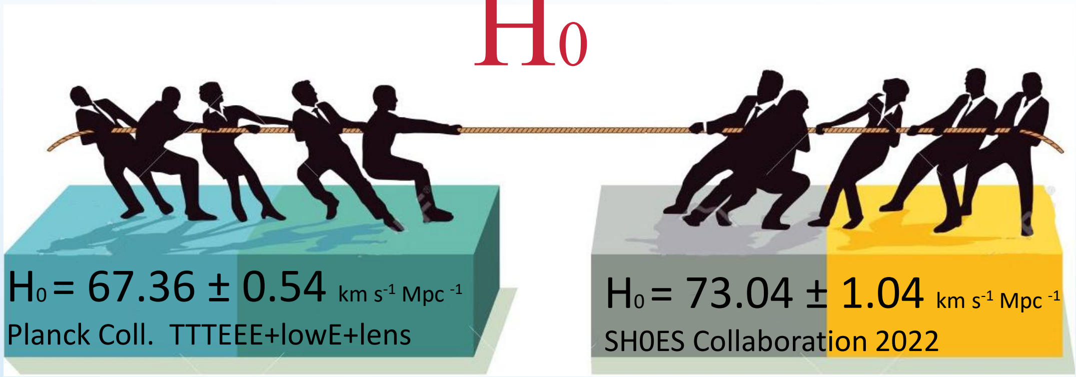
Annual Meeting QGSKY 2023 - Quantum Universe

OUTLINE

- why it is useful cosmography?
- what are pros and cons?
- Current research line in cosmography

H_0



$5\,\sigma$

Cosmography as a parametric approach to describe the data

→ addresses the problem in a **model-independent way** with the aim of obtaining important clues to be considered in the theory

As the standard cosmological model, Cosmography assumes the **cosmological principle**

Purely geometrical description of the Universe kinematic in which all the physics is hidden in the scale factor **a(t)**

$$a(t) = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} \frac{d^k a}{dt^k}\bigg|_{t=t_0} (t - t_0)^k$$

$$H(t) \equiv \frac{1}{a}\frac{da}{dt}$$

Hubble

$$q(t) \equiv -\frac{1}{aH^2}\frac{d^2a}{dt^2}$$

Deceleration

$$j(t) \equiv \frac{1}{aH^3}\frac{d^3a}{dt^3}$$

Jerk

$$s(t) \equiv \frac{1}{aH^4}\frac{d^4a}{dt^4}$$

Snap

$$D_L = (1+z)D(z)$$

Low redshift approximation

$$v(z) = H_0 D(z)$$

$$z \sim \frac{v(z)}{c}$$

$$D_L = (1+z)\frac{zc}{H_0}$$

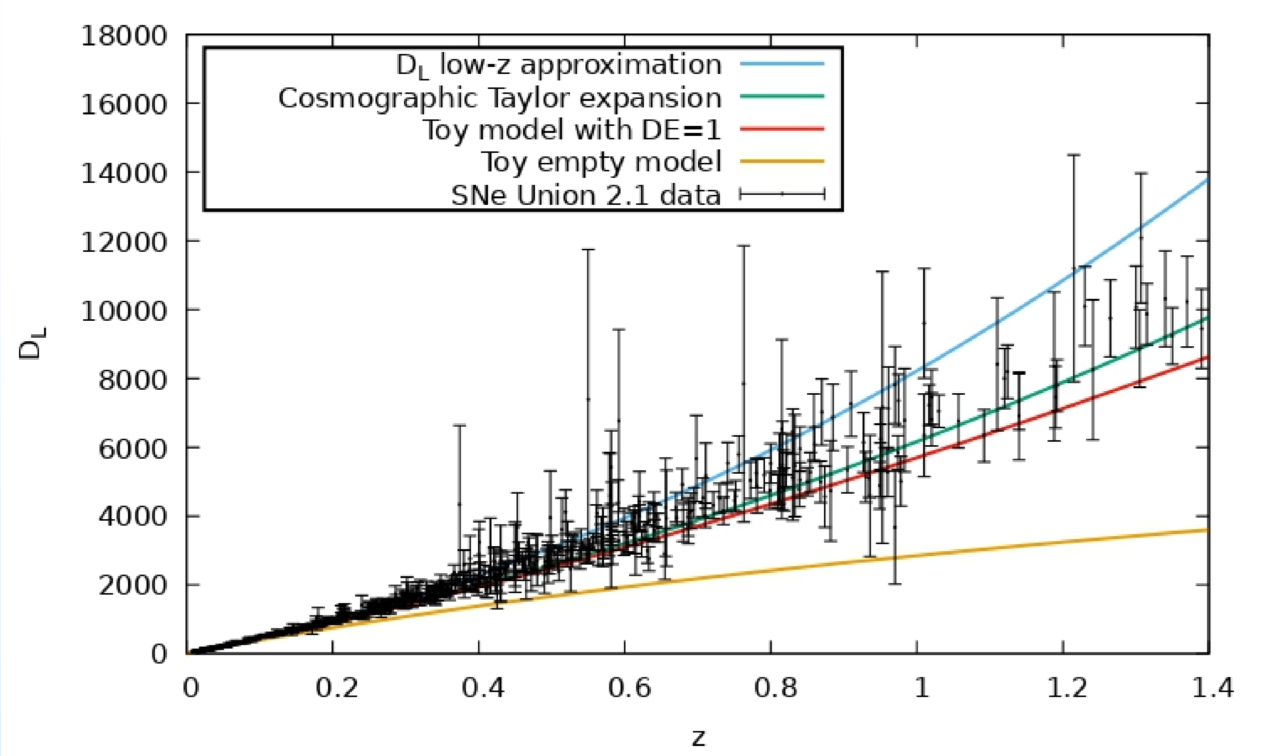
$$a(t) = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} \frac{d^k a}{dt^k}\bigg|_{t=t_0} (t-t_0)^k$$

$$H(t) \equiv \frac{1}{a}\frac{da}{dt} \qquad j(t) \equiv \frac{1}{aH^3}\frac{d^3a}{dt^3}$$

$$q(t) \equiv -\frac{1}{aH^2}\frac{d^2a}{dt^2} \qquad s(t) \equiv \frac{1}{aH^4}\frac{d^4a}{dt^4}$$

$$z = \frac{a(t_0)}{a(t_e)} - 1$$

$$D_L(z) = \frac{zc}{H_0}\Big[1 + \frac{z}{2}(1-q_0) - \frac{z^2}{6}(1 - q_0 - 3q_0^2 + j_0) + \frac{z^3}{24}(2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10q_0 j_0 + s_0) + \mathcal{O}(z^4)\Big]$$



$$D_L = (1+z)\frac{zc}{H_0}$$

$$D_L(z) = \frac{zc}{H_0}\left[1 + \frac{z}{2}(1-q_0) - \frac{z^2}{6}\left(1 - q_0 - 3q_0^2 + j_0\right) + \right.$$
$$\left. + \frac{z^3}{24}\left(2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10q_0 j_0 + s_0\right) + \mathcal{O}(z^4)\right]$$

$q_0=-1/2$
$j_0=s_0=0$

For a flat ΛCDM model:

$$D_L(z) = \frac{c}{H_0}(1+z)\int_0^z \frac{dz'}{\sqrt{\Omega_{M,0}(1+z')^3 + (1-\Omega_{M,0})}}$$

$$D_L(z) = \frac{c}{H_0}\left[z + \left(\frac{1-q_0}{2}\right)z^2 + \left(\frac{3q_0^2+q_0-1-j_0}{6}\right)z^3 - \left(\frac{15q_0^3+15q_0^2+2q_0-2-5j_0-s_0-10q_0j_0}{24}\right)z^4\right]$$

$$q_0 = \frac{3}{2}\Omega_{M,0} - 1,\ j_0 = 1,\ s_0 = 1 - \frac{9}{2}\Omega_{M,0}$$

From a 4th-order Taylor expansion at z=0 of both cosmographic and cosmological $D_L(z)$

Good news:

- If adopted expansion is sufficiently flexible, it is able to fit observational data with high accuracy
- Possibility to reduce the degeneracy of cosmological models
- Cosmographic parameters used to test any cosmological model

Issues:

- Arbitrary truncation limits the predictive power and may result in possible misleading outcomes

- A large number of cosmographic parameters makes their estimation difficult and introduces degeneracy among them

- To detect deviations from flat ΛCDM we need to explore high redshift data

  ➔ **Taylor show convergence issues for z≥1 !!**

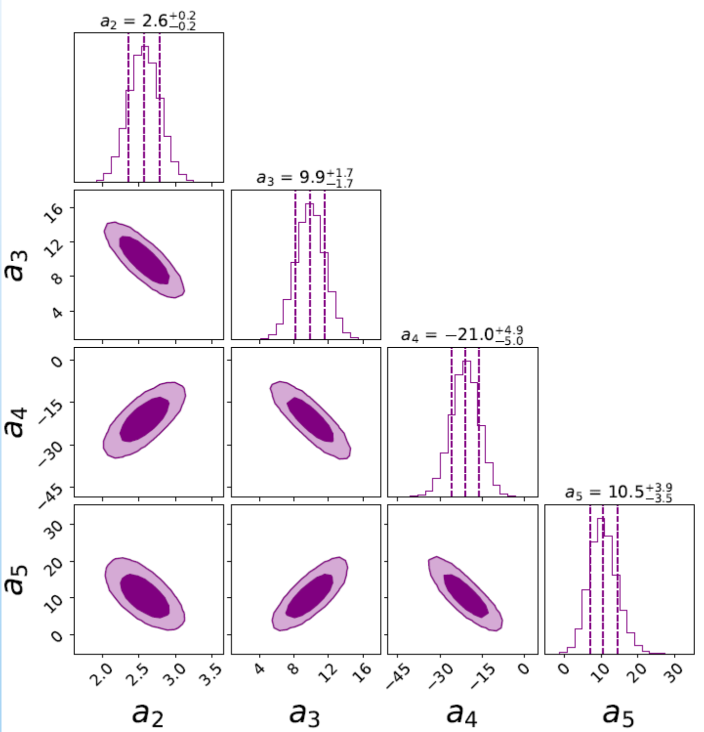
Possible solutions:

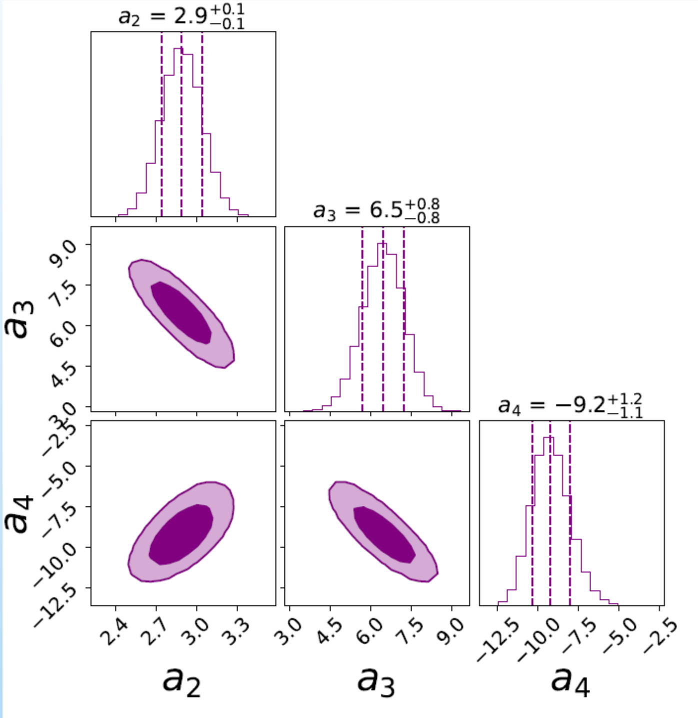
- orthogonal polynomials of logarithmic functions
- Rational polynomials
- Link cosmography with cosmology






NON-Orthogonal polynomials of logarithmic functions





Bargiacchi et al.
Astron.Astrophys.
649 (2021) A65

$$D_L = \frac{c}{H_0} \ln(10) \left[\log(1+z) + a_2 \log^2(1+z) + a_3 \log^3(1+z) + ..\right]$$

Orthogonal polynomials of logarithmic functions

$$D_L(z) = \frac{\ln(10)}{H_0}\Big\{\log(1+z) + a_2\log^2(1+z) + a_3\left[k_{32}\log^2(1+z) + \log^3(1+z)\right] +$$

$$+ a_4\left[k_{42}\log^2(1+z) + k_{43}\log^3(1+z) + \log^4(1+z)\right] + a_5\left[k_{52}\log^2(1+z) + k_{53}\log^3(1+z) + k_{54}\log^4(1+z) + \log^5(1+z)\right]\Big\}$$

Remove the correlation among coefficients

- A change in the truncation order of the series does not change the values of the cosmographic coefficients
- Allows to test the significance of a possible additional term in the expansion

$$D_L = \frac{c}{H_0}\ln(10)\left[\log(1+z) + a_2\log^2(1+z) + a_3\log^3(1+z) + ..\right]$$

$$D_L(z) = \frac{\ln(10)}{H_0}\Big\{\log(1+z) + a_2\log^2(1+z) + a_3\left[k_{32}\log^2(1+z) + \log^3(1+z)\right] +$$
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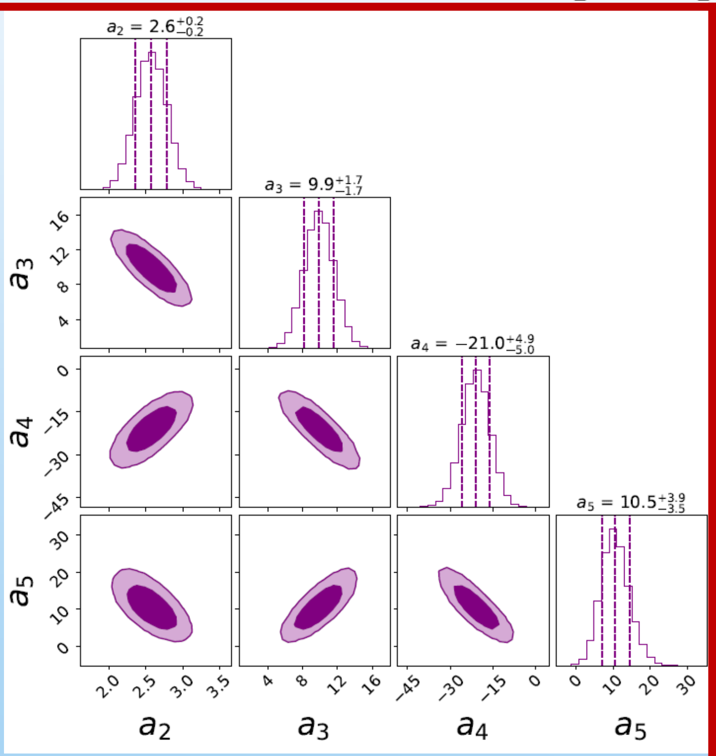

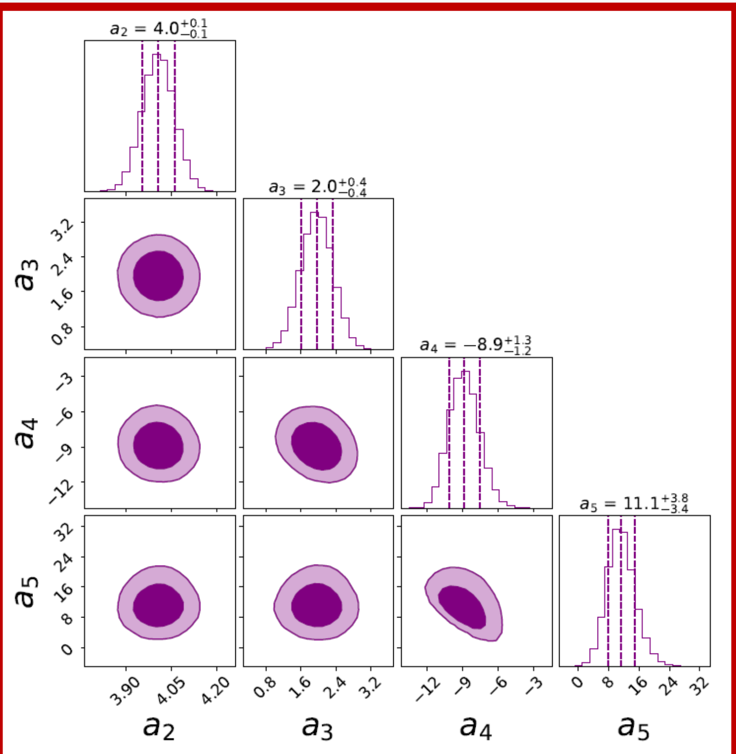

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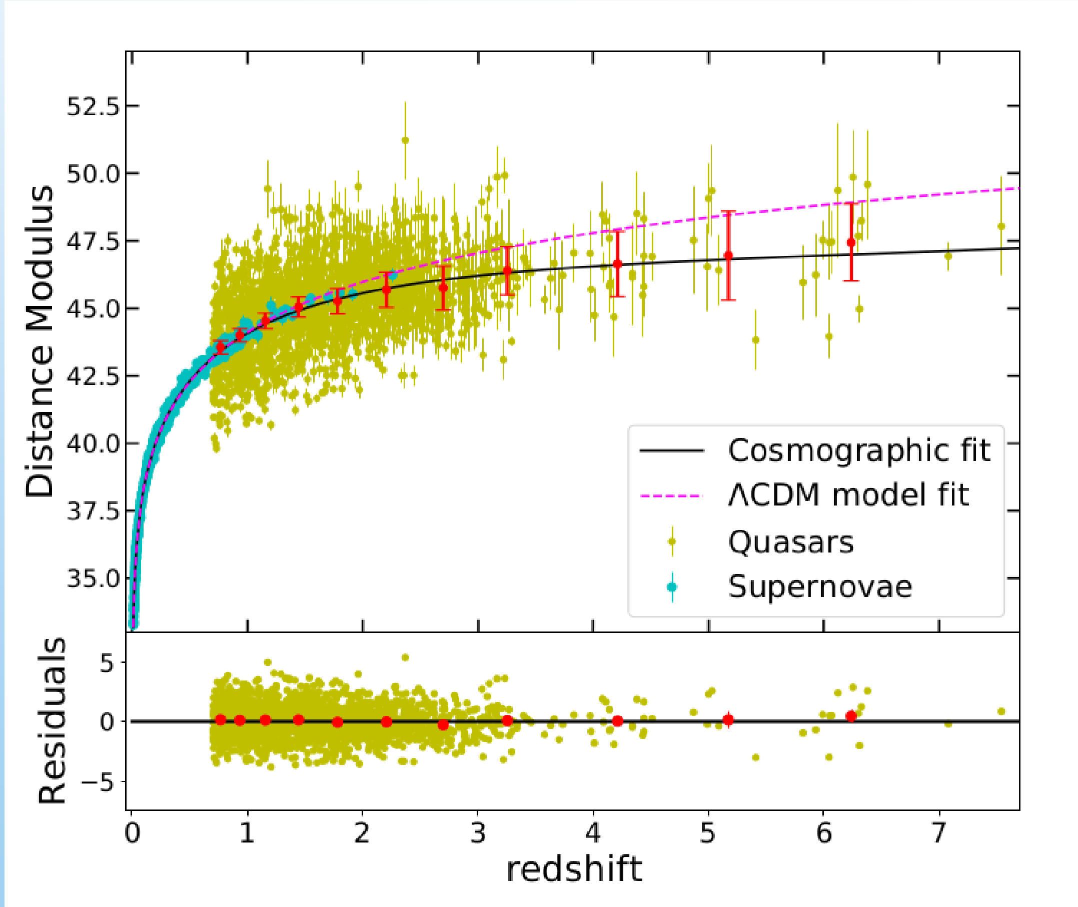
Bargiacchi et al. *Astron.Astrophys.* 649 (2021) A65



Fifth-order in the logarithmic polynomial is needed to fit data up to the maximum redshifts of quasars

A sixth-order would not be significant.

Possible solutions:

- ✓ orthogonal polynomials of logarithmic functions
- ➢ **Rational polynomials**
- ➢ Link cosmography with cosmology

Padè rational polynomial

Padè approximation: $$P_{n,m}(z) = \frac{\sum_{i=0}^{n} a_i z^i}{1 + \sum_{j=1}^{m} b_j z^j}$$

Standard Taylor series: $$\mathrm{f(z)} = \sum_{i=0}^{\infty} c_i z^i$$

Padè rational polynomial

Padè approximation: $$P_{n,m}(z) = \frac{\sum_{i=0}^{n} a_i z^i}{1+\sum_{j=1}^{m} b_j z^j}$$

$$P_{12}(z) = \frac{P_0 + P_1 z}{1 + Q_1 z + Q_2 z^2}$$

$$P_{22}(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$$P_{32}(z) = \frac{P_0 + P_1 z + P_2 z^2 + P_3 z^3}{1 + Q_1 z + Q_2 z^2}$$

$D_L = f(P_{nm})$ →

$$\begin{cases} P_{nm}(0) = D_L(0) \\ P'_{nm}(0) = D'_L(0) \\ \dots \\ P_{nm}^{(n+m)}(0) = D_L^{(n+m)}(0) \end{cases}$$

↑ PADE ↑ Taylor expansion

S. Capozziello, R. D'Agostino, and O. Luongo, Mon. Not. Roy. Astron. Soc. **494**, 2576 (2020), arXiv:2003.09341 [astro-ph.CO] .

Padè rational polynomial

$$P_{12}(z) = \frac{P_0 + P_1 z}{1 + Q_1 z + Q_2 z^2}$$

$$D_L(z) = \frac{c}{H_0}\left[z + \left(\frac{1-q_0}{2}\right)z^2 + \left(\frac{3q_0^2 + q_0 - 1 - j_0}{6}\right)z^3 - \left(\frac{15q_0^3 + 15q_0^2 + 2q_0 - 2 - 5j_0 - s_0 - 10q_0 j_0}{24}\right)z^4\right]$$

$$\begin{cases} P_{12}(0) = D_L(0) \\ P'_{12}(0) = D'_L(0) \\ P''_{12}(0) = D''_L(0) \\ P'''_{12}(0) = D'''_L(0) \end{cases}$$

$D_L = f(P_{21})$

$$P_{21}(z) = \frac{cz}{H_0}\frac{6(q_0 - 1) + z[-5 - 2j_0 + q_0(8 + 3q_0)]}{-2(3 + z + j_0 z) + 2q_0(3 + z + 3zq_0)}$$

Padè rational polynomial

Padè approximation: $$P_{n,m}(z) = \frac{\sum_{i=0}^{n} a_i z^i}{1 + \sum_{j=1}^{m} b_j z^j}$$

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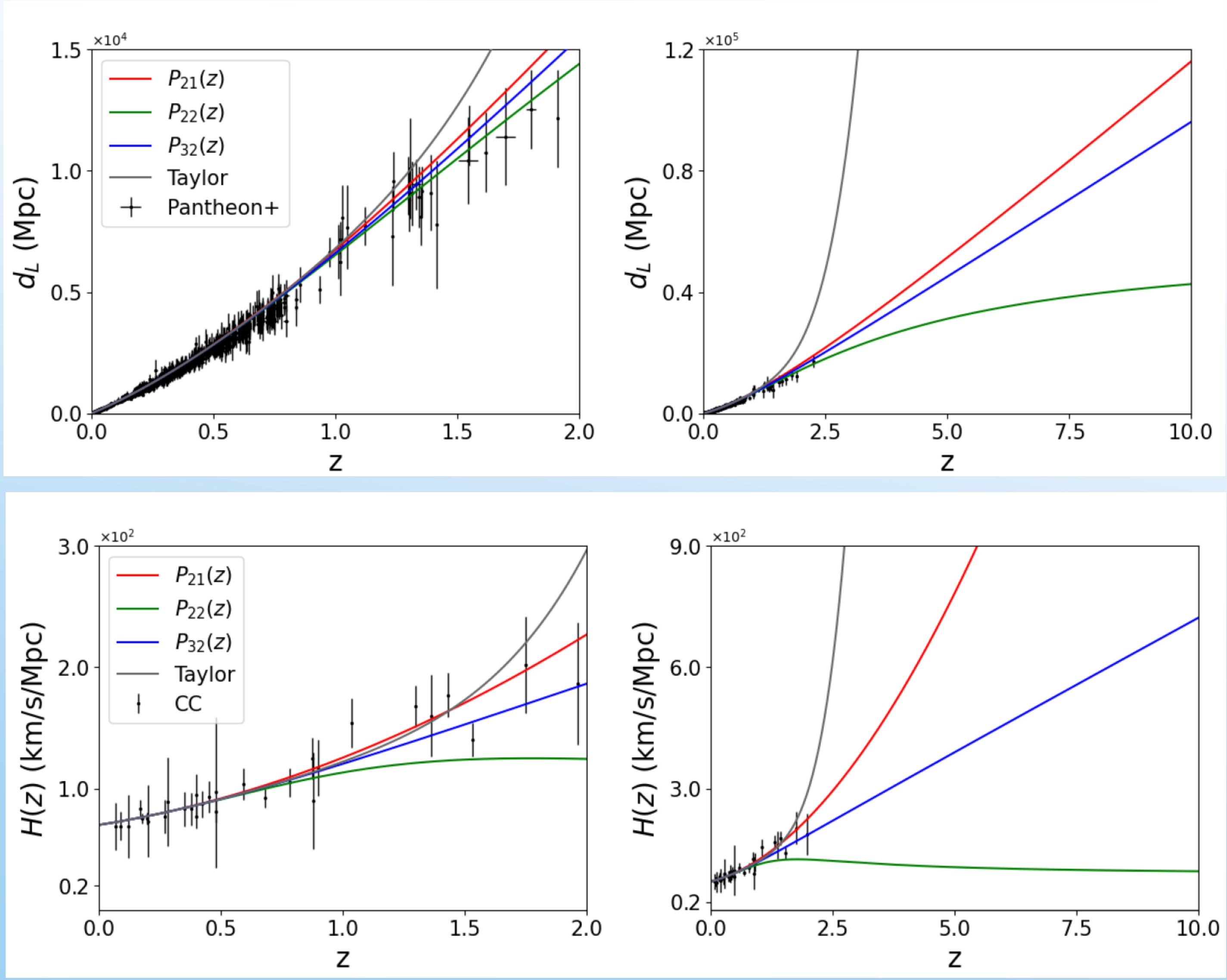
H(z) = f(P_{nm}) →

$$P_{nm}(0) = H(0)$$

$$P'_{nm}(0) = H'(0)$$

$$P_{nm}^{(n+m)}(0) = H^{(n+m)}(0)$$

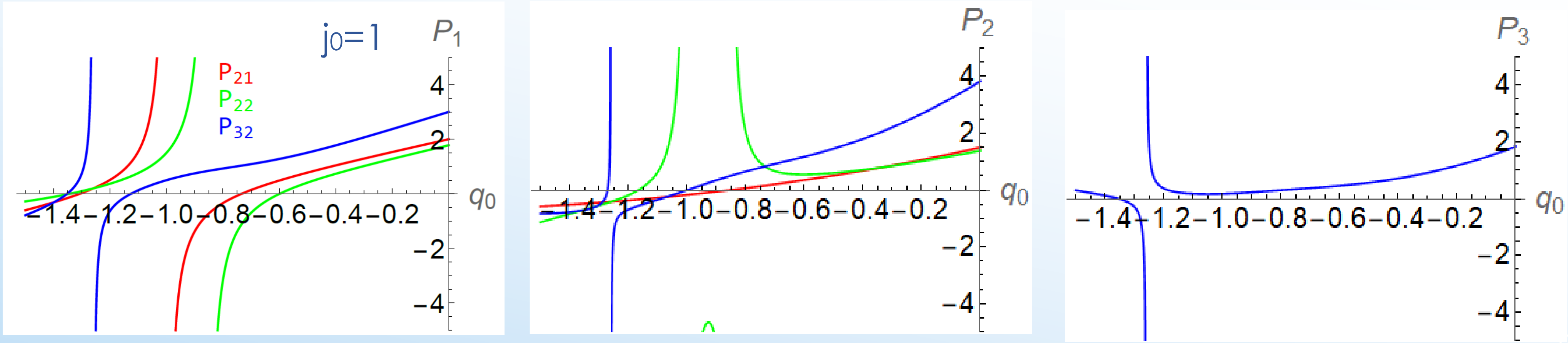
$$H(z) = H_0\{1 + (1+q_0)z + \frac{1}{2}(j_0 - q_0^2)z^2 - \frac{1}{6}[-3q_0^2 - 3q_0^3 + j_0(3+4q_0) + s_0]z^3 + \frac{1}{24}[-4j_0^2 + l_0 - 12q_0^2 - 24q_0^3 - 15q_0^4 + j_0(12 + 32q_0 + 25q_0^2) + 8s_0 + 7q_0 s_0]z^4 + \frac{1}{120}(p_0 + 15l_0 + 60(s_0 + j_0 - j_0^2 + 4j_0 q_0 + s_0 q_0^2 - q_0^2 - 3q_0^3) - 15s_0 j_0 + 11 l_0 q_0 + 105 s_0 q_0 - 70 j_0^2 q_0 + 375 j_0 q_0^2 + 210 j_0 q_0^3 - 225 q_0^4 - 105 q_0^5)z^5 + O(z^6)\}$$

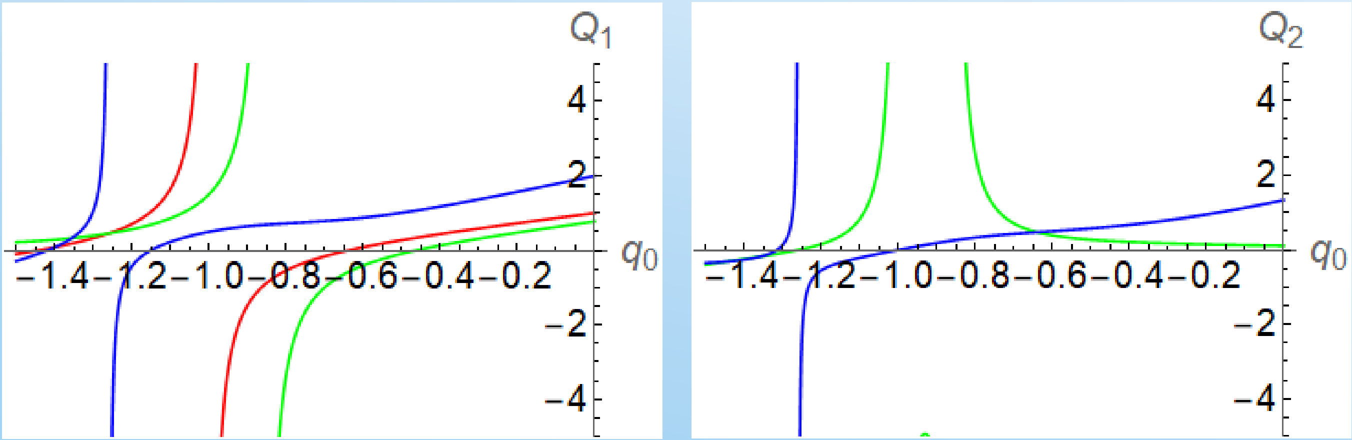


$q_0 = -0.55$
$j_0 = 1$
$l_0 = 0.685$
$s_0 = -0.35$
$p_0 = 1$

ΛCDM

Which is the best choice (stability, best performance-complexity ratio, ...) among these polynomials?





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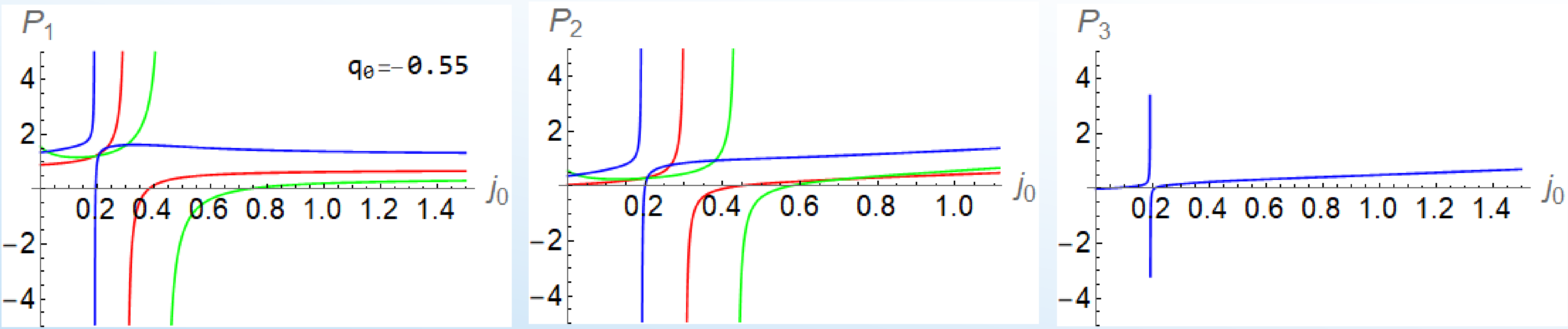

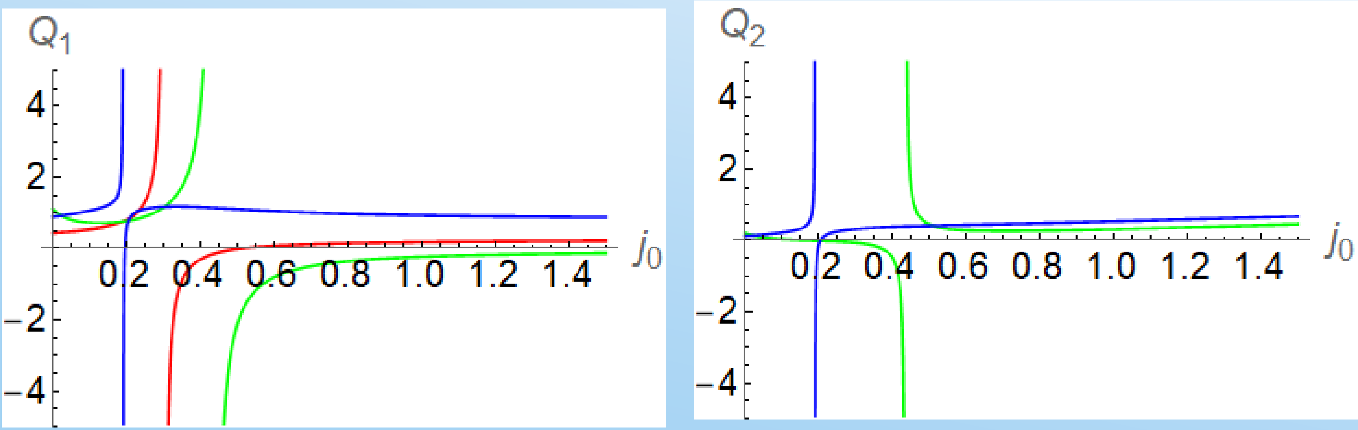

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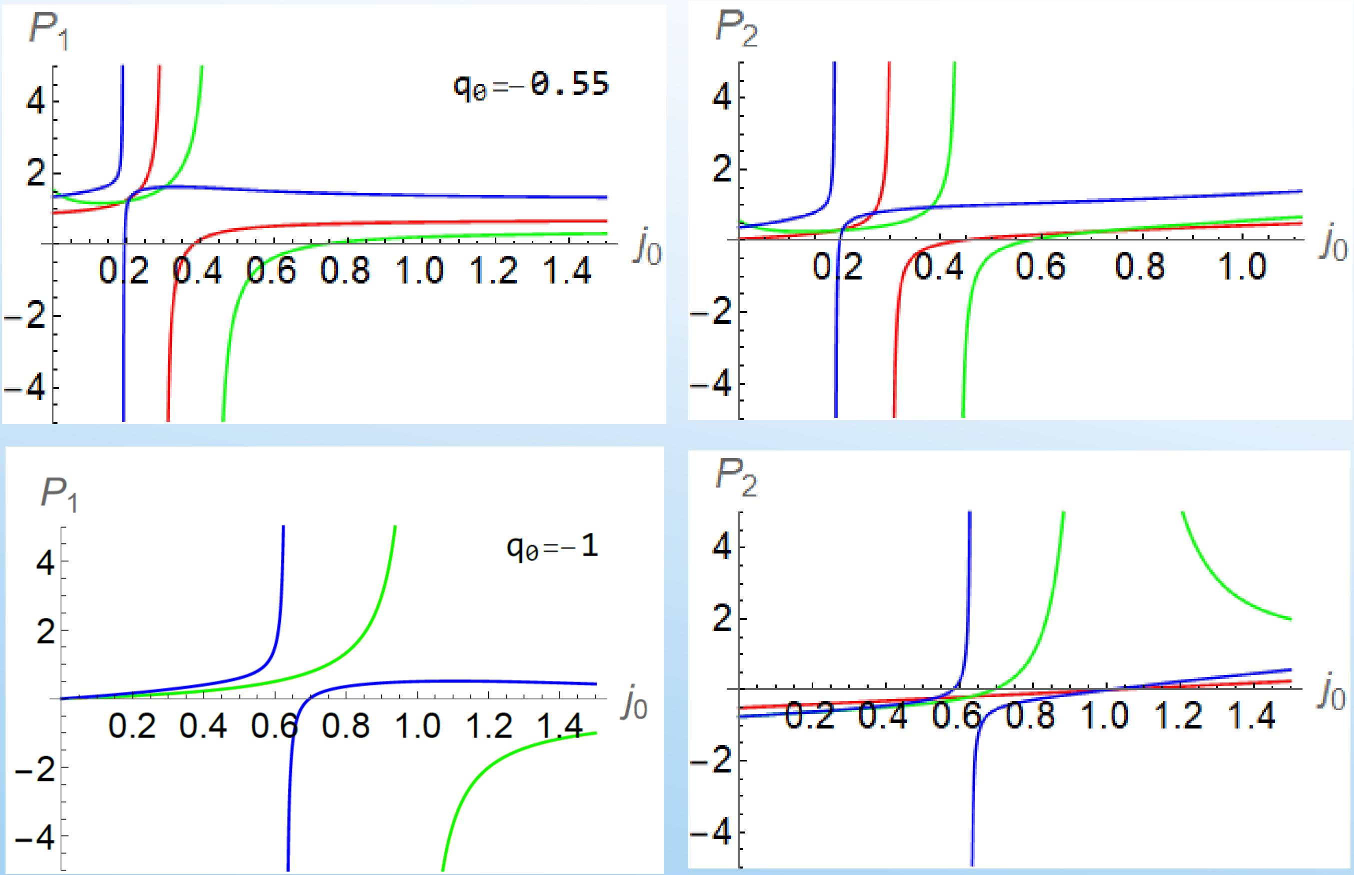

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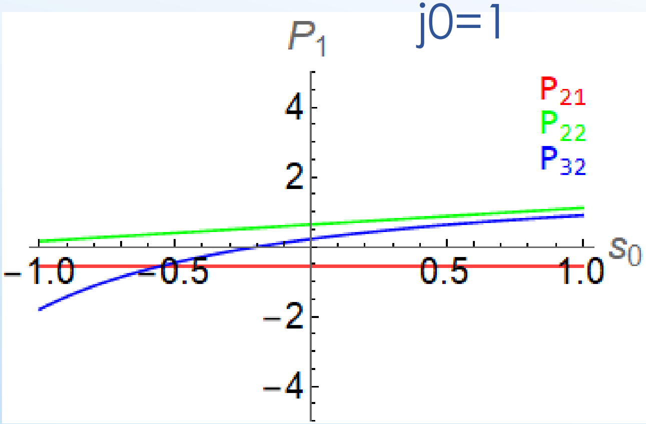
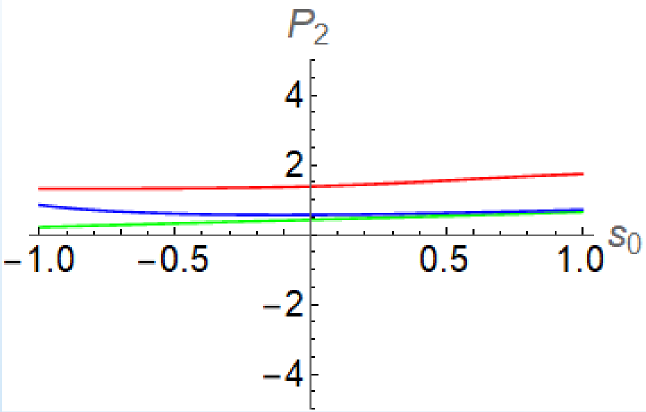
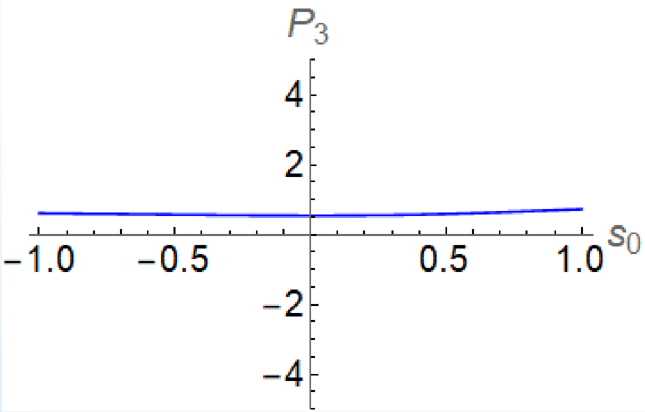
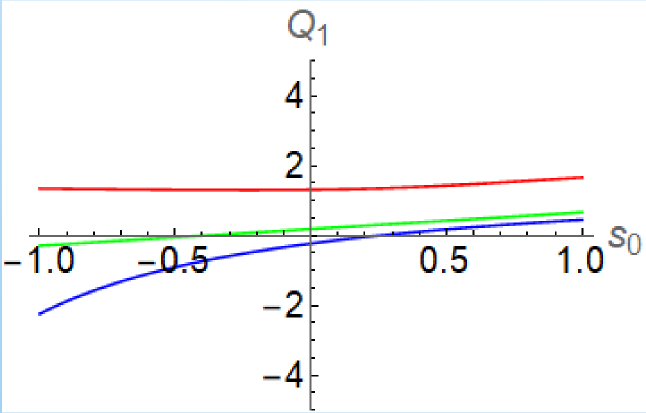
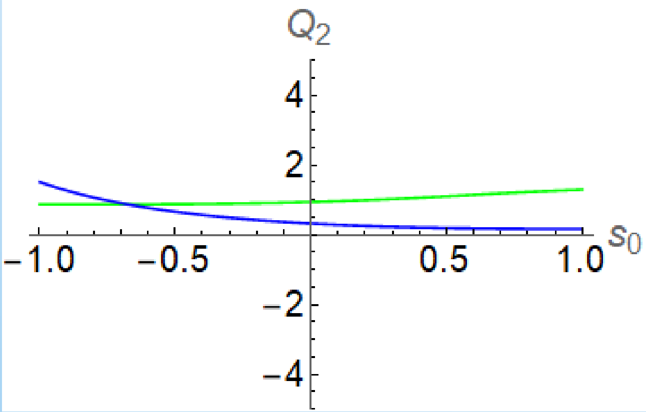
q0= -0.55
j0=1











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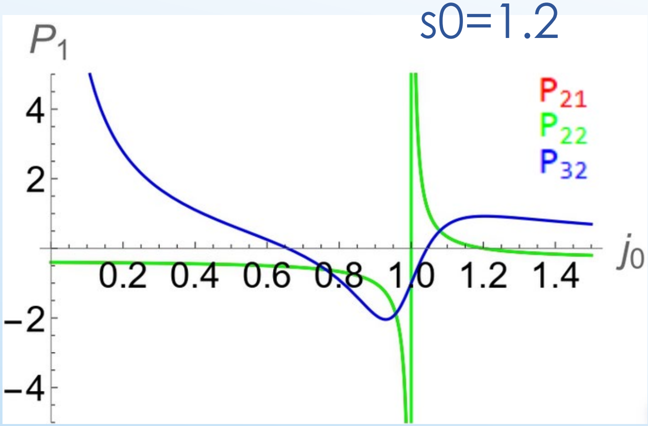
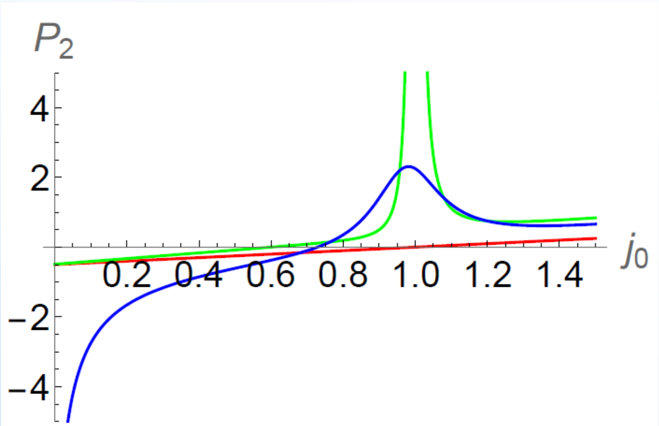
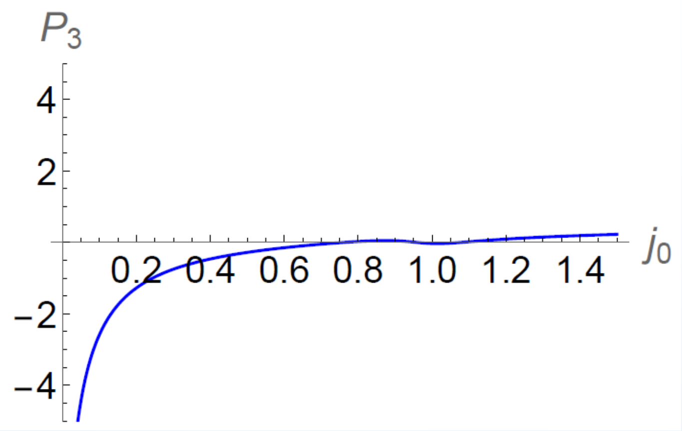
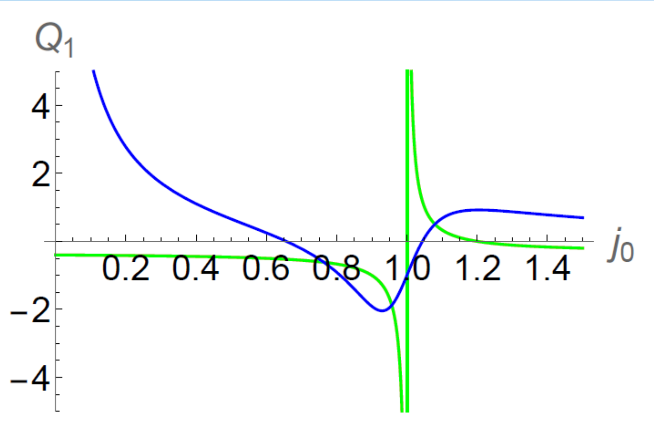
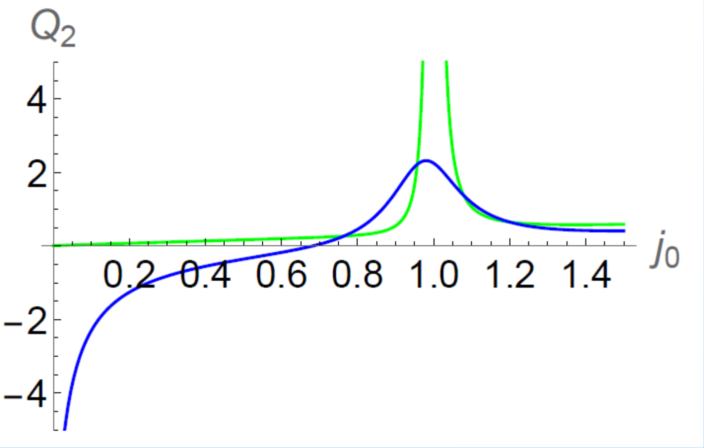
q0= -0.55
s0=1.2











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- Significant correlation between cosmographic parameter
- All works well at low redshift
- At high redshift, P21 and P32 converge better than P22
- P21 is simpler than P32, but P32 is more stable at very-high redshift
- P22 is more studied in literature

S. Capozziello, R. D'Agostino, and O. Luongo, Mon. Not. Roy. Astron. Soc. **494**, 2576 (2020), arXiv:2003.09341 [astro-ph.CO] .

S. Capozziello, R. D'Agostino, and O. Luongo, Int. J. Mod. Phys. D **28**, 1930016 (2019), arXiv:1904.01427 [gr-qc] .

S. Capozziello, R. D'Agostino, and O. Luongo, JCAP **05**, 008 (2018), arXiv:1709.08407 [gr-qc] .

M. Benetti and S. Capozziello, Journal of Cosmology and Astroparticle Physics **2019**, 008 (2019).

K. Dutta, Ruchika, A. Roy, A. A. Sen, and M. M. Sheikh-Jabbari, Gen. Rel. Grav. **52**, 15 (2020), arXiv:1808.06623 [astro-ph.CO] .

K. Dutta, A. Roy, Ruchika, A. A. Sen, and M. M. Sheikh-Jabbari, Phys. Rev. D **100**, 103501 (2019), arXiv:1908.07267 [astro-ph.CO] .

S. Capozziello, Ruchika, and A. A. Sen, Mon. Not. Roy. Astron. Soc. **484**, 4484 (2019), arXiv:1806.03943 [astro-ph.CO] .

Possible solutions:

- ✓ orthogonal polynomials of logarithmic functions
- ✓ Rational polynomials
- ➢ **Link cosmography with cosmology**

f(z)CDM model

Padè P_{22}

$$\frac{H(z)}{H_0} = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$$\frac{H(z)}{H_0} = \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda$$

ΛCDM

$$H(z)^2 = H_0^2 (\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_f f(z))$$

f(z)CDM

$$f(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

f(z)CDM model

$$\frac{H(z)}{H_0} = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$$q_0, j_0, s_0, ..$$

How can we find a relationship between the cosmographic coefficients?

$$\overline{q_0}, \overline{j_0}, \overline{s_0}, ..$$

$$H(z)^2 = H_0^2(\Omega_m(1+z)^3+\Omega_r(1+z)^4+\Omega_f f(z))$$

$$f(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

f(z)CDM model

$$\frac{H(z)}{H_0} = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

How can we find a relationship between the cosmographic coefficients?

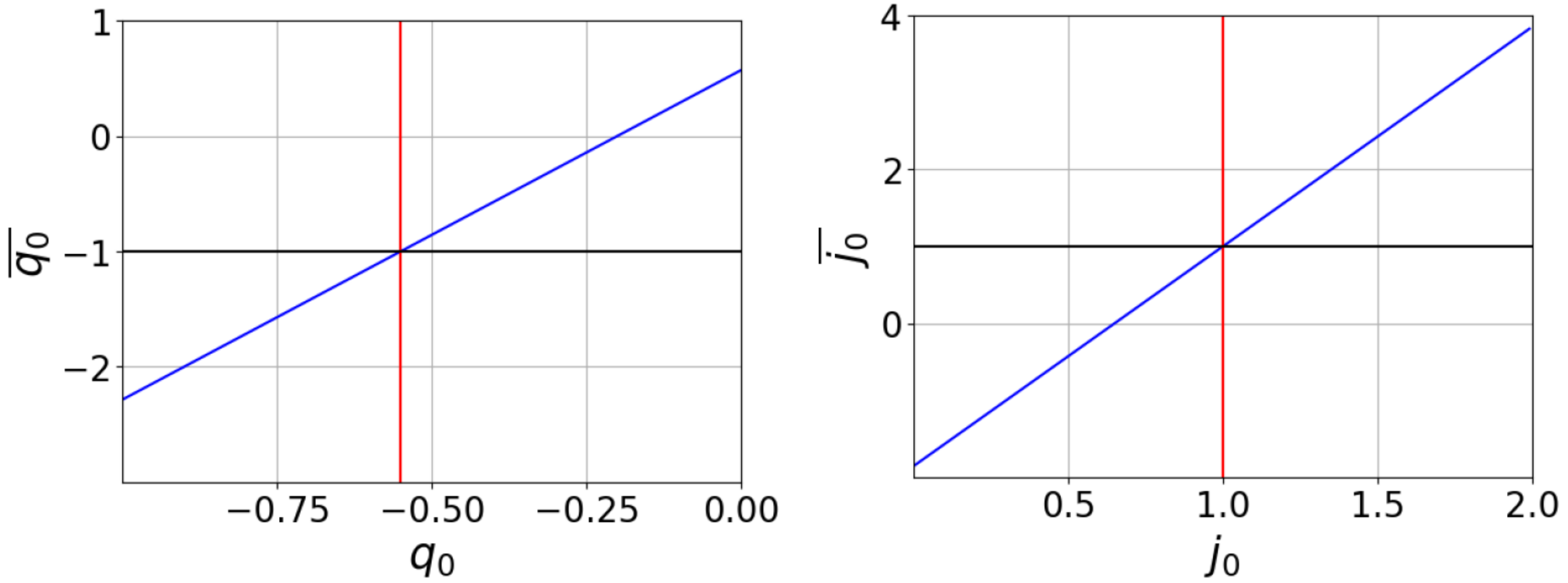
$$\frac{H'}{H_0}(0) = f_z'(0)$$

$$\frac{H''}{H_0}(0) = f_z''(0)$$

$$\frac{H^{(n+m)}}{H_0}(0) = f_z^{(n+m)}(0)$$

$$H(z)^2 = H_0^2(\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_f f(z))$$

$$f(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$



$$\overline{q_0} = \frac{-1 - 2q_0 + 2\Omega_m}{-1 + \Omega_m}$$

$$\overline{j_0} = \frac{3 + 4q_0^2 + q_0(8 - 12\Omega_m) - 2j_0(-1 + \Omega_m) - 12\Omega_m + 10\Omega_m^2}{(-1 + \Omega_m)^2}$$

PADÈ - P_{22}

Connecting early and late epochs by f(z)CDM cosmography
MB, S. Capozziello - JCAP 12 (2019) 008

- **Models**
- f(z)CDM model truncated to 2° order → q0
- f(z)CDM model truncated to 3° → q0 and j0
- f(z)CDM model truncated to 4° order → q0, j0, s0

$$H(t) \equiv \frac{1}{a}\frac{da}{dt}$$

$$q(t) \equiv -\frac{1}{aH^2}\frac{d^2a}{dt^2}$$

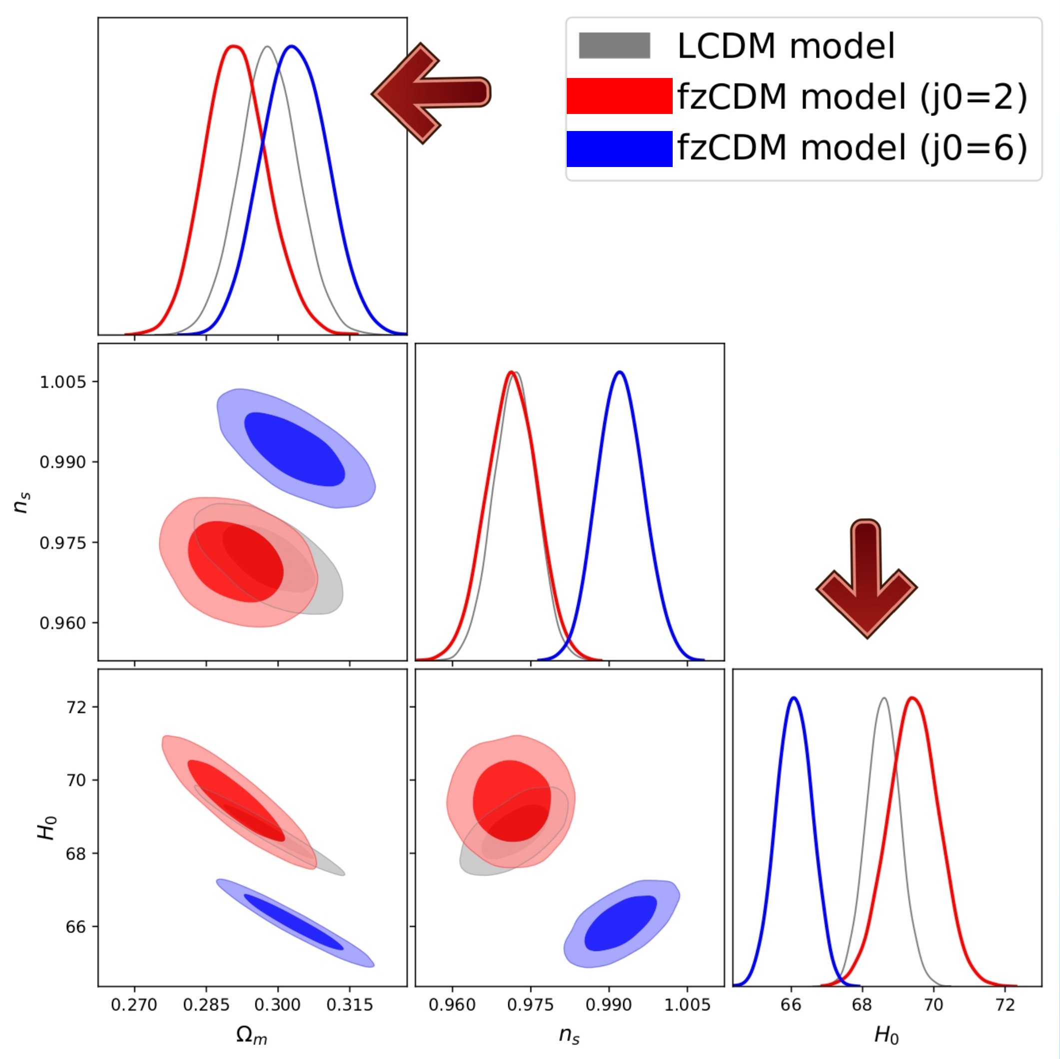
$$j(t) \equiv \frac{1}{aH^3}\frac{d^3a}{dt^3}$$

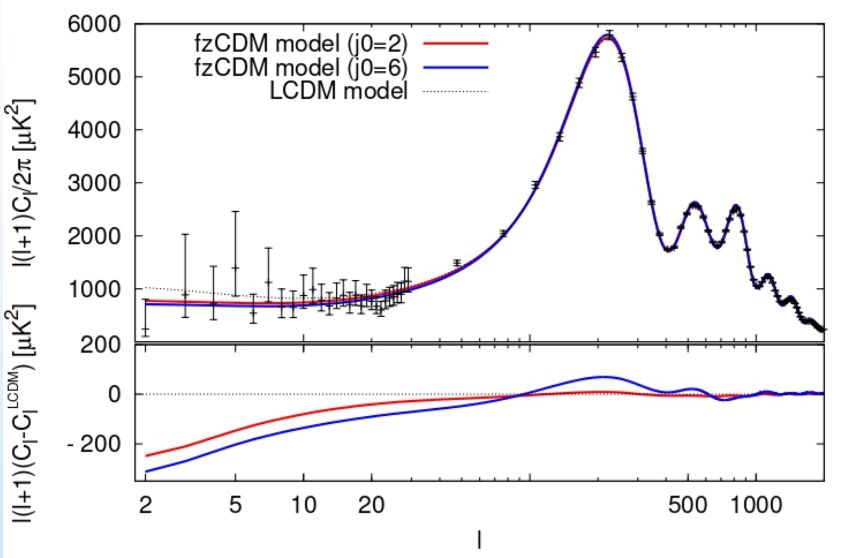
$$s(t) \equiv \frac{1}{aH^4}\frac{d^4a}{dt^4}$$

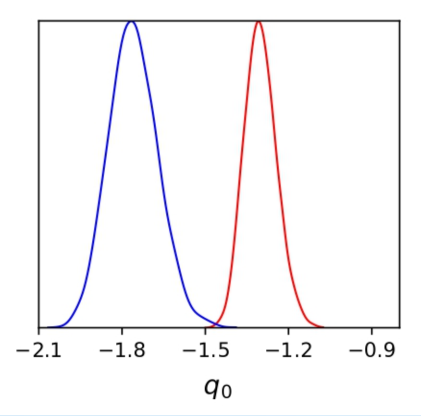
Base-dataset

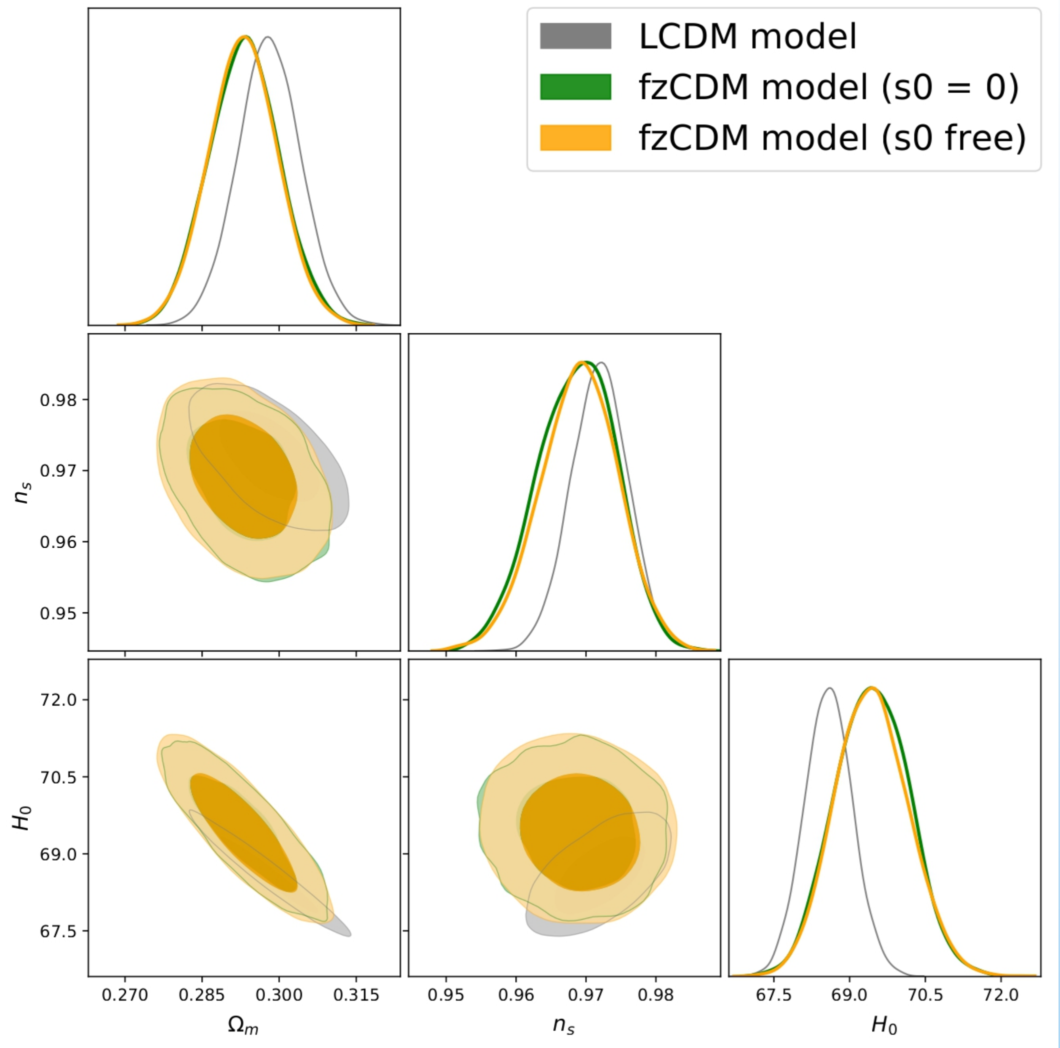
- Cosmic Microwave Background (CMB)
- Baryon Acoustic Oscillation (BAO)
- Supernovae Type Ia (Pantheon sample)
- Cosmic Clock (CC) data

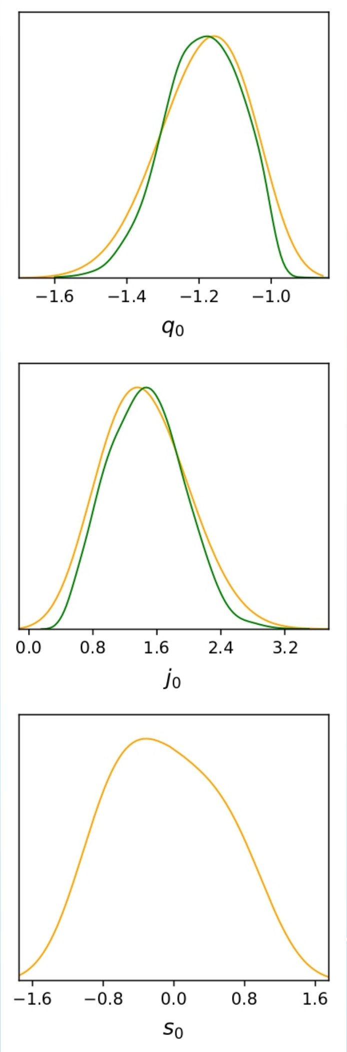
f(z)CDM model truncated to 2°











f(z)CDM model truncated t0 3°

$q_0 = -1.2 \pm 0.1$
$j_0 = 1.5 \pm 0.5$
$s_0 = 0$

f(z)CDM model truncated t0 4°

$q_0 = -1.2 \pm 0.1$
$j_0 = 1.5 \pm 0.5$
$s_0 = -0.1 \pm 0.6$

F(Z) CDM WITH PADÈ - P_{21} P_{22} P_{32}

Beyond ΛCDM with f (z)CDM - criticalities and solutions of Padè Cosmography. A. Turmina Petreca, MB, S. Capozziello Submitted in PDU, arxiv 2309.15711

$$H(z)^2 = H_0^2(\Omega_m(1+z)^3+\Omega_r(1+z)^4+\Omega_f f(z))$$

Base-dataset

- Cosmic Microwave Background (CMB)
- Baryon Acoustic Oscillation (BAO)
- Supernovae Type Ia (**Pantheon +**)
- Cosmic Clock (CC) data

$$P_{12}(z) = \frac{P_0 + P_1 z}{1 + Q_1 z + Q_2 z^2}$$

$$P_{22}(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$$P_{32}(z) = \frac{P_0 + P_1 z + P_2 z^2 + P_3 z^3}{1 + Q_1 z + Q_2 z^2}$$

$$P_{12}(z) = \frac{P_0 + P_1 z}{1 + Q_1 z + Q_2 z^2}$$

$$H^{P_{21}}(z) = H_0(3(j_0^2 - q_0^4) + 2q_0 s_0)z^2 + (-12q_0^3 + 2s_0)z(1+z) - 6q_0^2(1+z)^2 + 2j_0(3 + (6 + 7q_0)z + (3 + 7q_0 + q_0^2)z^2)/(-6q_0^3 z + 2s_0 z - 6q_0^2(1+z) + j_0(6 + (6 + 8q_0)z))$$

$$P_{22}(z) = \frac{P_0 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}$$

$$H^{P_{22}}(z) = H_0(24s_0 + 6l_0 z + 72s_0 z + 30j_0^3 z^2 + 6l_0 z^2 - 45q_0^6 z^2 + 48s_0 z^2 + 4s_0^2 z^2 - 90q_0^5 z(1+2z) - 18q_0^4(2 + 21z + 21z^2) + j_0^2(36 + 12(4 + 5q_0)z + (48 + 120q_0 - 5q_0^2)z^2) + 3q_0^3(-48 - 144z + (-96 + 5s_0)z^2) + 9q_0^2(-8 + 2(-8 + 3s_0)z + (-8 + l_0 + 12s_0)z^2) + 6q_0(l_0 z(1+2z) + s_0(4 + 23z + 23z^2)) + j_0(60q_0^4 z^2 + 90q_0^3 z(1+2z) + 6q_0^2(4 + 77z + 77z^2) + 7q_0(24 + 72z + (48 + 5s_0)z^2) + 3(24 + 4(12 + s_0)z + (24l_0 + 8s_0)z^2))/(24s_0 + 6l_0 z - 54q_0^5 z + 48s_0 z + 12j_0^3 z^2 - 9q_0^6 z^2 + 4s_0^2 z^2 - 18q_0^4(2 + 11z) + 3q_0^2(-24 + 2(-12 + 5s_0)z + l_0 z^2) + j_0^2(36 + 12(1 + 2q_0)z - 23q_0^2 z^2) - 3q_0^3(48 + 72z + s_0 z^2) + 6q_0(l_0 z + s_0(4 + 15z)) + j_0(72 + 66q_0^3 z + 12(6 + s_0)z - 3l_0 z^2 + 24q_0^4 z^2 + 6q_0^2(4 + 45z) + q_0(168 + 264z + 11s_0 z^2))))$$

$$P_{32}(z) = \frac{P_0 + P_1 z + P_2 z^2 + P_3 z^3}{1 + Q_1 z + Q_2 z^2}$$

$$\begin{aligned}
H^{P_{32}}(z) = cH_0\{&-540q_0^6 - 180q_0^3 s_0 + 240s_0^2 + 36p_0q_0^2 z - 1620q_0^6 z - 1620q_0^7 z - 540q_0^3 s_0 z - 180q_0^4 s_0 z + \\
&720s_0^2 z + 660q_0 s_0^2 z + 72p_0q_0^2 z^2 + 72p_0q_0^3 z^2 - 1620q_0^6 z^2 - 3240q_0^7 z^2 - 1215q_0^8 z^2 - 12p_0 s_0 z^2 - \\
&540q_0^3 s_0 z^2 - 360q_0^4 s_0 z^2 + 360q_0^5 s_0 z^2 + 720s_0^2 z^2 + 1320q_0 s_0^2 z^2 + 315q_0^2 s_0^2 z^2 + 36p_0q_0^2 z^3 + 72p_0q_0^3 z^3 \\
&+ 18p_0q_0^4 z^3 - 540q_0^6 z^3 - 1620q_0^7 z^3 - 1215q_0^8 z^3 - 135q_0^9 z^3 - 12p_0 s_0 z^3 - 12p_0q_0 s_0 z^3 - 180q_0^3 s_0 z^3 \\
&- 180q_0^4 s_0 z^3 + 360q_0^5 s_0 z^3 + 270q_0^6 s_0 z^3 + 240s_0^2 z^3 + 660q_0 s_0^2 z^3 + 315q_0^2 s_0^2 z^3 - 45q_0^3 s_0^2 z^3 - 40s_0^3 z^3 \\
&+ 15l_0^2 z^2(1 + z + q_0 z) + 60j_0^4 z^2(10 + (10 + 9q_0)z) + 10j_0^3(72 + 12(18 + 19q_0)z + 3(72 + 152q_0 \\
&+ 29q_0^2)z^2 + (72 + 228q_0 + 87q_0^2 - 82q_0^3 + 3s_0)z^3) + 6l_0(-12q_0^5 z^3 + 12q_0^4 z^2(1 + z) + 23q_0 s_0 z^2(1 \\
&+ z) + 66q_0^3 z(1 + z)^2 + 10s_0 z(1 + z)^2 + 3q_0^2(10 + 30z + 30z^2 + (10 + s_0)z^3)) + 6j_0(45q_0^7 z^3 + \\
&600q_0^6 z^2(1 + z) - 105q_0^3 s_0 z^2(1 + z) + 780q_0^5 z(1 + z)^2 - 120q_0^4(-2 - 6z - 6z^2 + (-2 + s_0)z^3) + \\
&l_0(-30 - 2(45 + 28q_0)z + (-90 - 112q_0 + 41q_0^2)z^2 + (-30 - 56q_0 + 41q_0^2 + 33q_0^3 + 10s_0)z^3) - \\
&2z(1 + z)(-25s_0^2 z + 3p_0(1 + z)) + 2q_0^2 z(-p_0 z^2 + 95s_0(1 + z)^2) + 2q_0(10s_0^2 z^3 - 7p_0 z^2(1 + z) + \\
&55s_0(1 + z)^3)) + 3j_0^2(75q_0^5 z^3 - 1125q_0^4 z^2(1 + z) - 1580q_0^3 z(1 + z)^2 + 20q_0^2(-23 - 69z - 69z^2 + \\
&(-23 + 10s_0)z^3) + 2q_0 z^2(-13l_0 z + 225s_0(1 + z)) + 2z(50s_0(1 + z)^2 - z(3p_0 z + 35l_0(1 + z)))))\}\times \\
&\times\{3(-180q_0^6 - 60q_0^3 s_0 + 80s_0^2 + 12p_0q_0^2 z - 360q_0^6 z - 360q_0^7 z - 120q_0^3 s_0 z + 160s_0^2 z + 140q_0 s_0^2 z + \\
&80j_0^4 z^2 + 5l_0^2 z^2 + 12p_0q_0^2 z^2 + 12p_0q_0^3 z^2 - 180q_0^6 z^2 - 360q_0^7 z^2 - 135q_0^8 z^2 - 4p_0 s_0 z^2 - 60q_0^3 s_0 z^2 + \\
&90q_0^5 s_0 z^2 + 80s_0^2 z^2 + 140q_0 s_0^2 z^2 + 5q_0^2 s_0^2 z^2 + 40j_0^3(6 + (12 + 13q_0)z + (6 + 13q_0 + 3q_0^2)z^2) + \\
&l_0(-18q_0^4 z^2 + 26q_0 s_0 z^2 + 72q_0^3 z(1 + z) + 20s_0 z(1 + z) + 60q_0^2(1 + z)^2) - 5j_0^2(8l_0 z^2 + 95q_0^4 z^2 - \\
&48q_0 s_0 z^2 + 224q_0^3 z(1 + z) - 20s_0 z(1 + z) + 92q_0^2(1 + z)^2) + 2j_0(225q_0^6 z^2 - 115q_0^3 s_0 z^2 + 540q_0^5 z(1 + \\
&z) + 80q_0^2 s_0 z(1 + z) + 240q_0^4(1 + z)^2 - 6z(p_0 + p_0 z - 5s_0^2 z) + l_0(-30 - 2(30 + 13q_0)z + (-30 - \\
&26q_0 + 37q_0^2)z^2) + 2q_0(-4p_0 z^2 + 55s_0(1 + z)^2)))\}^{-1}
\end{aligned}$$

Beyond ΛCDM with f (z)CDM - criticalities and solutions of Padè Cosmography.
A. Turmina Petreca, MB, S. Capozziello
Submitted in PDU
arxiv 2309.15711

	P_{21}	P_{22}	P_{32}
$100\Omega_b h^2$	2.240 ± 0.014	2.244 ± 0.016	2.240 ± 0.014
$\Omega_c h^2$	0.1195 ± 0.0010	0.1187 ± 0.0011	0.1196 ± 0.0012
Ω_m	0.3177 ± 0.0073	0.3207 ± 0.0086	0.3233 ± 0.0069
n_s	0.9661 ± 0.0038	0.9673 ± 0.0041	0.9656 ± 0.0045
H_0	67.00 ± 0.76	66.51 ± 0.84	66.45 ± 0.70
σ_8	0.8065 ± 0.0087	0.7980 ± 0.0084	0.8035 ± 0.0100
$\overline{q_0}$	-0.86 ± 0.06	-1.06 ± 0.12	-0.70 ± 0.09
$\overline{j_0}$	0.45 ± 0.17	1.71 ± 0.38	0.32 ± 0.24

Base-dataset

- Cosmic Microwave Background (CMB)
- Baryon Acoustic Oscillation (BAO)
- Supernovae Type Ia (Pantheon sample)
- Cosmic Clock (CC) data

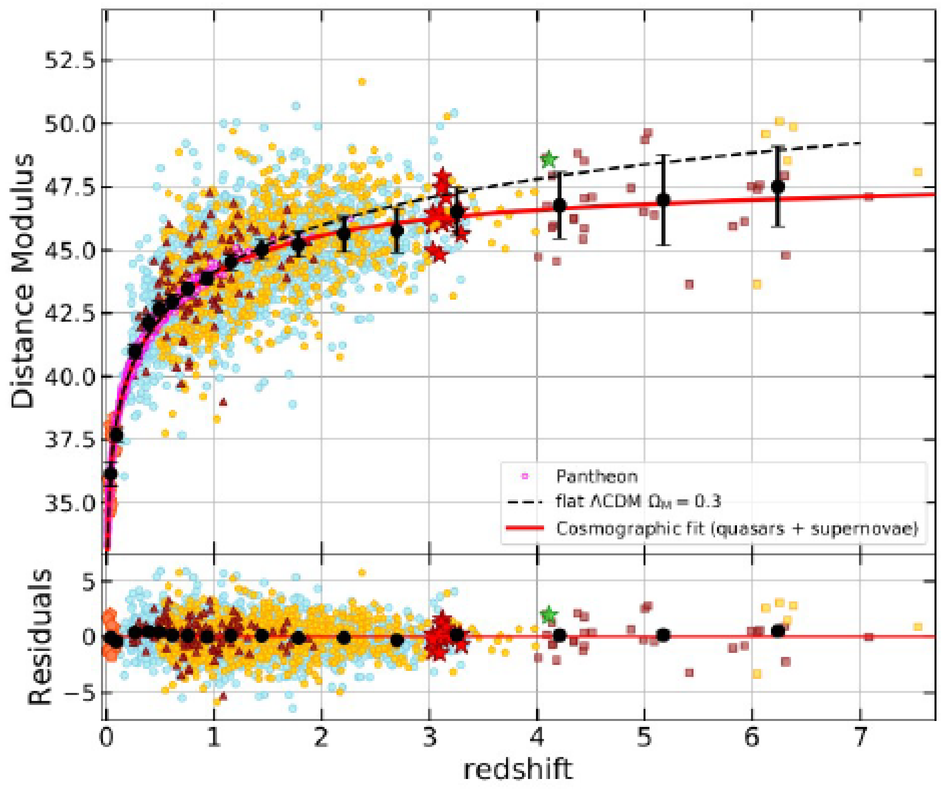
	P_{21}	P_{22}	P_{32}
$\overline{q_0}$	-0.79	-1.03	-0.69
$\overline{j_0}$	0.22	1.59	0.28
Ω_m	0.3126	0.3288	0.3159
q_0	-0.46	-0.52	-0.42
j_0	0.73	1.19	0.75

What next?

- Using P32, test QSO data from Lusso-Risaliti gold sample (2036 sources covering up to z = 7.54)

- Orthogonalyse Padè (?!?)

- Any other ideas??



Lusso et al. 2020

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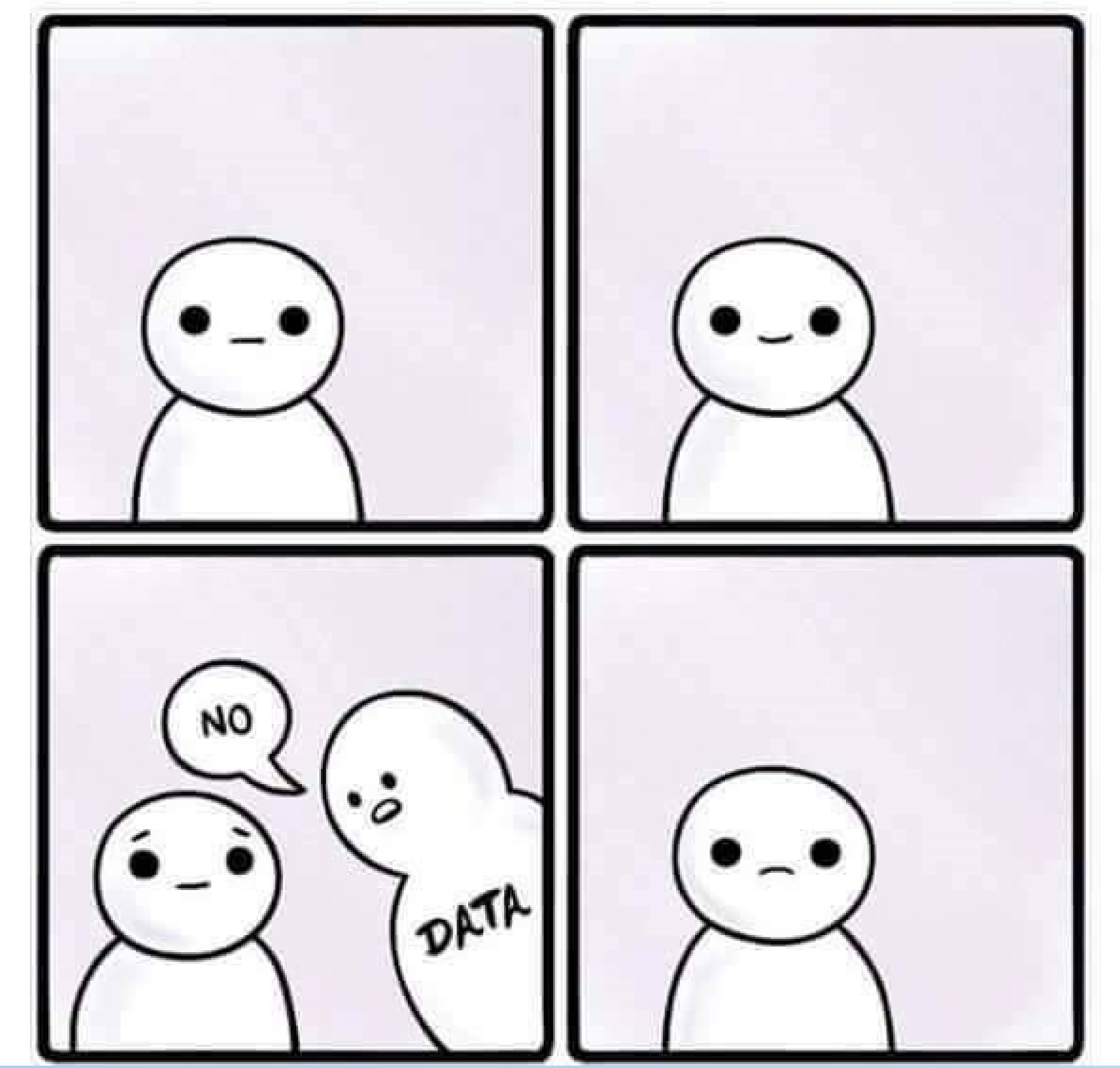

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