Kinetic approach of light nuclei in intermediate-energy heavy-ion collisions

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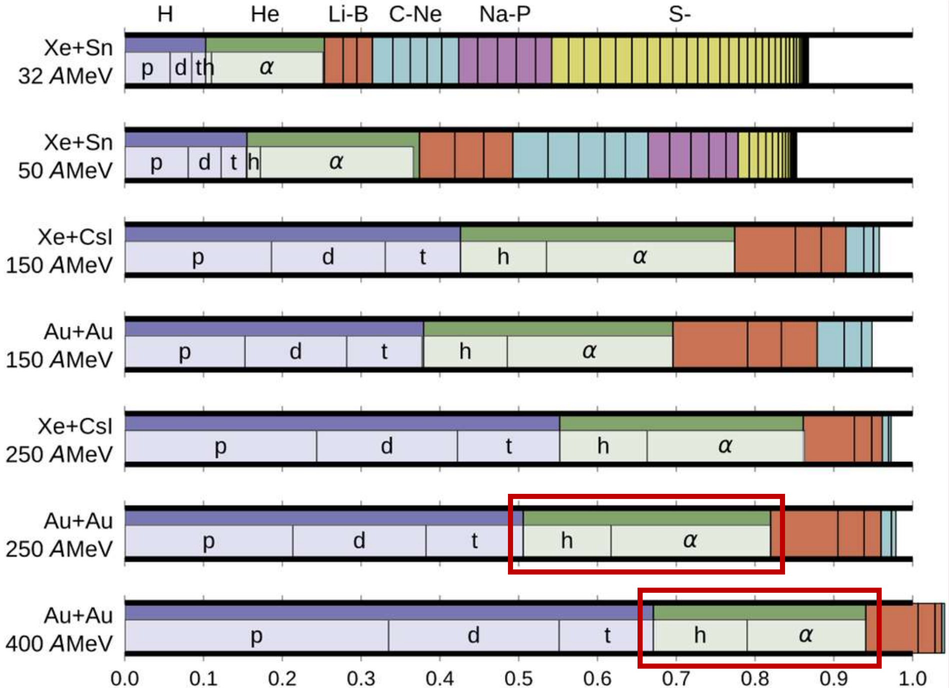
1 Light nuclei in heavy-ion collisions

2 Kinetic approach on light nuclei & FOPI measurements

3 α-particle fraction in warm and dense nuclear matter

Light nuclei in heavy-ion collisions



FOPI measurements

Light nuclei account for a large portion of the measured final state charged particles

- Their production mechanism
- Their effects on nucleon/pion observables
- They may provide more efficient probes of nuclear equation of state
- ……

One interesting feature is that more α–particles are produced than helium-3 (h).

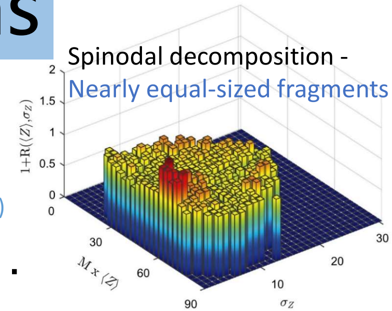
FOPI Collaboration, Nuclear Physics A 848 (2010) 366–427

A. Ono, Progress in Particle and Nuclear Physics 105 (2019) 139–179

Light nuclei in heavy-ion collisions

Cluster recognition based on the phase-space of nucleons

- Coalescing nucleons according to Δr and/or Δp
  e.g., A. S. Botvina, et. al, Physical Review C 103, 064602 (2021)
- Minimum spanning tree/plus most negative E_B
  e.g., FRIGA, A. Le Fèvre, et. al, Physical Review C 100, 034904 (2019)
- ……



Spinodal decomposition - Nearly equal-sized fragments

B. Borderie, et al., INDRA Collaboration, PRL86, 3252 (2001)
B. Borderie, et al., INDRA Collaboration, PLB782 (2018) 291–296

Dynamical treatment

Mean-field → **Spinodal decomposition**

Low incident energies

- e.g., Statistical mean-field (SMF) or Boltzmann–Langevin equation

M. Colonna, et. al, Nuclear Physics A 642 (1998) 449-460

→ Large fragments → Secondary decay e.g. SMM or GEMINI → Light nuclei

Interplay (Wang, Burrello, Colonna, in preparation)

Many-body correlations → **Many-body scatterings like $NNN \leftrightarrow Nd$**

- Anti-symmetrized molecular dynamics (AMD) (Below pion threshold)

A. Ono, Journal of Physics: Conference Series 420 (2013) 012103

- Kinetic approach/Boltzmann–Uehling–Uhlenbeck equation

Light nuclei with A = 2 (d) and A = 3 (t, h) have been included.

P. Danielewicz and G. F. Bertsch, Nuclear Physics A533, (1991) 712-748

→ Light nuclei

Light nuclei in kinetic approach

Kinetic equations are derived based on the closed time-path Green's function formulism

For example, in the deuteron case, the two-body Green's function G_2 satisfies an equation

$$G_2 = \mathcal{G}_2 + \frac{1}{4}\mathcal{G}_2 v G_2$$

The light nuclei are realized as poles of the many-body Green's function.
In the vicinity of the pole, we have

$$i\langle x|G_2^<(P,\Omega,R,T)|x'\rangle \sim \langle x|\phi(P,R,T)|\rangle\langle\phi(P,R,T)x'\rangle f_2(P,R,T)2\pi\delta[\Omega - E(P,R,T)]$$

$$i\langle x|G_2^>(P,\Omega,R,T)|x'\rangle \sim \langle x|\phi(P,R,T)|\rangle\langle\phi(P,R,T)x'\rangle [1+f_2(P,R,T)]2\pi\delta[\Omega - E(P,R,T)]$$

P. Danielewicz and G. F. Bertsch, Nuclear Physics A533, 712-748 (1991)

Finally leads to equations of the occupation number f_τ of light nuclei

$$(\partial_t + \vec{\nabla}_p \epsilon_\tau \cdot \vec{\nabla}_r - \vec{\nabla}_r \epsilon_\tau \cdot \vec{\nabla}_p) f_\tau = \mathcal{K}_\tau^<[f_n, f_p, f_d, \cdots](1 \pm f_\tau) - \mathcal{K}_\tau^>[f_n, f_p, f_d, \cdots] f_\tau, \quad \tau = n, p, d, t, h, \alpha$$

Light nuclei in kinetic approach

For example, the loss term of the α particle

$$
\begin{aligned}
K_\alpha^> f_\alpha =& \frac{\mathcal{S}_{5'} f_\alpha}{2E_\alpha} \int \prod_{i=1'}^{5'} \frac{\mathrm{d}\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{\mathrm{d}\vec{p}_N}{(2\pi\hbar)^3 2E_N} \overline{|\mathcal{M}_{N\alpha\to NNNNN}|^2} g_N f_N \prod_{i=1'}^{5'} (1 \pm f_i)(2\pi)^4 \delta^4(\sum_{i=1'}^{5'} p_i - p_N - p_\alpha) \\
&+ \frac{\mathcal{S}_{3'} f_\alpha}{2E_\alpha} \int \prod_{i=1'}^{3'} \frac{\mathrm{d}\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{\mathrm{d}\vec{p}_N}{(2\pi\hbar)^3 2E_N} \overline{|\mathcal{M}_{N\alpha\to NNt}|^2} g_N f_N \prod_{i=1'}^{3'} (1 \pm f_i)(2\pi)^4 \delta^4(\sum_{i=1'}^{3'} p_i - p_N - p_\alpha) + t \to h \\
&+ \frac{\mathcal{S}_{2'} f_\alpha}{2E_\alpha} \int \prod_{i=1'}^{2'} \frac{\mathrm{d}\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{\mathrm{d}\vec{p}_N}{(2\pi\hbar)^3 2E_N} \overline{|\mathcal{M}_{N\alpha\to dt}|^2} g_N f_N \prod_{i=1'}^{2'} (1 \pm f_i)(2\pi)^4 \delta^4(\sum_{i=1'}^{2'} p_i - p_N - p_\alpha) + t \to h \\
&+ \text{elastic part.}
\end{aligned}
$$

Light nuclei can be produced and dissociated through many-body scatterings (currently we have included the red ones)

- A = 2 $\pi NN \leftrightarrow \pi d$, $NNN \leftrightarrow Nd$
- A = 3 $\pi NNN \leftrightarrow \pi t(h)$, $\pi Nd \leftrightarrow \pi t(h)$, $NNNN \leftrightarrow Nt(h)$, $NNd \leftrightarrow Nt(h)$
- A = 4 $\pi NNNN \leftrightarrow \pi\alpha$, $\pi NNd \leftrightarrow \pi\alpha$, $\pi Nt(h) \leftrightarrow \pi\alpha$, $NNNNN \leftrightarrow N\alpha$, $NNNd \leftrightarrow N\alpha$, $NNt(h) \leftrightarrow N\alpha$, $dt(h) \leftrightarrow N\alpha$

- Many body **transition amplitudes** e.g., $\overline{|M_{Npn\leftrightarrow Nd}|^2}$
- The medium effect of light nuclei – **Mott effect**

R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, Z. Zhang, PRC108, L031601 (2023)

Impulse approximation

$$\overline{|\mathcal{M}_{Nd\to NNN}|^2} \approx F(\sqrt{s}) \sum_{\substack{\text{spectator}\\ \text{nucleons}}} |\langle \vec{k}|\phi_d\rangle|^2 \overline{|M_{NN\to NN}|^2}$$

Detailed balance

$$\overline{|\mathcal{M}_{NNN\to Nd}|^2}$$

Internal wave function of light nuclei

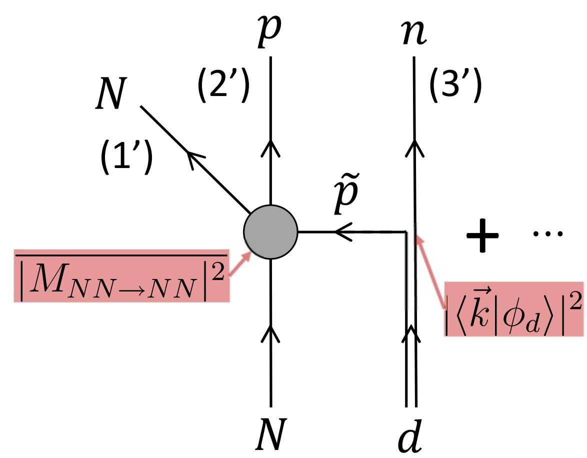
$\vec{k}$ are the standard Jacobian relative momenta

Under IA, $\sigma_{Nd}^{total} \sim \sigma_{Np}^{total} + \sigma_{Nn}^{total}$

$F(\sqrt{s})$ is a factor to 1) account for the inadequacy of IA, 2) exclude the elastic part of N-d from the total amplitude.

They should be determined by comparing with experimental N-d N-t N-α in-elastic cross sections.

In transport approach, under IA the scattering $NNN \leftrightarrow Nd$ can be divided into two subprocess

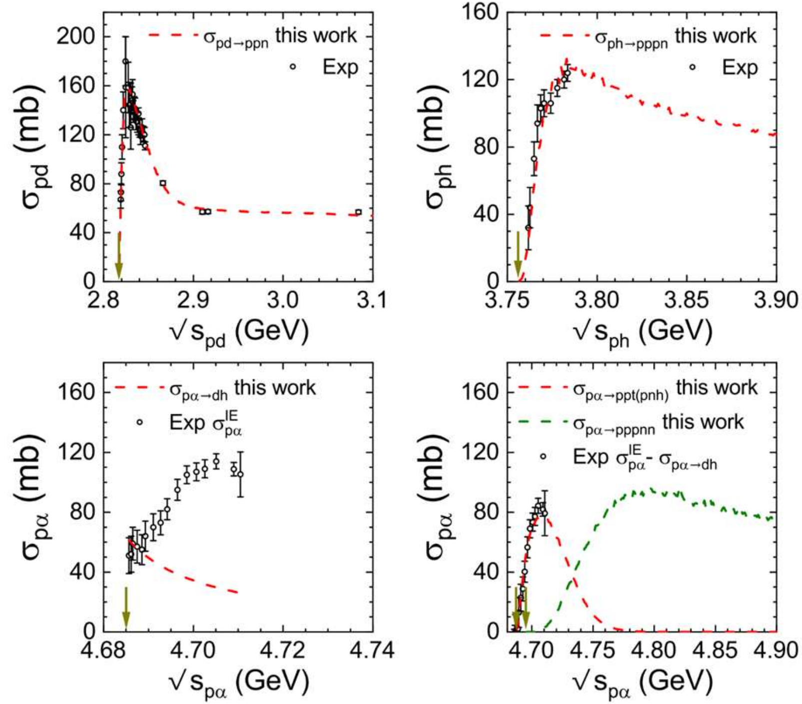


IA could ensure the **detailed balance condition** of many-body scatterings like $NNN \leftrightarrow Nd$

Cross sections

$$\overline{|\mathcal{M}_{N\alpha\to NNNNN}|^2} \approx \boxed{F(\sqrt{s})} \sum_{\substack{\text{spectator} \\ \text{nucleons}}} |\langle \vec{k}\vec{k}_\lambda\vec{k}_\mu|\phi_\alpha\rangle|^2 \overline{|M_{NN\to NN}|^2}$$

- A natural feature of F is it approaches to 1 as $\sqrt{s}$ increases. (For large incident energy, IA becomes very good, and the reaction is dominated by in-elastic channels)
- Different parameterizations of $F(\sqrt{s})$ for different many-body scattering channels are adopted to properly reproduce the experimental N-d, N-h and N-α inelastic cross sections.
- $N\alpha \leftrightarrow NNt(h)$ and $N\alpha \leftrightarrow dt(h)$ should be included to reproduce the experimental N-α inelastic cross sections at small $\sqrt{s}$. Assumption has to be made of the branching ratios of $N\alpha$ scattering



R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, Z. Zhang, PRC108, L031601 (2023)

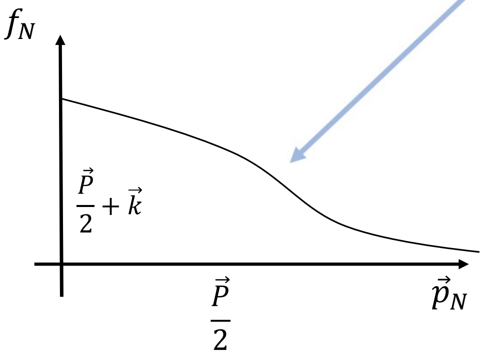
Mott effect

A light nucleus can not bind if its surrounding nucleon phase space is too dense

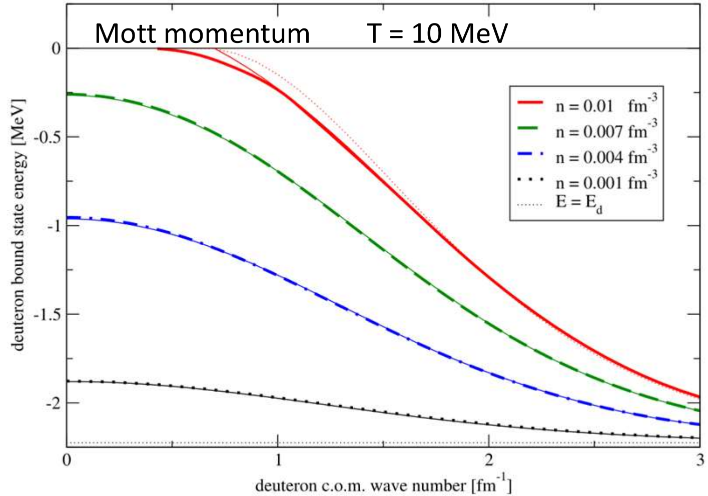
In-medium Schrodinger equation

$$\left[E_n(\tfrac{1}{2}\vec{P}+\vec{q})+E_p(\tfrac{1}{2}\vec{P}-\vec{q})\right]\Psi_{\vec{P}}(\vec{q})+\left[1-f_n(\tfrac{1}{2}\vec{P}+\vec{q})-f_p(\tfrac{1}{2}\vec{P}-\vec{q})\right]\int\frac{d\vec{q}'}{(2\pi\hbar)^3}\langle\vec{q}|v|\vec{q}'\rangle\Psi_{\vec{P}}(\vec{q}')=E(\vec{P})\Psi_{\vec{P}}(\vec{q})$$

The binding energy of a light nucleus (with momentum P) in nuclear medium, the Mott point is recognized as where the binding energy becomes negative.



Nucleon - Fermi distribution



G. Röpke, Nuclear Physics A 867 (2011) 66–80

Mott effect

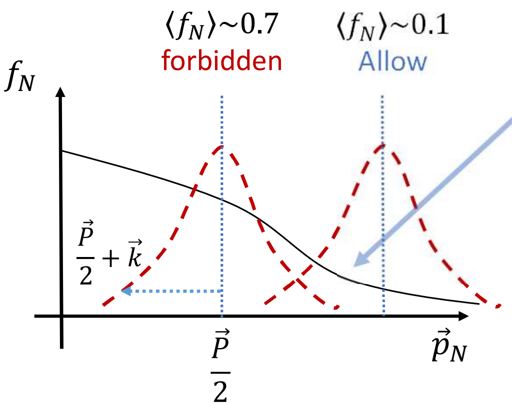
In kinetic approach

The Mott effect can be effectively introduced into the kinetic approach through a phase-space cutoff parameter.

$$\langle f_N \rangle_i(\vec{P}) \equiv \int f_N^{\mathrm{tot}}\left(\frac{\vec{P}}{A_i}+\vec{p}\right)|\phi_i(\vec{p})|^2 \mathrm{d}\vec{p} \leqslant f_{A=A_i}^{\mathrm{cut}}$$

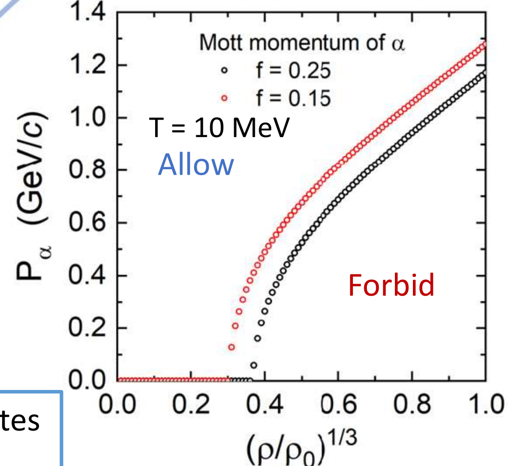
Momentum distribution of the constituent nucleon, related to the light-nuclei internal wave function

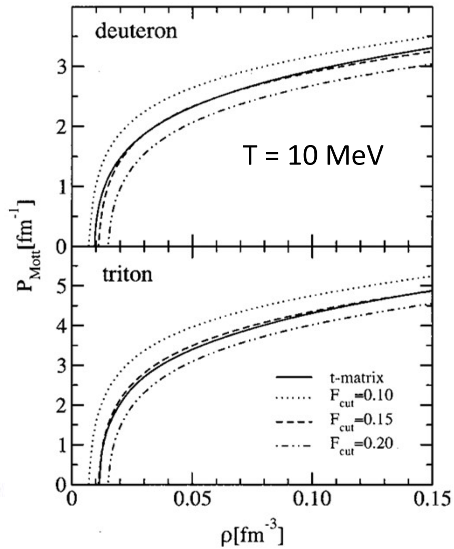
A larger $f_{A=A_i}^{cut}$ corresponds to a weaker Mott effect



$f_{A=2}^{cut}$, $f_{A=3}^{cut}$, and $f_{A=4}^{cut}$ can be treated as surrogates of the strength of light-nuclei Mott effect

In nuclear matter





C. Kuhrts, et. al., Physical Review C 63, 034605 (2001)

Light-nuclei yields

Central Au+Au collisions at 0.4*A* GeV

Multiplicities and charge balance for Au + Au at $E/A = 0.40$ GeV and $b_0 < 0.15$.

$Z=1$	106.0 ± 4.0	p	52.9 ± 2.7	$Z=2$	21.3 ± 1.9	^{3}He	9.4 ± 0.9
		d	34.2 ± 2.1			^{4}He	11.9 ± 1.1
		t	19.4 ± 1.8				
π^+	0.95 ± 0.08	π^-	2.80 ± 0.14				
		Li	3.5 ± 0.4	Be	0.84 ± 0.09		
		B	0.27 ± 0.03	C	0.096 ± 0.01		

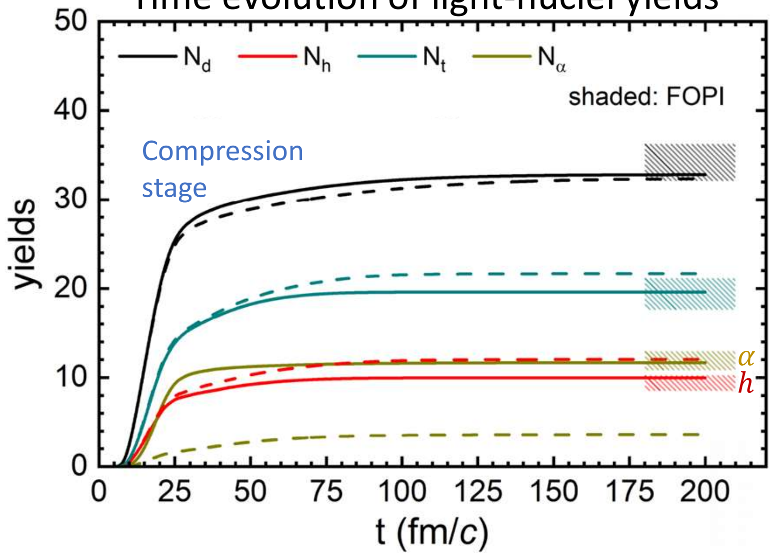
Charge balance LCP: 148.6 or 94.1%
Charge balance LCP + pions: 146.8 or 92.9%
Charge in HC (Z3–Z6): 15.77 ± 1.6 or (10.0 ± 1.0)%
Total balance: 102.9%

FOPI data at 0.4*A* GeV

To reproduce the measured light-nuclei yields (Note N_α is larger than N_h), one needs to use a larger $f_{A=4}^{cut}$ compared with $f_{A=3}^{cut}$, which means the Mott effect of α-particle should be weaker than that of triton/helium3.

A smaller $f_{A=4}^{cut}$ leads to a significant decrease of N_α.

Time evolution of light-nuclei yields



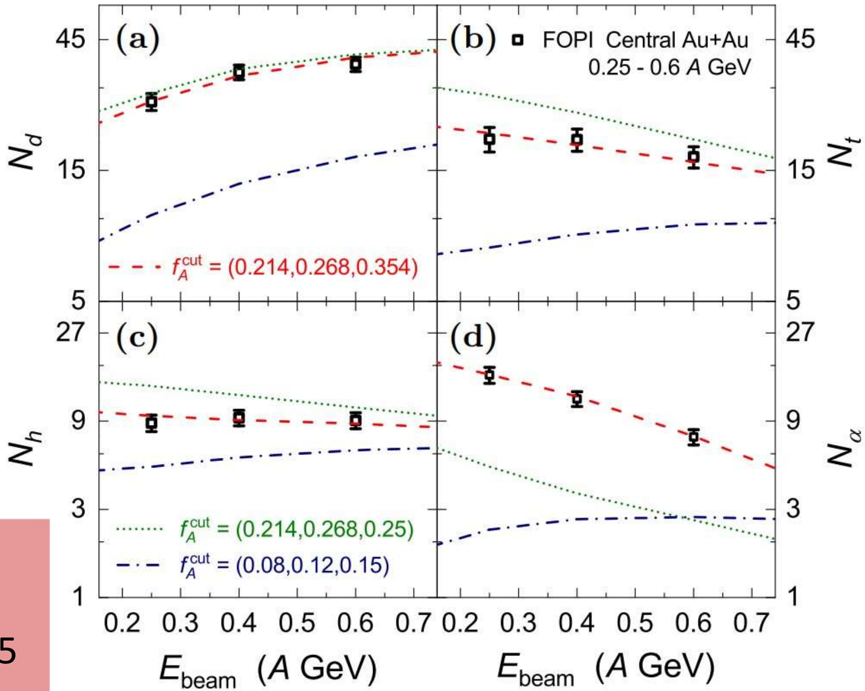
R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, Z. Zhang, PRC108, L031601 (2023)

Light-nuclei yields

Bayesian analysis of f_A^{cut} from light-nuclei yields in Au+Au central collisions at energies of 0.25*A* to 0.6*A* GeV.

- Influence of spinodal decomposition (larger fragments) in the low-energy region.
- Pion catalysis reactions, i.e., $\pi NN \leftrightarrow \pi d$, may contribute to the light nuclei yields at $E_{beam} > 0.6A$ GeV

The cutoff values $f_{A=2}^{cut} = 0.214$, $f_{A=3}^{cut} = 0.268$, and $f_{A=4}^{cut} = 0.354$ reasonably reproduce the FOPI data at energies of 0.25 to 0.6*A* GeV.



R. Wang, Z. Zhang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, in preparation

α-particle fraction

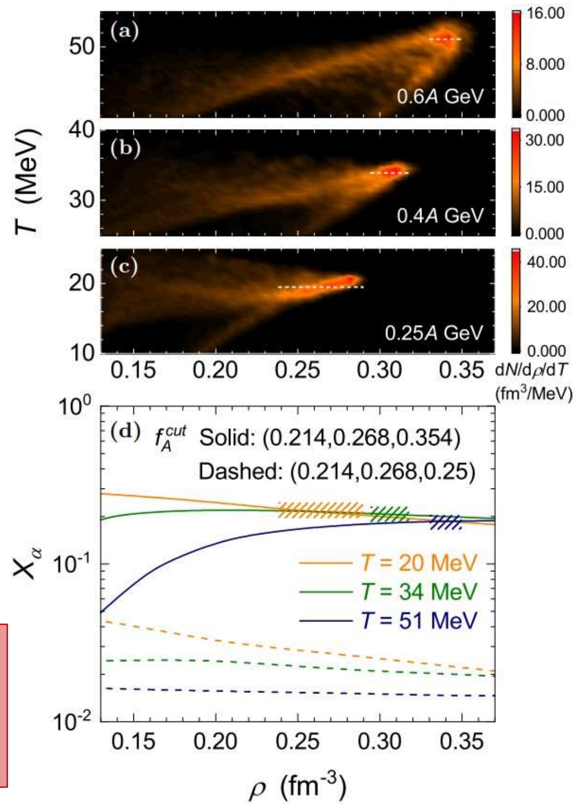
- According to the present kinetic approach, in intermediate-energy heavy-ion collisions, light nuclei are mainly formatted and freeze-out chemically at high densities ($NNNNN \leftrightarrow N\alpha$), especially for α-particles.

Collision rate $\sim\rho^5$

Contradict!

- It is generally thought that the dense nuclear matter can be regarded almost as a uniform nucleon liquid.

α-particle fraction $\sim$ 0.2 for in nuclear matter at around $\rho = 0.266\ \mathrm{fm}^{-3}$ with T = 19.5 MeV, $\rho = 0.306\ \mathrm{fm}^{-3}$ with T = 33.9 MeV, and $\rho = 0.340\ \mathrm{fm}^{-3}$ with T = 51.1 MeV.



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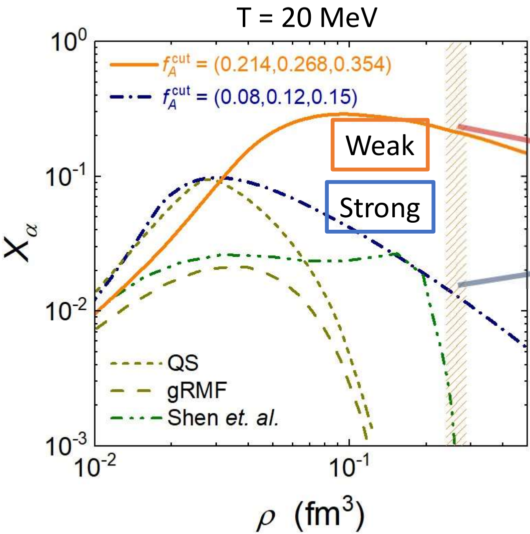
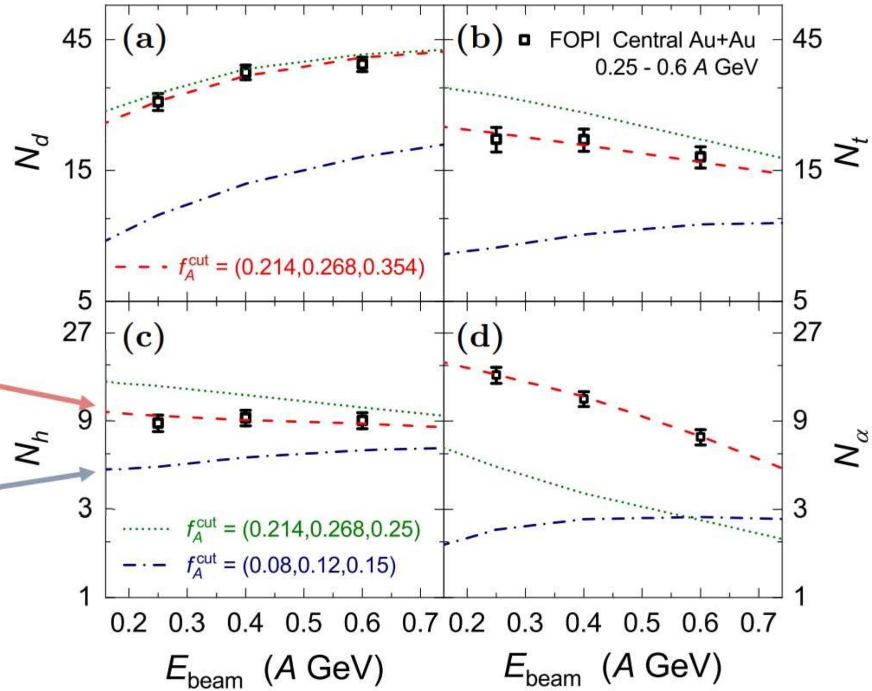
α-particle fraction

Comparison with other approaches

- QS & gRMF - Mott momentum from in-medium Schrodinger equation in low density regions
- Shen et. al - excluded volume prescription of α





R. Wang, Z. Zhang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, in preparation

Summary

- The FOPI data on light-nuclei yields can be reasonably reproduced within the present kinetic approach which incorporates dynamically all the light-nuclei (up to A=4) degrees of freedom.
- Our results indicate that the enhancement of α-particle yield is a consequence of its weaker Mott effect.
- Based on our approach, the FOPI data of light-nuclei yields indicate an unexpectedly high α-particle fraction in warm and dense nuclear matter, which challenges the usual thought.

Thank you