







University of Genova, 21/06/2023

Based on

arXiv: **2103.10440** [hep-th] with **LF Alday, C Behan, X Zhou** arXiv: **2305.01016** [hep-th] with **C Behan, S Chester**

Plan of the talk

Introduction







Long term goals

Solving an interacting QFT

Focus on Gauge theories (toy model for SM)

and **Conformal Field Theories** (chart space of QFTs)

with a certain amount of **Supersymmetry** (more tractable)

Understanding quantum gravity

Focus on **Superstring theory** (UV complete model)

in the low energy limit: **10d supergravity** *(Semiclassical)*

in particular **Supersymmetric solutions** (easier, interesting geometry)



Quantum gravity in d+1 dimensions

Quantum field theory in d dimensions

AdS/CFT Correspondence

String theory





Musical notes

Particles

Spectrum

Massless sector ($s \le 1$ for open strings, $s \le 2$ for closed strings) +

 ∞ tower of massive higher spin fields

UV-complete model of quantum gravity





4d N=4 Super Yang-Mills	$AdS_5 \times S^5$	
<i>SU</i> (<i>N</i>) gauge theory, adjoint fields	Solution of 10d IIB supergravity	
$A_{\mu}, \Phi^{A=1,,6}, \psi^{\alpha=1,,4}$	$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}(\text{matter})$	
$\mathscr{L}(A, \Phi, \psi)$ fixed by max susy	Preserves maximal susy	
<i>SO</i> (4,2) conformal symmetry	SO(4,2) isometry of AdS ₅	
$SU(4)_R$ global symmetry	SU(4) = SO(6) isometry of S ⁵	
Parameters g_{YM} , N	Parameters g_s , ℓ_s	
$g_s = \frac{g_{YM}^2}{4\pi^2}, \ell_s = \lambda^{-1/4} = (g_{YM}^2 N)^{-1/4}$		
Well-understood for		
't Hooft/planar limit	$g_s \rightarrow 0$ (no string loops)	
$g_{YM} \to 0, N \to \infty, \lambda \to \infty$	$\ell_s \rightarrow 0$ (no HD corrections)	

Conformal Bootstrap

Integrability

Planar 4d $\mathcal{N} = 4$ Super Yang-Mills

Localization

Holography

More on the correspondence

$$ds^{2}(AdS_{d+1}) = \frac{dz^{2} + ds^{2}(\mathbb{R}^{1,d-1})}{z^{2}} \quad ---$$

Conformal boundary at z = 0where the dual CFT lives

$$\left\langle e^{i\int_{\partial AdS}\bar{\phi}\mathcal{O}}\right\rangle_{CFT} = \int_{AdS} \left[d\phi \right] e^{iS[\phi]} \Big|_{\phi(\partial AdS) = \bar{\phi}}$$

CFT operators

Fields in AdS

Scalar

Conserved current

Stress tensor

Scalar

Gauge field

Graviton

CFT correlators



$$\left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4) \right\rangle = \frac{1}{x_{12}^{2\Delta} x_{34}^{2\Delta}} G(U, V) \qquad U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \quad V = \frac{x_{14}^2 x_{24}^2}{x_{13}^2 x_{24}^2}$$

The function $G(\boldsymbol{U},\boldsymbol{V})$:

Is a dynamical object (equivalent of 4-pt scattering amplitude) Contains information about the spectrum and OPE coefficients Must satisfy $V^{2\Delta}G(U, V) = U^{2\Delta}G(V, U)$: bootstrap equation

Graviton scattering



The string theory S-matrix





Conformal field theory from gravity

One would like to learn about **strongly-coupled QFTs**: very hard problem! We can focus on CFTs, but **analytic computations** are still challenging.

In AdS/CFT at large N we have a **weakly coupled supergravity description** in the bulk, which allows to access

- Strongly-coupled/non-Lagrangian CFTs
- Non-protected CFT data, otherwise hard even with Lagrangian

Gravity from conformal field theory

AdS/CFT can be used as a definition of **quantum gravity in AdS**.

We can use **CFT techniques** to study

- The effect of **quantum gravity** in AdS
- **Effective actions** of string/M-theory perturbatively

Scattering amplitudes program in AdS

For **scattering amplitudes in flat space**: rich story, plenty of interesting physical and mathematical **structures** hidden in Lagrangian description.

A natural question: **what happens in curved space?** Simplest case to look at is **AdS**.

• Generalizations of the **structures of flat space amplitudes**? *e.g.* color-kinematics duality, double copy, MHV limit, CHY formulae, ...

New hidden features for AdS amplitudes?
 e.g. Parisi-Sourlas dimensional reduction, hidden conformal symmetry, ...

How do we compute it?

In principle

Reduce \mathscr{L}_{10d} on internal space: gives **5d supergravity Lagrangian** [massless gravity + KK modes] Extract "**Feynman rules**" Add all the relevant **Witten diagrams** to obtain the result

In practice...

Effective type IIB Lagrangian on $AdS_5 \times S^5$: quartic interactions of graviton Kaluza-Klein modes

Arutyunov, Frolov hep-th/9912210

 $k_2 + k_2 - 2$. The non-renormalization of such correlation functions was proven in [54], and very recently checked to first order in perturbation theory in [55]. The non-renormalization theorem also implies the unabling of the corresponding functions of extended CPOs and, since it is not difficult to show that there is no exchange diagram in this case, the corresponding "next-toexternal" quartic couplings of scalars s¹ have to vanish too. It would be interesting to check this.

Note added

We have recently shown that the relevant part of the gauged $\mathcal{N} = 8$ 5-dimensional suggravity action coincides with the action for the scalar s_2 we found in the paper.

8 Appendix A

Here we collect the quartic couplings of the scalars s^{I} representing our main result. The couplings are given by sums of terms depending on various independent SO(6) tensors. To simplify the orsentation we sometimes use the following notations

> $x \equiv k_1$, $y \equiv k_2$, $t \equiv k_3$, $w \equiv k_4$, $z \equiv k_3$, $\delta = (x + 1)(y + 1)(t + 1)(w + 1)$.

e SO(6) tensors are given by tensors of $a_{lb}p_{lb}a_{ld}$, where $F(f_5)$ is a function of runsh the couplings with different func-

artic couplings of 4-derivative verti $(A_2)_{1,0,0,1,4}^{(0)} = \frac{1}{4\delta}f_2^2(a_{14}a_{235} - a_{132}a_{240})$ $(A_2)_{1,0,0,4,4}^{(0)} = -\frac{1}{4\delta}(3(f_1 + f_2 + f_3 + f_4))$ $(A_1)_{1,0,0,4,4}^{(0)} = -\frac{1}{4\delta}(f_1 - f_2)(f_3 - f_3)f_3$ $-\frac{1}{\delta}(f_1 + f_2 + f_3 + f_4 - 2)$ $(A_0)_{1,0,0,4,4}^{(0)} = \frac{21}{4\delta}(f_1 - f_2)(f_3 - f_3)a_{125}$

$$\begin{split} &10528r_{1}^{2}y+1024r_{2}^{2}y+80\\ &100r_{2}^{2}y+100r_{2}^{2}y+10r_{2}^{2}r_{2}^{2}y+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^{2}r_{2}^{2}r_{2}^{2}+10r_{2}^$$

 $\begin{array}{r} - 46764t^3x^4 + 6638t^4x^4 + 292t\\ - 66020t^3wx^4 + 406t^3wx^2 + 1\\ - 1158t^2wx^2 + 1466t^3wx^2 + 1\\ - 1196t^3wx^4 + 466t^3wx^2 + 1\\ + 24w^3x^4 + 4663t^3wx^2 + 1\\ + 24w^3x^4 + 4663t^3wx^2 + 1\\ + 592t^3wx^4 + 2920w^3x^3 + 2\\ - 272twx^3 + 2920wx^3 + 2\\ + 592t^2wx^3 + 146wx^3 + 2\\ + 352t^2wx^3 + 146wx^3 + 2\\ + 322twx^3 + 212tx^3 + 3\\ + 322twx^3 + 146wx^3 + 2\\ + 322twx^3 + 212tx^3 + 3\\ + 32tx^3 + 22tx^3 + 1\\ + 32tx^3 + 2tx^3 + 2\\ + 32tx^3 + 2\\ + 2tx^3 + 2\\ + 2tx^$

$$\begin{split} & 68021^2w_y^2 + 6940t^2w_y^2 + 1692t^4w_y^2 + 288^2w_y^2 - 17836w^2y \\ & 57456w_y^2 - 2724^2w_y^2 + 3021^2w_y^2 + 382t^2w_y^2 - 28000w^4y \\ & 5828w_y^2 + 1702t^2w_y^2 + 3802t^2w_y^2 - 132w_y^2 \\ & 1024tw_y^2 + 222t^2w_y^2 + 800w_y^2 - 96t^2x_y + 1236t^2x_y - 220300t^4x_y \\ & 57456t^2x_y - 41286tx_y + 131756t^2x_y + 370976t^2x_y - 220300t^4x_y \\ & 57456t^2x_y - 4126t^2x_y - 96t^2x_y + 1236tx_x + 21456tx_y + 2404018tw_x \\ & 4900t^2w_{xy} - 10256t^2w_{xy} - 4136t^2w_{xy} \\ & 4902t^2w_{xy} - 2112t^4w_{xy} - 384t^3w_{xy} \\ & 4902t^2w_{xy} - 2112t^4w_{xy} - 384t^3w_{xy} \\ & 57456t^2x_y + 130056t^2w_x + 130056t^2w_x - 128736t^2w^2x_y \\ & -128736t^2w_x^2 + 886t^2w^2x_y - 315786t^2w_x^2 - 128736t^2w^2x_y \\ & -12886tw^2x_y + 896t^2w_x^2 + 3400t^2w_x - 740868tw^2x_y - 128736t^2w^2y \\ & -11588t^4w^2x_y + 896t^4w^2x_y + 618t^4w^2x_y \\ & -1588t^4w^2x_y + 896t^4w^2x_y - 12432t^2w^2y \\ & -11588t^4w^2x_y + 680t^4w^2y - 2112t^2w^2x_y - 882t^4w^2x_y - 128736t^2x_y - 8630t^2x_y^2 \\ & -1584t^4w^2x_y - 868t^4w^2y - 11848t^2w^2x_y - 128736t^2x_y - 8630t^2w^2y \\ & -1588t^4w^2x_y - 868t^4w^2y - 11848t^2w^2x_y - 128736t^2x_y - 8630t^2w^2y \\ & -1588t^4w^2x_y - 868t^4w^2y - 11848t^2w^2y - 31736t^2x_y - 8630t^2w^2y \\ & -128t^2w^2y - 1300t^2w^2y - 310t^2w^2y - 31076w^2y - 31076t^2w^2y - 8630t^2w^2y \\ & -128t^2w^2y - 1300t^2w^2y - 310t^2w^2y - 31076w^2y - 31076w^2y - 31076w^2y - 8630t^2w^2y \\ & -128t^2w^2y - 1300t^2w^2y - 310t^2w^2y - 31076w^2y - 3$$

 $(S_0)_{i_1i_2j_4i_4}^{(0)} = \frac{7}{4\delta} (2f_1f_2 + 2f_3f_4 - (f_1 + f_2)(f_3 + f_4)) a_{120}a_{34}$ $(A_{-1})_{i_1i_2j_4i_4}^{(0)} = -\frac{12}{\delta} (f_1 - f_2)(f_3 - f_4)f_5^{-4}a_{120}a_{36}.$ $(A_{21})_{i_1i_2j_4i_4}^{(0)} = -\frac{3}{4} (f_5 - 1)^2 t_{120}t_{36}.$

uartic couplings of 2-derivative vertices $(A_4)^{(2)}_{l_1l_2l_3l_4l_4} = \frac{5}{488} f_8^4 (a_{445}a_{235} - a_{135}a_{245}).$ $(A_3)^{(2)}_{l_1l_2l_3l_4l_4} = \frac{5}{387} (k_1 - k_2)(k_3 - k_4) f_3^2 a_{123} a_{345}.$

 $\rangle_{l_1k_2l_3l_4}^{(2)} = \frac{1}{16\delta} \left(137 - 80(k_1 + k_2 + k_3 + k_4) + 2(f_1 + f_2 + f_3 + f_4) + 32(k_1k_2 + k_3k_4) + 24(k_1 + k_2)(k_3 + k_4) \right) f_5^3 a_{125}a_{345}.$

 $\sum_{i=1}^{2^{2}} \frac{(k_{1}-k_{2})(k_{3}-k_{4})}{i\xi} = \frac{(k_{1}-k_{2})(k_{3}-k_{4})}{i\xi} \left(40-12(k_{1}+k_{2}+k_{3}+k_{4})+2(f_{1}+f_{2}+f_{3}+f_{4})\right) + 2(f_{1}+f_{2}+f_{3}+f_{4})$

$$\begin{split} &+16(k_1k_2+k_3k_4)+(k_1+k_2)(k_3+k_4)\int_{t_1^2}^{t_2}a_{125}a_{36}.\\ (S_2)_{h,hhh}^{(2)}a_{h}=&\frac{1}{16\delta}\Big(-3741+2984t-342t^2-56t^3+31t^4+2984w-2272tw+376}\\ &+32k^2w^2+376tw^2+42t^2w^2-56w^3+128tw^3+31w^4+2984x^2+36tw^2+34t^2w^2-56w^3+128tw^3+31w^4+2984x^2+36tw^2+34t^2w^2+36tw^2+34t^2w^2+36tw^2+34tw^2+36tw^2+34tw^2+34tw^2+36tw^2+34tw$$

$$\begin{split} &-1760tx+144t^2x+88t^3x-1760wx+832twx+88t^2wx+144w^2x\\ &+88w^3x-342x^2+144tx^2+40t^2x^2+144wx^2+192twx^2+40w^2x^2\\ &+88tx^3+88wx^3+31x^4+2984y-1760ty+144t^2y+88t^3y-1760t \end{split}$$

$$\begin{split} \frac{1}{486} & \left(20079 - 537844 + 186606^2 + 40564^2 - 11971^4 + 1924^3 + 726^4 \right. \\ & \frac{1}{53784} + 596481w - 17792^4 w - 28164^3 w + 18061^4 w + 2564^3 w + 18060w^2 \right. \\ & \frac{1}{53784} + 72864^3 + 13414^3 w^2 + 2864^3 w^2 + 40564^3 - 28144^3 w^3 \right. \\ & \frac{1}{5364} + 1097w^3 + 1896w^2 + 1920w^4 + 2564w^3 + 7286^3 w^2 + 1097w^3 + 2864^3 w^2 + 1010^3 w^2 + 2866^3 w^2 + 10007w^2 + 1128^3 w^2 + 1010^3 w^2 + 1288w^2 + 1188w^2 + 1128w^2 + 2186w^2 - 11900w^2 + 8166^2 - 11000w^2 + 8117^2 w^2 + 1188w^2 + 1128w^2 + 2186w^2 - 11900w^2 + 38664^3 - 11001w^2 + 8117w^2 + 1128w^2 + 2188w^2 + 1188w^2 + 1188w^2 + 1010w^2 + 1017w^2 + 1017w^2 + 1287w^2 + 2050w^2 + 2080w^2 + 4000w^3 + 1010w^2 + 173w^2 + 2050w^2 + 4061w^2 - 255588w^2 + 460w^2 + 2186w^2 - 21000w^2 + 45680w^2 + 3568w^2 + 4666w^2 + 4188w^2 + 2188w^2 + 2188w^2 + 1188w^2 + 2180w^2 + 1188w^2 + 2188w^2 + 1188w^2 + 1188w^2 + 11900w^2 + 1286W^2 + 1188W^2 + 2188W^2 + 118W^2 + 1$$

$$\begin{split} 2006^{-1}w = 11000^{-1}w + 1281^{-1}w + 10022^{-1}w - 110640^{-1}w + 5627^{-1}w^{-1}\\ 9206^{-1}w + 11000^{-1}w + 1280^{-1}w^{-1} + 2380^{-1}w^{-1} + 2380^{-1}w^{-1}\\ 9206^{-1}w + 11000^{-1}w^{-1} + 1760^{-1}w^{-1} + 4100^{-1} + 1280^{-1}w^{-1} + 2320^{-1}w^{-1}\\ 17176^{-1} - 920122^{-1} - 58880^{-1}x^{-1} + 2818^{-1}w^{-1} + 1224^{-1}w^{-1} + 2820^{-1}\\ 20124xx^{-1} - 18480^{-1}w^{-1} + 1760^{-1}w^{-1} + 1280^{-1}w^{-1} + 1242^{-1}w^{-1} + 28380^{-1}x^{-1}\\ 8880^{-1}w^{-1} + 1360^{-1}w^{-1} + 11070^{-1}w^{-1} + 1840^{-1}w^{-1} + 1124^{-1}w^{-1} + 28380^{-1}x^{-1}\\ 8880^{-1}w^{-1} + 1240^{-1}w^{-1} + 1070^{-1}w^{-1} - 1880^{-1}w^{-1} + 1124^{-1}w^{-1} + 1222^{-1}w^{-1} + 1102^{-1}w^{-1} + 1124^{-1}w^{-1} + 1124^{-1}w^{-$$

$$\begin{split} + 1984t^*w + 880t^*w + 64t^*w + 14952t^*w^- - 3208t^*w^- - 11710t^*w^+ \\ + 7896t^*w^2 + 1706t^*w^2 + 926t^*w^2 + 21504tt^{-0} - 11710t^*w^+ + 168t^*w^+ \\ + 2544t^*w^3 + 320t^*w^3 - 10528tw^4 + 7896t^*w^4 + 2544t^*w^4 + 10t^*w^4 \\ + 1984tw^4 + 1776t^*w^4 + 320t^*w^3 + 880tw^4 + 92t^*w^4 + 64tw^3 \\ + 144288tx - 74770t^*x - 10752t^*x + 5264t^4x - 992t^*x - 440t^8 \\ - 32t^*x + 14428wx + 20752t^*x^2 - 30t^*w^4 + 92t^*w^4 + 40t^*w^4 \\ + 14428wx + 20752t^*x - 30t^*w^4 + 92t^*w^4 + 40t^*w^4 \\ + 14428wx + 20752t^*x - 30t^*w^4 + 92t^*w^4 + 40t^*w^4 \\ + 1084tw^4 + 100t^*w^4 + 20t^*w^4 + 90t^*w^4 + 10t^*w^4 \\ + 1084tw^4 + 10t^*w^4 + 10t^*w^4 + 10t^*w^4 + 10t^*w^4 \\ + 1084tw^4 + 10t^*w^4 + 10t^*w^4 + 10t^*w^4 + 10t^*w^4 \\ + 1084tw^4 + 10t^*w^4 + 10t^*w^4 + 10t^*w^4 + 10t^*w^4 + 10t^*w^4 \\ + 1084tw^4 + 10t^*w^4 +$$

 $\begin{array}{l} + 1984tw^5 + 1776t^2w^5 + 320t^3w^5 + 880tw^6 + 92t^2w^6 + 64tw^7 \\ + 144288tx - 74776t^2x - 10752t^3x + 5264t^4x - 992t^5x - 440t^6x \\ - 32t^7x + 144288wx + 26752t^2wx - 6400t^3wx + 1024t^4wx + 960t^5wx \\ + 96t^6wx - 74776w^2x + 26752tw^2x - 3520t^2w^2x + 2368t^3w^2x + 1104t^4w^2x \\ + 144t^5w^2x - 10752w^3x - 6400tw^3x + 2368t^2w^3x + 1024t^3w^3x - 112t^4w^3x \\ + 5264w^4x + 1024tw^4x + 1104t^2w^4x - 112t^3w^4x - 992w^5x + 960tw^5x \\ + 144t^2w^5x - 440w^6x + 96tw^6x - 32w^7x - 74776tx^2 + 26042t^2x^2 \\ + 5888t^3x^2 - 3948t^4x^2 - 888t^5x^2 - 46t^6x^2 - 74776wx^2 - 53504twx^2 \\ + 1760t^2wx^2 + 2560t^3wx^2 + 480t^4wx^2 + 26042w^2x^2 + 1760tw^2x^2 + 96t^3w^2x^2 \\ + 28t^4w^2x^2 + 5888w^3x^2 + 2560tw^3x^2 + 96t^2w^3x^2 - 256t^3w^3x^2 - 3948w^4x^2 \\ + 480tw^4x^2 + 28t^2w^4x^2 - 888w^5x^2 - 46w^6x^2 - 10752tx^3 + 5888t^2x^3 \\ - 832t^3x^3 - 1272t^4x^3 - 160t^5x^3 - 10752wx^3 + 12800twx^3 - 4928t^2wx^3 \\ - 512t^3wx^3 + 176t^4wx^3 + 5888w^2x^3 - 4928tw^2x^3 - 192t^2w^2x^3 + 128t^3w^2x^3 \\ \end{array}$

$$\begin{split} &+ 2320 x^2 r_y^2 + 1122^4 w_x^2 r_y^2 + 4000 e^{-1} k_y^2 \\ &+ 854 w_y^2 + 295 w_y^2 r_y^2 - 2000 r_y^2 + 853 w_y^2 + 1792 r_y^2 r_y^2 + 352 r_y^2 r_y^2 + 352 r_y^2 r_y^2 + 352 r_y^2 r_y^2 + 152 r_y^2 r_y^2 + 1024 r_y^2 - 254 r_y^2 r_y^2 + 152 r_y^2 r_y^2 + 1024 r_y^2 - 254 r_y^2 r_y^2 + 1024 r_y^2 - 254 r_y^2 r_y^2 + 1024 r_y^2 - 254 r_y^2 r_y^2 + 1024 r_y^2 - 256 r_y^2 + 1275 r_y^2 - 10052 r_y^2 - 996 w_y^2 - 3238 r_y^2 + 1160 r_y^2 + 1275 r_y^2 - 6075 6 r_y^2 + 1275 r_y^2 r_y^2 - 6075 6 r_y^2 + 2325 r_y^2 - 6075 6 r_y^2 + 1105 r_y^2 r_y^2 + 18632 r_y^2 r_y^2 - 2732 r_y^2 r_y^2 - 2175 r_y^2 r_y^2 + 10050 r_y^2 - 2152 r_y^2 r_y^2 - 13356 r_y^2 r_y^2 + 1055 r_y^2 r_y^2 - 9056 r_y^2 - 2532 r_y^2 r_y^2 - 2175 r_y^2 r_y^2 - 1055 r_y^2 r_y^2 + 10050 r_y^2 - 2152 r_y^2 r_y^2 - 21356 r_y^2 r_y^2 - 13756 r_y^2 + 1055 r_y^2 r_y^2 - 9056 r_y^2 - 213156 r_y^2 r_y^2 - 13756 r_y^2 r_y^2 - 13756 r_y^2 r_y^2 + 1055 r_y^2 + 292 r_y^2 r_y^2 - 1275 r_y^2 r_y^2 + 11576 r_y^2 r_y^2 - 21756 r_y^2 r_$$

$$\begin{split} & - \cos \log x \; y \; - \; 1.1000 ter \; y \; + \; 1.000 \; ter \; y \; - \; 1.0000 \; ter \; y \; - \; 1.0000\; ter \; y \; -$$

$$\begin{split} & 288 m_1^{10} + 224 m_1^{10} + 177012 xy^2 - 223000 tx_2y^4 - 08200 t^2 xy^4 \\ & 6400 t^2 x_2^4 + 650 t^2 x_2^4 + 680 t^2 x_2y^4 - 6800 t^2 x_2y^4 - 1158 t^2 x_2y^4 \\ & 1158 t^2 x_2y^4 + 800 t^2 x_2y^4 + 680 t^2 x_2y^4 - 6800 t^2 x_2y^4 - 1158 t^2 x_2y^4 \\ & 1158 t^2 x_2y^4 - 1580 t^2 x_2y^4 - 260 t^2 x_2y^4 - 6800 t^2 x_2y^4 - 1850 t^2 x_2y^4 \\ & 167 t^2 x_2y^4 + 036 t^2 x_2y^4 - 560 t^2 x_2y^4 - 6800 t^2 x_2y^4 - 1158 t^2 x_2y^4 \\ & 156 t^2 x_2y^4 - 156 t^2 x_2y^4 - 690 t^2 x_2y^4 + 6100 t^2 x_2^4 + 6400 t^2 x_2^4 \\ & 456 t^2 x_2y^4 - 156 t^2 x_2y^4 - 56 t^2 x_2y^4 - 6400 t^2 x_2^4 + 6400 t^2 x_2^4 \\ & 456 t^2 x_2y^4 - 156 t^2 x_2y^4 - 156 t^2 x_2y^4 - 156 t^2 x_2y^4 + 1510 tx_2^2 x_2^4 - 540 t^2 x_2^4 \\ & 456 t^2 x_2y^4 - 156 t^2 x_2y^4 - 106 t^2 x_2^4 y^4 - 6808 t^2 y^4 + 1672 t^2 x_2^4 \\ & 456 t^2 x_2y^4 - 105 t^2 x_2y^4 - 106 t^2 x_2^4 y^4 - 560 t^2 x_2^4 + 200 t^2 x_2^4 \\ & 456 t^2 x_2y^4 - 107 t^2 x_2y^4 + 6100 t^2 x_2^4 y^4 - 560 t^2 x_2^4 + 200 t^2 x_2^4 y^4 \\ & 3578 t^2 x_2 - 370 t^2 x_2^4 + 6100 t^2 x_2^4 + 1672 t^2 x_2^4 y^5 \\ & 7754 t^2 x_2y^4 + 172 t^2 x_2y^4 + 1000 t^2 x_2^4 + 200 t^2 x_2^4 - 3754 t t^2 y^2 \\ & 7754 t^2 x_2y^4 + 127 t^2 x_2y^4 + 1000 t^2 x_2^4 + 200 t^2 x_2^4 - 3754 t t^2 y^4 \\ & 7754 t^2 x_2y^4 + 127 t^2 x_2y^4 + 1000 t^2 x_2^4 + 200 t^2 x_2^4 - 30754 t^2 x_2^4 + 300 t^2 x_2^4 + 200 t^2 x_2^4 + 200$$

$$\begin{split} -& 1928^{11}\pi_{1}^{2}-1884^{12}\pi_{2}^{2}-1844^{12}\pi_{2}^{2}-1857222\pi_{2}^{2}-2567\pi_{2}^{2}\pi_{2}^{2}-982\pi_{2}^{2}\pi_{2}^{2}-384\pi_{1}^{2}\pi_{2}^{2}\\ -& 1907\pi_{2}^{2}\pi_{1}^{2}-1584\pi_{2}^{2}\pi_{2}^{2}-1584\pi_{2}^{2}\pi_{2}^{2}-2567\pi_{2}^{2}\pi_{2}^{2}-9288\pi_{2}^{2}\pi_{2}^{2}-3814\pi_{2}^{2}\pi_{2}^{2}\\ -& 2507\pi_{2}^{2}\pi_{1}^{2}-1584\pi_{2}^{2}\pi_{2}^{2}-152\pi_{2}^{2}\pi_{2}^{2}-512\pi_{2}^{2}\pi_{2}^{2}+3284\pi_{2}^{2}\pi_{2}^{2}-610\pi_{2}^{2}\pi_{2}^{2}\\ +& 2507\pi_{2}^{2}\pi_{1}^{2}-528\pi_{2}^{2}\pi_{2}^{2}\pi_{2}^{2}-52\pi_{2}^{2}\pi_{2}^{2}+324\pi_{2}^{2}\pi_{2}^{2}-610\pi_{2}^{2}\pi_{2}^{2}+384\pi_{2}^{2}\pi_{2}^{2}-616\pi_{2}^{2}\pi_{2}^{2}\\ -& 17506\pi_{2}^{2}\pi_{1}^{2}+3384\pi_{2}^{2}\pi_{1}^{2}+512\pi_{2}^{2}\pi_{2}^{2}\pi_{2}^{2}-52\pi_{2}^{2}\pi_{2}^{2}+9384\pi_{2}^{2}\pi_{2}^{2}-616\pi_{2}^{2}\pi_{2}^{2}\\ -& 17507\pi_{1}^{2}\pi_{1}^{2}-71028\pi_{1}^{2}-19028\pi_{1}^{2}-91052\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-10028\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-10028\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-10028\pi_{1}^{2}\pi_{1}^{2}-11088\pi_{1}^{2}\pi_{1}^{2}-10028\pi_{1}^{2}\pi_{1}^{2}-1208\pi_{1}^{2}$$

$$\begin{split} &+ 4992wxy^3 + 2432x^3wxy^3 + 84^{10}wxy^5 - 2224w^2xy^5 + 2432w^2xy^5 \\ &+ 1112x^3w^2xy^3 + 2406w^2xy^3 + 5324w^2xy^5 - 231788x^2y^5 \\ &- 7224x^2x^3y + 2206w^2x^3y + 502t^2x^2y^5 - 2212wx^2y^4 - 2432wx^2y^5 \\ &+ 1112x^2wx^2y + 2266w^2x^3y + 4000wx^2y^4 + 384wx^2y^5 - 3376x^2y^5 \\ &+ 0000x^2y^5 + 502t^2x^2y^4 + 4000wx^2y^4 + 3484wx^2y^5 - 3376x^2x^2y^5 \\ &+ 0000x^2y^5 + 502t^2x^2y^4 + 4000wx^2y^4 + 3484wx^2y^5 - 452wx^2y^5 \\ &+ 0200x^4y^4 + 204w^2y^4 + 416x^2y^4 + 245w^4y^5 - 20000xy^6 \\ &+ 5882wx^4y + 1102w^2xy^4 + 532w^2y^4 + 145x^2y^4 + 1720wx^2y^6 \\ &+ 5882wx^4y + 1720x^2xy^4 + 532w^2y^4 + 552wx^2y^6 - 22000xy^6 \\ &+ 5882wx^4y + 1720x^2xy^4 + 532w^2y^4 + 552wx^2y^6 + 512wx^2y^6 \\ &+ 5882wx^4y + 520x^2y^4 + 532wx^2y^6 + 525wx^2y^6 + 512wx^2y^6 \\ &+ 5684wx^2y^4 + 532w^2y^4 + 532wx^2y^6 + 532w^2y^6 + 110x^2y^6 \\ &+ 1720x^2y^4 + 58tx^2y^4 + 332wx^2y^6 + 688wx^2y^6 + 534w^2y^6 + 534w^2y^6 \\ &+ 1506y^2y^4 + 532w^2y^4 + 332wx^2y^6 + 132wx^2y^6 + 134w^2y^6 + 132w^2y^6 \\ &+ 1566w^2y^4 + 52wx^2y^4 + 332wx^2y^6 + 132w^2y^2 + 130w^2y^2 + 122wx^2y^6 \\ &+ 1566w^2y^4 + 532w^2y^4 + 332wx^2y^6 + 132w^2y^2 + 130w^2y^4 + 222^2xy^2 \\ &+ 1366w^2y^2 + 532w^2y^2 + 132w^2y + 132w^2y + 1024wy^2 + 222^2xy^2 \\ &+ 136w^2y^2 - 334wxy^2y + 222w^2y^2 + 134w^2y^4 + 222^2xy^2 \\ &+ 1266w^2y^2 + 32w^2y^2 + 238w^2y^2 + 132w^2y + 1024wy^2 + 222^2xy^2 \\ &+ 126w^2y^2 - 384wxy^2 + 228w^2y^2 + 138w^2y^4 + 222w^2y^2 \\ &+ 122wx^2y^2 - 384wxy^2 + 22w^2y^2 + 118wy^2 + 122w^2y^2 \\ &+ 122wx^2y^2 - 384wxy^2 + 22w^2y^2 + 138w^2y^4 + 122w^2y^2 \\ &+ 122wx^2y^2 - 384wxy^2 + 22w^2y^2 + 138w^2y^2 + 122w^2y^2 \\ &+ 122wx^2y^2 - 384wxy^2 + 22w^2y^2 + 138w^2y^2 + 122w^2y^2 \\ &+ 122wx^2y^2 - 384wxy^2 + 22w^2y^2 + 138w^2y^2 + 122w^2y^2 \\ &+ 122wx^2y^2 + 32wy^2 + 22w^2y^2 + 118wy^2 + 22w^2y^2 \\ &+ 122wx^2y^2 + 28w^2y^2 + 22w^2y^2 + 138w^2 + 22w^2y^2 \\ &+ 122wx^2y^2 + 28w^2y^2 + 22w^2y^2 + 118w^2y^2 + 22w^2y^2 \\ &+ 128w^2y^2 + 28w^2y^2 + 28w^2y^2 + 28w^2y^2 + 28w^2y^2 \\ &+ 128w^2y^2 + 28w^2y^2 + 28w^2y^2 + 28w^2y^2 + 28w^2y^2 \\ &+ 128w^2y^2 + 28w^2y^2 + 28w^2y^2 +$$

 $2640w^6x^3 + 352tw^6x^3 + 336w^7x^3 - 111795x^4 + 177012tx^4 - 94964t^2$

$$\begin{split} &(S_{-1})_{l_1 l_2 l_3 l_4}^{(0)} = \frac{2}{\delta} f_5^{-1} a_{125} a_{345} (k_1 - k_2) (k_3 - k_4) (f_1 - f_2) (f_3 - f_4) \\ &\times \left(-36 + 2(k_1 + k_2 + k_3 + k_4) + f_1 + f_2 + f_3 + f_4 - 2k_1 k_2 - 2k_3 k_4 \right) \end{split}$$

 $(S_{22})_{l_1 l_2 l_3 l_4}^{(0)} = \frac{(f_3 - 1)^2 t_{122} t_{345}}{2\delta} (k_1 - k_2) (k_3 - k_4) (f_1 + f_2 + f_3 + f_4) + 2(k_1 + k_2 + k_3 + k_4) - 2(k_1 k_2 + k_3 k_4) - 36).$

 $(S_{p3})^{(0)}_{I_1I_2I_3I_4} = -\frac{1}{\delta}f_5^3p_{125}p_{345}.$

$$\begin{split} &(S_2)_{i_1,i_2,i_3,i_4}^{(0)} = \frac{2}{g^2} f_2^2 p_{i22} p_{236}(k_1^2 + k_2^2 + k_3^2 + k_4^2 - 2(k_1 + k_2 + k_3 + k_4) + 2(k_1 k_2 + k_3 k_4) - 4). \\ &(S_4)_{i_1,i_2,i_3,i_4}^{(0)} = \frac{9 a_{123} a_{236}}{644(f_5 - 5)}(-1 + k_1 - k_2)(1 + k_1 - k_2)(3 + k_1 + k_2)(5 + k_1 + k_2) \\ &\times (-1 + k_3 - k_4)(1 + k_3 - k_4)(3 + k_3 + k_4)(5 + k_3 + k_4) \end{split}$$

× $(-1 + k_3 - k_4)(1 + k_3 - k_4)(3 + k_3 + k_4)(5 + k_3 + k_4)$ × $(2k_1^2 + 2k_2^2 + 2k_3^2 + 2k_4^2 + 4k_1k_2 + 4k_3k_4 - 4(k_1 + k_2 + k_3 + k_4) - 5).$

Tree level Witten diagram for φ^4 interaction with no derivatives, dual to $\langle \mathcal{O}_{\Delta=4} \mathcal{O}_{\Delta=4} \mathcal{O}_{\Delta=4} \mathcal{O}_{\Delta=4} \rangle$

$$\frac{1}{(z-z)^{6}}$$

$$2 \left(26 z^{2} - 13 z^{3} + 88 z z - 127 z^{2} z + 26 z^{3} z + 26 z^{2} - 127 z z^{2} + 88 z^{2} z^{2} - 13 z^{3} + 26 z z^{3}\right) + \frac{1}{(z-z)^{8}} 4 \left(\log[1-z] + \log[1-z]\right)$$

$$\left(25 z^{3} - 28 z^{4} + 6 z^{5} + 185 z^{2} z - 388 z^{3} z + 222 z^{4} z - 28 z^{5} z + 185 z z^{2} - 848 z^{2} z^{2} + 1032 z^{3} z^{2} - 388 z^{4} z^{2} + 25 z^{5} z^{2} + 25 z^{3} - 388 z z^{3} + 1032 z^{2} z^{3} - 848 z^{3} z^{3} + 185 z^{4} z^{3} - 28 z^{4} + 222 z z^{4} - 388 z^{2} z^{4} + 185 z^{3} z^{4} + 6 z^{5} - 28 z z^{5} + 25 z^{2} z^{5}\right) - \frac{1}{(z-z)^{8}} 4 \left(\log[z] + \log[z]\right) \left(-6 z^{4} + 3 z^{5} - 96 z^{3} z + 111 z^{4} z - 22 z^{5} z - 216 z^{2} z^{2} + 516 z^{3} z^{2} - 292 z^{4} z^{2} + 25 z^{5} z^{2} - 96 z z^{3} + 516 z^{2} z^{3} - 632 z^{3} z^{3} + 185 z^{4} z^{3} - 6 z^{4} + 111 z z^{4} - 292 z^{2} z^{4} + 185 z^{3} z^{4} + 3 z^{5} - 22 z z^{5} + 25 z^{2} z^{5}\right) + \frac{1}{(z-z)^{9}}$$

$$4 \left(6 z^{4} - 6 z^{5} + z^{6} + 96 z^{3} z - 162 z^{4} z + 72 z^{5} z - 6 z^{6} z + 216 z^{2} z^{2} - 672 z^{3} z^{2} + 603 z^{4} z^{2} - 162 z^{5} z^{2} + 6 z^{6} z^{2} + 96 z z^{3} - 672 z^{2} z^{3} + 1168 z^{3} z^{3} - 672 z^{4} z^{3} + 96 z^{5} z^{3} + 6 z^{4} - 162 z z^{4} + 603 z^{2} z^{4} - 672 z^{3} z^{4} + 216 z^{4} z^{4} - 6 z^{5} + 72 z z^{5} - 162 z^{2} z^{5} + 96 z^{3} z^{5} + z^{6} - 6 z z^{6} + 6 z^{2} z^{6}\right)$$

$$\left(\log[1-z] \log[z] - \log[z] \log[1-z] + \log[1-z] \log[2] - \log[2] - \log[1-z] \log[2] - \log[1-z] \log[2] - \log[2] - \log[2] - 2 \operatorname{Polylog}[2, z] - 2 \operatorname{Polylog}[2, z]\right)$$

New tools to make progress



Mellin space

$$\mathscr{G}(U,V) = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}} \Gamma\left[\Delta - \frac{s}{2}\right]^2 \Gamma\left[\Delta - \frac{t}{2}\right]^2 \Gamma\left[\Delta - \frac{u}{2}\right]^2 \mathscr{M}(s,t)$$



Contact diagrams:
$$\mathscr{L}_{AdS} \supset \partial^{2L} \phi^4$$

$$\mathcal{M}(s,t) = \operatorname{Poly}^{(L)}(s,t)$$

Exchange diagrams: $\mathscr{L}_{AdS} \supset \phi^3$ $\mathscr{M}^{(s)}_{\Delta,\ell}(s,t) = \sum_{m=0}^{\infty} \frac{Q^{(\ell)}_m(t,u)}{s - \Delta + \ell - 2m} + P^{(\ell-1)}(s,t)$

The conformal bootstrap

nsatz

with arbitrary coefficients to be fixed

Rich

Observable = \sum known fur

Use constraints from conformal symmetry, sometimes supersymmetry

Consistency conditions

Require physical properties, match with other known results, limits etc



How far can we get?

Tree level

Well understood

Loops

Results at 1 and 2 loops, complicated non-analytic structure. Up to **renormalization ambiguities**

String corrections

Contact terms: very simple results (polynomial). Up to **overall coefficients**

What about the free parameters?

Supersymmetric localization

For all 4d $\mathcal{N} = 2$ Lagrangian QFTs

$$Z = e^{-F} = \int [d\varphi] e^{iS[\varphi]} = \int [dU] e^{-\frac{8\pi^2}{g_{YM}^2}} tr U^2 |Z_{1-loop}(U)|^2 |Z_{inst}(U, g_{YM})|^2$$

Deformations couple to stress tensor/conserved currents

$$\partial_m^4 F = \int [\mathrm{d}U \mathrm{d}V] \, G(U, V)$$

Non-perturbative constraint on correlation functions

Gluons in AdS



Gluon scattering at tree level in models with 1/2-maximal supersymmetry

First example, across dimensions d=3,4,5,6, all KK modes

Localization at finite coupling for interesting 4d N=2 SCFT

Surprisingly simple partition function, including instantons

First two string corrections at finite string coupling *Analogue of Veneziano amplitude in* AdS₅



A simple D3-D7 system

D7 branes at \mathbb{Z}_2 **orientifold singularity**, constant (arbitrary) axio-dilaton \leftrightarrow F-theory on K3= T^4/\mathbb{Z}_2 , with D_4 singularities: *SO*(8) gauge group on D7 **Sen**

Probe with N D3 branes parallel to the D7:



AdS₅ × S^5/\mathbb{Z}_2 NH geometry, with D7 wrapping AdS₅ × S^3

Aharony, Fayyazuddin, Maldacena, Spalinski

Field theory description

Aharony, Banks, Douglas, Seiberg, Sonnenschein, Theisen, Yankielowicz

D7 branes SO(8) vector multiplet: $8d \ \mathcal{N} = 1 \ SYM$

World-volume theory on D3 branes

D3 (+O7) \rightarrow USp(2N) adjoint vector + 1 antisymmetric hyper **D7** \rightarrow 8 fundamental half-hypers, SO(8) flavor symmetry

For
$$N = 1$$
 $USp(2) \simeq SU(2) \rightarrow \mathcal{N} = 2$ SQCD

N.B. String theory background defined for any (constant) τ \rightarrow conformal manifold, Lagrangian description.

Gluons in AdS₅

Spectrum of AdS₅ fields:

The *supergravity* modes from the bulk $\rightarrow 1/4$ -BPS or longer reps (AdS graviton and KK modes)

The *vector* multiplets living on D7 branes $\rightarrow 1/2$ -BPS reps (AdS gluons and KK modes on S^3)

1/2-BPS multiplets of $4d \ \mathcal{N} = 2 \text{ SCFTs:}$ \mathcal{O}_2^A : flavor current multiplet $\leftrightarrow \text{ AdS}_5$ gluons \mathcal{O}_k^A : tower of short multiplets $\leftrightarrow \text{ KK}$ modes of D7 gluons on S^3

 $k \leftrightarrow \operatorname{rep} \mathbf{k} + \mathbf{1} \text{ of } SU(2)_R \text{, or spin } j_R = k/2$

Gluon 4-pt functions

 $\langle \mathcal{O}_2^A(1)\mathcal{O}_2^B(2)\mathcal{O}_2^C(3)\mathcal{O}_2^D(4) \rangle = \operatorname{Pref} \times G^{ABCD}(U,V;\alpha) = \operatorname{Pref} \times \sum_{r \in \operatorname{Adj} \times \operatorname{Adj}}$

$$\sum_{dj \times Adj} P_r^{ABCD} G_r(U, V; \alpha)$$

$$U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}, \quad \alpha = \frac{\langle y_1, y_3 \rangle \langle y_2, y_4 \rangle}{\langle y_1, y_2 \rangle \langle y_3, y_4 \rangle}$$

 $\mathbf{28} \otimes \mathbf{28} = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_{\mathrm{v}} \oplus \mathbf{35}_{\mathrm{c}} \oplus \mathbf{35}_{\mathrm{s}} \oplus \mathbf{300} \oplus \mathbf{350}$

Permuted by triality

All 4-point functions in the flavor current multiplet fixed by $G_r(U, V; \alpha)$ It also satisfies the **superconformal Ward identity**

 $G_r(U, V; \alpha) = G_r^{\text{protected}}(U, V; \alpha) + (1 - z\alpha)(1 - \bar{z}\alpha)\mathscr{G}_r(U, V)$

 $M_r(s, t; \alpha) = \Theta \circ \mathcal{M}_r(s, t)$

Couplings & central charges

In \mathscr{L}_{AdS} **gluons** have a self-coupling given by

$$g_{YM,AdS}^2 \sim \frac{1}{k} \sim \frac{1}{N}$$

And a coupling to **gravity** given by

$$\frac{1}{G_N^{(5)}} \sim \frac{1}{c} \sim \frac{1}{N^2}$$

Aharony, Pawelczyk, Theisen, Yankielowicz Blau, Narain, Gava

Gluon self-interactions dominate at leading order

 $\mathcal{O}(1/N)$: pure gauge theory in AdS₅ (+ string corrections)

 $\mathcal{O}(1/N^2)$: gluon loops and graviton exchange (+ string corrections)

Tree level

 $M = c_s M_s + c_t M_t + c_\mu M_\mu$

Alday, Behan, PF, Zhou

 $c_s = f^{I_1 I_2 J} f^{J I_3 I_4}$ $\mathsf{c}_t = f^{I_1 I_4 J} f^{J I_2 I_3}$ $\mathsf{c}_{\mu} = f^{I_1 I_3 J} f^{J I_4 I_2}$

Exchange: *full super-gluon multiplet*. Fixed requiring same polar part as superblocks. Equivalently: MRV limit.

Contact: degree 0 as in flat space YM theory. Fixed by SCWI.

Tree level, all dimensions

The same procedure can be applied to compute all $\langle \mathcal{O}_{k_1} \mathcal{O}_{k_2} \mathcal{O}_{k_3} \mathcal{O}_{k_4} \rangle$ in a **variety of theories with 8 (or 6) supercharges:**

\mathbf{SCFT}	Holographic description	Global symmetry
E-string: $6d \mathcal{N} = 1$	M5 on end-of-the-world M9 $$	$SU(2)_L \times SU(2)_R \times G_F$
Seiberg: $5d \mathcal{N} = 1$	D4-D8 system	$SU(2)_L \times SU(2)_R \times G_F$
$4d \mathcal{N} = 2$	D3 near F-theory singularities	$SU(2)_L \times SU(2)_R \times U(1)_r \times G_F$

SCFT	Holographic description
$4d \mathcal{N} = 4 \text{ SYM} + \text{flavors: } 4d \mathcal{N} = 2$	D7 wrapping $AdS_5 \times S^3 \subset AdS_5 \times S^5$
$3d \mathcal{N} = 6 \text{ ABJM} + \text{flavors: } 3d \mathcal{N} = 3$	D6 wrapping $AdS_4 \times \mathbb{RP}^3 \subset AdS_4 \times \mathbb{CP}^3$

Alday, Behan, PF, Zhou

Interesting features

No contact terms when using Polyakov-Regge blocks, Color-kinematics duality, Double copy relations (AdS₅), Hidden conformal symmetry in (AdS₅)

Gluon exchange

Alday, Behan, PF, Zhou

$$M = \frac{\mathsf{c}_{s}\,\mathsf{n}_{s}}{s-2} + \frac{\mathsf{c}_{t}\,\mathsf{n}_{t}}{t-2} + \frac{\mathsf{c}_{u}\,\mathsf{n}_{u}}{u-2}$$

 $n_s = 4 - u - s\alpha$ $n_t = (1 - \alpha)(u + s\alpha - 4(1 + \alpha))$ $n_u = \alpha(u + \alpha(s - 4))$

$$\mathbf{c}_s + \mathbf{c}_t + \mathbf{c}_u = 0 \qquad \mathbf{n}_s + \mathbf{n}_t + \mathbf{n}_u = 0$$

Equivalently:

$$\mathcal{M} \sim \frac{1}{k} \frac{1}{(4-s-t)} \left(\frac{\mathsf{c}_s}{s-2} - \frac{\mathsf{c}_t}{t-2} \right)$$

Contact terms

Alday, Behan, PF, Zhou

These are just **polynomials in** *s*, *t* on which we impose **crossing symmetry**

Reduced Mellin amplitude

$$\mathcal{M}_{r} = \frac{a_{0}}{\lambda'} + \frac{1}{(\lambda')^{3/2}}(a_{10}s + a_{01}t) + \frac{1}{(\lambda')^{2}}(a_{20}^{2} + a_{11}st + a_{02}t^{2}) + \dots$$
$$\partial^{4} \qquad \partial^{6} \qquad \partial^{8}$$

N.B. For $d \neq 4$ higher-derivative corrections also affect OPE coefficients between super-gluons: contact+exchange. Similar to AdS₇ × S⁴

1 loop

Alday, Bissi, Zhou

$$\mathscr{M} = \frac{1}{k^2} \left[\mathsf{d}_{st} \mathscr{B}_{st} + \mathsf{d}_{su} \mathscr{B}_{su} + \mathsf{d}_{tu} \mathscr{B}_{tu} \right]$$



Large s, t: 8d box integral in flat space

Renormalization ambiguity

Graviton exchange Alday, Bissi, Zhou $M = \delta^{AB} \delta^{CD} M_s + \delta^{AD} \delta^{BC} M_t + \delta^{AC} \delta^{BD} M_u$

 M_s : stress-tensor multiplet exchange + contact terms

As for gluon exchange, fixed using correct polar part + SCWI

$$\mathscr{M} = \frac{1}{c} \left[\frac{\delta^{AB} \delta^{CD}}{s-2} + \frac{\delta^{AD} \delta^{BC}}{t-2} + \frac{\delta^{AC} \delta^{BD}}{u-2} \right]$$

Note: $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$ can be computed in other dimensions as well, but $\langle \mathcal{O}_{k_1} \mathcal{O}_{k_2} \mathcal{O}_{k_3} \mathcal{O}_{k_4} \rangle$ is problematic due to degeneracies in exchanged reps



What are we missing?

Free parameters



Localization constraints

$\mathcal{N} = 2$ mass deformation $4d \ \mathcal{N} = 2$ flavor current multiplet: $(J_{\mu}, \Sigma, \phi_{(\alpha\beta)}, ...)$

 $\delta S \sim \int \mathrm{d}^4 x \sqrt{-g} \frac{m_A}{R_{S^4}} \phi^A + \dots$

For gauge theories, **supersymmetric localization** allows to compute

$$F(m^A) = \log Z(m^A) = \log \langle e^{i \,\delta S} \rangle$$

Taking four derivatives

$$\partial_{m^A} \partial_{m^B} \partial_{m^C} \partial_{m^D} F \Big|_{m=0} = \left\langle \int d^4 x_1 \sqrt{-g} \frac{1}{R_{S^4}} \phi^A(x_1) \dots \int d^4 x_4 \sqrt{-g} \frac{1}{R_{S^4}} \phi^D(x_4) \right\rangle + \dots$$

Integrated constraint Chester

Ward identities: all four-point functions can be expressed in terms of $\langle \phi^A \phi^B \phi^C \phi^D \rangle$

$$-\partial_{m^{A}}\partial_{m^{B}}\partial_{m^{C}}\partial_{m^{D}}(F - F^{\text{free}})\Big|_{m=0} = k^{2} \int dU dV \,\mathbf{f}(U, V) \sum_{r} P_{r}^{ABCD} \,\mathcal{G}_{r}^{\text{int}}(U, V)$$
$$\mathbf{f}(U, V) \sim \bar{D}_{1111}(U, V)$$

constraints = # quartic Casimirs of G_F

Note: $F = F(m^A, \tau, \overline{\tau}) \Rightarrow$ non-perturbative constraint in the complexified coupling constant

$G_F = SO(8)$

In our model $G_F = SO(8)$: 3 quartic Casimirs. Set $m_A = T_A^{ab}\mu_A$

3 independent constraints

$$\mathcal{F}_{v} = -4\partial_{\mu_{1}}^{2}\partial_{\mu_{2}}^{2}F|_{\mu=0}$$

$$\mathcal{F}_{c} = -\partial_{\mu_{1}}^{4} F|_{\mu=0} - \partial_{\mu_{1}}^{2} \partial_{\mu_{2}}^{2} F|_{\mu=0} + 2\partial_{\mu_{1}} \partial_{\mu_{2}} \partial_{\mu_{3}} \partial_{\mu_{4}} F|_{\mu=0}$$

$$\mathcal{F}_{s} = -\partial_{\mu_{1}}^{4} F|_{\mu=0} - \partial_{\mu_{1}}^{2} \partial_{\mu_{2}}^{2} F|_{\mu=0} - 2\partial_{\mu_{1}} \partial_{\mu_{2}} \partial_{\mu_{3}} \partial_{\mu_{4}} F|_{\mu=0}$$

Permuted by triality of SO(8)

Large *N*, large λ

[generalizing] Beccaria, Korchemsky, Tseytlin

Perturbative terms only

 $\log \lambda' \leftrightarrow \log(s)$ present in one-loop result in flat space limit $1/\lambda' \leftrightarrow$ first contact term ($F_{\mu\nu}^4$ terms in DBI action) The expressions are <u>exact</u> in $1/\lambda'$

Veneziano amplitude in AdS₅

At order 1/*N* we have the analogue of the Veneziano amplitude in AdS

Veneziano amplitude in flat space

$$\begin{aligned} \mathscr{A}^{V} &= \frac{F_{st}}{st} \operatorname{tr}(T^{A}T^{B}T^{C}T^{D}) + \frac{F_{su}}{su} \operatorname{tr}(T^{A}T^{B}T^{C}T^{D}) + \frac{F_{tu}}{tu} \operatorname{tr}(T^{A}T^{B}T^{C}T^{D}) \\ F_{st} &= \frac{\Gamma[1 - \alpha's]\Gamma[1 - \alpha't]}{\Gamma[1 - \alpha's - \alpha't]} \simeq 1 - st\zeta(2) (\alpha')^{2} + stu\zeta(3) (\alpha')^{3} + \dots \end{aligned}$$

 $(\alpha')^2$ term: matched with localization constraint alone. $(\alpha')^3$ term: localization constraint + flat space limit.

$$\mathcal{M} = \frac{\tilde{F}_{st}}{st} \operatorname{tr}(T^{A}T^{B}T^{C}T^{D}) + \frac{\tilde{F}_{su}}{su} \operatorname{tr}(T^{A}T^{B}T^{C}T^{D}) + \frac{\tilde{F}_{tu}}{tu} \operatorname{tr}(T^{A}T^{B}T^{C}T^{D})$$
$$\tilde{F}_{st} = \simeq 1 - \frac{24}{\lambda'} (s-2) (t-2) \zeta(2) + \frac{192}{(\lambda')^{3/2}} (s-2) (t-2) (u-2) \zeta(3) (\alpha')^{3} + \dots$$

Large N, finite g_{YM}

Including instantons at large N

 $\begin{aligned} \mathscr{F}_{v} &= g_{v}(\tau_{s}, \bar{\tau}_{s}) + \tilde{F}(N) = 8 \log[\sqrt{\tau_{2}} |\theta_{2}(\tau_{s})|^{2}] - 24 \log[\sqrt{\tau_{2}} |\eta(\tau_{s})|^{2}] + \tilde{F}(N) \\ \mathscr{F}_{c} &= g_{c}(\tau_{s}, \bar{\tau}_{s}) + \tilde{F}(N) = 8 \log[\sqrt{\tau_{2}} |\theta_{3}(\tau_{s})|^{2}] - 24 \log[\sqrt{\tau_{2}} |\eta(\tau_{s})|^{2}] + \tilde{F}(N) \\ \mathscr{F}_{s} &= g_{s}(\tau_{s}, \bar{\tau}_{s}) + \tilde{F}(N) = 8 \log[\sqrt{\tau_{2}} |\theta_{4}(\tau_{s})|^{2}] - 24 \log[\sqrt{\tau_{2}} |\eta(\tau_{s})|^{2}] + \tilde{F}(N) \end{aligned}$

$$\tilde{F}(N) = 8\log(2\pi N) + 4\left(\frac{1}{N} - \frac{7}{48N^2} + ...\right)$$

$$SL(2,\mathbb{Z}) action \iff SO(8) triality$$
$$\tau_{s} = \tau_{1} + i \tau_{2} = \frac{\theta_{YM}}{\pi} + \frac{8\pi i}{(g'_{YM})^{2}}$$
$$S: \quad \tau_{s} \to -1/\tau_{s} \qquad \mathbf{35}_{v} \leftrightarrow \mathbf{35}_{s}$$
$$T: \quad \tau_{s} \to \tau_{s} + 1 \qquad \mathbf{35}_{c} \leftrightarrow \mathbf{35}_{s}$$

Contact terms at finite g_{YM}

 $\mathcal{M}^{(\partial^4)}$: 4-derivative contact term, with 3 coefficients $c_i(\tau_s, \bar{\tau}_s)$

$$\mathcal{M}_{v}^{(\partial^{4})} = g_{v} + 8 \log(2\pi N) + \text{const}$$
$$\mathcal{M}_{c}^{(\partial^{4})} = g_{c} + 8 \log(2\pi N) + \text{const}$$
$$\mathcal{M}_{s}^{(\partial^{4})} = g_{s} + 8 \log(2\pi N) + \text{const}$$

 $\mathcal{M}^{(\partial^6)}$: 6-derivative contact term, with 5 coefficients $c_i(\tau_s, \bar{\tau}_s)$

$$\mathscr{M}^{(\partial^6)} = \frac{E_{3/2}(\tau, \bar{\tau})}{(s T_s^{ABCD} + t T_t^{ABCD} + u T_u^{ABCD})}$$

Non-holomorphic Eisenstein series: $4\tau_2^2 \partial_{\tau} \partial_{\bar{\tau}} E_r(\tau, \bar{\tau}) = r(r-1)E_r(\tau, \bar{\tau})$



Huang, Wang, Yang, Zhou

More bootstrap

Two-loops, $\langle k_1 k_2 k_3 k_4 \rangle$ at one loop, contact terms and graviton exchange, other dimensions/theories, higher-point functions...

Alday, Gonçalves, Zhou

Veneziano amplitude in AdS₅

Compute all string corrections to tree-level gluon scattering in $AdS_5 \times S^3$, along the lines of Shapiro-Virasoro in $AdS_5 \times S^5$ Alday, Hansen, Silva

CK duality and DC in AdS

So far only clear at tree level in AdS_5 , what about other dimensions and/or subleasing orders? **Zhou**

More with localization?

Bootstrap results available for other rank N 4d $\mathcal{N} = 2$ SCFTs, can we apply localization constraints? Behan More numerical bootstrap + localization for general 4d $\mathcal{N} = 2$ theories? Chester

Thank you for the attention!