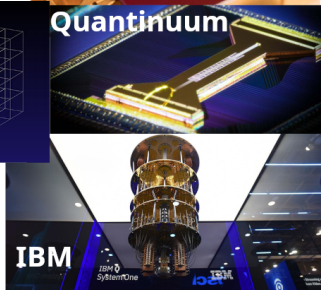
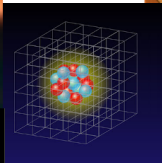
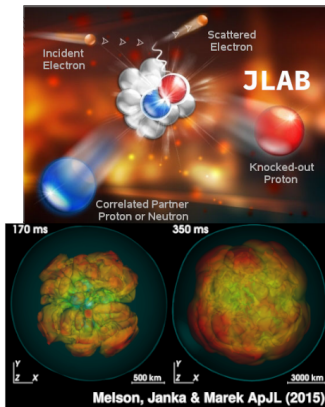


# Quantum Computing for Nuclear Physics

Alessandro Roggero



TNPI2023 - Cortona

12 Oct, 2023



# The need for ab-initio many-body dynamics in NP

- $\nu$  scattering for supernovae explosion and NS cooling
- capture reactions for crust heating and nucleosynthesis
- cross sections for dark-matter discovery and neutrino physics
- transport properties of neutron star matter for X-ray emission

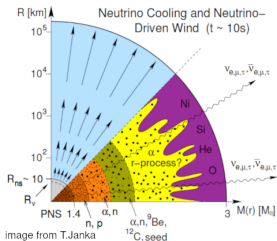


Image from T.Janka

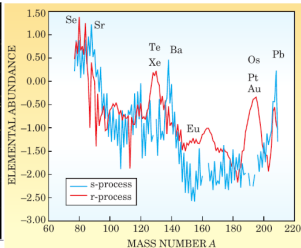
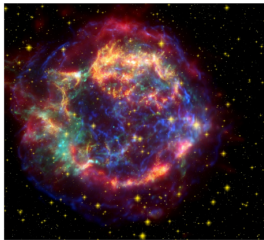


Image from Cowan & Thielemann (2004)



(C) Piro 2005

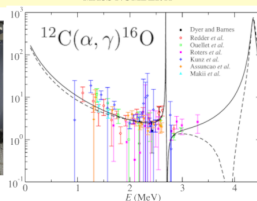
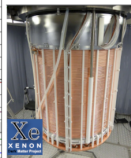
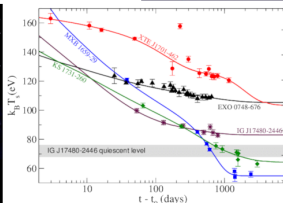
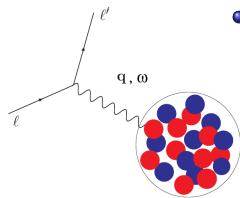


Image from Burro & Davis (2015)

## Inclusive cross section and the response function

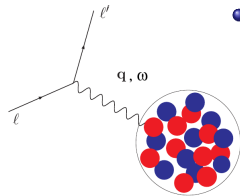


- cross section determined by the response function

$$R_O(\omega) = \sum_f \left| \langle f | \hat{O} | \Psi_0 \rangle \right|^2 \delta(\omega - E_f + E_0)$$

- excitation operator  $\hat{O}$  specifies the vertex

# Inclusive cross section and the response function



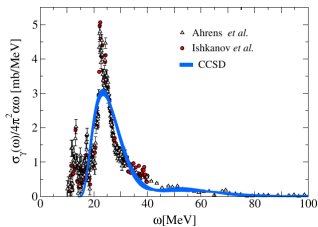
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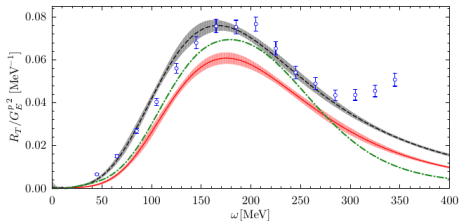
Extremely challenging classically for strongly correlated quantum systems

- dipole response of  $^{16}\text{O}$



Bacca et al. PRL(2013) LIT+CC

- quasi-elastic EM response of  $^{12}\text{C}$

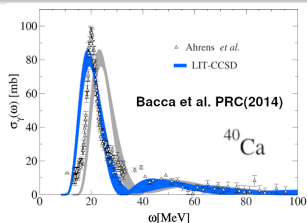


Lovato et al. PRL(2016) GFMC

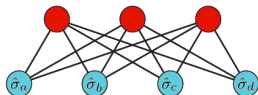
# Prospects for classical simulations of nuclear dynamics

## Quantum MC + Laplace/STA

- useful for quasi-elastic regime
- not yet accurate enough to go beyond  $A = 12$  (sign-problem)



Machine Learning ideas could help



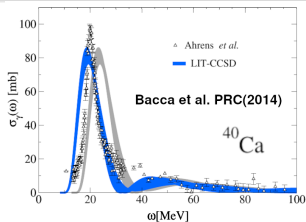
## Coupled Cluster + Lorentz/Gauss

- useful for low energy regime
- accuracy limited by inversion

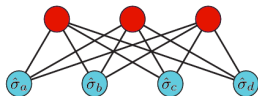
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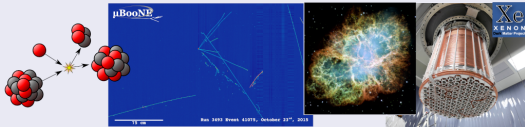


## Coupled Cluster + Lorentz/Gauss

- useful for low energy regime
- accuracy limited by inversion

## Some problems will still remain out of reach

- large open-shell nuclei
- exclusive cross-sections
- out of equilibrium



# Quantum Computing and Quantum Simulations

R.Feynman(1982) we can use a controllable quantum system to simulate the behaviour of another quantum system

**Quantum System  
we have control over**

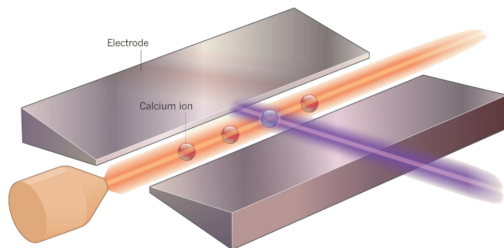


figure from E.Zohar

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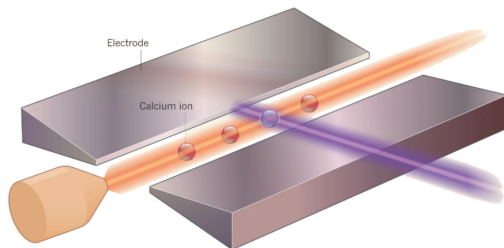
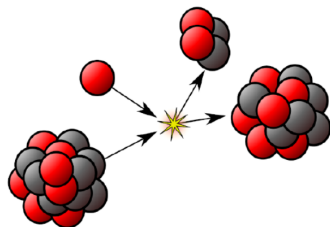


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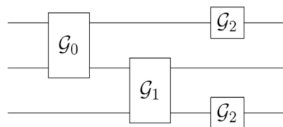




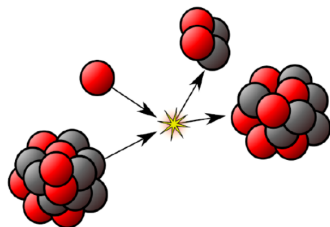
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# Black box model for a quantum computer



Blume-Kohout et al. (2013)

Box contains  $N$  qubits (2-level sys.)  
together with a set of buttons

- initial state preparation  $\rho$
- projective measurement  $\mathcal{M}$
- quantum operations  $G_k$

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We can build a **universal** black box  
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$$|\Psi(0)\rangle \rightarrow |\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

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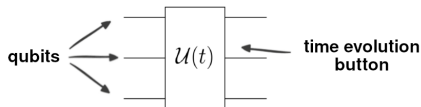
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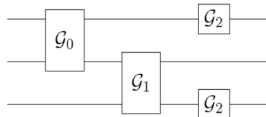
## Solovay–Kitaev Theorem

We can build a **universal** black box with only a **finite number** of buttons

Lloyd (1996) We can simulate time evolution of **local** Hamiltonians

- 1 discretize the physical problem
- 2 map physical states to bb states
- 3 push correct button sequence

$$|\Psi(0)\rangle \rightarrow |\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$



# First programmable quantum devices are here

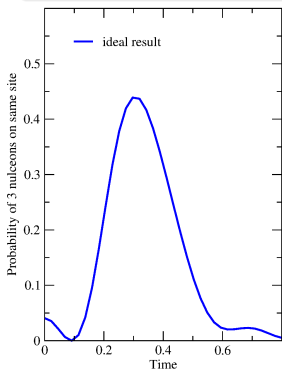
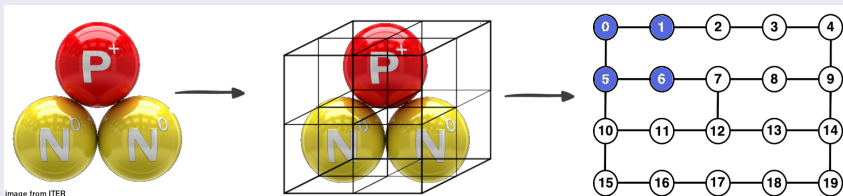


some figures from M.Savage



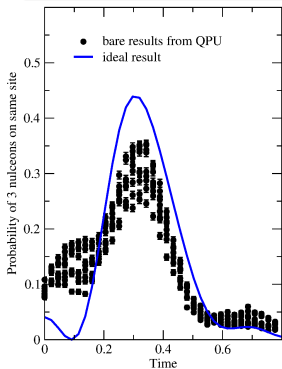
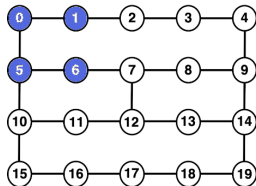
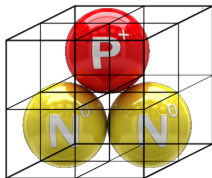
# Real time dynamics on current generation devices

AR, Li, Carlson, Gupta, Perdue PRD(2020)



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AR, Li, Carlson, Gupta, Perdue PRD(2020)



## Error sources

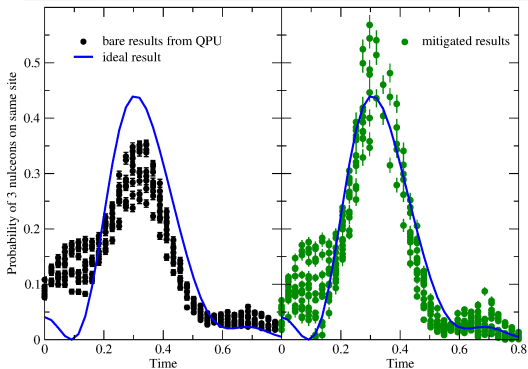
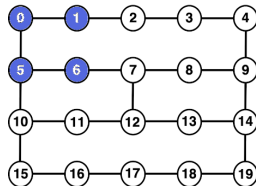
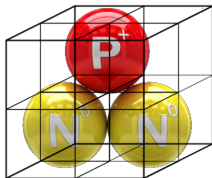
- decoherence (environment)
- imperfect calibration



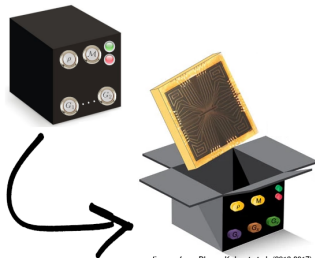
Blume-Kohout et al. (2013)

# Real time dynamics on current generation devices

AR, Li, Carlson, Gupta, Perdue PRD(2020)



● Error mitigation is crucial



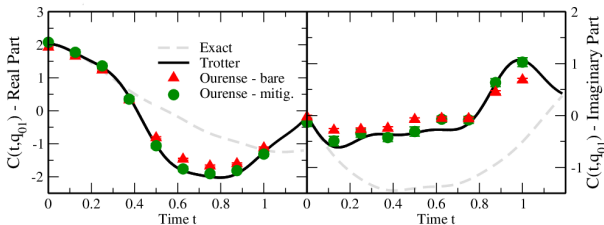
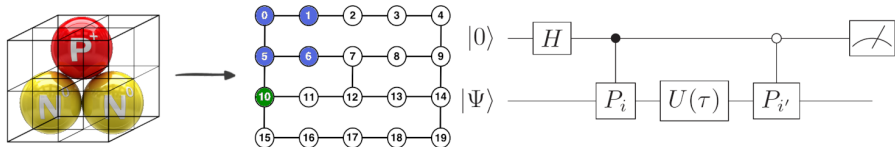
figures from Blume-Kohout et al. (2013,2017)

# Real time correlators on current generation devices

- First steps toward nuclear response: real-time correlators

$$R(\omega) = \int dt e^{i\omega t} C(t) \quad \text{with} \quad C(t) = \langle \Psi_0 | O(t) O(0) | \Psi_0 \rangle$$

- Can be done “easily” using one additional qubit (Somma et al. (2001))



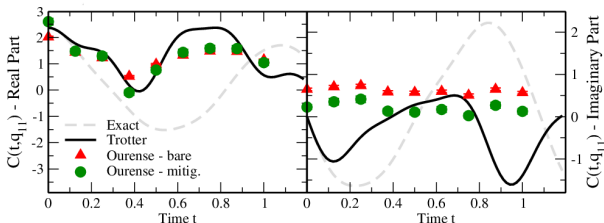
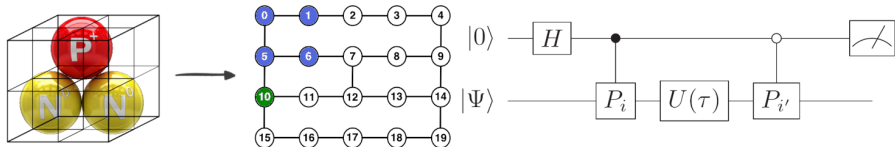
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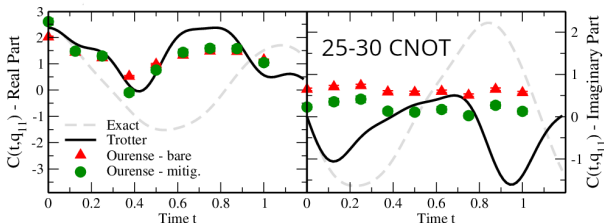
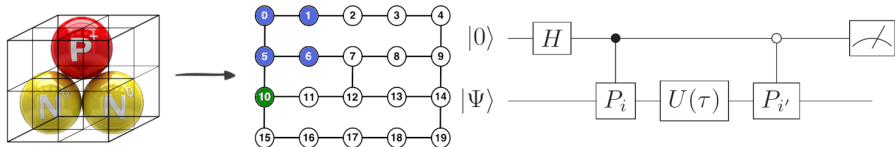
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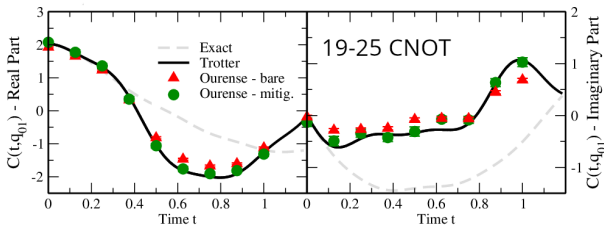
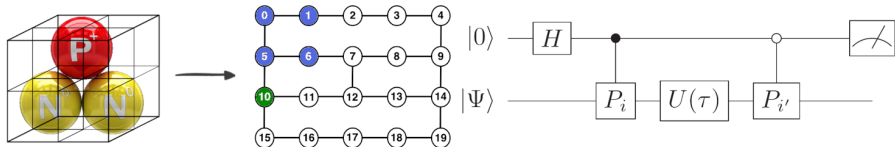
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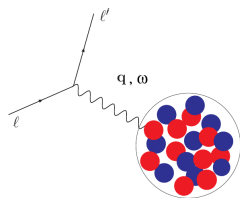
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Baroni, Carlson, Gupta, Li, Perdue, AR PRD(2022)

## Towards exclusive scattering using quantum computing



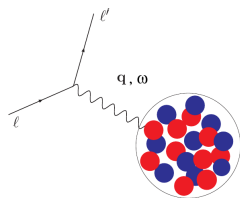
- response  $R(\omega) \Leftrightarrow$  probability for events at fixed  $\omega$
- exclusive x-sec  $\rightarrow$  events with specific final states

IDEA: prepare the following state on QC

$$|\Phi\rangle = \sum_{\omega} \sqrt{R(\omega)} |\omega\rangle \otimes |\psi_{\omega}\rangle$$



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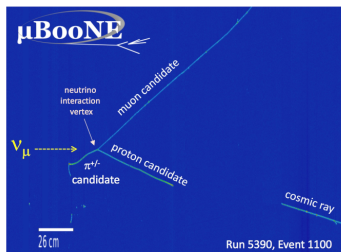
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- after measurement, the second register contains final states at  $\omega$ !

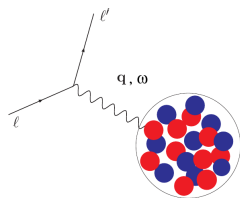


Blume-Kohout et al. (2013)



AR & Carlson PRC(2019)

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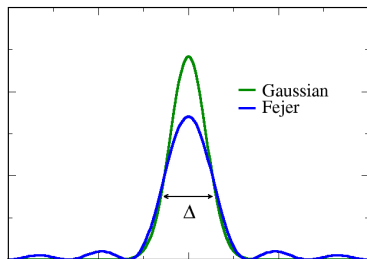
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- measurement of first register returns  $\omega$  with probability  $R(\omega)$ !
- after measurement, the second register contains final states at  $\omega$ !

Difficult to prepare  $|\Phi\rangle$  but we can prepare instead the following state

$$|\Phi_{\Delta}\rangle = \sum_{\omega} \sqrt{R_{\Delta}(\omega)} |\omega\rangle \otimes |\psi_{\omega}\rangle$$

with  $R_{\Delta}$  is an integral transform of the response with energy resolution  $\Delta$



AR & Carlson PRC(2019), AR PRA(2020)

# Prospects of impact of QC on Nuclear Physics

AR, Li, Carlson, Gupta, Perdue PRD(2020)

Cost estimates for realistic response in medium mass nuclei

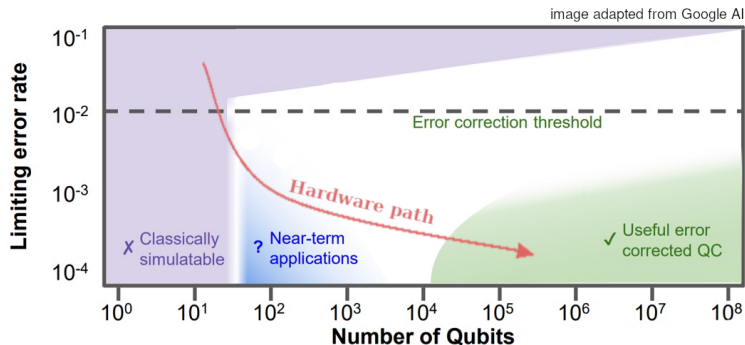
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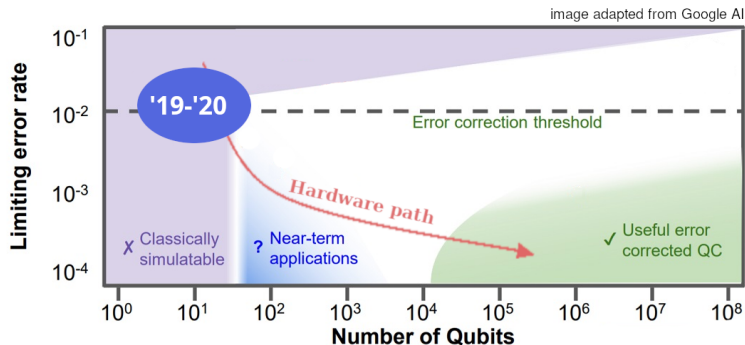


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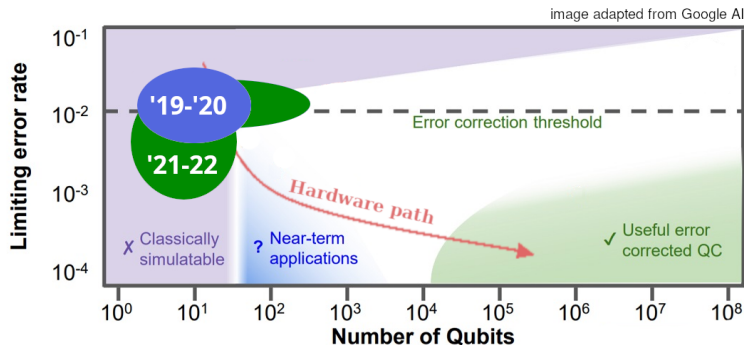


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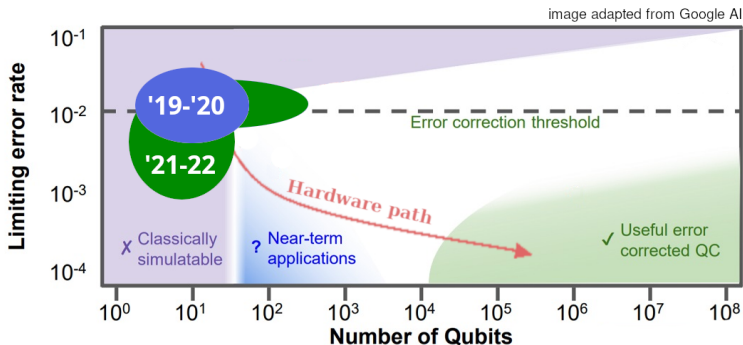


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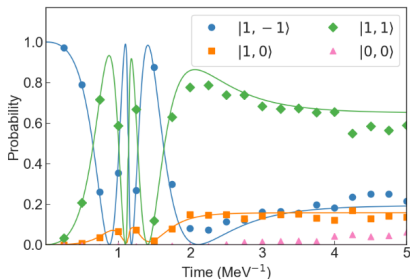
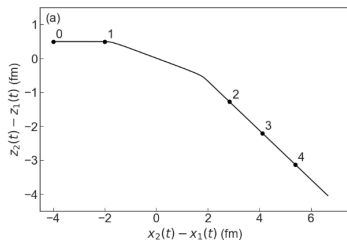
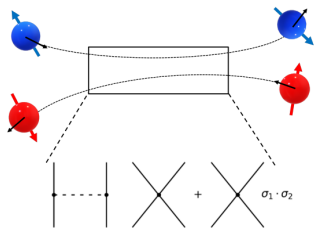
We need  $\approx 4000$  qubits and push the gate buttons  $\approx 10^6 - 10^8$  times



- Still possible to optimize further (other encodings need  $\approx 500$  qubits)
- Insights for classical methods could come before we have a large QC!

# Nuclear reactions in a semiclassical approach

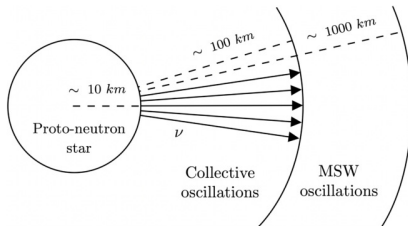
Turro, Chistolini, Hashim, King, Livingston, Wendt, Dubois, **Pederiva**, Quaglioni, Santiago, Siddiqi (2023)



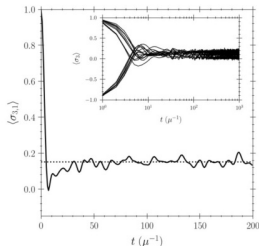
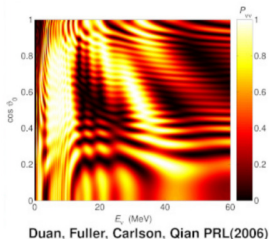


# Neutrino oscillations in astrophysical environments

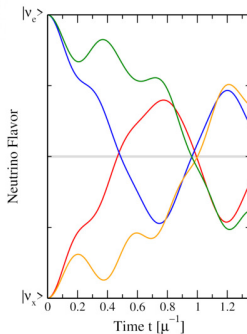
Fuller, Qian, Pantaleone, Sigl, Raffelt, Sawyer, Carlson, Duan, ...



see Valentina's talk



J. Martin, D. Neill, AR, H. Duan, J. Carlson  
arXiv:2307.16793

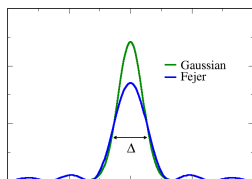


# Nuclear dynamics with quantum (inspired) computing?

We can prepare the following state

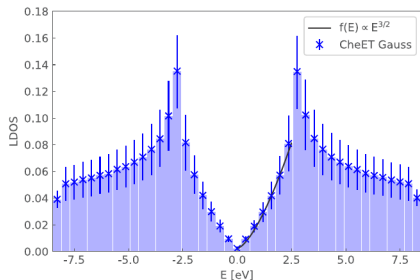
$$|\Phi_{\Delta}\rangle = \sum_{\omega} \sqrt{R_{\Delta}(\omega)} |\omega\rangle \otimes |\psi_{\omega}\rangle$$

with  $R_{\Delta}$  is an integral transform of the response with energy resolution  $\Delta$



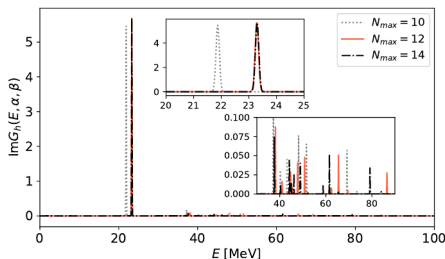
AR & Carlson PRC(2019), AR PRA(2020)

- Gaussian approach uses the fact that Chebyshev polynomials can be evaluated efficiently on quantum computers (Berry, Childs, Low, Chuang, ...)



Sobczyk, AR PRE(2022)

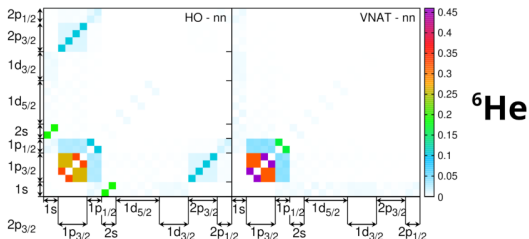
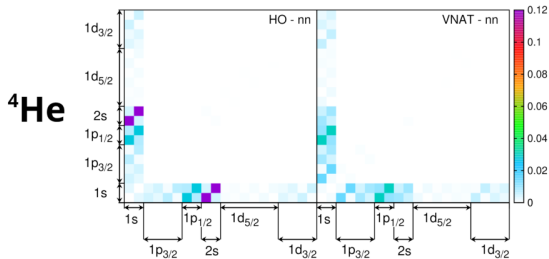
Alessandro Roggero



Sobczyk, Bacca, Hagen, Papenbrock PRC(2022)

# Entanglement structure in nuclei

Robin, Savage, Pillet PRC(2021)



# Summary & Conclusions

- Advances in theory and computing are opening the way to ab-initio calculation of equilibrium properties in the medium-mass region
- New ideas are needed to study nuclear dynamics in large open-shell nuclei, out-of-equilibrium processes and QCD at finite  $\mu$
- Quantum Computing has the potential to bridge this gap and increasingly better experimental test-beds are being built
- Error mitigation techniques will be critical to make the best use of these noisy near-term devices
- Early impact of QC on nuclear physics might come as insights into classical many-body methods and the role of entanglement

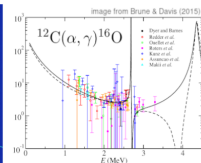
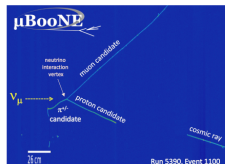


image from Chandra collab.

