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Gluon scattering in AdS from supersymmetric localization

University of Genova, 21/06/2023

Based on

arXiv: **2103.10440** [hep-th] with **LF Alday, C Behan, X Zhou**

arXiv: **2305.01016** [hep-th] with **C Behan, S Chester**

Plan of the talk

Introduction

Gluons in AdS

Introduction

Long term goals

Solving an interacting QFT

Focus on
Gauge theories
(toy model for SM)

and
Conformal Field Theories
(chart space of QFTs)

with a certain amount of
Supersymmetry
(more tractable)

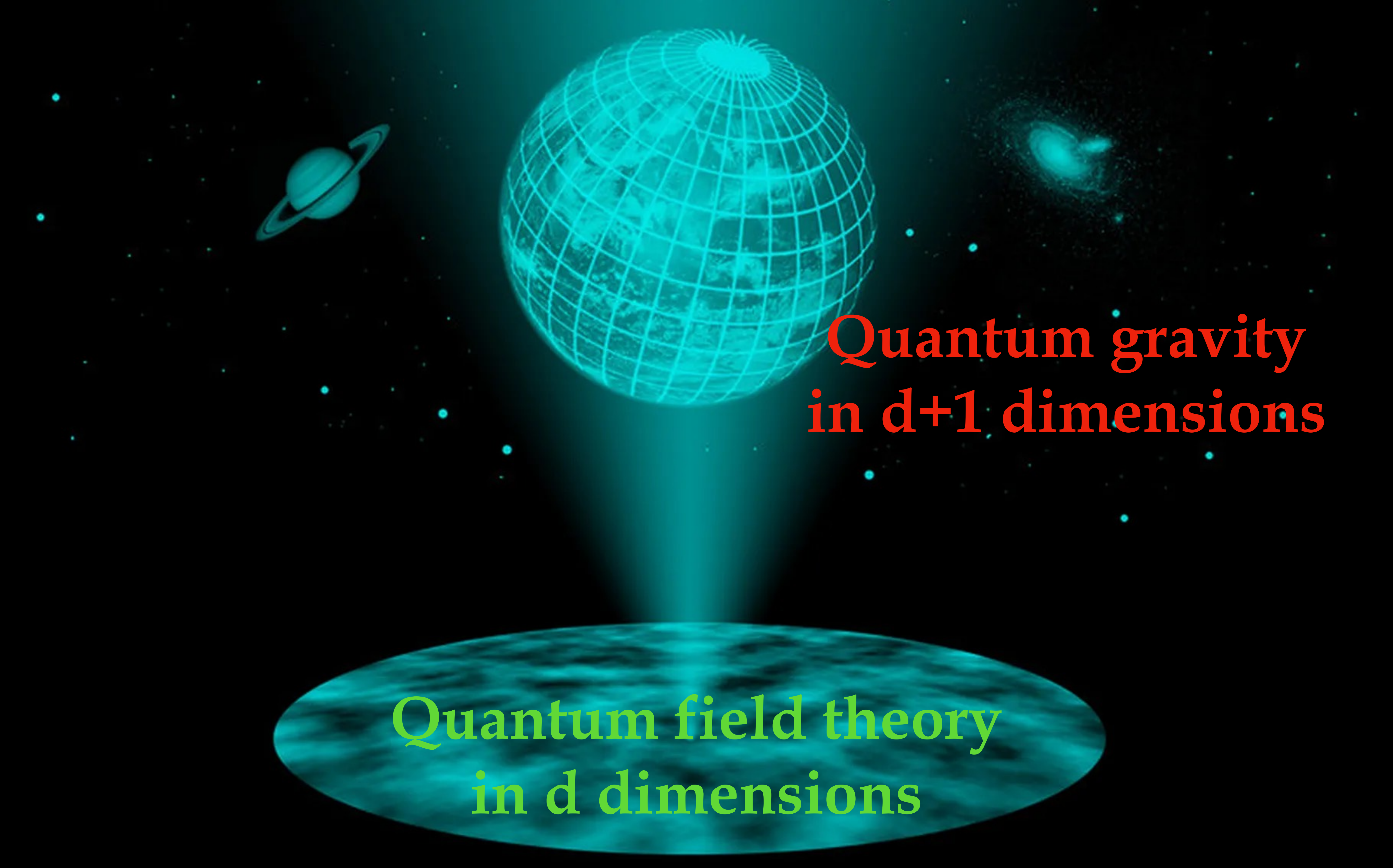
Understanding quantum gravity

Focus on
Superstring theory
(UV complete model)

in the low energy limit:
10d supergravity
(Semiclassical)

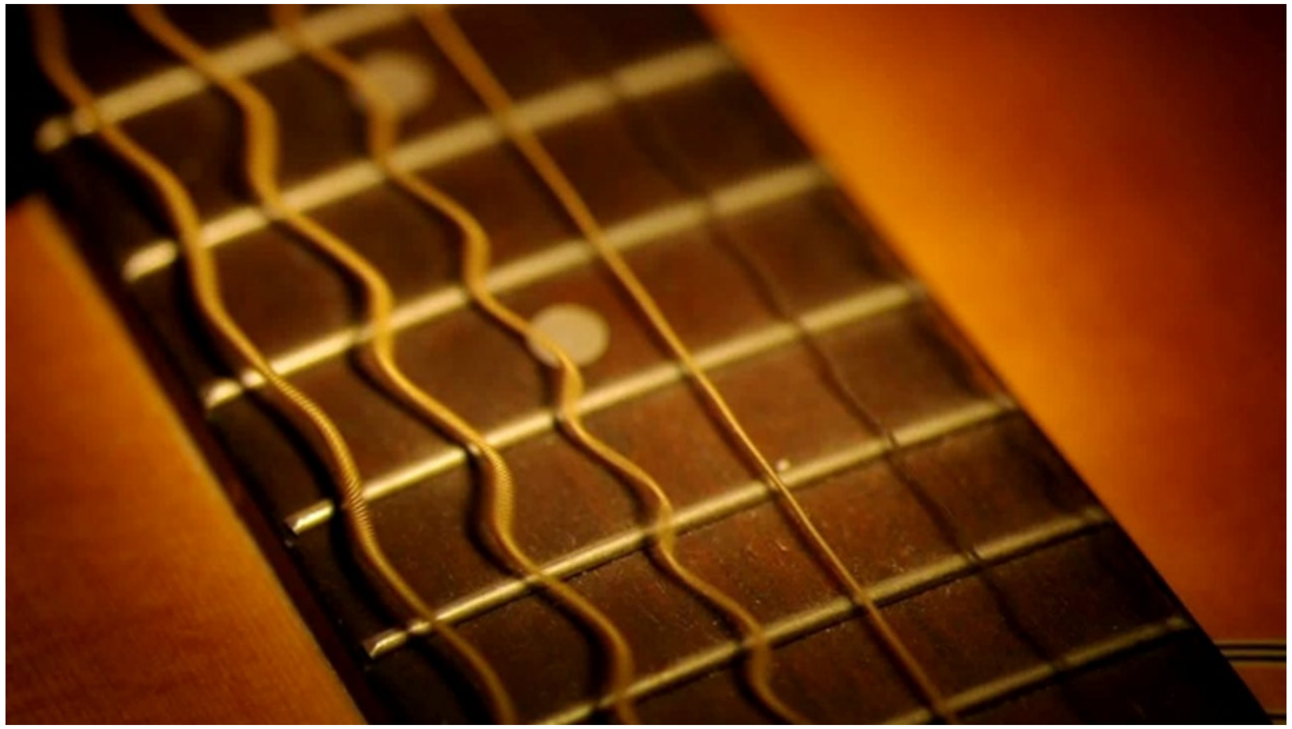
in particular
Supersymmetric solutions
(easier, interesting geometry)


STRONGLY COUPLED QFT
QUANTUM GRAVITY
Corporate needs you to find the differences between this picture and this picture.
They're the same picture.
imgflip.com

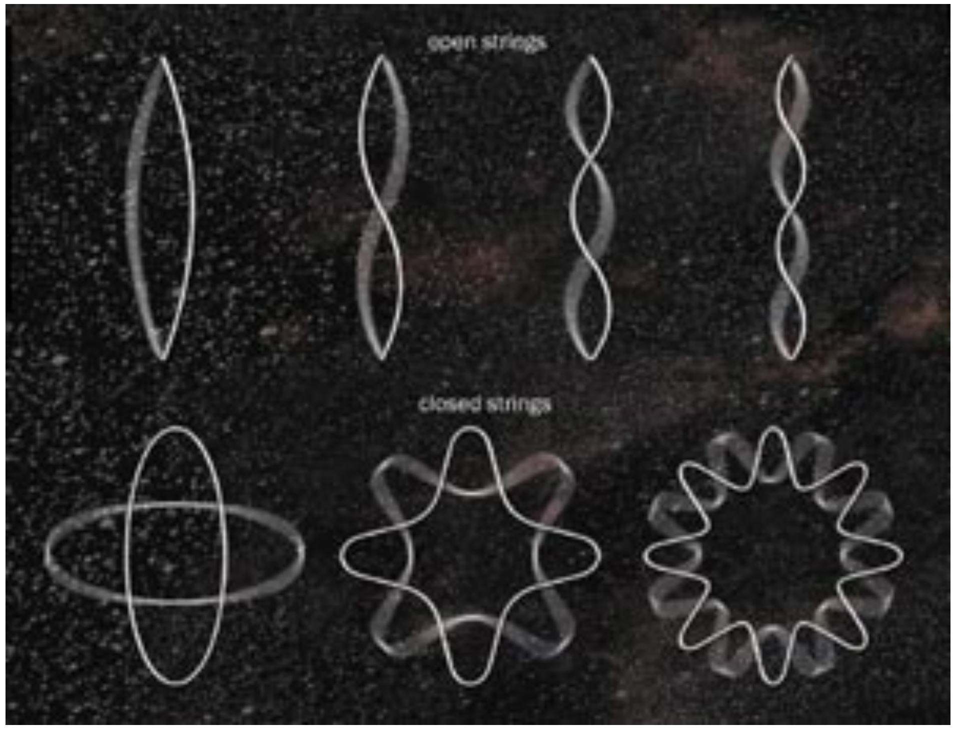


AdS/CFT Correspondence

String theory



Musical notes



Particles

Spectrum

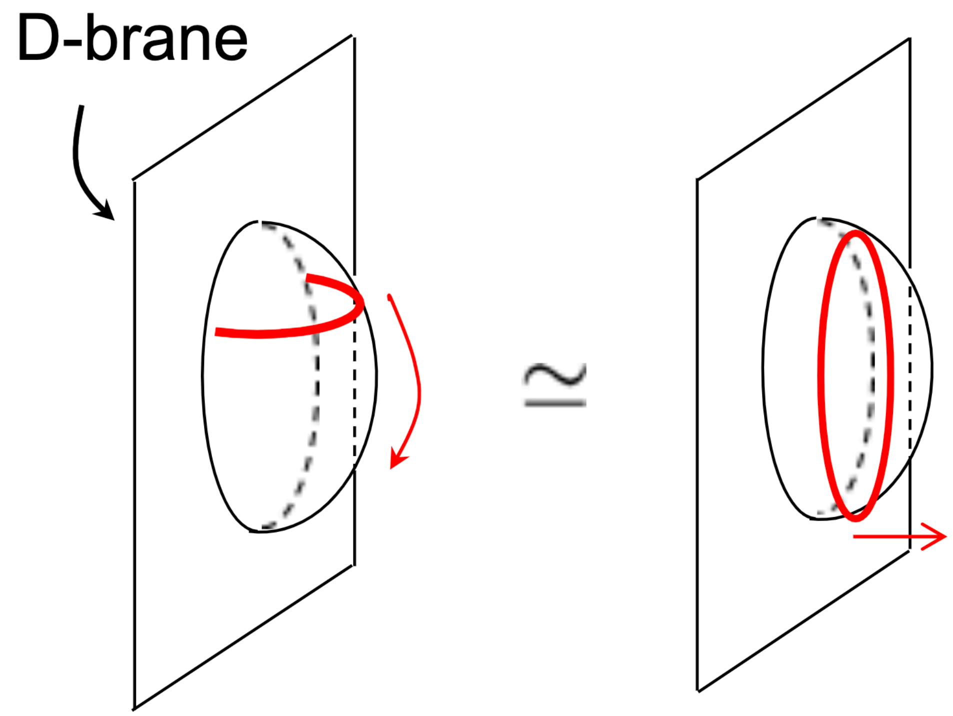
Massless sector ($s \leq 1$ for open strings, $s \leq 2$ for closed strings)

+

∞ tower of massive higher spin fields

UV-complete model of quantum gravity

Open strings
=
gauge theory



Closed strings
=
gravity

Gauge theory living on the brane

↕

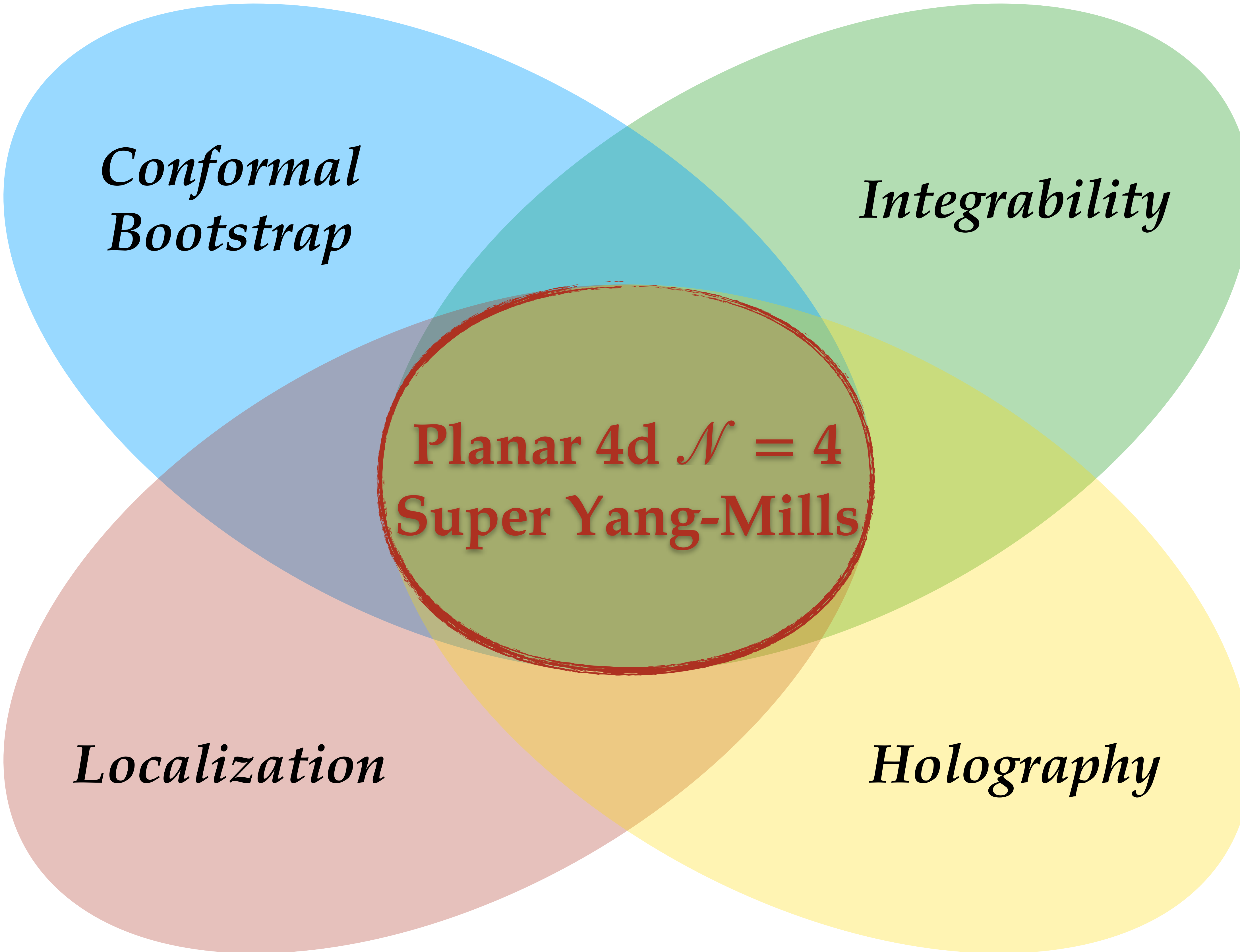
Propagation of gravity close to the brane

4d N=4 Super Yang-Mills	**$AdS_5 \times S^5$**
$SU(N)$ gauge theory, adjoint fields	Solution of 10d IIB supergravity
$A_\mu\,,\quad \Phi^{A=1,\ldots,6}\,,\quad \psi^{\alpha=1,\ldots,4}$	$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}(\text{matter})$
$\mathscr{L}(A,\Phi,\psi)$ fixed by max susy	Preserves maximal susy
$SO(4,2)$ conformal symmetry	$SO(4,2)$ isometry of AdS_5
$SU(4)_R$ global symmetry	$SU(4) = SO(6)$ isometry of S^5
Parameters $g_{YM}\,, N$	Parameters $g_s\,, \ell_s$

$$g_s = \frac{g_{YM}^2}{4\pi^2}\,,\quad \ell_s = \lambda^{-1/4} = (g_{YM}^2 N)^{-1/4}$$

Well-understood for

't Hooft/planar limit	
$g_{YM} \to 0\,,\quad N \to \infty\,,\quad \lambda \to \infty$	$g_s \to 0$ (no string loops) $\ell_s \to 0$ (no HD corrections)


Conformal Bootstrap
Integrability
Planar 4d $\mathcal{N} = 4$ Super Yang-Mills
Localization
Holography

More on the correspondence

$$\mathrm{d}s^2(\mathrm{AdS}_{d+1}) = \frac{\mathrm{d}z^2 + \mathrm{d}s^2(\mathbb{R}^{1,d-1})}{z^2} \longrightarrow$$

Conformal boundary at $z = 0$ where the dual CFT lives

$$\left\langle \mathrm{e}^{i\int_{\partial AdS}\bar{\phi}\mathcal{O}} \right\rangle_{CFT} = \int_{AdS} [\mathrm{d}\phi]\mathrm{e}^{i\,S[\phi]}\Big|_{\phi(\partial AdS)=\bar{\phi}}$$

CFT operators	**Fields in AdS**
Scalar	*Scalar*
Conserved current	*Gauge field*
Stress tensor	*Graviton*

CFT correlators

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \frac{1}{x_{12}^{2\Delta}}$$

Δ : conformal dimension

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\rangle = \frac{C_{123}}{x_{12}^{\Delta}x_{13}^{\Delta}x_{23}^{\Delta}}$$

C_{123} : OPE coefficients

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{1}{x_{12}^{2\Delta}x_{34}^{2\Delta}}G(U,V) \qquad U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \qquad V = \frac{x_{14}^2 x_{24}^2}{x_{13}^2 x_{24}^2}$$

The function G(U, V) :

Is a dynamical object (equivalent of 4-pt scattering amplitude)

Contains information about the spectrum and OPE coefficients

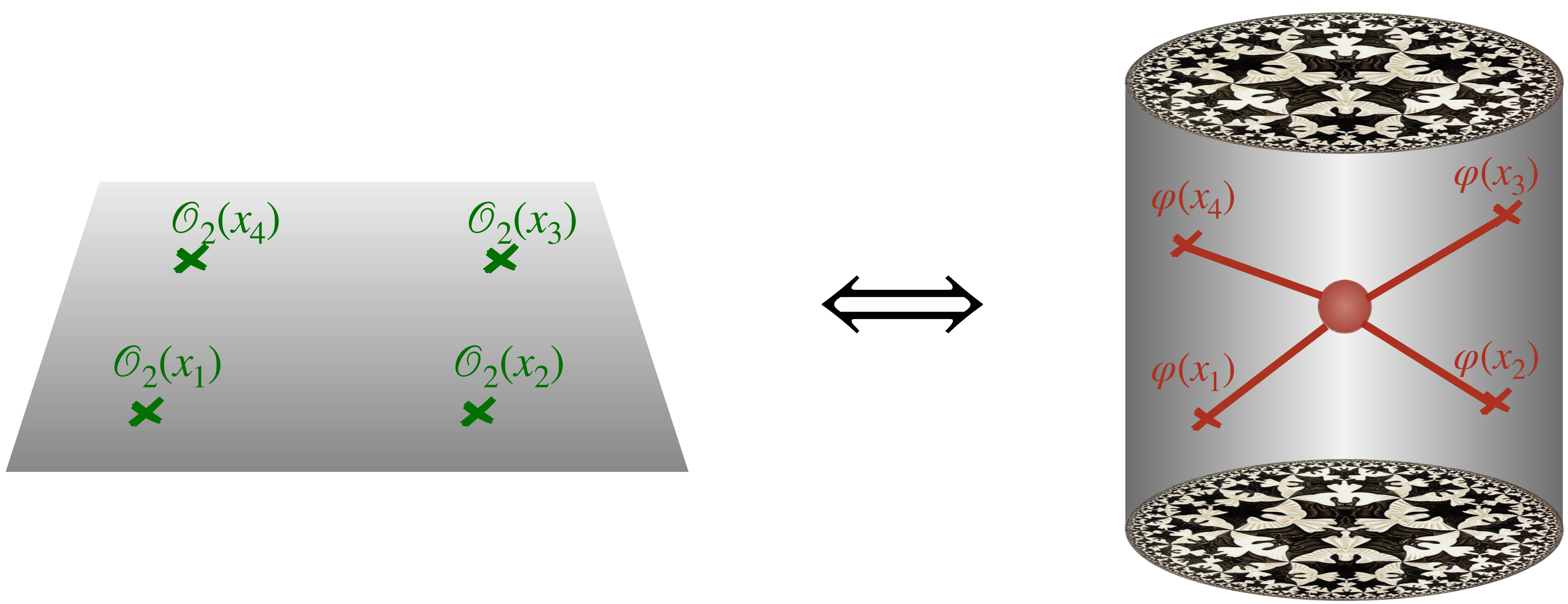
Must satisfy $V^{2\Delta}G(U,V) = U^{2\Delta}G(V,U)$ *: bootstrap equation*

Graviton scattering

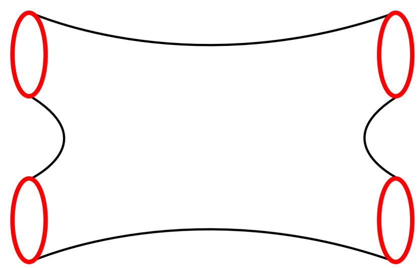
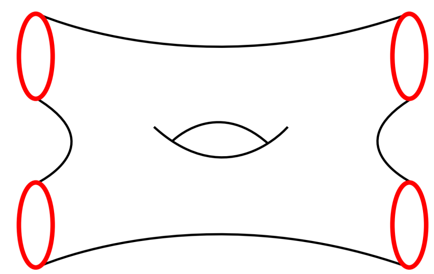
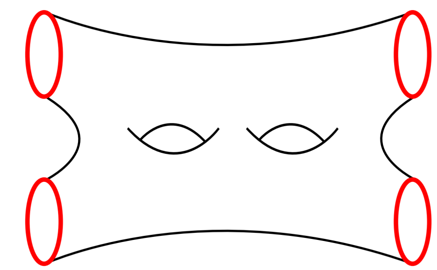
$\langle T_{\mu_1\nu_1} T_{\mu_2\nu_2} T_{\mu_3\nu_3} T_{\mu_4\nu_4} \rangle_{CFT}$ $\xrightarrow{\text{is fixed by susy in terms of}}$ $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle_{CFT}$

is dual to $\downarrow$ $\qquad\qquad$ $\downarrow$ *is dual to*

$\langle g_{\mu_1\nu_1} g_{\mu_2\nu_2} g_{\mu_3\nu_3} g_{\mu_4\nu_4} \rangle_{AdS}$ $\xrightarrow[\text{is fixed by susy in terms of}]{}$ $\langle \varphi\varphi\varphi\varphi \rangle_{AdS}$



The string theory S-matrix

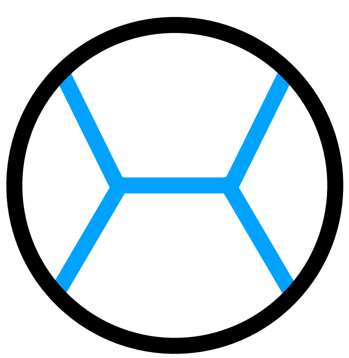
$\mathscr{A}(\alpha', g_s; s, t, u) =$  $+ g_s^2$  $+ g_s^4$  $+ \ldots$

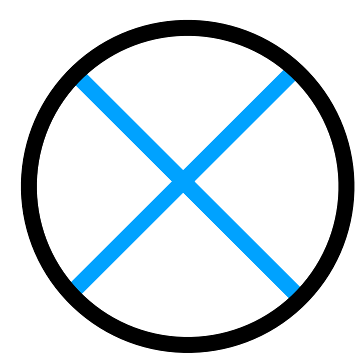
$$\mathscr{A}^{(g=0)}(\alpha'; s, t) = \frac{1}{s\,t\,u} + \ell_s^6 + \ell_s^{10}(s^2 + t^2 + u^2) + \ldots$$

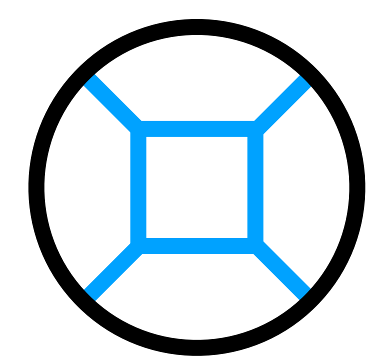
Loops
(Genus expansion)

2-derivative
Supergravity

Higher-derivative
String corrections







WAIT BUT WHY

Conformal field theory from gravity

One would like to learn about **strongly-coupled QFTs**: very hard problem!

We can focus on CFTs, but **analytic computations** are still challenging.

In AdS/CFT at large N we have a **weakly coupled supergravity description** in the bulk, which allows to access

- Strongly-coupled/**non-Lagrangian** CFTs
- **Non-protected CFT data**, otherwise hard even with Lagrangian

Gravity from conformal field theory

AdS/CFT can be used as a definition of **quantum gravity in AdS**.

We can use **CFT techniques** to study

- The effect of **quantum gravity** in AdS
- **Effective actions** of string/M-theory perturbatively

Scattering amplitudes program in AdS

For **scattering amplitudes in flat space**: rich story, plenty of interesting physical and mathematical **structures** hidden in Lagrangian description.

A natural question: **what happens in curved space?**
Simplest case to look at is **AdS**.

- Generalizations of the **structures of flat space amplitudes**?

  e.g. color-kinematics duality, double copy, MHV limit, CHY formulae, …

- **New hidden features** for AdS amplitudes?

  e.g. Parisi-Sourlas dimensional reduction, hidden conformal symmetry, …

How do we compute it?

In principle

Reduce $\mathscr{L}_{10d}$ on internal space: gives **5d supergravity Lagrangian**
[massless gravity + KK modes]

↓

Extract "**Feynman rules**"

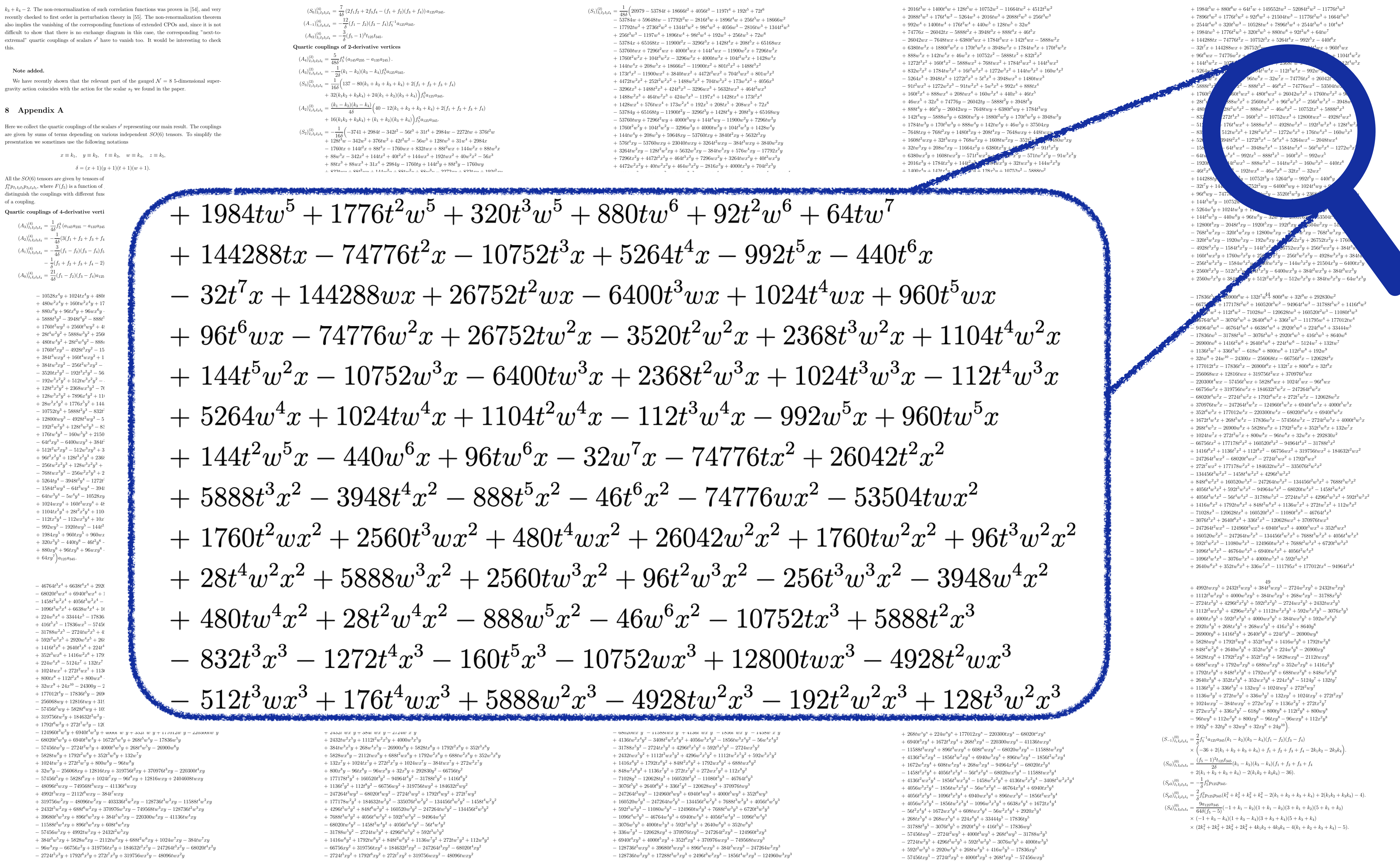
↓

Add all the relevant **Witten diagrams** to obtain the result

In practice…

Effective type IIB Lagrangian on $AdS_5 \times S^5$: quartic interactions of graviton Kaluza-Klein modes

Arutyunov, Frolov hep-th/9912210

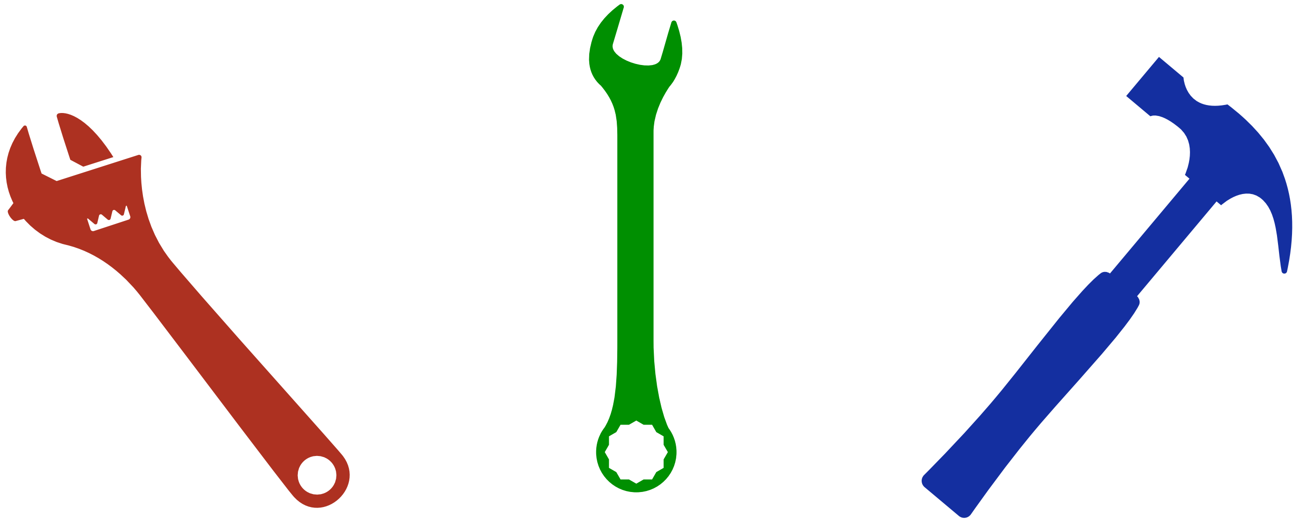


$$
\begin{aligned}
&+ 1984tw^5 + 1776t^2w^5 + 320t^3w^5 + 880tw^6 + 92t^2w^6 + 64tw^7 \\
&+ 144288tx - 74776t^2x - 10752t^3x + 5264t^4x - 992t^5x - 440t^6x \\
&- 32t^7x + 144288wx + 26752t^2wx - 6400t^3wx + 1024t^4wx + 960t^5wx \\
&+ 96t^6wx - 74776w^2x + 26752tw^2x - 3520t^2w^2x + 2368t^3w^2x + 1104t^4w^2x \\
&+ 144t^5w^2x - 10752w^3x - 6400tw^3x + 2368t^2w^3x + 1024t^3w^3x - 112t^4w^3x \\
&+ 5264w^4x + 1024tw^4x + 1104t^2w^4x - 112t^3w^4x - 992w^5x + 960tw^5x \\
&+ 144t^2w^5x - 440w^6x + 96tw^6x - 32w^7x - 74776tx^2 + 26042t^2x^2 \\
&+ 5888t^3x^2 - 3948t^4x^2 - 888t^5x^2 - 46t^6x^2 - 74776wx^2 - 53504twx^2 \\
&+ 1760t^2wx^2 + 2560t^3wx^2 + 480t^4wx^2 + 26042w^2x^2 + 1760tw^2x^2 + 96t^3w^2x^2 \\
&+ 28t^4w^2x^2 + 5888w^3x^2 + 2560tw^3x^2 + 96t^2w^3x^2 - 256t^3w^3x^2 - 3948w^4x^2 \\
&+ 480tw^4x^2 + 28t^2w^4x^2 - 888w^5x^2 - 46w^6x^2 - 10752tx^3 + 5888t^2x^3 \\
&- 832t^3x^3 - 1272t^4x^3 - 160t^5x^3 - 10752wx^3 + 12800twx^3 - 4928t^2wx^3 \\
&- 512t^3wx^3 + 176t^4wx^3 + 5888w^2x^3 - 4928tw^2x^3 - 192t^2w^2x^3 + 128t^3w^2x^3
\end{aligned}
$$

Tree level Witten diagram for φ^4 interaction with no derivatives, dual to $\langle \mathcal{O}_{\Delta=4}\, \mathcal{O}_{\Delta=4}\, \mathcal{O}_{\Delta=4}\, \mathcal{O}_{\Delta=4} \rangle$

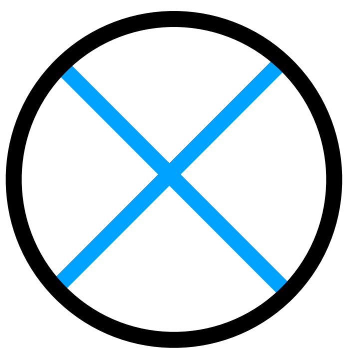
$$
\begin{aligned}
&\frac{1}{(z-\bar{z})^6}\, 2 \left(26 z^2 - 13 z^3 + 88 z \bar{z} - 127 z^2 \bar{z} + 26 z^3 \bar{z} + 26 \bar{z}^2 - 127 z \bar{z}^2 + 88 z^2 \bar{z}^2 - 13 \bar{z}^3 + 26 z \bar{z}^3\right) + \\
&\frac{1}{(z-\bar{z})^8} 4 \left(\mathrm{Log}[1-z] + \mathrm{Log}[1-\bar{z}]\right) \\
&\quad \left(25 z^3 - 28 z^4 + 6 z^5 + 185 z^2 \bar{z} - 388 z^3 \bar{z} + 222 z^4 \bar{z} - 28 z^5 \bar{z} + 185 z \bar{z}^2 - 848 z^2 \bar{z}^2 + \right. \\
&\qquad 1032 z^3 \bar{z}^2 - 388 z^4 \bar{z}^2 + 25 z^5 \bar{z}^2 + 25 \bar{z}^3 - 388 z \bar{z}^3 + 1032 z^2 \bar{z}^3 - 848 z^3 \bar{z}^3 + \\
&\qquad \left. 185 z^4 \bar{z}^3 - 28 \bar{z}^4 + 222 z \bar{z}^4 - 388 z^2 \bar{z}^4 + 185 z^3 \bar{z}^4 + 6 \bar{z}^5 - 28 z \bar{z}^5 + 25 z^2 \bar{z}^5\right) - \\
&\frac{1}{(z-\bar{z})^8} 4 \left(\mathrm{Log}[z] + \mathrm{Log}[\bar{z}]\right) \left(-6 z^4 + 3 z^5 - 96 z^3 \bar{z} + 111 z^4 \bar{z} - 22 z^5 \bar{z} - 216 z^2 \bar{z}^2 + \right. \\
&\qquad 516 z^3 \bar{z}^2 - 292 z^4 \bar{z}^2 + 25 z^5 \bar{z}^2 - 96 z \bar{z}^3 + 516 z^2 \bar{z}^3 - 632 z^3 \bar{z}^3 + 185 z^4 \bar{z}^3 - \\
&\qquad \left. 6 \bar{z}^4 + 111 z \bar{z}^4 - 292 z^2 \bar{z}^4 + 185 z^3 \bar{z}^4 + 3 \bar{z}^5 - 22 z \bar{z}^5 + 25 z^2 \bar{z}^5\right) + \frac{1}{(z-\bar{z})^9} \\
&4 \left(6 z^4 - 6 z^5 + z^6 + 96 z^3 \bar{z} - 162 z^4 \bar{z} + 72 z^5 \bar{z} - 6 z^6 \bar{z} + 216 z^2 \bar{z}^2 - 672 z^3 \bar{z}^2 + 603 z^4 \bar{z}^2 - \right. \\
&\qquad 162 z^5 \bar{z}^2 + 6 z^6 \bar{z}^2 + 96 z \bar{z}^3 - 672 z^2 \bar{z}^3 + 1168 z^3 \bar{z}^3 - 672 z^4 \bar{z}^3 + 96 z^5 \bar{z}^3 + 6 \bar{z}^4 - 162 z \bar{z}^4 + \\
&\qquad \left. 603 z^2 \bar{z}^4 - 672 z^3 \bar{z}^4 + 216 z^4 \bar{z}^4 - 6 \bar{z}^5 + 72 z \bar{z}^5 - 162 z^2 \bar{z}^5 + 96 z^3 \bar{z}^5 + \bar{z}^6 - 6 z \bar{z}^6 + 6 z^2 \bar{z}^6\right) \\
&\quad \left(\mathrm{Log}[1-z]\, \mathrm{Log}[z] - \mathrm{Log}[z]\, \mathrm{Log}[1-\bar{z}] + \mathrm{Log}[1-z]\, \mathrm{Log}[\bar{z}] - \right. \\
&\qquad \left. \mathrm{Log}[1-\bar{z}]\, \mathrm{Log}[\bar{z}] + 2\, \mathrm{PolyLog}[2, z] - 2\, \mathrm{PolyLog}[2, \bar{z}]\right)
\end{aligned}
$$

New tools to make progress



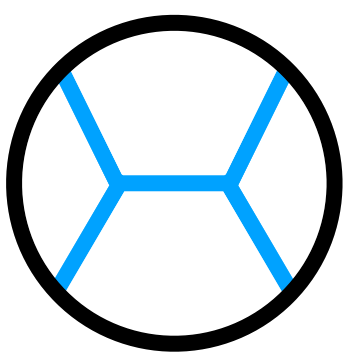
Mellin space

$$\mathcal{G}(U,V) = \int_{-i\infty}^{i\infty} \frac{ds dt}{(4\pi i)^2} U^{\frac{s}{2}} V^{\frac{t}{2}} \Gamma\left[\Delta - \frac{s}{2}\right]^2 \Gamma\left[\Delta - \frac{t}{2}\right]^2 \Gamma\left[\Delta - \frac{u}{2}\right]^2 \mathcal{M}(s,t)$$



Contact diagrams: $\mathcal{L}_{AdS} \supset \partial^{2L}\phi^4$

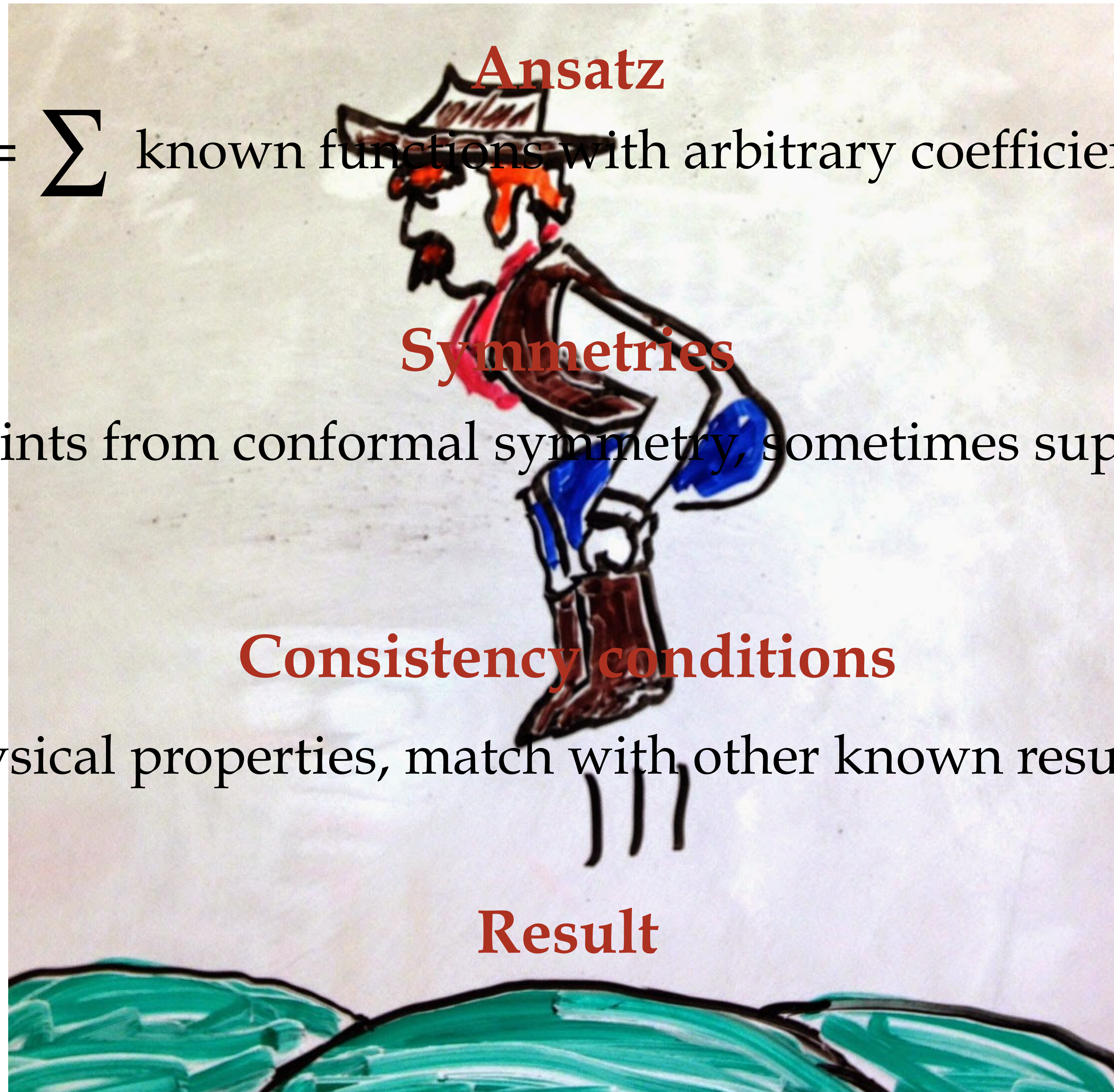
$$\mathcal{M}(s,t) = \mathrm{Poly}^{(L)}(s,t)$$



Exchange diagrams: $\mathcal{L}_{AdS} \supset \phi^3$

$$\mathcal{M}^{(s)}_{\Delta,\ell}(s,t) = \sum_{m=0}^{\infty} \frac{Q_m^{(\ell)}(t,u)}{s - \Delta + \ell - 2m} + P^{(\ell-1)}(s,t)$$

The conformal bootstrap



Ansatz

Observable = $\sum$ known functions with arbitrary coefficients to be fixed

Symmetries

Use constraints from conformal symmetry, sometimes supersymmetry

Consistency conditions

Require physical properties, match with other known results, limits etc

Result


How far can we get?

Tree level

Well understood

Loops

Results at 1 and 2 loops, complicated non-analytic structure.
Up to **renormalization ambiguities**

String corrections

Contact terms: very simple results (polynomial).
Up to **overall coefficients**

What about the free parameters?

Supersymmetric localization

For all 4d $\mathcal{N} = 2$ Lagrangian QFTs

$$Z = e^{-F} = \int [\mathrm{d}\varphi]\, e^{i\,S[\varphi]} = \int [\mathrm{d}U]\, e^{-\frac{8\pi^2}{g_{YM}^2}\mathrm{tr}\,U^2}\, |Z_{\text{1-loop}}(U)|^2\, |Z_{\text{inst}}(U, g_{YM})|^2$$

Deformations couple to stress tensor/conserved currents

$$\partial_m^4 F = \int [\mathrm{d}U \mathrm{d}V]\, G(U, V)$$

Non-perturbative constraint on correlation functions

Gluons in AdS

Summary

Gluon scattering at tree level in models with 1/2-maximal supersymmetry

First example, across dimensions d=3,4,5,6, all KK modes

Localization at finite coupling for interesting 4d N=2 SCFT

Surprisingly simple partition function, including instantons

First two string corrections at finite string coupling

Analogue of Veneziano amplitude in AdS_5

A simple D3-D7 system

D7 branes at $\mathbb{Z}_2$ **orientifold singularity**, constant (arbitrary) axio-dilaton
$\leftrightarrow$ F-theory on K3=$T^4/\mathbb{Z}_2$, with D_4 singularities: $SO(8)$ gauge group on D7

Sen

Probe with N D3 branes parallel to the D7:

AdS_5 $\quad\quad$ $S^5/\mathbb{Z}_2$

	0	1	2	3	4	5	6	7	8	9
D3	$\times$	$\times$	$\times$	$\times$						
D7	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		

$S^3 \subset S^5/\mathbb{Z}_2$

$\mathrm{AdS}_5 \times S^5/\mathbb{Z}_2$ NH geometry, with D7 wrapping $\mathrm{AdS}_5 \times S^3$

Aharony, Fayyazuddin, Maldacena, Spalinski

Field theory description

Aharony, Banks, Douglas, Seiberg, Sonnenschein, Theisen, Yankielowicz

D7 branes

SO(8) vector multiplet: $8d\ \mathcal{N} = 1$ *SYM*

World-volume theory on D3 branes

D3 (+O7) → $USp(2N)$ *adjoint vector + 1 antisymmetric hyper*

D7 → *8 fundamental half-hypers, SO(8) flavor symmetry*

For $N = 1$ $USp(2) \simeq SU(2) \quad \rightarrow \quad \mathcal{N} = 2$ SQCD

N.B. String theory background defined for any (constant) τ
→ conformal manifold, Lagrangian description.

Gluons in AdS_5

Spectrum of AdS_5 fields:

The *supergravity* modes from the bulk → 1/4-BPS or longer reps
(AdS graviton and KK modes)

The *vector* multiplets living on D7 branes → 1/2-BPS reps
(AdS gluons and KK modes on S^3)

1/2-BPS multiplets of $4d\ \mathcal{N}=2$ SCFTs:
$\mathcal{O}_2^A$: flavor current multiplet ↔ AdS_5 gluons
$\mathcal{O}_k^A$: tower of short multiplets ↔ KK modes of D7 gluons on S^3

$k \leftrightarrow$ rep $\mathbf{k+1}$ of $SU(2)_R$, or spin $j_R = k/2$

Gluon 4-pt functions

$$\langle \mathcal{O}_2^A(1)\mathcal{O}_2^B(2)\mathcal{O}_2^C(3)\mathcal{O}_2^D(4)\rangle = \text{Pref} \times G^{ABCD}(U,V;\alpha) = \text{Pref} \times \sum_{r \in \text{Adj}\times\text{Adj}} P_r^{ABCD}\, G_r(U,V;\alpha)$$

$$U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}, \quad \alpha = \frac{\langle y_1, y_3\rangle\langle y_2, y_4\rangle}{\langle y_1, y_2\rangle\langle y_3, y_4\rangle}$$

$$\mathbf{28} \otimes \mathbf{28} = \mathbf{1} \oplus \mathbf{28} \oplus \underbrace{\mathbf{35_v} \oplus \mathbf{35_c} \oplus \mathbf{35_s}}_{\textit{Permuted by triality}} \oplus \mathbf{300} \oplus \mathbf{350}$$

All 4-point functions in the flavor current multiplet fixed by $G_r(U,V;\alpha)$
It also satisfies the **superconformal Ward identity**

$$G_r(U,V;\alpha) = G_r^{\text{protected}}(U,V;\alpha) + (1 - z\alpha)(1 - \bar{z}\alpha)\mathcal{G}_r(U,V)$$

$$M_r(s,t;\alpha) = \Theta \circ \mathcal{M}_r(s,t)$$

Couplings & central charges

In $\mathscr{L}_{\mathrm{AdS}}$ **gluons** have a self-coupling given by

$$g^2_{YM,AdS} \sim \frac{1}{k} \sim \frac{1}{N}$$

And a coupling to **gravity** given by

$$\frac{1}{G_N^{(5)}} \sim \frac{1}{c} \sim \frac{1}{N^2}$$

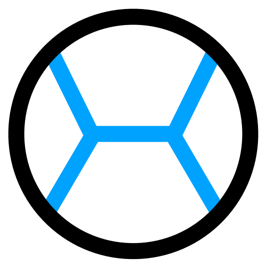
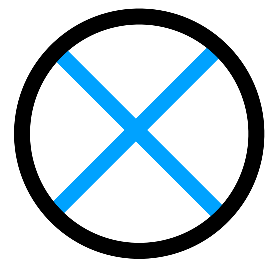
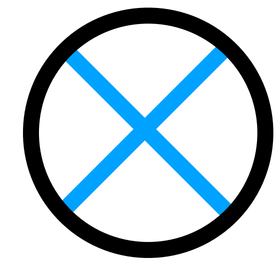
Aharony, Pawelczyk, Theisen, Yankielowicz
Blau, Narain, Gava

Gluon self-interactions dominate at leading order

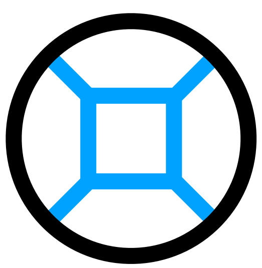
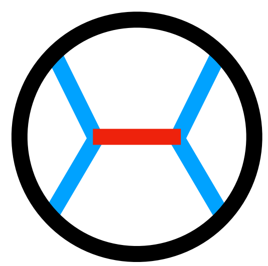
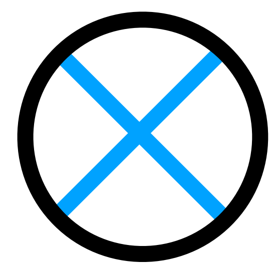
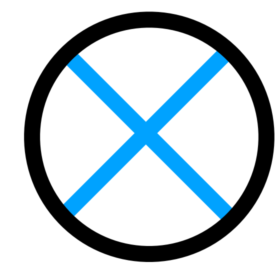
$\mathcal{O}(1/N)$: pure gauge theory in AdS_5 (+ string corrections)

$\mathcal{O}(1/N^2)$: gluon loops and graviton exchange (+ string corrections)

Perturbative expansion

$\mathscr{M} = \frac{1}{N}\Big($  $+ \frac{1}{\lambda'}$  $+ \frac{1}{(\lambda')^{3/2}}$  $+ \cdots \Big)$

Gluon tree   *Contact* (∂^4) *Contact* (∂^6)

$+\frac{1}{N^2}\Big($  $+$  $+$  $+ \log \lambda'$  $+ \cdots \Big)$

Gluon loop   *Graviton exchange*   *Contact* (∂^4) *Contact* (∂^4)

$+\mathcal{O}(1/N^3)$

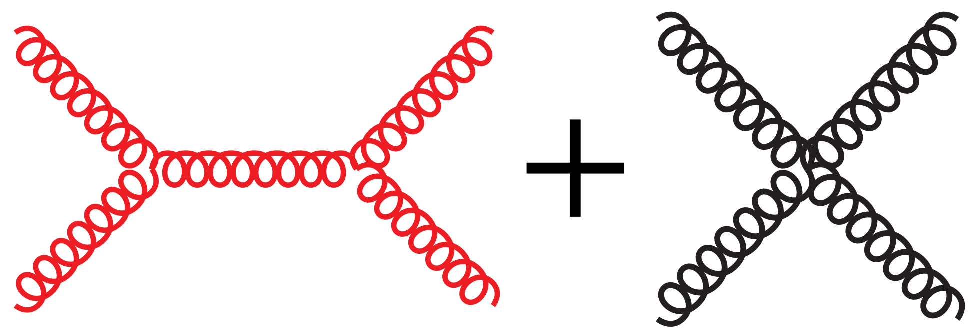
Tree level

Alday, Behan, PF, Zhou

$$\mathsf{c}_s = f^{I_1 I_2 J} f^{J I_3 I_4}$$
$$\mathsf{c}_t = f^{I_1 I_4 J} f^{J I_2 I_3}$$
$$\mathsf{c}_u = f^{I_1 I_3 J} f^{J I_4 I_2}$$

$$M = \mathsf{c}_s M_s + \mathsf{c}_t M_t + \mathsf{c}_u M_u$$

$$M_s \sim$$



Exchange: *full super-gluon multiplet*.
Fixed requiring same polar part as superblocks. Equivalently: MRV limit.

Contact: degree 0 as in flat space YM theory. Fixed by SCWI.

Tree level, all dimensions

The same procedure can be applied to compute all $\langle \mathcal{O}_{k_1}\mathcal{O}_{k_2}\mathcal{O}_{k_3}\mathcal{O}_{k_4}\rangle$ in a **variety of theories with 8 (or 6) supercharges:**

SCFT	Holographic description	Global symmetry
E-string: $6d$ $\mathcal{N}=1$	M5 on end-of-the-world M9	$SU(2)_L \times SU(2)_R \times G_F$
Seiberg: $5d$ $\mathcal{N}=1$	D4-D8 system	$SU(2)_L \times SU(2)_R \times G_F$
$4d$ $\mathcal{N}=2$	D3 near F-theory singularities	$SU(2)_L \times SU(2)_R \times U(1)_r \times G_F$

SCFT	Holographic description
$4d$ $\mathcal{N}=4$ SYM + flavors: $4d$ $\mathcal{N}=2$	D7 wrapping $AdS_5 \times S^3 \subset AdS_5 \times S^5$
$3d$ $\mathcal{N}=6$ ABJM + flavors: $3d$ $\mathcal{N}=3$	D6 wrapping $AdS_4 \times \mathbb{RP}^3 \subset AdS_4 \times \mathbb{CP}^3$

Alday, Behan, PF, Zhou

Interesting features

No contact terms when using Polyakov-Regge blocks,
Color-kinematics duality, Double copy relations (AdS_5),
Hidden conformal symmetry in (AdS_5)

Gluon exchange

Alday, Behan, PF, Zhou

$$M = \frac{\mathtt{c}_s \mathtt{n}_s}{s-2} + \frac{\mathtt{c}_t \mathtt{n}_t}{t-2} + \frac{\mathtt{c}_u \mathtt{n}_u}{u-2}$$

$\mathtt{n}_s = 4 - u - s\alpha$ $\quad$ $\mathtt{n}_t = (1-\alpha)(u + s\alpha - 4(1+\alpha))$ $\quad$ $\mathtt{n}_u = \alpha(u + \alpha(s-4))$

$$\mathtt{c}_s + \mathtt{c}_t + \mathtt{c}_u = 0 \qquad\qquad \mathtt{n}_s + \mathtt{n}_t + \mathtt{n}_u = 0$$

Equivalently:

$$\mathscr{M} \sim \frac{1}{k} \frac{1}{(4-s-t)} \left(\frac{\mathtt{c}_s}{s-2} - \frac{\mathtt{c}_t}{t-2} \right)$$

Contact terms

Alday, Behan, PF, Zhou

These are just **polynomials in** s, t on which we impose **crossing symmetry**

Reduced Mellin amplitude

$$\mathscr{M}_r = \frac{a_0}{\lambda'} + \frac{1}{(\lambda')^{3/2}}(a_{10}s + a_{01}t) + \frac{1}{(\lambda')^2}(a_{20}^2 + a_{11}st + a_{02}t^2) + \ldots$$

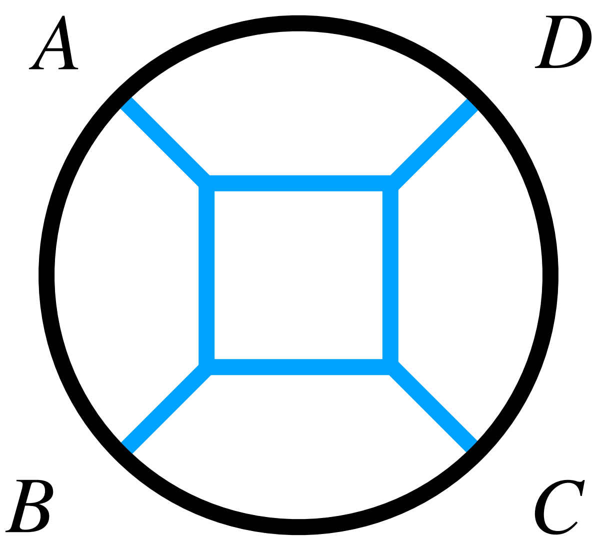
∂^4 ∂^6 ∂^8

N.B. For $d \neq 4$ higher-derivative corrections also affect OPE coefficients between super-gluons: contact+exchange. Similar to $\text{AdS}_7 \times S^4$

1 loop

Alday, Bissi, Zhou

$$\mathcal{M} = \frac{1}{k^2}\left[\mathtt{d}_{st}\,\mathcal{B}_{st} + \mathtt{d}_{su}\,\mathcal{B}_{su} + \mathtt{d}_{tu}\,\mathcal{B}_{tu}\right]$$



$$\mathtt{d}_{st} = f^{IAJ} f^{JBK} f^{KCL} f^{LDI}$$

$$\mathcal{B}_{st} = \sum_{m,n=2}^{\infty} \frac{c_{mn}}{(s-2m)(t-2n)} + a$$

Large s, t: 8d box integral in flat space

Renormalization ambiguity

Graviton exchange

Alday, Bissi, Zhou

$$M = \delta^{AB}\delta^{CD}\, M_s + \delta^{AD}\delta^{BC}\, M_t + \delta^{AC}\delta^{BD}\, M_u$$

M_s : *stress-tensor multiplet exchange + contact terms*

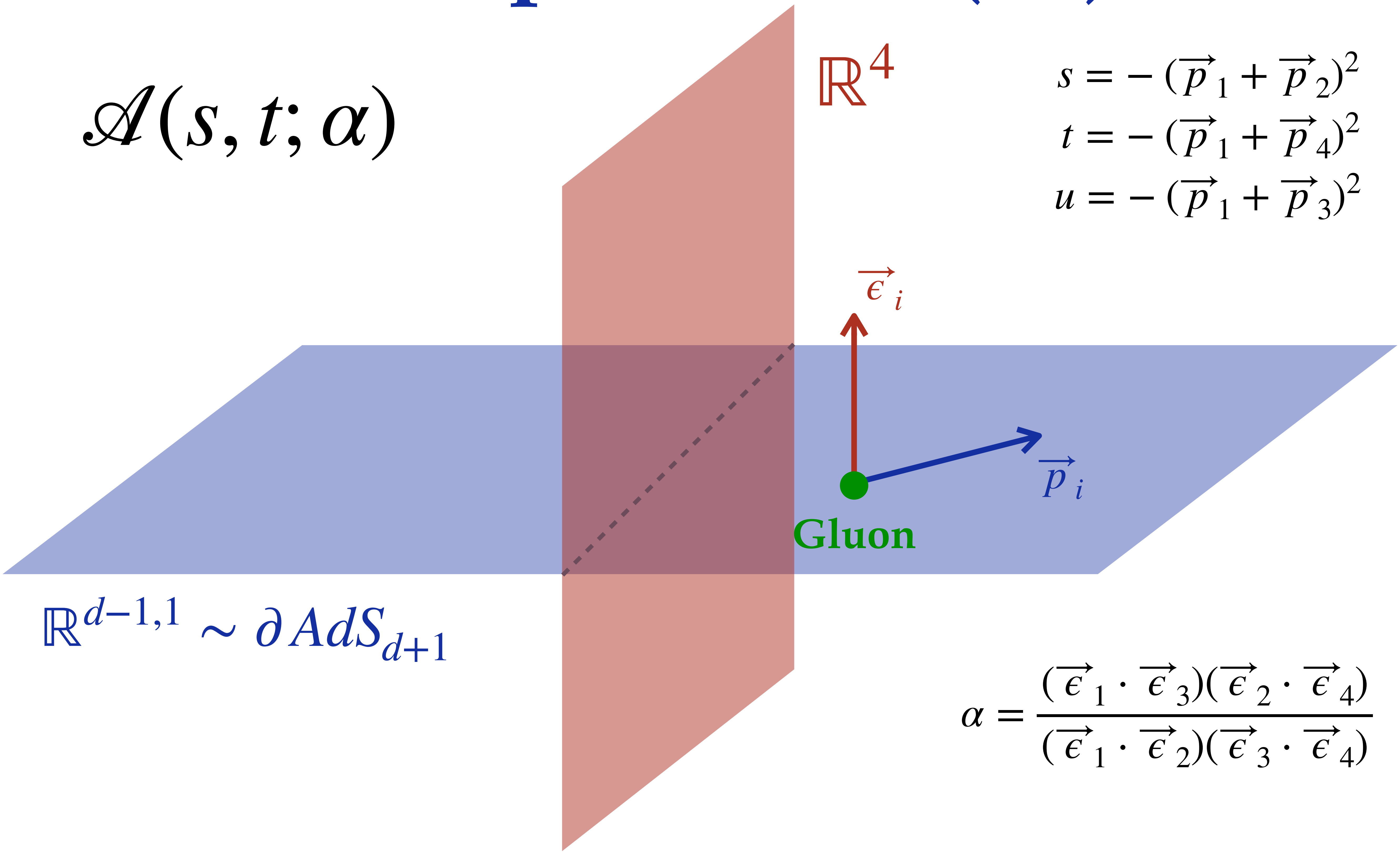
As for gluon exchange, fixed using **correct polar part + SCWI**

$$\mathscr{M} = \frac{1}{c}\left[\frac{\delta^{AB}\delta^{CD}}{s-2} + \frac{\delta^{AD}\delta^{BC}}{t-2} + \frac{\delta^{AC}\delta^{BD}}{u-2}\right]$$

Note: $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2 \rangle$ can be computed in other dimensions as well, but $\langle \mathcal{O}_{k_1}\mathcal{O}_{k_2}\mathcal{O}_{k_3}\mathcal{O}_{k_4} \rangle$ is problematic due to degeneracies in exchanged reps

Flat space limit (8d)

$$\mathscr{A}(s,t;\alpha)$$

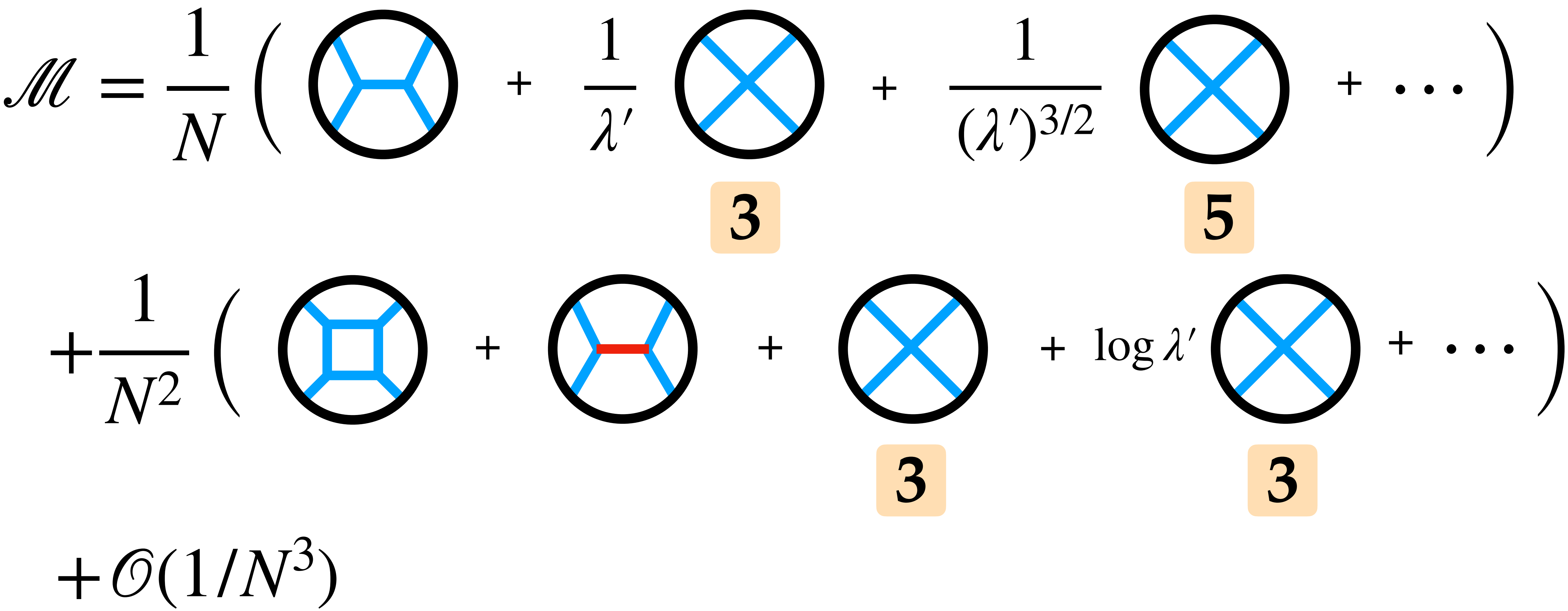


$$s = -(\vec{p}_1 + \vec{p}_2)^2$$
$$t = -(\vec{p}_1 + \vec{p}_4)^2$$
$$u = -(\vec{p}_1 + \vec{p}_3)^2$$

$$\alpha = \frac{(\vec{\epsilon}_1 \cdot \vec{\epsilon}_3)(\vec{\epsilon}_2 \cdot \vec{\epsilon}_4)}{(\vec{\epsilon}_1 \cdot \vec{\epsilon}_2)(\vec{\epsilon}_3 \cdot \vec{\epsilon}_4)}$$

What are we missing?

Free parameters



Localization constraints

$\mathcal{N} = 2$ mass deformation

Chester

$4d$ $\mathcal{N} = 2$ flavor current multiplet: $(J_\mu, \Sigma, \phi_{(\alpha\beta)}, \ldots)$

$$\delta S \sim \int \mathrm{d}^4 x \sqrt{-g}\, \frac{m_A}{R_{S^4}} \phi^A + \ldots$$

For gauge theories, **supersymmetric localization** allows to compute

$$F(m^A) = \log Z(m^A) = \log \langle \mathrm{e}^{i\,\delta S} \rangle$$

Taking four derivatives

$$\partial_{m^A}\partial_{m^B}\partial_{m^C}\partial_{m^D} F \Big|_{m=0} = \left\langle \int \mathrm{d}^4 x_1 \sqrt{-g}\, \frac{1}{R_{S^4}} \phi^A(x_1) \ldots \int \mathrm{d}^4 x_4 \sqrt{-g}\, \frac{1}{R_{S^4}} \phi^D(x_4) \right\rangle + \ldots$$

Integrated constraint

Chester

Ward identities: all four-point functions can be expressed in terms of $\langle \phi^A \phi^B \phi^C \phi^D \rangle$

$$-\partial_{m^A}\partial_{m^B}\partial_{m^C}\partial_{m^D}(F - F^{\text{free}})\Big|_{m=0} = k^2 \int \mathrm{d}U\mathrm{d}V\, \mathbf{f}(U,V) \sum_r P_r^{ABCD}\, \mathcal{G}_r^{\text{int}}(U,V)$$

$$\mathbf{f}(U,V) \sim \bar{D}_{1111}(U,V)$$

 # constraints = # quartic Casimirs of G_F

Note: $F = F(m^A, \tau, \bar{\tau}) \;\Rightarrow\;$ non-perturbative constraint in the complexified coupling constant

$G_F = SO(8)$

In our model $G_F = SO(8)$: 3 quartic Casimirs. Set $m_A = T_A^{ab}\mu_A$

3 independent constraints

$$\mathscr{F}_v = -4\partial^2_{\mu_1}\partial^2_{\mu_2}F|_{\mu=0}$$

$$\mathscr{F}_c = -\partial^4_{\mu_1}F|_{\mu=0} - \partial^2_{\mu_1}\partial^2_{\mu_2}F|_{\mu=0} + 2\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4}F|_{\mu=0}$$

$$\mathscr{F}_s = -\partial^4_{\mu_1}F|_{\mu=0} - \partial^2_{\mu_1}\partial^2_{\mu_2}F|_{\mu=0} - 2\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4}F|_{\mu=0}$$

Permuted by triality of $SO(8)$

Large N, large λ

[generalizing] Beccaria, Korchemsky, Tseytlin

Perturbative terms only

$$\mathcal{F}_v = 8\log\lambda' + 4\left(\frac{1}{N} - \frac{7}{48N^2} + \ldots\right)$$

$$\mathcal{F}_c = \mathcal{F}_s = \frac{32\pi^2}{\lambda'}N + 8\log(\lambda'/4) + 4\left(\frac{1}{N} - \frac{7}{48N^2} + \ldots\right)$$

$$\frac{1}{(g'_{YM})^2} = \frac{1}{g_{YM}^2} + \frac{\log 2}{2\pi^2}$$

$\log\lambda' \leftrightarrow \log(s)$ *present in one-loop result in flat space limit*

$1/\lambda' \leftrightarrow$ *first contact term ($F_{\mu\nu}^4$ terms in DBI action)*

The expressions are exact in $1/\lambda'$

Veneziano amplitude in AdS_5

At order $1/N$ we have the analogue of the Veneziano amplitude in AdS

Veneziano amplitude in flat space

$$\mathcal{A}^V = \frac{F_{st}}{st}\,\mathrm{tr}(T^AT^BT^CT^D) + \frac{F_{su}}{su}\,\mathrm{tr}(T^AT^BT^CT^D) + \frac{F_{tu}}{tu}\,\mathrm{tr}(T^AT^BT^CT^D)$$

$$F_{st} = \frac{\Gamma[1-\alpha' s]\Gamma[1-\alpha' t]}{\Gamma[1-\alpha' s-\alpha' t]} \simeq 1 - s\,t\,\zeta(2)\,(\alpha')^2 + s\,t\,u\,\zeta(3)\,(\alpha')^3 + \ldots$$

$(\alpha')^2$ **term:** matched with localization constraint alone.

$(\alpha')^3$ **term:** localization constraint + flat space limit.

$$\mathcal{M} = \frac{\tilde{F}_{st}}{st}\,\mathrm{tr}(T^AT^BT^CT^D) + \frac{\tilde{F}_{su}}{su}\,\mathrm{tr}(T^AT^BT^CT^D) + \frac{\tilde{F}_{tu}}{tu}\,\mathrm{tr}(T^AT^BT^CT^D)$$

$$\tilde{F}_{st} = \simeq 1 - \frac{24}{\lambda'}\,(s-2)\,(t-2)\,\zeta(2) + \frac{192}{(\lambda')^{3/2}}(s-2)\,(t-2)\,(u-2)\,\zeta(3)\,(\alpha')^3 + \ldots$$

Large N, finite g_{YM}

Including instantons at large N

$$\mathcal{F}_v = g_v(\tau_s, \bar{\tau}_s) + \tilde{F}(N) = 8\log[\sqrt{\tau_2}\,|\,\theta_2(\tau_s)\,|^2] - 24\log[\sqrt{\tau_2}\,|\,\eta(\tau_s)\,|^2] + \tilde{F}(N)$$

$$\mathcal{F}_c = g_c(\tau_s, \bar{\tau}_s) + \tilde{F}(N) = 8\log[\sqrt{\tau_2}\,|\,\theta_3(\tau_s)\,|^2] - 24\log[\sqrt{\tau_2}\,|\,\eta(\tau_s)\,|^2] + \tilde{F}(N)$$

$$\mathcal{F}_s = g_s(\tau_s, \bar{\tau}_s) + \tilde{F}(N) = 8\log[\sqrt{\tau_2}\,|\,\theta_4(\tau_s)\,|^2] - 24\log[\sqrt{\tau_2}\,|\,\eta(\tau_s)\,|^2] + \tilde{F}(N)$$

$$\tilde{F}(N) = 8\log(2\pi N) + 4\left(\frac{1}{N} - \frac{7}{48N^2} + \dots\right)$$

$$SL(2,\mathbb{Z})\ \textit{action} \ \leftrightarrow\ SO(8)\ \textit{triality}$$

$$\tau_s = \tau_1 + i\,\tau_2 = \frac{\theta_{YM}}{\pi} + \frac{8\pi i}{(g'_{YM})^2}$$

$$S: \quad \tau_s \to -1/\tau_s \qquad \mathbf{35}_v \leftrightarrow \mathbf{35}_s$$

$$T: \quad \tau_s \to \tau_s + 1 \qquad \mathbf{35}_c \leftrightarrow \mathbf{35}_s$$

Contact terms at finite g_{YM}

$\mathcal{M}^{(\partial^4)}$: 4-derivative contact term, with 3 coefficients $c_i(\tau_s, \bar{\tau}_s)$

$$\mathcal{M}_v^{(\partial^4)} = g_v + 8\log(2\pi N) + \text{const}$$
$$\mathcal{M}_c^{(\partial^4)} = g_c + 8\log(2\pi N) + \text{const}$$
$$\mathcal{M}_s^{(\partial^4)} = g_s + 8\log(2\pi N) + \text{const}$$

$\mathcal{M}^{(\partial^6)}$: 6-derivative contact term, with 5 coefficients $c_i(\tau_s, \bar{\tau}_s)$

$$\mathcal{M}^{(\partial^6)} = E_{3/2}(\tau, \bar{\tau})\,(s\,T_s^{ABCD} + t\,T_t^{ABCD} + u\,T_u^{ABCD})$$

Non-holomorphic Eisenstein series:

$$4\tau_2^2 \partial_\tau \partial_{\bar{\tau}} E_r(\tau, \bar{\tau}) = r(r-1) E_r(\tau, \bar{\tau})$$

Outlook

Huang, Wang, Yang, Zhou

More bootstrap

Two-loops, $\langle k_1 k_2 k_3 k_4 \rangle$ at one loop, contact terms and graviton exchange, other dimensions/theories, higher-point functions…

Alday, Gonçalves, Zhou

Veneziano amplitude in AdS_5

Compute all string corrections to tree-level gluon scattering in $AdS_5 \times S^3$, along the lines of Shapiro-Virasoro in $AdS_5 \times S^5$

Alday, Hansen, Silva

CK duality and DC in AdS

So far only clear at tree level in AdS_5, what about other dimensions and/or subleasing orders? Zhou

More with localization?

Bootstrap results available for other rank N $4d$ $\mathcal{N} = 2$ SCFTs, can we apply localization constraints? Behan

More numerical bootstrap + localization for general $4d$ $\mathcal{N} = 2$ theories?

Chester

Thank you
for the attention!