

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

**cross section and error updates**

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# recent evaluations of IBD

**Characteristics, mutual agreement, IBD cross section values**



# 1999

## Vogel-Beacom

a systematic inclusion of small effects, relevant in the region below  $E_\nu < 60 \text{ MeV}$  as, weak magnetism and recoil (first discussed in 30s, till Gell-Mann, PR 1958).

several useful analytical results; discussion of supernova pointing

PHYSICAL REVIEW D, VOLUME 60, 053003

### Angular distribution of neutron inverse beta decay, $\bar{\nu}_e + p \rightarrow e^+ + n$

P. Vogel\* and J. F. Beacom†

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(Received 1 April 1999; published 27 July 1999)

The reaction  $\bar{\nu}_e + p \rightarrow e^+ + n$  is very important for low-energy ( $E_\nu \lesssim 60 \text{ MeV}$ ) antineutrino experiments. In this paper we calculate the positron angular distribution, which at low energies is slightly backward. We show that weak magnetism and recoil corrections have a large effect on the angular distribution, making it isotropic at about 15 MeV and slightly forward at higher energies. We also show that the behavior of the cross section and the angular distribution can be well understood analytically for  $E_\nu \lesssim 60 \text{ MeV}$  by calculating to  $\mathcal{O}(1/M)$ , where  $M$  is the nucleon mass. The correct angular distribution is useful for separating  $\bar{\nu}_e + p \rightarrow e^+ + n$  events from other reactions and detector backgrounds, as well as for possible localization of the source (e.g., a supernova) direction. We comment on how similar corrections appear for the lepton angular distributions in the deuteron breakup reactions  $\bar{\nu}_e + d \rightarrow e^+ + n + n$  and  $\nu_e + d \rightarrow e^- + p + p$ . Finally, in the reaction  $\bar{\nu}_e + p \rightarrow e^+ + n$ , the angular distribution of the outgoing neutrons is strongly forward peaked, leading to a measurable separation in positron and neutron detection points, also potentially useful for rejecting backgrounds or locating the source direction. [S0556-2821(99)04015-1]



## A. Differential cross section: expansion in powers of $1/M$

We begin with the matrix element of the form

$$\mathcal{M} = \frac{G_F \cos \theta_C}{\sqrt{2}} \left[ \bar{u}_n \left( \gamma_\mu f - \gamma_\mu \gamma_5 g - \frac{if_2}{2M} \sigma_{\mu\nu} q^\nu \right) u_p \right] \times [\bar{\nu}_\nu \gamma^\mu (1 - \gamma_5) \nu_e] , \quad (4)$$



**2002**

**Strumia-FV**

an "exact" expression  
based on the 4 known  
form factors. virtually  
valid at all energies

includes a pedantic  
comparison with  
previous calculations  
and an estimate of the  
uncertainty





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# Precise quasielastic neutrino/nucleon cross-section

Alessandro Strumia<sup>a</sup>  , Francesco Vissani<sup>b</sup>

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## Abstract

Quasielastic antineutrino/proton and neutrino/neutron scatterings can be well approximated by simple formulae, valid around MeV or GeV energies. We obtain a single expression valid in the whole range, and discuss its relevance for studies of supernova neutrinos, which reach intermediate energies.



Table 2  
Percentage difference between our full result and various approximations for  $\bar{\nu}_e$  (above) and  $\nu_e$  (below) total cross-sections. A negative (positive) sign means that a certain cross-section is an over(under)-estimate. It is easy to implement approximations made with  $\star\star\star$ , while implementing those marked with a  $\star$  is not much simpler than performing a full computation

$E_\nu$ , MeV		ease	2.5	5	10	20	40	80	160
Percentage difference in $\sigma(\bar{\nu}_e p \rightarrow n \bar{e})$									
(1)	Naïve	$\star\star\star$	−3.9	−5.8	−9.9	−19	−38	−84	−210
(2)	Naïve+	$\star\star\star$	0	0.3	−0.2	0.4	0.2	0.5	−0.9
(3)	Vogel and Beacom	$\star\star$	0	0	0.3	1.2	5.6	28	150
(4)	NLO in $E_\nu/m_p$	$\star$	0	0	0	0	0.1	1.5	13
(5)	Horowitz	$\star\star$	−370	−83	−32	−14	−6.4	−3.0	−1.3
(6)	Llewellyn-Smith+	$\star$	−13	−2.1	−0.5	−0.1	0	0	0
(7)	LS + VB	$\star$	0.5	0.1	0	0	0	0	0

Very good agreement with Vogel and Beacom for  $E_\nu < 60$  MeV; note that the two implementations are equally demanding.



# estimated uncertainty

## low energy region - high energy region

### 3.2. Overall uncertainty

We now discuss how accurate our full expressions for the cross-sections are.

The axial coupling  $g_1(0)$  is measured from neutron decay.<sup>4</sup> Different experimental determinations do not fully agree, therefore we conservatively increased the error. Newer measurements, performed with a higher neutron polarization than older ones, are consistent and agree on  $g_1(0)/f_1(0) = -1.272 \pm 0.002$  when older determinations are discarded—a value slightly different from the one quoted in Section 2. Isospin-breaking corrections to  $f_1(0) = 1$  are negligible [15].

In conclusion, at low energy  $\sigma(\bar{\nu}_e p)$  has an overall 0.4% uncertainty, which is adequate for present experiments. The ratio between the measured and the no-oscillation reactor  $\bar{\nu}_e$  flux is  $1.01 \pm 2.8\%$  (stat)  $\pm 2.7\%$  (syst) at

The above discussion shows why it is difficult to assess the uncertainty on  $g_1$  and  $g_2$ . Optimistically assuming that (1) or (2) is right, it is negligible. On the other side, a pessimistic estimate can be obtained by using  $M_{A_1}$  in place of  $M_A$ : the total  $\bar{\nu}_e p$  cross-section increases by  $0.4\% \times (E_\nu/50 \text{ MeV})^2$  for  $E_\nu \lesssim 200 \text{ MeV}$ . The shift remains relatively small because, as shown in Section 2, the  $t$ -dependence of the form factors affects  $\bar{\nu}_e p$  only at NNLO in  $E_\nu/m_p$ .



# why an updated cross-section and error assessment?

*the two cross sections are in good agreement and they are quite accurate: an error of 0.4% as PLB2002 matches the statistical error of a sample of **60,000** events*

- however, Daya Bay has collected already **3.5 million** events (*60 times*) and similarly, other reactor antineutrino experiments
- JUNO will collect **180,000** events after 6 years (*3 times*)
- Super-Kamiokande (and JUNO) will collect **5,000** events from a future galactic supernova, a number that scales as  $(10 \text{ kpc} / D)^2$ . For Hyper-Kamiokande, multiply by a factor of **10**



2022

Ricciardi-Vignaroli-FV

objective: assess better  
the **uncertainty** of  
expectations

updating of relevant  
parameters, testing with  
the neutron decay rate


verification of the  
significance of “second-  
class currents”

# An accurate evaluation of electron (anti-)neutrino scattering on nucleons

[Giulia Ricciardi](#), [Nataschia Vignaroli](#)  & [Francesco Vissani](#)

[Journal of High Energy Physics](#) **2022**, Article number: 212 (2022) | [Cite this article](#)

**133** Accesses | **3** Citations | **1** Altmetric | [Metrics](#)

 A [preprint version](#) of the article is available at arXiv.

## ABSTRACT

We discuss as accurately as possible the cross section of quasi-elastic scattering of electron (anti-)neutrinos on nucleons, also known as inverse beta decay in the case of antineutrinos. We focus on the moderate energy range from a few MeV up to hundreds of MeV, which includes neutrinos from reactors and supernovae. We assess the uncertainty on the cross section, which is relevant to experimental advances and increasingly large statistical samples. We estimate the effects of second-class currents, showing that they are small and negligible for current applications.



### 2.1.1 The form factors

One possible formulation of the most general matrix element of the charged weak current between proton and neutron states, of 4-momenta  $p_p$  and  $p_n$  respectively, is

$$\mathcal{J}_\mu = \bar{u}_n \left( f_1 \gamma_\mu + g_1 \gamma_\mu \gamma_5 + i f_2 \sigma_{\mu\nu} \frac{q^\nu}{2M} + g_2 \frac{q_\mu}{M} \gamma_5 + f_3 \frac{q_\mu}{M} + i g_3 \sigma_{\mu\nu} \frac{q^\nu}{2M} \gamma_5 \right) u_p \quad (2.1)$$

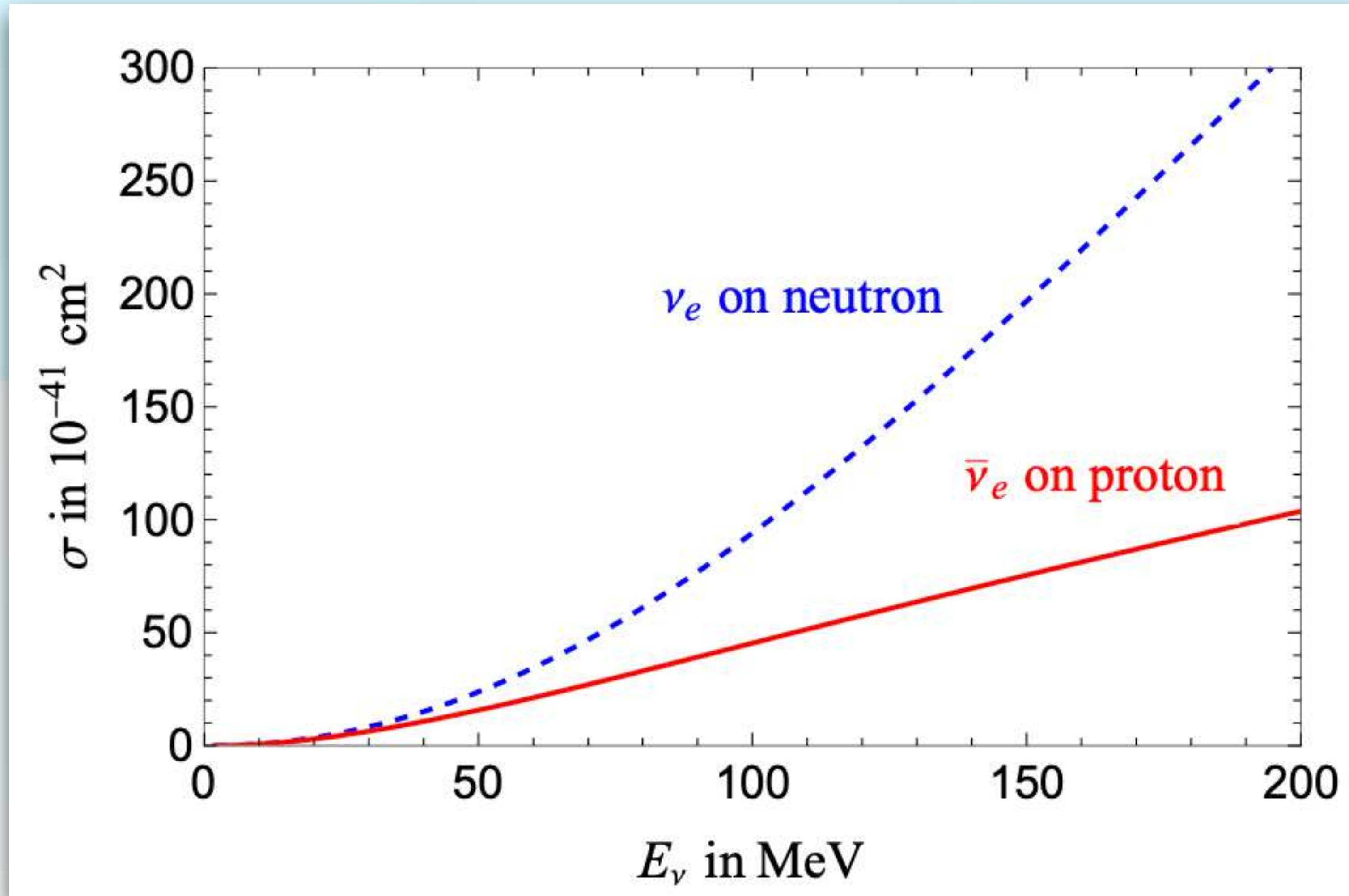
The normalisation mass scale is  $M = (m_n + m_p)/2$ . The form factors  $f_1$ ,  $f_2$  and  $f_3$  are generally referred to, respectively, as vector, weak magnetism and scalar. The terms including them represent the vector part of the current. The terms including  $g_1$ ,  $g_2$  and  $g_3$  represent the axial part of the current. These six dimensionless form factors are Lorentz invariant, and in general depend upon the four-momentum transfer squared  $t = q^2 = -Q^2$ , where  $q = p_n - p_p$ .

- There are various way to rewrite this current, due to Gordon identity.
- $f_3$  and  $g_3$  are second class currents, expected to be small; we use *Day & McFarland, PRD 86, 2012* to estimate the phenomenologically maximum value.



# results 1: the updated cross section

result: second-class currents, even at maximum value, give a **negligible** contribution





# what is the accuracy of the IBD cross section?

**quantitative discussion of the uncertainty; neutron decay as a test; axial radius**



*leading uncertainties are due to input parameters:*

- $V_{ud}$  - namely,  $\cos \theta_C$  - and the parameter  $\lambda$  ,
- the axial mass - or, the axial radius,

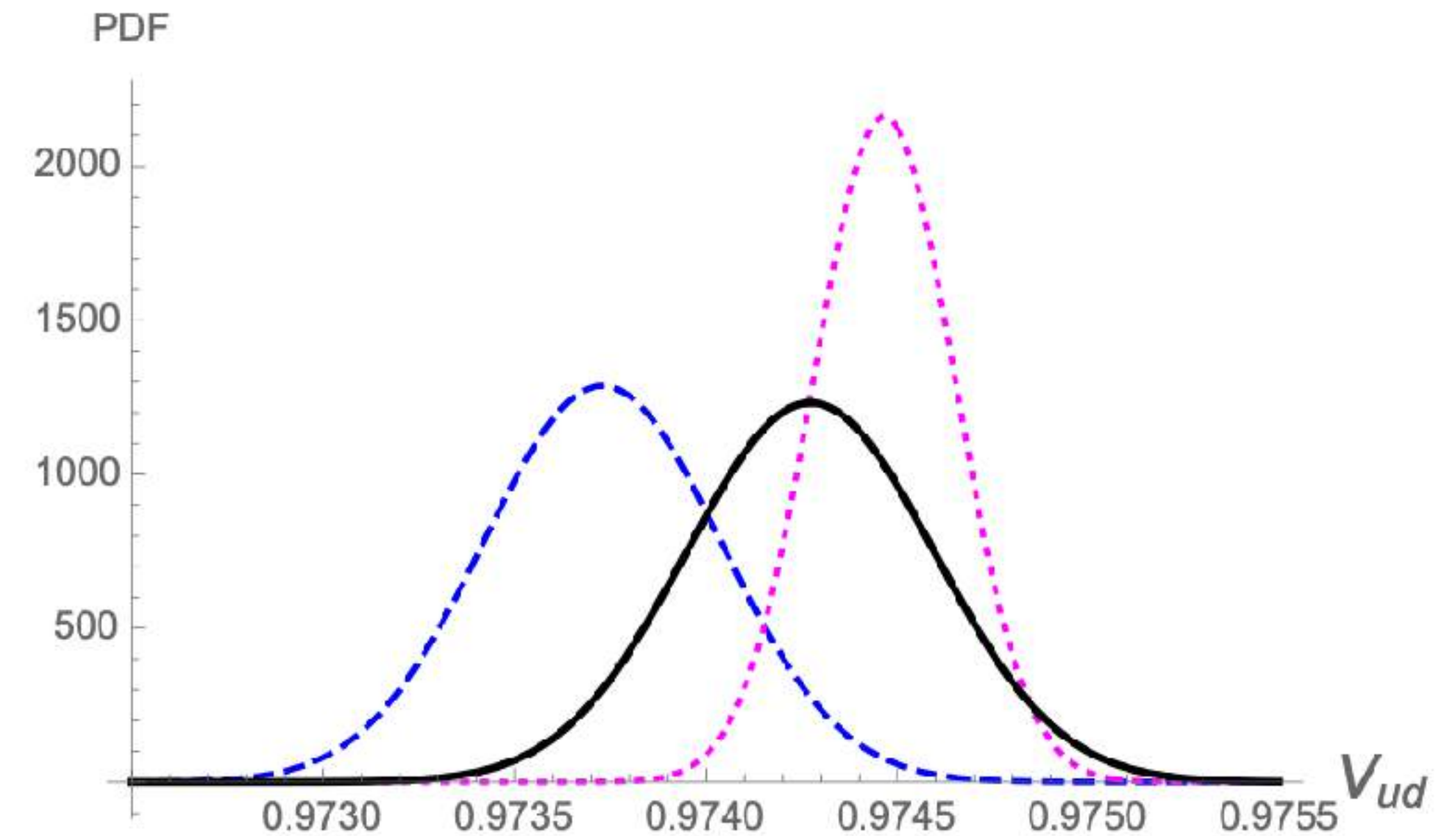
*at low and high energies, respectively.*



$$V_{ud}$$

=the cosine of the Cabibbo angle

- For the **superaligned transitions**, we use *Hardy & Towner, PRD 102 (2020)*
- Using the **unitarity of CKM matrix**, we can estimate  $V_{ud}$  from  $V_{us}$  and  $V_{ub}$ , following *PDG 2020*
- The two results are not in perfect agreement; thus, we include the scale factor  $S = \sqrt{\chi^2/(N - 1)} = 2.0$  for a conservative estimation of the uncertainty





$$\lambda$$

**=the zero momentum transfer  $g_1(q^2)$**

★ eight measurements with polarized neutron decay

★ most recent one (PERKEO-III) is very precise

★ *Czarnecki, Marciano & Sirlin, PRL 120 (2018)*

suggest to omit pre-2002 ones

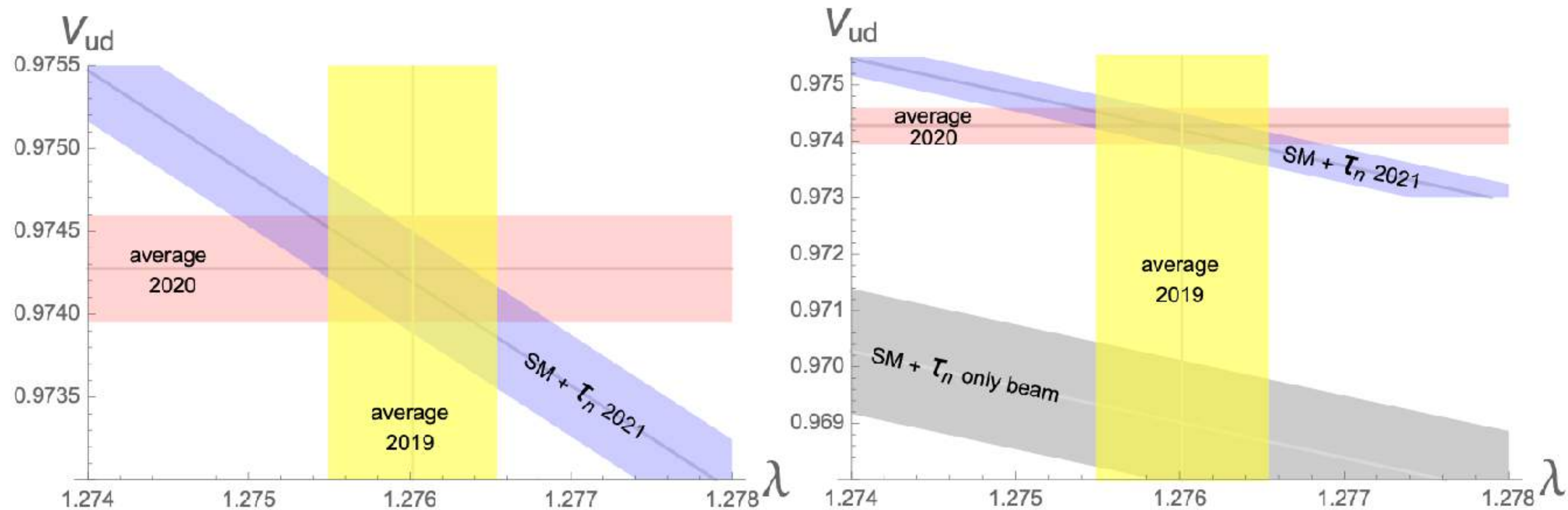
★ we prefer to include them, enlarging  $S = 2$

*result within  $1\sigma$  from most recent & global average*



# the neutron decay constraint

## compatibility test



**Figure 2.** Left: illustration of the compatibility, within the SM, among the determinations of  $\lambda$ ,  $V_{ud}$  and  $\tau_n(\text{tot})$ . Right: enlargement of the parameter region to include the prediction of the correlation  $\lambda - V_{ud}$  (gray band) that follows from the SM assuming the correctness of measurement  $\tau_n(\text{beam})$ : this is incompatible with the determinations of  $\lambda$  and  $V_{ud}$ .



“A priori, it would be possible to hypothesize an **additional neutron decay channel** into undetected particles, which would shorten the total average lifetime — a possible way out, recently attempted.

This would require an agreement between the prediction and the **exclusive** measurement, namely  $\tau_n(\text{beam})$ .

This is not what is observed: the predicted value  $\tau_n(\text{SM})$  - a function of  $V_{ud}$  and  $\lambda$  - agrees with the **inclusive** measurement  $\tau_n(\text{tot})$  instead.”

**there is no simple theoretical way out; the first suspect becomes an unknown systematic error**



# summary of low energy uncertainties

## conservative and standard error propagation

### 3.1.4 Procedures for assessing the uncertainty on the cross section

At this point in the discussion, we can evaluate the uncertainty on the  $\sigma$  cross section. By calculating the derivatives with respect to the parameters of interest, at the point of maximum likelihood,

$$\vec{\xi} = \left( \frac{\partial \sigma}{\partial V_{\text{ud}}}, \frac{\partial \sigma}{\partial \lambda} \right) \Big|_{\text{best fit}} \quad (3.8)$$

we find the uncertainty from the formula

$$\delta \sigma = \sqrt{\vec{\xi}^t \Sigma^2 \vec{\xi}} \quad \text{where} \quad \Sigma^2 = \begin{pmatrix} (\delta V_{\text{ud}})^2 & \rho \delta V_{\text{ud}} \delta \lambda \\ \rho \delta V_{\text{ud}} \delta \lambda & (\delta \lambda)^2 \end{pmatrix} \quad (3.9)$$

We conclude that  $\delta \sigma = 0.1 \%$  , i.e. 4 times better than 2002  
(or half as much if we had included the neutron decay data, that we prefer to use as a test)

$r_A$  **or**  $M_A$

parameterization of  $g_1(q^2)/g_1(0)$

★ at GeV energies,  $g_1(t)/g_1(0) = 1/(1 - t/M_A^2)^2$  gives good results. But at low energies, it is more unbiased to use the linear expansion:  $g_1(t)/g_1(0) = 1 + (r_A^2 \cdot t)/6$

★ a global fit, based on the assumed double-dipole, gives  $M_A = 1014 \pm 14$  MeV. This corresponds to  $r_A^2 = 0.455 \pm 0.013$  fm<sup>2</sup>, supported by electro-pion production data

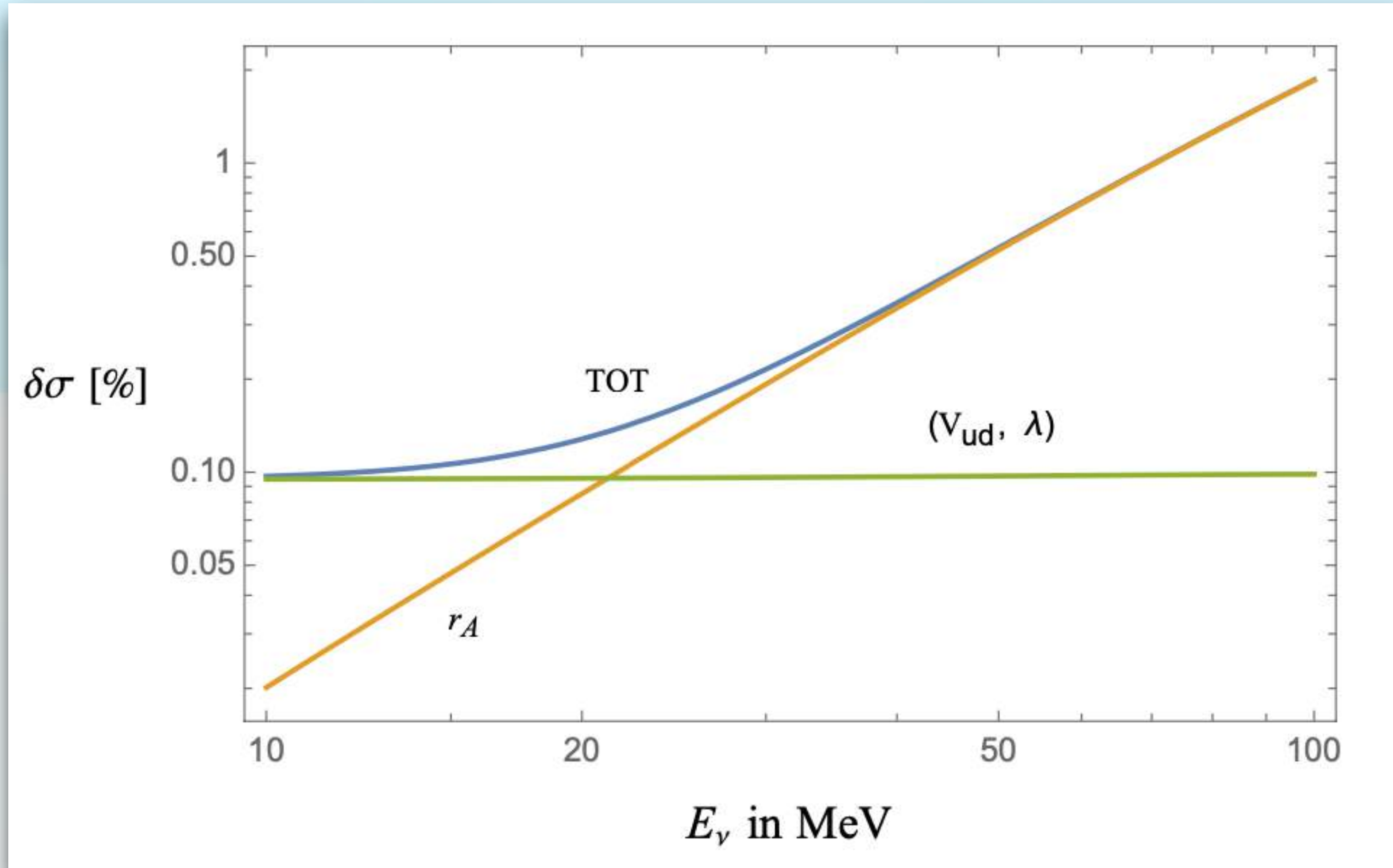
★ an analysis **that does not assume double-dipole** finds instead  $r_A^2 = 0.46 \pm 0.12$  fm<sup>2</sup>. We use this to estimate a conservative error on the cross section

*compare Bodek et al EPJC 2008 and Hill et al, PRD 2018*



# results 2: the cross section uncertainty

the low energy and the high energy uncertainties sum in quadrature



# summary and discussion

*The cross section of the IBD is well known.*

*To perform its maintenance, all we need is a set of consolidated theoretical concepts  
and, most importantly, reliable measurements of the key parameters.*



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- ★ the cross section depends critically upon  $V_{ud} = \cos \theta_C$  ,  $g_1(0) = \lambda$  ,  $r_A^2 \sim 12/M_A^2$  ;
- ★ the uncertainty is small (0.1 % ) at low energies,  $1.1 \% \left( \frac{E_\nu}{50 \text{ MeV}} \right)^2$  at high ones;
- ★ second class currents are not expected to give a significant contribution.

# summary and discussion

*The cross section of the IBD is well known.*

*To perform its maintenance, all we need is a set of consolidated theoretical concepts  
and, most importantly, reliable measurements of the key parameters.*

*how to clarify / improve?*

- ★ need to understand the reason of discrepancy in  $\tau_n$  - measurements.
- ★ need to decrease the uncertainty due to  $r_A^2$  — i.e. we need refine the description of the axial form factor in the 100 MeV range.



# Thanks for the attention!