

# A Catania-Goa (with a piece of Jyvaskyla)'s tale of charm and beauty

*Heavy quarks in heavy ion collisions*

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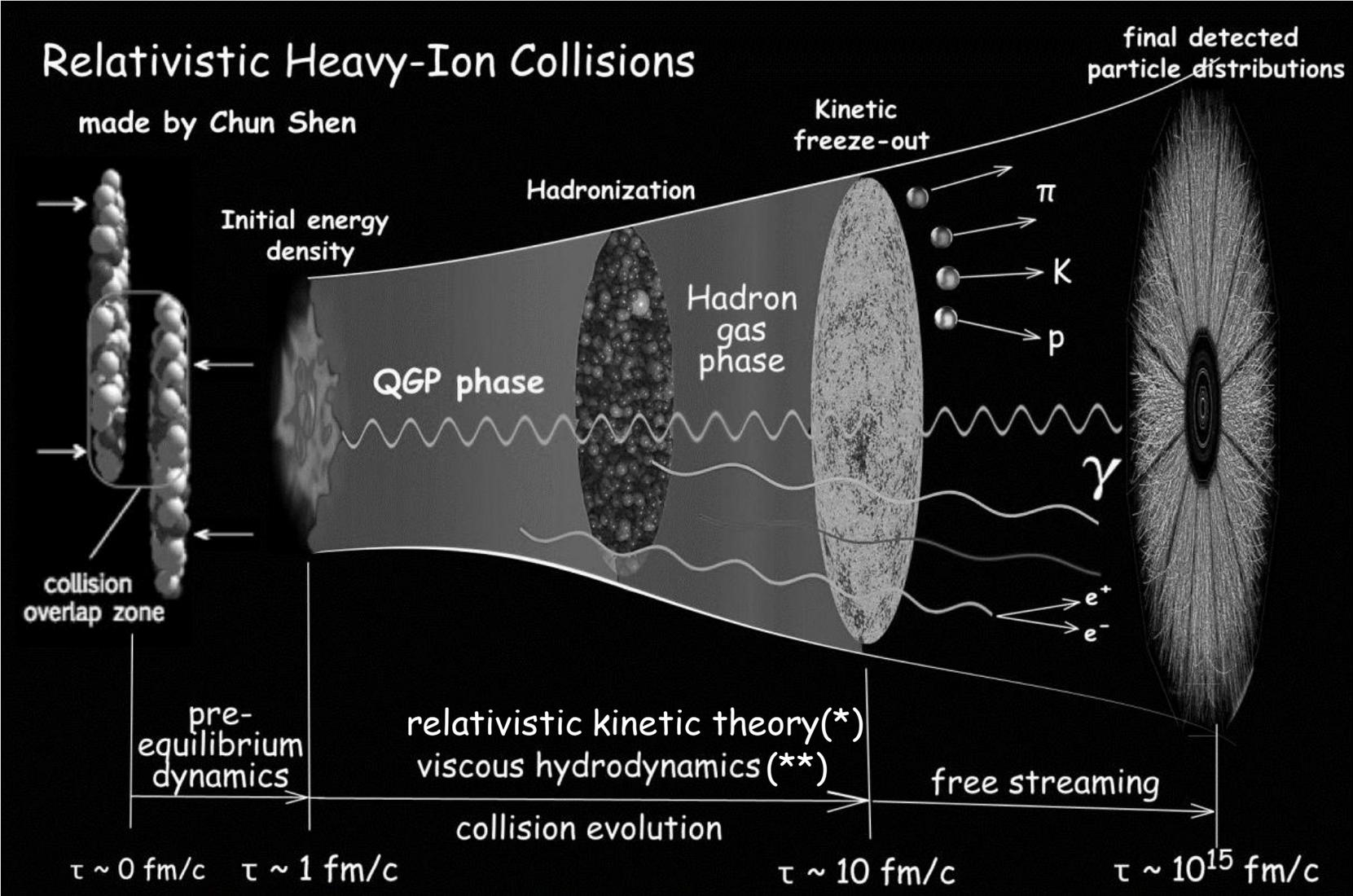
Catania's group: Vincenzo Greco, Gabriele Parisi, Maria Lucia Sambataro, Salvatore Plumari, Lucia Oliva, Nugara, Minissale, Angelo Asta

Goa's group: Santosh K. Das, Pooja, Jay Prakash, Yosouf Jamal

The Jyvaskyla's bit: Dana Avramescu

- ❖ A user-friendly introduction to the early stage of relativistic heavy ion collisions (RHICs): the *Glasma*, and its *evolution*
- ❖ *Heavy quarks* in the evolving Glasma
- ❖ *Heavy quarks* in the quark-gluon plasma
- ❖ *Hadronization of heavy quarks* in RHICs
- ❖ Conclusions





(\*)Talk by Nugara

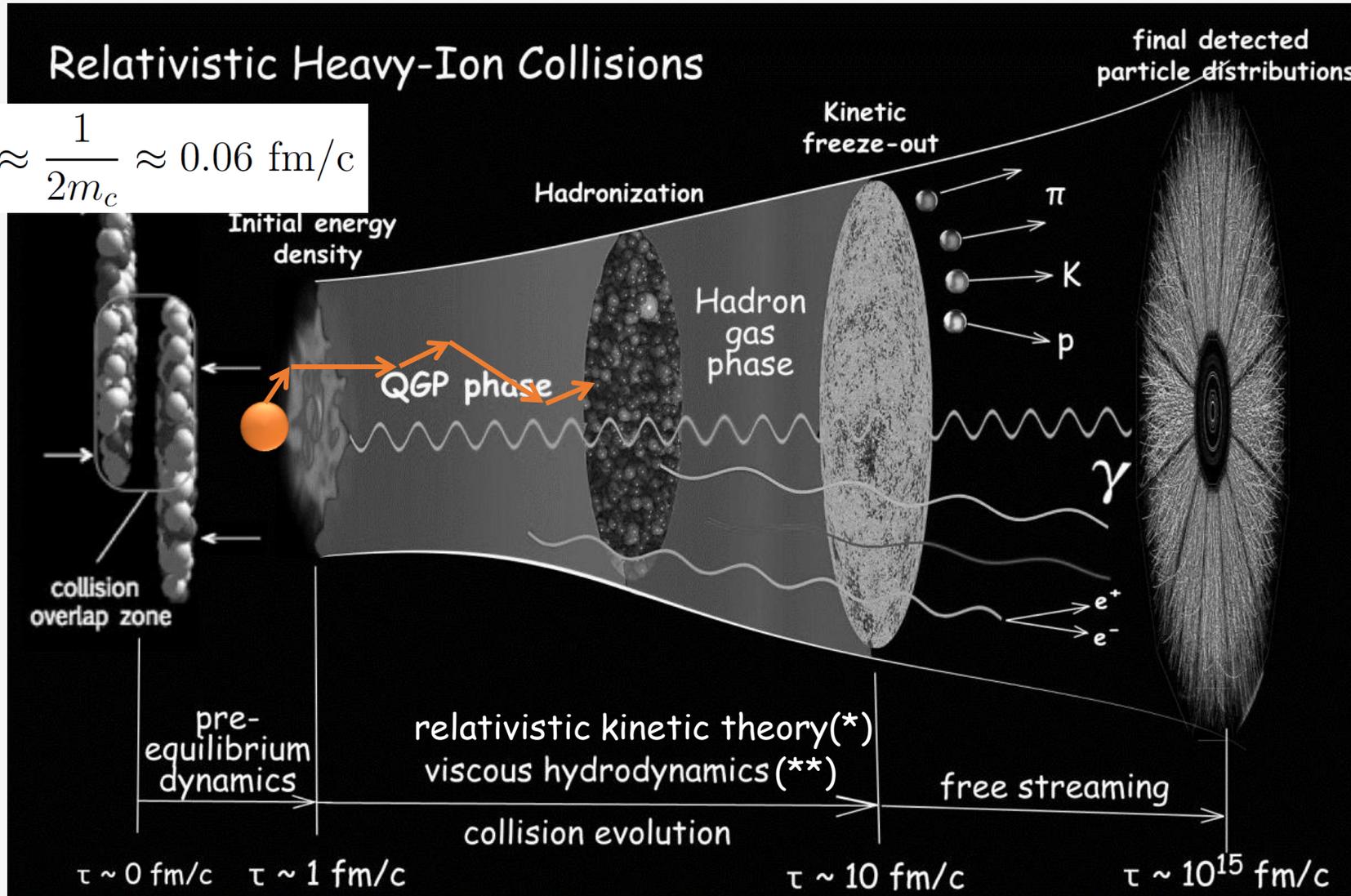
(\*\*)Talk by Edoardo Grossi

# Heavy quarks, c and b, in RHICs

## Relativistic Heavy-Ion Collisions

$$t_{\text{formation}} \approx \frac{1}{2m_c} \approx 0.06 \text{ fm}/c$$

HQs can probe the entire evolution of the medium, from the *early stage* up to hadronization

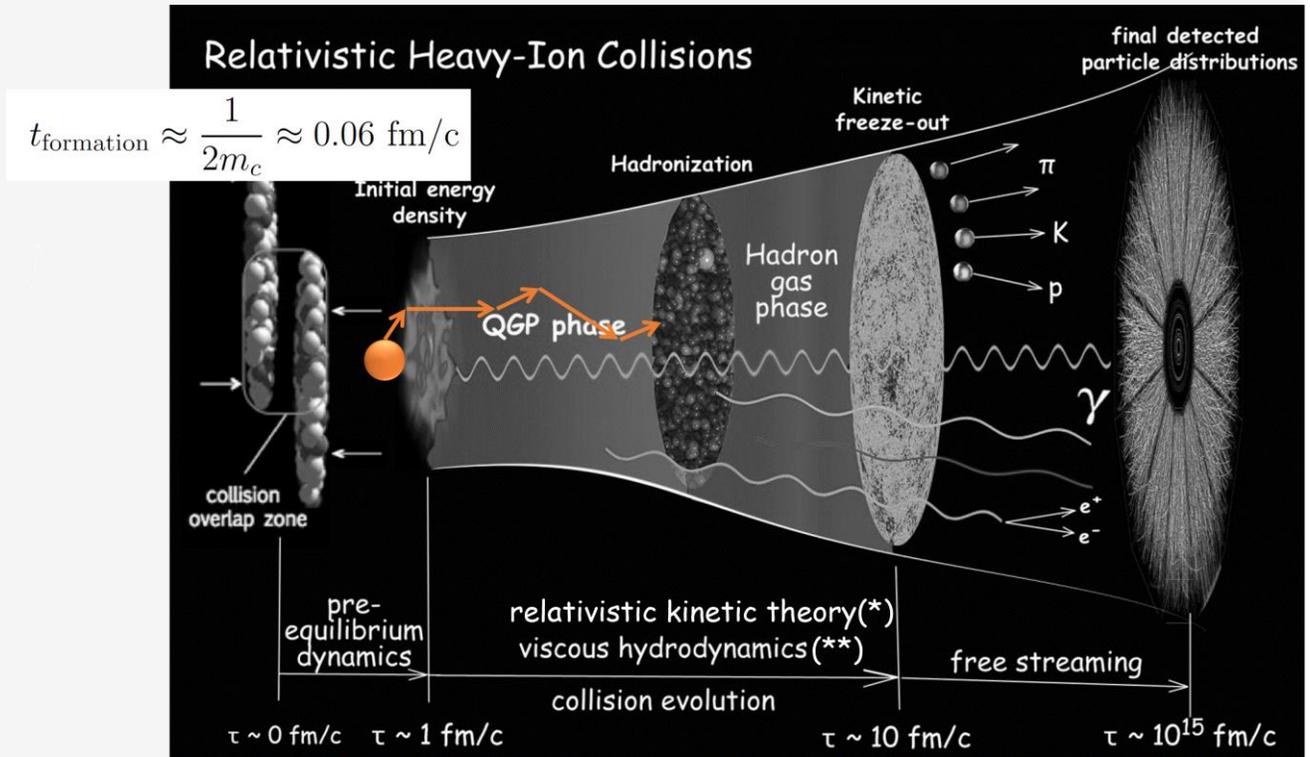


$\tau \sim 0 \text{ fm}/c$   $\tau \sim 1 \text{ fm}/c$

$\tau \sim 10 \text{ fm}/c$

$\tau \sim 10^{15} \text{ fm}/c$

# Heavy quarks, c and b, in RHICs



*It is meaningful to study the evolution of HQs:*

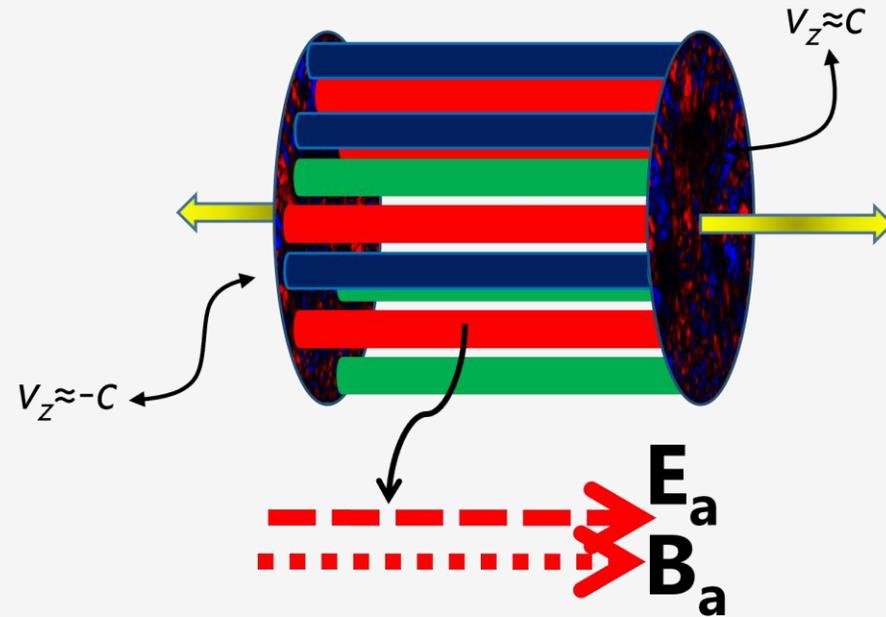
- In the early stage (this talk)*
- In the QGP (this talk, Martinez's talk)*

*And eventually analyze the possible hadronization mechanisms*

# Early stage: strong gluon fields and the Glasma

Many gluons in the early stage

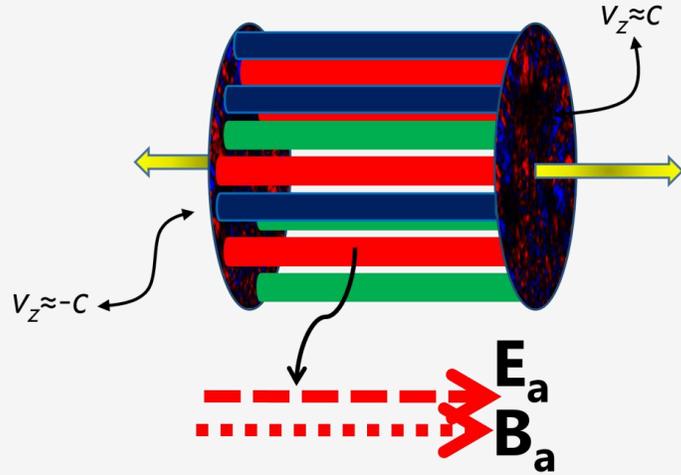
Useful, easy description in terms of **classical, intense fields**, rather than in terms of one-particle states



*Glasma* (\*)

Glasma: *initial condition* for the medium produced in RHICs.

Early stage: strong gluon fields and the Glasma



*Strength of initial fields:  $O(Q_s^2)$*   
*Initial energy density:  $O(Q_s^4)$*

$Q_s$ : saturation scale

$Q_s$  is the only energy scale in this model

$$Q_s \approx 1 - 3 \text{ GeV } (*)$$

# Evolving the Glasma: classical Yang-Mills (CYM) equations

Due to the large density the gluon field behaves like a classical field:

*Dynamics is governed by classical EoMs, namely the classical Yang-Mills (CYM) equations(\*)*.

$$(D^\mu F_{\mu\nu})^a = 0.$$

$$\partial_\tau E_i = \frac{1}{\tau} \mathcal{D}_\eta F_{\eta i} + \tau \mathcal{D}_j F_{ji},$$

$$\partial_\tau E_\eta = \frac{1}{\tau} \mathcal{D}_j F_{j\eta},$$

$$E_i = \tau \partial_\tau A_i,$$

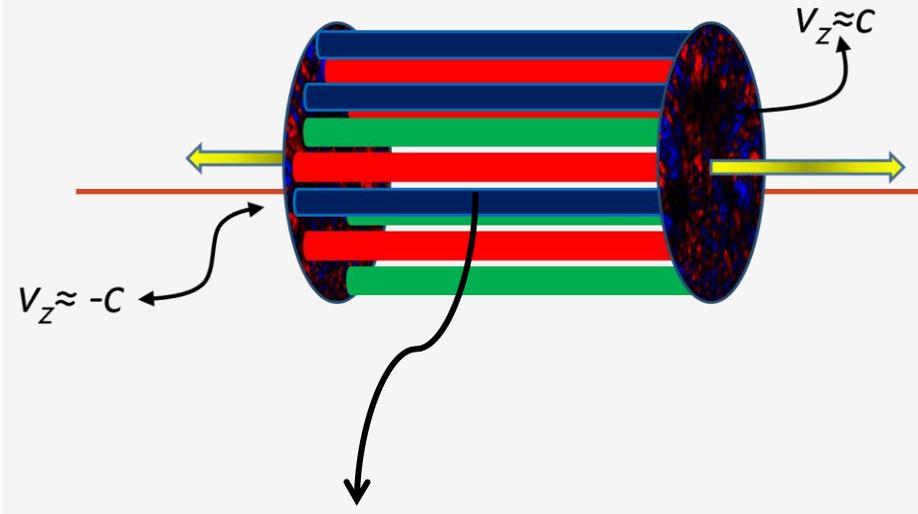
$$E_\eta = \frac{1}{\tau} \partial_\tau A_\eta.$$

$$\tau = \sqrt{t^2 - z^2}$$
$$\eta = \frac{1}{2} \log \left( \frac{t+z}{t-z} \right)$$

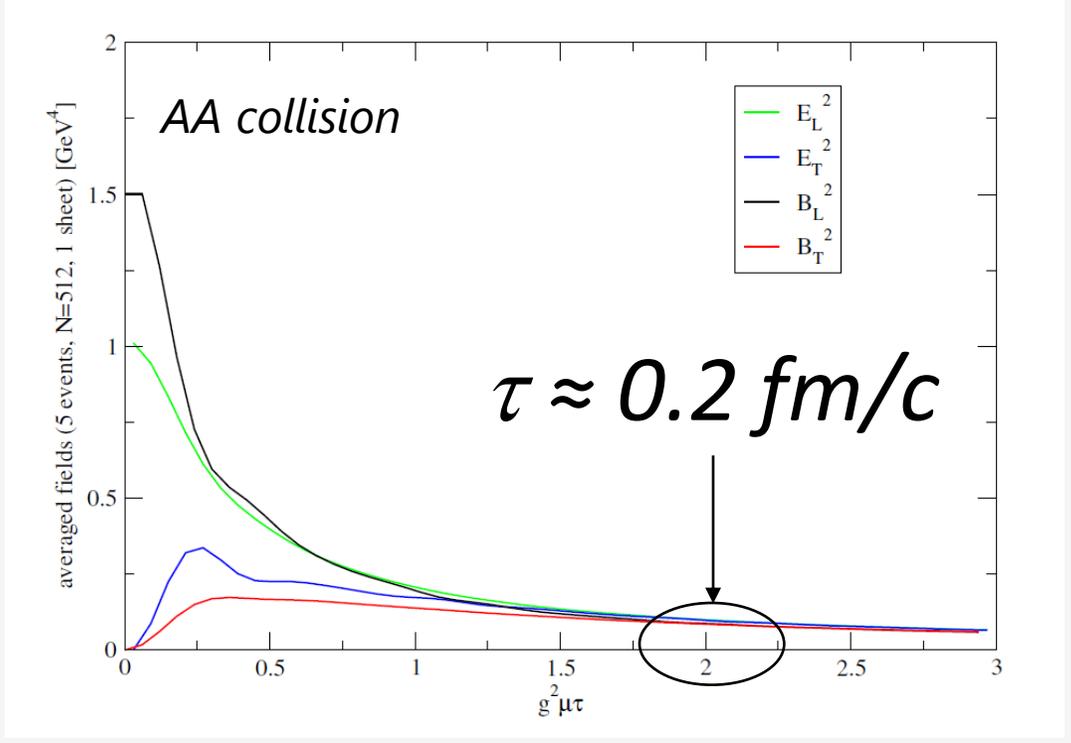
*Evolution of the system is studied assuming the Glasma initial condition, and evolving this condition by virtue of the CYM equations.*

(\*)See the talk by Gabriele Parisi for details on how these equations are implemented and solved.

# Evolving fields



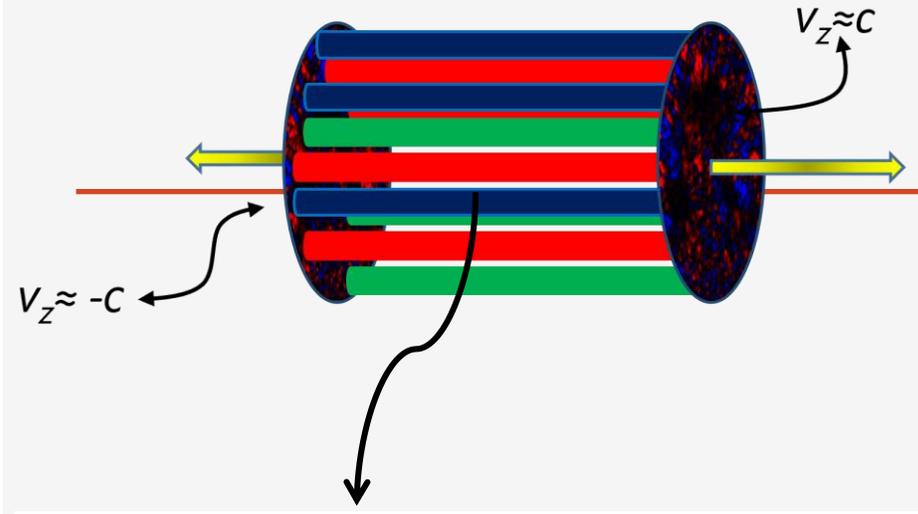
$$\varepsilon = \text{Tr} [E_L^2 + E_T^2 + B_L^2 + B_T^2]$$



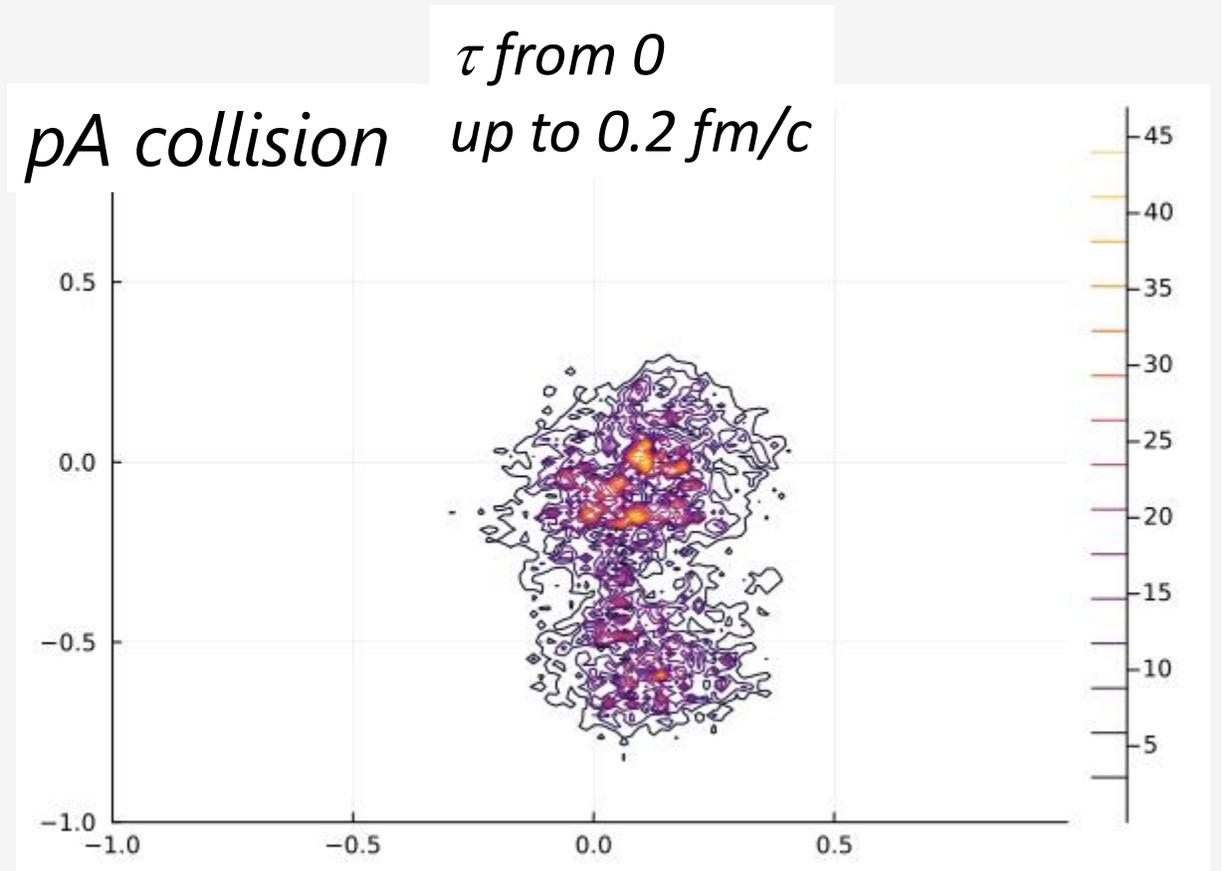
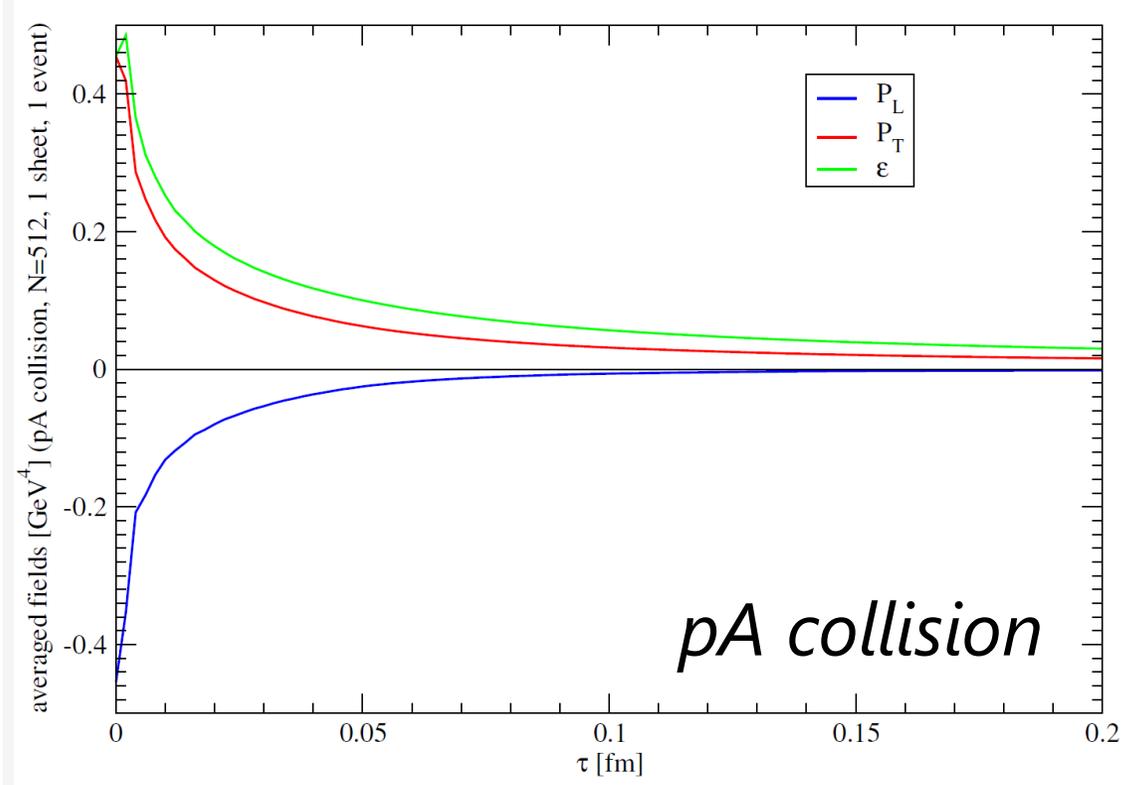
$$\left. \frac{dE_a^x}{dt} \right|_{t=0^+} = \partial_y B_z^a + f_{abc} A_y^b B_z^c$$

Formation time of transverse fields:  
 $Q_s \tau \approx 1$  namely  $\tau \approx 0.1 \text{ fm/c}$

# The evolution of the energy density

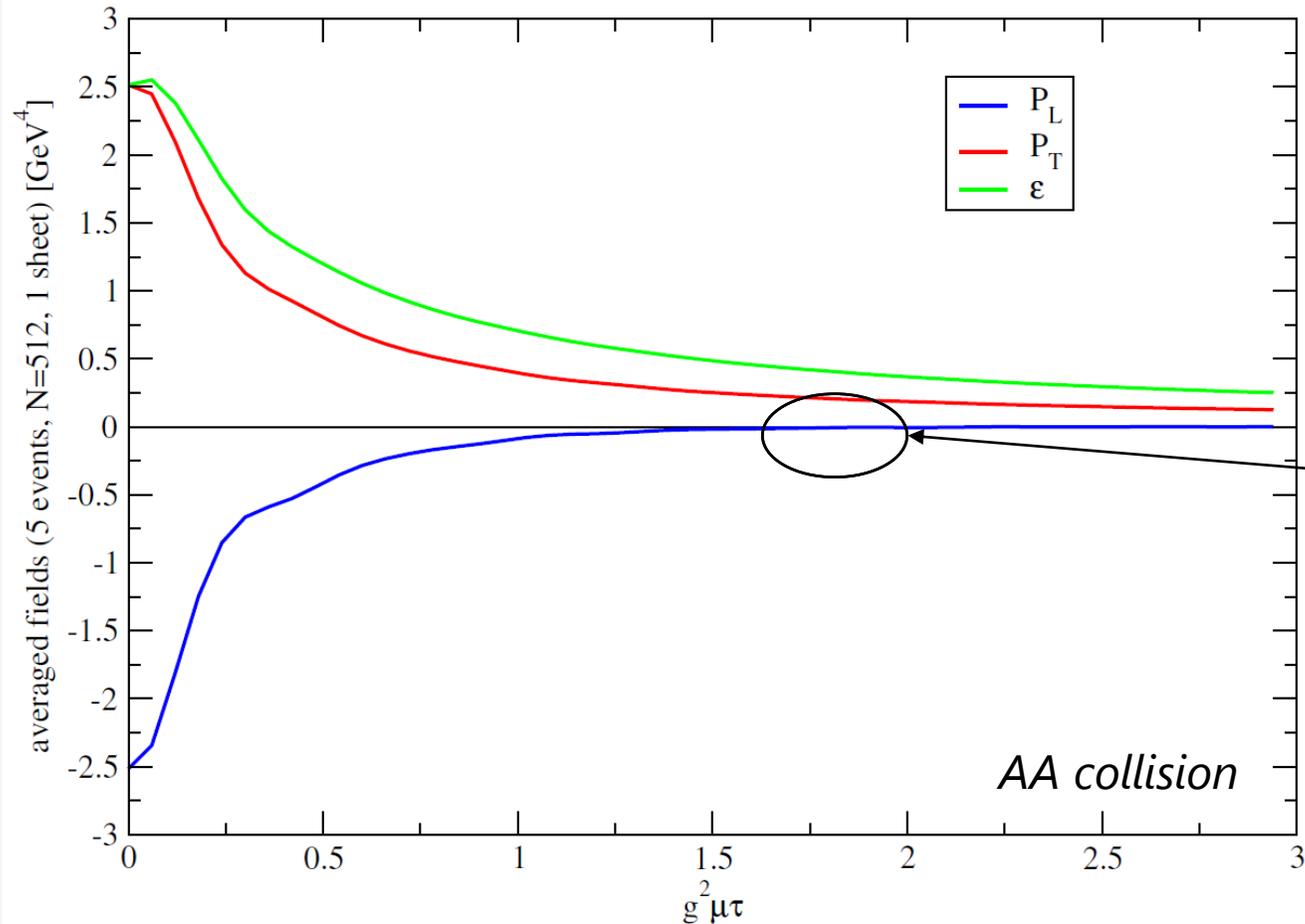
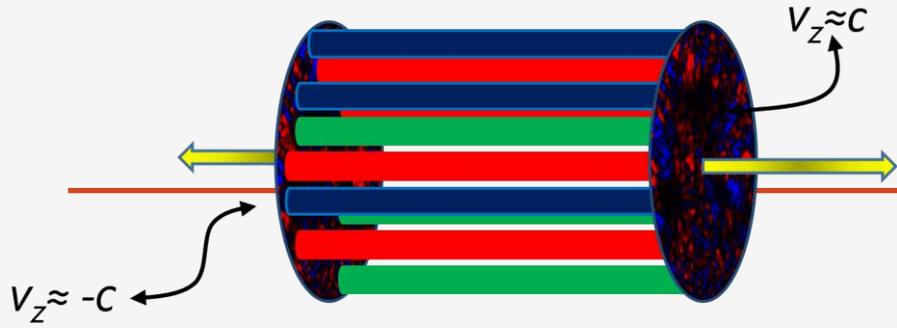


$$\varepsilon = \text{Tr} [E_L^2 + E_T^2 + B_L^2 + B_T^2]$$



Animation: courtesy of Gabriele Parisi

# The free streaming regime, AA collisions



$$\epsilon = \text{Tr} [E_L^2 + E_T^2 + B_L^2 + B_T^2]$$

$$P_L = \text{Tr} [-E_L^2 - B_L^2 + E_T^2 + B_T^2]$$

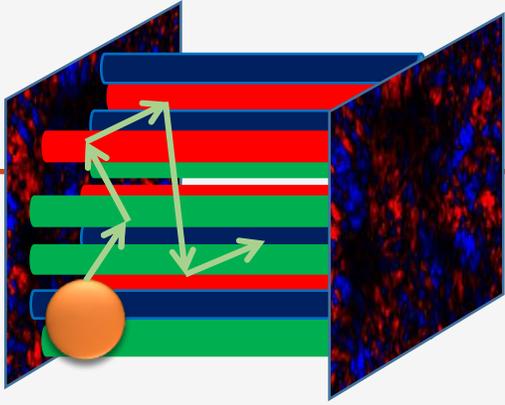
$$P_T = \text{Tr} [E_L^2 + B_L^2]$$

$$\tau \approx 0.2 \text{ fm}/c$$

Longitudinal pressure *vanishes*(\*)

$P_L \neq P_T$ : the system is quite anisotropic. This anisotropy affects observables, e.g. those of the heavy quarks.

(\*)Free streaming regime. See the talk by Gabriele Parisi



# Heavy quarks (HGs) probing of the evolving Glasma

$$t_{\text{formation}} \approx \frac{1}{2m_c} \approx 0.06 \text{ fm}/c$$

Relativistic kinetic theory of HQs in Glasma

$$\frac{dx_i}{dt} = \frac{p_i}{E} \quad E = \sqrt{\mathbf{p}^2 + m^2}$$

$$\mathbf{v} \equiv \frac{\mathbf{p}}{E} \quad (\text{Relativistic}) \text{ Velocity}$$

$$E \frac{dp_i}{dt} = gQ_a F_{i\nu}^a p^\nu$$

$$\frac{d\mathbf{p}}{dt} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) \quad \text{Lorentz force}$$

$$E \frac{dQ_a}{dt} = -gQ_c \varepsilon^{cba} \mathbf{A}_b \cdot \mathbf{p}_c$$

Wong (1979), Heinz (1985)

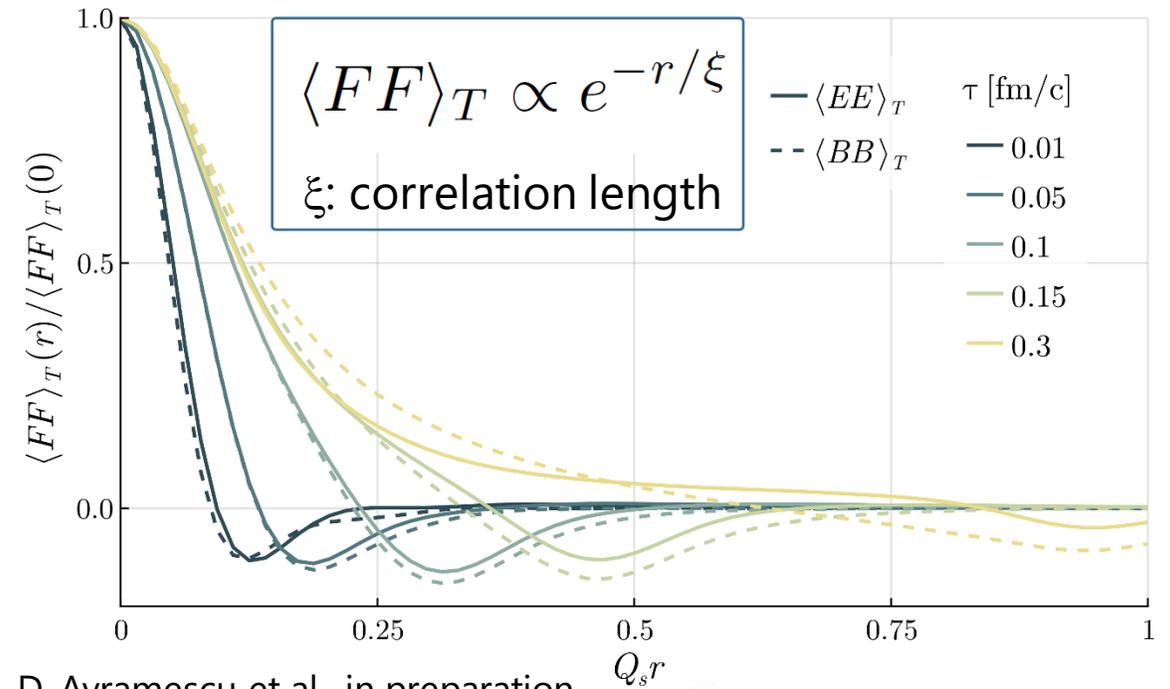
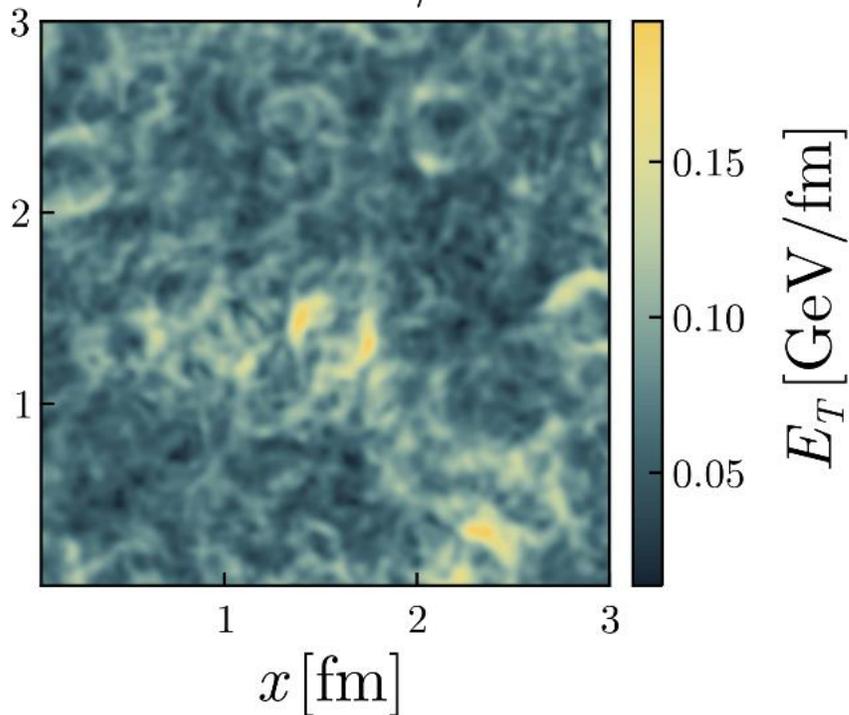
$$D_\mu J_a^\mu = 0 \quad \text{Gauge-invariant conservation of the color current carried by charm quarks + gluons}$$

$$J_a^\mu = \bar{c} \gamma^\mu T_a c$$

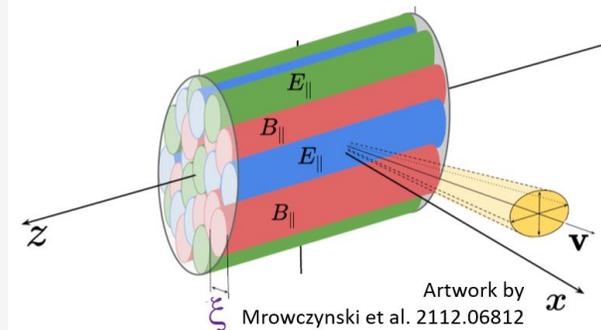
Equations of motion of heavy quarks are solved in the background given by the evolving Glasma fields

# The colored filaments of the Glasma

Fields arrange in *correlation domains*, aka *filaments*, of transverse area  $\approx \xi^2$ :  
 $\xi^2 = \mathcal{O}(1/Q_s^2)$

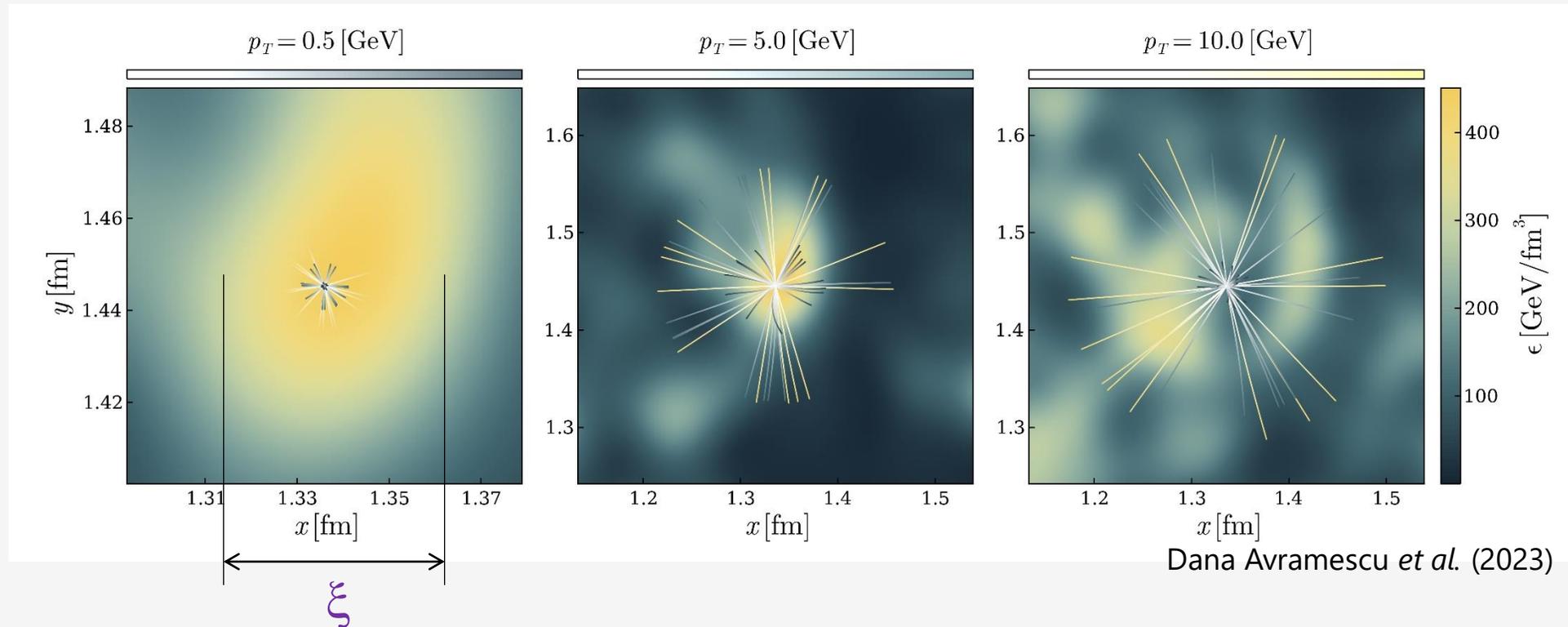


D. Avramescu et al., in preparation



Artwork by  
 Mrowczynski et al. 2112.06812

# Heavy quarks inside color filaments

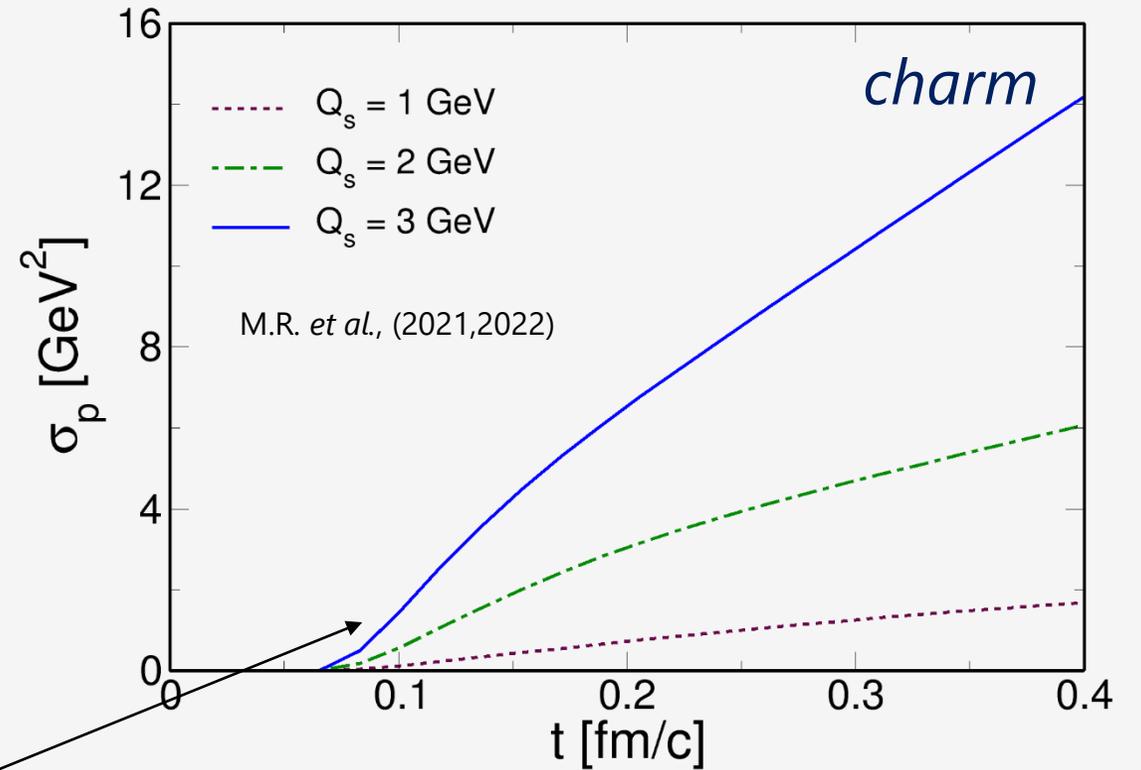


*Slow color charges spend some time within one single filament: diffusion in a coherent field, rather than in a random medium.*

*The force exerted on these charges is time-correlated.*

# Momentum broadening of charm quarks

$$\sigma_p = \langle p - \langle p \rangle \rangle^2$$



*Diffusion in the color filament*

*Standard Brownian motion*

• *Early time:  $\sigma_p \approx Q_s D t^2$*

• *Later time:  $\sigma_p \approx 2 D t$*

*D: momentum diffusion coefficient*

# Comparison with perturbative QCD-Langevin

## Standard Brownian motion

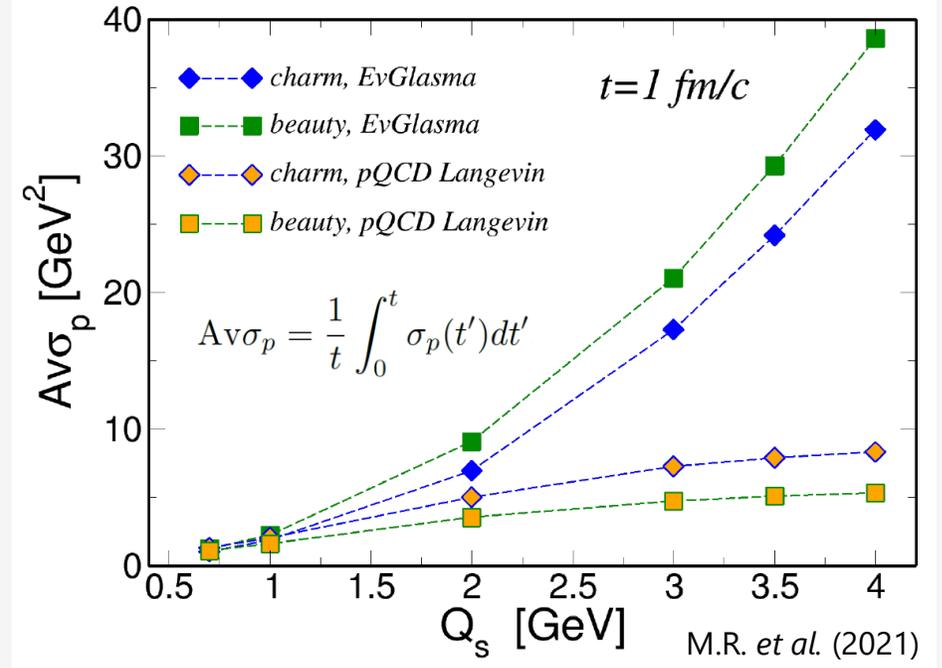
$$\frac{dp}{dt} = -\gamma p + \xi$$

energy loss

$$\langle \xi(t_1)\xi(t_2) \rangle = 2D\delta(t_1 - t_2)$$

random kicks from the medium  
(momentum diffusion)

$$\sigma_p = \langle p - \langle p \rangle \rangle^2$$

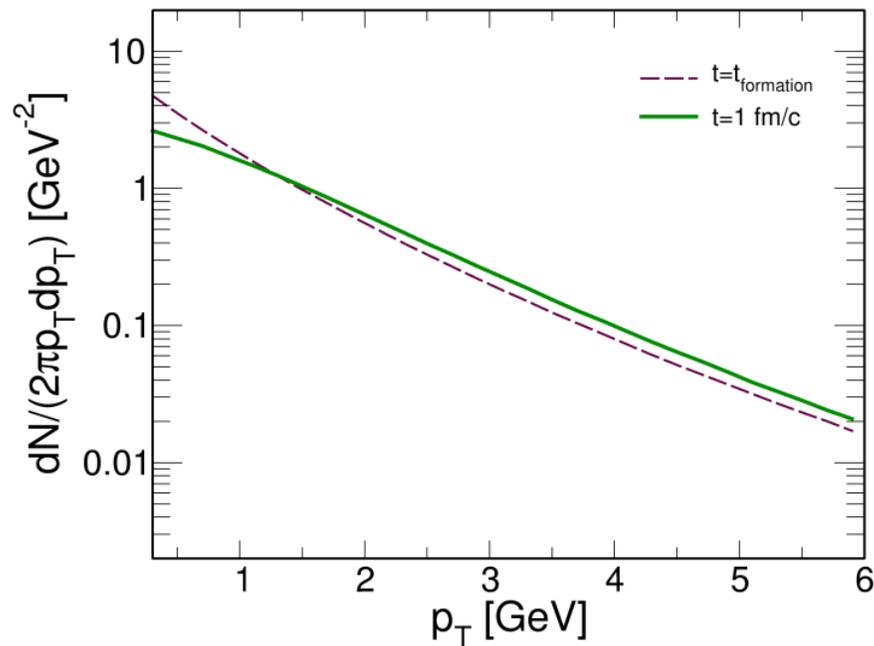


*Average diffusion coefficient of HQs in Glasma agrees with pQCD for small values of  $Q_s$  (diluted Glasma).*

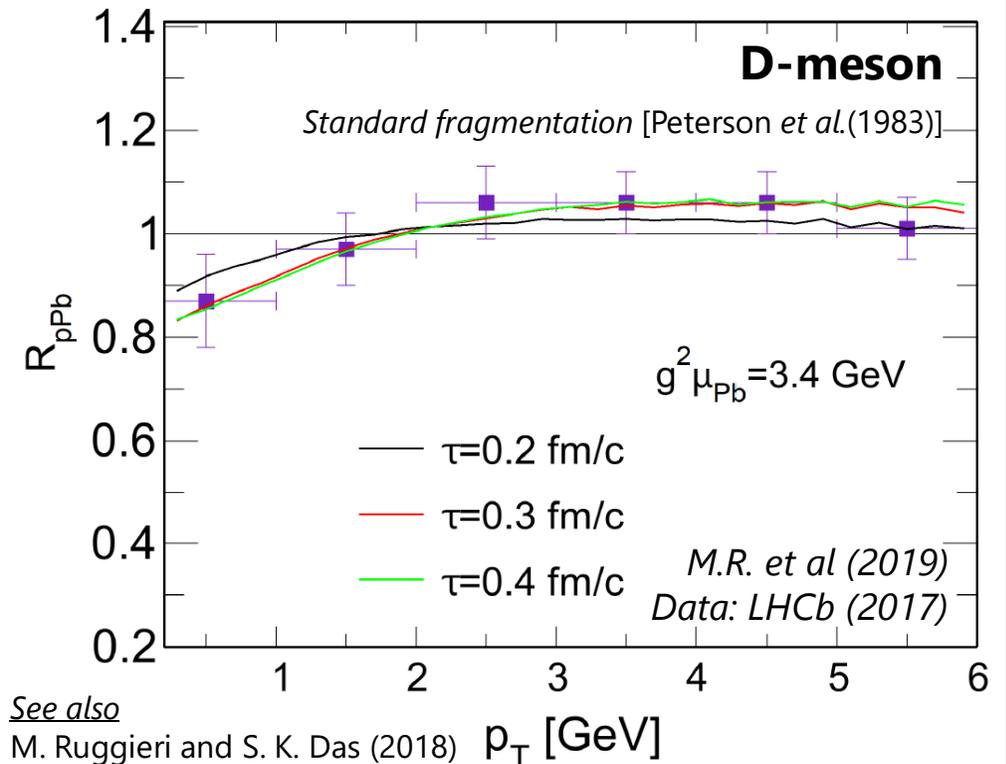
## Initial distribution

From perturbative QCD, aka **FONLL** [Cacciari et al. (2001, 2012)]

**p-Pb @ 5.02 TeV**



$$R_{pPb} = \frac{(dN/d^2p_T)_{\text{final}}}{(dN/d^2p_T)_{pQCD}}$$



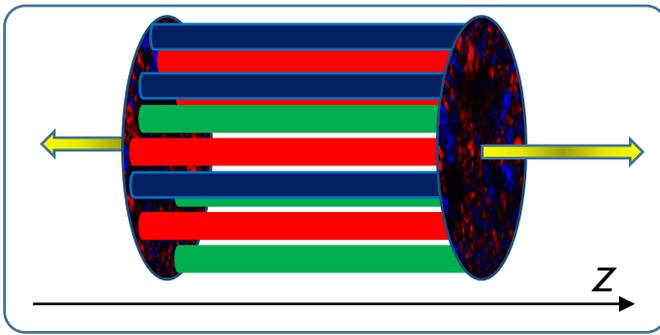
**$R_{pPb} \neq 1$**

Interaction with the fields created  
by the collision

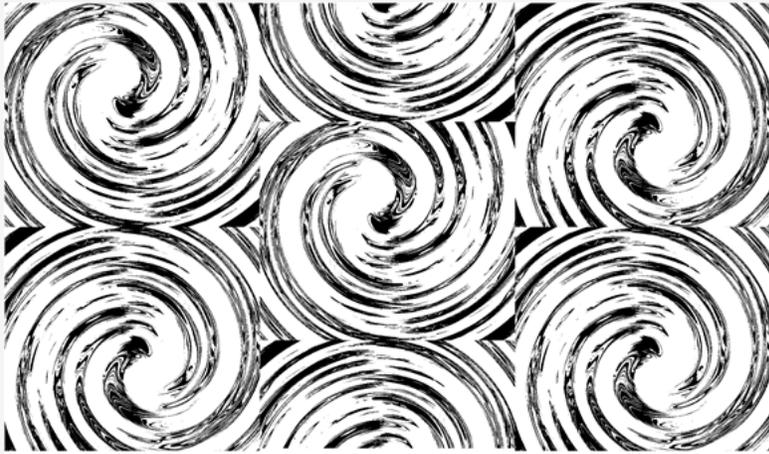
See also

M. Ruggieri and S. K. Das (2018)

# Glasma-induced polarization of c and b



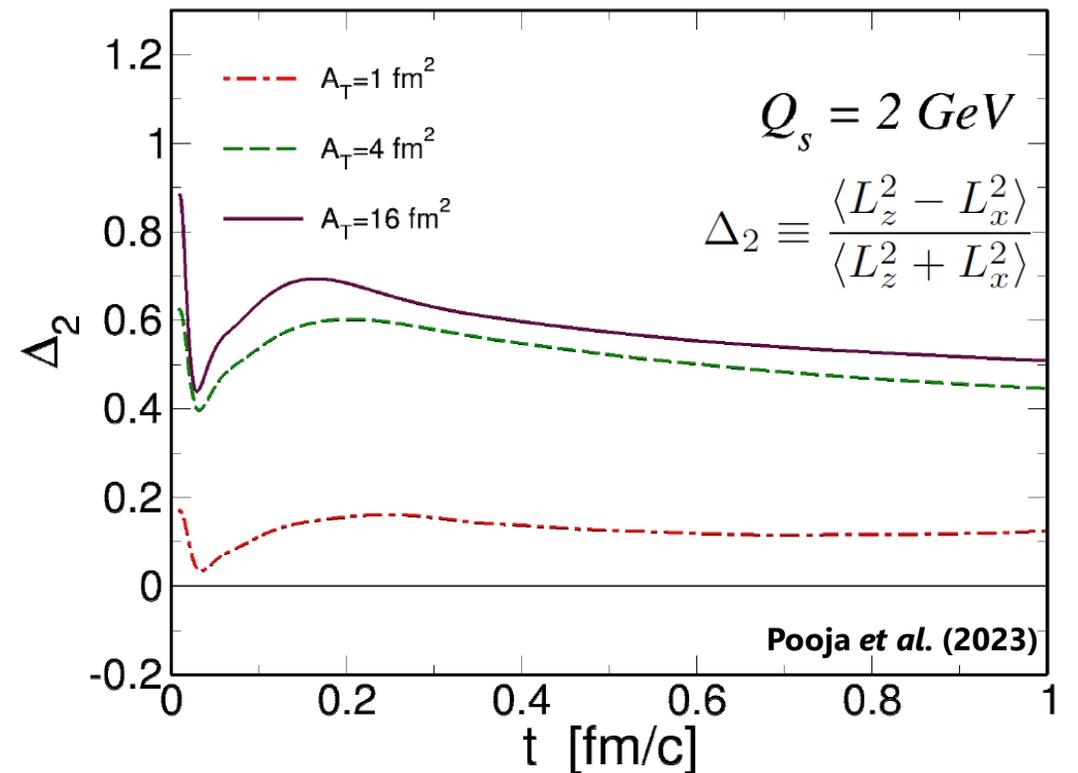
*Polarization* of c and b, along the longitudinal direction



Naively: glasma induces vortex-like motion of c and b around color filaments in the transverse plane

*Anisotropic distribution of the momentum*

[Pooja et al. (2023), Ipp et al. (2020), Avramescu et al. (2023)]  
as well as of *angular momentum* [Pooja et al. (2023)]



# Heavy quarks in the quark-gluon plasma: relativistic transport theory

$$p^\mu \partial_\mu f_q(x, p) + m(x) \partial_\mu^x m(x) \partial_p^\mu f_q(x, p) = C[f_q, f_g]$$

$$p^\mu \partial_\mu f_g(x, p) + m(x) \partial_\mu^x m(x) \partial_p^\mu f_g(x, p) = C[f_q, f_g]$$

Free-streaming

field interaction

$$\varepsilon - 3p \neq 0$$

Collision term

gauged to some  $\eta/s \neq 0$

*QGP evolution*

Relativistic transport theory

Equivalent to hydrodynamics for  $\eta/s \approx 0.1$

*HQs evolution in the QGP*

$$p^\mu \partial_\mu f_Q(x, p) = C[f_q, f_g, f_Q]$$

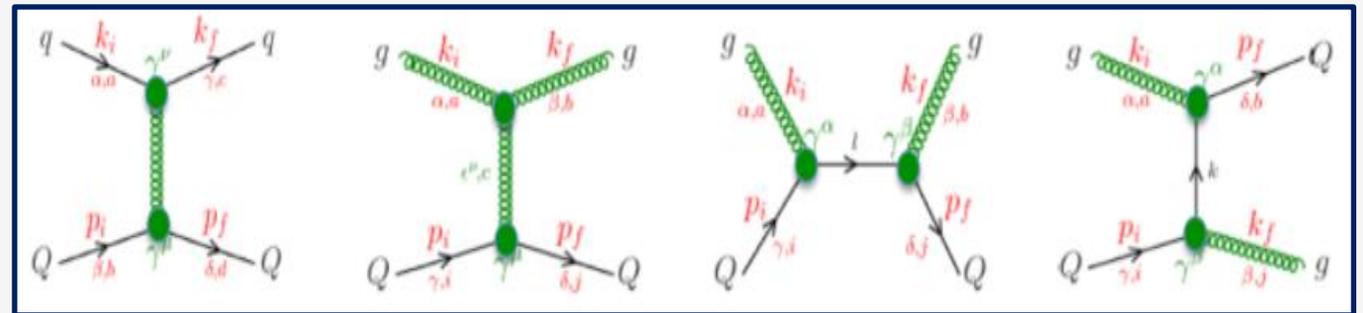
$$C[f_q, f_g, f_Q] = \frac{1}{2E_1} \int \frac{d^3 p_2}{2E_2 (2\pi)^3} \int \frac{d^3 p_1'}{2E_1' (2\pi)^3}$$

$$\times [f_Q(p_1') f_{q,g}(p_2') - f_Q(p_1) f_{q,g}(p_2)]$$

$$\times |M_{(q,g) \rightarrow Q}(p_1 p_2 \rightarrow p_1' p_2')|$$

$$\times (2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2')$$

Computed within a quasiparticles model



**HADRONIZATION of HQs: hybrid Coalescence + fragmentation**

# The quasiparticle model (QPM) for the quark-gluon plasma

**Non perturbative dynamics** → M scattering matrices (q,g → Q) evaluated by Quasi-Particle Model fit to **lattice QCD thermodynamics**

$N_f=2+1$   
Bulk:  
u,d,s

$$m_g^2(T) = \frac{2N_c}{N_c^2 - 1} g^2(T) T^2$$

$$m_q^2(T) = \frac{1}{N_c} g^2(T) T^2$$



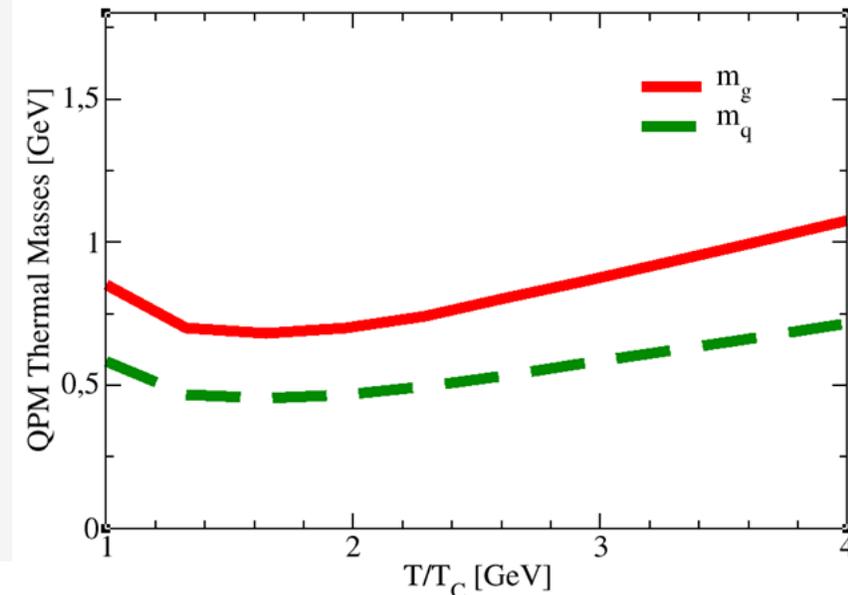
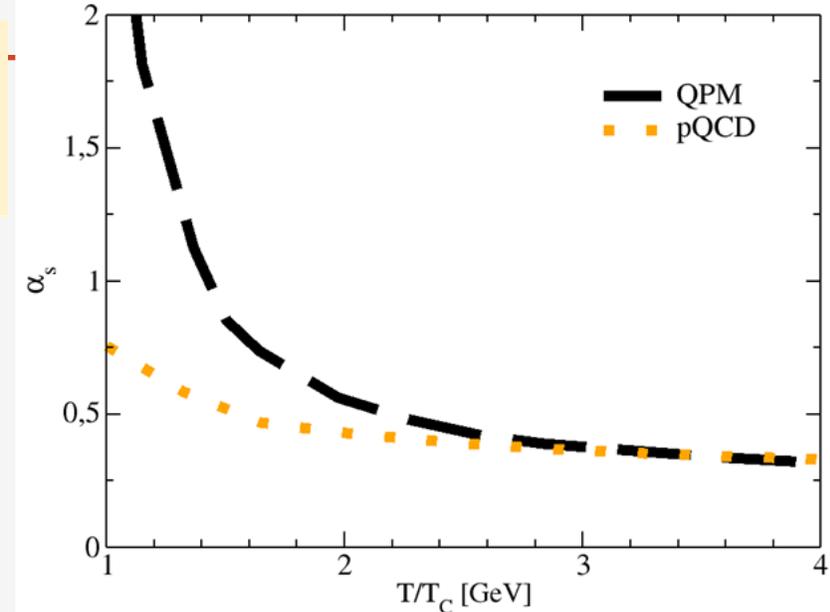
**Thermal masses of gluons and light quarks**

$g(T)$  from a fit to  $\epsilon$  from lQCD data → good reproduction of  $P$ ,  $\epsilon-3P$

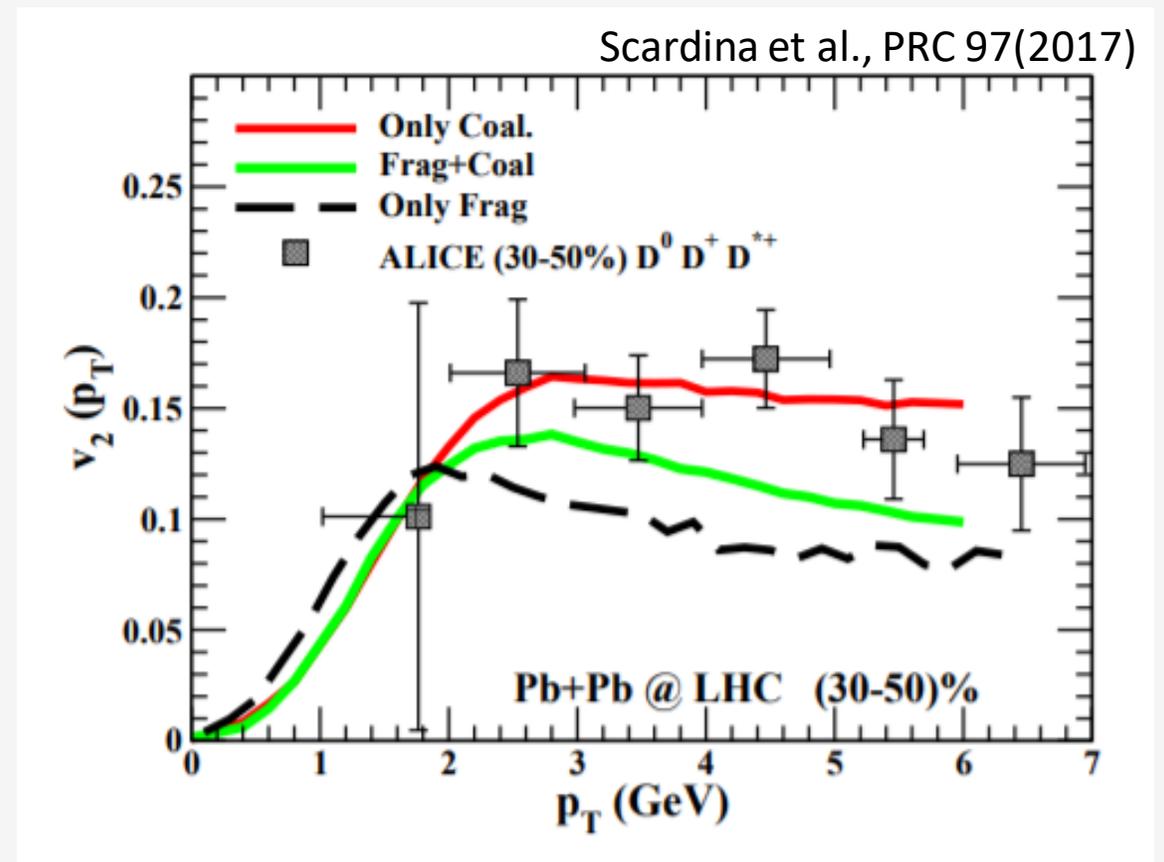
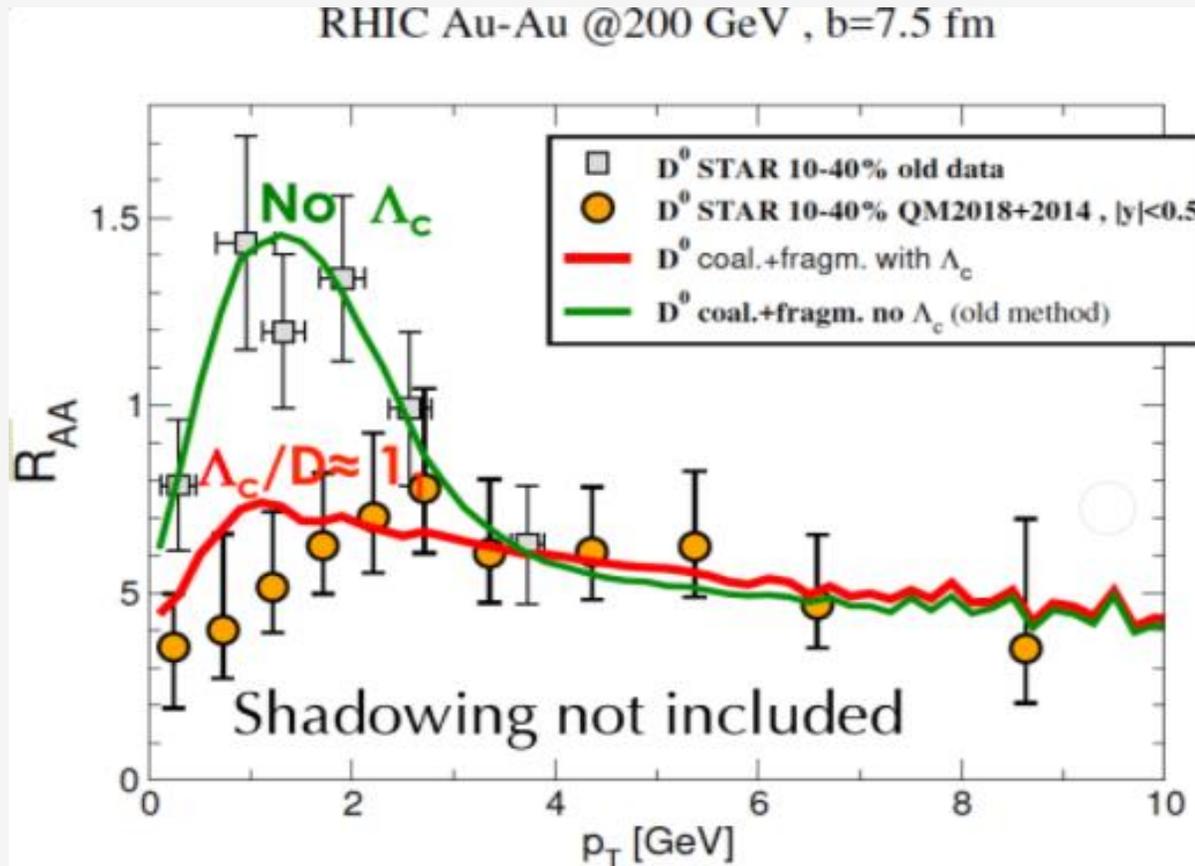
$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f) \ln \left[ \lambda \left( \frac{T}{T_c} - \frac{T_s}{T_c} \right) \right]^2}$$

$\lambda=2.6$   
 $T_s=0.57 T_c$

**Larger than pQCD especially as  $T \rightarrow T_c$**

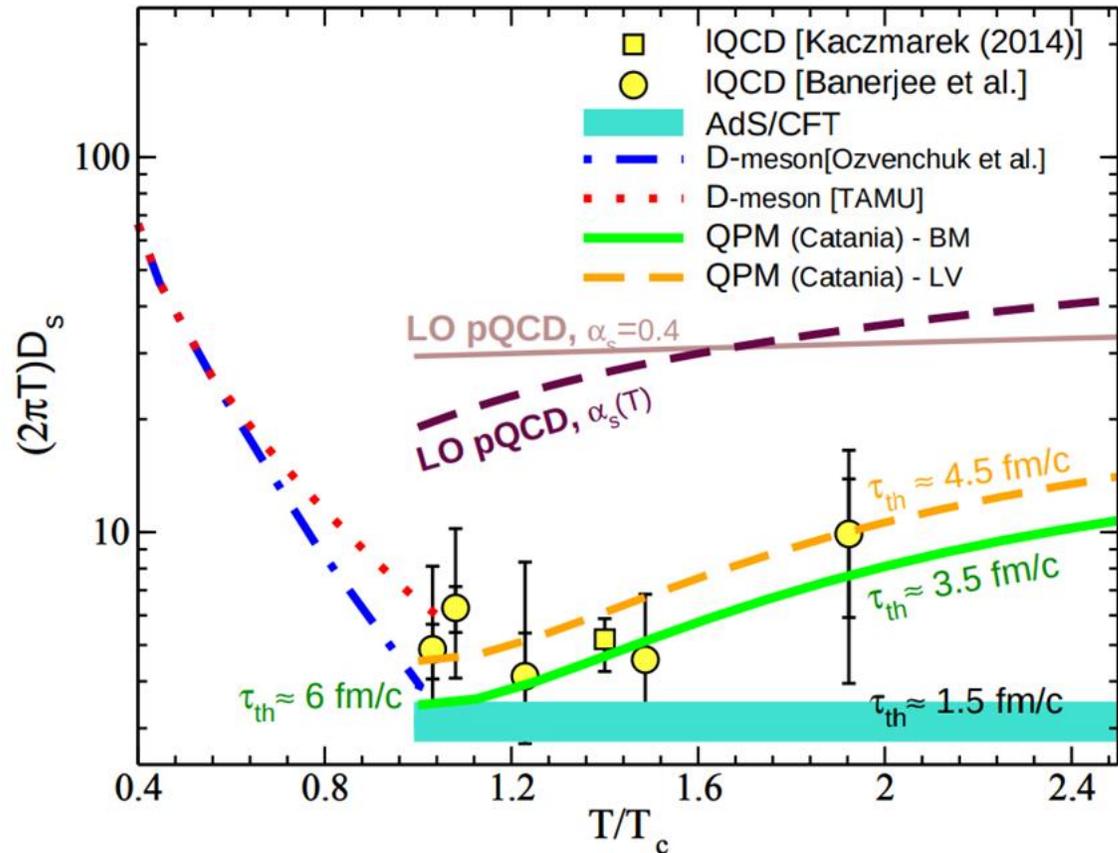


# Nuclear modification factor and elliptic flow of heavy mesons



Good description of  $R_{AA}$  and  $v_2$  at the RHIC and the LHC energies

# Spatial diffusion coefficient, $D_s$ , of charm in QGP



$R_{AA}(p_T)$  and  $v_2(p_T)$  (computed within relativistic transport, in agreement with experimental data)



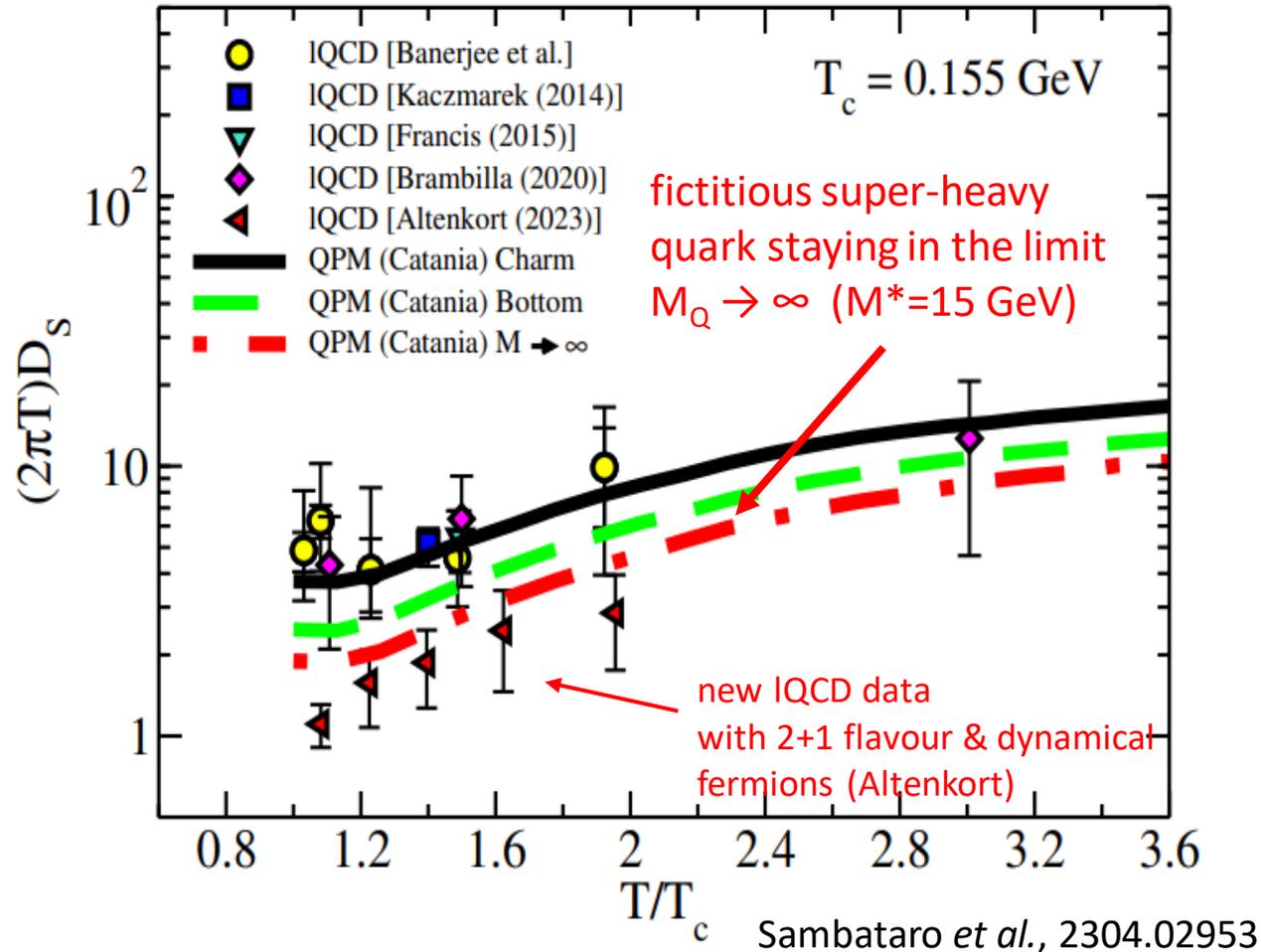
thermalization time,  $\tau_{th}$ , of  $c$

## Reviews:

- F. Prino and R. Rapp, JPG(2019)
- X. Dong and V. Greco, Prog.Part.Nucl.Phys. (2019)
- Jiaying Zhao et al., arXiv:2005.08277

$$\tau_{th} \leftarrow \frac{M}{2\pi T^2} (2\pi T D_s) \cong 1.8 \frac{2\pi T D_s}{(T/T_c)^2} \text{ fm/c}$$

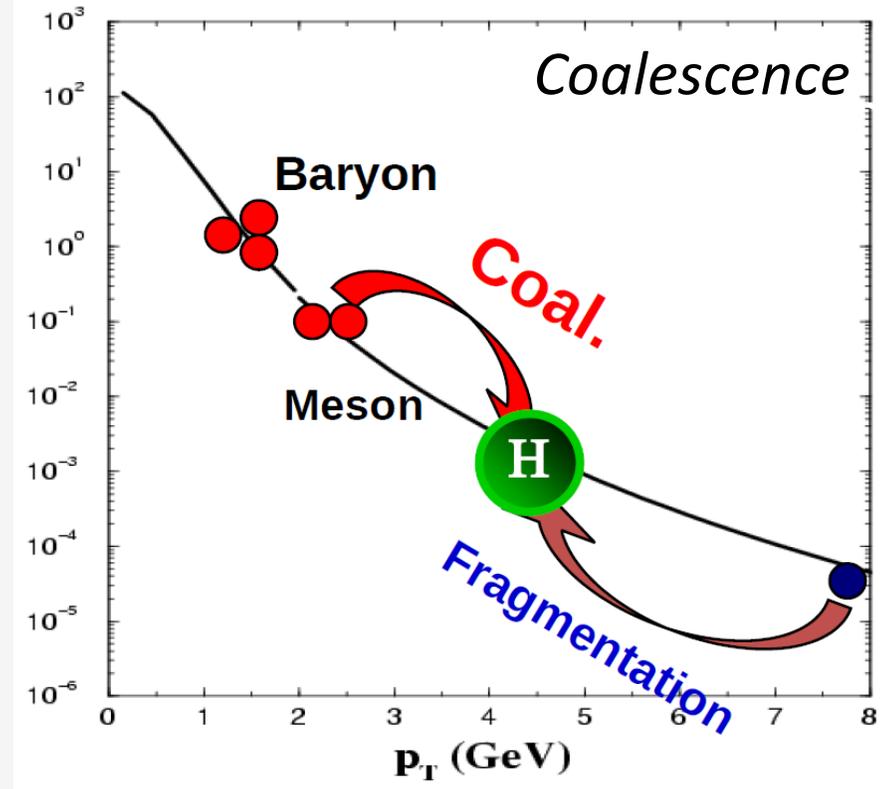
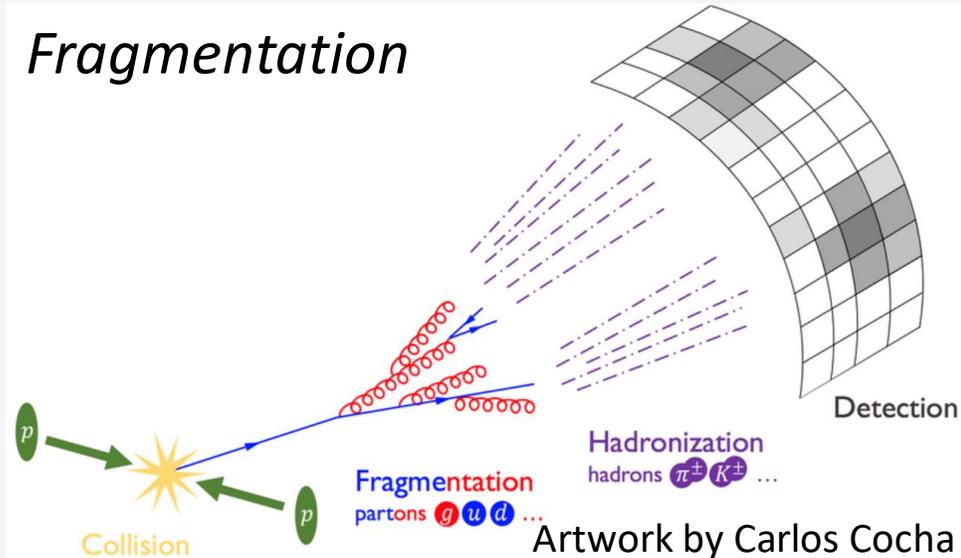
# Spatial diffusion coefficient, $D_s$ , of beauty in QGP



From  $D_s$  we obtain ( $T$  in  $1-2T_c$ ):

- $\tau_{th}(c) \sim 5 \text{ fm}/c$
- $\tau_{th}(b) \sim 11 \text{ fm}/c$

# Hadronization of charm in heavy ion collisions: coalescence plus fragmentation

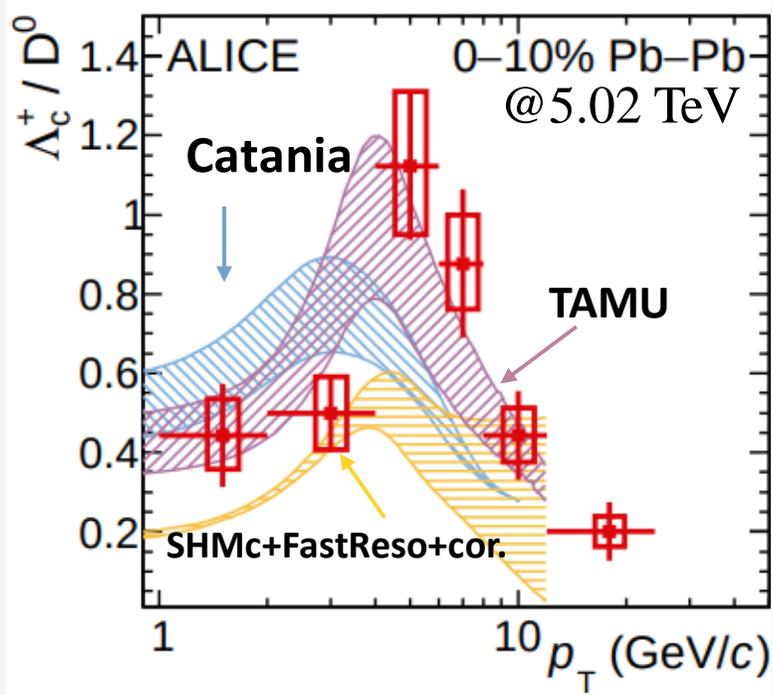


Coalescence consists in the recombination of quarks which *sit close in phase space*:

- 1 quark and 1 antiquark recombine to form a meson
- 3 quarks recombine to form a baryon

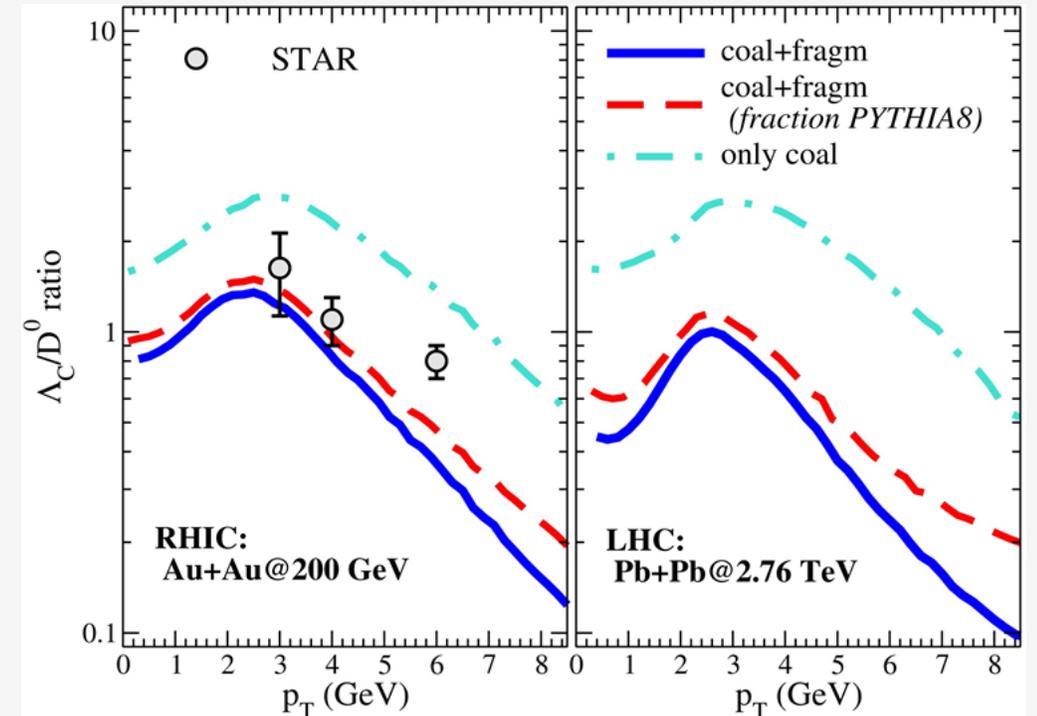
Typically coalescence enhances hadrons production at intermediate  $p_T$ .

# Hadronization of charm in heavy ion collisions: coalescence plus fragmentation



ALICE Coll. arXiv:2112.08156v1

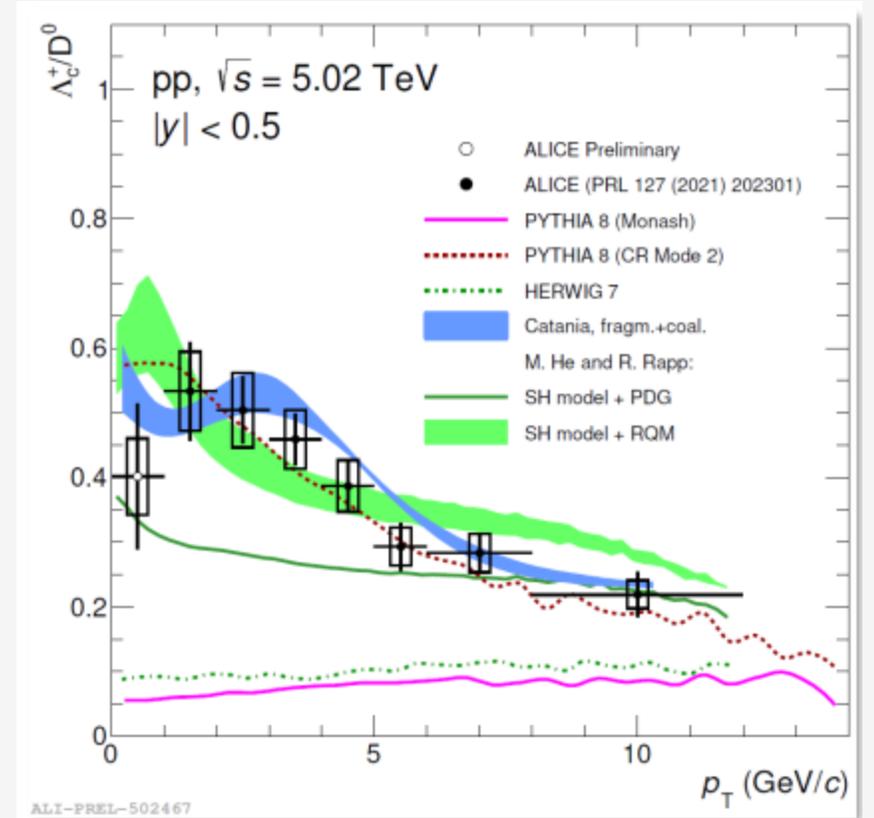
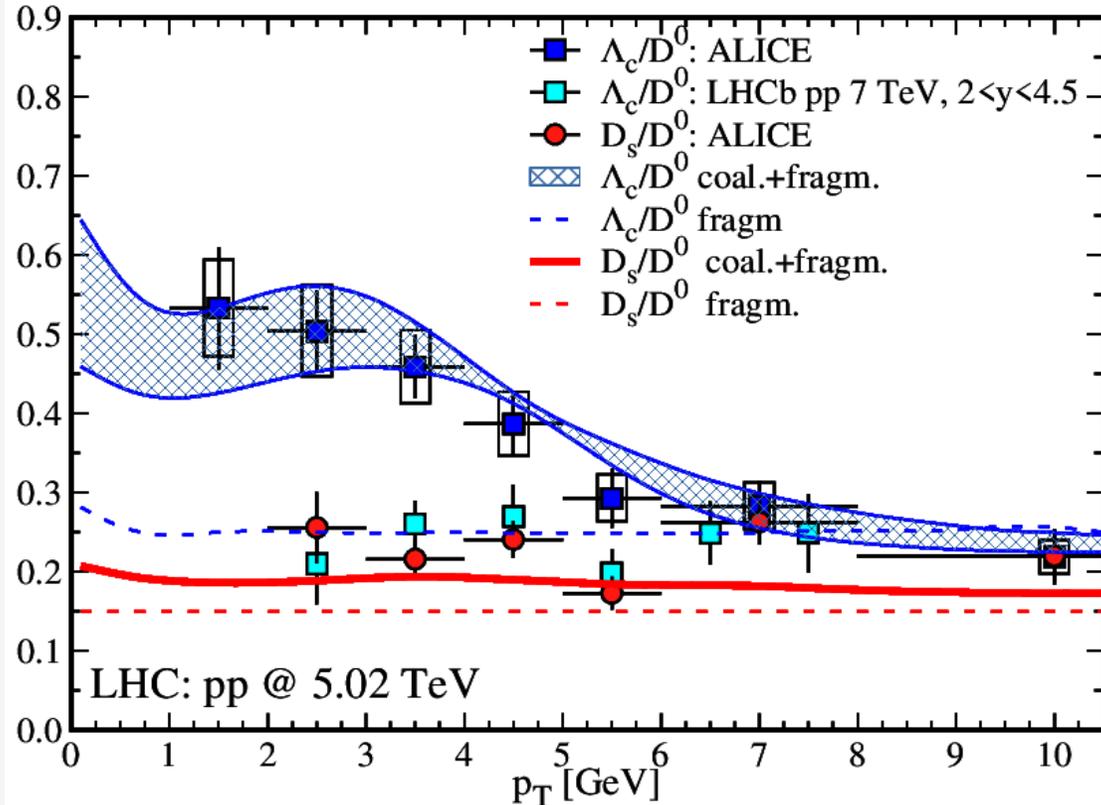
STAR Coll., Phys.Rev.Lett. 124 (2020) 17, 172301



S. Plumari, V. Minissale et al., Eur. Phys. J. C78 no. 4, (2018) 348

# Hadronization of charm in proton-proton collisions: coalescence plus fragmentation

V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 136622



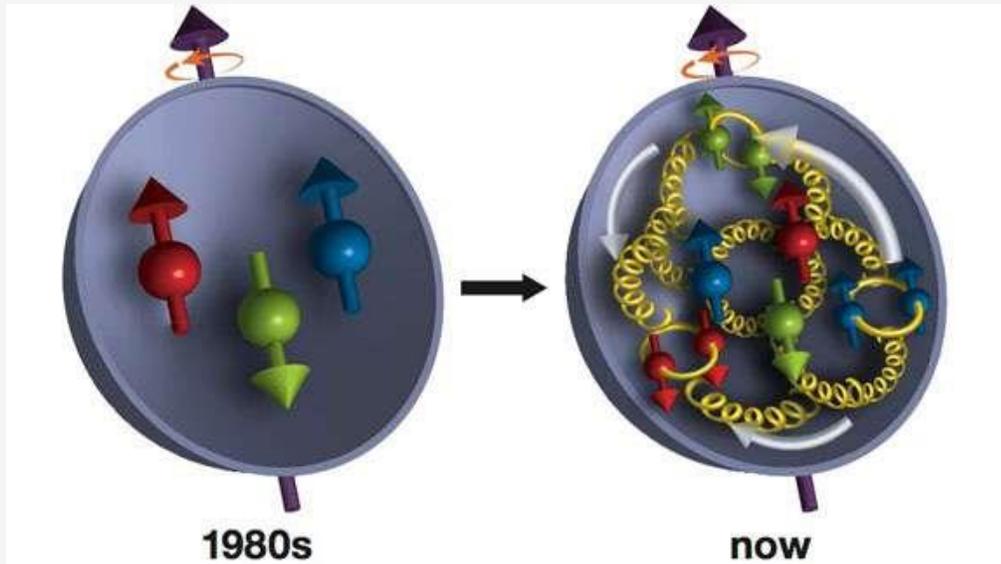
ALICE,  
Phys.Rev.Lett. 127 (2021) 20, 202301

# Conclusions

- ❖ Heavy quarks (HQs) excellent probes of the entire evolution of the medium created in relativistic heavy ion collisions
- ❖ *Heavy quarks* in the evolving Glasma: anomalous diffusion, anisotropic diffusion, polarization
- ❖ *Heavy quarks* in the quark-gluon plasma: relativistic transport theory, thermalization time, spatial diffusion coefficient for c and b
- ❖ *Hadronization of heavy quarks* in RHICs: coalescence plus fragmentation, effects in AA and pp collisions

# Appendix

# Gluons dominate the proton wave function at small x



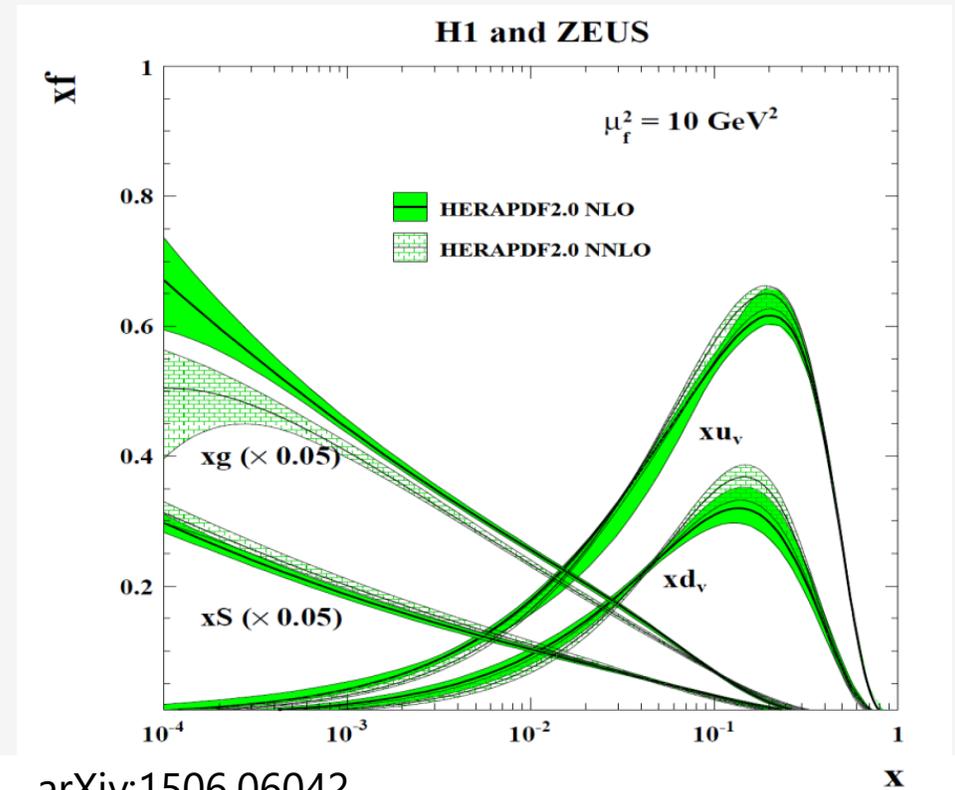
$x$ : parton momentum/nucleon momentum

Valence quarks (uud):  $x \approx 1$

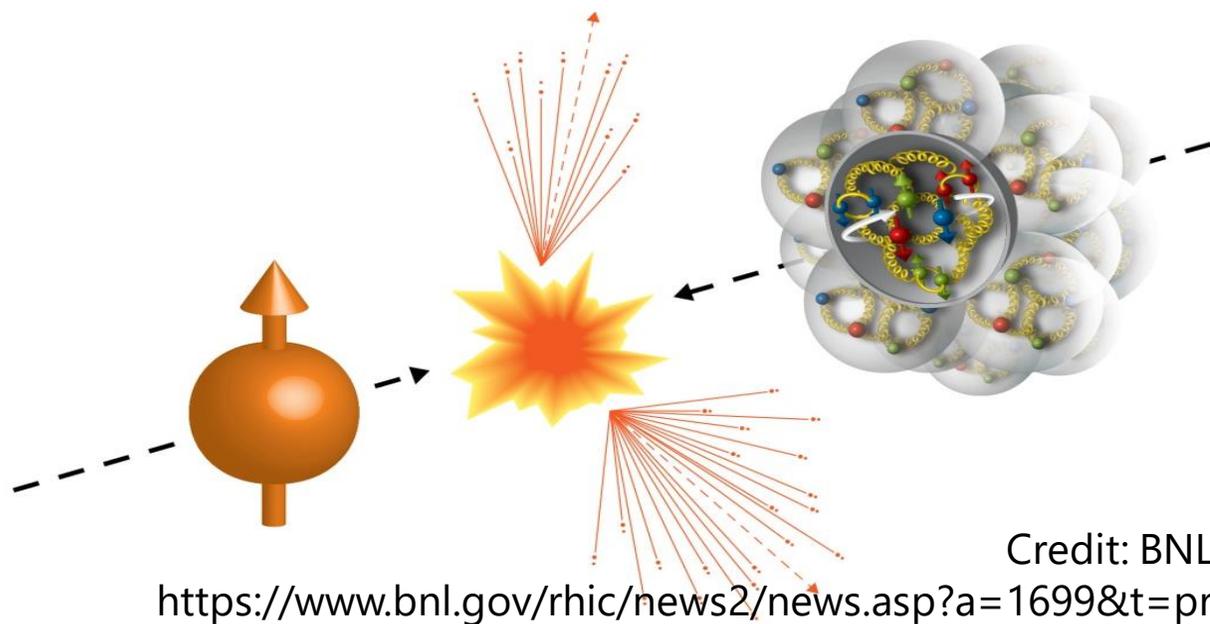
*Small-x content of the proton*

Sea quarks+antiquarks

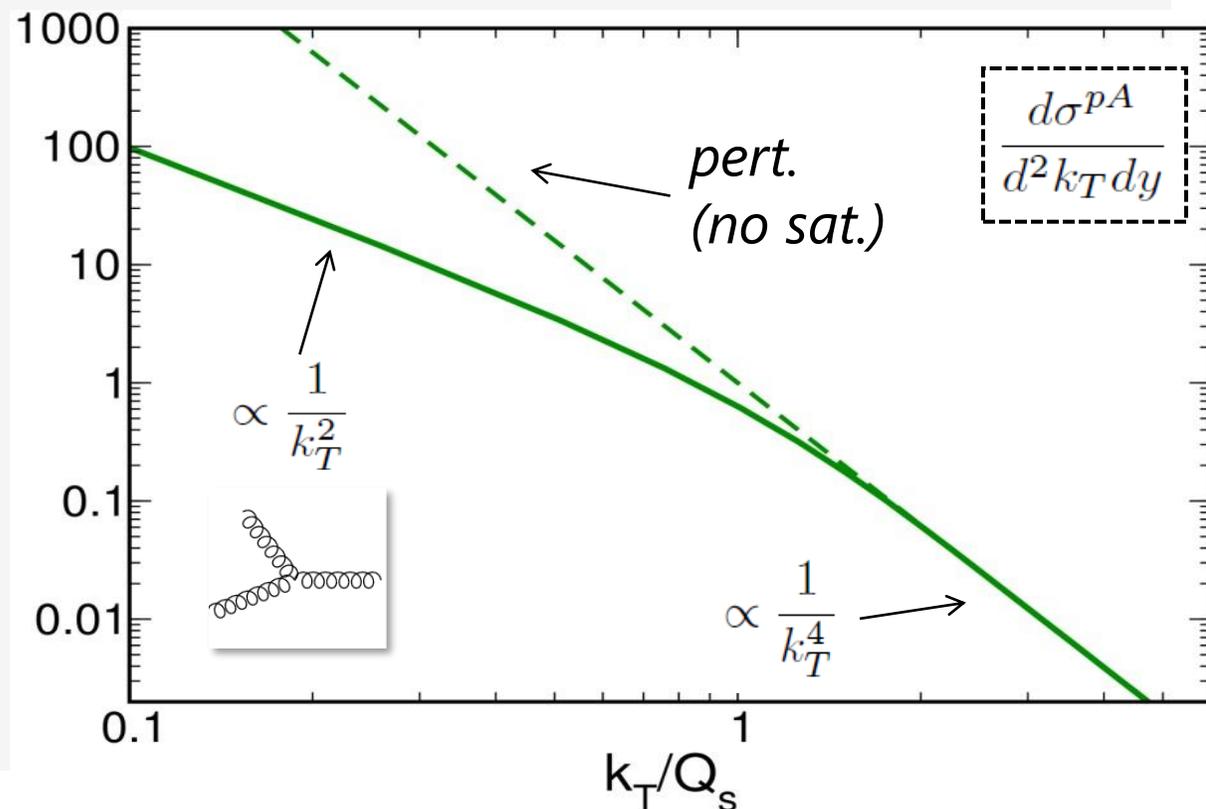
Sea gluons



Glucos dominate the proton wave function at small  $x$



*Large gluon density:  
 Gluon recombination*



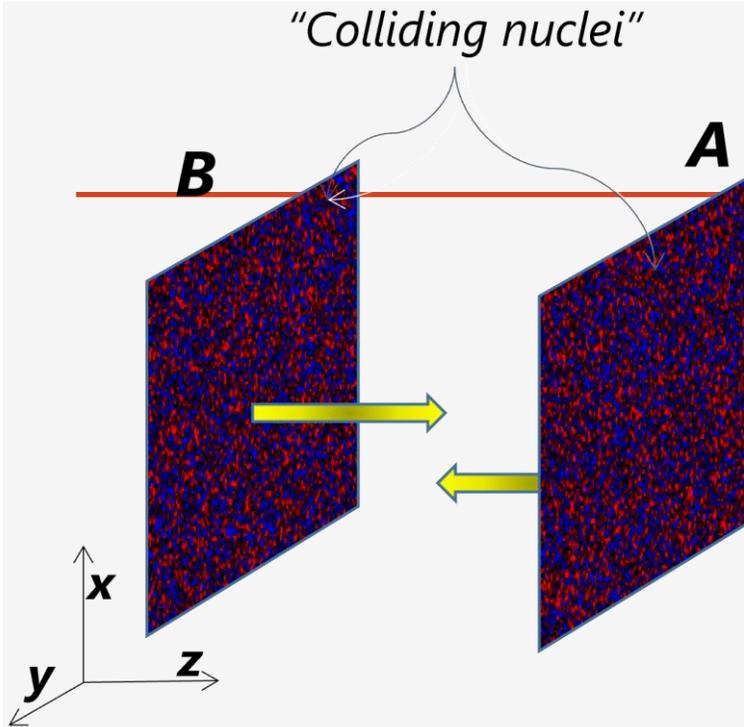
## Saturation

*Gluon production is suppressed due to the abundance of the  $2 \rightarrow 1$  processes.*

## Saturation scale, $Q_s$

*Momentum scale at which saturation becomes important*

# The MV model for the sources of the Glasma



**Model of sources of the classical gluon fields (MV)**  
*Uncorrelated color density fluctuations on the two nuclei.*

McLerran and Venugopalan (1996)  
Kovchegov (1996)

**Gaussian distribution of color charges (\*)**

$$\langle \rho^a(\mathbf{x}_T) \rangle = 0,$$

$$\langle \rho_A^a(\mathbf{x}_T) \rho_A^b(\mathbf{y}_T) \rangle = (g\mu_A)^2 \delta^{ab} \delta^{(2)}(\mathbf{x}_T - \mathbf{y}_T)$$

**$g^2\mu \approx Q_s$ : saturation scale** Lappi (2008)

$$Q_s \approx 1 - 3 \text{ GeV}$$

(\*)See Parisi's talk for more details

# The MV model for the sources of the Glasma: the proton case

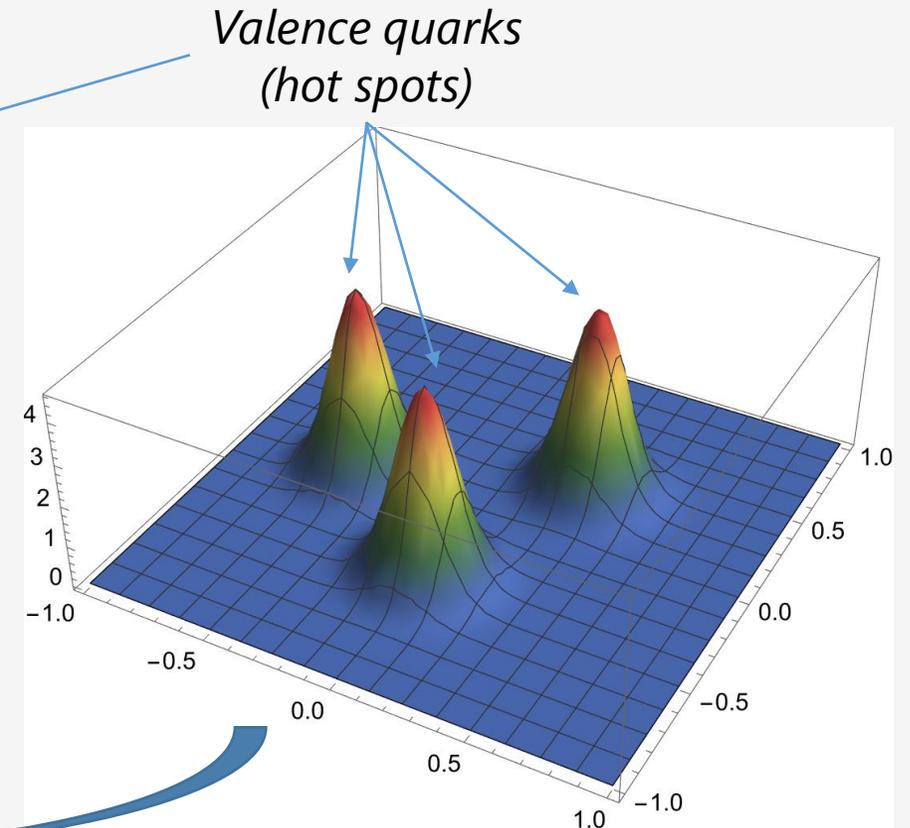
## Hot spots model

Color charges mostly near the valence quarks in the proton

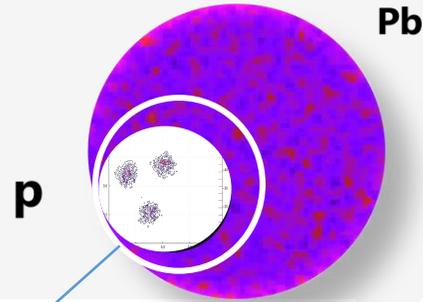
$$T_p(\mathbf{x}_T) = \frac{1}{3} \sum_{i=1}^3 \frac{1}{2\pi B_q} \exp\left(-\frac{(\mathbf{x}_T - \mathbf{x}_i)^2}{2B_q}\right)$$

## Gaussian distribution of color charges

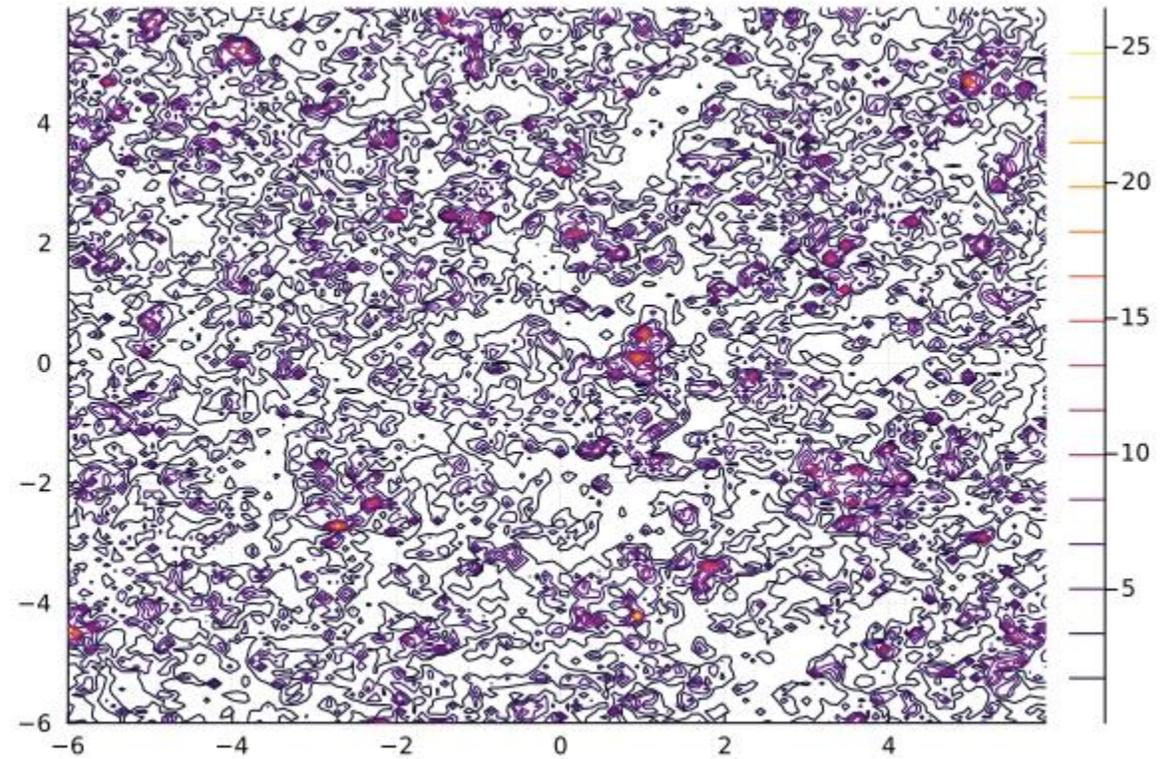
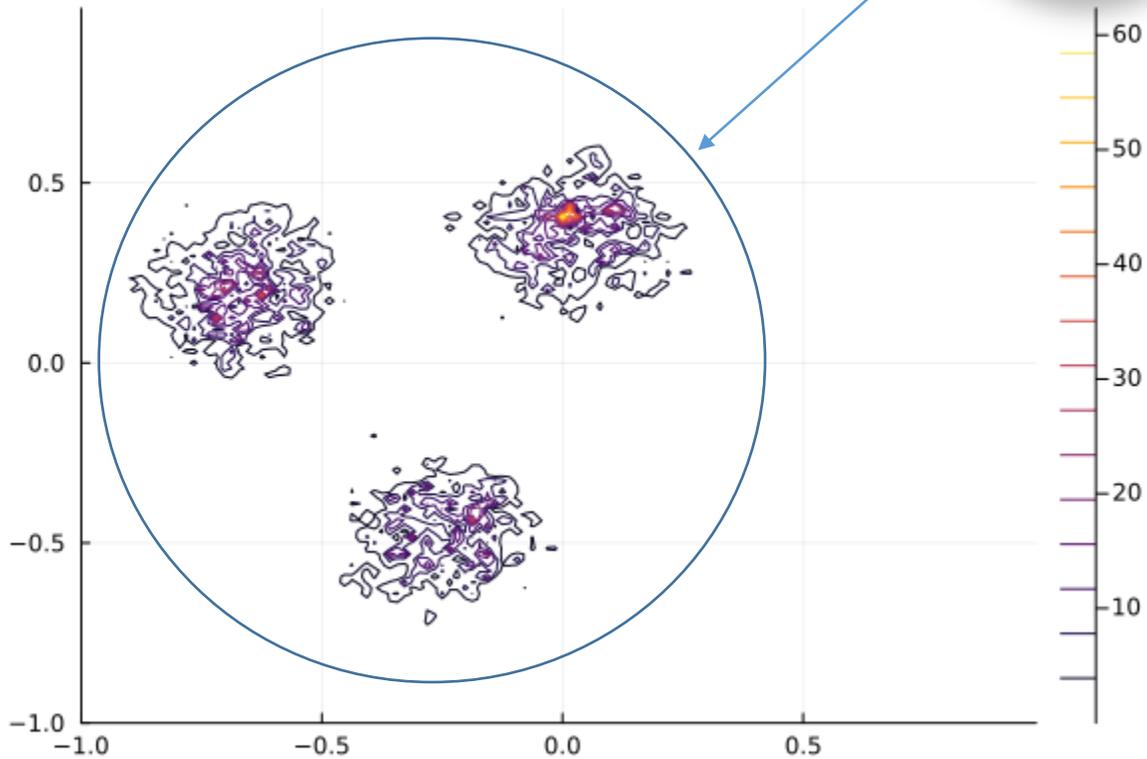
$$\langle \rho^a(\mathbf{x}_T) \rho^b(\mathbf{y}_T) \rangle = g^2 \mu^2(x, \mathbf{x}_T) \delta^{ab} \delta^{(2)}(\mathbf{x}_T - \mathbf{y}_T)$$



# A closer look at the Glasma: the initial energy density



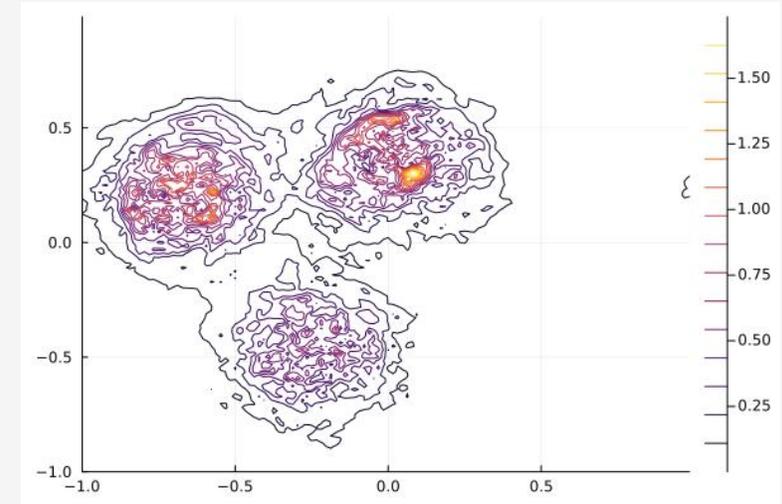
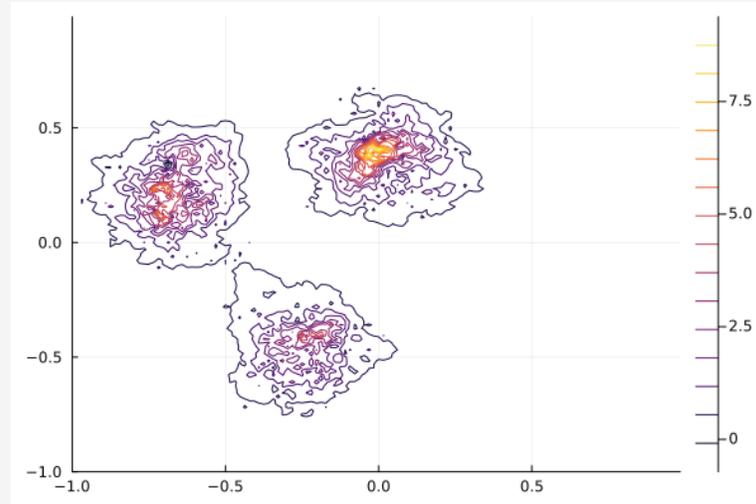
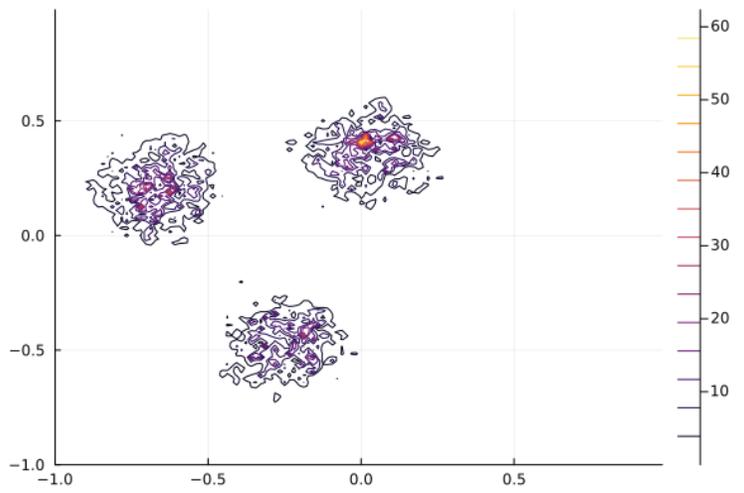
$$\varepsilon = \text{Tr} [E_L^2 + E_T^2 + B_L^2 + B_T^2]$$



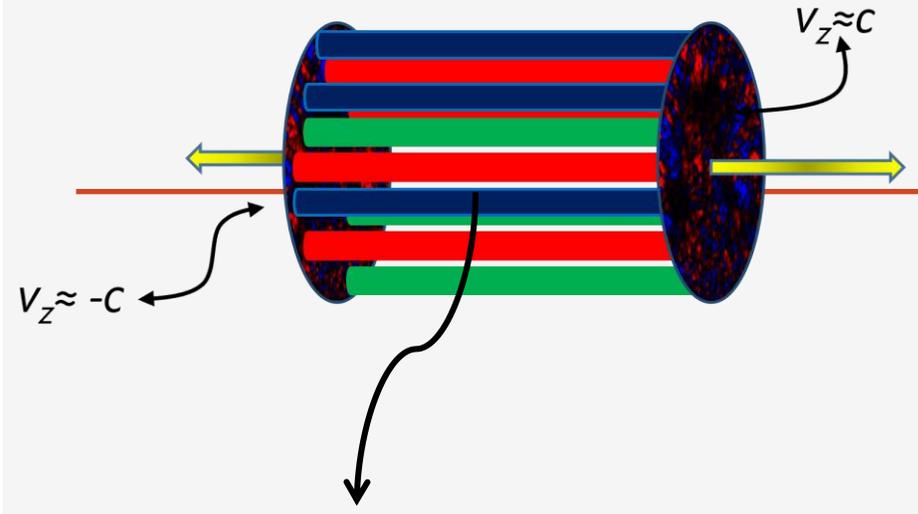
# Expansion in the transverse plane: the p-Pb case

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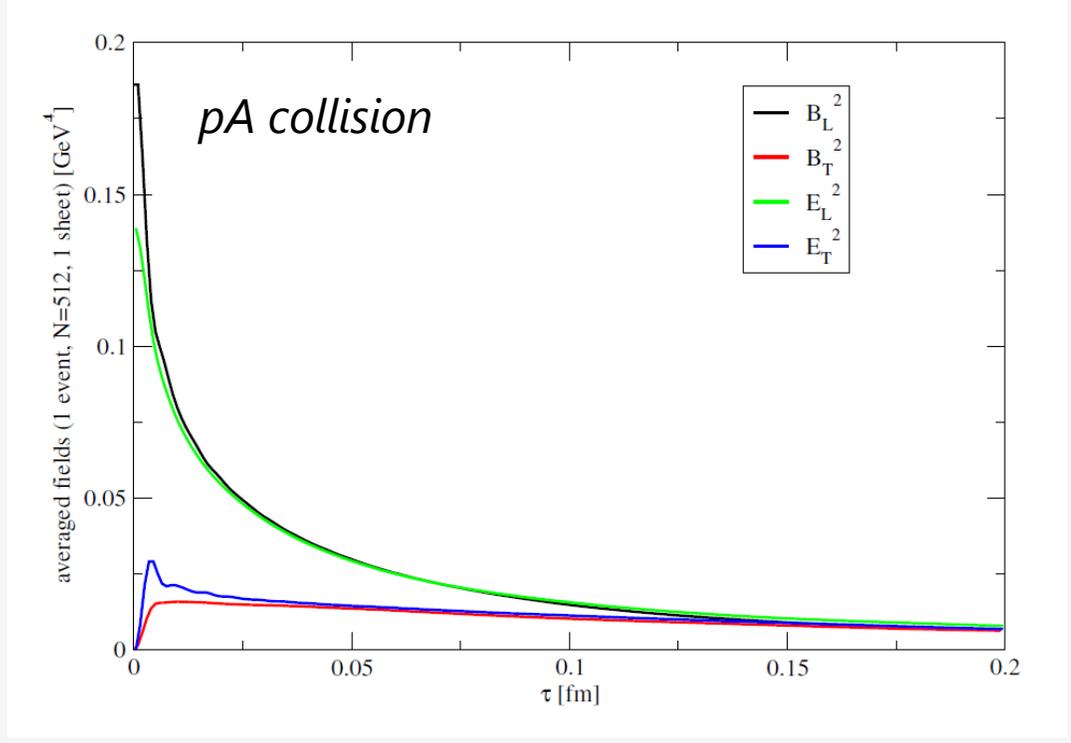
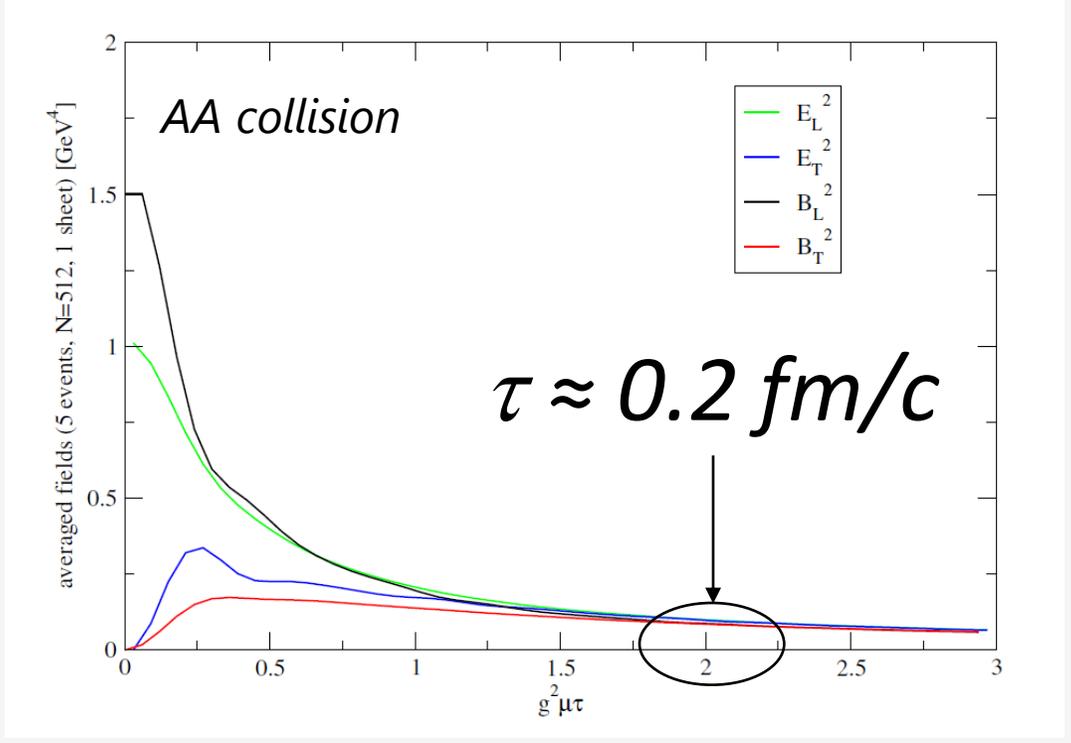
$$\varepsilon = \text{Tr} [E_L^2 + E_T^2 + B_L^2 + B_T^2]$$



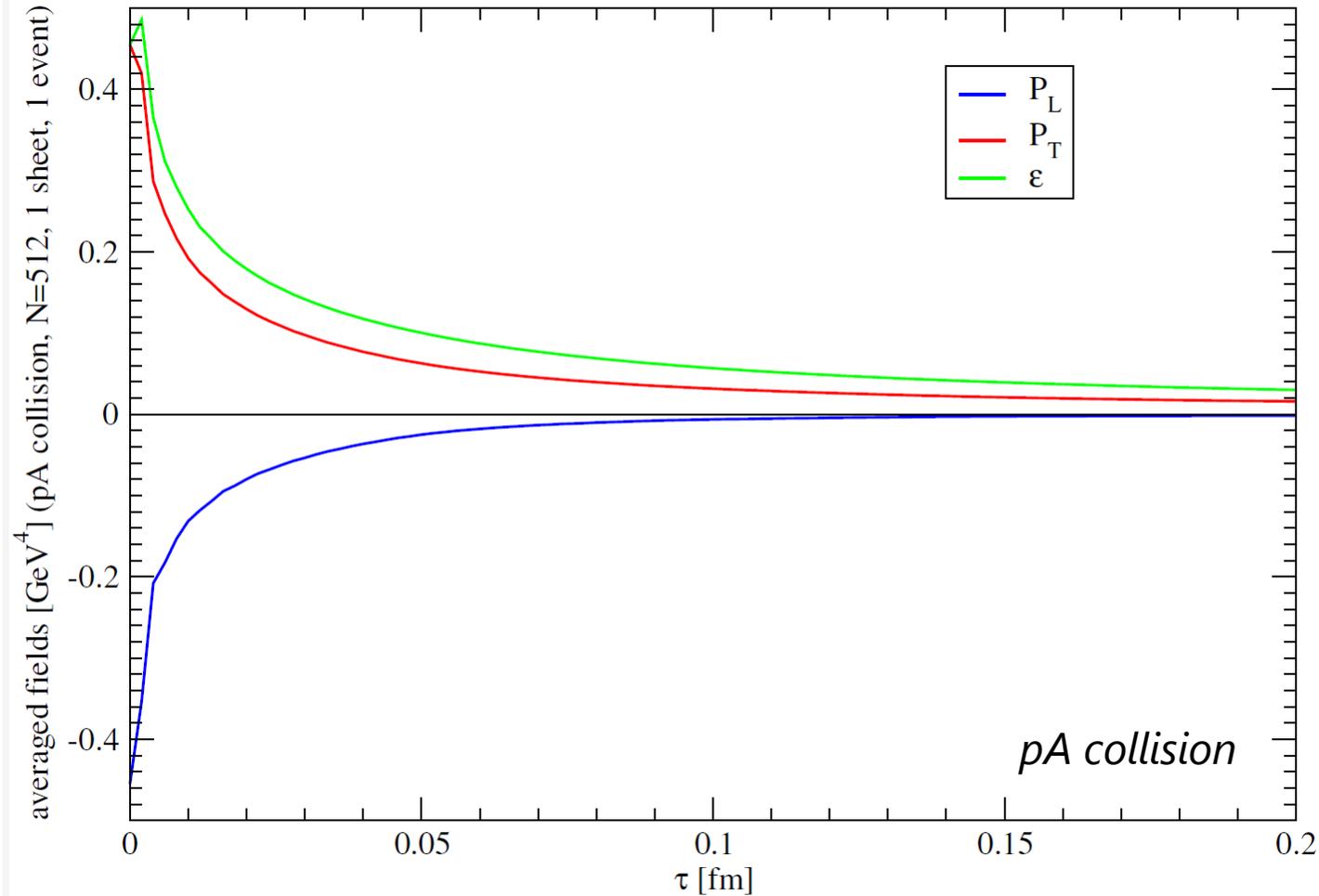
# Evolving fields



$$\varepsilon = \text{Tr} [E_L^2 + E_T^2 + B_L^2 + B_T^2]$$



# The free streaming regime, pA collisions



$$\epsilon = \text{Tr} [E_L^2 + E_T^2 + B_L^2 + B_T^2]$$

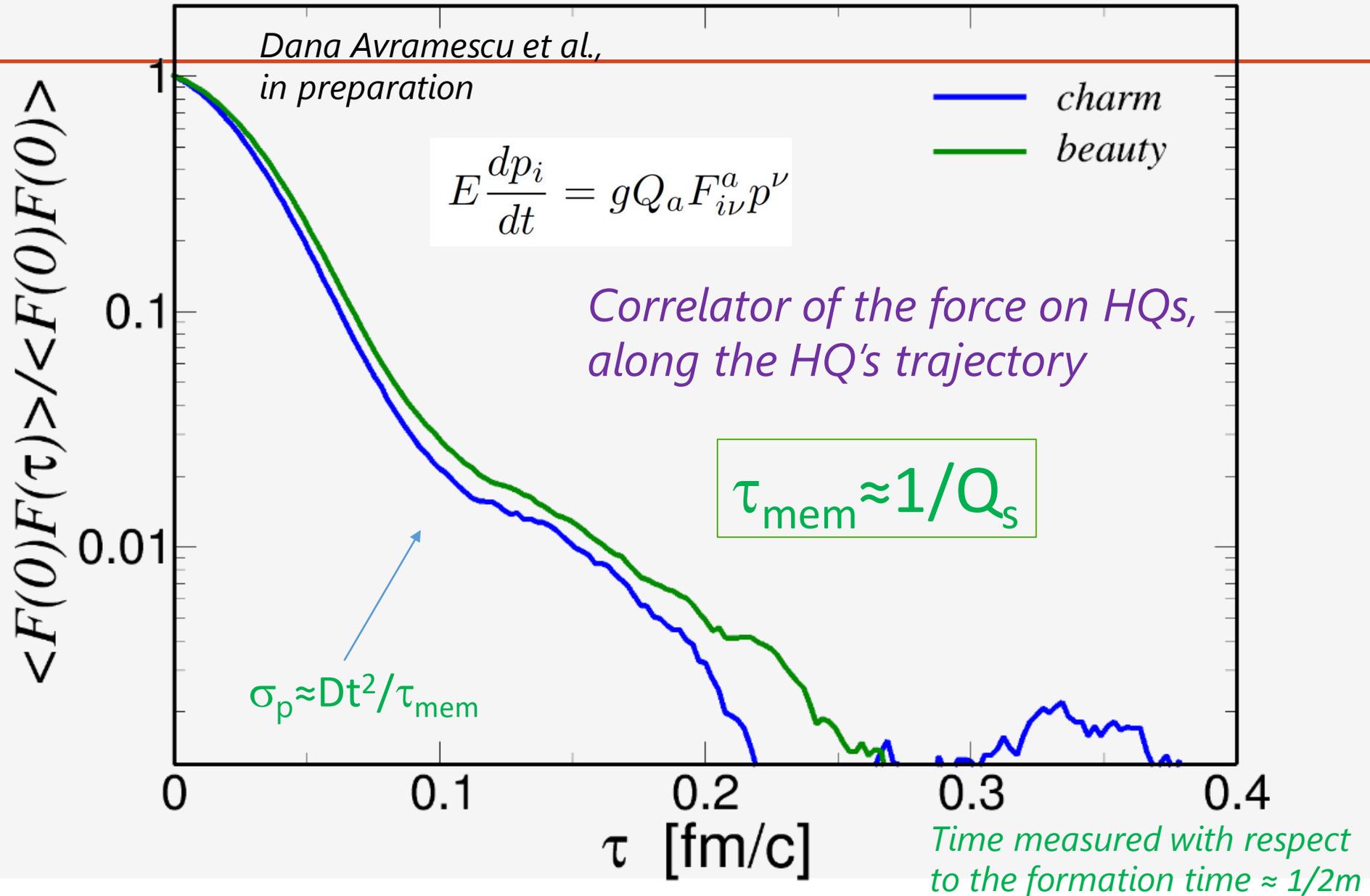
$$P_L = \text{Tr} [-E_L^2 - B_L^2 + E_T^2 + B_T^2]$$

$$P_T = \text{Tr} [E_L^2 + B_L^2]$$

$$\tau \approx 0.2 \text{ fm}/c$$

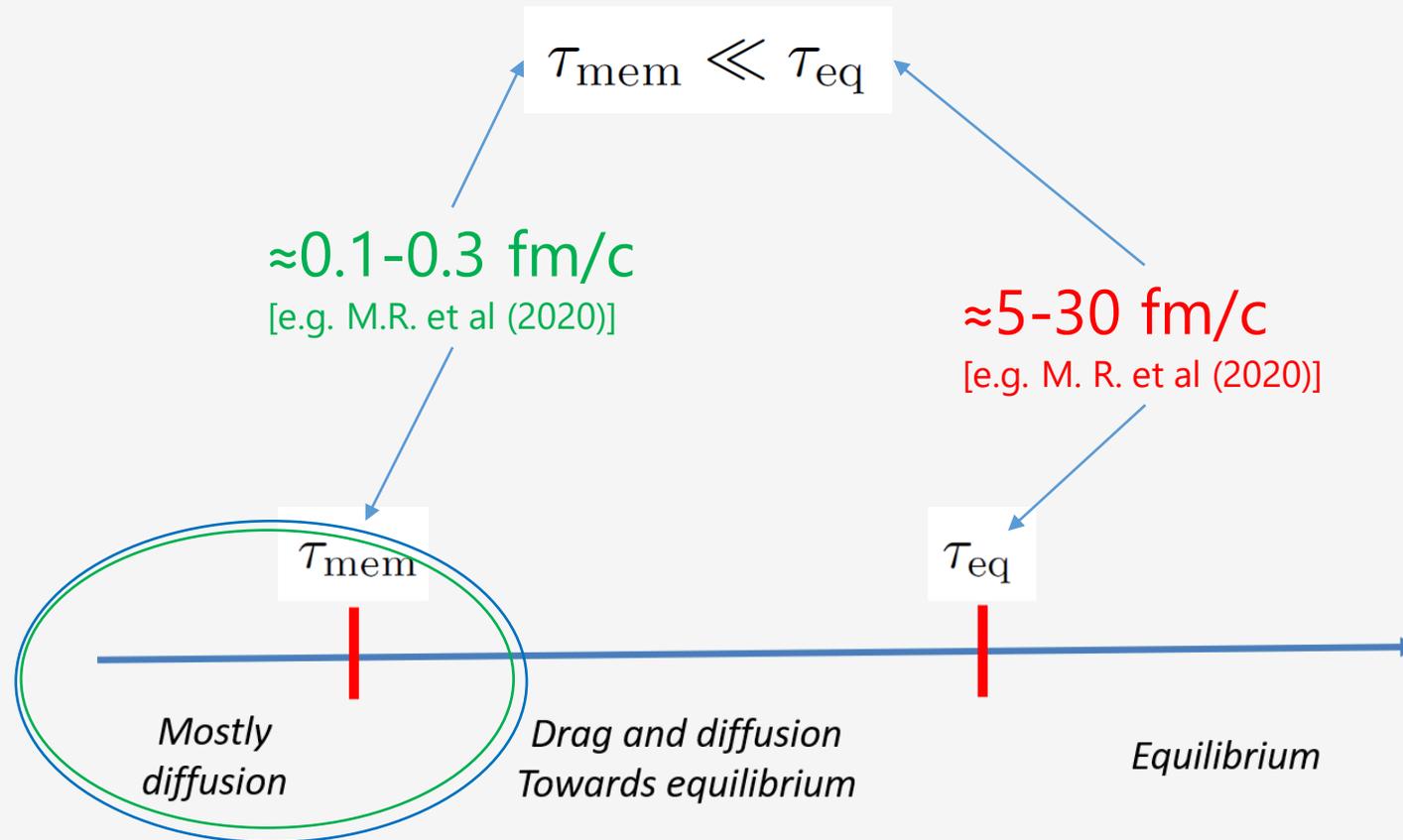
Longitudinal pressure *vanishes*

# Memory for the HQs diffusion in EvGlasma



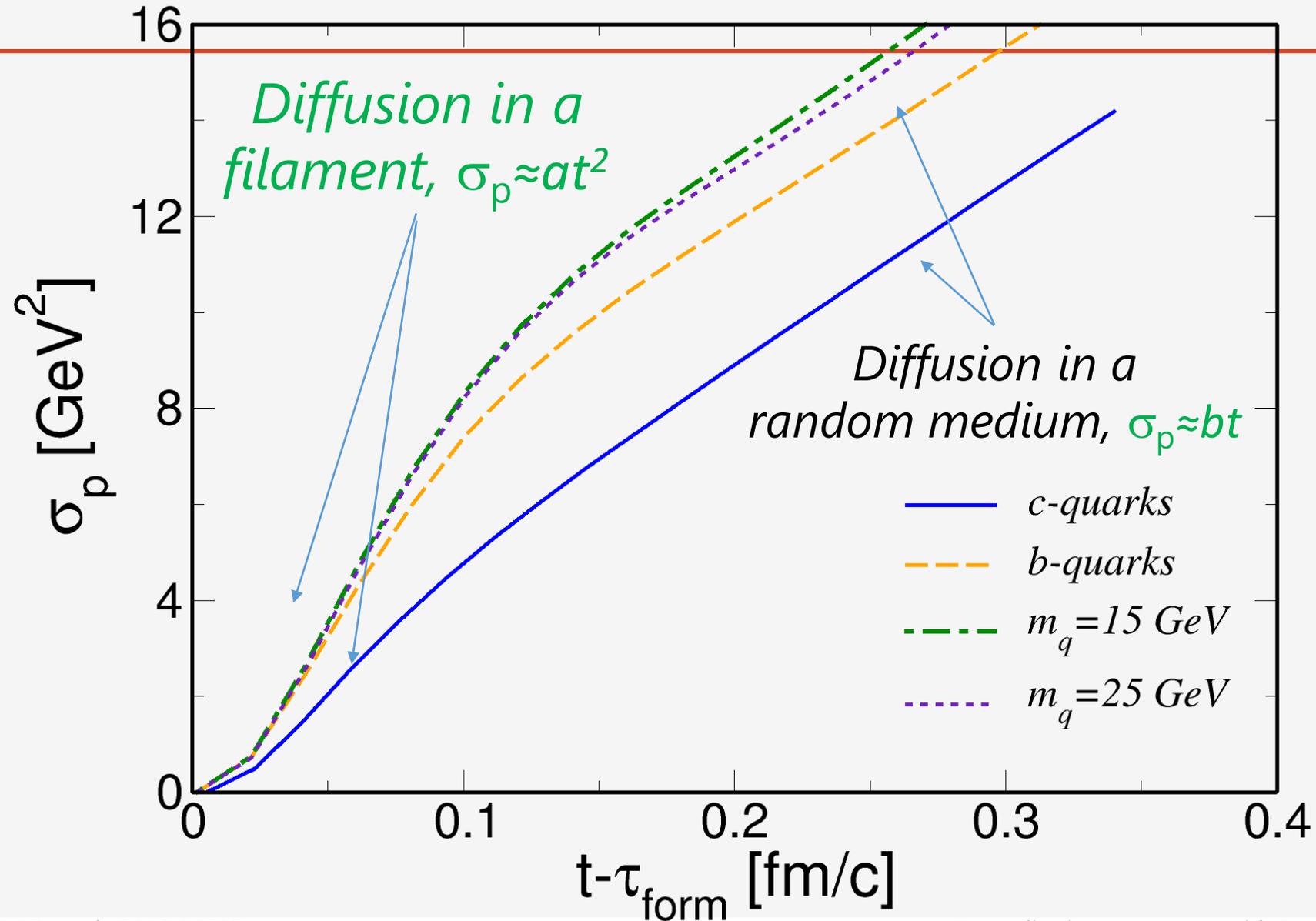
# Heavy quarks in Glasma: diffusion-dominated motion

EvGlasma lifetime  $\approx$  QGP thermalization time  $\approx 0.3\text{-}0.6$  fm/c

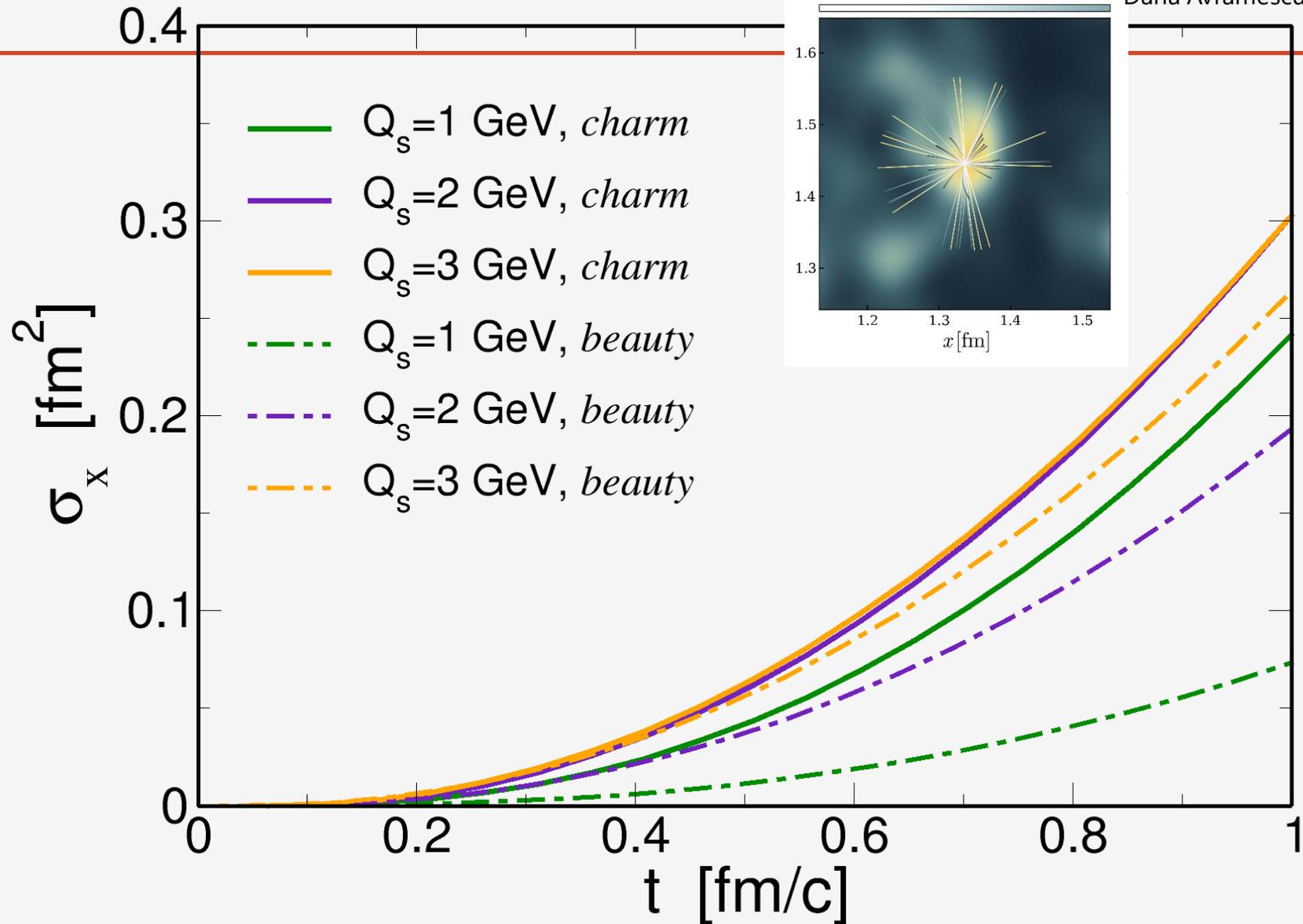


- Diffusion dominates because of the large thermalization time
- Memory leads to nonlinear evolution of  $\sigma_p$

# Momentum broadening: effect of HQ's mass



# Coordinates spreading vs HQ mass

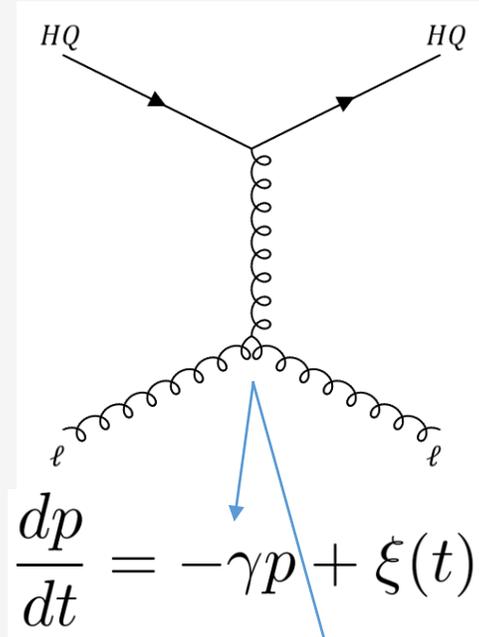


# Comparison with pQCD-Langevin

We prepare a bath of gluons *at temperature  $T$* , with the *same energy density* of the EvGlasma, and study the *diffusion of heavy quarks in this bath with Langevin equation*.

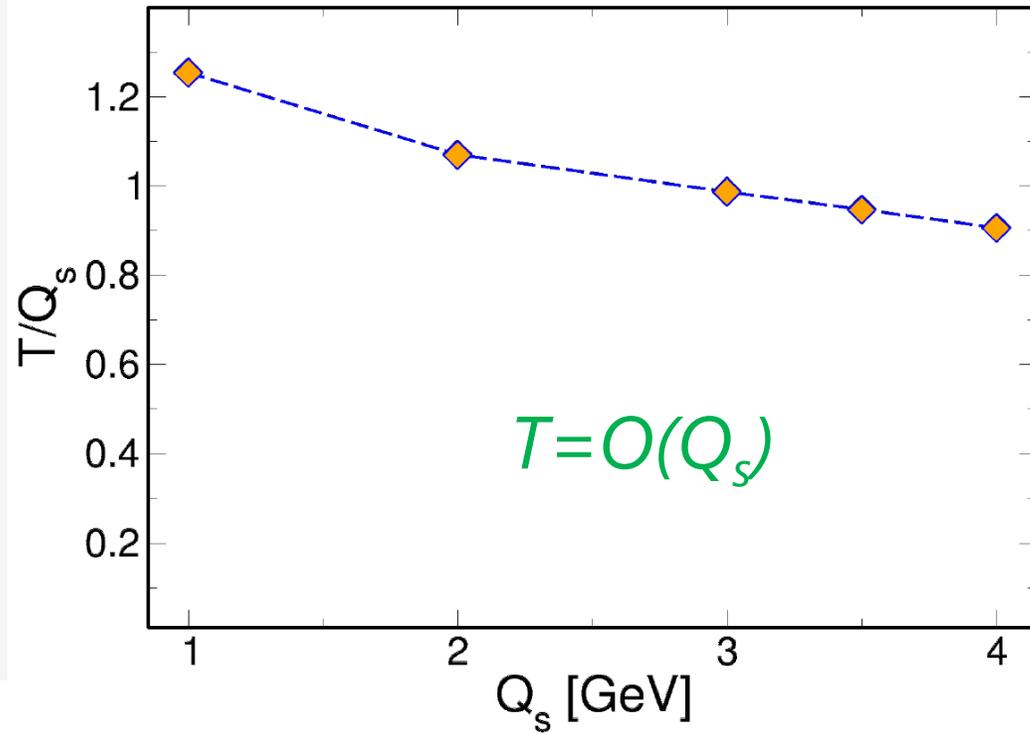
$$\varepsilon = 2(N_c^2 - 1) \int \frac{d^3p}{(2\pi)^3} \frac{p}{e^{\beta p} - 1} = \frac{(N_c^2 - 1)\pi^2 T^4}{15}$$

$$T = 1/\beta$$



$$\langle \xi(t_1) \xi(t_2) \rangle = 2\mathcal{D}f(t_1 - t_2)$$

$$f(t_1 - t_2) = \delta(t_1 - t_2)$$



# Comparison with pQCD-Langevin

We prepare a bath of gluons *at temperature*  $T$ , with the *same energy density* of the EvGlasma, and study the *diffusion of heavy quarks in this bath with Langevin equation*.

$$\varepsilon = 2(N_c^2 - 1) \int \frac{d^3p}{(2\pi)^3} \frac{p}{e^{\beta p} - 1} = \frac{(N_c^2 - 1)\pi^2 T^4}{15}$$

$$T = 1/\beta$$

$$\frac{dp}{dt} = -\gamma p + \xi(t)$$

*Why do we want to do this:*

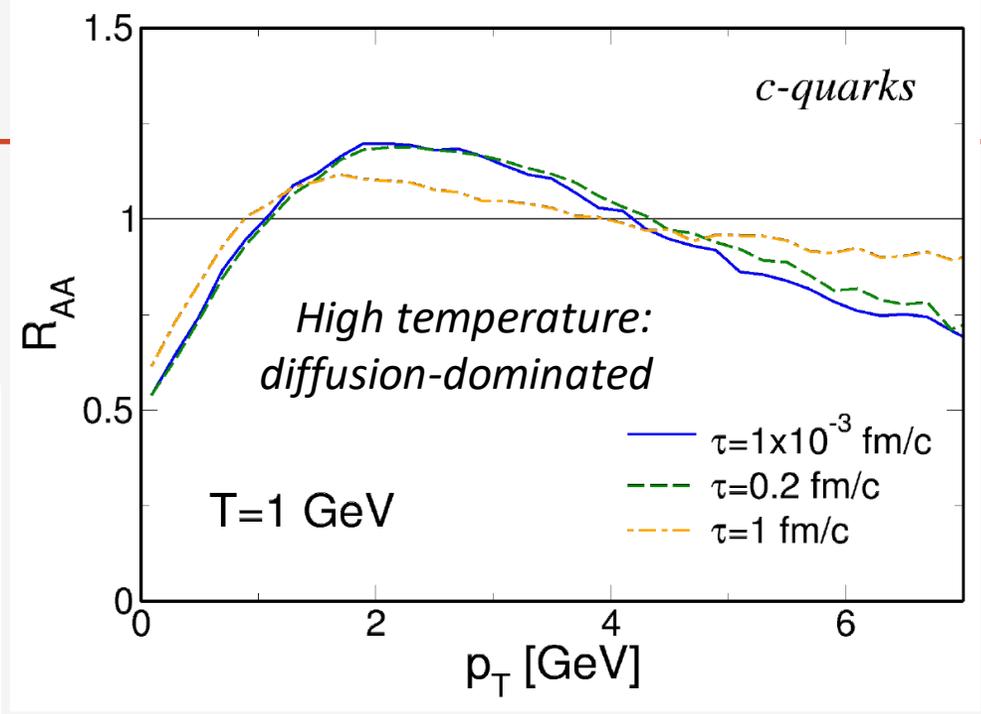
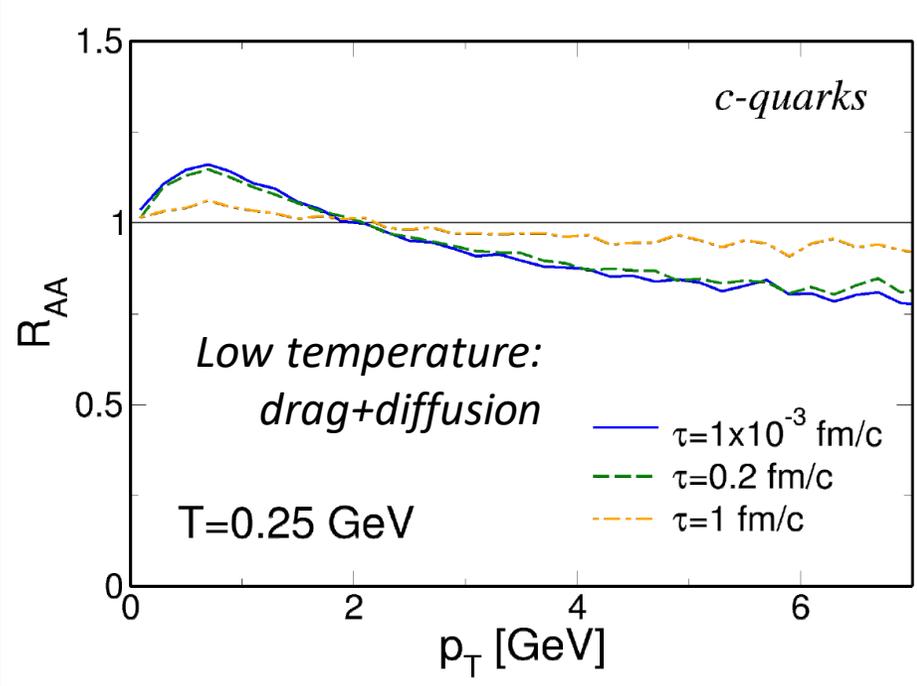
- Check if Langevin eqs, used abundantly in the HQs literature, can be used to reproduce the diffusion of HQs in the EvGlasma.

*Qualitatively* we already know there are differences.

*Quantitatively* it is worth checking if momentum broadening obtained within the two schemes is comparable.



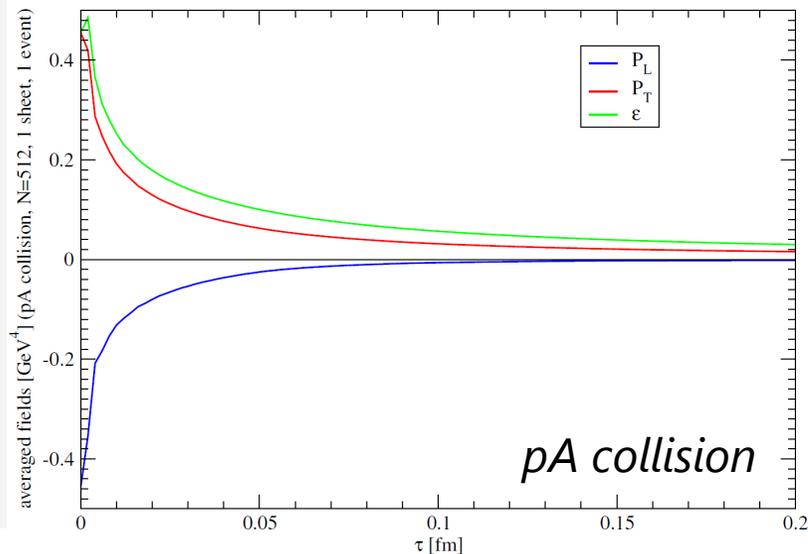
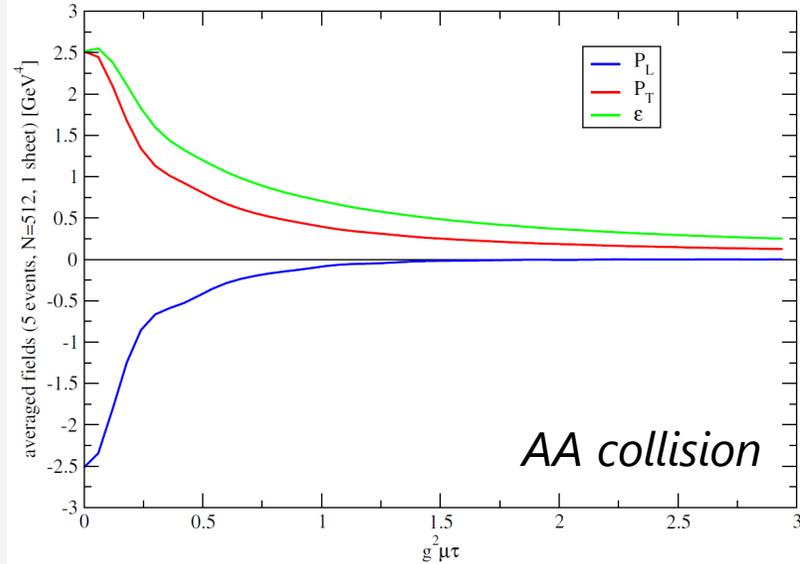
# Impact on RAA



- Initial spectrum is tilted by the combined effect of*
- *Diffusion-dominated evolution*
  - *Memory slowing down momentum broadening*



# The free streaming regime



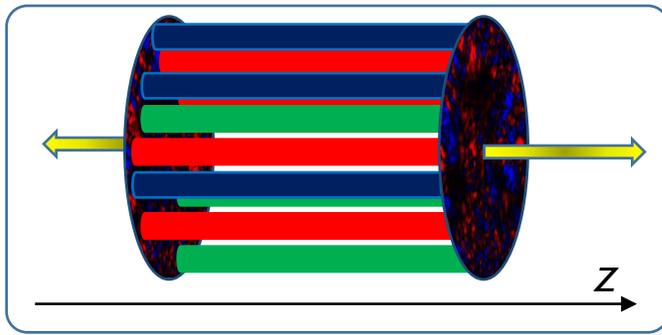
- *Fields dilute: description in terms of gluons, and relativistic kinetic theory, possible (\*)*

- *$P_L \neq P_T$ : the system is quite anisotropic. This anisotropy affects observables, e.g. those of the heavy quarks.*

*Relativistic transport can easily handle this type of initialization.*

(\*) See the talk by Gabriele Parisi

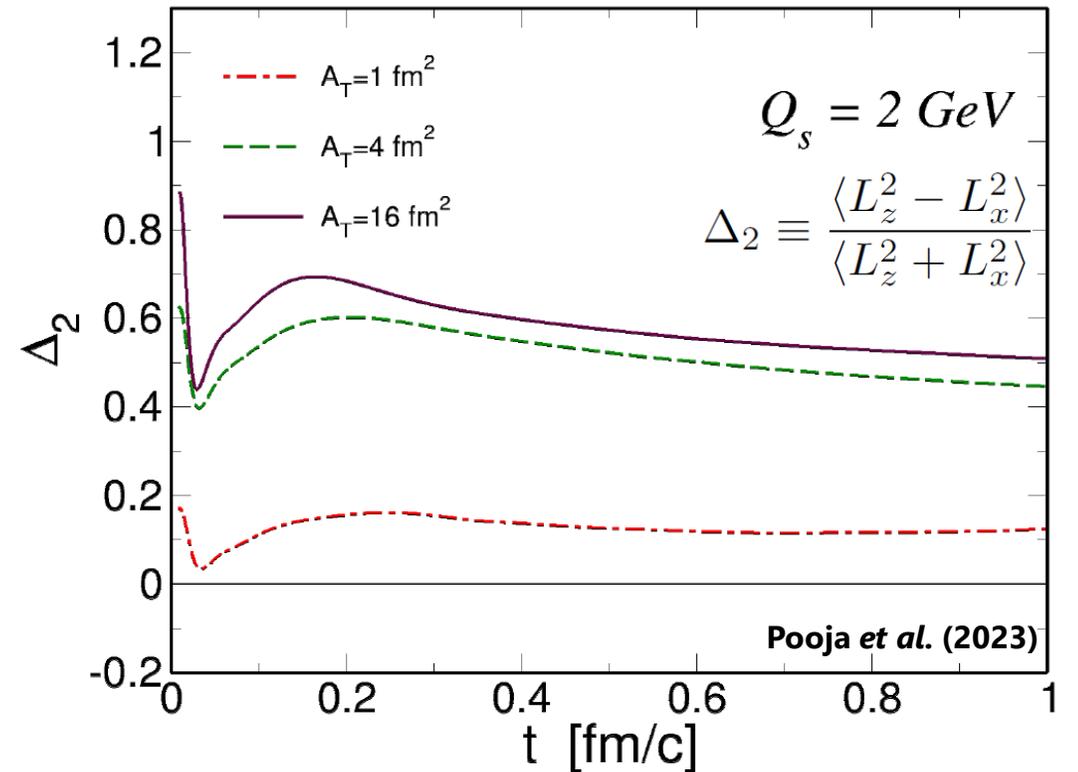
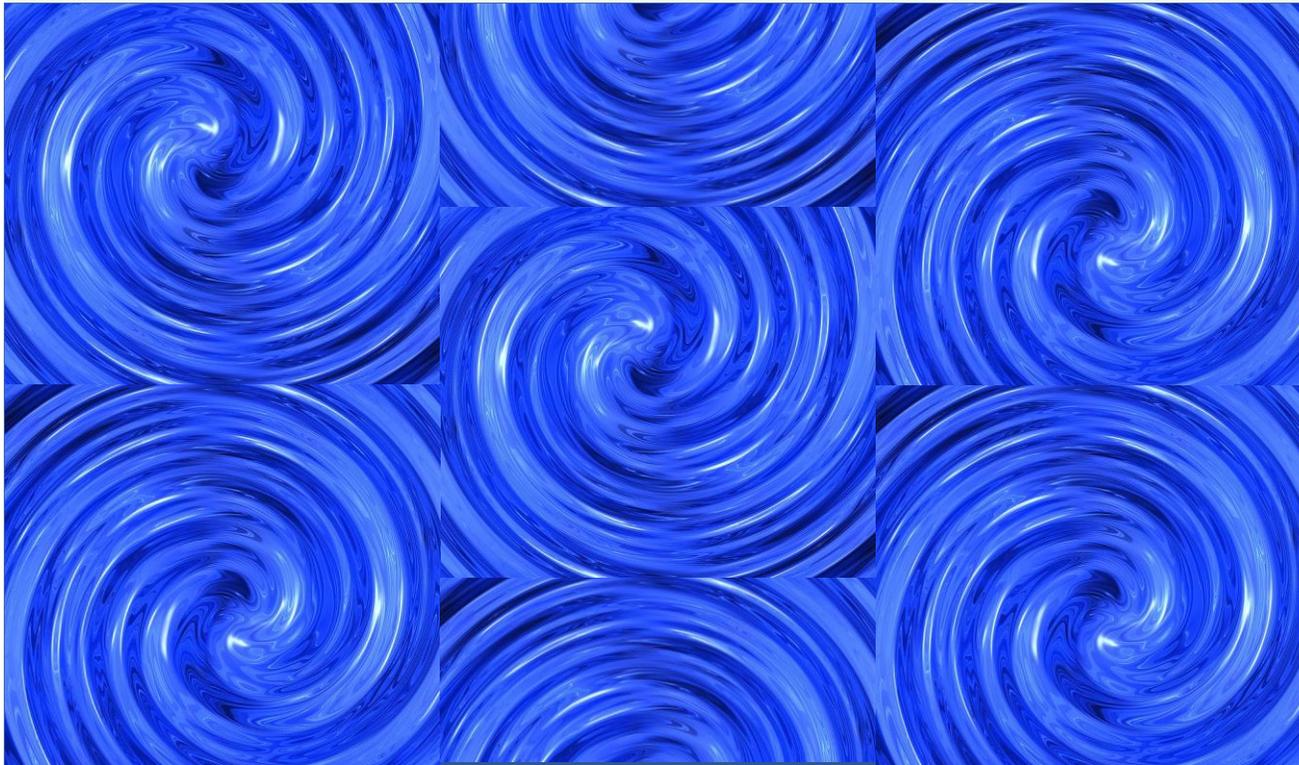
# Glasma-induced polarization of c and b



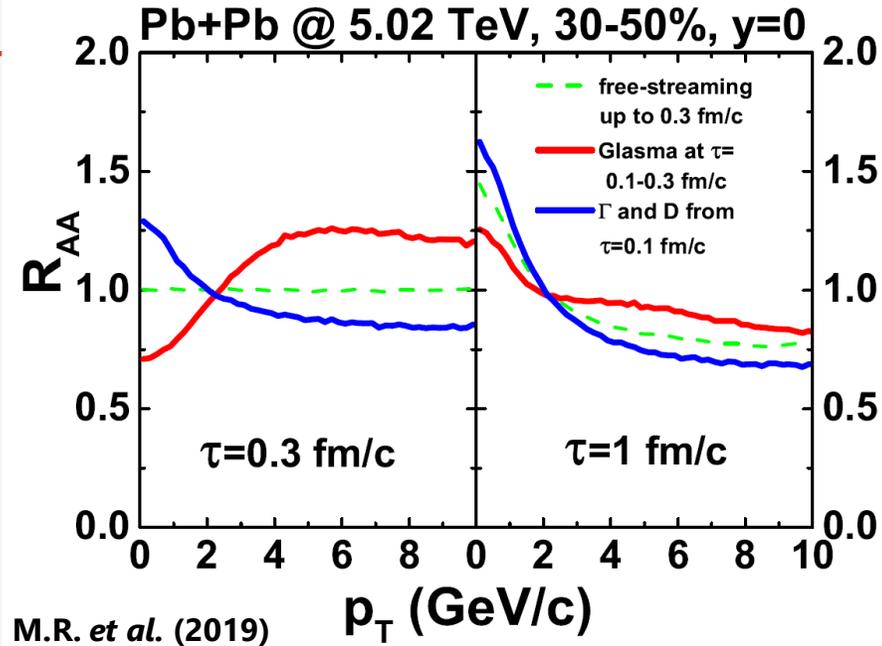
*Polarization* of c and b, along the longitudinal direction

*Anisotropic distribution of the momentum*

[Pooja et al. (2023), Ipp et al. (2020), Avramescu et al. (2023)]  
as well as of *angular momentum* [Pooja et al. (2023)]



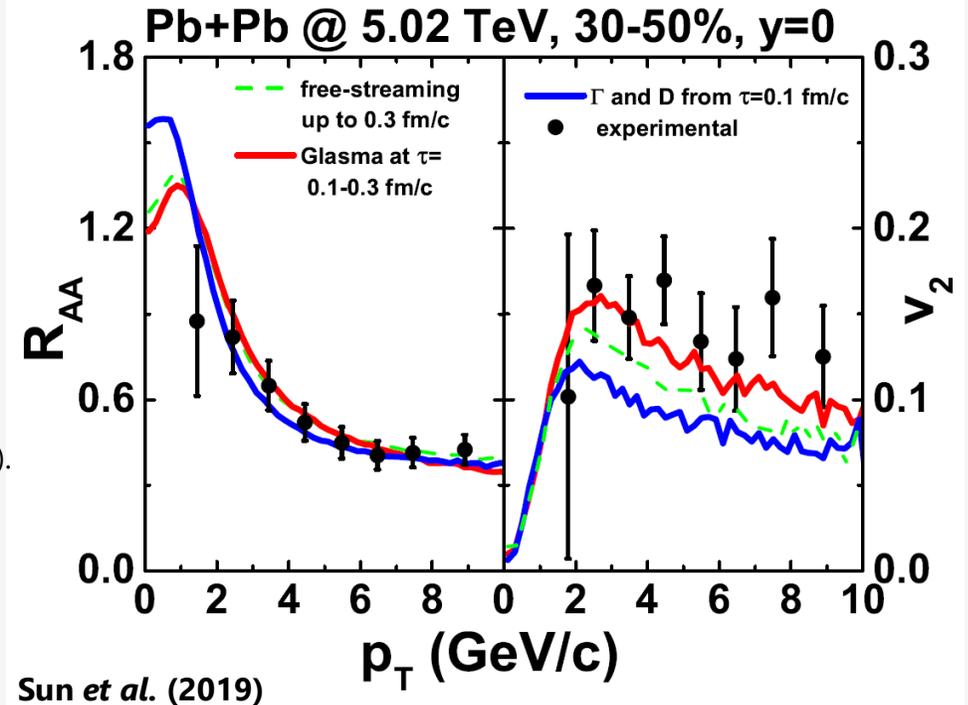
# The effect on the elliptic flow in Pb-Pb



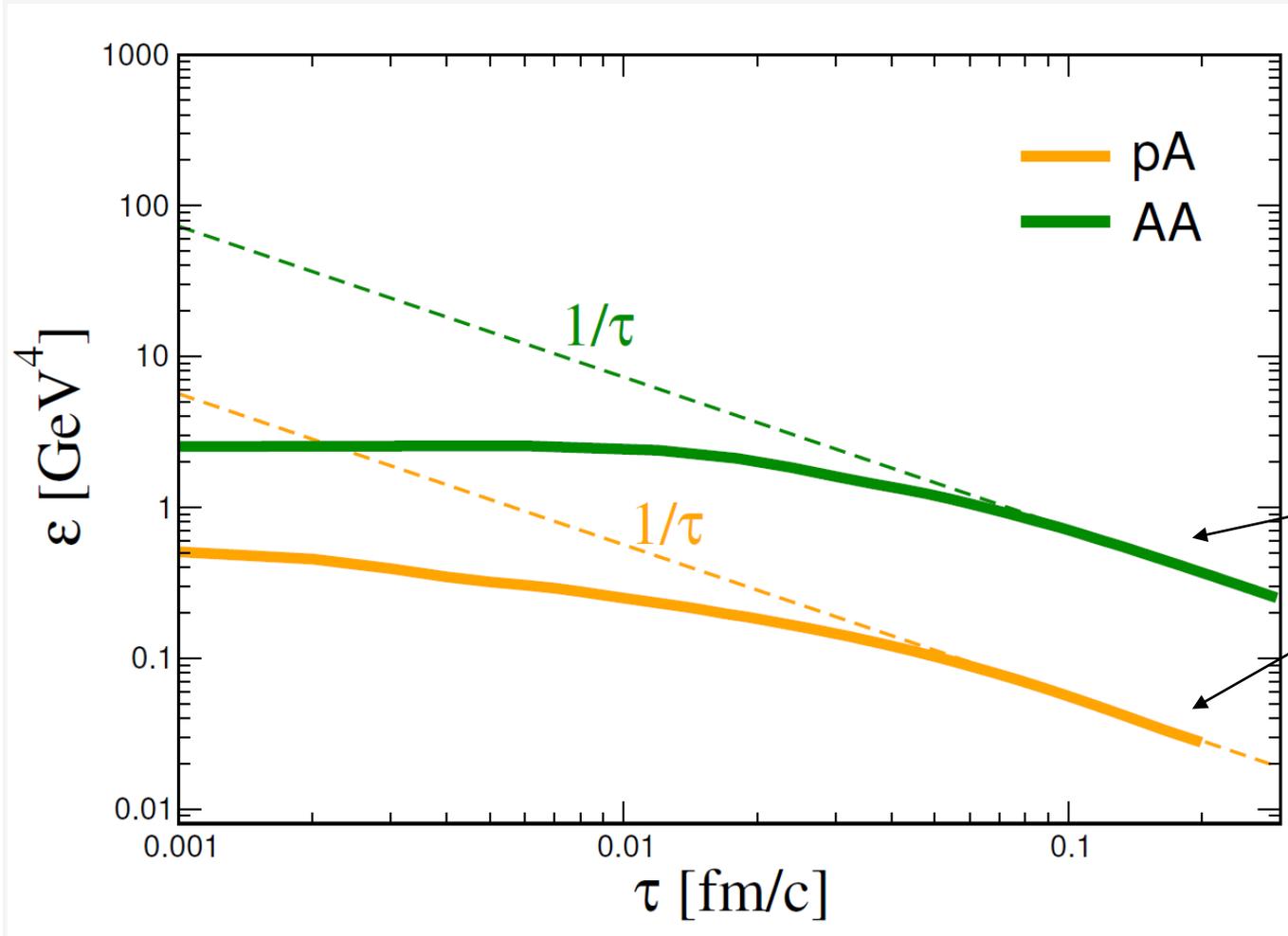
Data: ALICE, NPA 967 (2017), PRL 120 (2018).

*Diffusion in the early stage helps to describe simultaneously the RAA and the  $v_2$ .*

- Diffusion in the early stage
- Evolution in the QGP



# The free streaming regime



$$\varepsilon = \text{Tr} [E_L^2 + E_T^2 + B_L^2 + B_T^2]$$

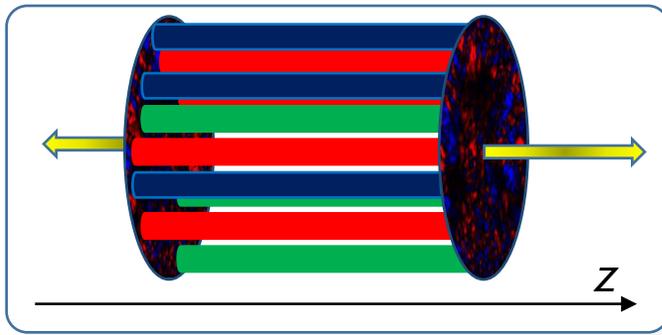
Longitudinal expansion  
with *zero pressure* (aka free streaming):

$$\frac{d\varepsilon}{\varepsilon} = -\frac{d\tau}{\tau}$$

- *Fields are diluted*: description in terms of gluons, and relativistic kinetic theory, is possible in this regime(\*)

(\*)See the talk by Gabriele Parisi

# Glasma-induced polarization of c and b



*Polarization* of c and b, along the longitudinal direction



*Naively: glasma induces vortex-like motion of c and b around color filaments in the transverse plane*

*Anisotropic distribution of the **momentum***

*[Pooja et al. (2023), Ipp et al. (2020), Avramescu et al. (2023)]  
as well as of **angular momentum** [Pooja et al. (2023)]*

