Effective field theory for the inspiral of a compact binary system

Workshop on EOB and Amplitudes for gravitational systems
Università di Bologna

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Outline of the presentation

1. Construction of the effective field theory for the inspiral of a non-relativistic compact binary system
2. Derivation of the relevant Feynman rules
3. Evaluation of diagrams contributing to the conservative sector

Outline of the idea

- Our goal is to describe the dynamics of a binary system which evolves under the influence of the gravitational interaction.

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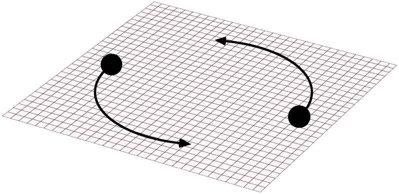
- An example, relevant for gravitational wave observations, is black holes and neutron stars inspiraling towards each other.



Outline of the idea

- In full generality the dynamics of a binary system are prescribed by the most fundamental theory, so for example by general relativity coupled to the two compact objects:

$$S_{tot}[\{x_a^\mu\}, g_{\mu\nu}] = -2\Lambda^2 \int \mathrm{d}^4x \sqrt{-g}\left(R - \frac{1}{2}\Gamma^\mu \Gamma_\mu\right) - \sum_{a=1}^{2} m_a \int \mathrm{d}\sigma_a \sqrt{g_{\mu\nu}(x_a)\frac{\mathrm{d}x_a^\mu}{\mathrm{d}\sigma}\frac{\mathrm{d}x_a^\nu}{\mathrm{d}\sigma}} + \dots$$

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- Yet in practice we would like to work with an *effective action* S_{eff} which is a function only of the positions $\vec{x}_1$, $\vec{x}_2$ of the two compact objects, i.e. at the Newtonian level:

$$S_{eff}[\{x_a^\mu\}] = \int \mathrm{d}t\, L = \int \mathrm{d}t\,(T - V) = \int \mathrm{d}t\,\left(\frac{1}{2}\,m_1\,v_1^2 + \frac{1}{2}\,m_2\,v_2^2 + G\,\frac{m_1 m_2}{r}\right).$$

Outline of the idea

- To achieve our goal of obtaining the *effective action* $S_{eff}[\{x_a^\mu\}]$ which describes the dynamics of only the compact objects, we can start from the full action and *integrate out* the gravitational degrees of freedom; schematically:

$$e^{iS_{eff}[\{x_a^\mu\}]} = \int Dg_{\mu\nu}\, e^{iS_{tot}[\{x_a^\mu\}, g_{\mu\nu}]} \ .$$

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- This path-integration may performed perturbatively, by summing over the relevant Feynman diagrams.
- In practice, regarding the case of non-relativistic binary systems, this idea has been first implemented in an effective field theory framework by *Goldberger and Rothstein (2006)*.

Why an EFT approach to binary dynamics?

- Considering only the tree level diagrams yields *fully classical results*.
- The diagrammatic approach allows to employ modern multi-loop quantum field theory techniques, to obtain *state-of-the-art results*.
- The effective field theory construction allows to consistently and systematically include spin and finite size effects, which are fundamental to obtain *accurate predictions*.

Dimensional regularization

In order to evaluate Feynman integrals, and to deal with divergencies appearing at higher orders, we employ the *dimensional regularization* scheme for the spatial dimensions, recovering the physical limit with

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The gravitational coupling then reads

$$\Lambda = \frac{1}{\sqrt{32\pi G}}\,\mu^{\frac{d-3}{2}} \longrightarrow \frac{1}{\sqrt{32\pi G}} + \mathcal{O}(d-3) \ .$$

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$$\boldsymbol{v}^2 \sim \frac{\boldsymbol{G}\, m}{r} \, .$$

Construction of the EFT for a NR binary system

Preliminaries

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$$\boldsymbol{v}^2 \sim \frac{\boldsymbol{G}\, m}{r} \,.$$
- Each of the two compact objects is extremely heavy, with typical momentum $\mathrm{p} \sim m\mathrm{v}$: then their interaction with the gravitons, of typical momenta $\mathrm{k} \lesssim \frac{1}{r}$, induces a recoil of the order $\frac{|\mathrm{k}|}{|\mathrm{p}|} \sim \frac{\hbar}{L} \ll 1$. Therefore we can *treat the compact objects as background non-dynamical* (non propagating) *sources*: our EFT instead will revolve around integrating out the gravitational degrees of freedom.

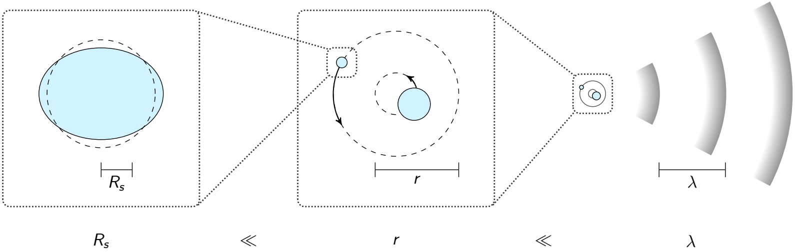
Construction of the EFT for a NR binary system

Separation of scales

The assumption of small velocities, $v \ll 1$, implies the hierarchy of scales:

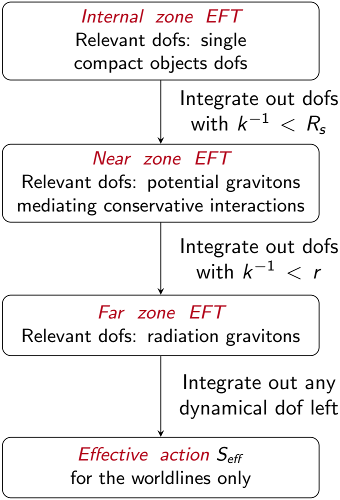
$$R_s \sim r\,v^2 \ll r \;,$$
$$r \sim \lambda\,v \ll \lambda \;;$$

which let us clearly separate an *internal zone*, a *near zone* (or potential zone) and a *far zone* (or radiation zone).
Then we'll consider *three different effective theories*, one for each zone, to describe the relevant dynamics.



Construction of the EFT for a NR binary system

Tower of EFTs



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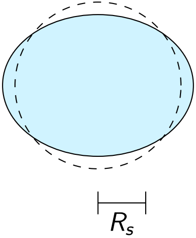

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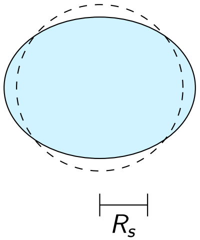
  - single compact objects (simplifying assumption: spinless and spherically symmetric)



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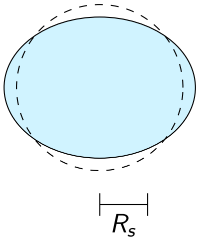
- Relevant dofs:
  - gravitational field $g_{\mu\nu}$;
  - single compact objects (simplifying assumption: spinless and spherically symmetric)
- Symmetries:
  - diffeomorphism invariance $x^\mu \to x'^\mu(x)$;
  - worldline reparametrization invariance $\sigma \to \sigma'(\sigma)$;
  - $SO(3)$ invariance;



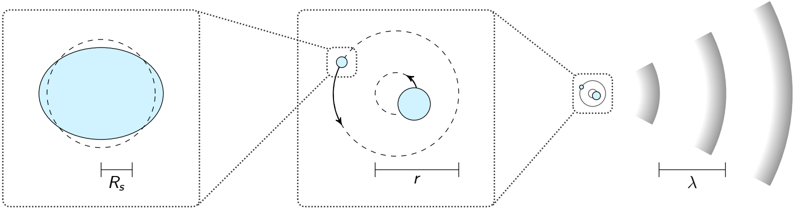
Writing down all operators compatible with the symmetries we find the *worldline effective theory*:

$$S_{eff\,,worldline}[x^\mu, g_{\mu\nu}] = S_{EH}[g_{\mu\nu}] + S_{pp}[x^\mu, g_{\mu\nu}]$$

$$S_{EH}[\{x_a^\mu\}, g_{\mu\nu}] = -2\Lambda^2 \int \mathrm{d}^{d+1}x \sqrt{-g}\, R \qquad (\Lambda^2 = \frac{1}{32\pi G})$$

$$S_{pp}[x^\mu, g_{\mu\nu}] = -m \int \mathrm{d}\tau + c_R \int \mathrm{d}\tau R^{(L)}(x(\tau)) + c_V \int \mathrm{d}\tau R^{(L)}_{\mu\nu}(x(\tau)) \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}(\tau) \frac{\mathrm{d}x^\nu}{\mathrm{d}\tau}(\tau) + \dots$$

Construction of the EFT for a NR binary system



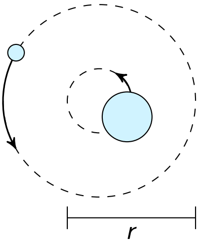
$$R_s \ll r \ll \lambda$$

Construction of the EFT for a NR binary system

Internal zone EFT

The degrees of freedom of the *internal zone EFT* are:

- gravitational field $g_{\mu\nu}$ with $k^{-1} > R_s$;
- two compact objects, so two worldlines x_a^μ, $a = 1, 2$.



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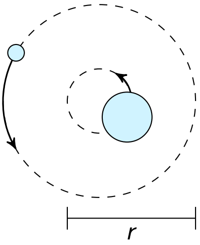

Then the effective action which describes the dynamics at this scale is given by, adding the harmonic gauge fixing term:

$$S_{near,UV}[x^\mu, g_{\mu\nu}] = S_{EH}[g_{\mu\nu}] + S_{GF}[g_{\mu\nu}] + \sum_{a=1}^{2} S_{pp}[x_a^\mu, g_{\mu\nu}]$$

$$S_{GF}[g_{\mu\nu}] = \Lambda^2 \int \mathrm{d}^{d+1}x \sqrt{-g}\, g_{\mu\nu}\Gamma^\mu\Gamma^\nu$$

$$\Gamma^\mu = \Gamma^\mu_{\alpha\beta} g^{\alpha\beta} = -\frac{1}{\sqrt{-g}}\partial_\nu(\sqrt{-g} g^{\mu\nu})$$

Construction of the EFT for a NR binary system

Internal zone EFT – Kol-Smolkin parametrization of the metric

For convenience we introduce the *Kol-Smolkin* parametrization of the metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad h_{\mu\nu} \leftrightarrow \phi, \boldsymbol{A}, \boldsymbol{\sigma}$$

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$$g_{\mu\nu} = e^{2\frac{\phi}{\Lambda}} \begin{pmatrix} 1 & -\frac{\boldsymbol{A}_j}{\Lambda} \\ -\frac{\boldsymbol{A}_i}{\Lambda} & \frac{\boldsymbol{A}_i}{\Lambda}\frac{\boldsymbol{A}_j}{\Lambda} - e^{-c_d\frac{\phi}{\Lambda}}\gamma_{ij} \end{pmatrix} , \qquad \gamma_{ij} \equiv \delta_{ij} + \frac{\boldsymbol{\sigma}_{ij}}{\Lambda} ,$$

$$c_d \equiv 2\frac{(d-1)}{(d-2)} \xrightarrow{d\to 3} 4 .$$

Construction of the EFT for a NR binary system

Internal zone EFT – Preferred coordinate frame

We choose a coordinate frame in which the typical velocity of the system is small, $v \ll 1$.

$$\frac{\mathrm{d}x_a^\mu}{\mathrm{d}t} = (1, \mathrm{v}_a) \ , \qquad v_a^i \equiv \frac{\mathrm{d}x_a^i}{\mathrm{d}t} \ ,$$

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and imposing the Kol-Smolkin parametrization as well the point-particle action becomes:

$$\begin{aligned} S_{pp,a}^{(PP)}[x_a^\mu, \boldsymbol{\phi}, \boldsymbol{A}_i, \boldsymbol{\sigma}_{ij}] &= -m_a \int \mathrm{d}t \mathrm{d}^{d+1}x \sqrt{g_{\mu\nu}(x) \frac{\mathrm{d}x^\mu}{\mathrm{d}t} \frac{\mathrm{d}x^\nu}{\mathrm{d}t}} \delta^{(4)}(x^\mu - x_a^\mu(t)) \\ &= -m_a \int \mathrm{d}t \, e^{\frac{\boldsymbol{\phi}}{\Lambda}} \left[1 - e^{-c_d \frac{\boldsymbol{\phi}}{\Lambda}} v_a^2 - 2\frac{\boldsymbol{A}_i}{\Lambda} v_a^i + \left(\frac{\boldsymbol{A}_i}{\Lambda} \frac{\boldsymbol{A}_j}{\Lambda} - e^{-c_d \frac{\boldsymbol{\phi}}{\Lambda}} \frac{\boldsymbol{\sigma}_{ij}}{\Lambda}\right) v_a^i v_a^j\right]^{\frac{1}{2}} \Big|_{x=x(t)} . \end{aligned}$$

Imposing the Kol-Smolkin parametrization in the purely gravitational *bulk action* (Einstein-Hilbert with harmonic gauge fixing):

$$
\begin{aligned}
S_{bulk}[\phi, \boldsymbol{A}_i, \boldsymbol{\sigma}_{ij}] &= S_{EH}[\phi, \boldsymbol{A}_i, \boldsymbol{\sigma}_{ij}] + S_{GF}[\phi, \boldsymbol{A}_i, \boldsymbol{\sigma}_{ij}] \\
&= -2\Lambda^2 \int \mathrm{d}^{d+1}x \sqrt{-g}\left(R - \frac{1}{2}\Gamma^\mu \Gamma_\mu\right) \\
&\supset \int \mathrm{d}^{d+1}x \,\Big[\left(c_d \dot{\phi}^2 - c_d\, \partial^i \phi \partial_i \phi\right) + \left(\partial_j \boldsymbol{A}_i \partial^j \boldsymbol{A}^i - \partial_i \boldsymbol{A}_j \partial^j \boldsymbol{A}^i + (\partial_i \boldsymbol{A}^i)^2 - \dot{\boldsymbol{A}}_i \dot{\boldsymbol{A}}^i\right) \\
&\quad + \frac{1}{4}\left(\partial^j \boldsymbol{\sigma}^i_i \partial_j \boldsymbol{\sigma}^k_k + 4(\partial_i \boldsymbol{\sigma}^{ij} \partial_k \boldsymbol{\sigma}^k_j - \partial_j \boldsymbol{\sigma}_{ik} \partial^k \boldsymbol{\sigma}^{ij}) - 2\,\partial_k \boldsymbol{\sigma}_{ij} \partial^k \boldsymbol{\sigma}^{ij} + (\dot{\boldsymbol{\sigma}}^i_i)^2 - 2\,\dot{\boldsymbol{\sigma}}_{ij} \dot{\boldsymbol{\sigma}}^{ij}\right) \\
&\quad + 4\left(\dot{\phi} \partial_i \boldsymbol{A}^i - \partial_i \phi \dot{\boldsymbol{A}}_i\right) + \left(2\,(\dot{\boldsymbol{A}}^i \partial_j \boldsymbol{\sigma}^j_i - \partial^j \boldsymbol{A}^i \dot{\boldsymbol{\sigma}}_{ij}) + (\partial_i \boldsymbol{A}^i \dot{\boldsymbol{\sigma}}^j_j - \dot{\boldsymbol{A}}^i \partial_i \boldsymbol{\sigma}^j_j)\right) \\
&\quad + \frac{1}{2\Lambda}\Big(c_d (2\, \boldsymbol{\sigma}_{ij} \partial^i \phi \partial^j \phi - \boldsymbol{\sigma}^j_j \partial^i \phi \partial_i \phi) + 2\, c_d\, (\phi\, \partial_j \boldsymbol{A}_i \partial^j \boldsymbol{A}^i - \phi\, \partial_i \boldsymbol{A}_j \partial^j \boldsymbol{A}^i) \\
&\quad + 2\, c_d\, (\phi (\partial_i \boldsymbol{A}^i)^2 - 2\, \dot{\phi} \boldsymbol{A}^i \partial^i \phi) - 2\, c_d^2\, \phi (\dot{\phi})^2\Big)\Big] + \dots \,,
\end{aligned}
$$

A key point of the binary EFT construction is the separation of gravitational modes in *potential* and *radiation* fields:

$$h_{\mu\nu} = H_{\mu\nu} + \bar{h}_{\mu\nu}$$

with radiation graviton, scaling as

$$\partial_0 \bar{h}_{\mu\nu} \sim \frac{v}{r} \bar{h}_{\mu\nu}, \qquad \partial_i \bar{h}_{\mu\nu} \sim \frac{v}{r} \bar{h}_{\mu\nu}$$

and off-shell potential gravitons, scaling as:

$$\partial_0 H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}, \qquad \partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu}$$

Construction of the EFT for a NR binary system

Internal zone EFT – Potential and radiative modes separation

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$$\phi \to \phi + \bar{\phi}$$

$$A \to A + \bar{A}$$

$$\sigma \to \sigma + \bar{\sigma}$$

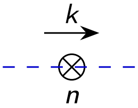
Potential fields		Radiation fields	
ϕ		$\bar{\phi}$	
A		$\bar{A}$	
σ		$\bar{\sigma}$	

For the off-shell potential gravitons the scaling:

$$\partial_0 H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}, \qquad \partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu}$$

allows to expand the propagator in homogeneous powers of v^2 (PN expansion parameter):

$$\frac{1}{k^2 + i\epsilon} = \frac{1}{(k_0)^2 - |\mathrm{k}|^2 + i\epsilon} \overset{k^0 \ll |\mathrm{k}|}{=} -\frac{1}{|\mathrm{k}|^2} \frac{1}{1 - \frac{(k^0)^2}{|\mathrm{k}|^2}} = -\frac{1}{|\mathrm{k}|^2} \sum_{n=0}^{+\infty} \left(\underbrace{\frac{(k^0)^2}{|\mathrm{k}|^2}}_{v^2} \right)^n ,$$



We'll now focus on the near zone EFT, which yields conservative contribution to the dynamics of the binary.

$$S_{near,IR}[\{x_a^\mu\}, \bar{\phi}, \bar{A}_i, \bar{\sigma}_{ij}] = \sum_{a=1}^{2} S_{pp,a}^{(kin)}[x_a^\mu] \underbrace{-i \log \left(\int D\phi \, DA_i \, D\sigma_{ij} \; e^{i\tilde{S}_{near,UV}[\{x_a^\mu\},\phi,A_i,\sigma_{ij},\bar{\phi},\bar{A}_i,\bar{\sigma}_{ij}]} \right)}_{\equiv S_{cons}[\{x_a^\mu\}] + S_{eff}^{rad}[\{x_a^\mu\},\bar{\phi},\bar{A}_i,\bar{\sigma}_{ij}]} ,$$

Construction of the EFT for a NR binary system
Internal zone EFT

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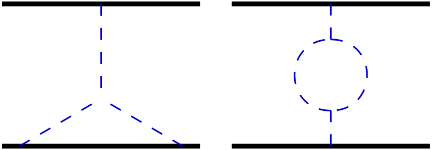
To obtain the classical contributions to the dynamics we sum over the connected vacuum diagrams without graviton loops.

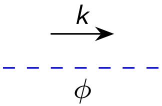
$$iS_{eff} = \sum \mathcal{A}$$



Feynman rules for the conservative sector

Propagator of the potential field ϕ

$$
\begin{aligned}
S_{bulk}[\phi, A_i, \sigma_{ij}] \supset \int \mathrm{d}^{d+1}x \, \Big[& \Big(\boldsymbol{c_d \dot{\phi}^2 - c_d \, \partial^i \phi \partial_i \phi} \Big) + \Big(\partial_j A_i \partial^j A^i - \partial_i A_j \partial^j A^i + (\partial_i A^i)^2 - \dot{A}_i \dot{A}^i \Big) \\
& + \frac{1}{4} \left(\partial^j \sigma^i_i \partial_j \sigma^k_k + 4(\partial_i \sigma^{ij} \partial_k \sigma^k_j - \partial_j \sigma_{ik} \partial^k \sigma^{ij}) - 2 \, \partial_k \sigma_{ij} \partial^k \sigma^{ij} + (\dot{\sigma}^i_i)^2 - 2 \, \dot{\sigma}_{ij} \dot{\sigma}^{ij} \right) \\
& + \frac{1}{2\Lambda} \Big(c_d (2 \, \sigma_{ij} \partial^i \phi \partial^j \phi - \sigma^j_j \partial^i \phi \partial_i \phi) + 2 \, c_d \, (\phi \, \partial_j A_i \partial^j A^i - \phi \, \partial_i A_j \partial^j A^i) \\
& + 2 \, c_d \, (\phi (\partial_i A^i)^2 - 2 \, \dot{\phi} A^i \partial^i \phi) - 2 \, c_d^2 \, \phi (\dot{\phi})^2 \Big) \Big] \; ,
\end{aligned}
$$



$$
= \frac{1}{2c_d} \frac{i}{k^2 + i\epsilon}
$$

Feynman rules for the conservative sector

Propagator of the potential field A_i

$$
\begin{aligned}
S_{bulk}[\phi, A_i, \sigma_{ij}] \supset \int \mathrm{d}^{d+1}x \, \Big[\Big(c_d \dot{\phi}^2 - c_d \, \partial^i \phi \partial_i \phi \Big) &+ \Big(\boldsymbol{\partial_j A_i \partial^j A^i - \partial_i A_j \partial^j A^i + (\partial_i A^i)^2 - \dot{A}_i \dot{A}^i} \Big) \\
&+ \frac{1}{4} \left(\partial^j \sigma^i_i \partial_j \sigma^k_k + 4(\partial_i \sigma^{ij} \partial_k \sigma^k_j - \partial_j \sigma_{ik} \partial^k \sigma^{ij}) - 2 \, \partial_k \sigma_{ij} \partial^k \sigma^{ij} + (\dot{\sigma}^i_i)^2 - 2 \, \dot{\sigma}_{ij} \dot{\sigma}^{ij} \right) \\
&+ \frac{1}{2\Lambda} \Big(c_d (2 \, \sigma_{ij} \partial^i \phi \partial^j \phi - \sigma^j_j \partial^i \phi \partial_i \phi) + 2 \, c_d \, (\phi \, \partial_j A_i \partial^j A^i - \phi \, \partial_i A_j \partial^j A^i) \\
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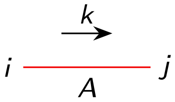
$$

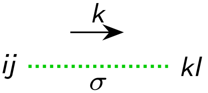


$$
= -\frac{\delta_{ij}}{2} \frac{i}{k^2 + i\epsilon} \, .
$$

Feynman rules for the conservative sector

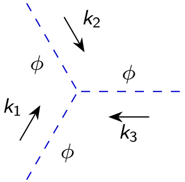
Propagator of the potential field σ_{ij}

$$
\begin{aligned}
S_{bulk}[\phi, A_i, \sigma_{ij}] \supset \int \mathrm{d}^{d+1}x \, \Big[& \Big(c_d \dot{\phi}^2 - c_d \, \partial^i \phi \partial_i \phi \Big) + \Big(\partial_j A_i \partial^j A^i - \partial_i A_j \partial^j A^i + (\partial_i A^i)^2 - \dot{A}_i \dot{A}^i \Big) \\
& + \frac{1}{4} \left(\partial^j \sigma^i_i \partial_j \sigma^k_k + 4(\partial_i \sigma^{ij} \partial_k \sigma^k_j - \partial_j \sigma_{ik} \partial^k \sigma^{ij}) - 2 \, \partial_k \sigma_{ij} \partial^k \sigma^{ij} + (\dot{\sigma}^i_i)^2 - 2 \, \dot{\sigma}_{ij} \dot{\sigma}^{ij} \right) \\
& + \frac{1}{2\Lambda} \Big(c_d (2 \, \sigma_{ij} \partial^i \phi \partial^j \phi - \sigma^j_j \partial^i \phi \partial_i \phi) + 2 \, c_d \, (\phi \, \partial_j A_i \partial^j A^i - \phi \, \partial_i A_j \partial^j A^i) \\
& + 2 \, c_d \, (\phi (\partial_i A^i)^2 - 2 \, \dot{\phi} A^i \partial^i \phi) - 2 \, c_d^2 \, \phi (\dot{\phi})^2 \Big) \Big] \ ,
\end{aligned}
$$



$$
= \frac{1}{2} \frac{i}{k^2 + i\epsilon} \left(-\frac{2}{d-2} \delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right) \ .
$$

$$
\begin{aligned}
S_{bulk}[\phi, A_i, \sigma_{ij}] \supset \int \mathrm{d}^{d+1}x \, \Big[& \Big(c_d \dot{\phi}^2 - c_d \, \partial^i \phi \partial_i \phi \Big) + \Big(\partial_j A_i \partial^j A^i - \partial_i A_j \partial^j A^i + (\partial_i A^i)^2 - \dot{A}_i \dot{A}^i \Big) \\
& + \frac{1}{4} \left(\partial^j \sigma^i_i \partial_j \sigma^k_k + 4(\partial_i \sigma^{ij} \partial_k \sigma^k_j - \partial_j \sigma_{ik} \partial^k \sigma^{ij}) - 2 \, \partial_k \sigma_{ij} \partial^k \sigma^{ij} + (\dot{\sigma}^i_i)^2 - 2 \, \dot{\sigma}_{ij} \dot{\sigma}^{ij} \right) \\
& + \frac{1}{2\Lambda} \Big(c_d (2 \, \sigma_{ij} \partial^i \phi \partial^j \phi - \sigma^j_j \partial^i \phi \partial_i \phi) + 2 \, c_d \, (\phi \, \partial_j A_i \partial^j A^i - \phi \, \partial_i A_j \partial^j A^i) \\
& + 2 \, c_d \, (\phi (\partial_i A^i)^2 - 2 \, \dot{\phi} A^i \partial^i \phi) - 2 \, c_d^2 \, \phi (\dot{\phi})^2 \Big) \Big] \ ,
\end{aligned}
$$



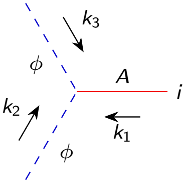
$$
= i \, (2\pi)^{d+1} \delta^{(d+1)}(k_1 + k_2 + k_3) \, \frac{2 c_d^2}{\Lambda} \left(k_2^0 k_3^0 + k_1^0 k_3^0 + k_1^0 k_2^0 \right) .
$$

Scaling: $L^{-\frac{1}{2}} \, v^4$

Feynman rules for the conservative sector

Gravitational self interactions – $\phi^2 A$ interaction vertex

$$S_{bulk}[\phi, A_i, \sigma_{ij}] \supset \int \mathrm{d}^{d+1}x \left[\left(c_d \dot{\phi}^2 - c_d\, \partial^i \phi \partial_i \phi \right) + \left(\partial_j A_i \partial^j A^i - \partial_i A_j \partial^j A^i + (\partial_i A^i)^2 - \dot{A}_i \dot{A}^i \right) \right.$$
$$+ \frac{1}{4} \left(\partial^i \sigma^j_i \partial_j \sigma^k_k + 4(\partial_i \sigma^{ij} \partial_k \sigma^k_j - \partial_j \sigma_{ik} \partial^k \sigma^{ij}) - 2\, \partial_k \sigma_{ij} \partial^k \sigma^{ij} + (\dot{\sigma}^i_i)^2 - 2\, \dot{\sigma}_{ij} \dot{\sigma}^{ij} \right)$$
$$+ \frac{1}{2\Lambda} \left(c_d (2\, \sigma_{ij} \partial^i \phi \partial^j \phi - \sigma^j_j \partial^i \phi \partial_i \phi) + 2\, c_d\, (\phi\, \partial_j A_i \partial^j A^i - \phi\, \partial_i A_j \partial^j A^i) \right.$$
$$\left. \left. + 2\, c_d\, \phi (\partial_i A^i)^2 - 4 c_d\, \dot{\phi} A^i \partial^i \phi - 2\, c_d^2\, \phi (\dot{\phi})^2 \right) \right] \, ,$$



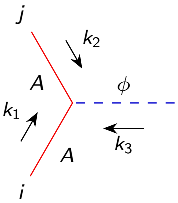
$$= -i\, (2\pi)^{d+1} \delta^{(d+1)}(k_1 + k_2 + k_3)\, \frac{2\, c_d}{\Lambda} \left(k_2^0 k_3^i + k_3^0 k_2^i \right) \, .$$

Scaling: $L^{-\frac{1}{2}}\, v^3$

Feynman rules for the conservative sector

Gravitational self interactions – ϕA^2 interaction vertex

$$
\begin{aligned}
S_{bulk}[\phi, A_i, \sigma_{ij}] \supset \int \mathrm{d}^{d+1}x \, \Bigg[\left(c_d \dot{\phi}^2 - c_d \, \partial^i \phi \partial_i \phi \right) + \left(\partial_j A_i \partial^j A^i - \partial_i A_j \partial^j A^i + (\partial_i A^i)^2 - \dot{A}_i \dot{A}^i \right) \\
+ \frac{1}{4} \left(\partial^j \sigma^i_i \partial_j \sigma^k_k + 4(\partial_i \sigma^{ij} \partial_k \sigma^k_j - \partial_j \sigma_{ik} \partial^k \sigma^{ij}) - 2 \, \partial_k \sigma_{ij} \partial^k \sigma^{ij} + (\dot{\sigma}^i_i)^2 - 2 \, \dot{\sigma}_{ij} \dot{\sigma}^{ij} \right) \\
+ \frac{1}{2\Lambda} \Big(c_d (2 \, \sigma_{ij} \partial^i \phi \partial^j \phi - \sigma^j_j \partial^i \phi \partial_i \phi) + 2 \, \boldsymbol{c_d} \, (\boldsymbol{\phi \, \partial_j A_i \partial^j A^i - \phi \, \partial_i A_j \partial^j A^i}) \\
+ 2 \, \boldsymbol{c_d} \, \boldsymbol{\phi (\partial_i A^i)}^2 - 4 c_d \, \dot{\phi} A^i \partial^i \phi - 2 \, c_d^2 \, \phi (\dot{\phi})^2 \Big) \Bigg] \; ,
\end{aligned}
$$



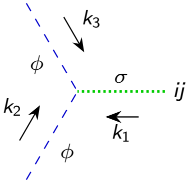
$$
= -i \, (2\pi)^{d+1} \delta^{(d+1)}(k_1 + k_2 + k_3) \, \frac{2 \, c_d}{\Lambda} \left(\delta^{ij} (k_1 \cdot k_2) + k_1^i k_2^j - k_1^j k_2^i \right) \; .
$$

Scaling: $L^{-\frac{1}{2}} \, v^2$

Feynman rules for the conservative sector

Gravitational self interactions – $\phi^2\sigma$ interaction vertex

$$S_{bulk}[\phi, A_i, \sigma_{ij}] \supset \int \mathrm{d}^{d+1}x \left[\left(c_d \dot{\phi}^2 - c_d\, \partial^i\phi\partial_i\phi \right) + \left(\partial_j A_i \partial^j A^i - \partial_i A_j \partial^j A^i + (\partial_i A^i)^2 - \dot{A}_i \dot{A}^i \right) \right.$$
$$+ \frac{1}{4} \left(\partial^j \sigma^i_i \partial_j \sigma^k_k + 4(\partial_i \sigma^{ij} \partial_k \sigma^k_j - \partial_j \sigma_{ik} \partial^k \sigma^{ij}) - 2\, \partial_k \sigma_{ij} \partial^k \sigma^{ij} + (\dot{\sigma}^i_i)^2 - 2\, \dot{\sigma}_{ij} \dot{\sigma}^{ij} \right)$$
$$+ \frac{1}{2\Lambda} \left(\boldsymbol{c_d (2\, \sigma_{ij} \partial^i\phi \partial^j\phi - \sigma^j_j \partial^i\phi \partial_i\phi)} + 2\, c_d \left(\phi\, \partial_j A_i \partial^j A^i - \phi\, \partial_i A_j \partial^j A^i \right) \right.$$
$$\left. \left. + 2\, c_d\, \phi(\partial_i A^i)^2 - 4 c_d\, \dot{\phi} A^i \partial^i \phi - 2\, c_d^2\, \phi(\dot{\phi})^2 \right) \right] ,$$



$$= i\,(2\pi)^{d+1} \delta^{(d+1)}(k_1 + k_2 + k_3)\, \frac{c_d}{\Lambda} \left(\delta^{ij} (\mathrm{k}_2 \cdot \mathrm{k}_3) - \left(k_2^i k_3^j + k_2^j k_3^i \right) \right) .$$

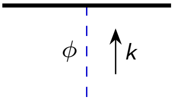
Scaling: $L^{-\frac{1}{2}}\, v^2$

Feynman rules for the conservative sector

Worldline-gravity interactions – Point particle action

$$
\begin{aligned}
S_{pp}^{(PP)}[x_a^\mu, \phi, A_i, \sigma_{ij}] &= -\sum_{a=1}^{2} m_a \int \mathrm{d}t\, \mathrm{d}^{d+1}x\ \delta^{(d+1)}(x - x_a(t)) \left(e^{\frac{\Phi}{\Lambda}} \left[(1 - \frac{A_i}{\Lambda} v_a^i)^2 - e^{-c_d \frac{\phi}{\Lambda}} v_a^2 - e^{-c_d \frac{\phi}{\Lambda}} (\frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j) \right]^{\frac{1}{2}} \right) \\
&= -\sum_{a=1}^{2} m_a \int \mathrm{d}t\, \mathrm{d}^{d+1}x\ \delta^{(d+1)}(x - x_a(t)) \left(\sqrt{1 - v_a^2} + \left(\frac{2 + (-2 + c_d)\, v_a^2}{2\sqrt{1 - v_a^2}} \right) \frac{\phi}{\Lambda} \right. \\
&\quad + \left(-\frac{1}{\sqrt{1 - v_a^2}} \right) \frac{A_i}{\Lambda} v_a^i + \left(-\frac{1}{2\sqrt{1 - v_a^2}} \right) \frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j + \left(\frac{-2 + (2 + c_d)\, v_a^2}{2\,(1 - v_a^2)^{3/2}} \right) \frac{\phi}{\Lambda} \frac{A_i}{\Lambda} v_a^i \\
&\quad + \left(-\frac{2 + c_d\,(-2 + v_a^2) - 2v_a^2}{4\,(1 - v_a^2)^{3/2}} \right) \frac{\phi}{\Lambda} \frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j + \left(-\frac{1}{2\,(1 - v_a^2)^{3/2}} \right) \frac{A_i}{\Lambda} v_a^i \frac{\sigma_{jk}}{\Lambda} v_a^j v_a^k \\
&\quad + \frac{1}{2} \left(\frac{4 + v_a^2\,(-8 - 2\,(-2 + c_d)\, c_d + (-2 + c_d)^2 v_a^2)}{4\,(1 - v_a^2)^{3/2}} \right) \frac{\phi^2}{\Lambda^2} + \ldots
\end{aligned}
$$

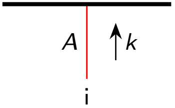
$$
\begin{aligned}
S_{pp}^{(PP)}[x_a^\mu, \phi, A_i, \sigma_{ij}] = -\sum_{a=1}^{2} m_a \int \mathrm{d}t\, \mathrm{d}^{d+1}x\; \delta^{(d+1)}(x - x_a(t)) \Bigg(\left(\frac{2 + (-2 + c_d)\, v_a^2}{2\sqrt{1 - v_a^2}} \right) \frac{\phi}{\Lambda} \\
+ \left(-\frac{1}{\sqrt{1 - v_a^2}} \right) \frac{A_i}{\Lambda} v_a^i + \left(-\frac{1}{2\sqrt{1 - v_a^2}} \right) \frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j + \left(\frac{-2 + (2 + c_d)\, v_a^2}{2\,(1 - v_a^2)^{3/2}} \right) \frac{\phi}{\Lambda} \frac{A_i}{\Lambda} v_a^i \\
+ \left(-\frac{2 + c_d\,(-2 + v_a^2) - 2v_a^2}{4\,(1 - v_a^2)^{3/2}} \right) \frac{\phi}{\Lambda} \frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j + \left(-\frac{1}{2\,(1 - v_a^2)^{3/2}} \right) \frac{A_i}{\Lambda} v_a^i \frac{\sigma_{jk}}{\Lambda} v_a^j v_a^k \\
+ \frac{1}{2} \left(\frac{4 + v_a^2\,(-8 - 2\,(-2 + c_d)\, c_d + (-2 + c_d)^2 v_a^2)}{4\,(1 - v_a^2)^{3/2}} \right) \frac{\phi^2}{\Lambda^2} + \dots
\end{aligned}
$$



$$
= -i \sum_{a=1}^{2} \frac{m_a}{\Lambda} \int \mathrm{d}t\; e^{-ikx_a(t)} \left(\frac{2 + (-2 + c_d)\, v_a^2}{2\sqrt{1 - v_a^2}} \right)
$$

LO scaling: $L^{\frac{1}{2}}\, v^0$

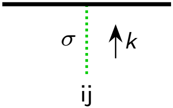
$$S_{pp}^{(PP)}[x_a^\mu,\phi,A_i,\sigma_{ij}] = -\sum_{a=1}^{2} m_a \int \mathrm{d}t\, \mathrm{d}^{d+1}x\ \delta^{(d+1)}(x-x_a(t)) \left(\left(\frac{2+(-2+c_d)\,v_a^2}{2\sqrt{1-v_a^2}} \right) \frac{\phi}{\Lambda} \right.$$
$$+ \left(-\frac{1}{\sqrt{1-v_a^2}} \right) \frac{A_i}{\Lambda} v_a^i + \left(-\frac{1}{2\sqrt{1-v_a^2}} \right) \frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j + \left(\frac{-2+(2+c_d)\,v_a^2}{2\,(1-v_a^2)^{3/2}} \right) \frac{\phi}{\Lambda} \frac{A_i}{\Lambda} v_a^i$$
$$+ \left(-\frac{2+c_d\,(-2+v_a^2)-2v_a^2}{4\,(1-v_a^2)^{3/2}} \right) \frac{\phi}{\Lambda} \frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j + \left(-\frac{1}{2\,(1-v_a^2)^{3/2}} \right) \frac{A_i}{\Lambda} v_a^i \frac{\sigma_{jk}}{\Lambda} v_a^j v_a^k$$
$$\left. + \frac{1}{2} \left(\frac{4+v_a^2\,(-8-2\,(-2+c_d)\,c_d+(-2+c_d)^2 v_a^2)}{4\,(1-v_a^2)^{3/2}} \right) \frac{\phi^2}{\Lambda^2} + \dots \right.$$



$$= -i \sum_{a=1}^{2} \frac{m_a}{\Lambda} \int \mathrm{d}t\ e^{-ikx_a(t)} \left(-\frac{1}{\sqrt{1-v_a^2}} \right) v_a^i$$

LO scaling: $L^{\frac{1}{2}}\ v^1$

$$S_{pp}^{(PP)}[x_a^\mu,\phi,A_i,\sigma_{ij}] = -\sum_{a=1}^{2} m_a \int \mathrm{d}t\,\mathrm{d}^{d+1}x\;\delta^{(d+1)}(x-x_a(t))\,\Bigg(\left(\frac{2+(-2+c_d)\,v_a^2}{2\sqrt{1-v_a^2}}\right)\frac{\phi}{\Lambda}$$
$$+\left(-\frac{1}{\sqrt{1-v_a^2}}\right)\frac{A_i}{\Lambda}v_a^i+\left(-\frac{1}{2\sqrt{1-v_a^2}}\right)\frac{\sigma_{ij}}{\Lambda}v_a^iv_a^j+\left(\frac{-2+(2+c_d)\,v_a^2}{2\,(1-v_a^2)^{3/2}}\right)\frac{\phi}{\Lambda}\frac{A_i}{\Lambda}v_a^i$$
$$+\left(-\frac{2+c_d\,(-2+v_a^2)-2v_a^2}{4\,(1-v_a^2)^{3/2}}\right)\frac{\phi}{\Lambda}\frac{\sigma_{ij}}{\Lambda}v_a^iv_a^j+\left(-\frac{1}{2\,(1-v_a^2)^{3/2}}\right)\frac{A_i}{\Lambda}v_a^i\frac{\sigma_{jk}}{\Lambda}v_a^jv_a^k$$
$$+\frac{1}{2}\left(\frac{4+v_a^2\,(-8-2\,(-2+c_d)\,c_d+(-2+c_d)^2v_a^2)}{4\,(1-v_a^2)^{3/2}}\right)\frac{\phi^2}{\Lambda^2}+\dots$$



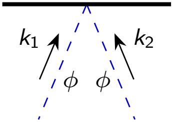
$$= -i\sum_{a=1}^{2}\frac{m_a}{\Lambda}\int \mathrm{d}t\; e^{-ikx_a(t)}\left(-\frac{1}{2\sqrt{1-v_a^2}}\right)v_a^iv_a^j$$

LO scaling: $L^{\frac{1}{2}}\,v^2$

Feynman rules for the conservative sector

Worldline-gravity interactions – worldline-ϕ^2

$$S_{pp}^{(PP)}[x_a^\mu,\phi,A_i,\sigma_{ij}] = -\sum_{a=1}^{2} m_a \int \mathrm{d}t\, \mathrm{d}^{d+1}x\ \delta^{(d+1)}(x - x_a(t)) \left(\left(\frac{2 + (-2 + c_d)\, v_a^2}{2\sqrt{1 - v_a^2}} \right) \frac{\phi}{\Lambda} \right.$$
$$+ \left(-\frac{1}{\sqrt{1 - v_a^2}} \right) \frac{A_i}{\Lambda} v_a^i + \left(-\frac{1}{2\sqrt{1 - v_a^2}} \right) \frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j + \left(\frac{-2 + (2 + c_d)\, v_a^2}{2\,(1 - v_a^2)^{3/2}} \right) \frac{\phi}{\Lambda} \frac{A_i}{\Lambda} v_a^i$$
$$+ \left(-\frac{2 + c_d\,(-2 + v_a^2) - 2v_a^2}{4\,(1 - v_a^2)^{3/2}} \right) \frac{\phi}{\Lambda} \frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j + \left(-\frac{1}{2\,(1 - v_a^2)^{3/2}} \right) \frac{A_i}{\Lambda} v_a^i \frac{\sigma_{jk}}{\Lambda} v_a^j v_a^k$$
$$\left. + \frac{1}{2} \left(\frac{4 + v_a^2\,(-8 - 2\,(-2 + c_d)\,c_d + (-2 + c_d)^2 v_a^2)}{4\,(1 - v_a^2)^{3/2}} \right) \frac{\phi^2}{\Lambda^2} + \dots \right.$$



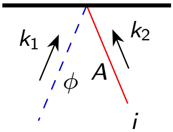
$$= -i \sum_{a=1}^{2} \frac{m_a}{\Lambda^2} \int \mathrm{d}t\ e^{-i(k_1+k_2)x_a(t)} \left[\frac{1}{4\,(1 - v_a^2)^{3/2}} \cdot \right.$$
$$\left. \cdot \left(4 + v_a^2 \left(-8 - 2\,(-2 + c_d)\,c_d + (-2 + c_d)^2 v_a^2 \right) \right) \right]$$

LO scaling: $L^0\, v^2$

Feynman rules for the conservative sector

Worldline-gravity interactions – worldline-ϕ-A

$$S_{pp}^{(PP)}[x_a^\mu,\phi,A_i,\sigma_{ij}] = -\sum_{a=1}^{2} m_a \int \mathrm{d}t\,\mathrm{d}^{d+1}x\;\delta^{(d+1)}(x-x_a(t))\left(\left(\frac{2+(-2+c_d)\,v_a^2}{2\sqrt{1-v_a^2}}\right)\frac{\phi}{\Lambda}\right.$$
$$+\left(-\frac{1}{\sqrt{1-v_a^2}}\right)\frac{A_i}{\Lambda}v_a^i+\left(-\frac{1}{2\sqrt{1-v_a^2}}\right)\frac{\sigma_{ij}}{\Lambda}v_a^iv_a^j+\left(\frac{-2+(2+c_d)\,v_a^2}{2\,(1-v_a^2)^{3/2}}\right)\frac{\phi}{\Lambda}\frac{A_i}{\Lambda}v_a^i$$
$$+\left(-\frac{2+c_d\,(-2+v_a^2)-2v_a^2}{4\,(1-v_a^2)^{3/2}}\right)\frac{\phi}{\Lambda}\frac{\sigma_{ij}}{\Lambda}v_a^iv_a^j+\left(-\frac{1}{2\,(1-v_a^2)^{3/2}}\right)\frac{A_i}{\Lambda}v_a^i\frac{\sigma_{jk}}{\Lambda}v_a^jv_a^k$$
$$\left.+\frac{1}{2}\left(\frac{4+v_a^2\,(-8-2\,(-2+c_d)\,c_d+(-2+c_d)^2v_a^2)}{4\,(1-v_a^2)^{3/2}}\right)\frac{\phi^2}{\Lambda^2}+\dots\right.$$



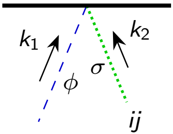
$$= -i\sum_{a=1}^{2}\frac{m_a}{\Lambda^2}\int \mathrm{d}t\; e^{-i(k_1+k_2)x_a(t)}\left(\frac{-2+(2+c_d)\,v_a^2}{2\,(1-v_a^2)^{3/2}}\right)v_a^i$$

LO scaling: $L^0\,v^3$

Feynman rules for the conservative sector

Worldline-gravity interactions – worldline-ϕ-σ

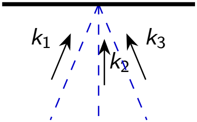
$$S_{pp}^{(PP)}[x_a^\mu,\phi,A_i,\sigma_{ij}] = -\sum_{a=1}^{2} m_a \int \mathrm{d}t\,\mathrm{d}^{d+1}x\;\delta^{(d+1)}(x-x_a(t))\left(\left(\frac{2+(-2+c_d)\,v_a^2}{2\sqrt{1-v_a^2}}\right)\frac{\phi}{\Lambda}\right.$$
$$+\left(-\frac{1}{\sqrt{1-v_a^2}}\right)\frac{A_i}{\Lambda}v_a^i+\left(-\frac{1}{2\sqrt{1-v_a^2}}\right)\frac{\sigma_{ij}}{\Lambda}v_a^i v_a^j+\left(\frac{-2+(2+c_d)\,v_a^2}{2\,(1-v_a^2)^{3/2}}\right)\frac{\phi}{\Lambda}\frac{A_i}{\Lambda}v_a^i$$
$$+\left(-\frac{2+c_d\,(-2+v_a^2)-2v_a^2}{4\,(1-v_a^2)^{3/2}}\right)\frac{\phi}{\Lambda}\frac{\sigma_{ij}}{\Lambda}v_a^i v_a^j+\left(-\frac{1}{2\,(1-v_a^2)^{3/2}}\right)\frac{A_i}{\Lambda}v_a^i\frac{\sigma_{jk}}{\Lambda}v_a^j v_a^k$$
$$\left.+\frac{1}{2}\left(\frac{4+v_a^2\,(-8-2\,(-2+c_d)\,c_d+(-2+c_d)^2 v_a^2)}{4\,(1-v_a^2)^{3/2}}\right)\frac{\phi^2}{\Lambda^2}+\dots\right.$$



$$= -i\sum_{a=1}^{2}\frac{m_a}{\Lambda^2}\int \mathrm{d}t\; e^{-i(k_1+k_2)x_a(t)}\left[-\frac{1}{4\,(1-v_a^2)^{3/2}}\cdot\left(2+c_d\left(-2+v_a^2\right)-2v_a^2\right)v_a^i v_a^j\right]$$

LO scaling: $L^0\,v^4$

$$S_{pp}^{(PP)}[x_a^\mu, \phi, A_i, \sigma_{ij}] = -\sum_{a=1}^{2} m_a \int \mathrm{d}t\, \mathrm{d}^{d+1}x\ \delta^{(d+1)}(x - x_a(t)) \left(\left(\frac{2 + (-2 + c_d)\, v_a^2}{2\sqrt{1 - v_a^2}} \right) \frac{\phi}{\Lambda} \right.$$
$$+ \left(-\frac{1}{\sqrt{1 - v_a^2}} \right) \frac{A_i}{\Lambda} v_a^i + \left(-\frac{1}{2\sqrt{1 - v_a^2}} \right) \frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j + \left(\frac{-2 + (2 + c_d)\, v_a^2}{2\,(1 - v_a^2)^{3/2}} \right) \frac{\phi}{\Lambda} \frac{A_i}{\Lambda} v_a^i$$
$$+ \left(-\frac{2 + c_d\,(-2 + v_a^2) - 2v_a^2}{4\,(1 - v_a^2)^{3/2}} \right) \frac{\phi}{\Lambda} \frac{\sigma_{ij}}{\Lambda} v_a^i v_a^j + \left(-\frac{1}{2\,(1 - v_a^2)^{3/2}} \right) \frac{A_i}{\Lambda} v_a^i \frac{\sigma_{jk}}{\Lambda} v_a^j v_a^k$$
$$\left. + \frac{1}{2} \left(\frac{4 + v_a^2\,(-8 - 2\,(-2 + c_d)\, c_d + (-2 + c_d)^2 v_a^2)}{4\,(1 - v_a^2)^{3/2}} \right) \frac{\phi^2}{\Lambda^2} + \dots \right.$$



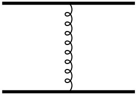
$$= -i \sum_{a=1}^{2} \frac{m_a}{\Lambda^3} \int \mathrm{d}t\ e^{-i(k_1 + k_2 + k_3) x_a(t)} \left[\frac{1}{8\,(1 - v_a^2)^{5/2}} \cdot \right.$$
$$\cdot \left(8 + v_a^2 \left(-24 + 4c_d\,(3 + (-3 + c_d)\, c_d) + 24 v_a^2 \right. \right.$$
$$\left. \left. \left. -2c_d\,(12 + (-9 + c_d)\, c_d)\, v_a^2 + (-2 + c_d)^3 \left(v_a^2 \right)^2 \right) \right) \right]$$

LO scaling: $L^{-\frac{1}{2}}\ v^4$

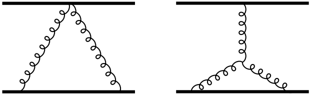
G^n topologies

G^1, G^2 and G^3 topologies

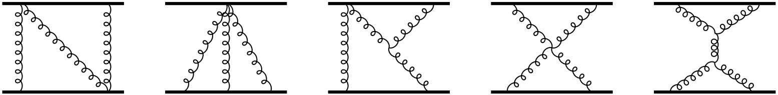
Each worldline vertex increases by one unit the order in G of the corresponding contribution.



(a)



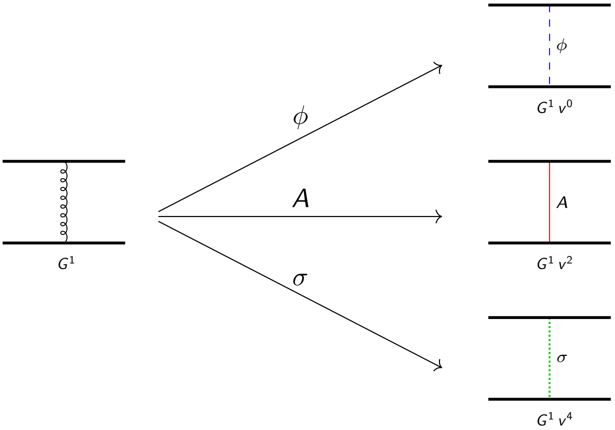
(a) (b)



(a) (b) (c) (d) (e)

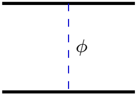
Conservative post-Newtonian diagrams

The G^n topologies have no definite scaling in the post-Newtonian expansion parameter (v^2): to obtain it we substitute the ϕ, A_i and σ_{ij} fields.



Conservative post-Newtonian diagrams

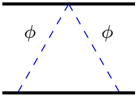
Leading order – 0 PN



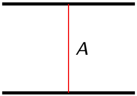
(0a) – $G^1\, v^0$

Conservative post-Newtonian diagrams

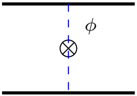
Next-to-leading order – 1 PN



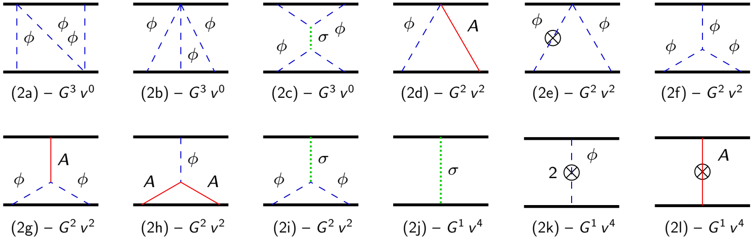
(1a) – $G^2\, v^0$

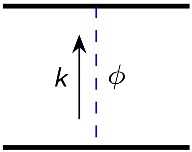


(1b) – $G^1\, v^2$



(1c) – $G^1\, v^2$

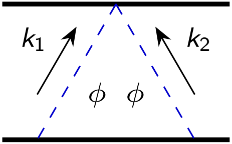

(2a) – $G^3\, v^0$
(2b) – $G^3\, v^0$
(2c) – $G^3\, v^0$
(2d) – $G^2\, v^2$
(2e) – $G^2\, v^2$
(2f) – $G^2\, v^2$
(2g) – $G^2\, v^2$
(2h) – $G^2\, v^2$
(2i) – $G^2\, v^2$
(2j) – $G^1\, v^4$
(2k) – $G^1\, v^4$
(2l) – $G^1\, v^4$



$$
\begin{aligned}
&= \frac{1}{2}\int \frac{\mathrm{d}^{d+1}k}{(2\pi)^{d+1}} \left(-i\frac{1}{\Lambda}\int \mathrm{d}t\; m_1\; e^{-ikx_1(t)}\right)\left(\frac{1}{2c_d}\,\frac{i}{k^2+i\epsilon}\right)\left(-i\frac{1}{\Lambda}\int \mathrm{d}t'\; m_2\; e^{ikx_2(t')}\right) \\
&= -\frac{i}{4c_d\Lambda^2} m_1 m_2 \int \mathrm{d}t\mathrm{d}t' \int \frac{\mathrm{d}^{d+1}k}{(2\pi)^{d+1}}\; e^{-ik(x_1(t)-x_2(t'))}\,\frac{1}{k^2+i\epsilon} + (1 \leftrightarrow 2) \\
&= \frac{i}{4c_d\Lambda^2} m_1 m_2 \int \mathrm{d}t\mathrm{d}t' \int \frac{\mathrm{d}^{d+1}k}{(2\pi)^{d+1}}\; e^{-ik(x_1(t)-x_2(t'))}\,\frac{1}{|\boldsymbol{k}|^2} + (1 \leftrightarrow 2) + \mathcal{O}\left(L\, v^2\right)\;.
\end{aligned}
$$

$$
\begin{aligned}
S_{eff} &= \frac{1}{4c_d\Lambda^2} m_1 m_2 \int \frac{\mathrm{d}^d k}{(2\pi)^d} \int \mathrm{d}t \int \mathrm{d}t' \underbrace{\int \frac{\mathrm{d}k^0}{(2\pi)} e^{-ik^0(t-t')}}_{=\delta(t'-t)} e^{i\mathbf{k}\cdot(\mathbf{x}_1(t)-\mathbf{x}_2(t'))} \frac{1}{|\mathbf{k}|^2} + (1\leftrightarrow 2) + \mathcal{O}\left(L\, v^2\right) \\
&= \frac{1}{4c_d\Lambda^2} m_1 m_2 \int \mathrm{d}t \underbrace{\int \frac{\mathrm{d}^d k}{(2\pi)^d}\; e^{i\mathbf{k}\cdot\mathbf{r}} \frac{1}{|\mathbf{k}|^2}}_{\equiv I_F(d,1)[\mathbf{r}]]} + (1\leftrightarrow 2) + \mathcal{O}\left(L\, v^2\right) \\
&= \frac{1}{4c_d\Lambda^2} m_1 m_2 \int \mathrm{d}t\; \frac{\Gamma\left(\frac{d}{2}-1\right)}{(4\pi)^{\frac{d}{2}}\Gamma(1)} \left(\frac{|\mathbf{r}|}{2}\right)^{2-d} + (1\leftrightarrow 2) + \mathcal{O}\left(L\, v^2\right) \\
&= \frac{32\pi G}{8} m_1 m_2 \int \mathrm{d}t\; \frac{1}{8\pi}\frac{1}{r} + (1\leftrightarrow 2) + \mathcal{O}\left(L\, v^2\right) + \mathcal{O}\left(d-3\right) \\
&= \int \mathrm{d}t\; \frac{Gm_1 m_2}{r} + \mathcal{O}\left(L\, v^2\right) \quad (d=3)
\end{aligned}
$$

$$
S_{eff} = \int \mathrm{d}t\; T - V \implies V_{0PN} = -\frac{Gm_1 m_2}{r}
$$



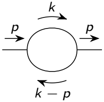
$$
= \frac{1}{2} \int \frac{\mathrm{d}^{d+1} k_1}{(2\pi)^{d+1}} \int \frac{\mathrm{d}^{d+1} k_2}{(2\pi)^{d+1}} \left(-i \frac{m_2}{\Lambda} \int \mathrm{d}t_1 \left(\frac{2 + (-2 + c_d)\, v_2^2}{2\sqrt{1 - v_2^2}} \right) e^{i k_1 x_2(t_1)} \right)
$$

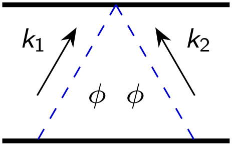
$$
\left(-\frac{1}{2c_d} \frac{i}{|k_1|^2} \right) \left(-i \frac{m_1}{\Lambda^2} \int \mathrm{d}t_2 \; e^{-i(k_1+k_2) x_1(t_2)} \right.
$$

$$
\left. \times \left[\frac{\left(4 + v_1^2 \left(-8 - 2\left(-2 + c_d\right) c_d + \left(-2 + c_d\right)^2 v_1^2\right)\right)}{4\left(1 - v_1^2\right)^{3/2}} \right] \right)
$$

$$
\left(-\frac{1}{2c_d} \frac{i}{|k_2^2|} \right) \left(-i \frac{m_2}{\Lambda} \int \mathrm{d}t_3 \left(\frac{2 + (-2 + c_d)\, v_2^2}{2\sqrt{1 - v_2^2}} \right) e^{i k_2 x_2(t_3)} \right) .
$$

$$
\begin{aligned}
&= -\frac{1}{2}\frac{i}{4c_d^2}\frac{m_1 m_2^2}{\Lambda^4}\int \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}t_3 \int \frac{\mathrm{d}^{d+1}k_1}{(2\pi)^{d+1}} \int \frac{\mathrm{d}^{d+1}k_2}{(2\pi)^{d+1}} \Bigg[\frac{1}{|k_1|^2}\frac{1}{|k_2^2|} \\
&\qquad \times e^{-i[k_1(x_2(t_2)-x_1(t_1))+k_2(x_2(t_2)-x_1(t_3))]} \Bigg] + \mathcal{O}\left(L\, v^4\right) \\
&= -\frac{1}{2}\frac{i}{4c_d^2}\frac{m_1 m_2^2}{\Lambda^4}\int \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}t_3 \Bigg[\underbrace{\int \frac{\mathrm{d}k_1^0}{2\pi} e^{-ik_1^0(t_2-t_1)}}_{=\delta(t_2-t_1)} \underbrace{\int \frac{\mathrm{d}k_2^0}{2\pi} e^{-ik_2^0(t_2-t_3)}}_{=\delta(t_2-t_3)} \\
&\qquad \times \int \frac{\mathrm{d}^d \mathrm{k}_1}{(2\pi)^d}\frac{1}{|\mathrm{k}_1|^2} e^{i\mathrm{k}_1\cdot(\mathrm{x}_1(t_2)-\mathrm{x}_2(t_1))} \int \frac{\mathrm{d}^d \mathrm{k}_2}{(2\pi)^d}\frac{1}{|\mathrm{k}_2|^2} e^{i\mathrm{k}_2(\mathrm{x}_1(t_2)-\mathrm{x}_2(t_3))} \Bigg] + \mathcal{O}\left(L\, v^4\right) \\
&= -\frac{i}{8c_d^2}\frac{m_1 m_2^2}{\Lambda^4}\int \mathrm{d}t \int \frac{\mathrm{d}^d \mathrm{k}_1}{(2\pi)^d} \int \frac{\mathrm{d}^d \mathrm{k}_2}{(2\pi)^d} \frac{1}{|\mathrm{k}_1|^2}\frac{1}{|\mathrm{k}_2|^2} e^{i(\mathrm{k}_1+\mathrm{k}_2)\cdot\mathrm{r}} + \mathcal{O}\left(L\, v^4\right) \\
&= -\frac{i}{8c_d^2}\frac{m_1 m_2^2}{\Lambda^4}\int \mathrm{d}t \int \frac{\mathrm{d}^d \mathrm{p}}{(2\pi)^d} e^{i\mathrm{p}\cdot\mathrm{r}} \underbrace{\int \frac{\mathrm{d}^d \mathrm{k}}{(2\pi)^d}\frac{1}{|\mathrm{k}|^2}\frac{1}{|\mathrm{k}-\mathrm{p}|^2}} + \mathcal{O}\left(L\, v^4\right)
\end{aligned}
$$



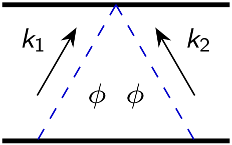


$$= -\frac{i}{8c_d^2}\frac{m_1 m_2^2}{\Lambda^4}\int \mathrm{d}t \int \frac{\mathrm{d}^d \mathrm{p}}{(2\pi)^d} e^{i\mathrm{p}\cdot\mathrm{r}} \int \frac{\mathrm{d}^d \mathrm{k}}{(2\pi)^d}\frac{1}{|\mathrm{k}|^2}\frac{1}{|\mathrm{k}-\mathrm{p}|^2} + \mathcal{O}\left(L\, v^4\right)$$

$$= -\frac{i}{8c_d^2}\frac{m_1 m_2^2}{\Lambda^4}\int \mathrm{d}t \int \frac{\mathrm{d}^d \mathrm{p}}{(2\pi)^d} e^{i\mathrm{p}\cdot\mathrm{r}} \left(\frac{1}{(4\pi)^{\frac{d}{2}}} \frac{\Gamma\left(2-\frac{d}{2}\right)\left(\Gamma\left(\frac{d}{2}-1\right)\right)^2}{\Gamma(d-2)} |\mathrm{p}|^{d-4} \right) + \mathcal{O}\left(L\, v^4\right)$$

$$= -\frac{i}{8c_d^2}\frac{m_1 m_2^2}{\Lambda^4}\int \mathrm{d}t \left(\frac{1}{16\pi^d}\left(\Gamma\left(\frac{d}{2}-1\right)\right)^2 \frac{1}{r^{2d-4}} \right) + \mathcal{O}\left(L\, v^4\right)$$

$$= -i\int \mathrm{d}t \frac{1}{2}\frac{G^2\, m_1 m_2^2}{r^2} + \mathcal{O}\left(L\, v^4\right) + \mathcal{O}\left(d-3\right)$$



$$= -\frac{i}{8c_d^2}\frac{m_1 m_2^2}{\Lambda^4}\int \mathrm{d}t \int \frac{\mathrm{d}^d \mathrm{p}}{(2\pi)^d} e^{i\mathrm{p}\cdot\mathrm{r}} \int \frac{\mathrm{d}^d \mathrm{k}}{(2\pi)^d}\frac{1}{|\mathrm{k}|^2}\frac{1}{|\mathrm{k}-\mathrm{p}|^2} + \mathcal{O}\left(L\,v^4\right)$$

$$= -\frac{i}{8c_d^2}\frac{m_1 m_2^2}{\Lambda^4}\int \mathrm{d}t \int \frac{\mathrm{d}^d \mathrm{p}}{(2\pi)^d} e^{i\mathrm{p}\cdot\mathrm{r}} \left(\frac{1}{(4\pi)^{\frac{d}{2}}}\frac{\Gamma\left(2-\frac{d}{2}\right)\left(\Gamma\left(\frac{d}{2}-1\right)\right)^2}{\Gamma(d-2)}|\mathrm{p}|^{d-4}\right) + \mathcal{O}\left(L\,v^4\right)$$

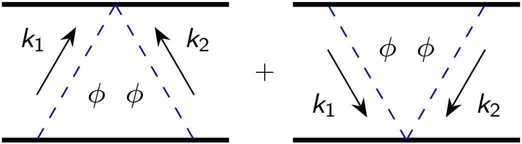
$$= -\frac{i}{8c_d^2}\frac{m_1 m_2^2}{\Lambda^4}\int \mathrm{d}t \left(\frac{1}{16\pi^d}\left(\Gamma\left(\frac{d}{2}-1\right)\right)^2\frac{1}{r^{2d-4}}\right) + \mathcal{O}\left(L\,v^4\right)$$

$$= -i\int \mathrm{d}t\,\frac{1}{2}\frac{G^2\,m_1 m_2^2}{r^2} + \mathcal{O}\left(L\,v^4\right) + \mathcal{O}\left(d-3\right)$$

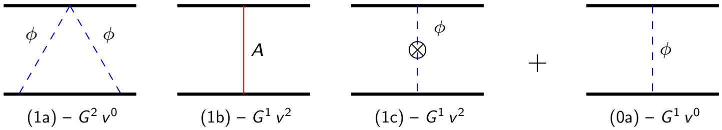
$$\Delta V = \frac{1}{2}\frac{G^2\,m_1 m_2^2}{r^2}$$

Conservative post-Newtonian diagrams

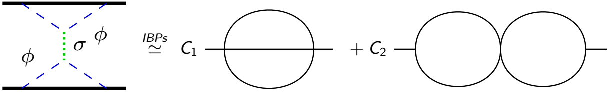
1PN calculation



$$\Delta V = \frac{1}{2}\frac{G^2\, m_1 m_2^2}{r^2} + \frac{1}{2}\frac{G^2\, m_1^2 m_2}{r^2}$$



$$\Delta V_{1PN} = -\frac{Gm_1 m_2}{2r}\left(-G\frac{(m_1+m_2)}{r} - 7\,(\mathrm{v}_1\cdot\mathrm{v}_2) + 3\left(v_1^2+v_2^2\right) - (\mathrm{v}_1\cdot\hat{\mathrm{r}})\,(\mathrm{v}_2\cdot\hat{\mathrm{r}})\right)$$



ϕ
σ
ϕ
$\overset{IBPs}{\simeq}$
C_1
$+ C_2$

Thank you for your attention!