Inferring neutron star properties using a simultaneous EM and CW observation

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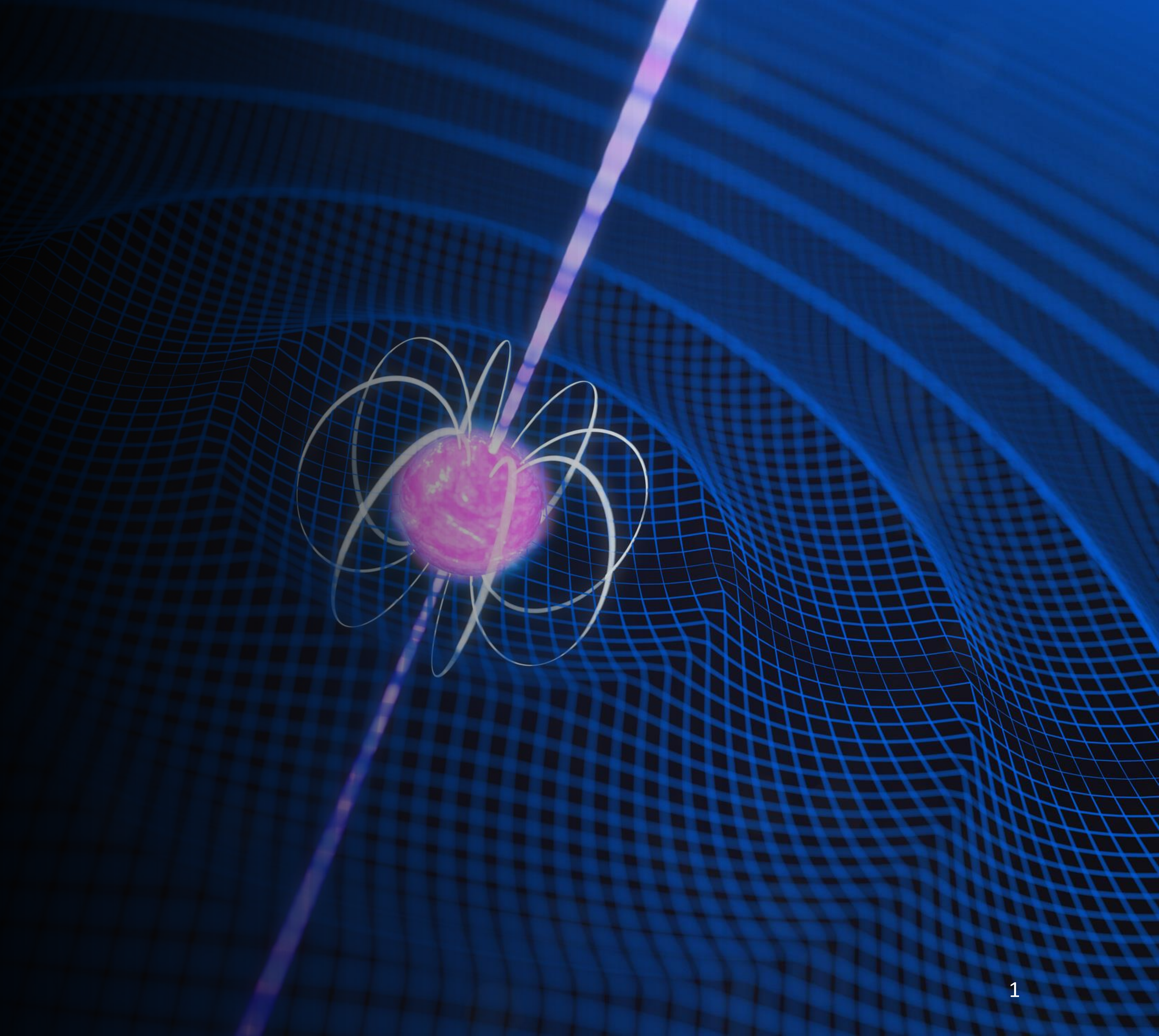

Image credit: Carl Knox/OzGrav/Swinburne-University

The emoji abstract

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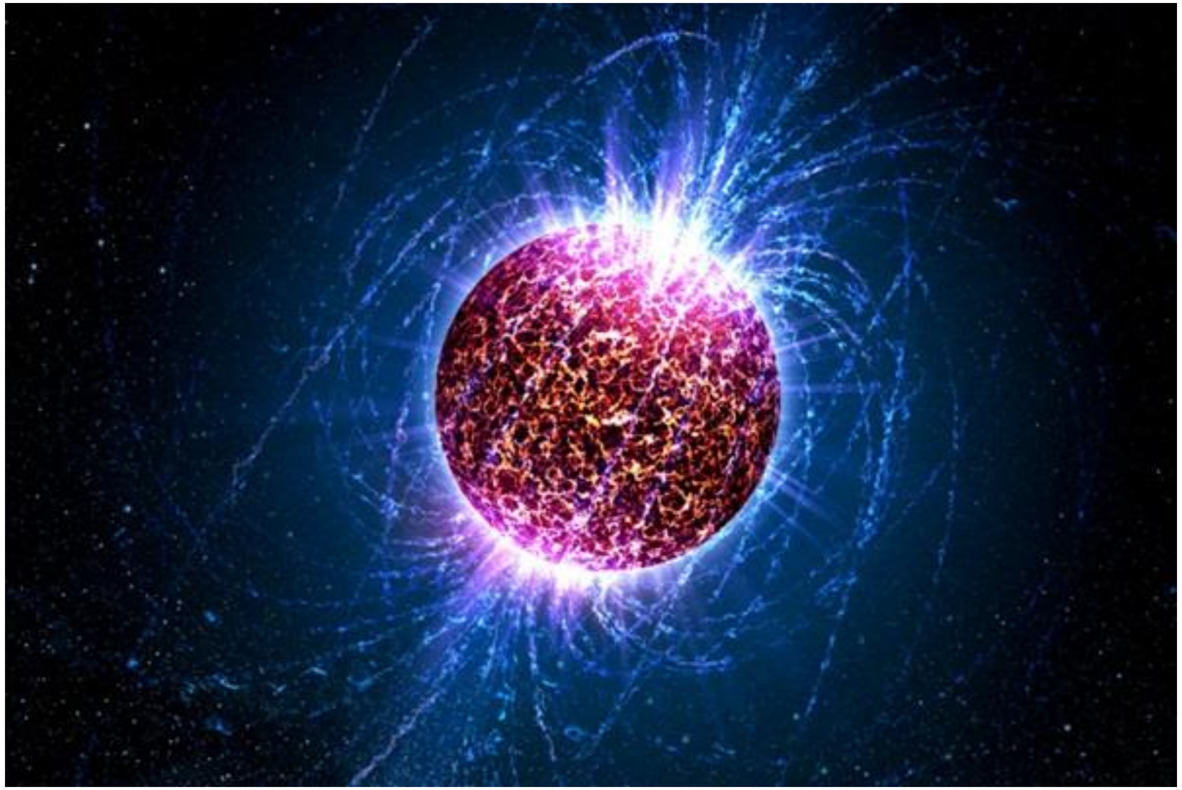
Talk structure

🌟: Neutron stars
🌊: Gravitational waves

🔢 😵‍💫 🌌: Theoretical work
💻 📊: Simulational work

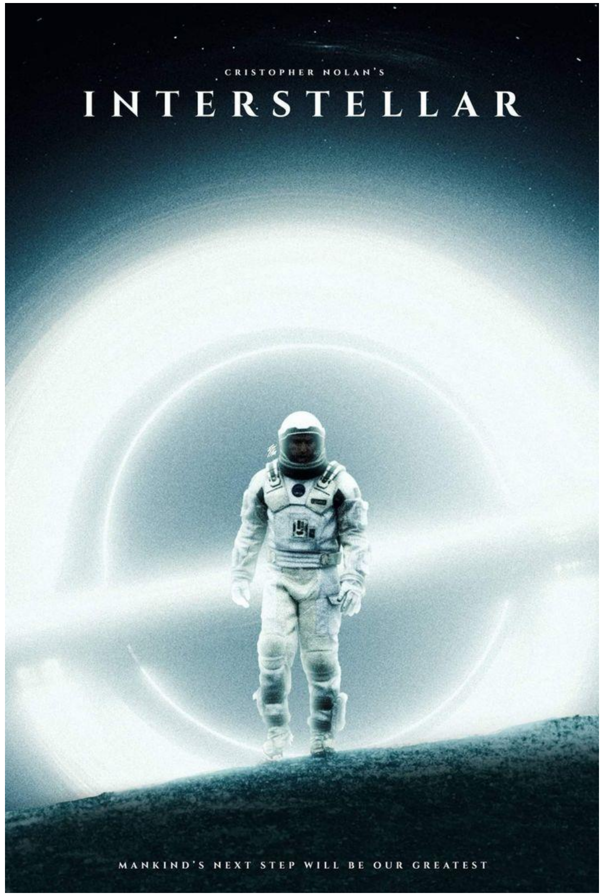
Motivation: Neutron Stars

- Collapsed cores of massive stars
- Extremely high densities and pressures
- Pulsars



✴ > ● Motivation: Why not black holes?

- You can actually see it's interior physics 👀
- No weird quantum effects* 👻
- Particle physicts can have fun as well 🎉



Interstellar doesn't even make any sense!!!?!?!

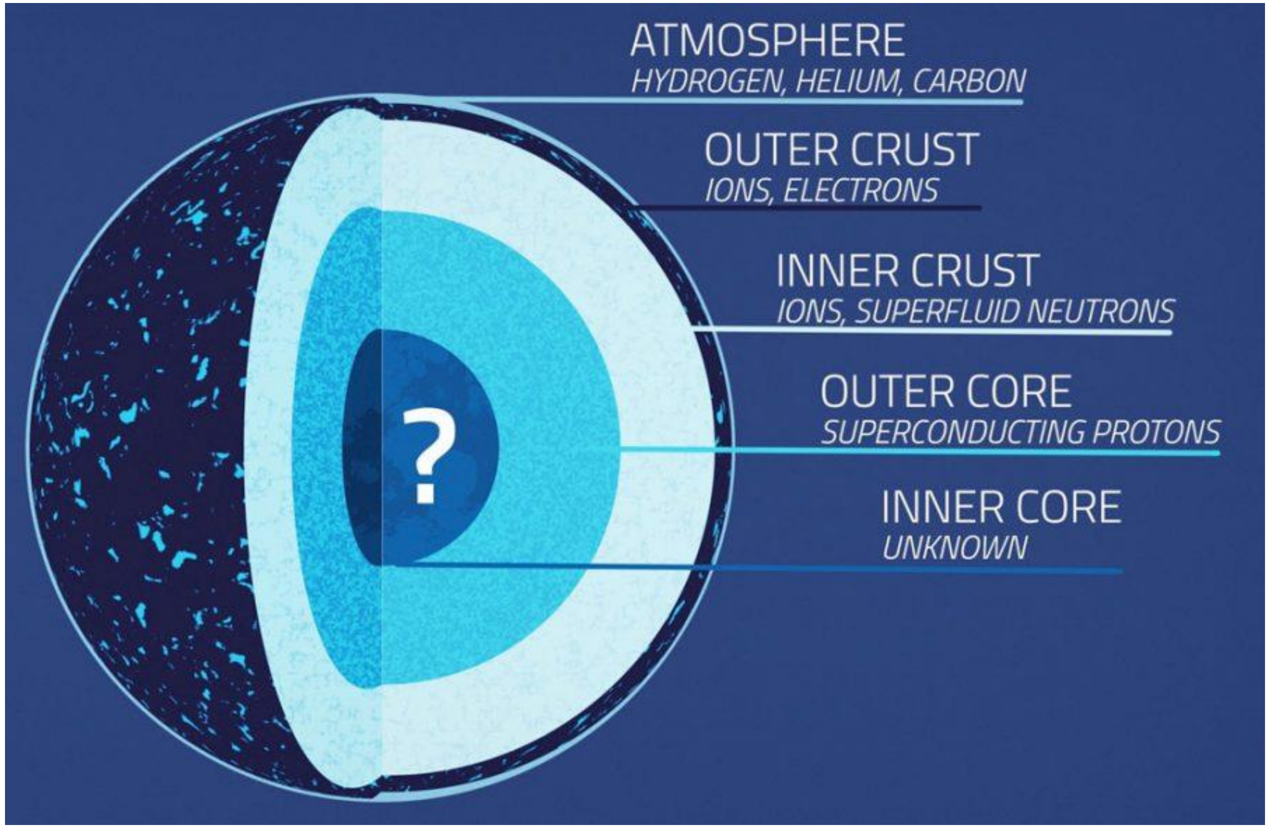
Motivation:
What's the problem?

- We don't anything

❓💥 Motivation: What's the problem?

- We don't anything 😬
  - Mass
  - Radius
  - Magnetic field

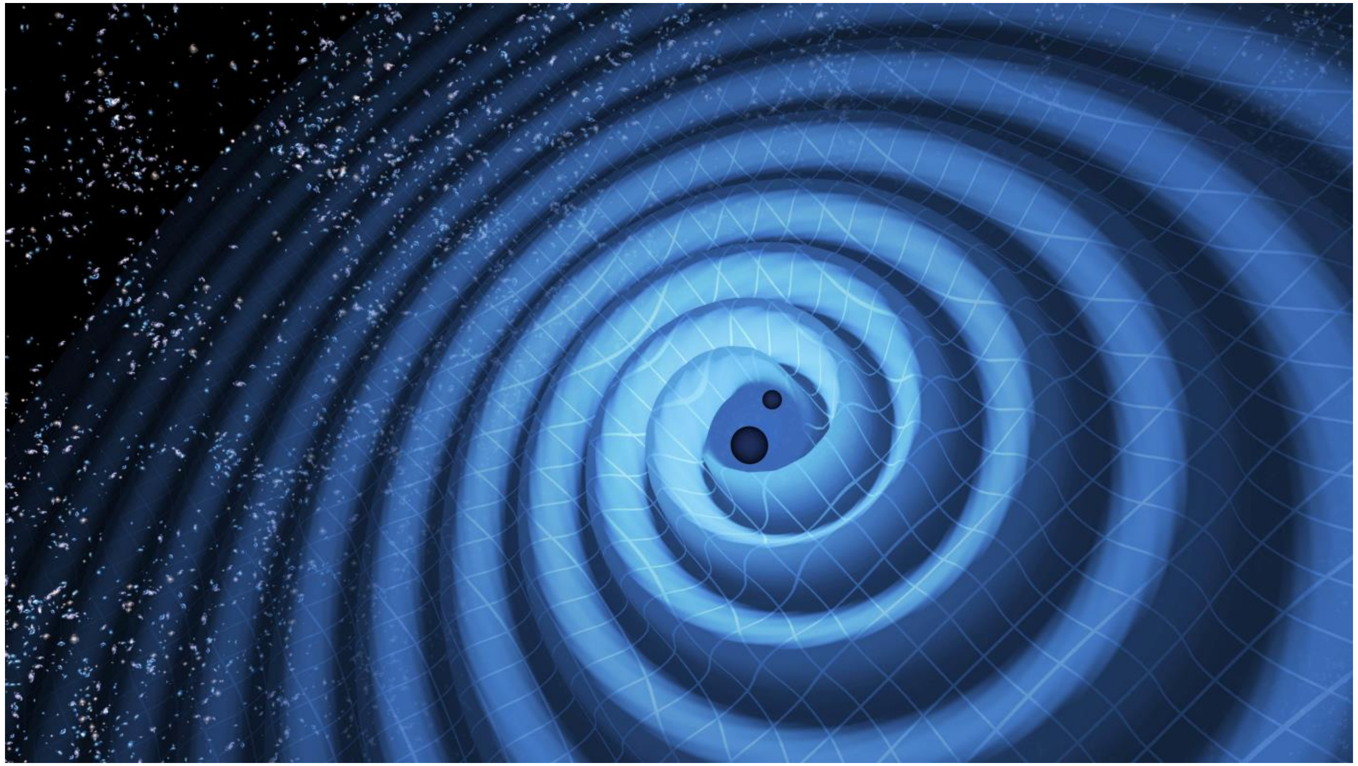
- Can't probe interior physics with EM 😎



When we don't even pretend to know what's going on

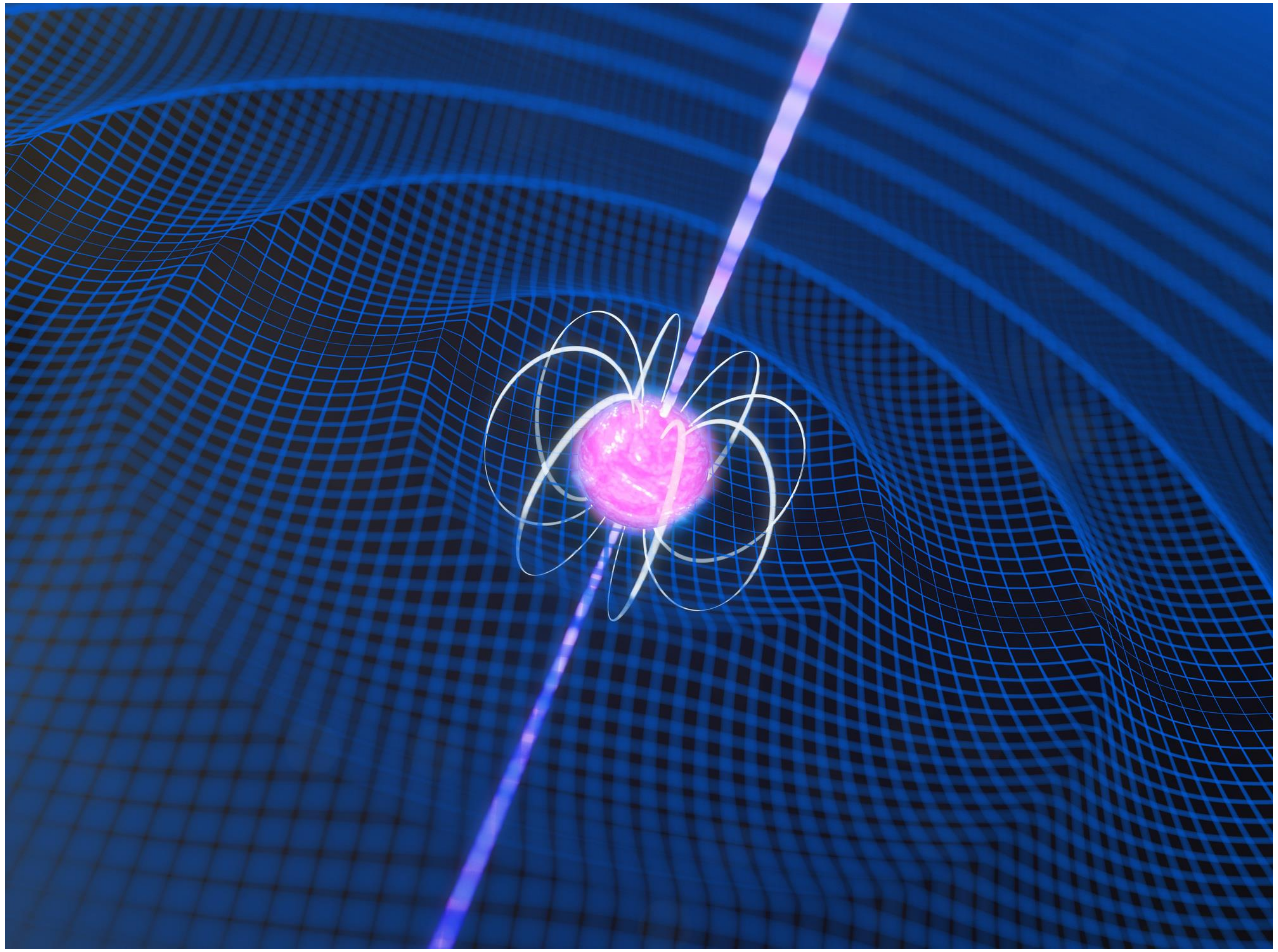
Gravitational waves

- Emitted by heavy things accelerating*
- All known detections are Compact Binary Coalescences (CBCs)
- Neutron stars have been detected
- Why is this (still) not enough

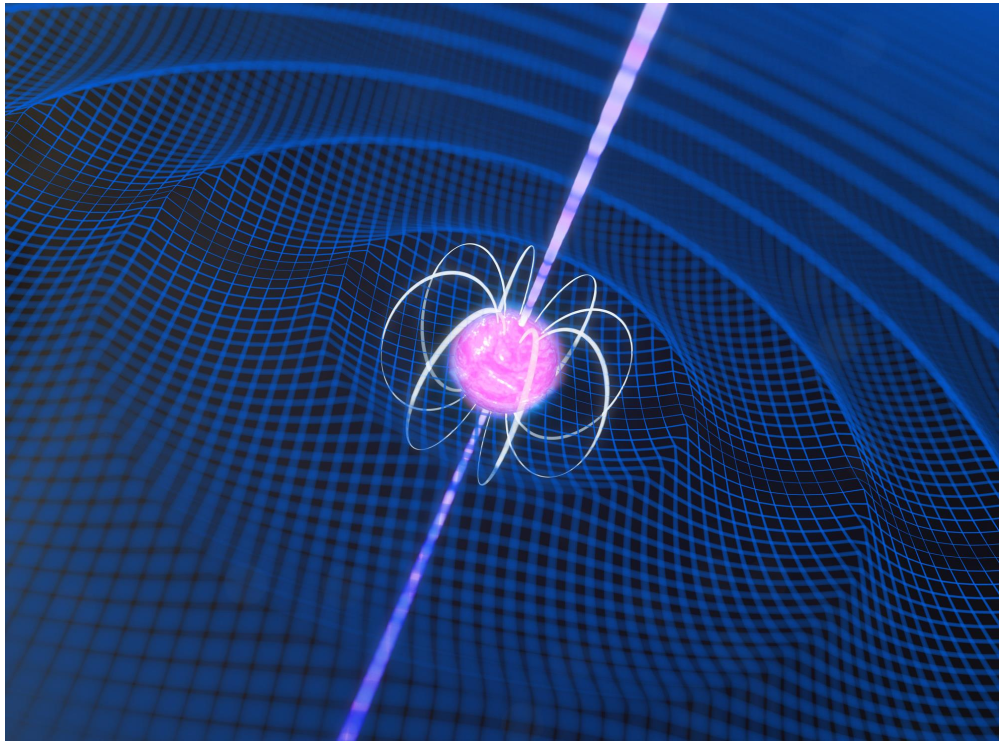


Continuous gravitational waves

- Quasi-monochromatic, quasi-continuous
- Emitted by isolated neutron stars
- No signal has been detected yet



The conceptual idea:



🔢😵💫: Theoretic work

Measured parameters

- Spindown parameters - CW

$$f, \dot{f}, \ddot{f}$$

- Characteristic strain amplitude – CW

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{\epsilon I_{zz} f^2}{D}$$

- Distance – EM

$$D$$

Deriving equations

$$\left(\frac{dE}{dt}\right)_{EM} + \left(\frac{dE}{dt}\right)_{GW} = -\left(\frac{dE}{dt}\right)_{rot}$$

$$\frac{2m_p^2\pi^4 f^4}{3c^3\mu_0} + \frac{32GI_{zz}^2\pi^6\epsilon^2 f^6}{5c^5} = -I_{zz}\pi^2 f\dot{f}\,.$$

$$\dot{f} = -\frac{K_{EM}\, m_p^2\, f^3}{I_{zz}} - K_{GW} I_{zz}\epsilon^2 f^5\,.$$

$$\ddot{f} = -\frac{3K_{EM}\, m_p^2 f^2 \dot{f}}{I_{zz}} - 5I_{zz}K_{GW}\epsilon^2 f^4 \dot{f}$$

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$$h_0 = \frac{16\pi^2 G}{c^4}\frac{\epsilon I_{zz} f^2}{D}$$

Result

$$I_{zz} = \frac{K_{GW}\, c^8\, D^2\, h_0^2\, f\, \dot{f}}{8\pi^4\, G^2\, (3\dot{f}^2 - f\,\ddot{f})}$$

$$\epsilon = \frac{2\pi^2 G(3\dot{f}^2 - f\,\ddot{f})}{K_{GW}\, c^4 h_0\, D\, f^3\, \dot{f}}$$

$$m_p = \frac{c^4\, h_0\, D\, \sqrt{K_{GW}(-5\dot{f}^2 + f\,\ddot{f})}}{4\pi^2 G f \sqrt{K_{EM}(3\dot{f}^2 - f\,\ddot{f})}}\,.$$

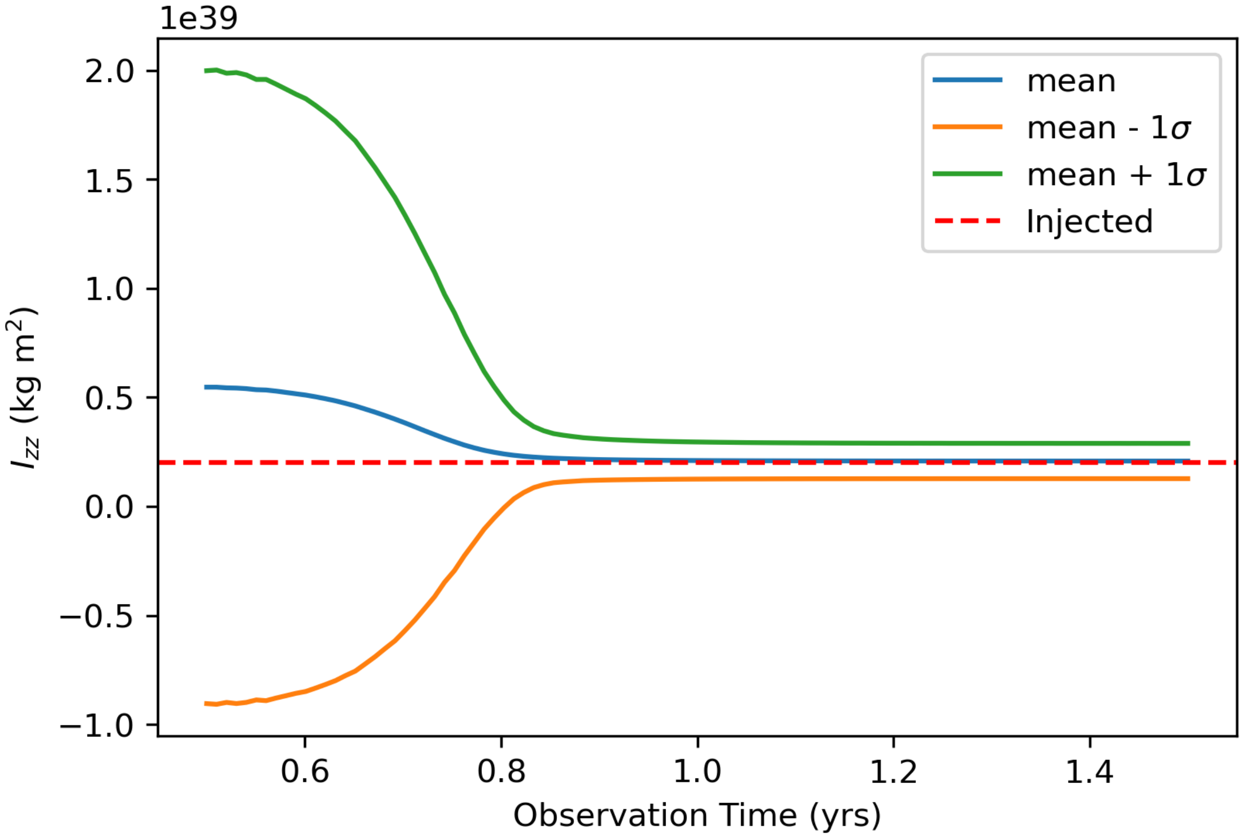
💻📊 Data analysis:
Are the errors small enough to actually constrain NS physics?

💻 Monte Carlo simulations

- Injections of NS parameters
- Model an observation as a sample from a Gaussian distribution
- Use the observed values to infer the NS parameters
- Calculate the error between inferred and injected parameters

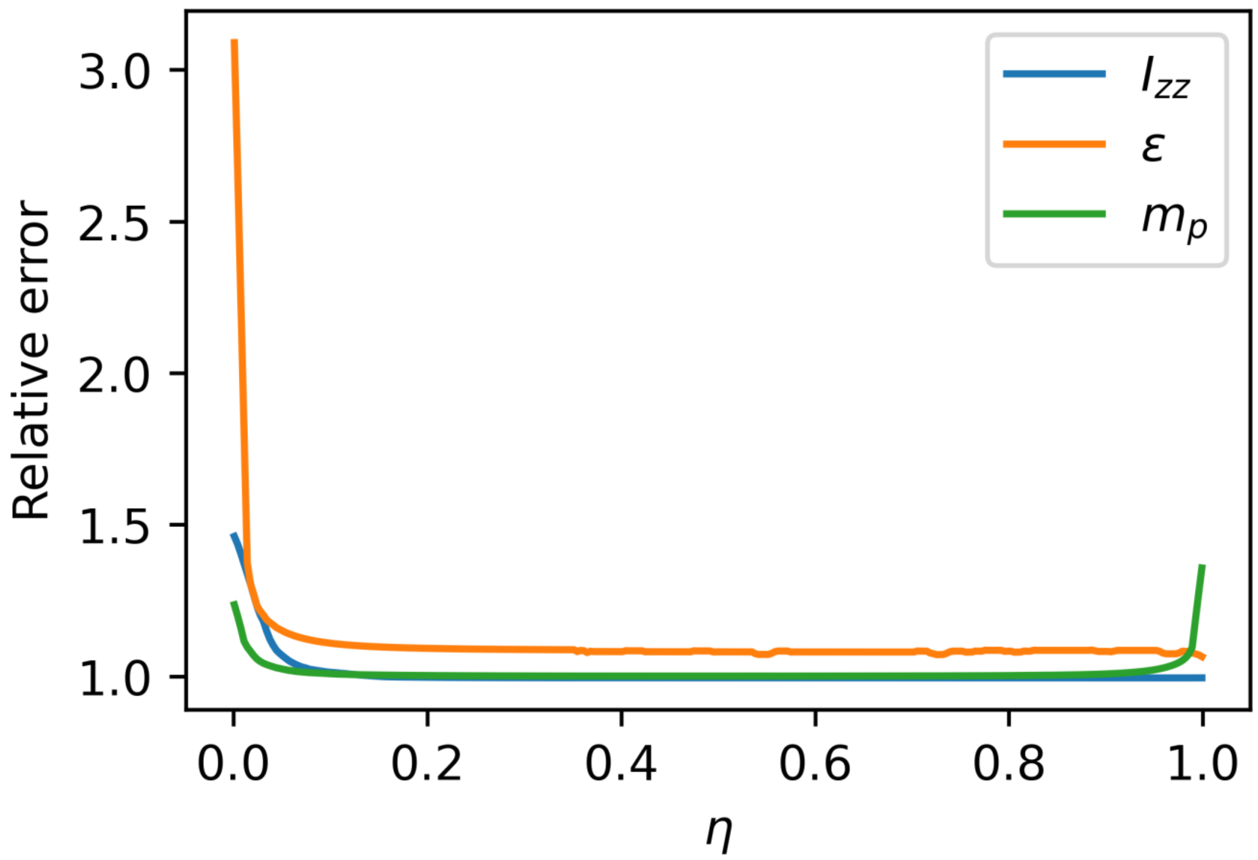
Results: for a single NS

- NS with Izz = 2e38
- Error scales with observation time
- Initial error in the mean value – unphysical simulations
- Saturated due to distance uncertainty



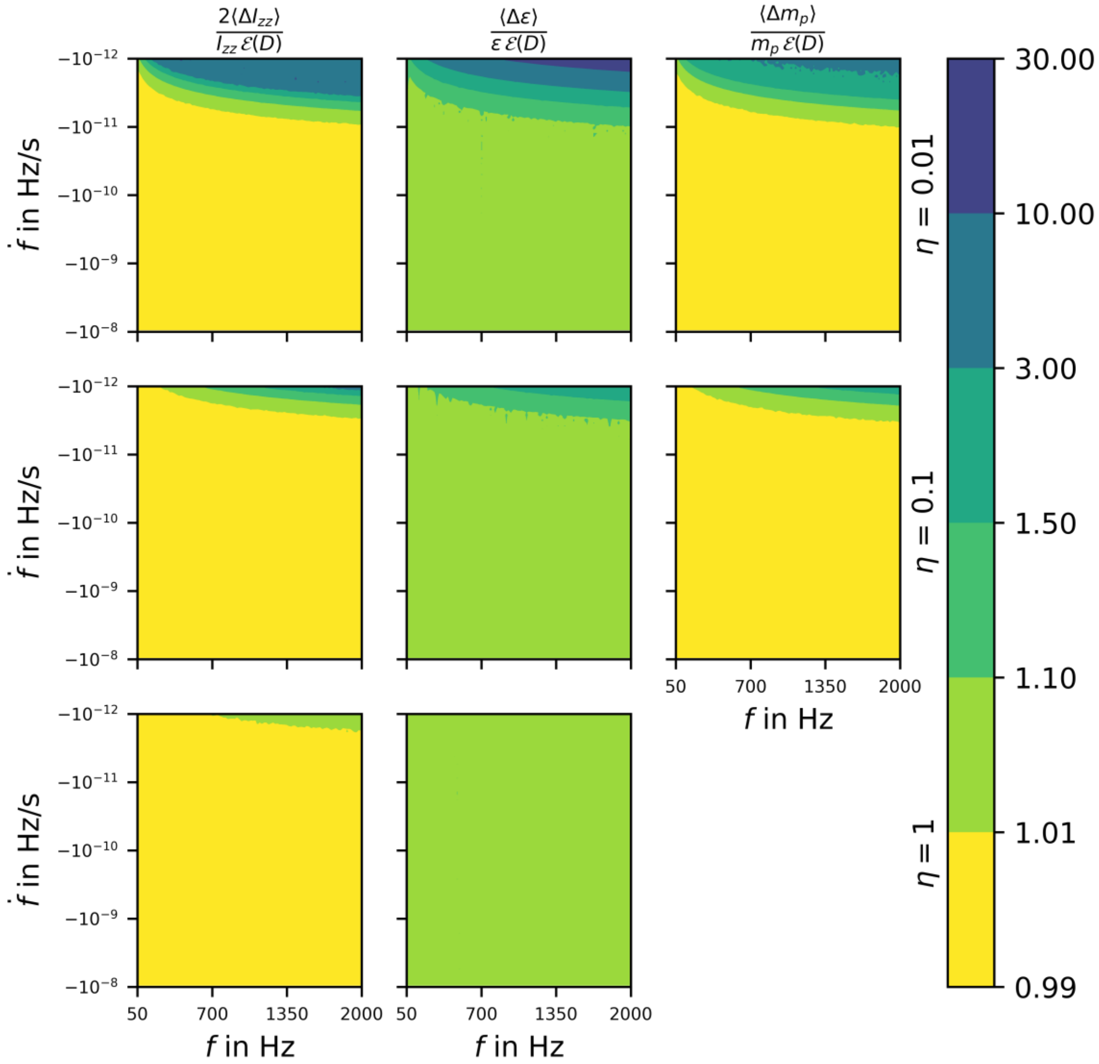
Results: eta dependence

- $\eta = \frac{L_{GW}}{L_{tot}}$
- Relative error removes the saturated error
- Sensitivity depth = 30
- Inference errors are larger for quiet GW emitters



Results: spindown dependence

- For T = 1yr
- Fractional error in inferred parameters with saturated error normalised out
- Dependence on spindown



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Supplementary slides

Summary

- Novel theoretical framework to infer NS parameters using a simultaneous observation
- Error mostly dominated by contribution of distance uncertainty
- Some regions in the parameter space where error from the continuous wave parameters is larger
- Future work: Bayesian parameter estimation

What does the error depend on?

- Observation time: T
- Frequency: f
- First frequency derivative $\dot{f}$
- Second frequency derivative
- Sensitivity depth: THIS HAS A TYPO $\mathbb{D} = S_h/h_0$
- Fractional uncertainty in distance:

$\sigma(D)/D$

Results: sensitivity depth dependence

- Sensitivity depth $\propto \frac{1}{\rho^2}$
- Stability time = time to reach within 1% of saturated error
- Eta dependency
- Non-linear relationship