Blackhole Shadows as probes to Modified Gravity and Electrodynamics

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Observation of the light deflection during a solar eclipse in 1919 in Brazil was the first experimental confirmation of a prediction from the general theory of relativity.

The great interest of the general public in the shadow of a black hole has its reason in the fact that it gives us a visual impression of how a black hole looks like. In addition, there is also a high scientific relevance of shadow observations, in particular because they can be used for distinguishing different types of black holes from each other.

Here we will show that this shadow can also be used to probe modified electrodynamics as well as modified gravity.

The Shadow Radius

The shadow silhouette is given by the photon sphere whose radius is given by

$$r_{\text{sh}} = \frac{r_{\text{ph}}}{\sqrt{A(r_{\text{ph}})}}. \tag{1}$$

for metrics like,

$$ds^2 = A(r)dt^2 - \frac{dr^2}{B(r)} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{2}$$

Our aim is to use the experimental data for the shadow radius to constraint parameters in modified theories of gravity/electrodynamics.

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Recently[1] has been shown that Podolsky's Electrodynamics on curved space-time is described by two possible independent invariants beyond the usual Maxwell term. These invariants are equivalent to promote the flat space lagrangian by a minimal coupling with the gravitational field plus a non-minimal coupling term, such that the lagrangian for the gauge field in curved spaces gets the following final form:

$$\begin{aligned}\mathcal{L}_m = & - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + \frac{(a^2+2b^2)}{2}\nabla_\beta F^{\alpha\beta}\nabla_\gamma F_\alpha{}^\gamma \\ & + b^2\left(R_{\sigma\beta}F^{\sigma\alpha}F_\alpha{}^\beta + R_{\alpha\sigma\beta\gamma}F^{\sigma\gamma}F^{\alpha\beta}\right). \end{aligned} \quad (3)$$

[1] R. R. Cuzinatto, C. A. M. de Melo, L. G. Medeiros, B. M. Pimentel and P. J. Pompeia, Bopp–Podolsky black holes and the no-hair theorem, Eur. Phys. J. C 78, 43 (2018). arXiv:1706.09455

Adding the Einstein-Hilbert term for the graviational field, the corresponding field equations are

$$\nabla_\nu \left[F^{\mu\nu} - \left(a^2 + 2b^2\right) H^{\mu\nu} + 2b^2 S^{\mu\nu}\right] = 0, \tag{4}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu} = \; 8\pi \left(T^M_{\mu\nu} + T^a_{\mu\nu} + T^b_{\mu\nu}\right), \tag{5}$$

where

$$H^{\mu\nu} \equiv \nabla^\mu K^\nu - \nabla^\nu K^\mu, \tag{6}$$

$$S^{\mu\nu} \equiv F^{\mu\sigma} R_\sigma{}^\nu - F^{\nu\sigma} R_\sigma{}^\mu + 2R^\mu{}_\sigma{}^\nu{}_\beta F^{\beta\sigma}, \tag{7}$$

with $K^\mu \equiv \nabla_\gamma F^{\mu\gamma}$.

The components of the energy-momentum tensor are given by

$$T^{M}_{\mu\nu} = \frac{1}{4\pi}\left[F_{\mu\sigma}F^{\sigma}{}_{\nu} + g_{\mu\nu}\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}\right], \tag{8}$$

$$\begin{aligned} T^{a}_{\mu\nu} &= \frac{a^2}{4\pi}\left[g_{\mu\nu}F_{\beta}{}^{\gamma}\nabla_{\gamma}K^{\beta} + \frac{g_{\mu\nu}}{2}K^{\beta}K_{\beta}\right. \\ &\left. + \; 2F_{(\mu}{}^{\alpha}\nabla_{\nu)}K_{\alpha} - 2F_{(\mu}{}^{\alpha}\nabla_{\alpha}K_{\nu)} - K_{\mu}K_{\nu}\right], \end{aligned} \tag{9}$$

$$\begin{aligned} T^{b}_{\mu\nu} &= \frac{b^2}{2\pi}\left[-\frac{1}{4}g_{\mu\nu}\nabla^{\beta}F^{\alpha\gamma}\nabla_{\beta}F_{\alpha\gamma} + F^{\gamma}_{(\mu}\nabla^{\beta}\nabla_{\beta}F_{\nu)\gamma}\right. \\ &\left. + \; F_{\gamma(\mu}\nabla_{\beta}\nabla_{\nu)}F^{\beta\gamma} - \nabla_{\beta}\left(F_{\gamma}{}^{\beta}\nabla_{(\mu}F_{\nu)}{}^{\gamma}\right)\right]. \end{aligned} \tag{10}$$

The notation (...) indicates symmetrization with respect to indices $\mu\nu$.

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Perturbative solution

From the spherically symmetric Ansatz in the (t, r, θ, ϕ) coordinates, i.e.,

$$ds^2 = A(r)dt^2 - \frac{dr^2}{B(r)} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{11}$$

an approximate solution will be available if we assume

$$A(r) = A_0(r) + A_1(r), \tag{12}$$

$$B(r) = B_0(r) + B_1(r), \tag{13}$$

$$E(r) = E_0(r) + E_1(r). \tag{14}$$

As we are looking for a small deviation from the Reissner–Nordström geometry, then we assume the following input

$$A_0(r) = B_0(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \quad \text{and} \quad E_0(r) = \frac{Q}{r^2}. \tag{15}$$

Perturbative Solution

Under suitable boundary conditions, the perturbative solution is:

$$A_1(r) = \frac{2b^2Q^2}{r^4}\left(1 - \frac{3M}{r} + \frac{9Q^2}{5r^2}\right), \tag{16}$$

$$B_1(r) = -\frac{2b^2Q^2}{r^4}\left(2 - \frac{3M}{r} + \frac{6Q^2}{5r^2}\right), \tag{17}$$

$$E_1(r) = -\frac{b^2Q}{r^5}\left(8M - \frac{11Q^2}{r}\right). \tag{18}$$

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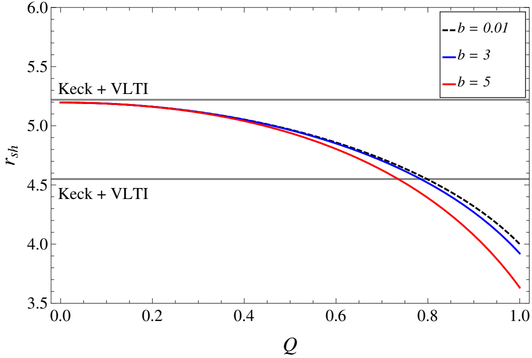
Sgr A* Radius

Survey	M $(10^6 M_\odot)$	D(kpc)
Keck[2]	3.975 ± 0.058	7.959 ± 0.059
VLTI[3]	4.297 ± 0.012	8.277 ± 0.009



[2] T. Do, *et al.* Science **365**, no.6454, 664-668 (2019). arXiv:1907.10731

[3] R. Abuter *et al.* Astron.Astrophys. **657**, L12 (2022). arXiv:2112.07478

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Co-varying physical couplings (CPC) is framework for modified gravity which contemplate the possibility for two or all the couplings in the set $\{c, G, \Lambda\}$ varying at the same time but not arbitrarily. The CPC framework is a modified gravity alternative wherein Einstein field equations hold, i.e.,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{19}$$

while the Newtonian gravitational coupling G, the spacetime causality coupling c, and the cosmological parameter Λ are not genuine constants but otherwise are allowed to vary with respect to the spacetime coordinates x^μ, that is to say, $G = G(x^\mu)$, $c = c(x^\mu)$, and $\Lambda = \Lambda(x^\mu)$.

Using the metric compatibility condition $\nabla_\mu g^{\mu\nu} = 0$, and assuming covariant conservation of the energy momentum tensor,

$$\nabla_\mu T^{\mu\nu} = 0, \tag{20}$$

one has:

$$\left(\frac{\partial_\mu G}{G} - 4\frac{\partial_\mu c}{c}\right)\frac{8\pi G}{c^4}T^{\mu\nu} - (\partial_\mu \Lambda)\, g^{\mu\nu} = 0. \tag{21}$$

This relation is called the general constraint (GC). It entangles the eventual variations of $\{c, G, \Lambda\}$, thus the name CPC of our framework. The relation between the luminosity distance d_L and the proper distance d_p in the CPC context is

$$d_L(z) = \left(\frac{c_e}{c_0}\right)(1+z)\, d_p(z), \tag{22}$$

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The McVittie geometry[4] describes a black hole in an expanding universe.

$$ds^2 = -\left(\frac{1-\mu(t,r)}{1+\mu(t,r)}\right)^2 c^2dt^2 + a(t)^2\left(1+\mu(t,r)\right)^4 \times \left(dr^2 + r^2 d\Omega^2\right), \tag{23}$$

where

$$\mu(t,r) = \frac{GM}{2c^2 r a(t)}, \tag{24}$$

$a(t)$ is the scale factor, and M is the black hole mass.

[4] McVittie G. C., 1933, Mon. Not. Roy. Astron. Soc., 93, 325

Angular Diameter Distance

The angular diameter distance in terms of the redshift z in the CPC framework adopted here is

$$d_A(z) = \frac{c_0}{a_0 H_0 (1+z)} \int_0^z \frac{\phi_c(z) dz}{E(z)}, \tag{25}$$

where the normalized Hubble parameter $E(z)$ reads

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0})\phi_\Lambda(z)\phi_c(z)^2}. \tag{26}$$

where

$$\phi_\Lambda = \phi_{\Lambda,0} + \phi_{\Lambda,1}\left(1 - \frac{a}{a_0}\right) = 1 + \phi_{\Lambda,1}\left(\frac{z}{1+z}\right), \tag{27}$$

$$\phi_c(z) = \left[1 - \frac{(1-\Omega_{m,0})\, a_0^4 \phi_{\Lambda,1}}{4\Omega_{m,0}}\left(1 - (1+z)^{-4}\right)\right]^{-\frac{1}{2}}. \tag{28}$$

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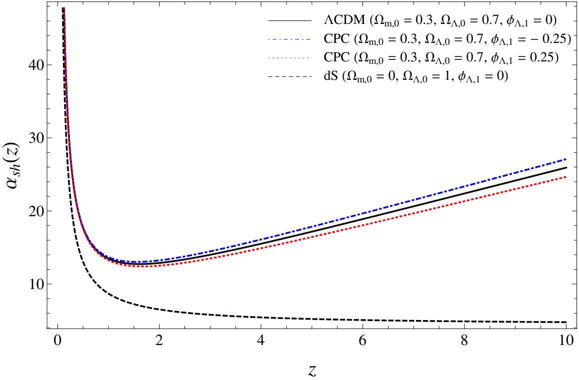

Figure: In the CPC framework shadows could appear smaller or larger compared with the ΛCDM model. For $\Omega_{m,0} > 0$, both models show that the angular radius is unbounded as the redshift increases. We adopted $c_0 = G_0 = a_0 = H_0 = M = 1$.

Constraining CPC

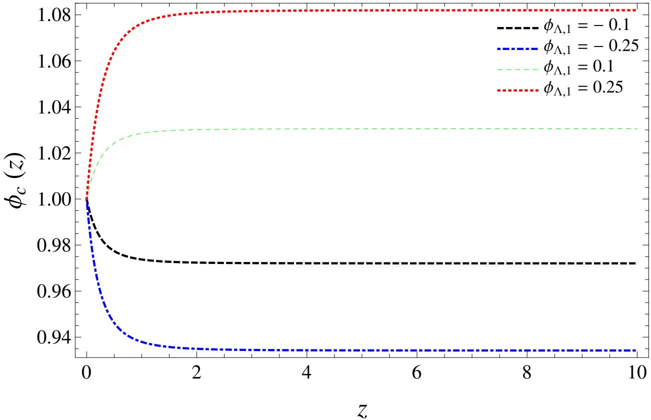

Figure: Deviation of the speed of light, Eq. (28), for different values of $\phi_{\Lambda,1}$ and redshift z. For negative values, the speed of light had its value increased during the cosmic expansion. For positive values, one has the contrary situation. We adopted $a_0 = 1$ and $\Omega_{m,0} = 0.3$.

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- We show, for the first time, that the radius of black hole shadows can be used to constraint or even ruled out modified theories of gravity and electrodynamics.
- The method was applied to CPC modified gravity and Podolsky eletrodynamics, but in principle can be used for any other modified theory of gravity or electrodynamics.
- More data is necessary to fix better limits in the parameters of each of these cases.