Trento Institute for Fundamental Physics and Applications

Quantum Science and Technology in Trento

Digital Quantum Computing for Collective Neutrino Oscillations

MOdeling Nuclear STructure and REactions

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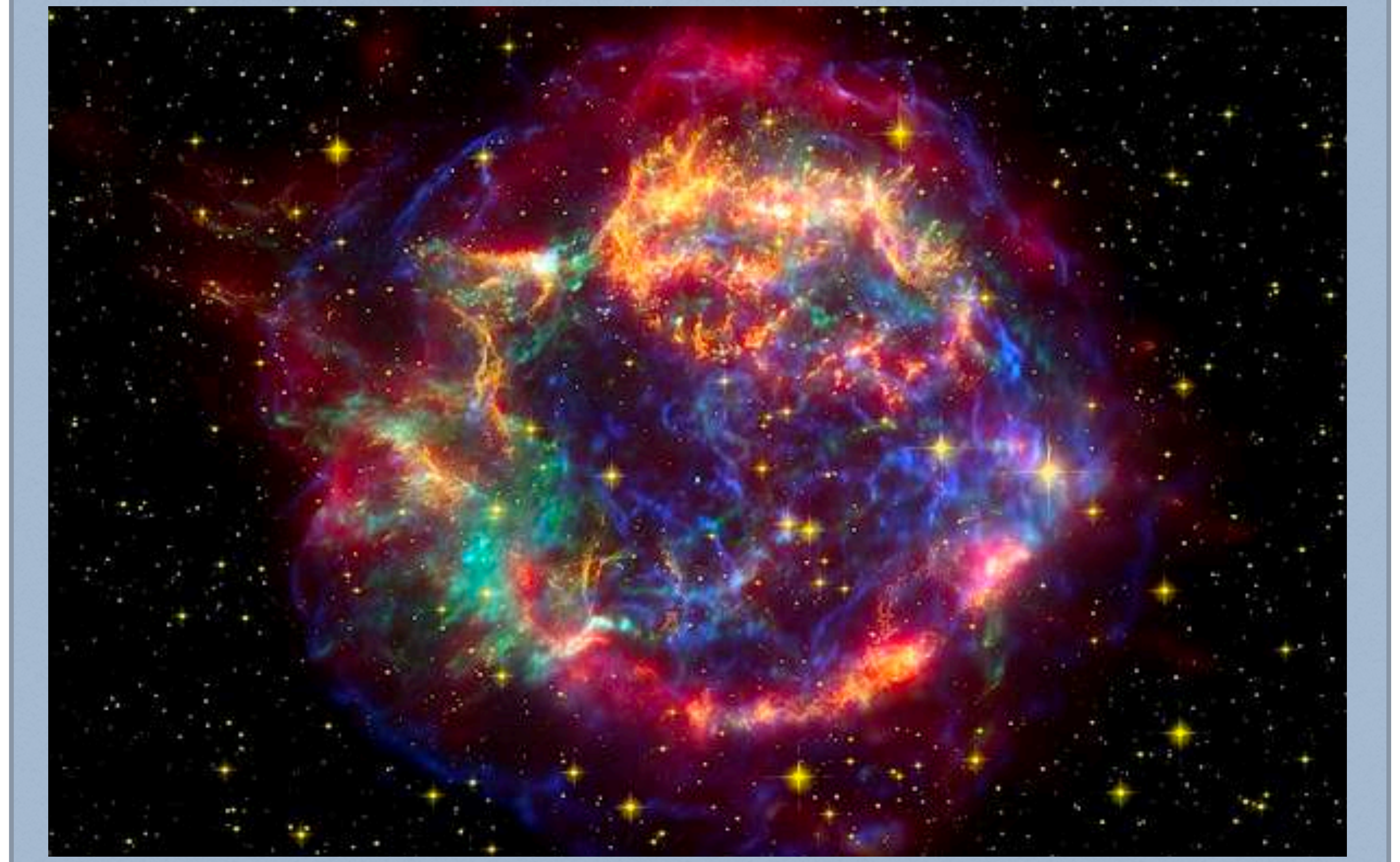
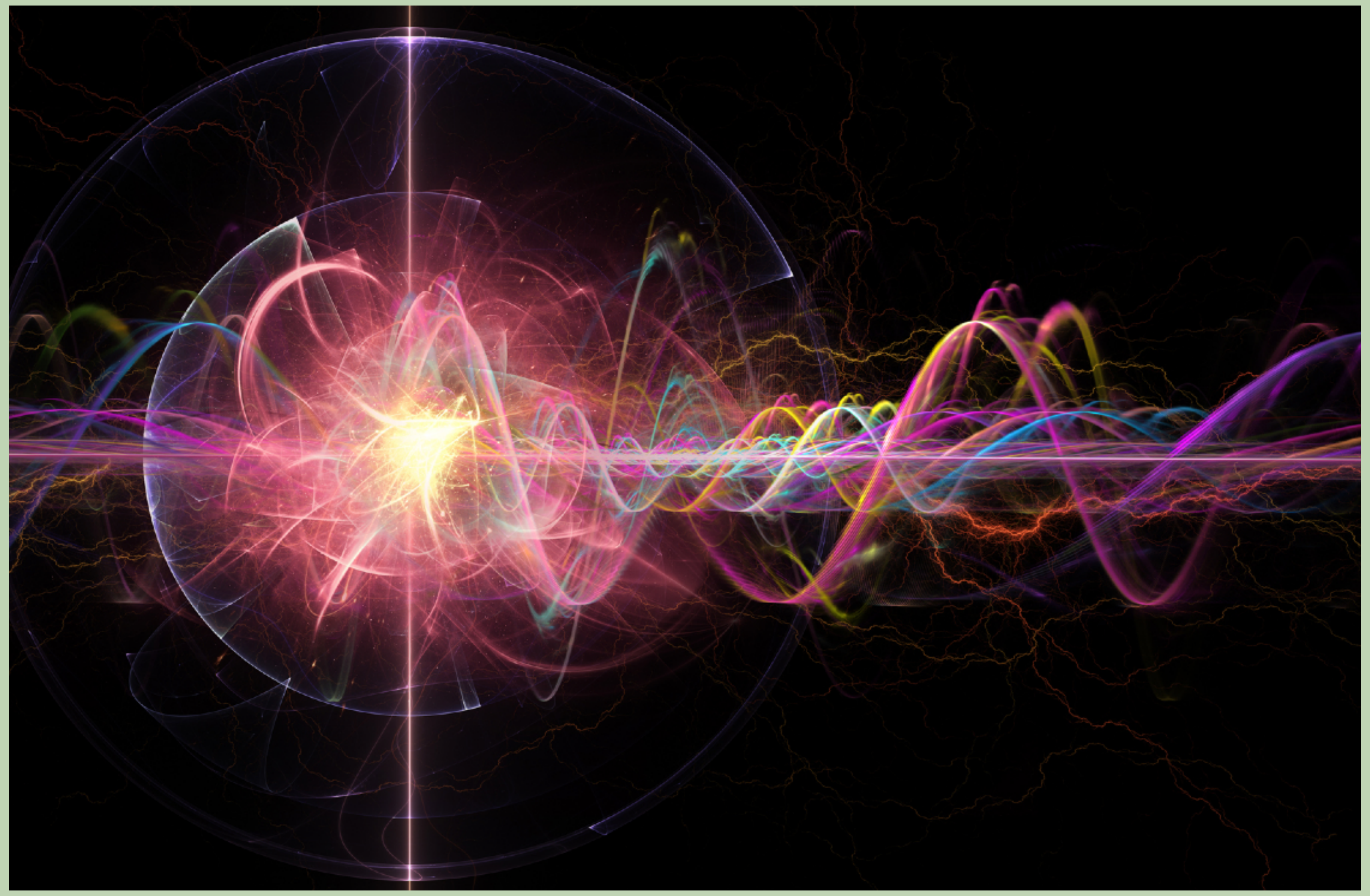
Outline

Motivation and physics



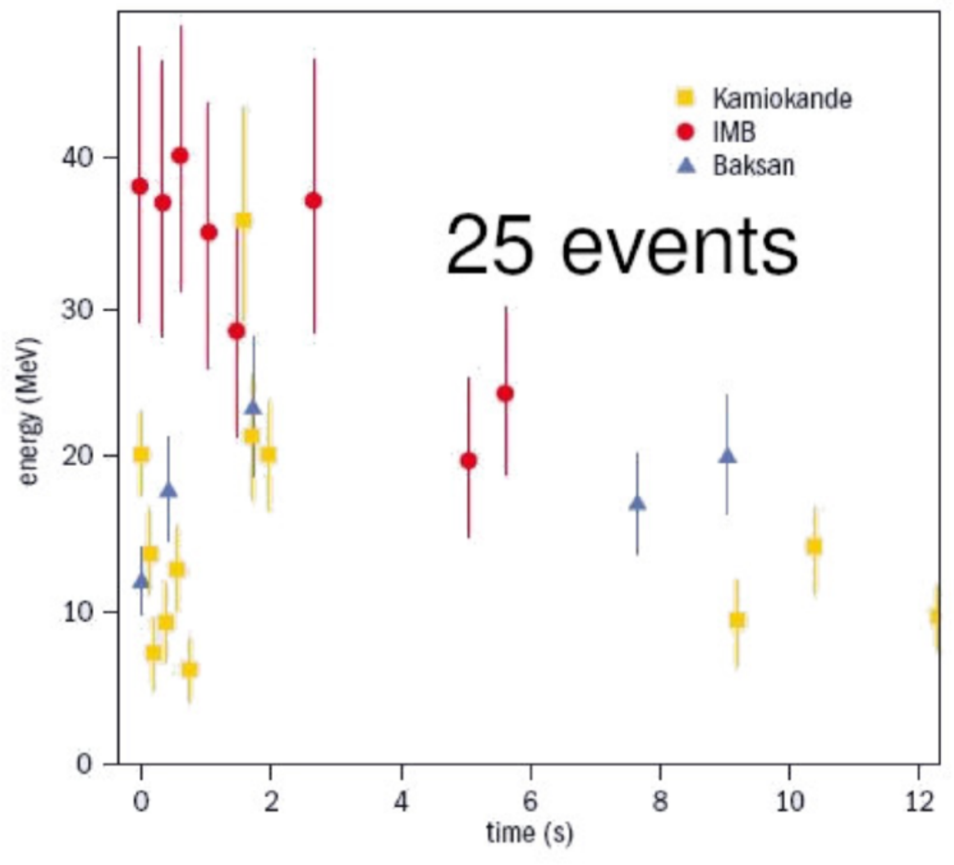
Quantum Computing Simulation

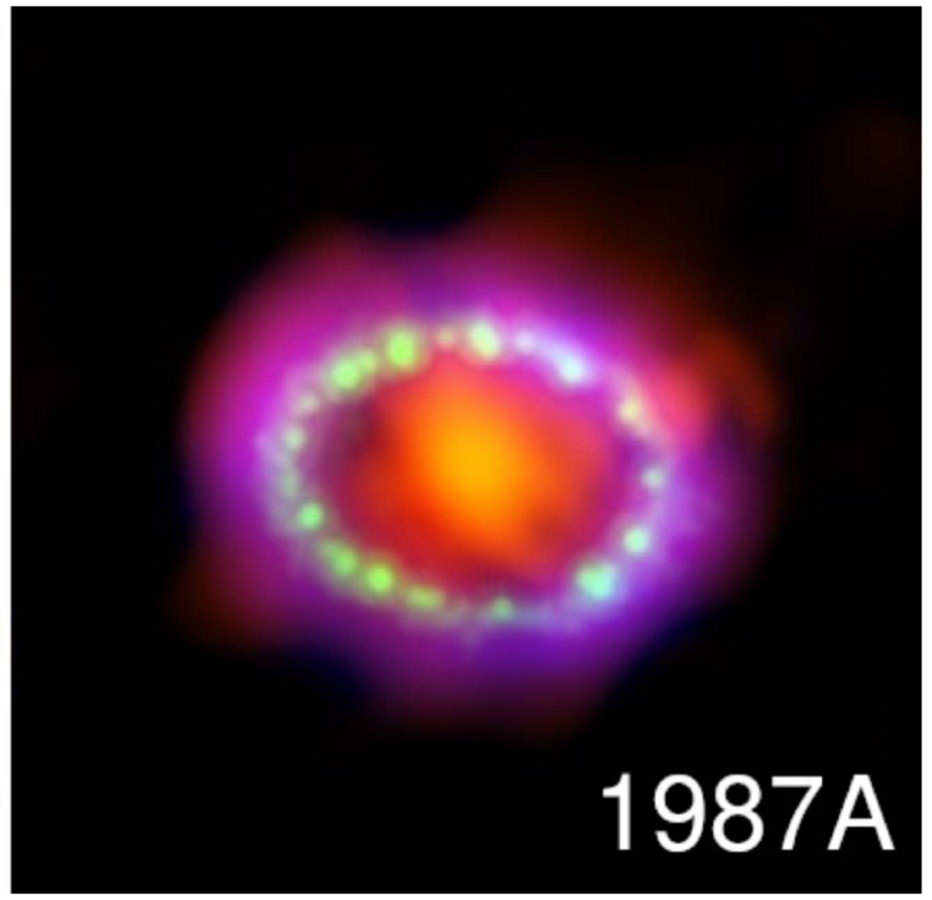
Results



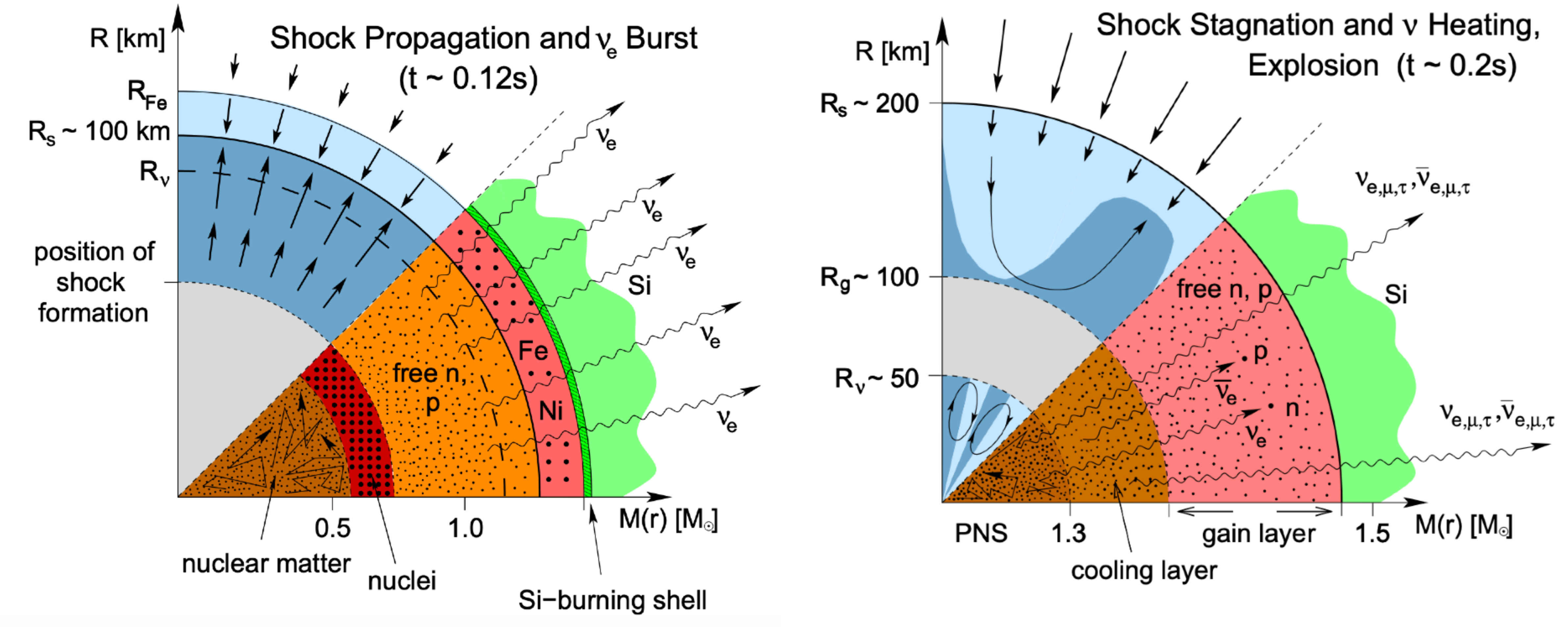
WHY WE CARE ABOUT SUPERNOVAE NEUTRINOS

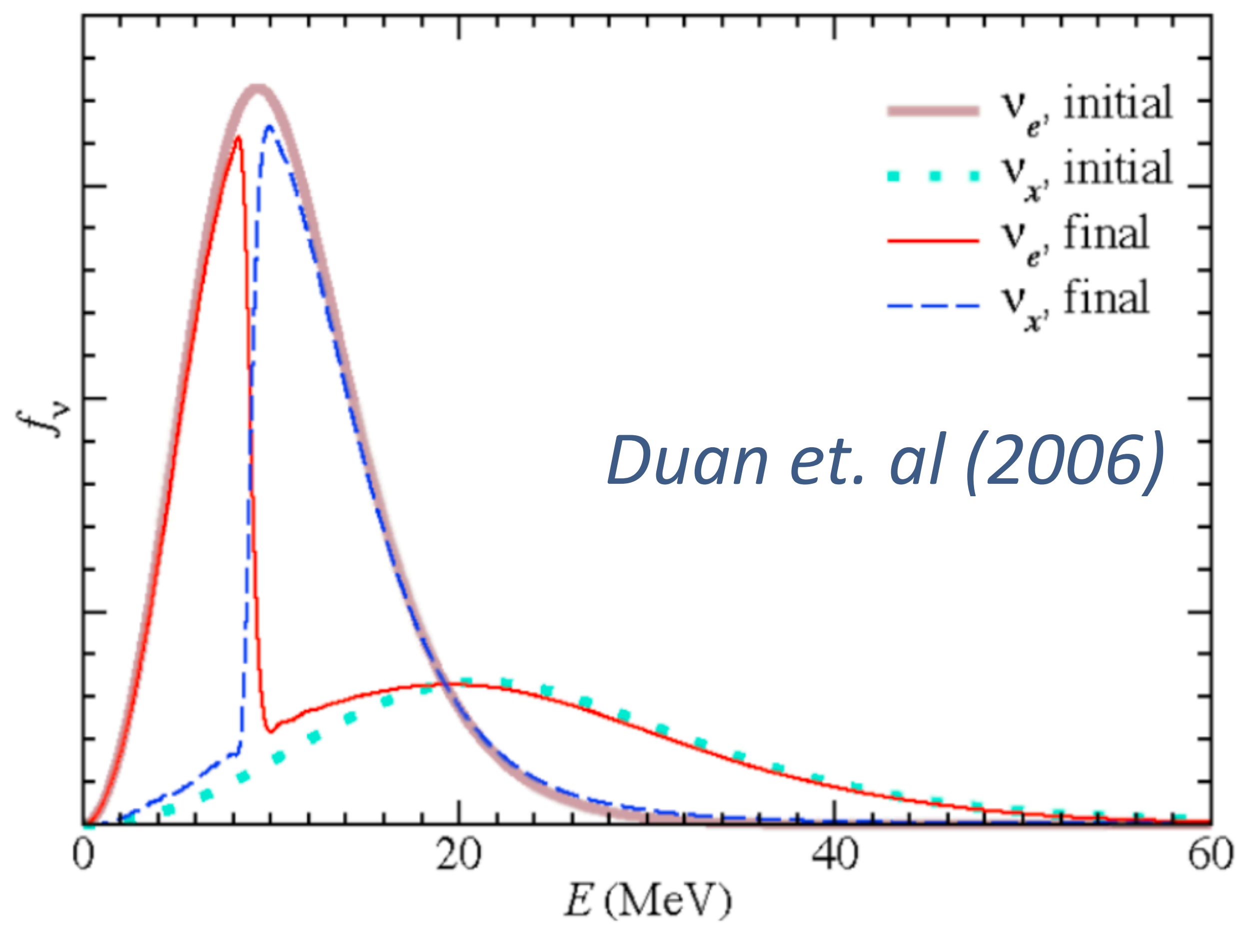
- Neutrinos are **messengers of information** of physics under extreme conditions
- Neutrinos **influence** the supernovae **explosion**
  - Efficient energy transport away from the shock region (neutrino burst)
  - Energy deposition to revive the stalled shock (explosion)
- Weak interaction process, electron capture, β decay are **flavor-dependent**
  - $p + e^- \longrightarrow n + \nu_e$
  - $p \longrightarrow n + e^+ + \nu_e$
- **Spectral splits** can happen at some distance from the emission sphere



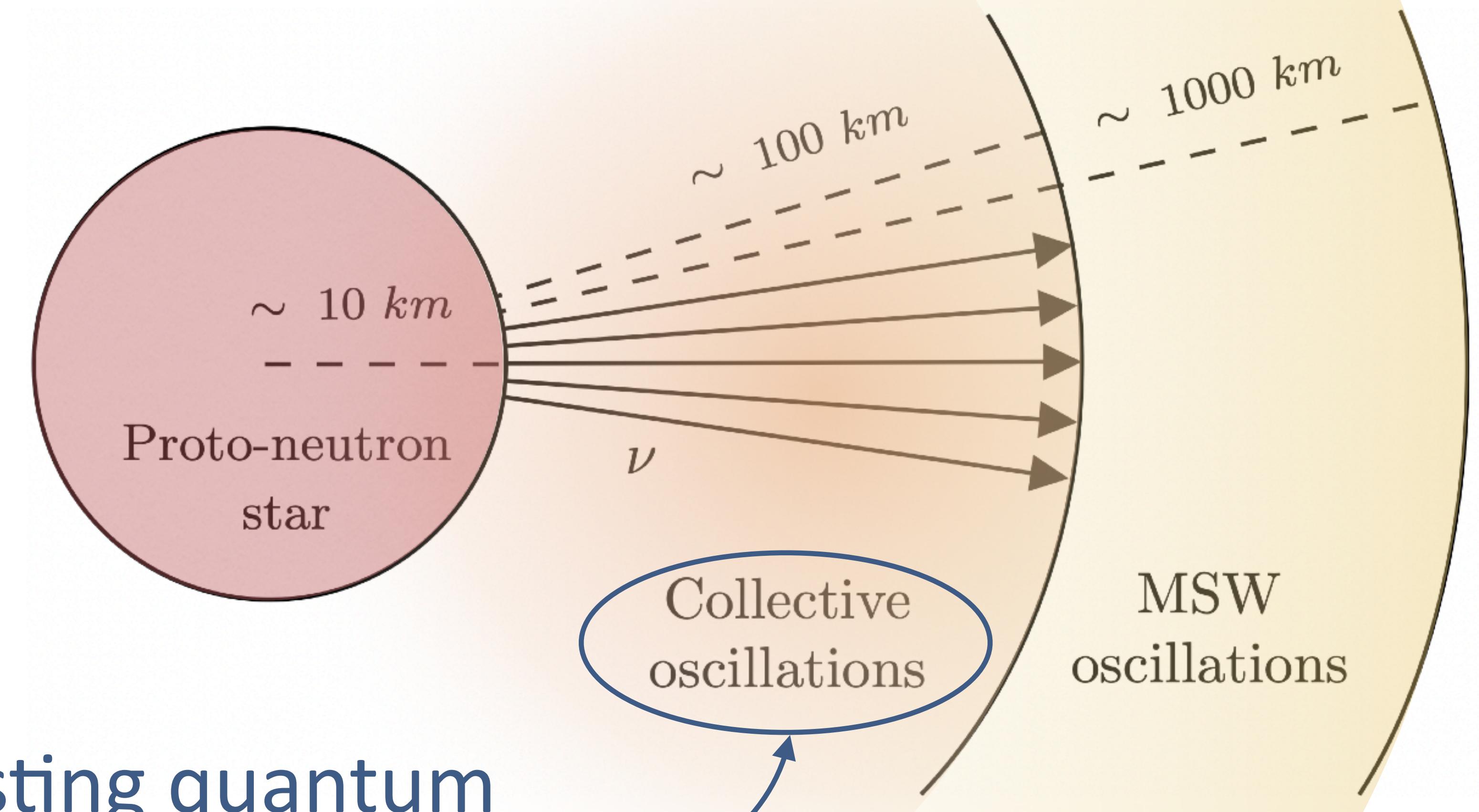


Janka et al. (2007)





Duan et. al (2006)



Interesting quantum many-body problem governed by weak interaction

- Massive stars $M \geq 8\,M_{\odot}$ explode releasing a huge amount of energy and neutrinos $\sim 10^{58}$

- Flavor Hamiltonian of many-neutrino system

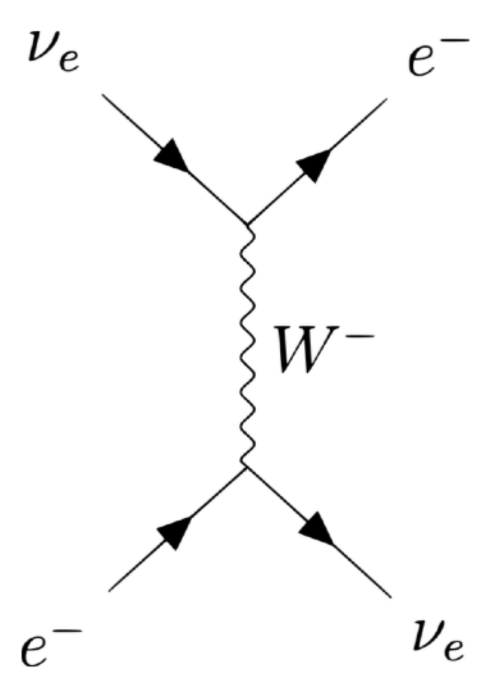
$$H = H_{vac} + H_{\nu e} + H_{\nu\nu}$$

Vacuum:

Mass eigenstates $\neq$ flavor eigenstates

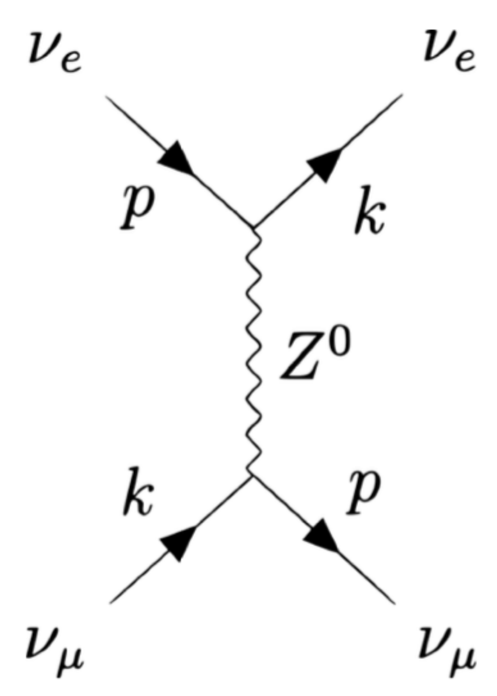
MSW:

Scattering with matter



$\nu\nu$-interaction:

Forward scattering



Two-flavor Hamiltonian (SU(2) model)

$$H = H_{vac} + H_{\nu\nu}$$

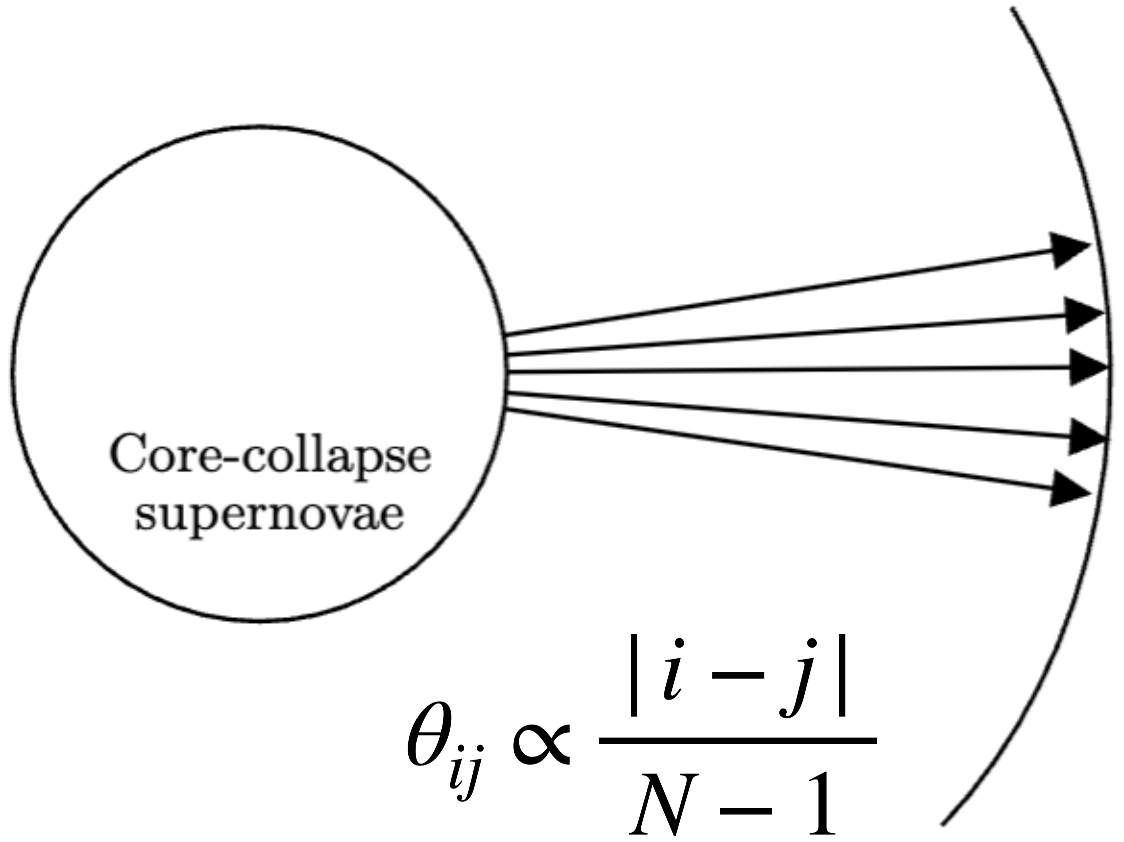
1-body term

$$H_{vac} = \Delta \sum_{i=1}^{N} \vec{b} \cdot \vec{\sigma}_i$$

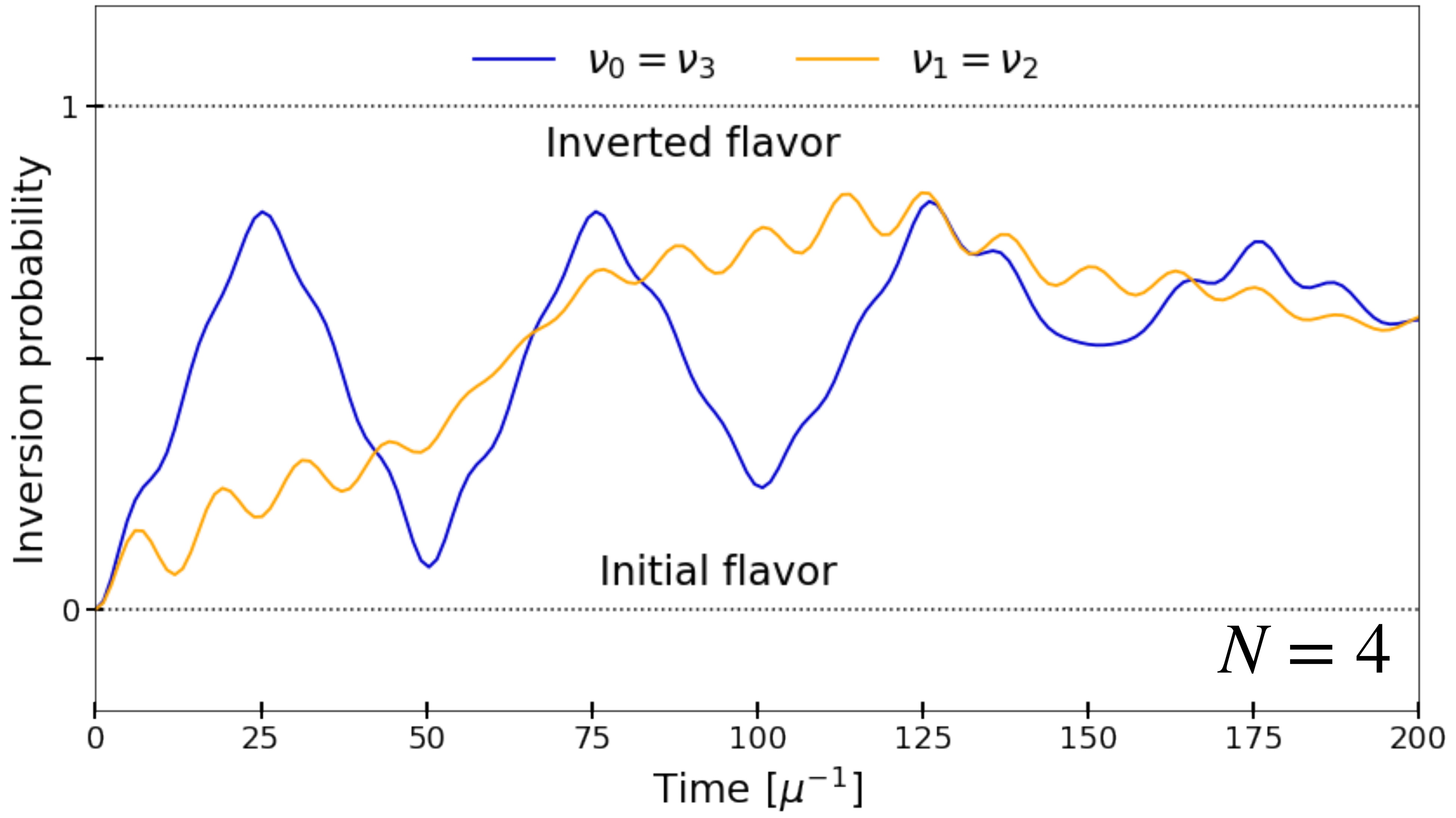
2-body term

$$H_{\nu\nu} = \frac{\mu}{N} \sum_{i<j}^{N} J_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- The flavor state of a neutrino is a **flavor isospin** $|\nu\rangle = \alpha|\nu_e\rangle + \beta|\nu_x\rangle$
- Rewrite the model using the $\mathfrak{su}(2)$ algebra



$$\theta_{ij} \propto \frac{|i-j|}{N-1}$$



- Initial state $|\psi_0\rangle = |\nu_e\rangle \otimes |\nu_e\rangle \otimes |\nu_x\rangle \otimes |\nu_x\rangle$
- Evolved state $|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle$
- $\langle \nu_e | Z | \nu_e \rangle = 1$ and $\langle \nu_x | Z | \nu_x \rangle = -1$
- Measure the probability to be in the inverted flavor as a function of time

$$P_{inv}^{(i)}(t) = \frac{|\langle Z_i(0)\rangle - \langle Z_i(t)\rangle|}{2}$$

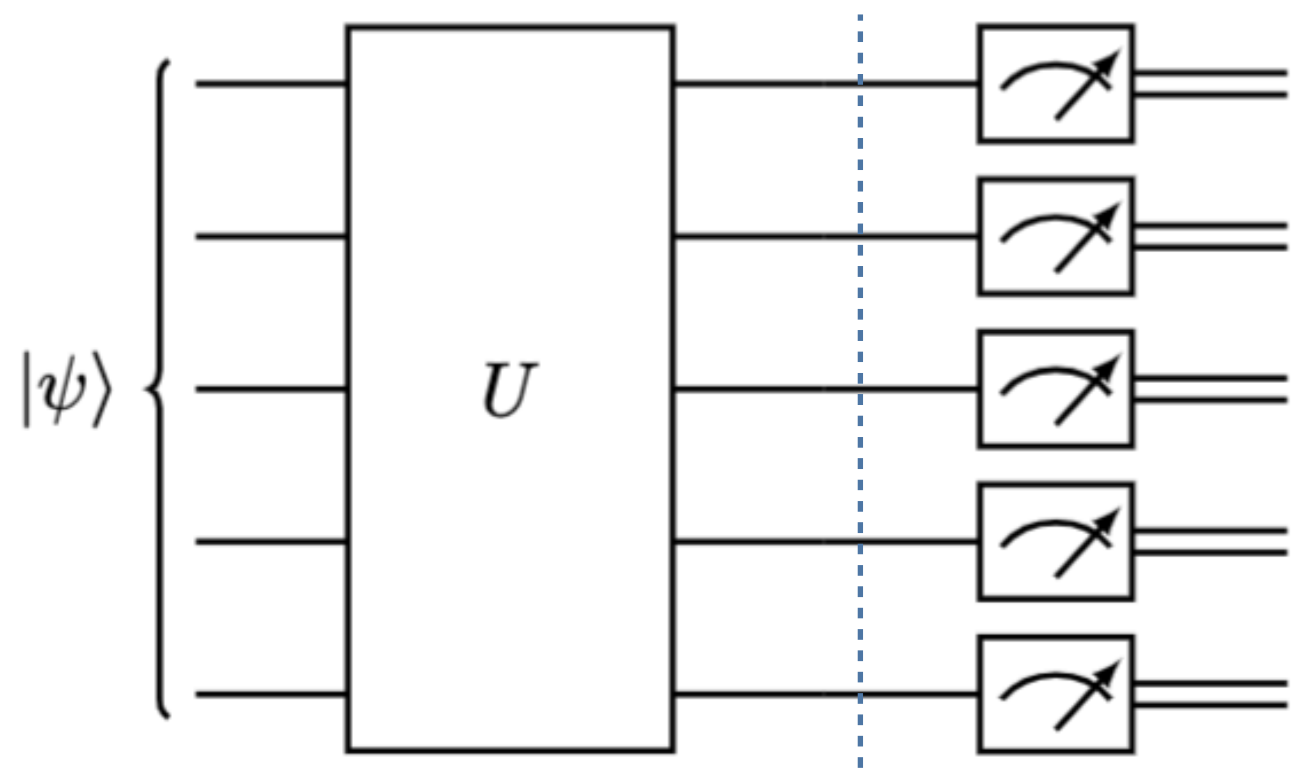
QUANTUM ALGORITHM

A quantum algorithm is a **unitary transformation** from an initial qubit state into a final one $U : |\psi\rangle \longmapsto |\psi'\rangle = U|\psi\rangle$

1. Initialization of qubit state

2. Unitary transformation

3. Measurement



The measurements allow us to extract information about final states and at the same time collapses the wave function. Classical register is represented by classical wires

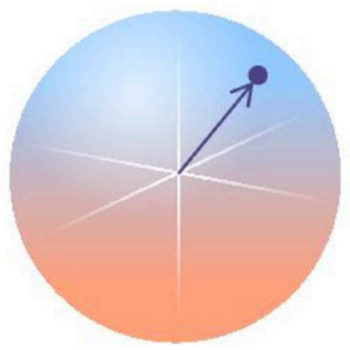
Involved qubits in the quantum register are represented by quantum wires

Final qubit state $|\psi'\rangle = U|\psi\rangle$
where $U \in SU(2^n)$ implies that
- it **preserves the norm** $\| |\psi\rangle \| = \| |\psi'\rangle \|$
- and is **reversible.**

A QC can store an exponential amount of information without an exponentially large amount of resources and can predict the time evolution of a many-body quantum system naturally taking into account all quantum features.

1° ingredient:
Encoding map

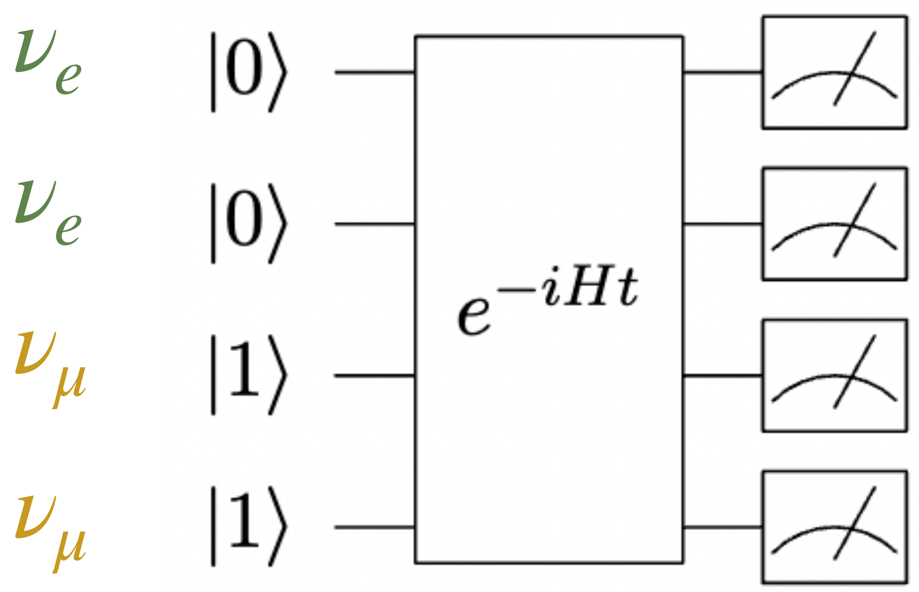
$|\nu_e\rangle \mapsto |0\rangle$



$|\nu_x\rangle \mapsto |1\rangle$

- $\mathscr{H}_{neutrinos} = \mathscr{H}_{qubits} = (\mathbb{C}^2)^{\otimes n}$
- Two-flavor approximation $|\nu\rangle = \alpha|\nu_e\rangle + \beta|\nu_x\rangle$
- Qubit state $|\nu\rangle = \alpha|0\rangle + \beta|1\rangle$
- N neutrinos encoded into N qubits

2° ingredient:
Unitary implementation

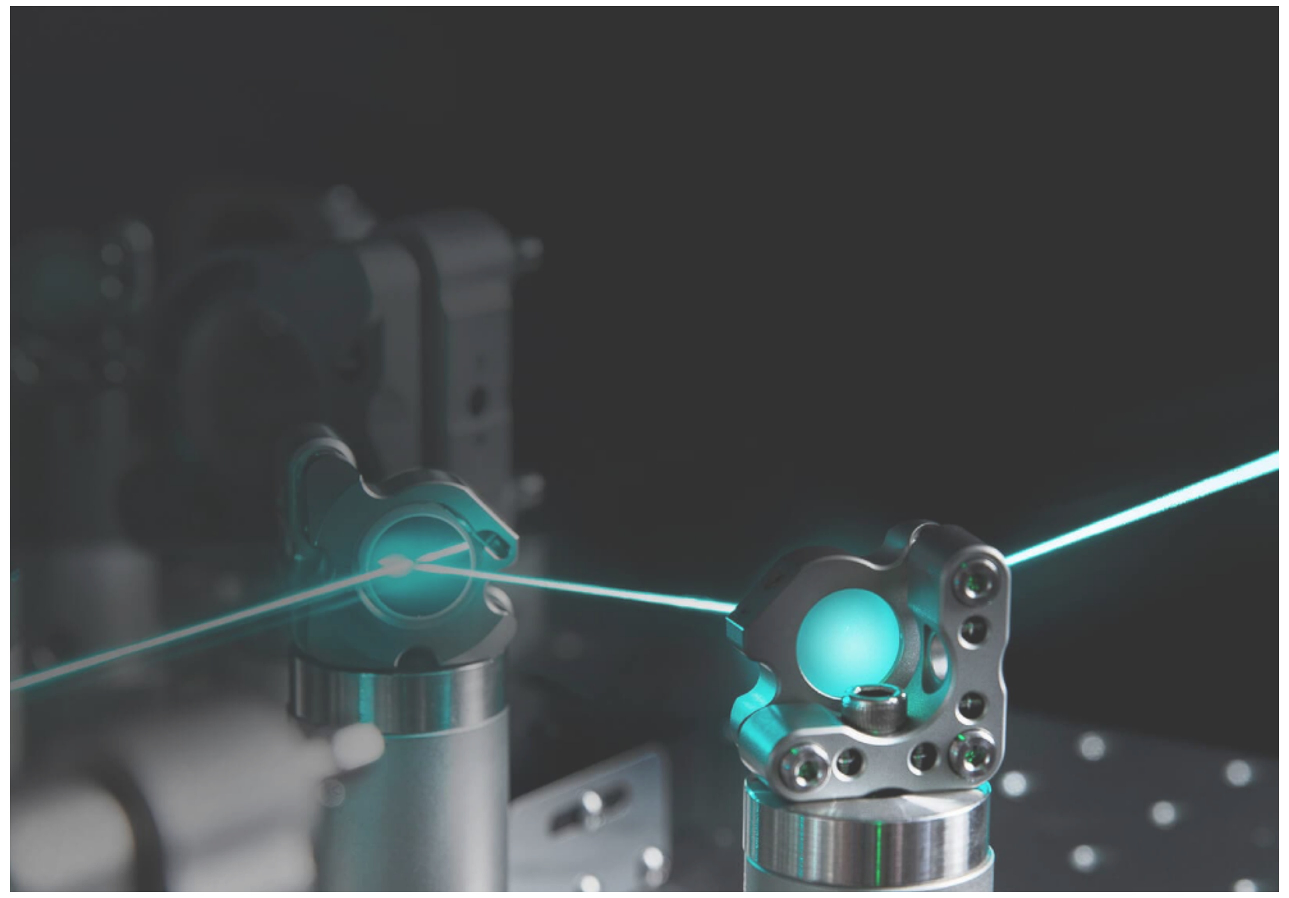


- Implement $U(t) = e^{-iHt}$ as a quantum algorithm
- **Quantum gate decomposition**
- Exponential number of operations in general…
    - we need to optimize it!

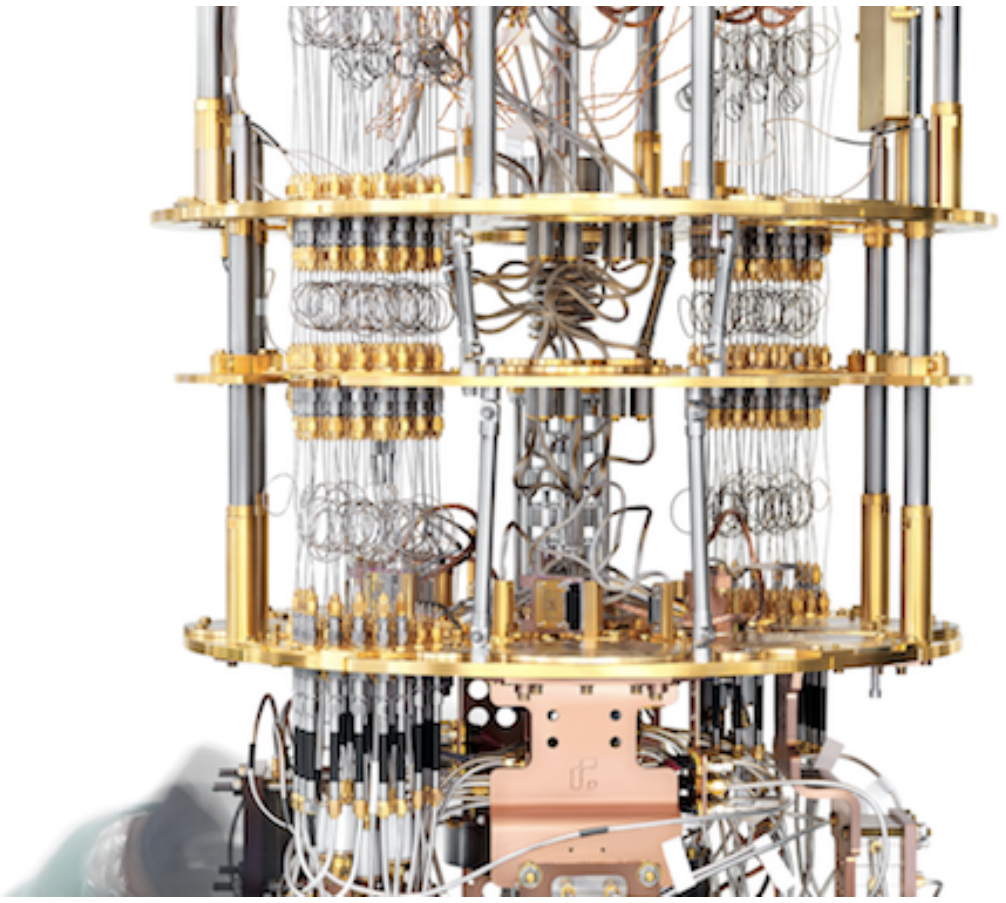
- Different **qubit**
  - Superconductive circuit
  - Trapped ions
- Different universal **gate set**
  - Circuit optimization
  - More control on what we are running
- Different qubit **connectivity**
  - Linear
  - All - to - all
  - Etc…

! **Trapped ions are perfect for the collective neutrino problem**

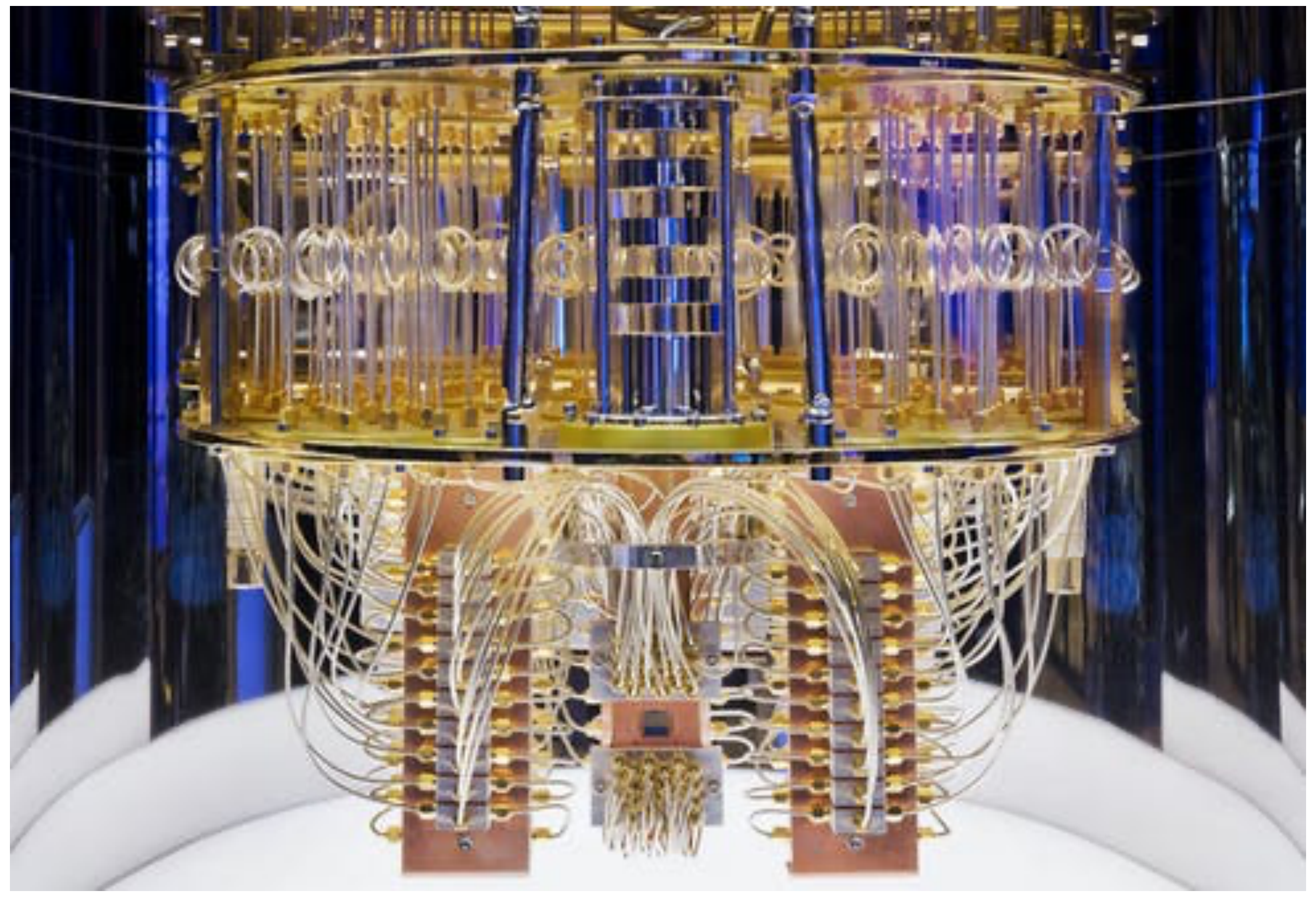
$$H_{\nu\nu} = \frac{\mu}{N}\sum_{i<j}^{N} J_{ij}\vec{\sigma}_i \cdot \vec{\sigma}_j$$



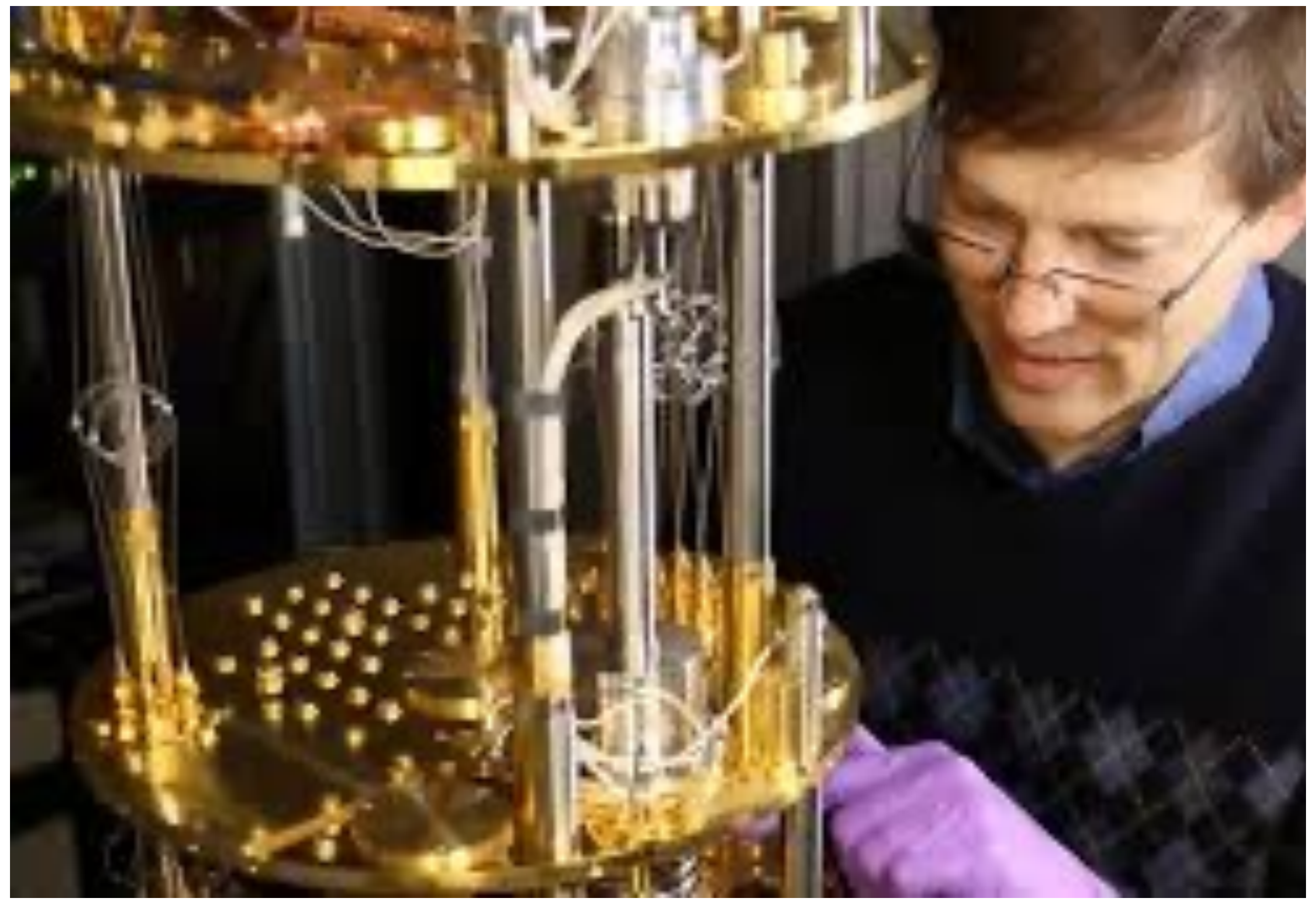
Honeywell Quantum



Rigetti Quantum

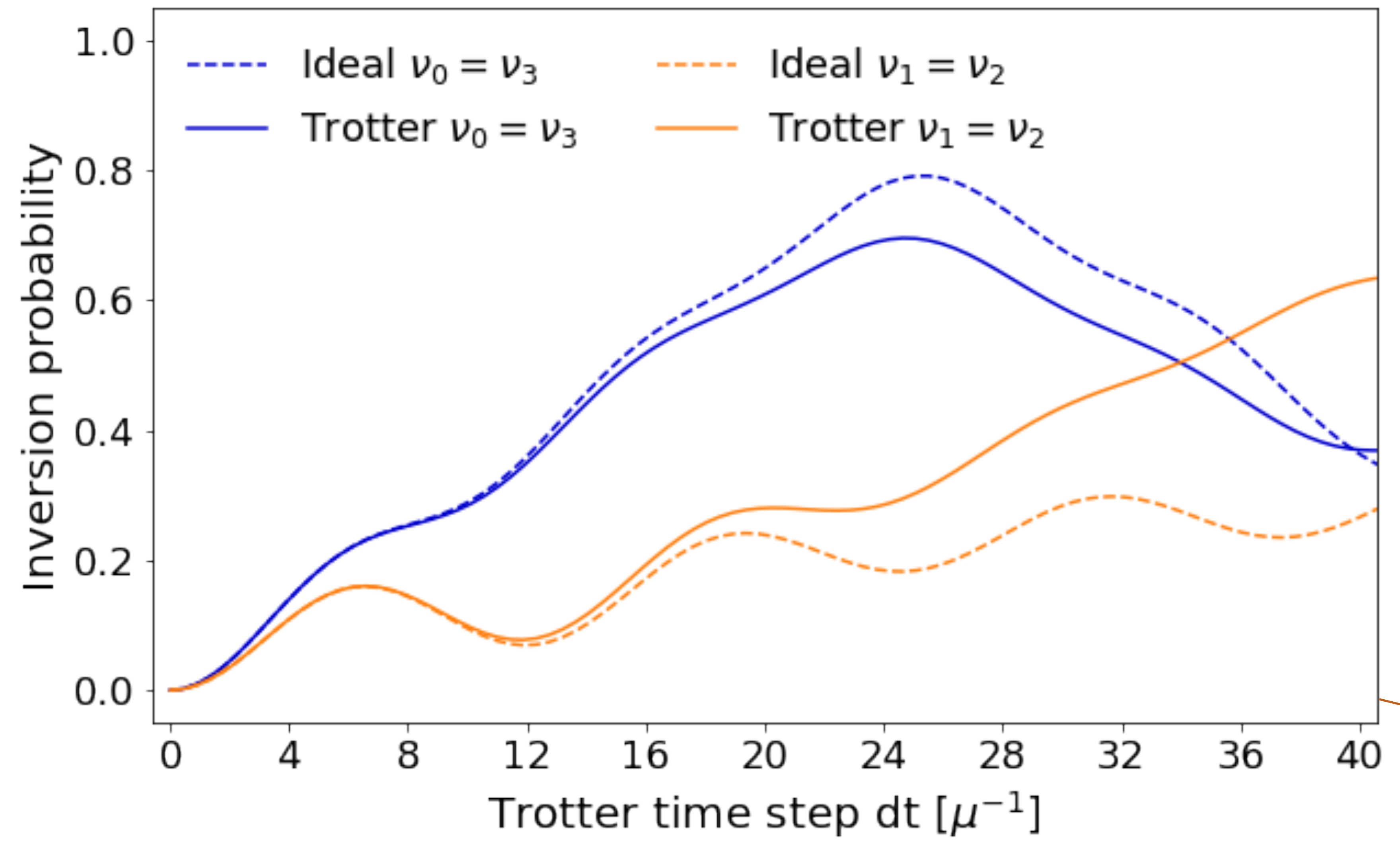


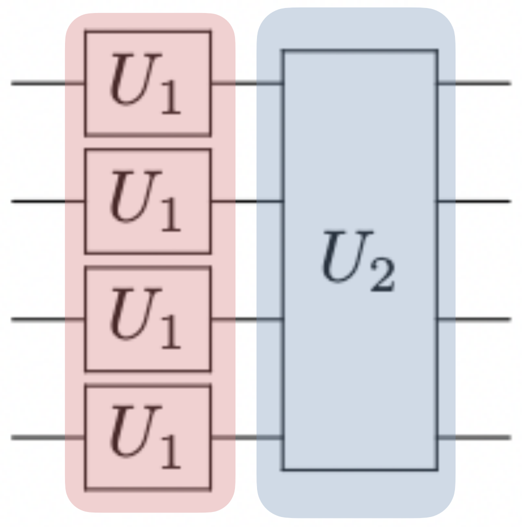
IBM Quantum



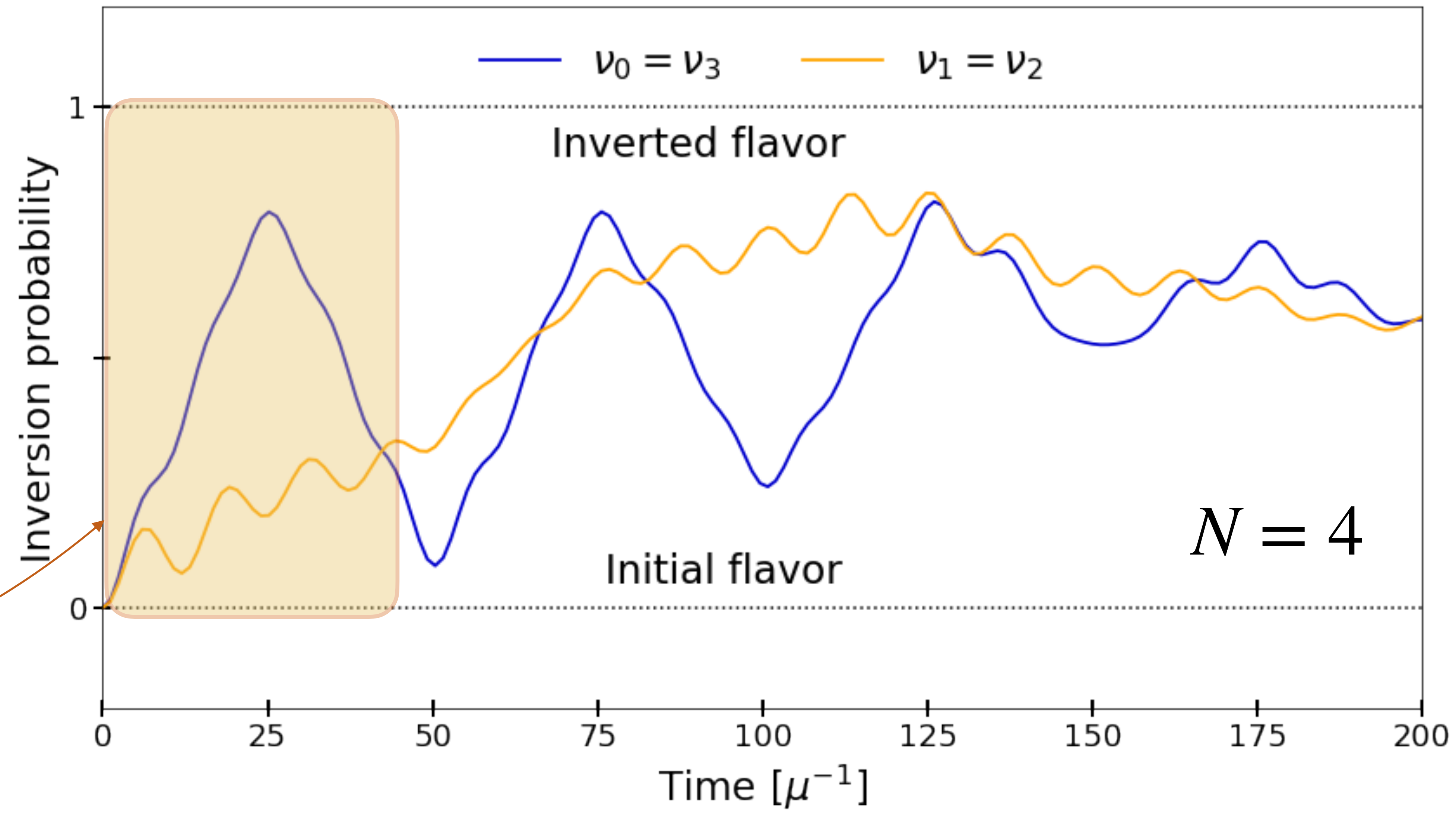
LLNL testbed

RESULTS: SINGLE TROTTER STEP

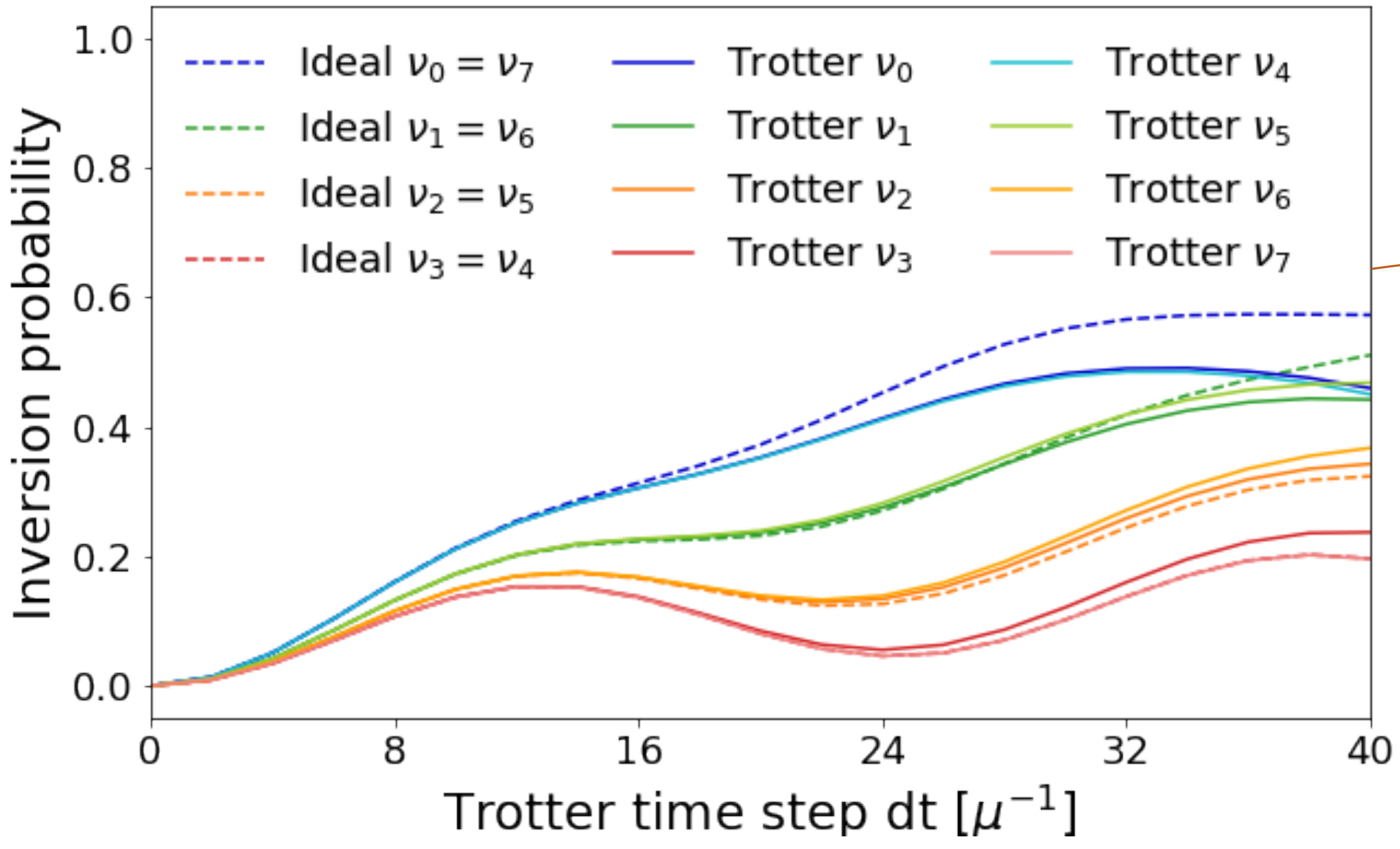


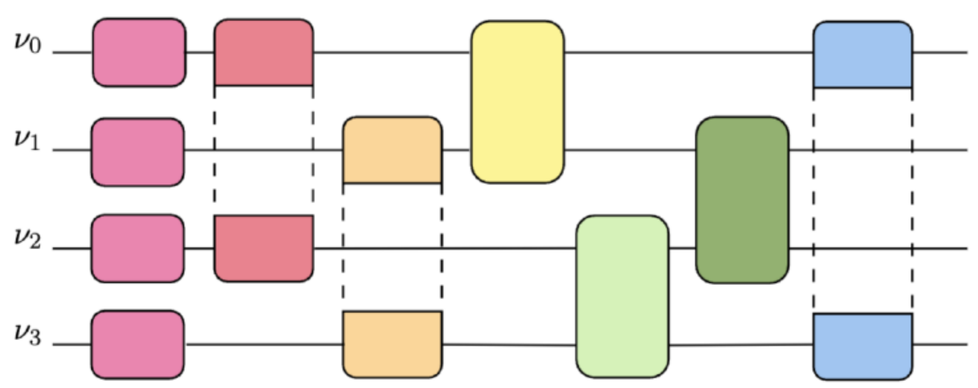


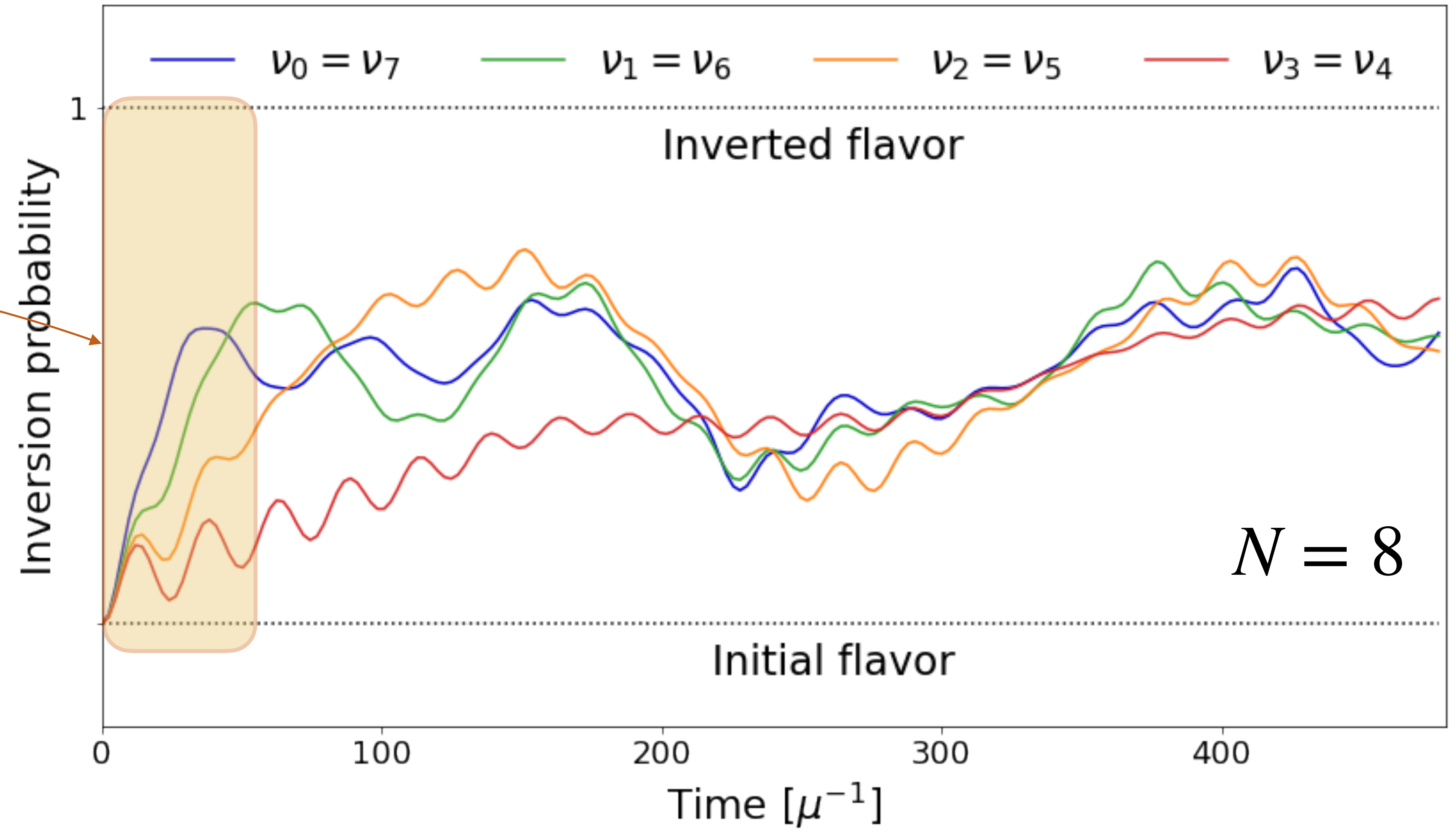
$\varepsilon \sim \mathcal{O}(dt^2)$



$$U_2(t) \approx \prod_{i<j}^{N} e^{-J_{ij}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j dt}$$







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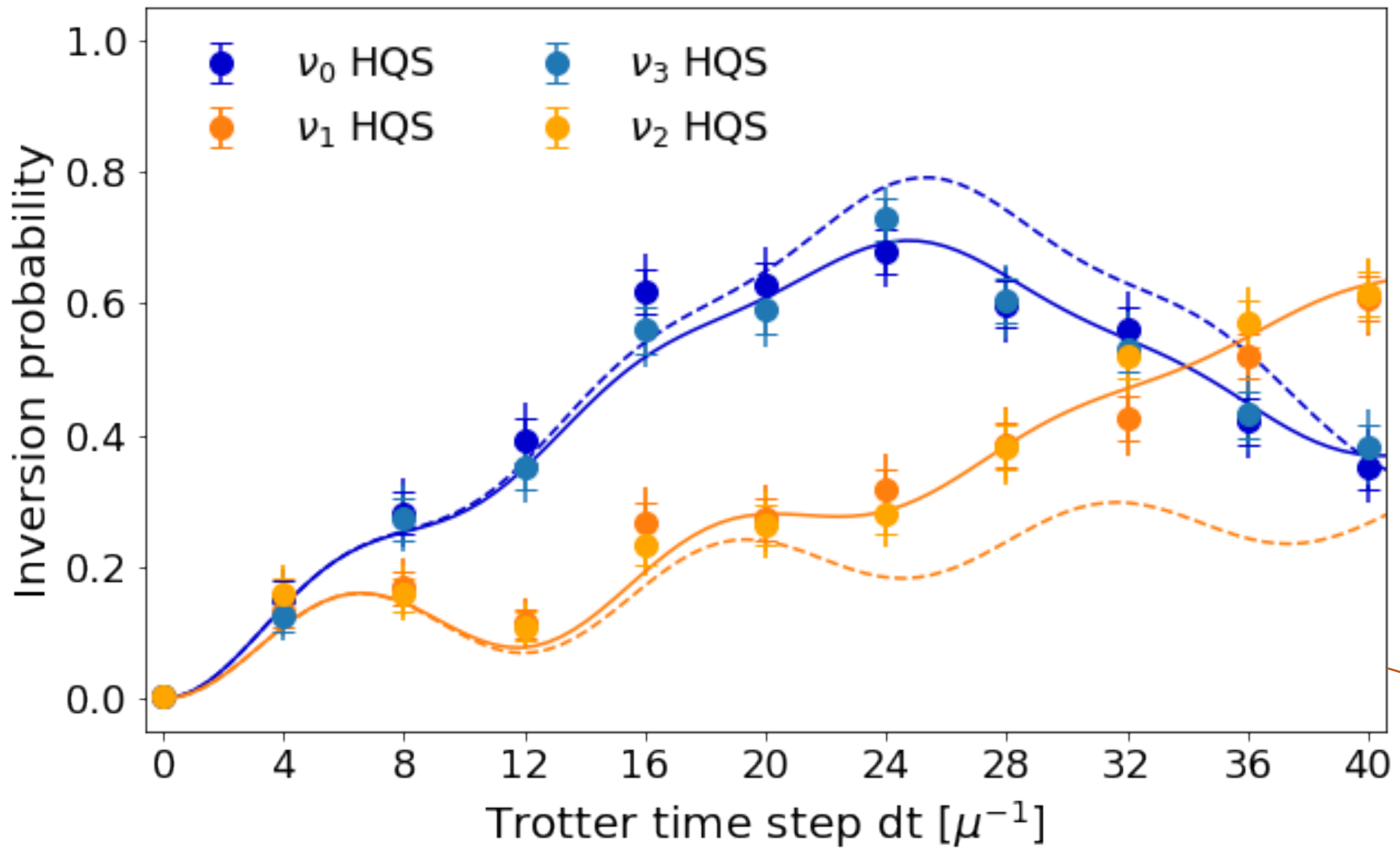

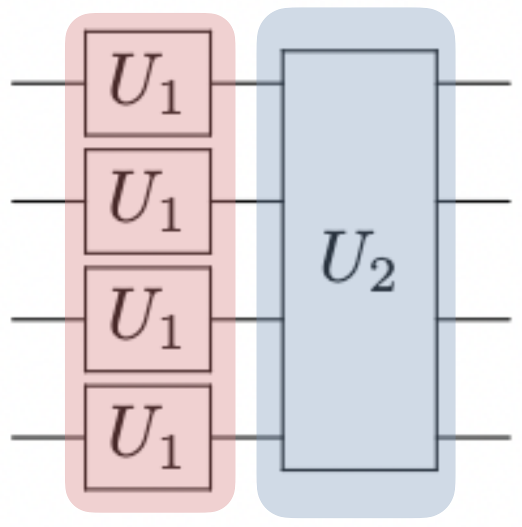

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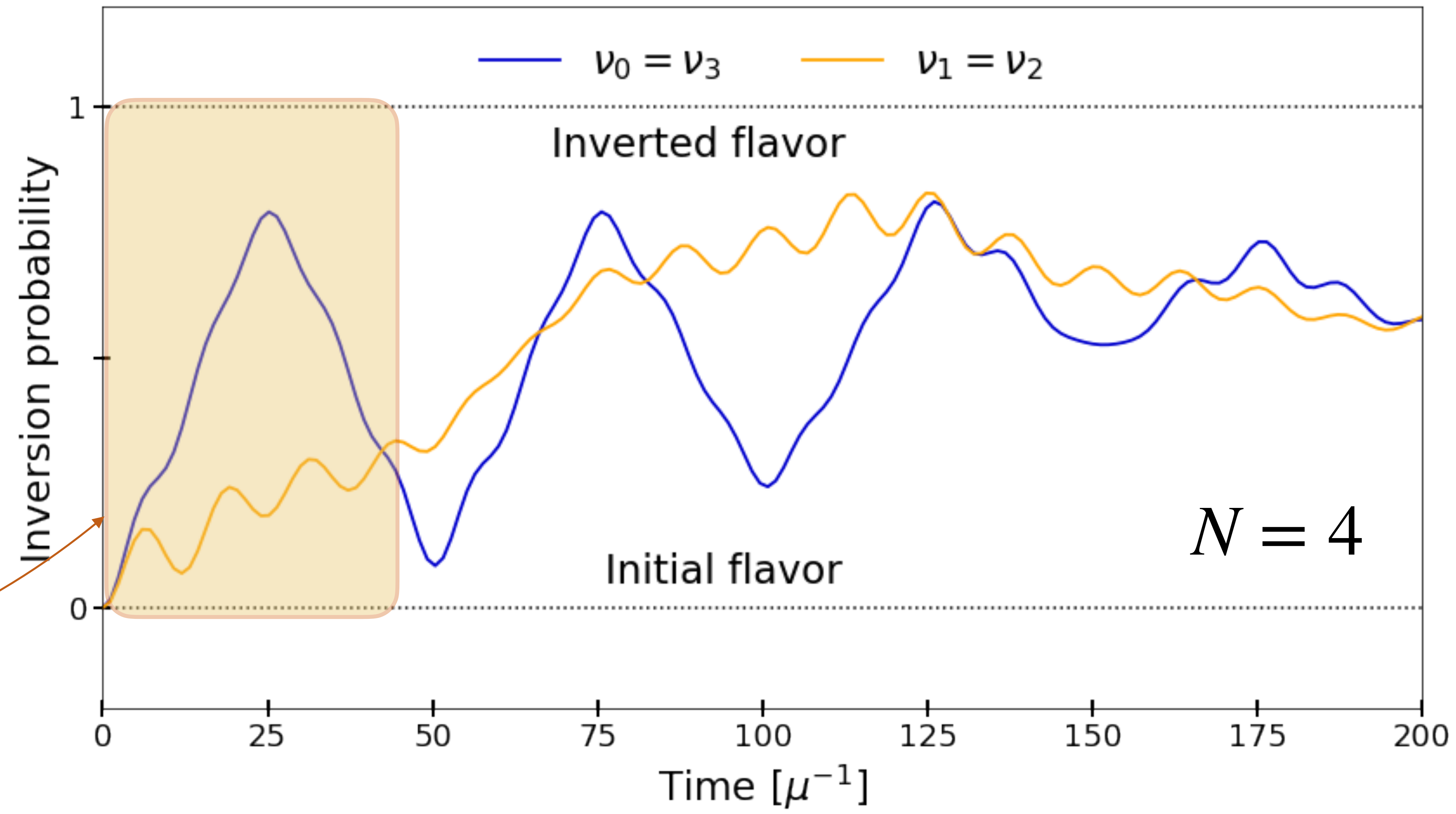
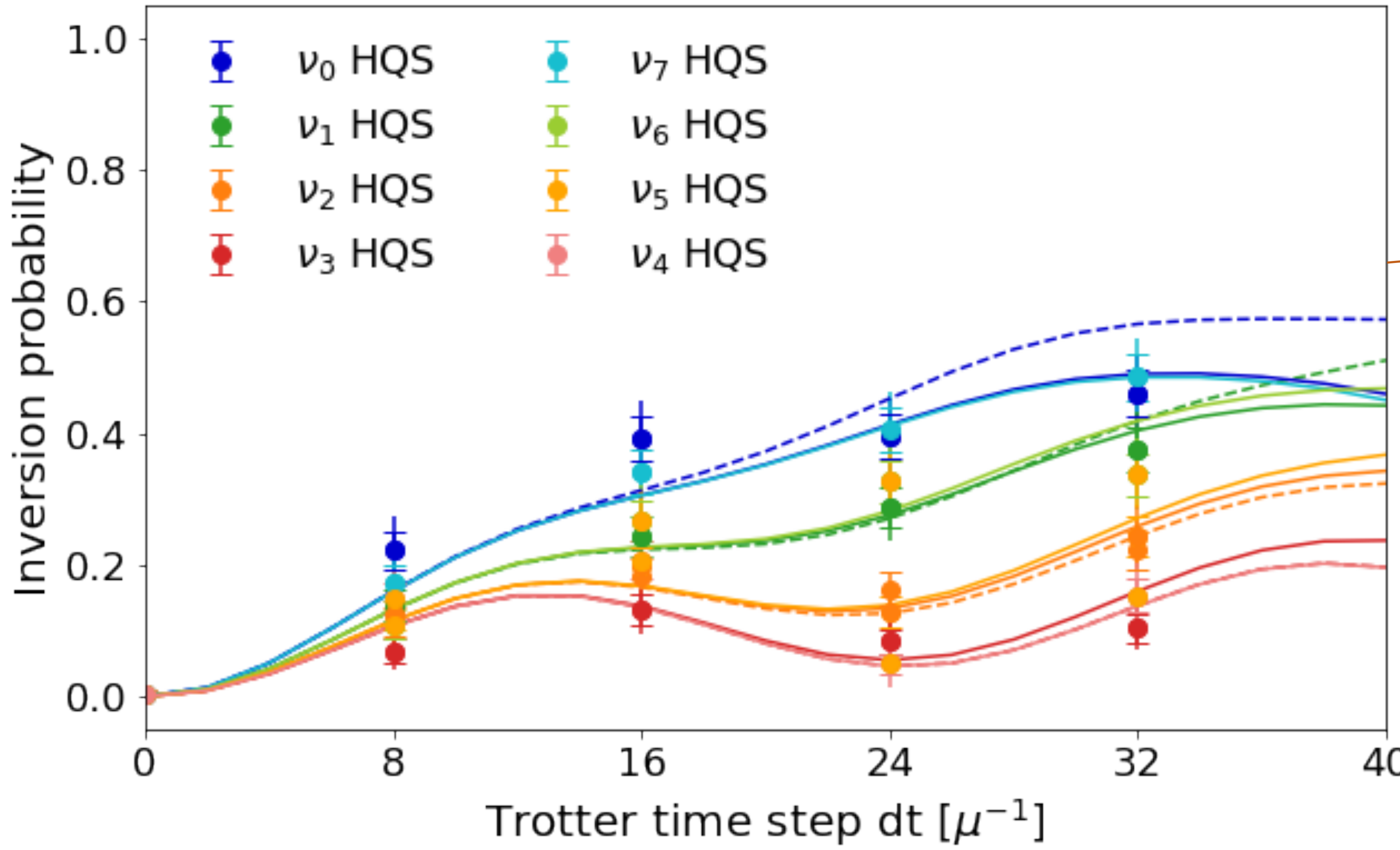
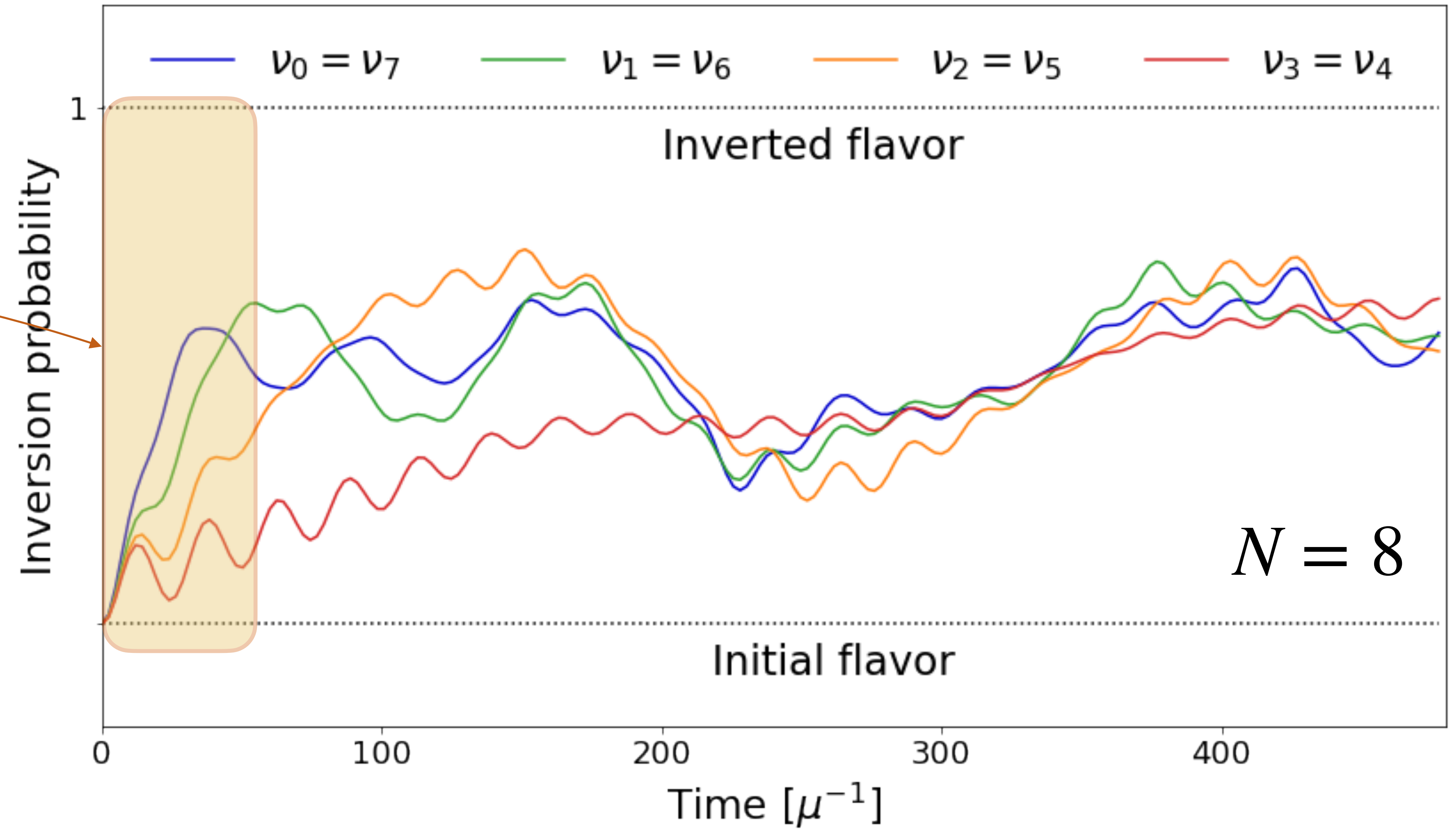


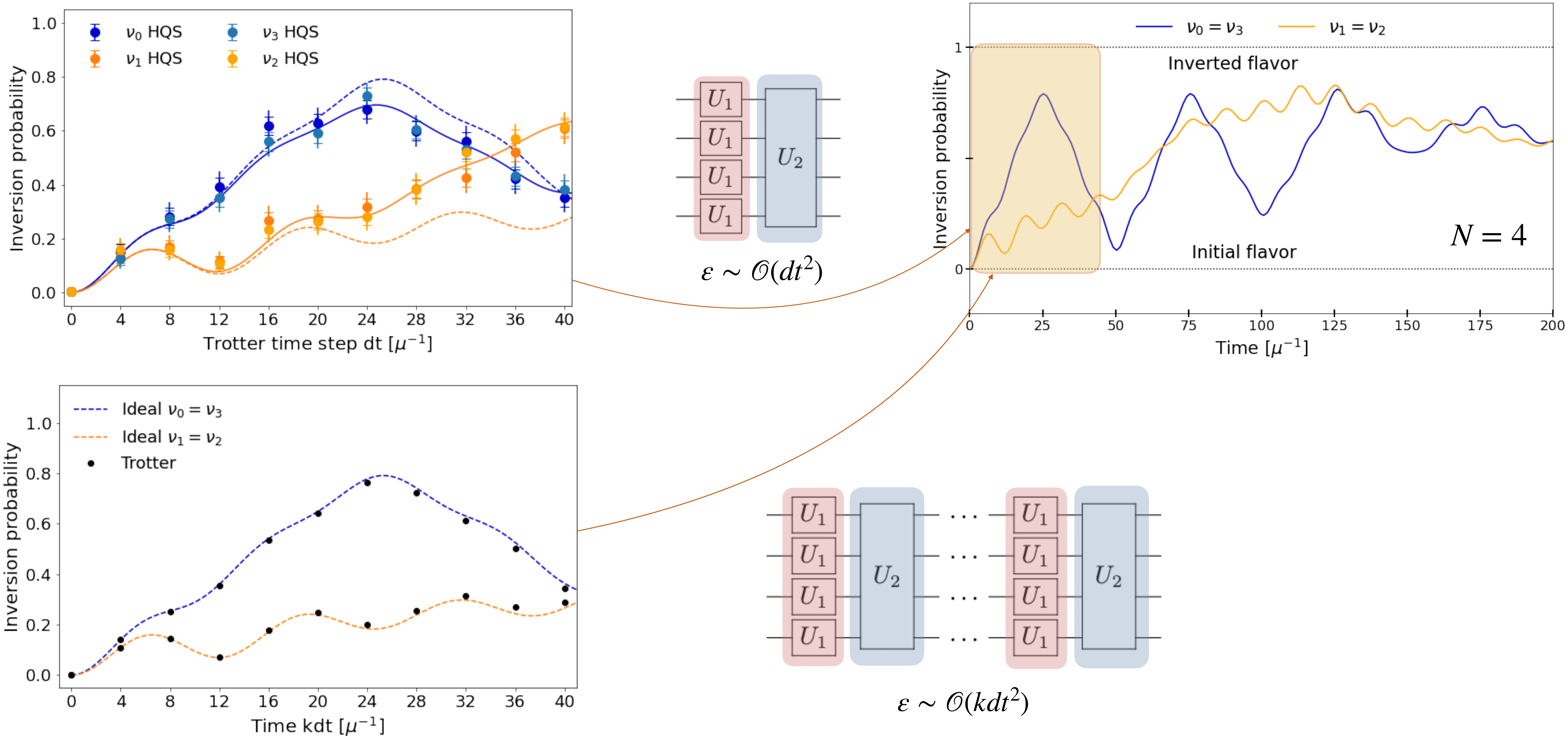


error bar = 68 %



V. Amitrano et. al. Phys. Rev. D 107, 023007 (2023)

RESULTS: MULTIPLE TROTTER STEPS



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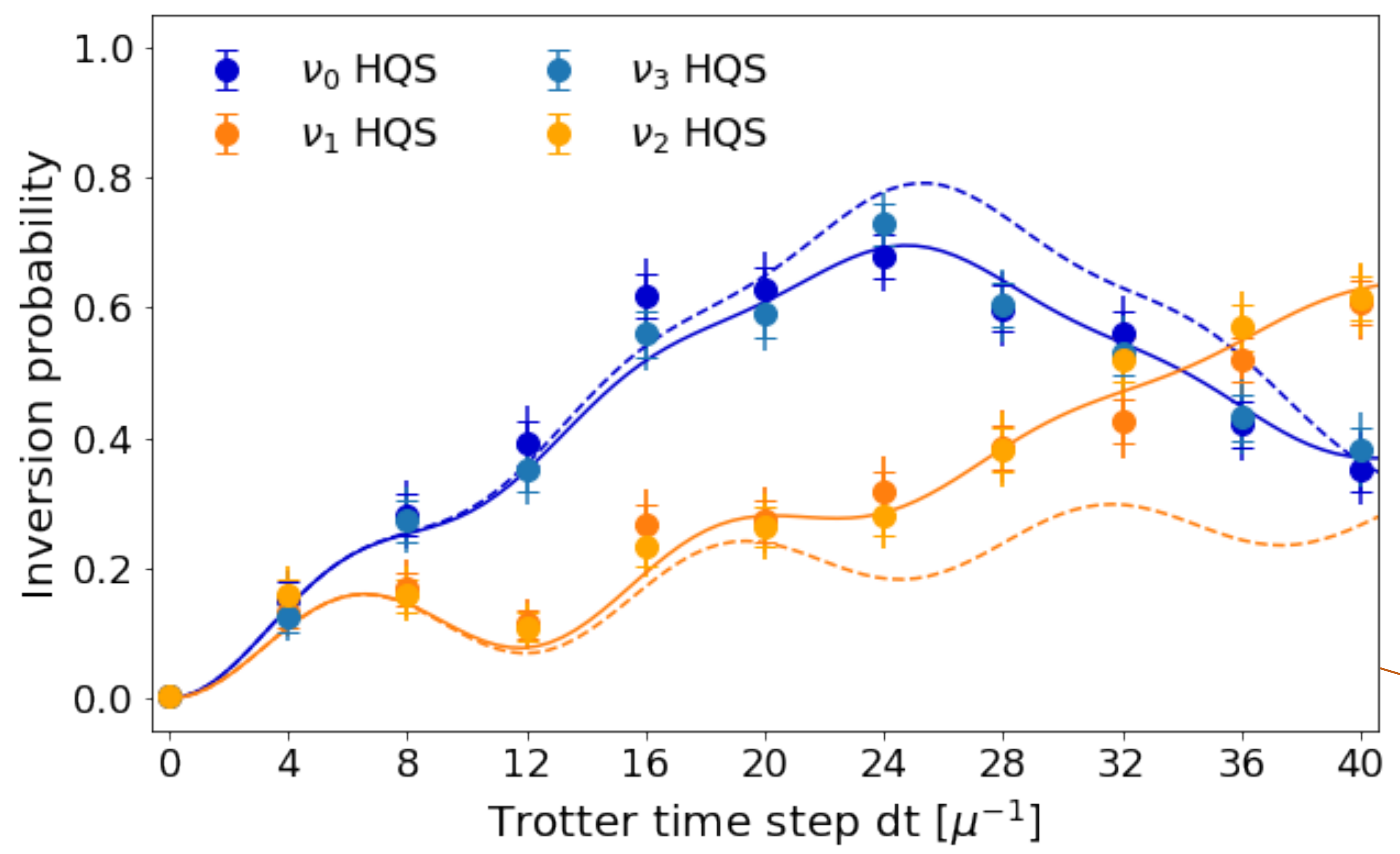
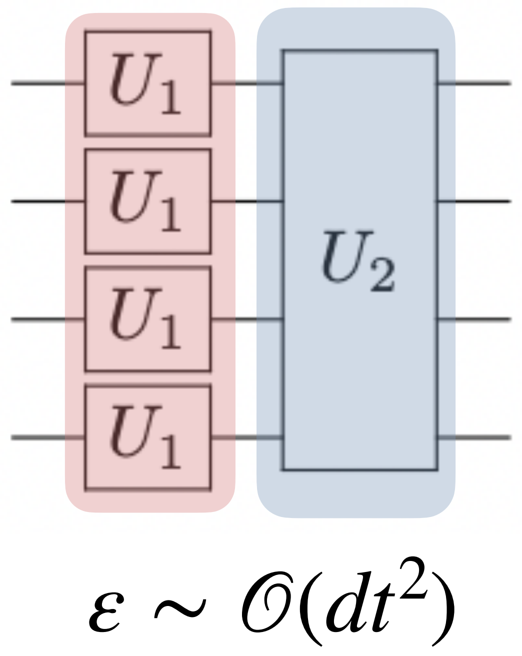
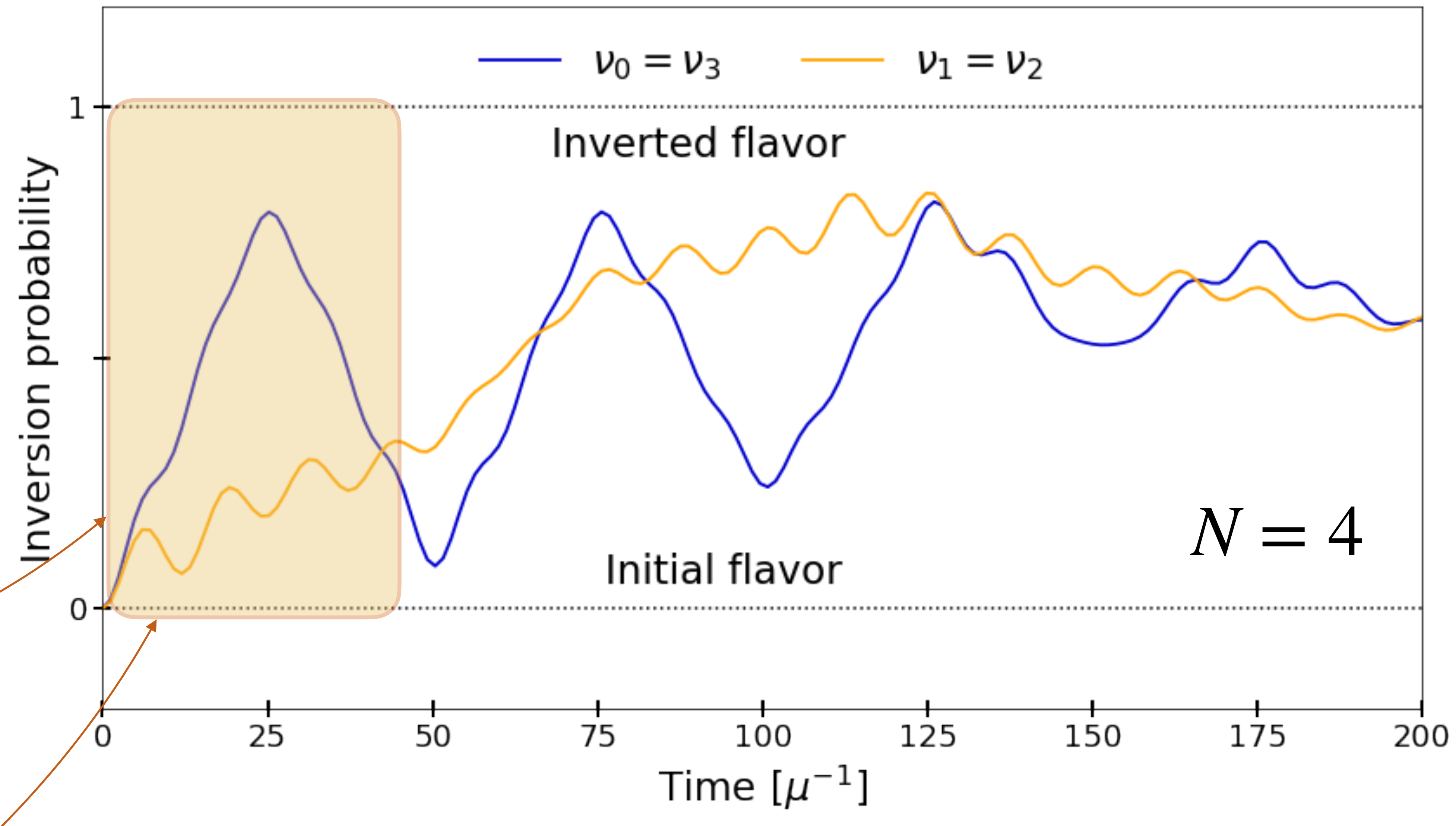
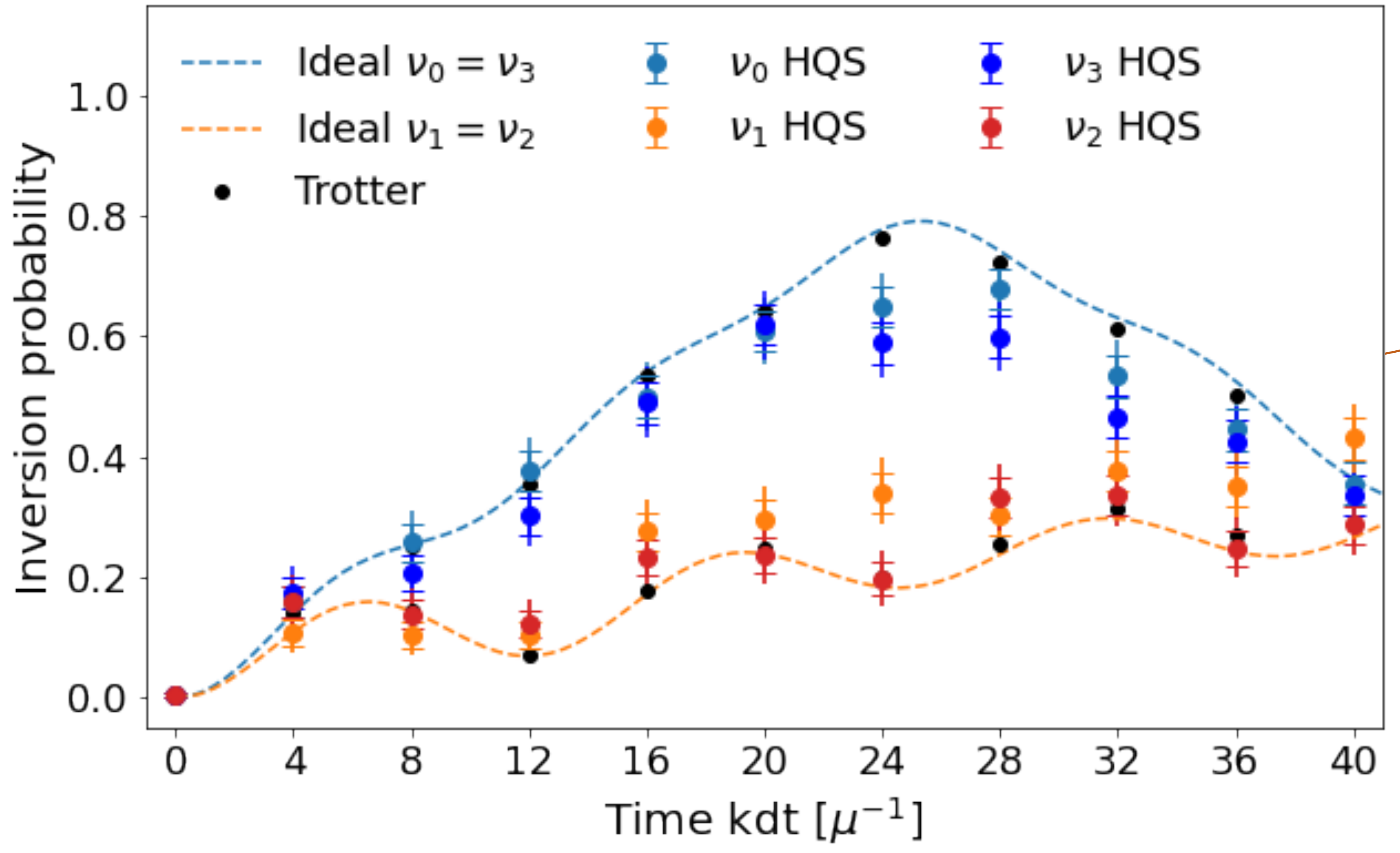
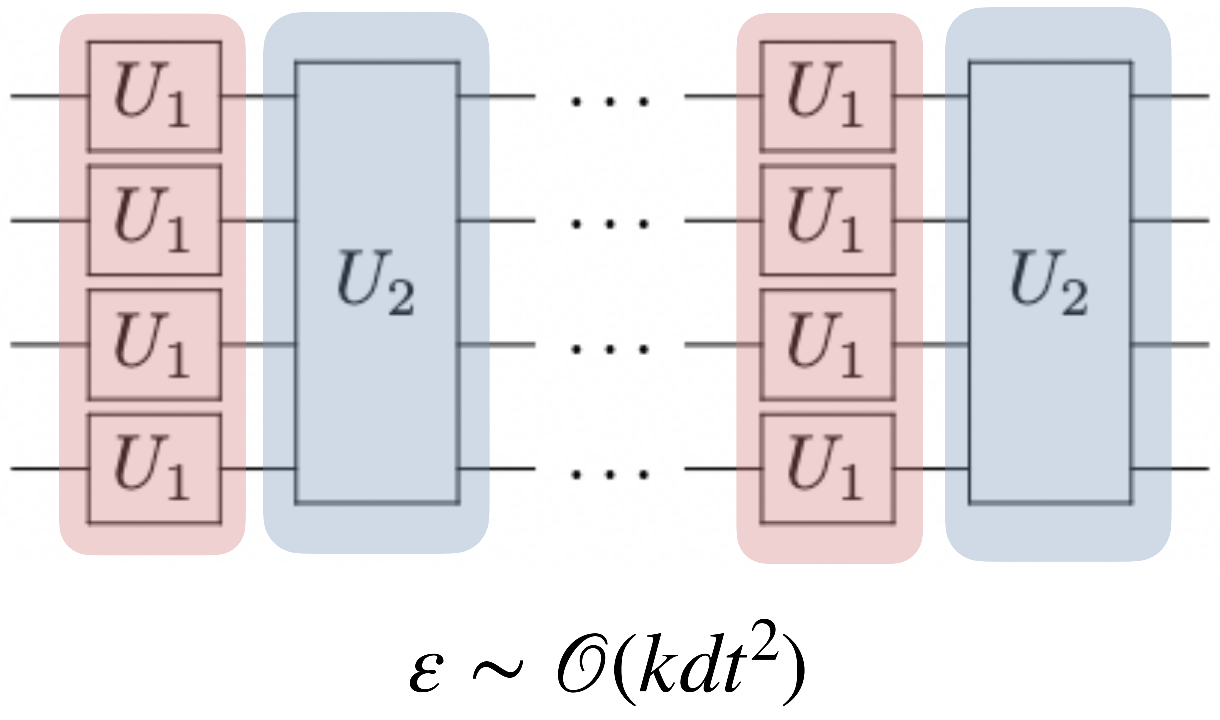


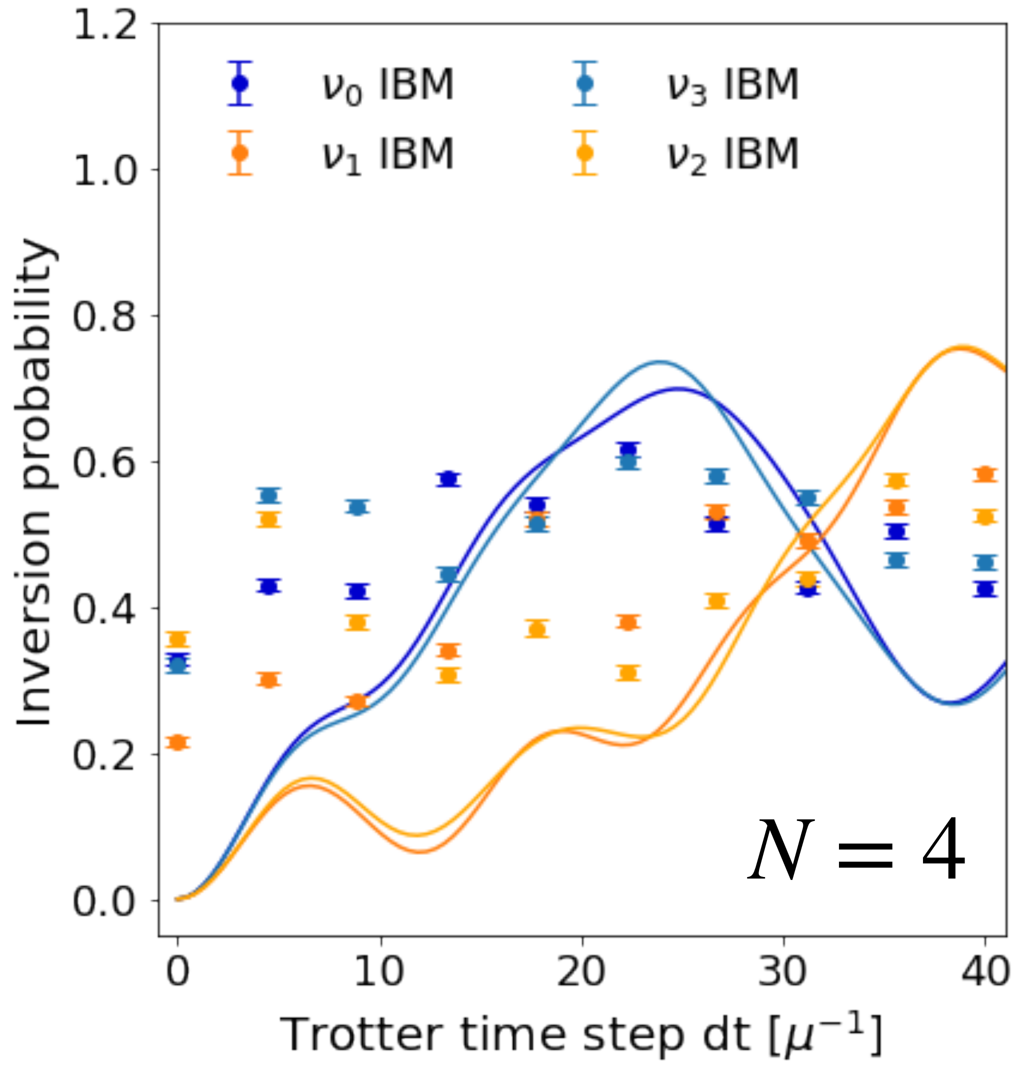


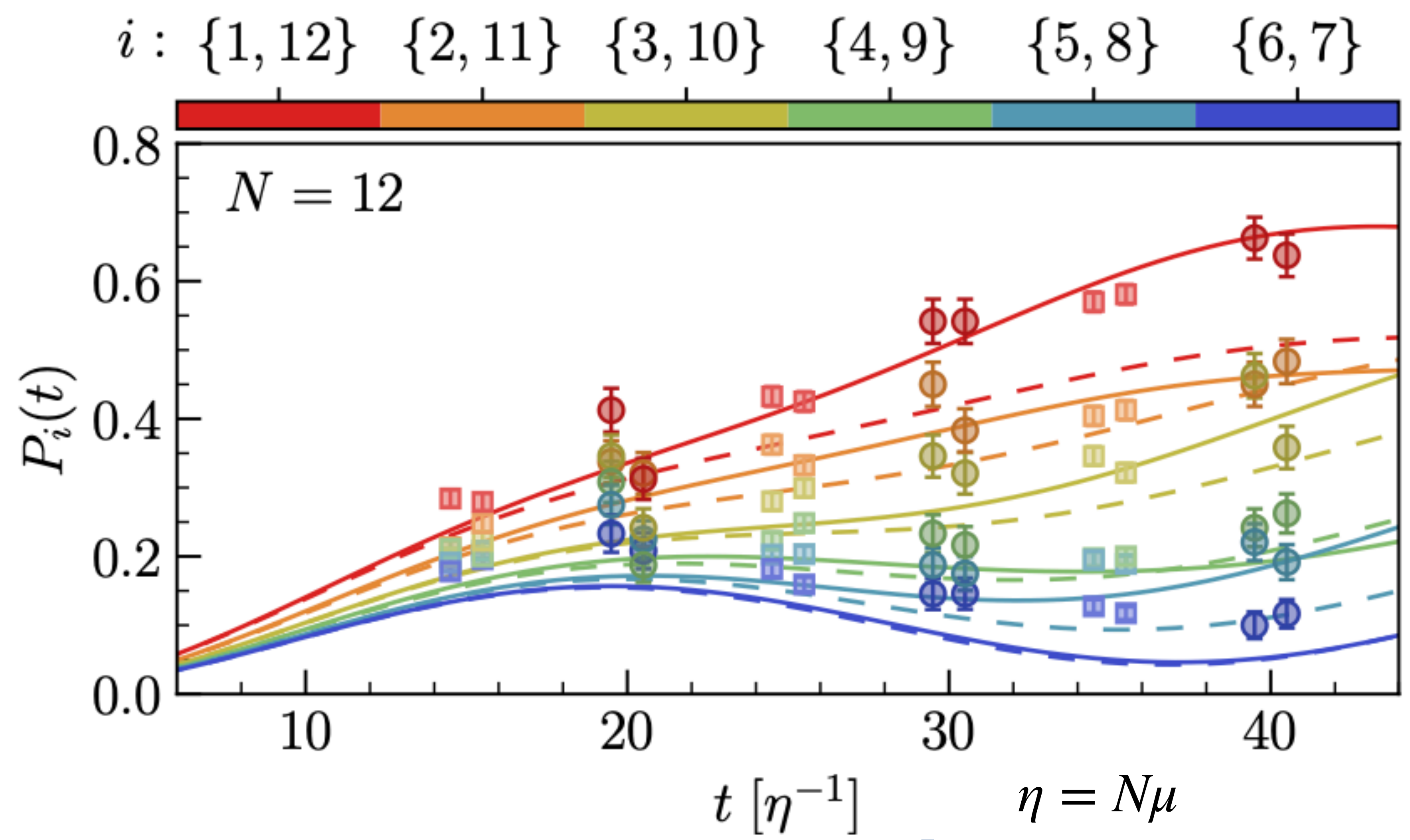






RESULTS

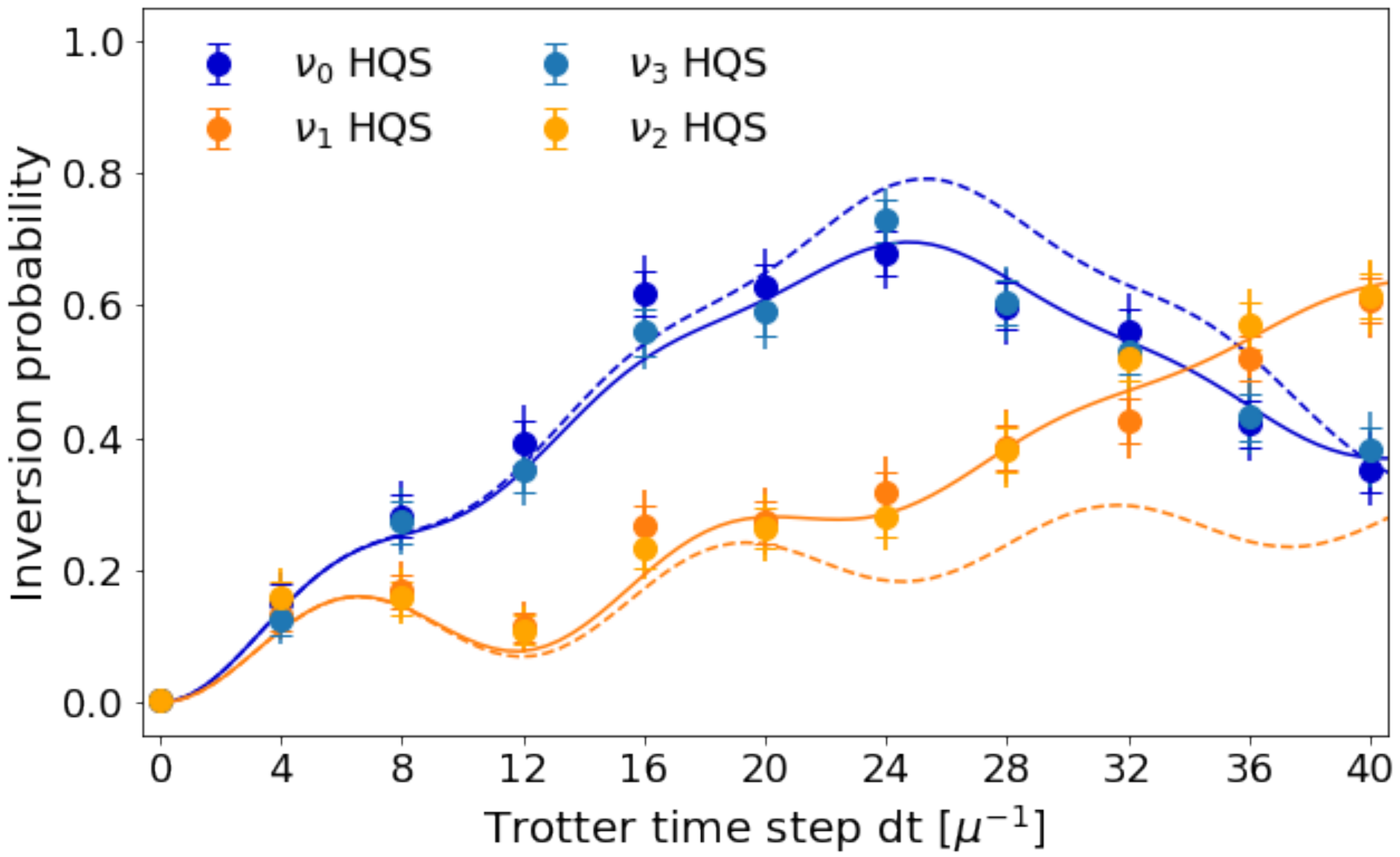




B. Hall et. al. (2021)

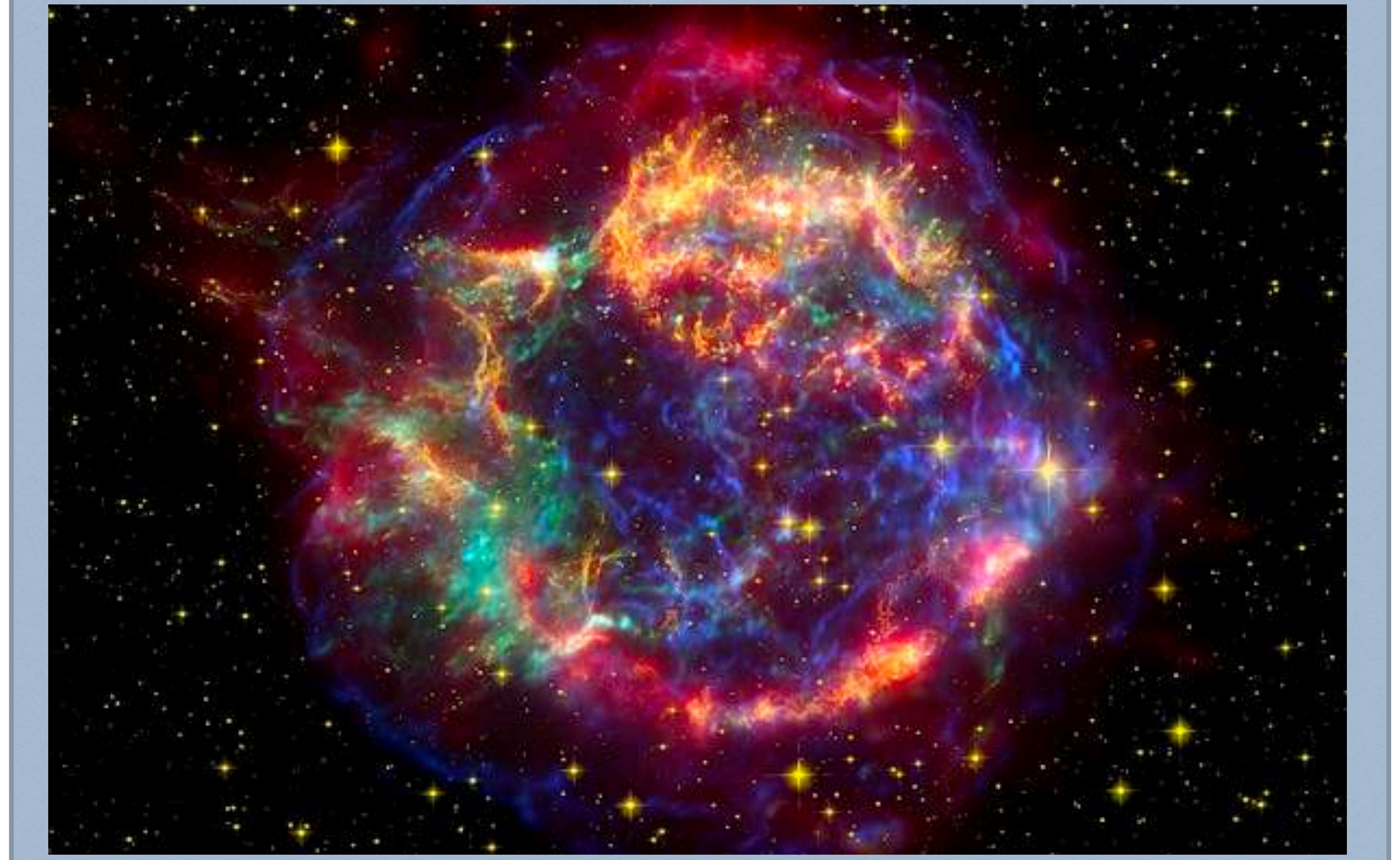
V. Amitrano et. al. (2023)

M. Illa and M. J. Savage (2023)



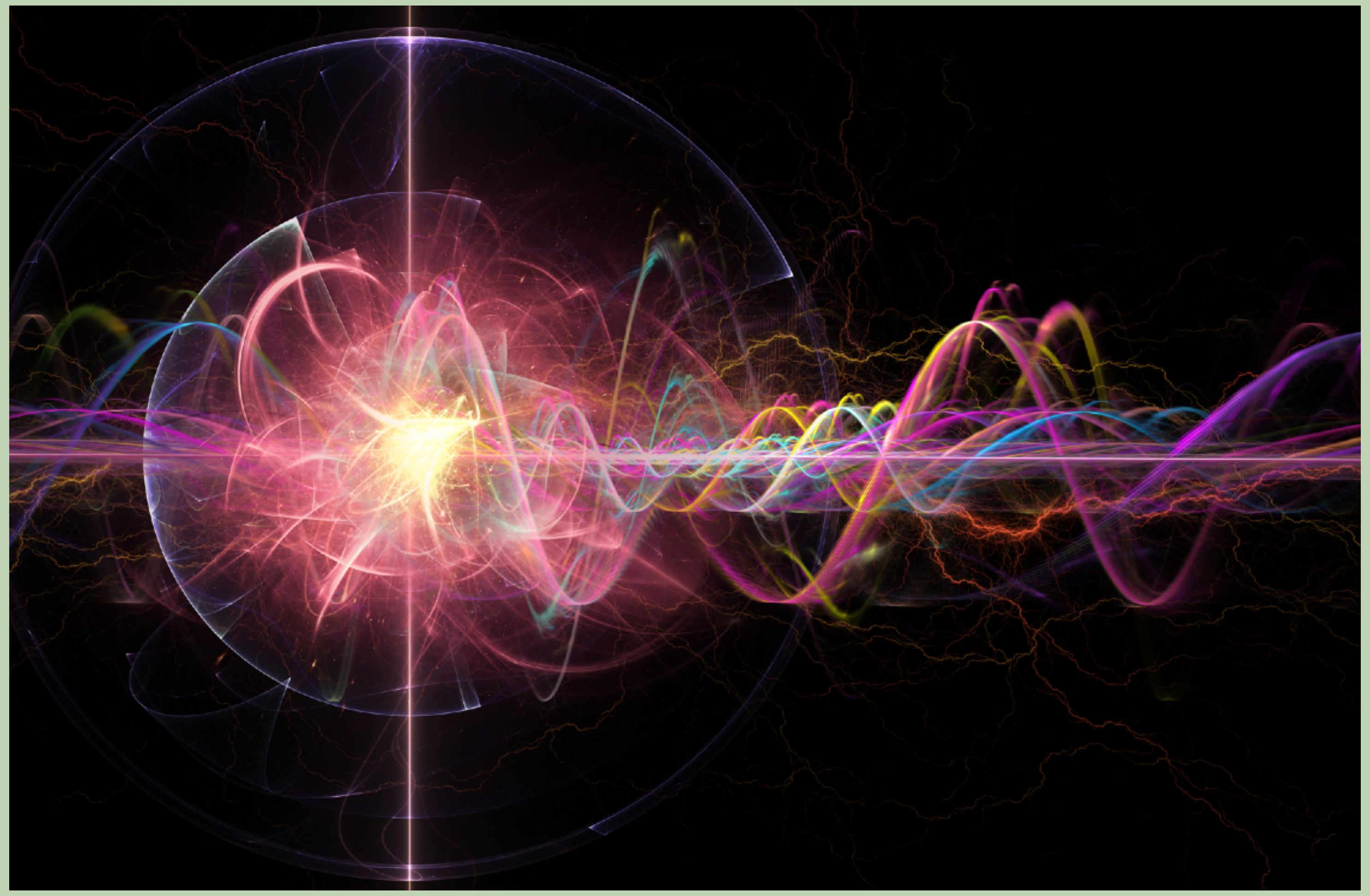
Outline

Motivation and physics



Quantum Computing Simulation

Results



- Flavor dynamics is crucial to describe many effects in core-collapse supernovae

- QC necessary for full dynamics simulation of collective neutrino oscillations

- Results are very promising
- We can increase the number of simulated neutrinos

Thank you for your attention

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