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Decoding a black hole metric from the interferometric pattern of relativistic images

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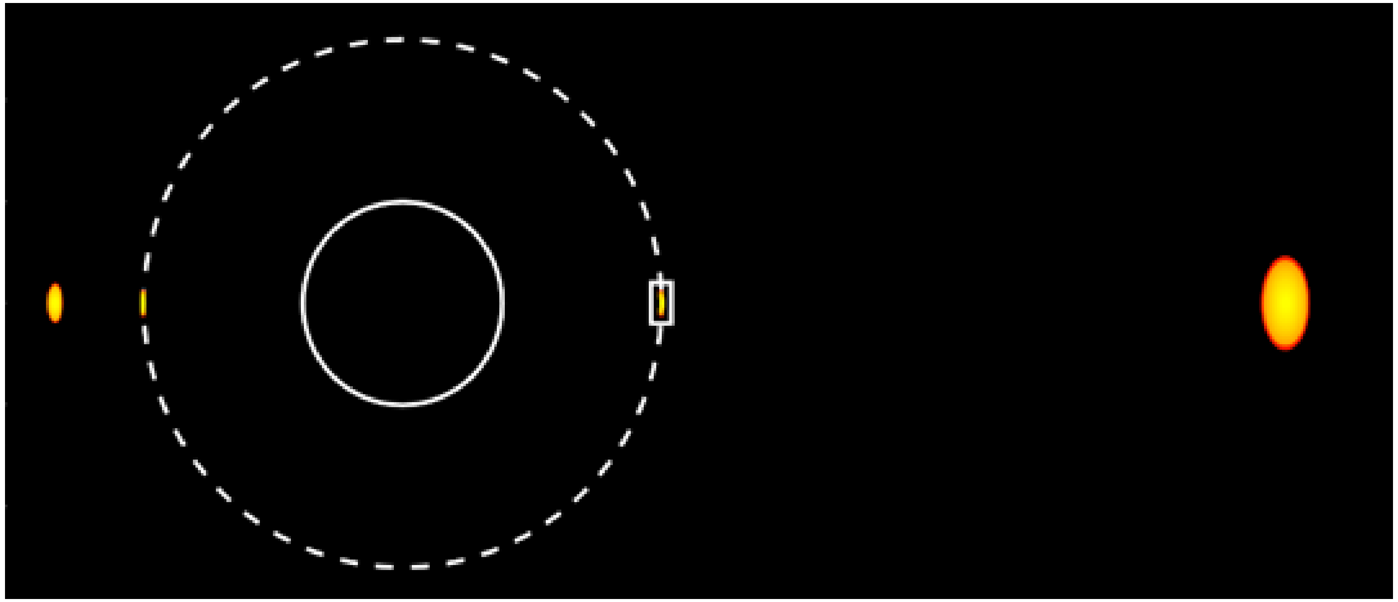

Annual Meeting QGSKY – Quantum Universe
Genova, 5 – 6 ottobre 2023

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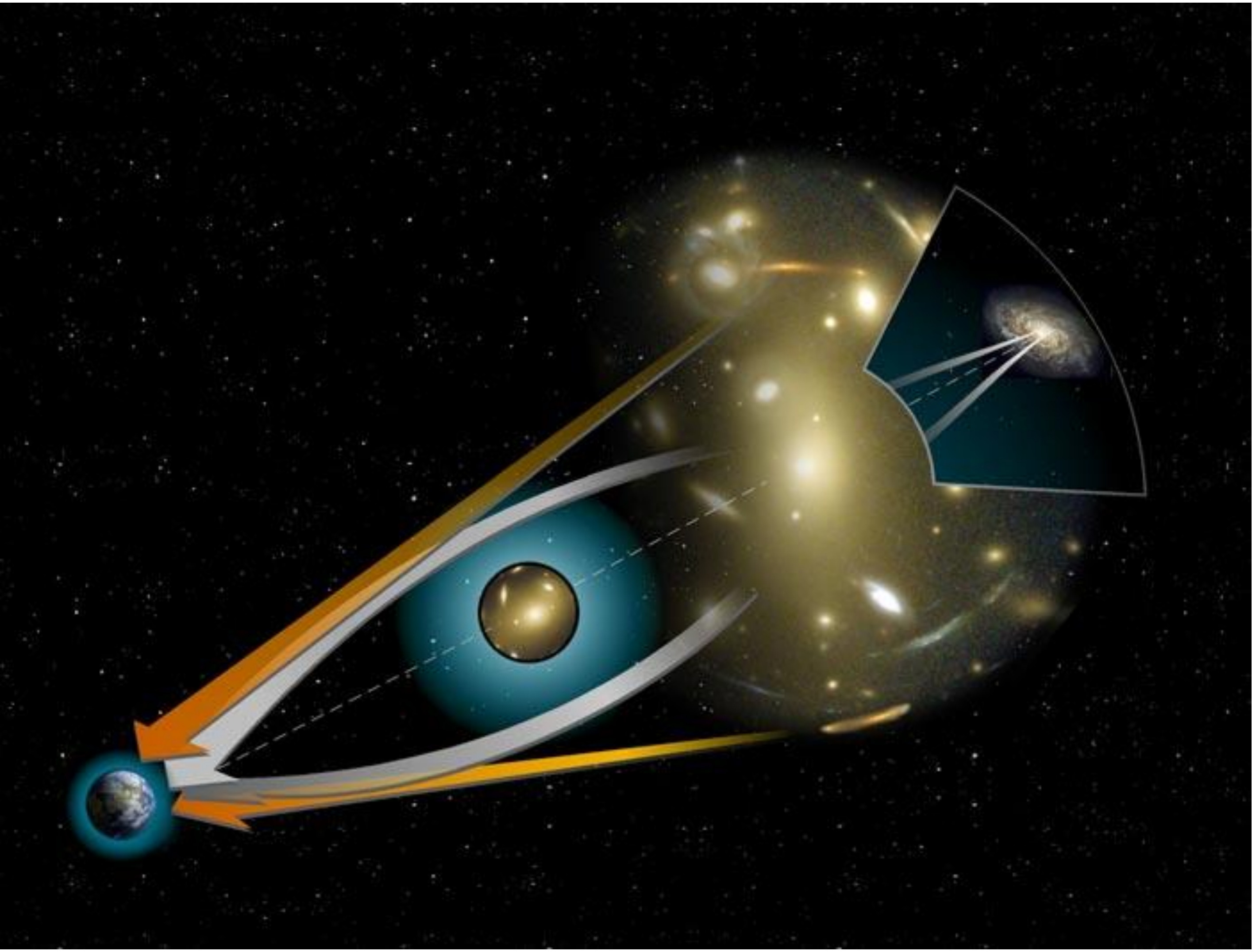
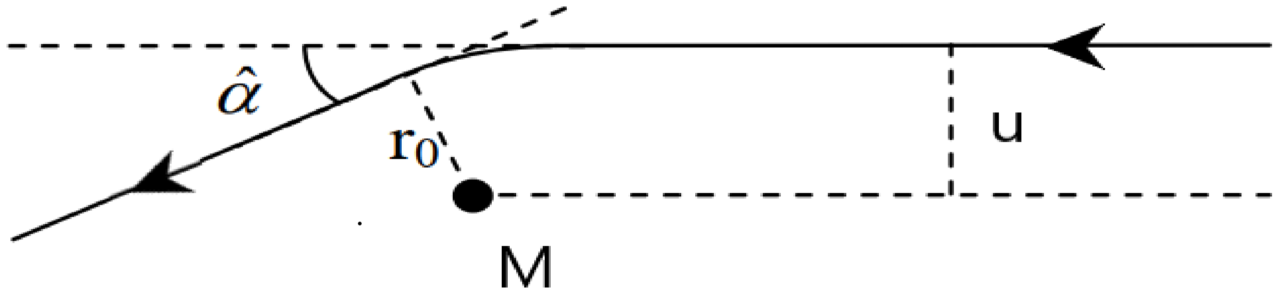


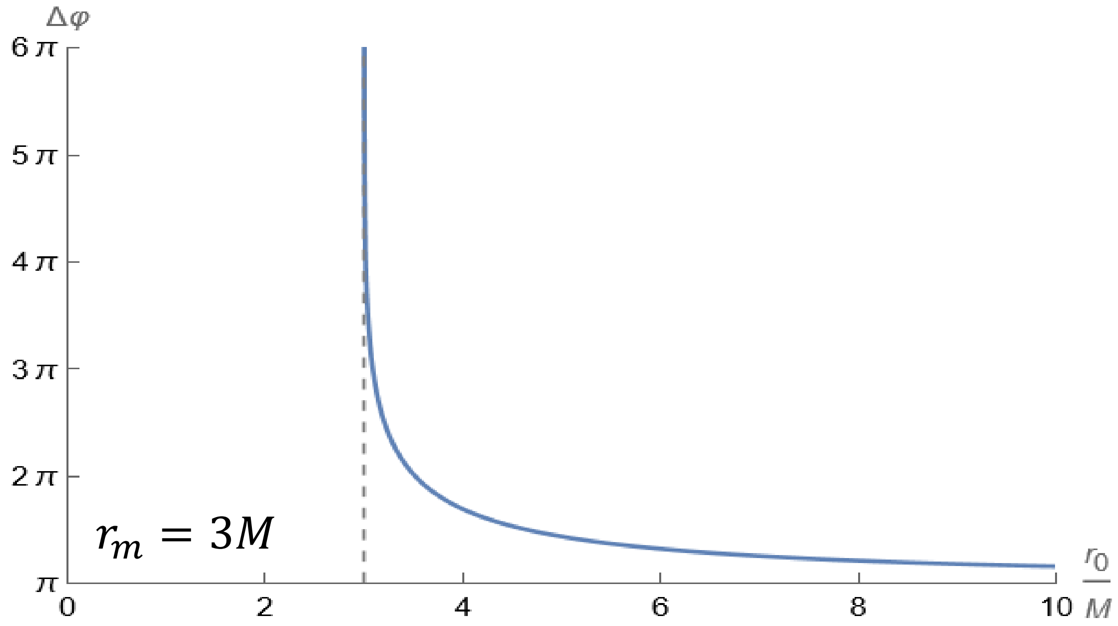
Gravitational lens-full - Lente gravitazionale - Wikipedia

- The gravitational lensing is the effect of deflection of light rays due to the presence of a mass.
- After the explanation of the precession of the perihelion of Mercury, this is the second experimental confirmation of GR.
- It’s a powerful instrument in different fields of Astrophysics.
- The entire theory of lensing is developed within the limit of small deflection but lately, gravitational lensing of compact objects is gaining a great importance.

Gravitational lensing 2

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2\vartheta\, d\phi^2$$





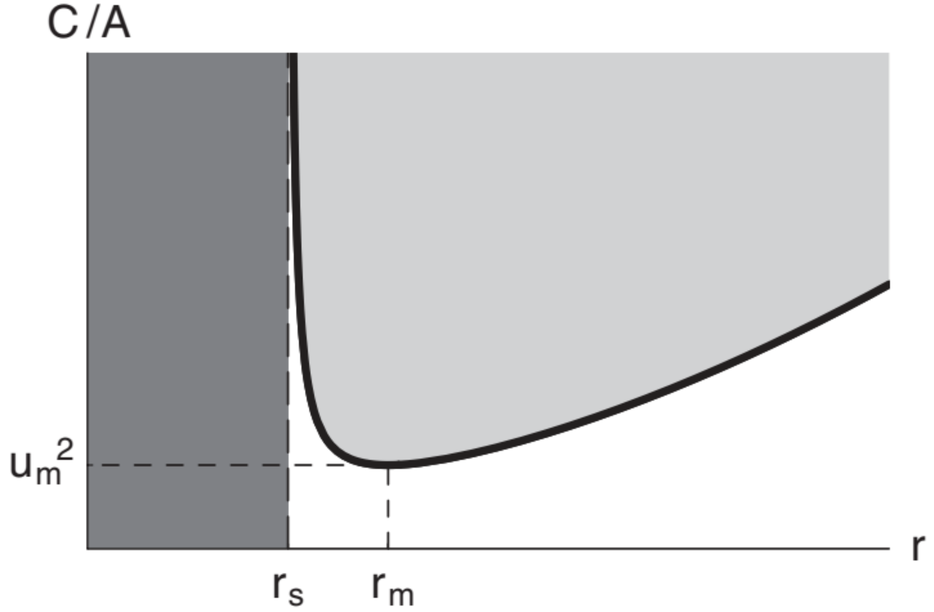
$$\Delta\phi = 2\int_{r_0}^{+\infty} \frac{1}{r\sqrt{\frac{r^2}{r_0^2}\left(1 - \frac{2M}{r_0}\right) - \left(1 - \frac{2M}{r}\right)}}\, dr$$

$$r_0 >> 2M \qquad \Delta\phi = \pi + \frac{4M}{r_0}$$

Gravitational lensing 3

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)(d\vartheta^2 + \sin^2\vartheta d\phi^2)$$

$$\Delta\phi = \int_{r_0}^{r_S} u\sqrt{\frac{B(r)}{C(r)\left[C(r)/A(r) - u^2\right]}}dr + \int_{r_0}^{r_o} u\sqrt{\frac{B(r)}{C(r)\left[C(r)/A(r) - u^2\right]}}dr$$



Bozza, V., and G. Scarpetta. "Strong deflection limit of black hole gravitational lensing with arbitrary source distances." *Physical Review D* 76.8 (2007): 083008.

$$u_m^2 = \frac{C(r_m)}{A(r_m)}$$

The deflection angle diverges

Strong deflection limit 1

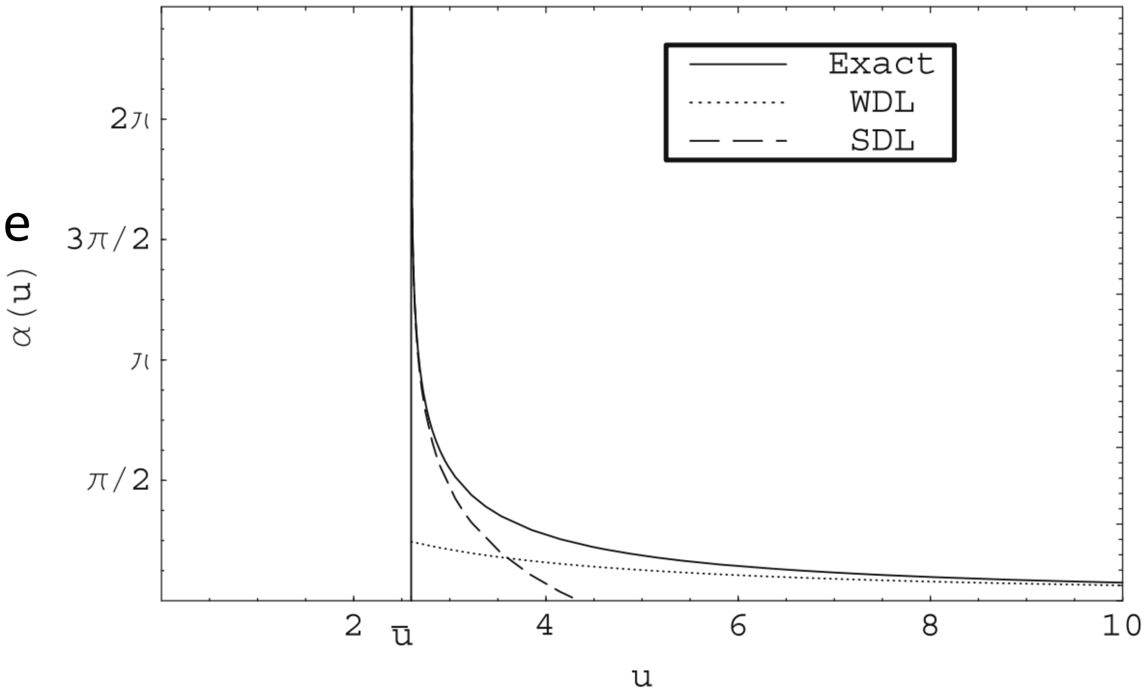
The SDL is a procedure that allows to provide a simple analytical solution of the deflection angle near the divergence.

$$r_0 = r_m(1+\delta) \quad \wedge \quad u = u_{cr}(1+\epsilon) \quad \Rightarrow \quad \Delta\phi = -a \log \epsilon + \mathrm{b}$$

Let us indicate with n the number of half orbit of the black hole performed by the photon

$$\Delta\phi = \phi_o - \phi_s + n\pi$$

We only consider relativistic images with $n \geq 2$



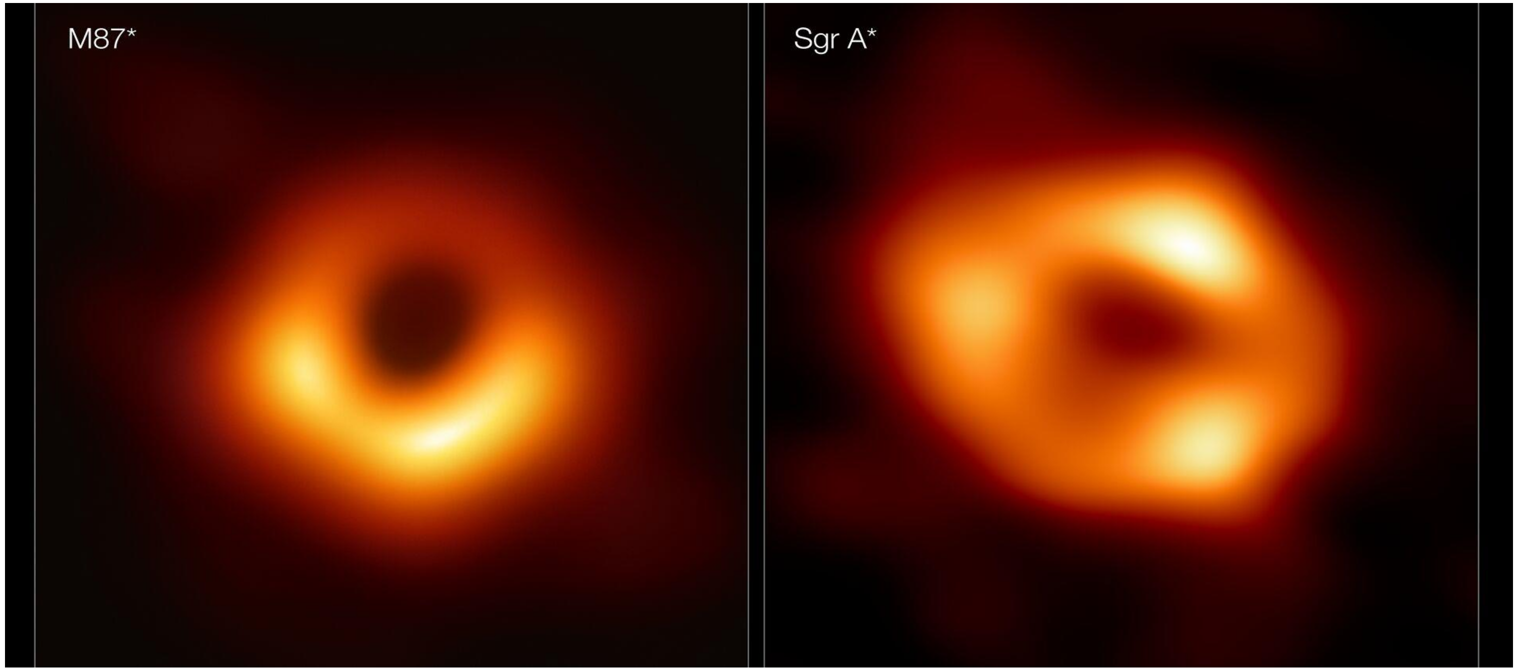
Bozza, V. (2010). Gravitational lensing by black holes. *General Relativity and Gravitation*, *42*, 2269-2300.

Strong deflection limit 2

An observer at infinity will measure the angular separation between the direction of arrival of the photon and the direction of the black hole simply as $\theta = {}^{u}/_{r_o}$

$$\theta_n = \frac{u_{cr}}{r_o}\left(1 + \frac{2\beta_m \eta_s}{u_{cr}^2}\ e^{\frac{b_s + b_o - \Delta\phi}{a}}\right) = \theta_m\left(1 + \frac{2\beta_m \eta_s}{u_{cr}^2}\ e^{\frac{b_s + b_o - \phi_o + \phi_s - n\pi}{a}}\right)$$

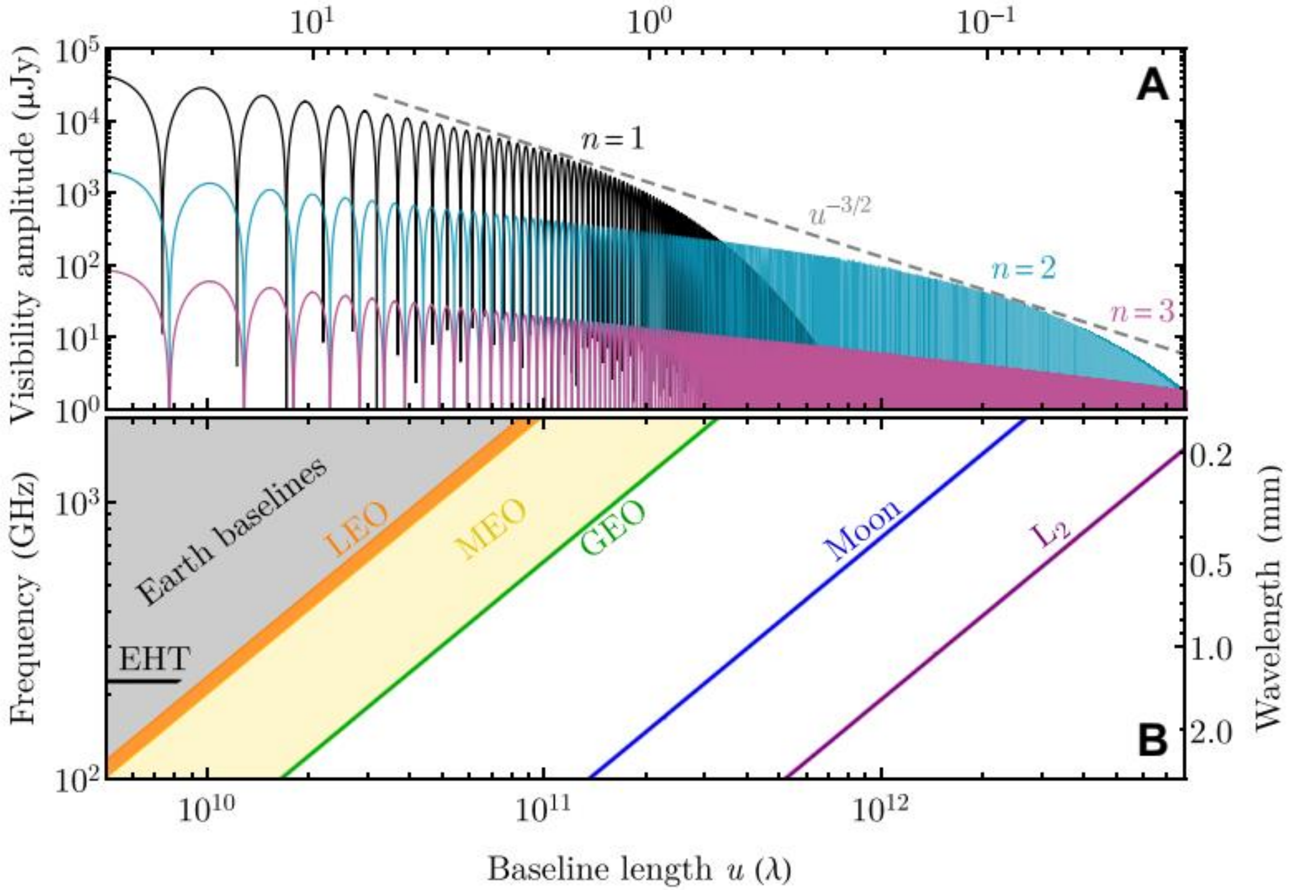
All images are created externally a region of angular radius θ_m called the shadow since this regione will remain obscure.



ESOblog - Spot the difference: Imaging Sagittarius A* and M87* | ESO

Universal interferometric signature of a black hole's photon rings

Johnson, Michael D., et al. "Universal interferometric signatures of a black hole's photon ring." *Science advances* 6.12 (2020): eaaz1310



$$V(\mathbf{u}) = \int I(\mathbf{x})\, e^{-2\pi i \mathbf{u}\cdot\mathbf{x}}\, d^2\mathbf{x}$$

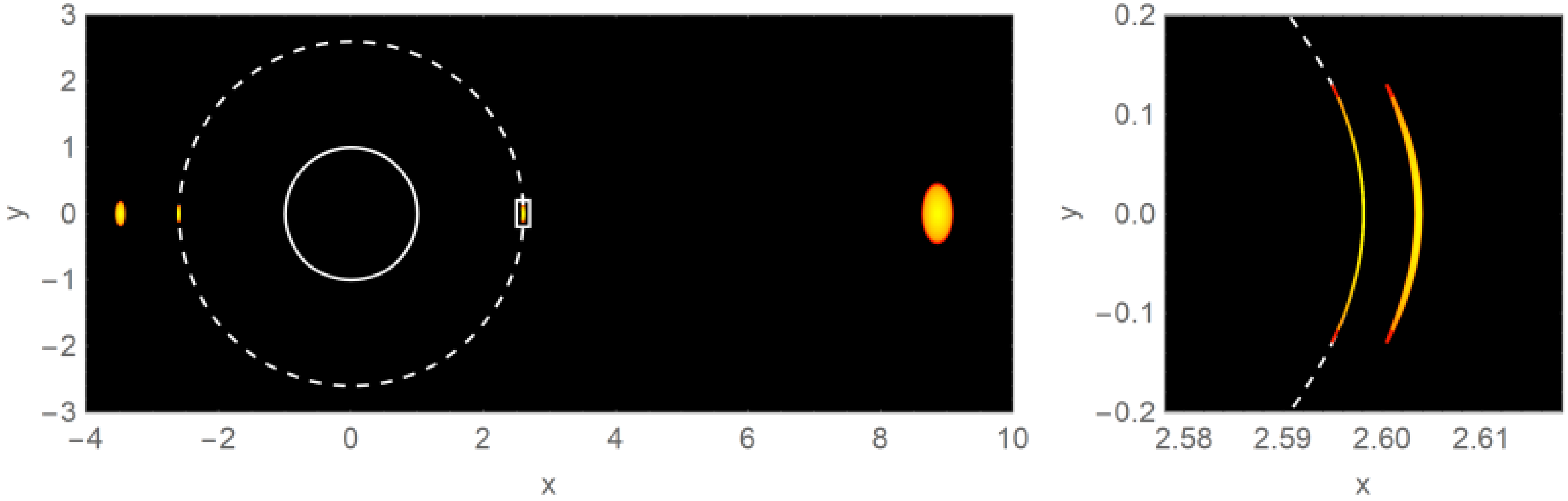
$$I(\rho, \varphi_\rho) = \frac{1}{\pi d}\delta\left(\rho - \frac{d}{2}\right) \sum_{m=-\infty}^{\infty} \beta_m e^{im\varphi_\rho}$$

$$V(u, \varphi_u) = \sum_{m=-\infty}^{\infty} \beta_m J_m(\pi d u)\, e^{im(\varphi_u - \pi/2)}$$

Interferometric signature of relativistic images 1

SCAN ME

Aratore, F., & Bozza, V. (2021). Decoding a black hole metric from the interferometric pattern of the relativistic images of a compact source. *Journal of Cosmology and Astroparticle Physics*, *2021*(10), 054.



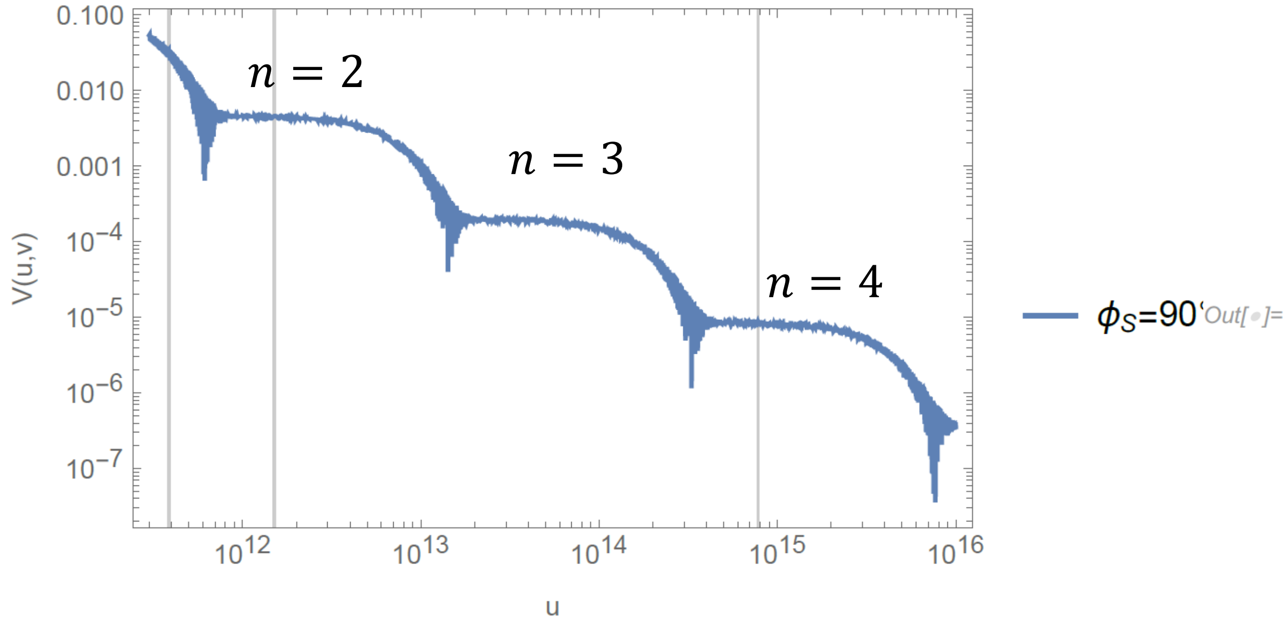
SCAN ME

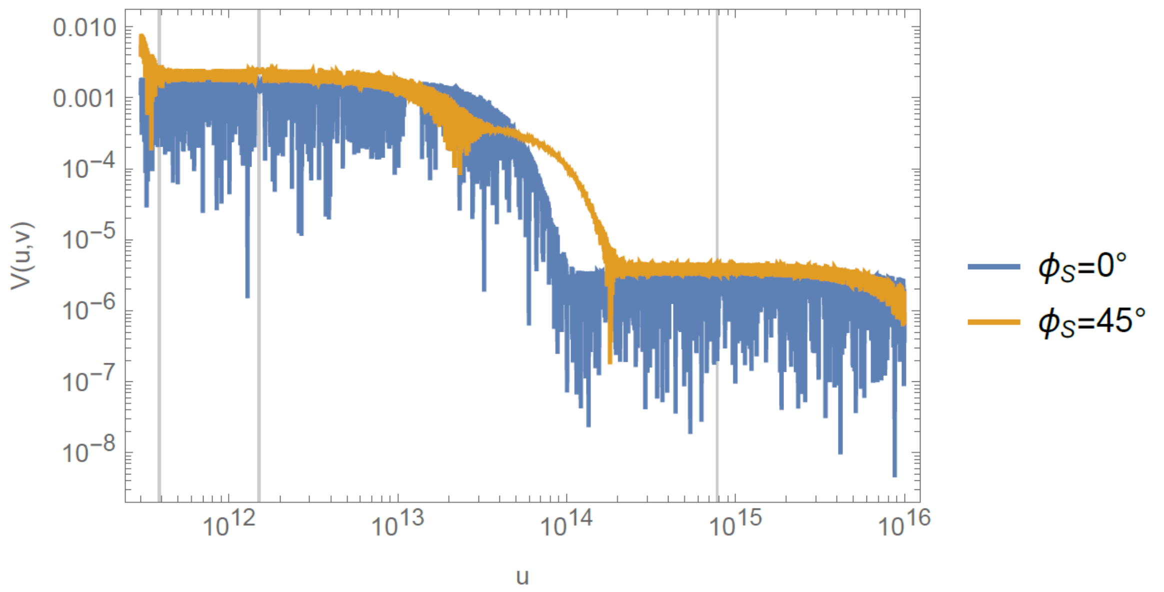
Interferometric signature of relativistic images 2

Aratore, F., & Bozza, V. (2021). Decoding a black hole metric from the interferometric pattern of the relativistic images of a compact source. *Journal of Cosmology and Astroparticle Physics*, *2021*(10), 054.

$$I(x,y) = I_0 \sum_{p=\pm 1} \sum_{n=1}^{+\infty} e^{-\frac{(x - p\theta_{n,p})^2}{2\Delta\theta_{n,p}^2}} e^{-\frac{y^2}{2\theta_{n,p}^2 \Delta\vartheta_S^2}}$$

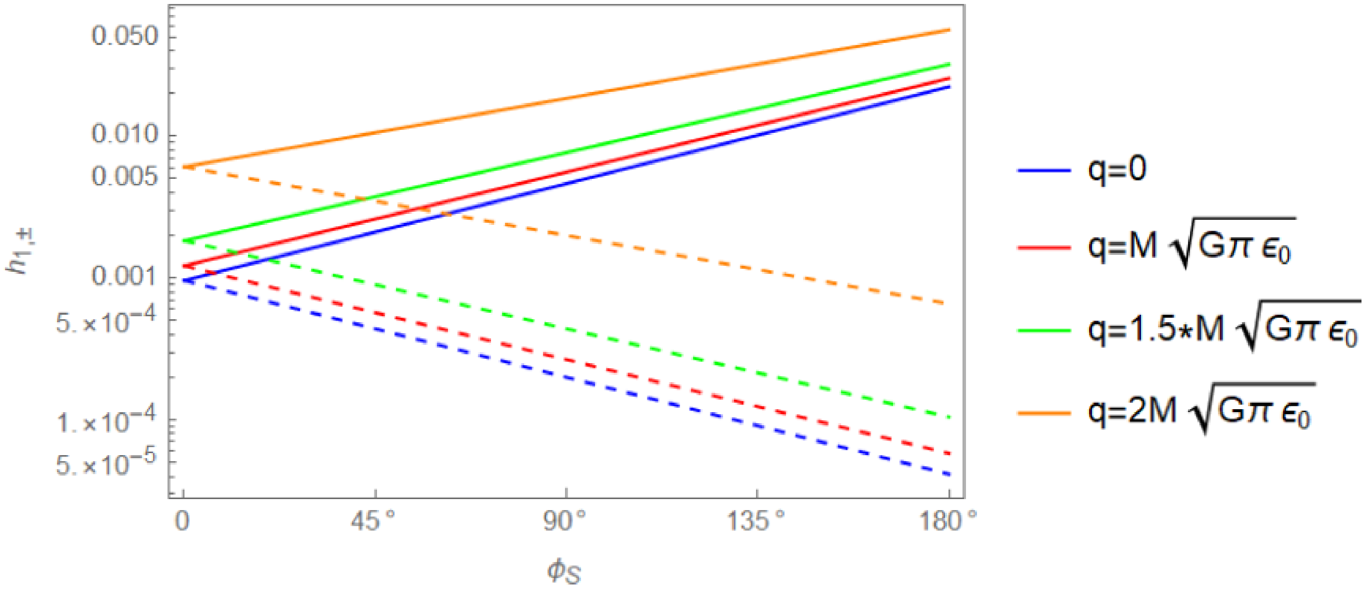
$$V(u,v) = N(v) \sum_{p=\pm 1} \sum_{n=1}^{+\infty} \epsilon_{n,p}\, e^{-2\pi^2 \Delta\theta_{n,p}^2 u^2} e^{-2\pi \imath p \theta_{n,p} u}$$

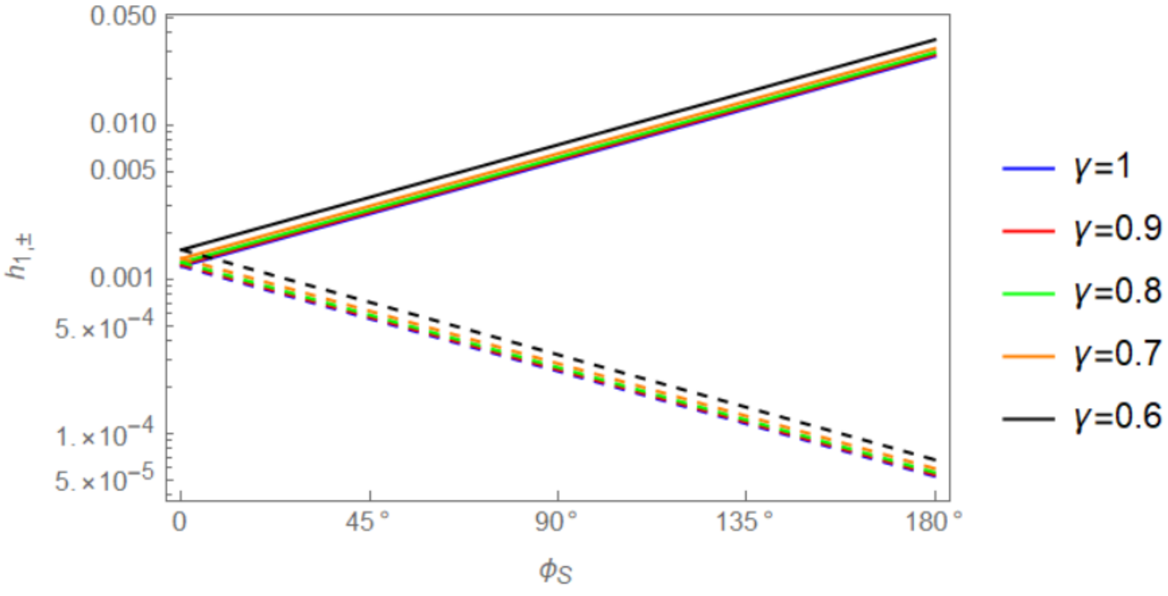




Interferometric signature of relativistic images 3

Aratore, F., & Bozza, V. (2021). Decoding a black hole metric from the interferometric pattern of the relativistic images of a compact source. *Journal of Cosmology and Astroparticle Physics*, *2021*(10), 054.





Interferometric signature of relativistic images 4

SCAN ME

Aratore, F., & Bozza, V. (2021). Decoding a black hole metric from the interferometric pattern of the relativistic images of a compact source. *Journal of Cosmology and Astroparticle Physics*, *2021*(10), 054.

$$a = \frac{2\pi}{\ln\left(h_{n,+}/h_{n+1,+}\right)}$$

$$\eta_O \eta_S e^{\frac{b-2n\pi}{a}} = \frac{2(\nu_{n,+} - \nu_{n,-})}{\nu_{n,-}\left(e^{\frac{\phi_S}{a}} + e^{-\frac{\phi_S}{a}}\right) - \nu_{n,+}\left(e^{\frac{\phi_S - 2\pi}{a}} + e^{-\frac{\phi_S}{a}}\right)}$$

$$\theta_m = \frac{1}{4}\left\{e^{\frac{2\pi}{a}}\left[\nu_{n,-} + e^{-\frac{2\phi_S}{a}}\left(\nu_{n,-} - \nu_{n,+}\right)\right] - \nu_{n,+}\right\}\left(\coth\frac{\pi}{a} - 1\right)$$

$$\phi_S = \frac{a}{2}\ln\left(h_{n,+}/h_{n,-}\right)$$

Rotating black hole 1

Kerr metric

$$ds^2 = \frac{\Delta - a^2\sin^2\theta}{\rho^2}dt^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 +$$

$$-\frac{(r^2+a^2)^2 - a^2\Delta\sin^2\theta}{\rho^2}\sin^2\theta d\phi^2 + \frac{2ar\sin^2\theta}{\rho^2}dtd\phi$$

$$\int \frac{dr}{\sqrt{R}} = \int \frac{d\theta}{\sqrt{\Theta}}$$

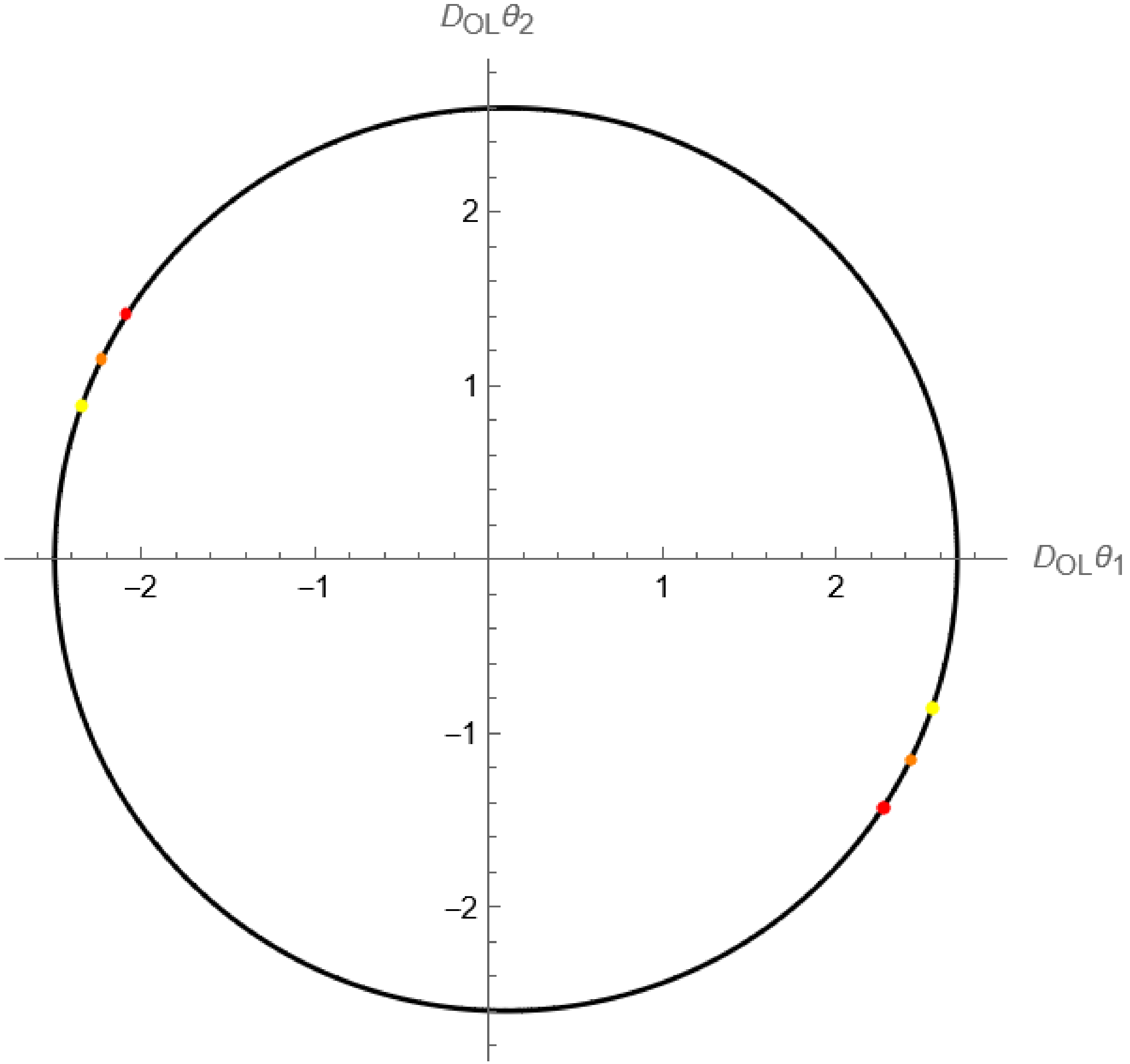
$$\phi_f - \phi_i = a\int \frac{r^2+a^2-aJ}{\Delta\sqrt{R}}dr - a\int \frac{dr}{\sqrt{R}} + J\int \frac{\csc^2\theta}{\sqrt{\Theta}}d\theta$$

Kerr geodesics

$$t = \int \frac{r^2}{\sqrt{R}}dr + a^2\int \frac{\cos^2\theta}{\sqrt{\Theta}}d\theta + \int \frac{r(r^2+a^2-aJ)}{\Delta\sqrt{R}}dr$$

Rotating black hole 2

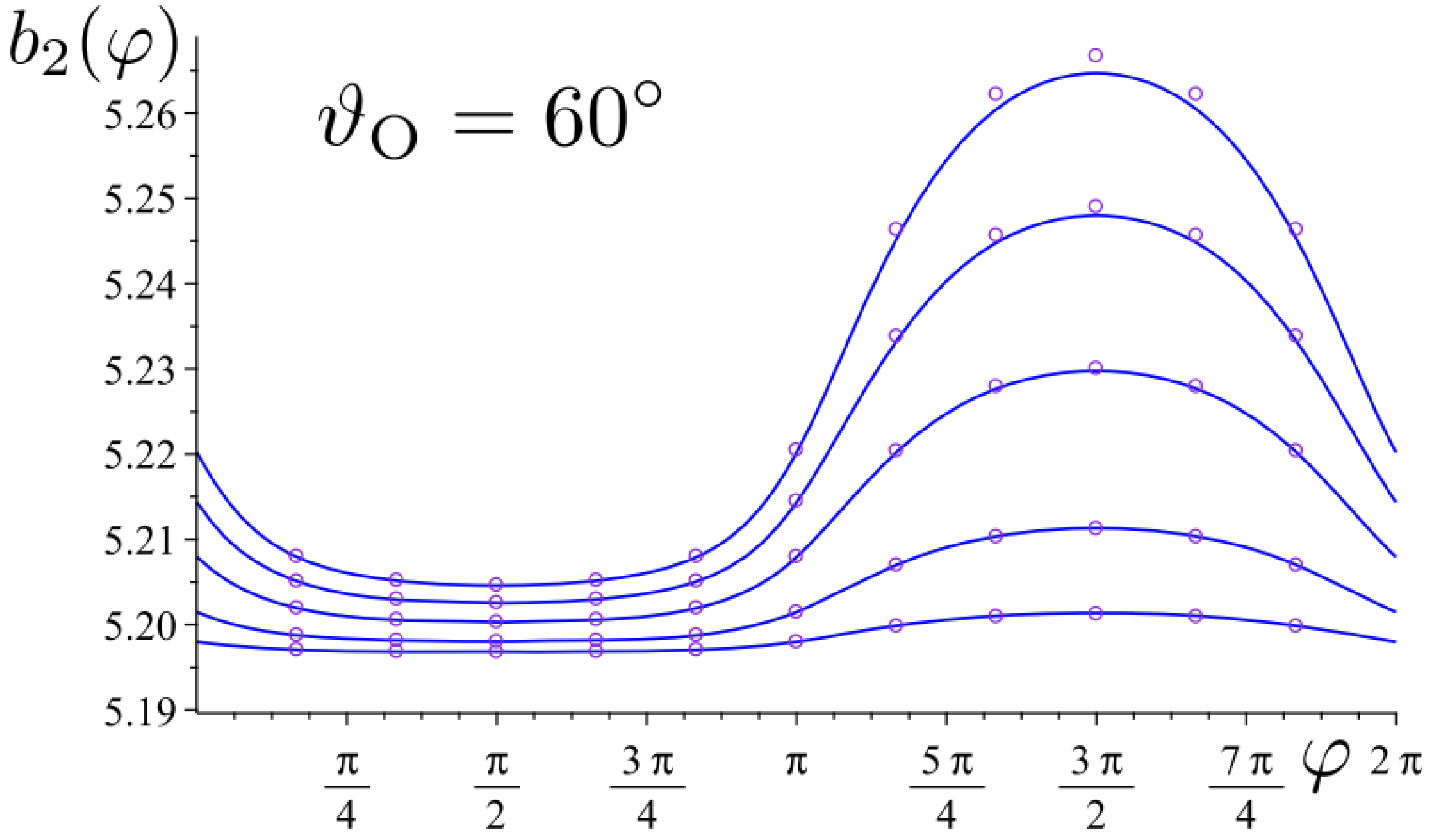
$$\begin{cases} \alpha = \alpha_0 + a\ \alpha_1 \\ \delta = \delta_0 + a\ \delta_1 \\ \Delta t = \Delta t_0 + a\ \Delta t_1 \end{cases}$$



Emission ring 1

Tsupko, Oleg Yu. "Shape of higher-order images of equatorial emission rings around a Schwarzschild black hole: Analytical description with polar curves." ***Physical Review D*** **106.6 (2022): 064033.**

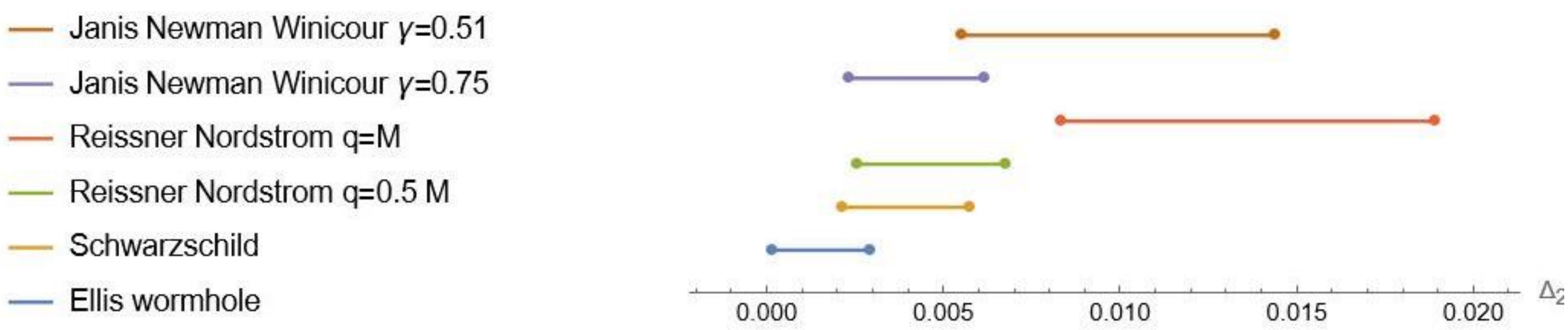
$$b_n(\varphi) = 3\sqrt{3}m\left\{1 + f(r_S)\exp\left[-(n+1)\pi + \arccos\left(\frac{\sin\varphi}{\sqrt{\sin^2\varphi + \cot^2\vartheta_O}}\right)\right]\right\},$$



Emission ring 2

$$b_n(\varphi) = b_{cr}\left\{1 + \frac{2\beta_{\rm ph}}{b_{\rm cr}^2}\left(1 - \frac{r_{\rm ph}}{r_s}\right) e^{\frac{k_o + k_s - (n+1)\pi}{a}} \exp\left[\frac{1}{\tilde{a}} \arccos \frac{\sin\varphi}{\sqrt{\sin^2\varphi + \cot^2\theta_o}}\right]\right\}$$

$$\Delta_n = \frac{b_n - b_{n+1}}{b_{\rm cr}}$$



Conclusion

- The SDL provide a complete analitycal derivation of all the main feature of the problem;
- The systems considered do not depend on the physical environment around the black hole and thus less model-dependent and provide a clear study of the metric;
- There are interesting perspectives from space interferometry in mm and MIR bands,