Gravitational Wave Astronomy and Precision Gravity

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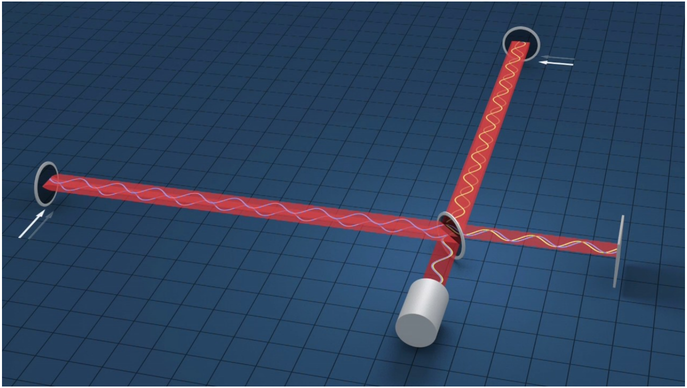
Dec 5th, 2023 - Poço de Caldas

LIGO and Virgo (+KAGRA): very precise rulers



Robert Hurt (Caltech)

Light intensity $\propto$ light travel difference in perpendicular arms
Effective optical path increased by factor $N \sim 500$ via Fabry-Perot cavities

Phase shift $\Delta\phi \sim 10^{-8}$ can be measured $\sim 2\pi N \Delta L/\lambda \to \Delta L \sim 10^{-15}/N$ m

Almost omnidirectional detectors

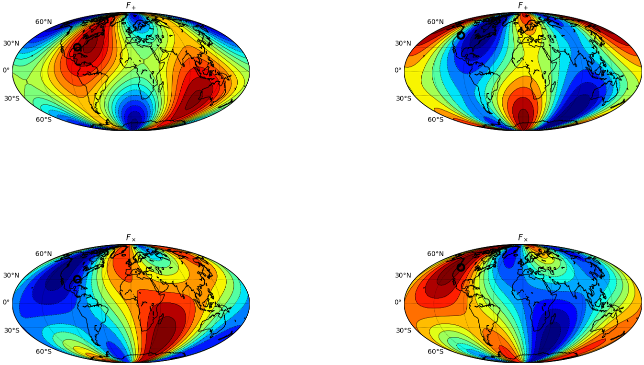
Detectors measure h_{det}: linear combination $F_+ h_+ + F_\times h_\times$

L H

-1 0 1

F_+

$F_\times$



$h_{+,\times}$ depend on source

pattern functions $F_{+,\times}$ depend on orientation source/detector

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L V

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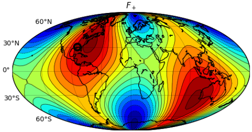
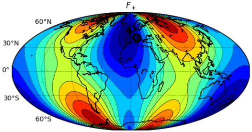
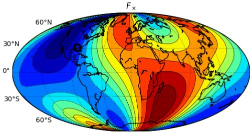
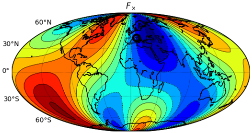




$F_\times$





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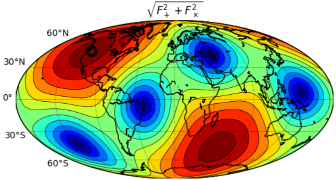
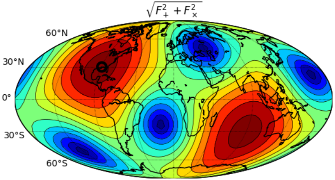
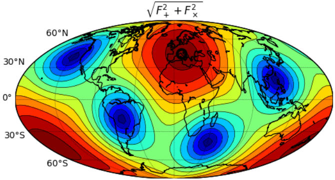
Pattern functions: $\sqrt{F_+^2 + F_\times^2}$







The LIGO and Virgo observatories

- Observation run **O1** Sept '15 - Jan '16 $\sim$ 130 days, with 49.6 days of actual data, PRX (2016) 4, 041014, 2 detectors, 3BBH
- **O2** Dec. '16 – Jul'17 2 det's + Aug '17 3 det's 3(+4) BBH + 1BNS in double (triple) coinc.
- **O3a**: 3 detectors, Apr - Sep 2019, 39 dets
- **O3b**: Nov 1st – Mar 27th 2020 $\rightarrow$ 90 detections
- **O4a**: Ongoing (since May 24th)

Wave generation: localized sources

Einstein formula relates h_{ij} to the source quadrupole moment Q_{ij}

$$
\begin{aligned}
Q_{ij} &= \int d^3x\rho\left(x_ix_j - \frac{1}{3}\delta_{ij}x^2\right), \qquad v^2 \simeq G_N M/r\,, \quad \eta \equiv m_1m_2/M^2 \\
h_{ij} &\sim g(\theta_{LN})\frac{2G_N}{D}\frac{d^2Q_{ij}}{dt^2} \simeq \frac{2G_N\eta M v^2}{D}\cos(2\phi(t))
\end{aligned}
$$

$$
\begin{aligned}
f &= 2\mathrm{kHz}\left(\frac{r}{30\mathrm{Km}}\right)^{-3/2}\left(\frac{M}{3M_\odot}\right)^{1/2} < f_{Max} \simeq 12kHz\left(\frac{M}{3M_\odot}\right)^{-1} \\
v &= 0.3\left(\frac{f}{1\mathrm{kHz}}\right)^{1/3}\left(\frac{M}{M_\odot}\right)^{1/3} < \frac{1}{\sqrt{6}}
\end{aligned}
$$

Geometric factor $g(\theta_{LN})$ takes account of transversality projection (angular momentum L of the binary, observation direction N)

$$
\begin{aligned}
h_+ &\sim \frac{1+\cos^2(\theta_{LN})}{2}\eta\frac{Mv^2}{D}\cos\phi(t_s/M,\eta,S_i^2/m_i^4,\ldots) \\
h_\times &\sim \cos(\theta_{LN})\,\eta\frac{Mv^2}{D}\sin\phi(t_s/M,\eta,\ldots)
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Amplitudes of 2 polarizations modulated by θ_{LN} ($h \nearrow$ for $\theta_{LN} \searrow_0$), never both vanishing unlike dipolar motion for the electromagnetic case

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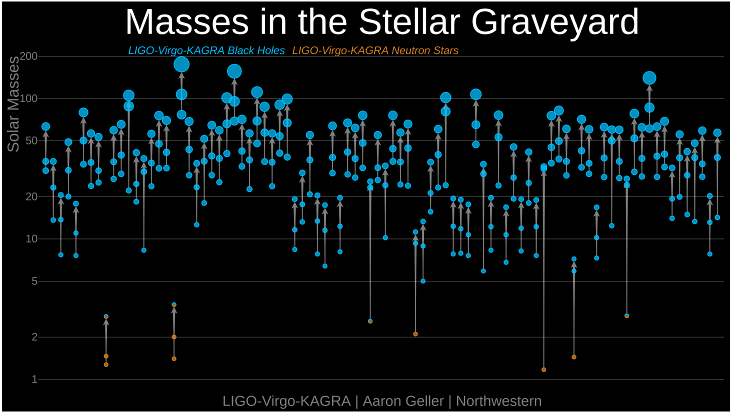
h sensitive to red-shifted masses $M \to M(1+z) \equiv \mathcal{M}$

Stellar ($< 100 M_\odot$) compact object with known masses



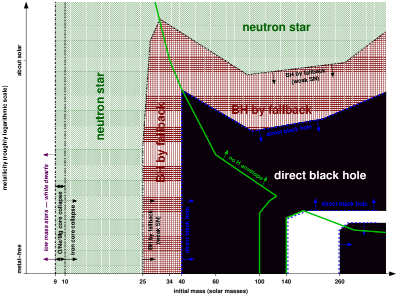
Frequency 10-10^3 Hz determines size of sources

Remnant of GW190521 first Intermediate Mass Black Hole ($> 10^2 M_\odot$)

SuperMassive BHs $\gtrsim 10^5 M_\odot$ (up to $10^9 M_\odot$)

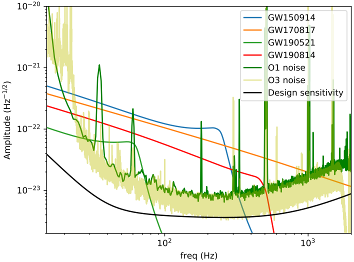
Solar Mass Black Holes

1st Mass gap: $2M_\odot < M_{BH} \lesssim 5M_\odot$
SN explosion prevents BH formation
2nd Mass gap: $50M_\odot < M_{BH} \lesssim 150M_\odot$
Pair Instability Super Nova
($\gamma \to e^+e^-$ drops pressure)
Are the (would be) gaps populated?



Heger+, Astrophys.J. 591 (2003) 288-300

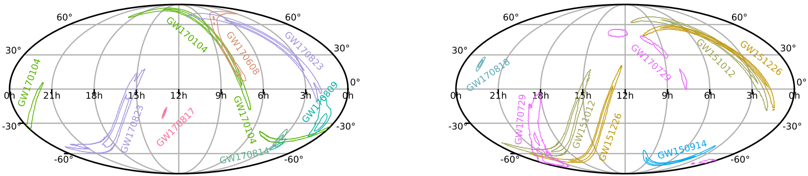
Sample wfs vs. detector's noise



Other groups searched for GWs in O1/2/3a public data:

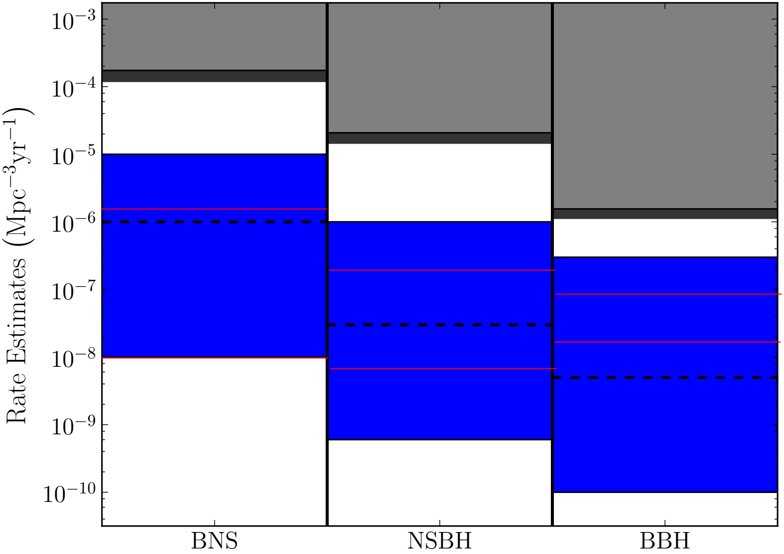
A. Nitz+ Ap.J. 891 123 ('19)
T.Venumadhav+ PRD 101 ('20)

Sky localization



Distances between 40 Mpc and $\sim$ 5 Gpc ($\pm$20%)
(Milky Way's size $\sim 30\mathrm{kpc}$)

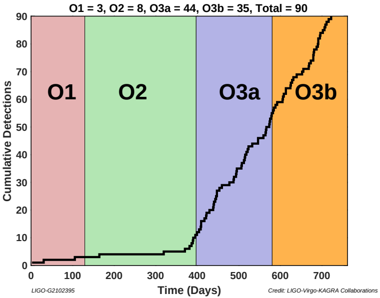
Image by Leo Singer, http://www.ligo.org

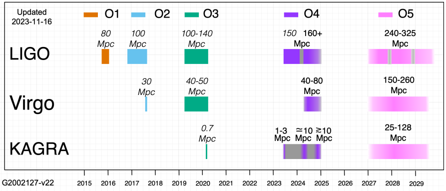


Astro predictions, measures from O1/O2/O3. Galaxy density $\sim 2 \times 10^{-2} Mpc^{-3}$

LIGO/Virgo CQG ('10) 27 173001, PRX ('16), APJ (2016), PRL 119 ('17)

LIGO/Virgo/KAGRA's prospects





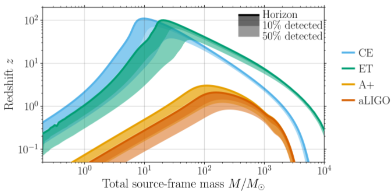
Future with ET and LISA looks very loud

Future 3rd generetaion detectors (Einstein Telescope, Cosmic Explore)/space telescope LISA will detect CBC signals with SNR $10 - 10^2$, with few golden events with SNR $\sim 10^3$.

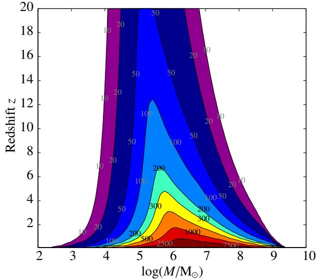
Templates few % accurate OK for characterising a source with SNR $O(10)$ (typical for LIGO/Virgo)

for SNR $\sim 10^3$ residual after extracting that source will have SNR $\sim O(10)$

1. baising parameter estimation
2. contaminating the extraction of additional sources.

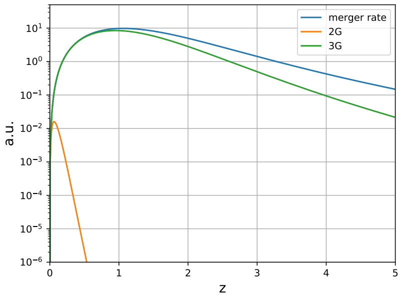


from arXiv:1902.09485

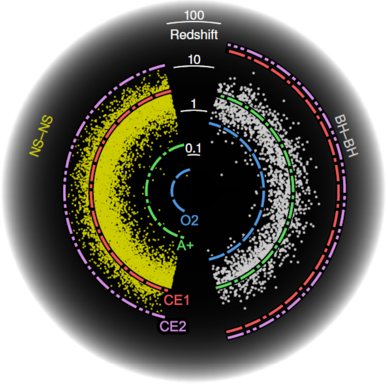


from arXiv:1201.3621

How many more?



Leandro, Marra, RS PRD '21



arXiv:1903.04615

New lines of research

- Precision gravity
- General Relativity tests in strong gravity
- "Hairiness" of black holes
- Universe expansion history (standard sirens)
- Phase transitions in the Early Universe
- Black hole mass function
- How binary systems form and how frequent are they? (star formation rate)
- Fate of a massive star
- Probe the the interior of a neutron star
- Understand the life of a pulsar and its evolution
- Dark matter and Gravitational Waves (modification of compact object and/or their environment)
- Data analysis challenges for signals with $SNR \sim 100$
- Central Core of Galaxies, Massive Black Holes and their role in Galaxy Formation
- Multi-messenger astronomy

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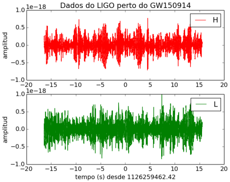
The importance of theoretical modeling



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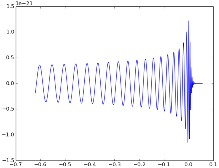

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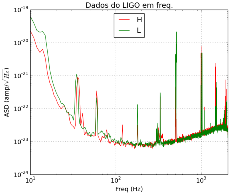
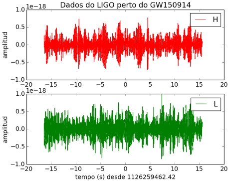


2

The importance of theoretical modeling

2



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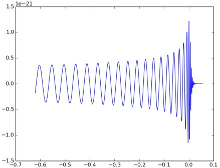

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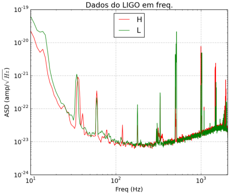

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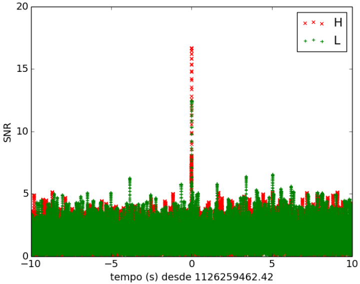

Data from https://losc.ligo.org/events/GW150914/

Fundamental GR: inspiral analytic model

Inspiral $h = A\cos(\phi(t))$ $\qquad \frac{\dot{A}}{A} \ll \dot{\phi}$

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \eta = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\begin{aligned} E(v) &= -\frac{1}{2}\eta M v^2 \left(1 + \#(\eta, S_i/m_i^2)v^2 + \#(\eta, S_i/m_i^2)v^4 + \dots\right) \\ P(v) \equiv -\frac{dE}{dt} &= \frac{32}{5G_N}v^{10}\left(1 + \#(\eta, S_i/m_i^2)v^2 + \#(\eta, S_i/m_i^2)v^3 + \dots\right) \end{aligned}$$

E(v)(P(v)) known up to 3(3.5)PN

$$\begin{aligned} \frac{1}{2\pi}\phi(T) &= \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)}dv \\ &\sim \int \left(1 + \#(\eta, S_i/m_i^2)v^2 + \dots + \#(\eta, S_i/m_i^2)v^6 + \dots\right)\frac{dv}{v^6} \end{aligned}$$

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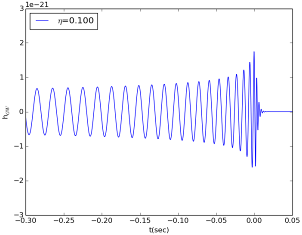
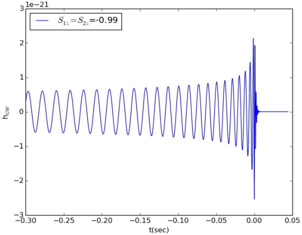
PN Coefficients (absorption $\sim v^8$, tidal $\sim v^{10}$)

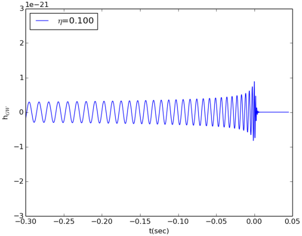
Looking for source fingerprints

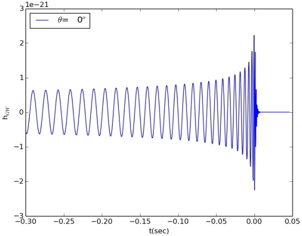
M_c fixed





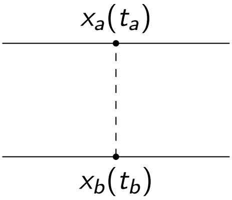
M fixed





1PM potential

Out of different ways of computing 2-body Post-Minkowskian expansion e.g. 1PM $O(G_N^1)$ potential gravity coupled to particle world-lines:



$$V_{PM}^{(1)}(x_a^\mu - x_b^\mu) = G_N T^a_{\mu\nu} T^b_{\rho\sigma} \Delta^{\mu\nu,\rho\sigma} \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik^\mu(x_{a\mu}-x_{b\mu})}}{|\mathrm{k}|^2 - k_0^2 + \epsilon\,\mathrm{terms}}$$

PM at higher orders

It is a formidable task to go to higher order: complete result so far at 3PM $O(G_N^3)$ (2PM beyond Newtonian interaction) with "particle physics" approach by

1. *Scattering amplitude* method by Bern, Cheung, Roiban, Shen, Solon PRL '19 and partial result at 4PM by

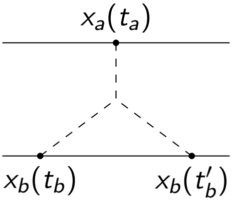
   Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng 2101.07254
2. *EFT+Boundary2Bound* by Kälin, Porto PRL '20

and up to 4PM $O(G_N^4)$ via the

3. "syncretic" *TuttiFrutti* method initiated by Bini, Damour, Geralico, PRL '20

PM gets complicated

At higher order things rapidly complicate



$$\begin{aligned} V^{(2PM)} &\supset G_N^2 m_1 m_2^2 \int d^4 p\, e^{ip_\mu(x_a^\mu(t_a)-x_b^\mu(t_b))} \frac{p^\alpha p^\beta}{p^2} \int d^4 k \frac{e^{ik^\mu(x_b^\mu(t_b)-x_b^\mu(t'_b))}}{(p-k)^2 k^2} \\ &= G_N^2 m_1 m_2^2 \int d^4 p\, e^{ip_\mu(x_a^\mu(t_a)-x_b^\mu(t_b))} \frac{p^\alpha p^\beta}{p^2} \Delta(p^\mu(x_{2\mu}(t_b)-x_{2\mu}(t'_b))) \end{aligned}$$

These kinds of “conservative” diagrams computed up to 4PM order

PN simplifies: Near

- Near Zone: consider $|\mathrm{k}| \gg k_0$, with $\frac{k_0^2}{|\mathrm{k}|^2} \sim v^2$

$$V_{PN-Near} = \int \frac{dk_0}{2\pi} \frac{d^3k}{(2\pi)^3} e^{ik_0(t_a-t_b)} \frac{e^{i\mathrm{k}\cdot(\mathrm{x}_a-\mathrm{x}_b)}}{|\mathrm{k}|^2} \left(1 + \frac{k_0^2}{|\mathrm{k}|^2} + \cdots \not\epsilon\right)$$

k_0 dependence factorizes $\rightarrow \int dk_0 k_0^{2n} e^{ik_0 t_{ab}} \sim \frac{d^n}{dt^n}\delta^{(2n)}(t_{ab})$
Near-Zone approximation $V_{PM} - V_{PN-Near}$ under control for $k_0^2 < |\mathrm{k}|^2$
Effects for $k_0 \simeq |\mathrm{k}|$ are missed: internal gravitons going on-shell

PN Simplifies: Far

- Far Zone, expand the numerator:

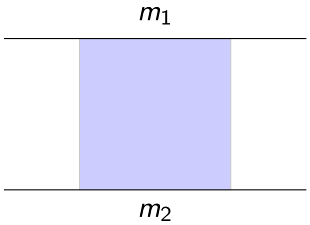
$$V_{PN-Far} \propto \int \frac{d^4k}{(2\pi)^4} e^{ik_0 t_{12}} \sum_n \frac{(i\mathrm{k}\cdot \mathrm{x}_{ab})^n}{n} \frac{1}{|\mathrm{k}|^2 - k_0^2 \pm iak_0(A,R)}$$

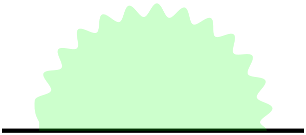
$$G_{A,R} = -\frac{1}{4\pi}\frac{\delta(t\pm r)}{r} \qquad \tilde{G}^*_{A,R}(k_0) = \tilde{G}_{A,R}(-k_0)$$

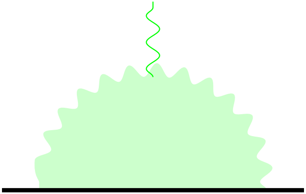
1. From individual world-lines to multipole expansions $Q_{i_1 \dots i_n}$, $L_{i_1 \dots i_n}$ with small parameter approximation $\mathrm{k}\cdot\mathrm{x}_{ab} \sim \frac{v}{r} \times r = v$
2. Time-symmetric process determined by $G_R + G_A$ (see later in this talk)
3. Longitudinal modes are present in Far Zone too, sourced by M, P_i, L_i

- Old friend of particle physicists: method of regions

Near vs. Far zone graphs (topology)







And 1 pt diagrams $\rightarrow$ radiation
In this talk only conservative sector
No radiation to ∞

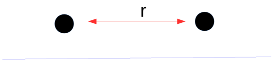
Interactions at high loop $\rightarrow$ lengthy expressions, double copy?

See e.g. Goldberger+ '18, Chen '18, Almeida+Foffa+RS '20

Fundamental aspects of the EFT formalism

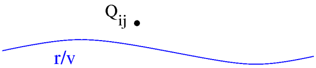
NRGR & PN approximation to GR: Small expansion parameter v, related to metric perturbation $v^2 \sim \frac{GM}{r}$

Near zone, $D \sim r$



Describe conservative dynamics

Far zone, $D \gtrsim \lambda = r/v$

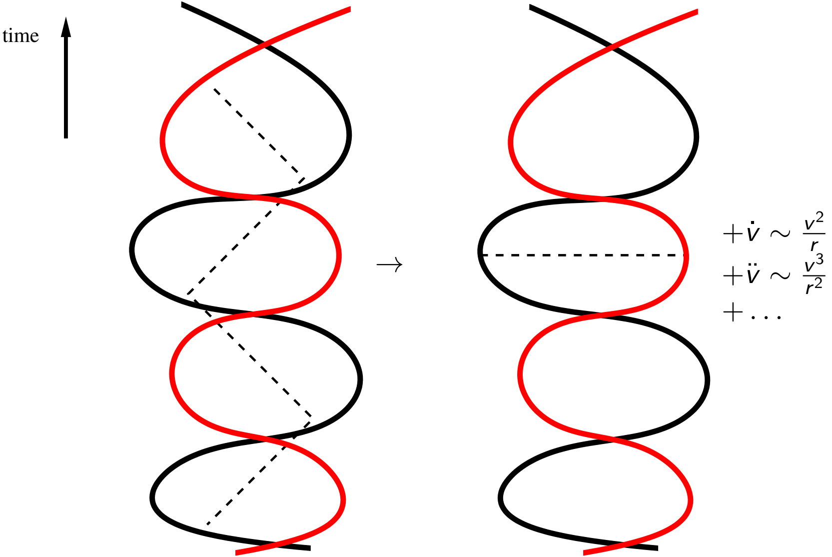


conservative + dissipative

EFT framework pioneered by W. Goldberger and I. Rothstein, PRD '06

for spin R. Porto PRD '06

NR zone trades knowledge over the full trajectory with knowledge of all derivatives of the trajectory at equal time (PN approximation)



Summary: 2 body dynamics expansions (spin-less)

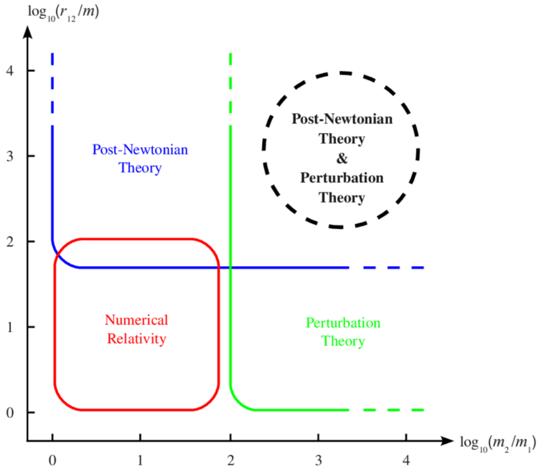
Post-Minkowskian expansion parameter is $G_N M/r$, vs PN expansion

$$\mathcal{L} = -Mc^2 + \frac{\mu v^2}{2} + \frac{GM\mu}{r} + \frac{1}{c^2}[\dots] + \frac{1}{c^4}[\dots]$$

Terms known so far

		N	1PN	2PN	3PN	4PN	5PN	...
0PM	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM					$1/r^4$	v^2/r^4	v^4/r^4	...
5PM						$1/r^5$	v^2/r^5	...
...							$1/r^6$(!)	...

Different approximation methods



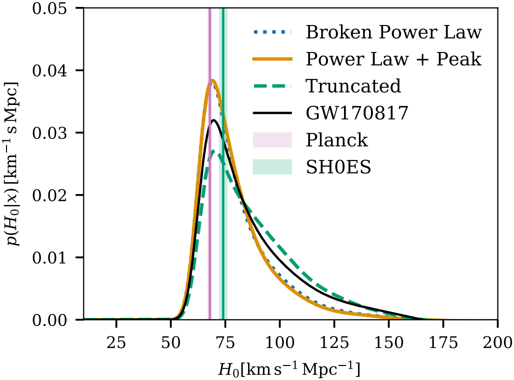
Blanchet et al. arXiv:1007.2614

Bini, Damour, Geralico in PRL ('19)+ completed 4PM dynamics from various input

Numerical relativity solution are expensive for large separation (large orbital scale) and large mass ratios (long dynamical evolution time)

The importance to know distance and redshift

Luminosity distance vs. redshift: $D_L H_0 = z + O(z^2)$

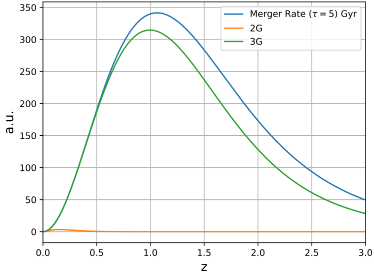


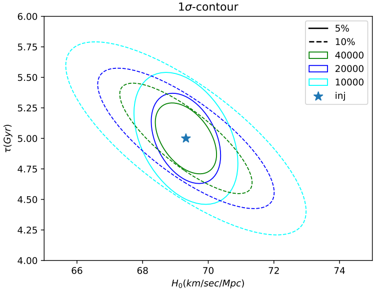
H_0 determination from EM bright 1 standard candle and 46 dark ones, short-circuiting with galaxy survey catalog GLADE+ Dálya et al. arXiv:2110.06184

LIGO/Virgo/KAGRA arXiv:2111.03604

Black sirens

Information also stored in black sirens if *statistical distribution* of merger known (with hyper-parameter τ)





Worst prior knowledge of the redshift distribution (modeling merger rate with more hyper-parameters) degrades predictive power of cosmo pars
Opportunity: fit cosmology and population property

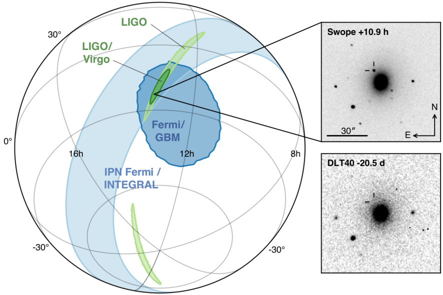
H. Leandro, V. Marra, RS PRD '21

Conclusion

- Gravitational Wave Astronomy is a young and fast growing science, its impact will go beyond astronomy
- Field theory methods to solve the 2-body problem in GR are being used as efficient tools for computations from first-principle
- Divergences well understood (indeed highly constrained hence helpful for sanity checks)
- For future developments going to higher order will lead to new master integrals, stumbling block for any perturbative method (PN, PM. . .)

GW170817

- GW trigger on Aug 17th, 2017, ended at 12h 41' 04.4" UTC, first in in LIGO Hanford, then confirmed as a triple coincidence → localized in an area of ∼ 28 deg^2
- GRB trigger from Fermi-GBM 1.7" after
- first optical image 10.87 hr afterwards by One-Meter Two Hemisphere team with Swope telescope at Las Campanas Observatory in Chile
- X obtained by the X-Ray Telescope on Swift after 14.9 h (NuSTAR 16.8 h)
- radio (∼ 3, 6 GHz) by VLA 16 days after GW event



LIGO/Virgo & Partner Astronomy groups, Astrophys.J. 848 (2017) no.2, L12