Recent developments in β–decay studies within the Nuclear Shell Model

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Outline

- Renormalization of στ matrix elements
- Origin of the quenching of στ matrix elements
- Details of the Calculation
- fp-shell results
- fpg-shell results
- $2\nu\beta\beta$–decay NME
- Conclusions and perspectives

Outline

- Renormalization of $\sigma\tau$ matrix elements

Renormalization of στ matrix elements

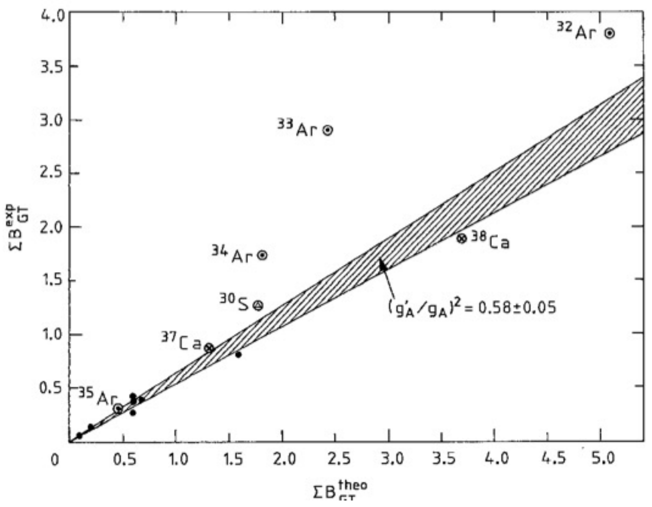
Gamow-Teller transitions (β-decay, EC, 2νββ,charge-exchange) are hindered from expected values based on sum rules derived by nuclear structure models.

Quenching of στ matrix elements is quite a general phenomenon in nuclear-structure physics.

$$g_A = g_A^{eff} = q\, g_A$$

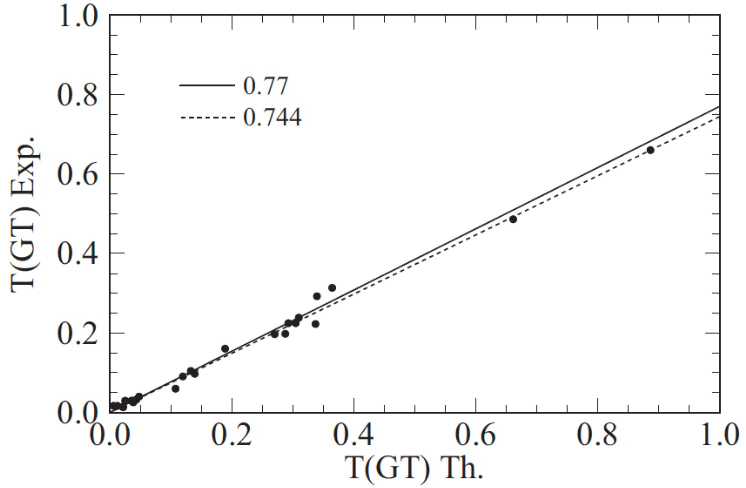
Renormalization of στ matrix elements

Z. Phys. A - Atomic Nuclei 332, 413417 (1989)



$$g_A = g_A^{eff} = q\ g_A$$

Martinez-Pinedo et al. PRC53 2602(1996)

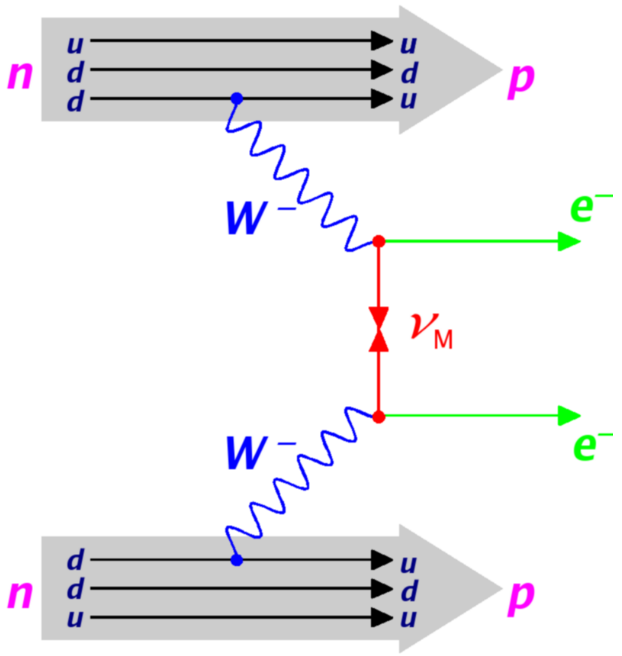


Suhonen, ACTA PHYSICA POLONICA B, 3 (2018)

Mass range	g_A^{eff}
Full $0p$ shell	$1.03^{+0.03}_{-0.02}$
$0p$–low $1s0d$ shell	1.18 ± 0.05
Full $1s0d$ shell	$0.96^{+0.03}_{-0.02}$
	1.0
$A = 41$–50 ($1p0f$ shell)	$0.937^{+0.019}_{-0.018}$
$1p0f$ shell	0.98
^{56}Ni	0.71
$A = 52$–67 ($1p0f$ shell)	$0.838^{+0.021}_{-0.020}$
$A = 67$–80 ($0f_{5/2}1p0g_{9/2}$ shell)	0.869 ± 0.019
$A = 63$–96 ($1p0f0g1d2s$ shell)	0.8
$A = 76$–82 ($1p0f0g_{9/2}$ shell)	0.76
$A = 90$–97 ($1p0f0g1d2s$ shell)	0.60
^{100}Sn	0.52
$A = 128$–130 ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.72
$A = 130$–136 ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.94
$A = 136$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.57

Quenching of στ matrix elements & 0νββ decay

The inverse of the 0νββ-decay half-life is proportional to the squared nuclear matrix element (NME). This evidences the relevance to calculate the NME ($M^{0\nu}$)



$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\,|M^{0\nu}|^2 \langle m_\nu \rangle^2 \propto g_A^4$$

- $G^{0\nu}$ → phase space factor
- $\langle m_\nu \rangle = |\sum_k m_k U_{ek}|$, effective mass of the Majorana neutrino
  U_{ek} being the lepton mixing matrix

Outline

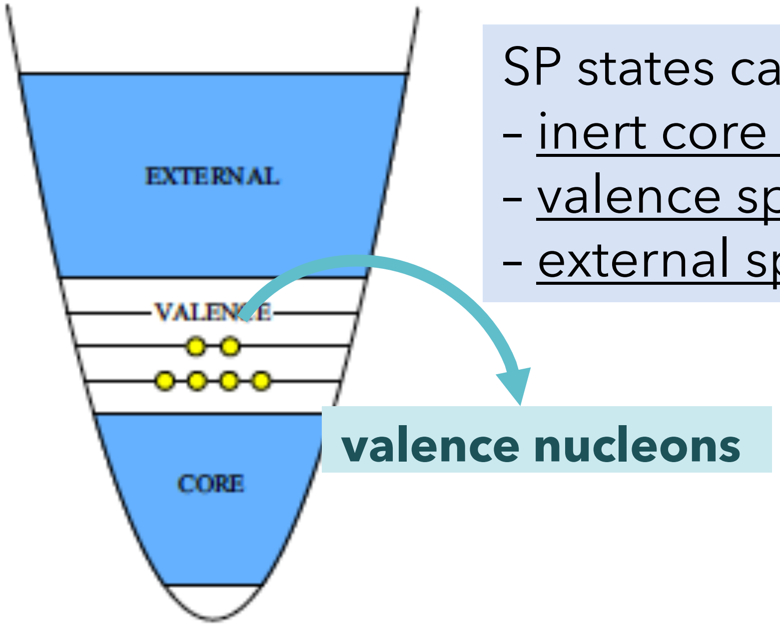
- Renormalization of $\sigma\tau$ matrix elements
- Origin of the quenching of $\sigma\tau$ matrix elements

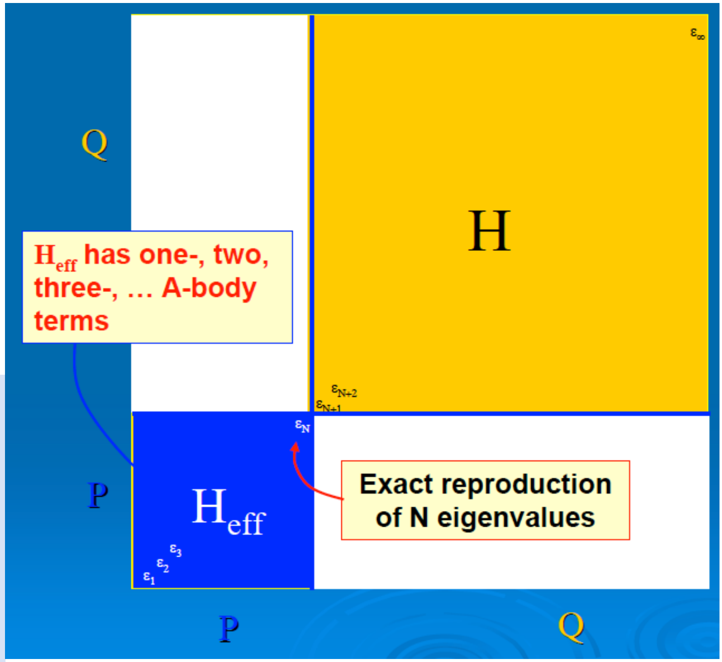
Quenching of στ matrix elements: theory

Two main sources:

1) LIMITED MODEL SPACE

SP states can be grouped into 3 sub-spaces, well separated in energy:
- inert core (*completely filled levels with neutrons and protons paired to J=0*)
- valence space (*partially filled levels*)
- external space (*empty levels*)

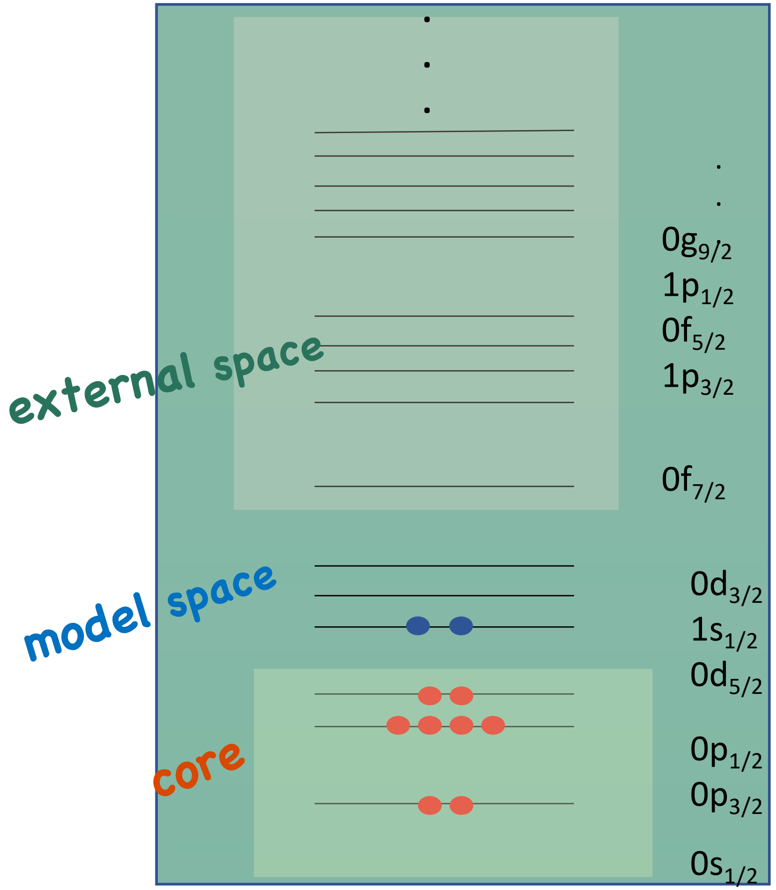




The only active degrees of freedom are given by nucleons inside the valence space (*valence nucleons*) while excitations of core nucleons and valence nucleons in the external space are "*frozen*" or, more properly, "not taken into account explicitly"

Realistic Shell-Model

Shell model: A well-established approach to obtain a microscopic description of both collective and single-particle properties of nuclei



The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.

$$V_{NN}\ (+V_{NNN})$$
$$\downarrow$$
$$MANY\ BODY\ PERTURBATION\ THEORY$$
$$\downarrow$$
$$H_{eff}$$

Realistic shell-model calculations starting from a nuclear Hamiltonian and decay operators derived consistently

Realistic Shell-Model

$$H \rightarrow H_{eff}$$

$$H|\psi_\nu\rangle = E_\nu|\psi_\nu\rangle \rightarrow H_{eff}|\varphi_\alpha\rangle = E_\nu|\varphi_\alpha\rangle$$

$|\varphi_\alpha\rangle$ = eigenvectors obtained diagonalizing H_{eff} in the reducedmodel space $|\varphi_\alpha\rangle$ = P $|\psi_\nu\rangle$

$$\langle\varphi_\nu|\Theta|\varphi_\lambda\rangle \neq \langle\Psi_\nu|\Theta|\Psi_\lambda\rangle$$

We then require an effective operator Θ_{eff} defined as follows

$$\Theta_{eff} = \sum_{\nu\lambda} |\varphi_\nu\rangle\langle\Psi_\nu|\Theta|\Psi_\lambda\rangle\langle\varphi_\lambda| \qquad \langle\varphi_\nu|\Theta_{eff}|\varphi_\lambda\rangle = \langle\Psi_\nu|\Theta|\Psi_\lambda\rangle$$

$$\Theta^{GT} = g_A\sigma\tau^{\pm} \rightarrow \Theta^{GT}_{eff} = g_A^{eff}\sigma\tau^{\pm}$$

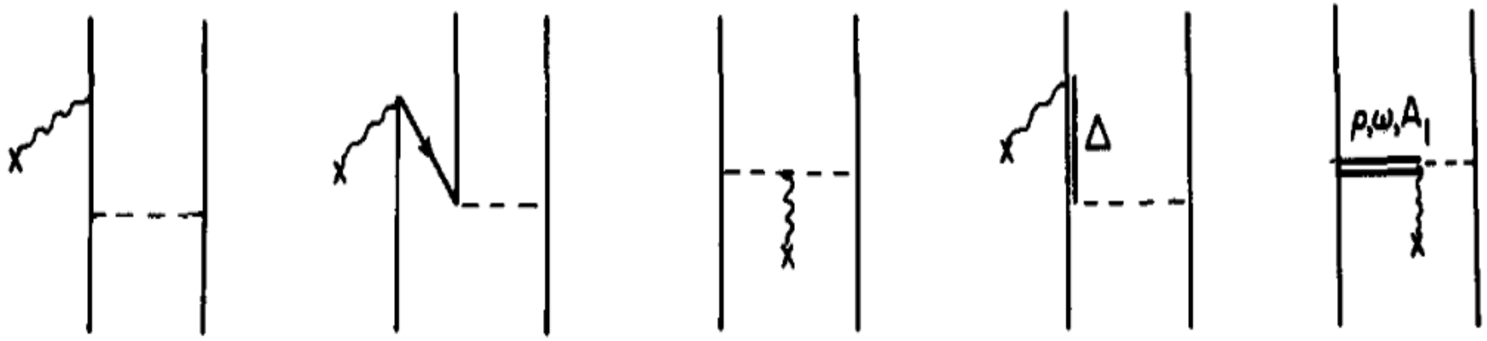
Quenching of στ matrix elements: theory

Two main sources:

2) NON-NUCLEONIC DEGREES OF FREEDOM

Processes in which the weak probe prompts a meson to be exchanged between two nucleons

→ meson-exchange two-body currents (2BC)



H. Hyuga and A. Arima, J. Phys. Soc. Jpn. Suppl. 34, 538 (1973)

Quenching of $\sigma\tau$ matrix elements: meson exchange currents

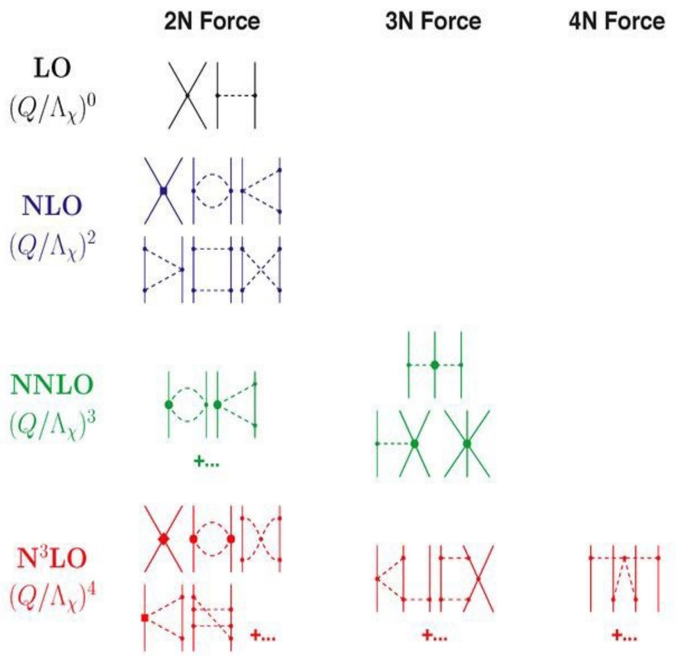
In the 80s starting from OBEP models two-nucleon meson-exchange current operators have been constructed consistently as required by the continuity equation for vector currents and the PCAC.

Nowadays, EFT provides a powerful approach where both nuclear potentials and two-body electroweak currents (2BC) may be consistently constructed, the latter appearing as subleading corrections to the one-body GT operator $\boldsymbol{\sigma\tau}^{\pm}$

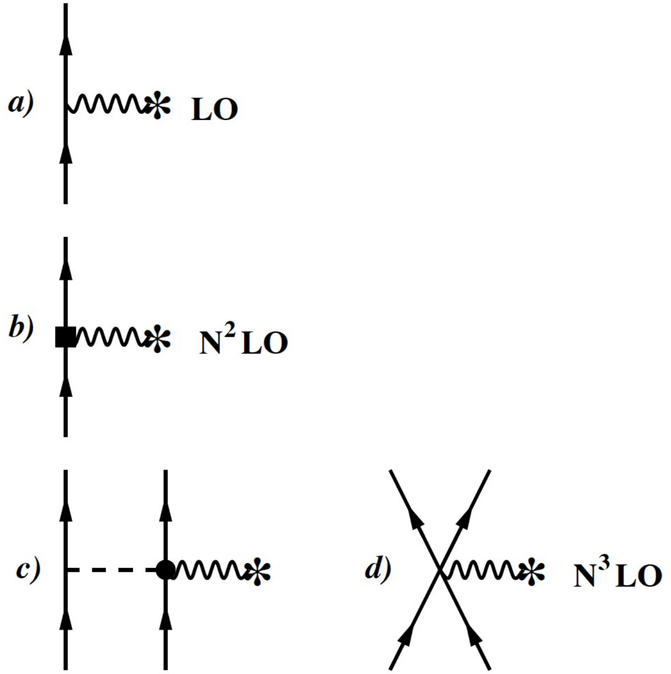
Quenching of $\sigma\tau$ matrix elements: meson exchange currents

Nowadays, EFT provides a powerful approach where both nuclear potentials and two electroweak currents may be consistently constructed, the latter appearing as subleading corrections to the one-body GT operator $\boldsymbol{\sigma\tau}^{\pm}$

Nuclear potential



Electroweak axial currents



The axial current J_A

The matrix elements of the axial current J_A are derived through a chiral expansion up to N3LO, and employing the same LECs as in 2NF and 3NF

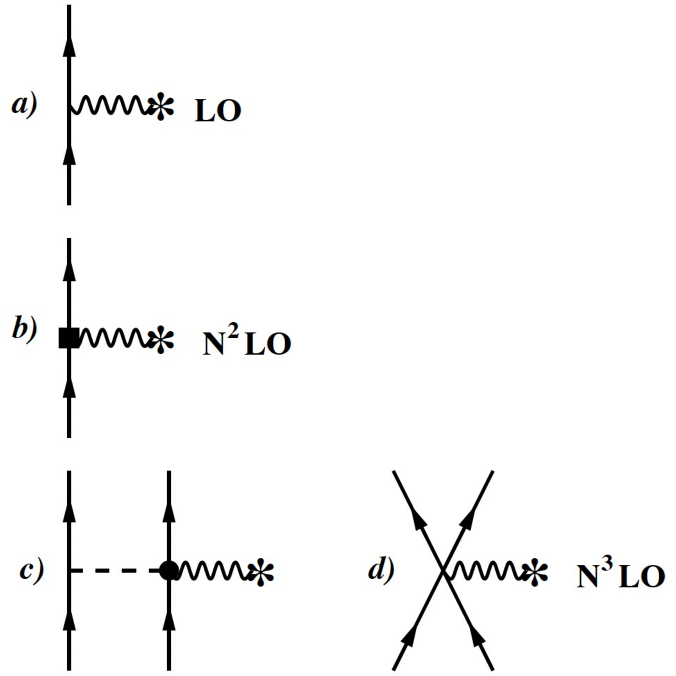
$$J_A^{LO} = -g_A \sum_i \sigma_i \tau_{i,\pm}$$

$$J_A^{N^2LO} = \frac{g_A}{2m_N^2} \sum_i K_i \times (\sigma_i \; x \; K_i) \, \tau_{i,\pm}$$

$$J_A^{N^3LO}(1PE;\boldsymbol{k}) = \sum_{i<j} \frac{g_A}{2f_\pi^2} \Big\{ 4c_3\, \tau_j \boldsymbol{k} + \left(\tau_i \times \tau_j \right)_\pm \times \left[\left(c_4 + \frac{1}{4m} \sigma_i \times \boldsymbol{k} - \frac{i}{2m} K_i \right) \right] \Big\} \, \sigma_j \cdot \boldsymbol{k} \, \frac{1}{\omega_k^2}$$

$$J_A^{N^3LO}(CT;\boldsymbol{k}) = \sum_{i<j} z_0 \left(\tau_i \times \tau_j \right)_\pm (\sigma_i \times \sigma_j)$$

$$z_0 = \frac{g_A}{2f_\pi^2 m_N^2} \left[\frac{m_N}{4 g_A \Lambda_\chi} c_D + \frac{m_N}{3} (c_3 + 2c_4) + \frac{1}{6} \right]$$

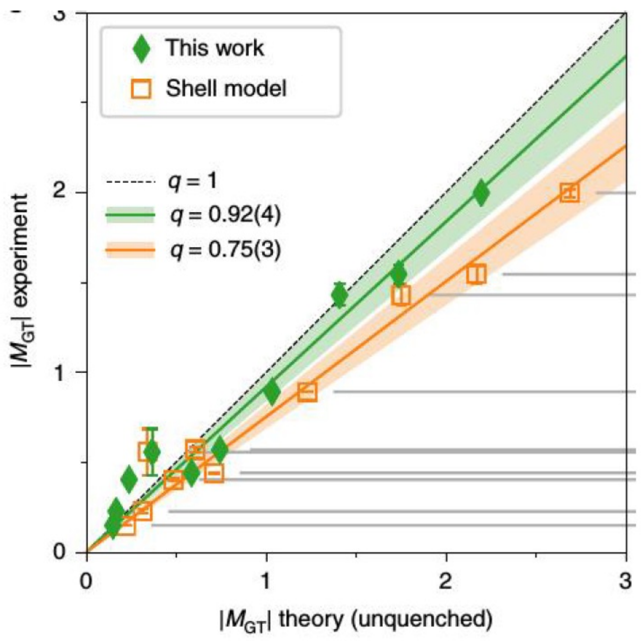


A. Baroni, L. Girlanda, S. Pastore, R. Schiavilla, and M. Viviani, Phys. Rev. C 93, 015501 (2016)

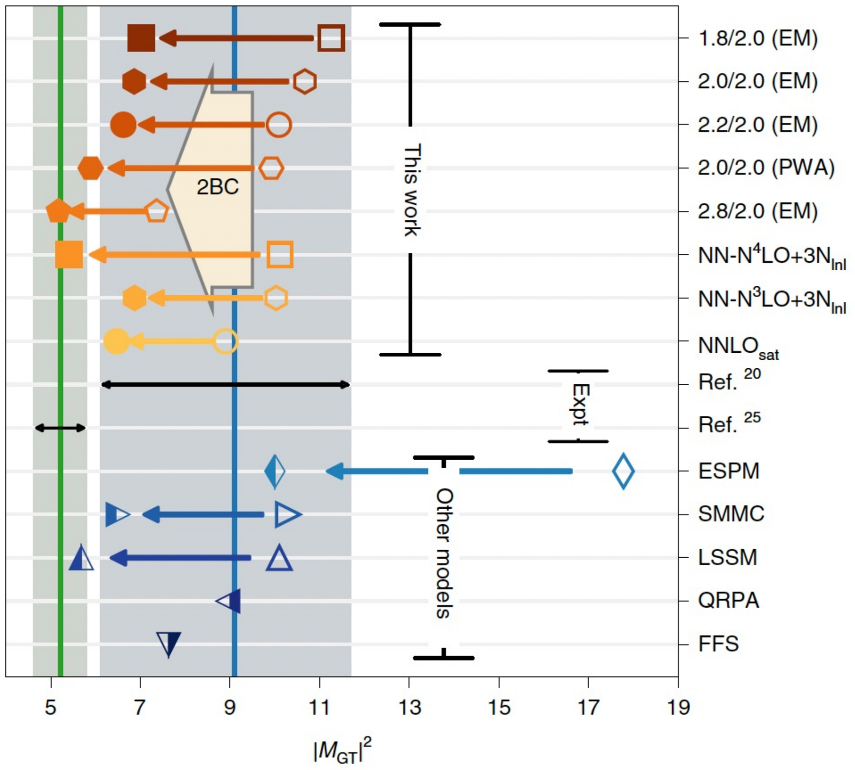
Two-body e.w. currents effects: light nuclei

The contribution of 2BC improves systematically the agreement between theory and experiment

In-Medium SRG



Gysbers et al. Nature Phys. 15 428 (2019)



A proper treatment of nuclear correlations and consistency between GT two-body currents and 3N forces, derived in terms of ChPT, describes microscopically the "quenching puzzle"

Outline

- Renormalization of στ matrix elements
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- Details of the Calculation

Details of the Calculations

- Nuclear Hamiltonian: Entem-Machleidt N3LO two-body potential plus N2LO three-body potential (Λ = 500 MeV)
- Axial current J_A calculated at N3LO in ChPT
- Heff obtained calculating the Q-box up to the 3rd order in V_{NN} (up to 2p-2h core excitations) and up to the 1st order in V_{NNN}
- Effective operators are consistently derived by way of the MBPT
- fp-shell nuclei: four proton and neutron orbitals outside ^{40}Ca: 0f7/2, 0f5/2, 1p3/2, 1p1/2
- fpg-shell nuclei: four proton and neutron orbitals outside ^{56}Ni: 0f5/2, 1p3/2, 1p1/2, 0g9/2

RSM calculations, starting from ChPT two- and three-body potentials and two-body meson-exchange currents for spectroscopic and spin-isospin dependent observables of ^{48}Ca, ^{76}Ge, ^{82}Se

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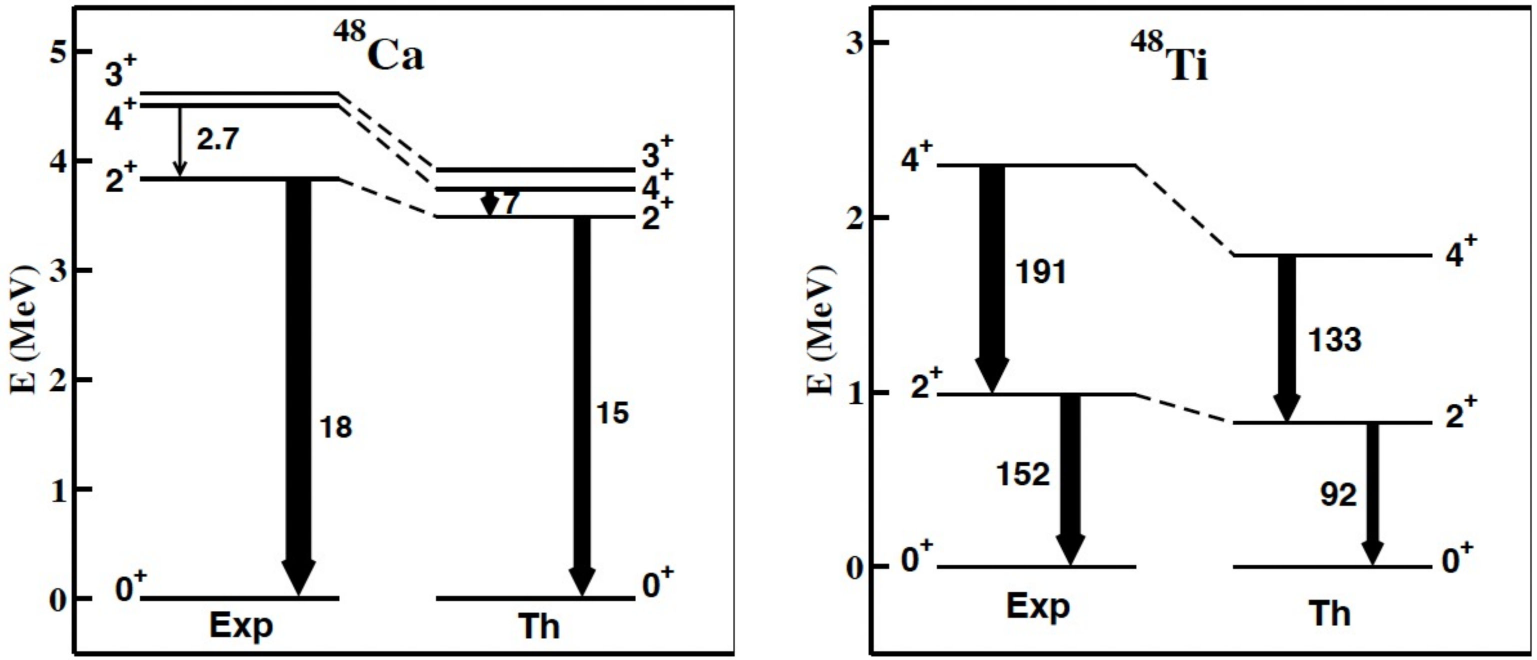
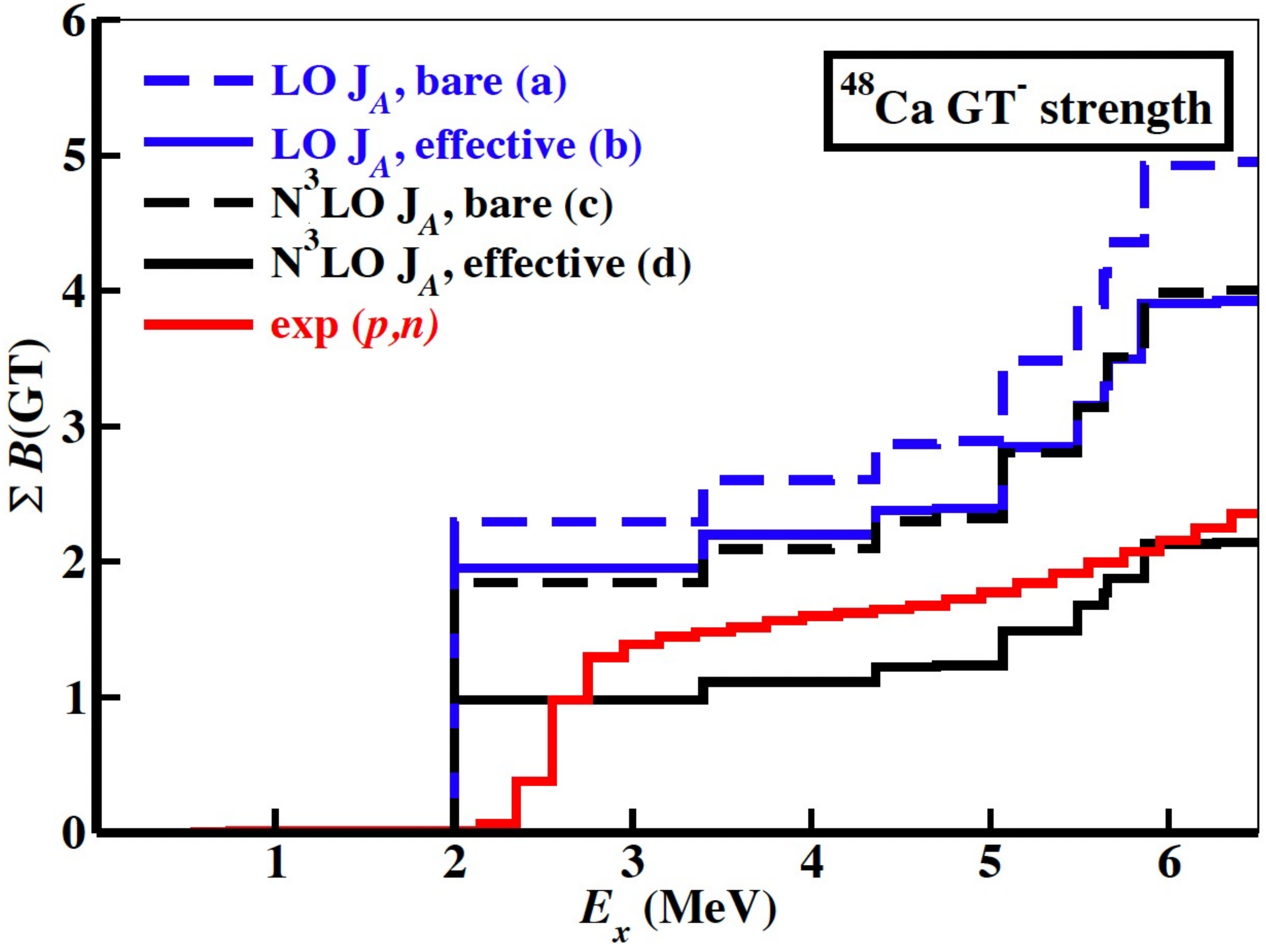
48Ca: Spectroscopy



48Ca: GT Strength

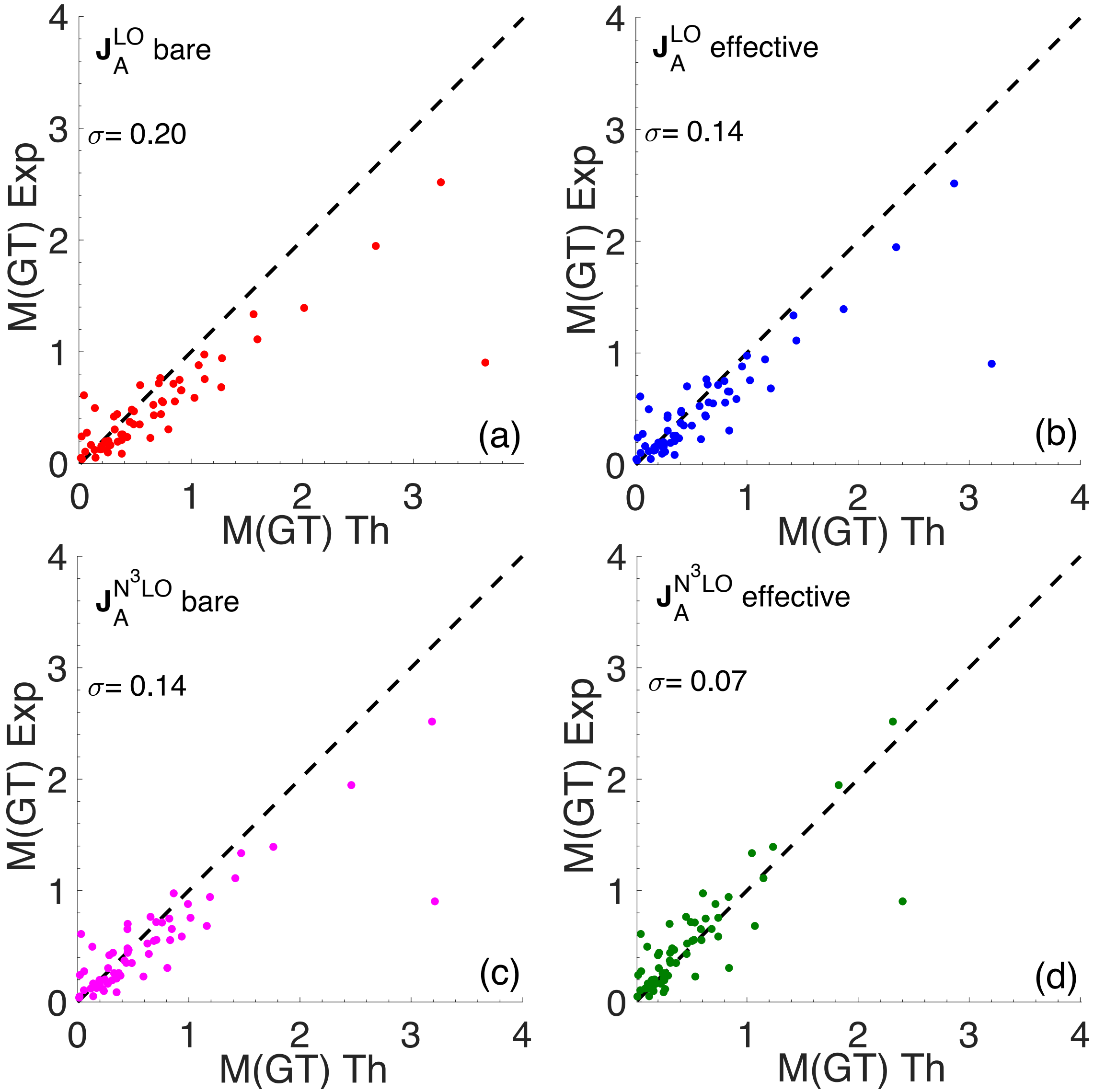


Charge exchange experiments

$$\left[\frac{d\sigma}{d\Omega}(q=0)\right] = \hat{\sigma} B_{exp}(GT)$$

$$B_{th}(GT) = \frac{\left|\langle \varphi_f \| J_A \| \varphi_i \rangle\right|^2}{2J_i + 1}$$

GT fp shell Nuclei



GT matrix elements of 60 experimental decays of 43 fp-shell nuclei

$$\sigma = \sqrt{\frac{\sum_i (x_i - \widehat{x_i})^2}{n}}$$

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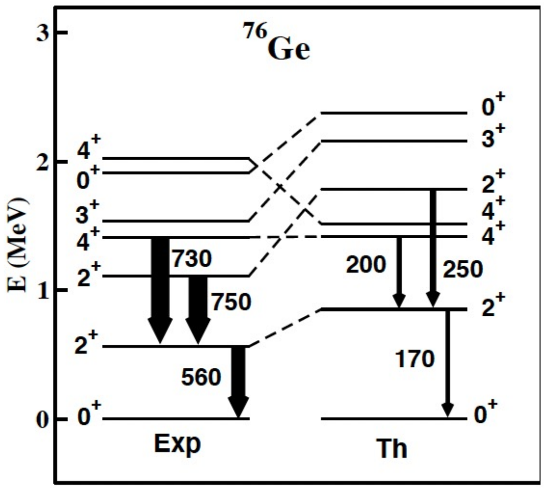
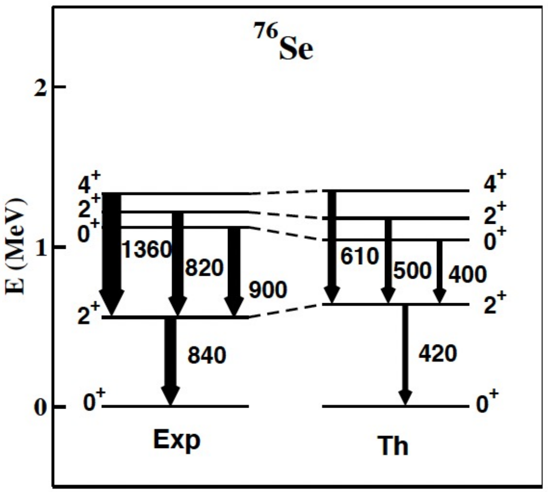
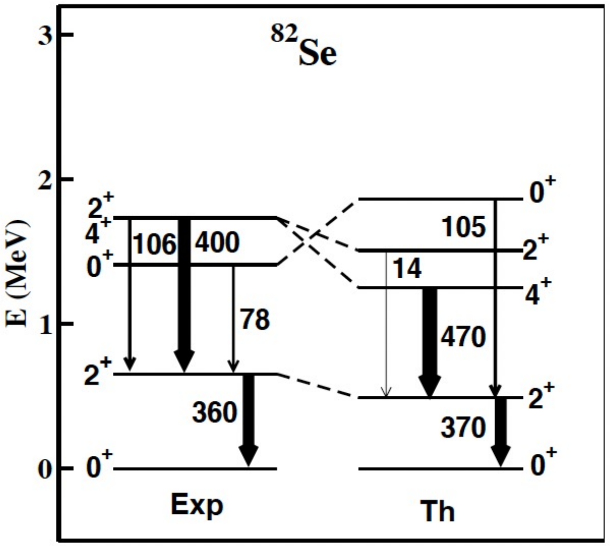
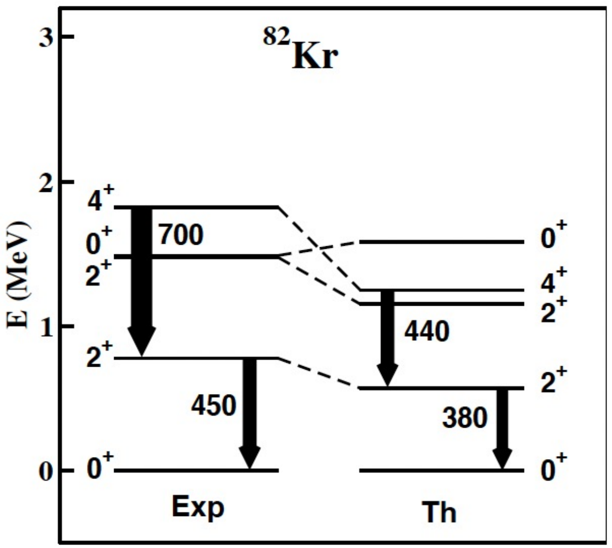
fpg shell nuclei: Spectroscopy

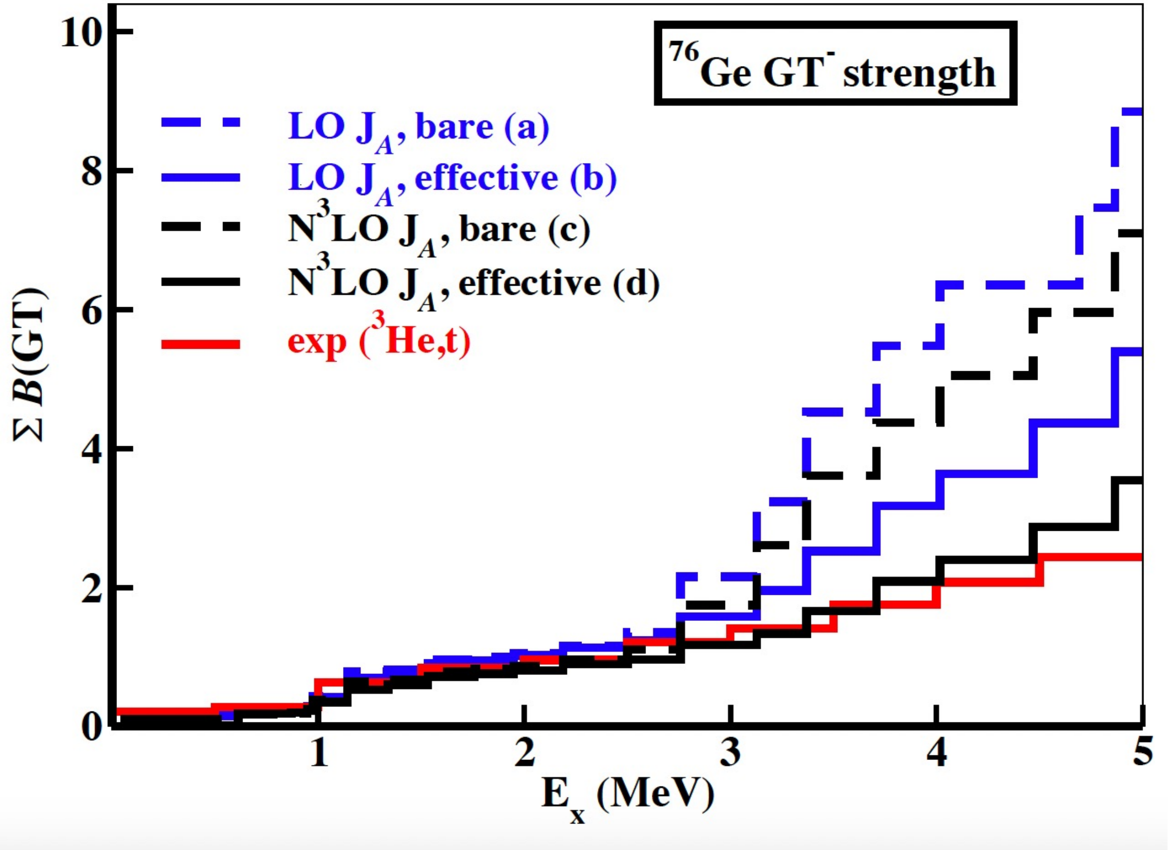


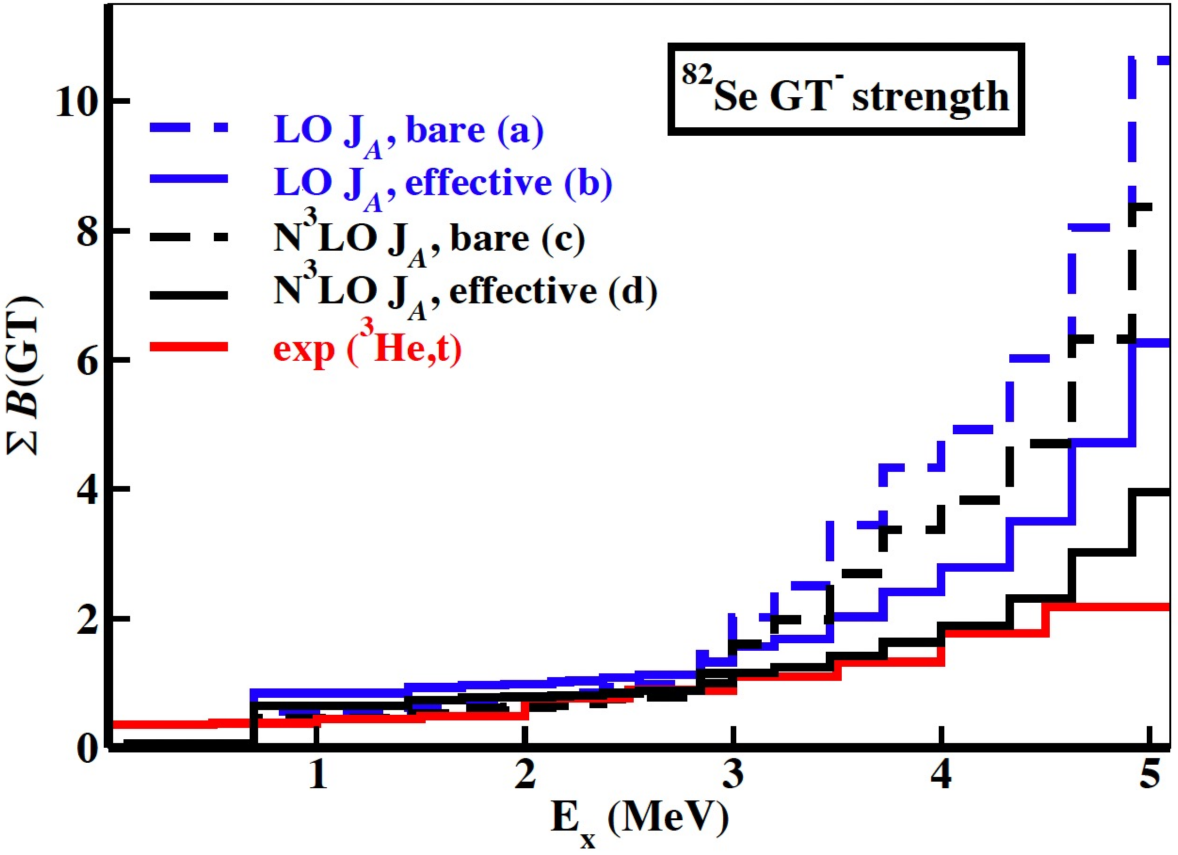






Fpg shell nuclei: GT strength





Charge exchange experiments

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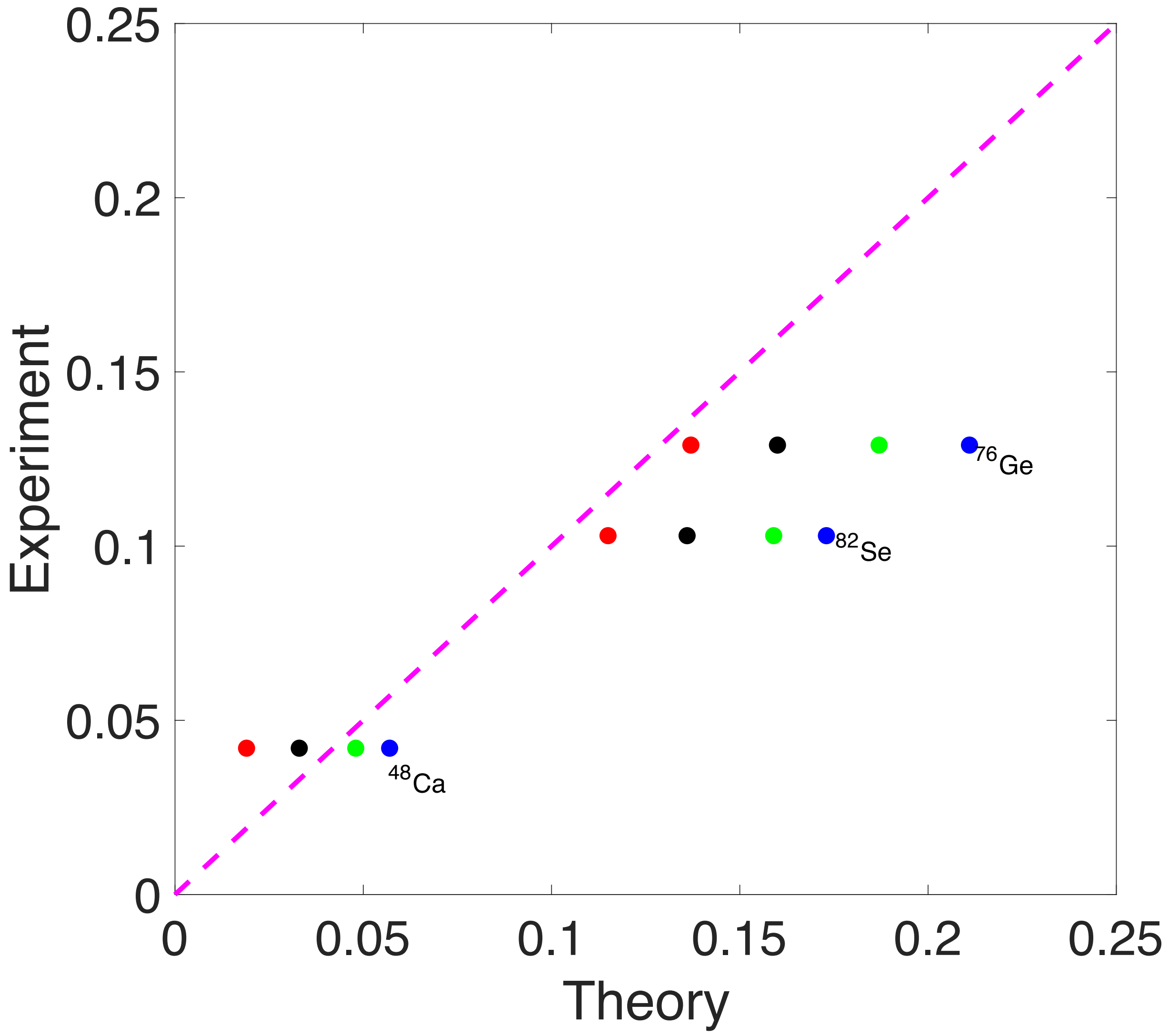
2νββ decay



$$M_{GT}^{2\nu} = \sum_k \frac{\langle 0_f^+ || \vec{\sigma} \cdot \tau^- || k \rangle \langle k || \vec{\sigma} \cdot \tau^- || 0_i^+ \rangle}{E_k + E_0}$$

Blue: bare J_A at LO in ChPT (namely the GT operator g_A)

Green: effective J_A at LO in ChPT

Black: bare J_A at N3LO in ChPT

Red effective J_A at N3LO in ChPT

Conclusions and persectives

Conclusions

- Correlations + electroweak 2BC ➡ quite good description of $\sigma\tau$ observables
- 2BC introduce ~ 20% reduction of GT matrix elements

Perspectives

- Meson-exchange two-body currents for the M1 transitions
- Calculations for heavier-mass systems (100Mo, 130Te, 136Xe)
- Calculating $0\nu\beta\beta$ decay $M^{0\nu}$ including also the LO contact term

Collaborators

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